# Threshold and kinematic effects for the exotic resonance－like structures 

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## Outline

1. General features about the threshold phenomena in hadron spectra?
2. Definition of anomalous triangle singularity (ATS)
3. Recognition of ATS in physical processes in the study of exotic threshold structures
4. Summary

## Charmonium Spectrum




New charmoniumlike states, i.e. "XYZ" states, are observed in experiment

## Charmonium Spectrum



## Expected \& Unexpected:

- States below openflavor thres. are well established.
- A large number of excited states, i.e. XYZ states, cannot be accommodated by the conventional QM!
- Full LQCD simulations are unavailable.


## Some urgent questions:

- Do we have indisputable evidence for exotic hadrons?

- Do we have efficient and sufficient criteria for identifying exotic hadrons?
-- e.g. genuine states vs. kinematic effects ?
-Where to look for exotic hadrons?
- ... ...


## Low-lying thresholds

Low-lying (Narrow) Charm Meson Pair Thresholds


## How the potential QM is broken down



- The effect of vacuum polarization due to dynamical quark pair creation may be manifested by the strong coupling to open thresholds and compensated by that of the hadron loops, i.e. coupled-channel effects.
E. Eichten et al., PRD17, 3090 (1987)
B.-Q. Li and K.-T. Chao, Phys. Rev. D79, 094004 (2009);
T. Barnes and E. Swanson, Phys.Rev. C77, 055206 (2008)


## Typical processes where the open threshold coupled channels can play a role



$$
D_{s 1}(2460)-D_{s 1}(2536)
$$

The mass shift in charmonia and charmed mesons, E.Eichten et al., PRD17(1987)3090
X.-G. Wu and Q. Zhao, PRD85, 034040 (2012)

## Special kinematic effects from anomalous triangle singularity (ATS)

$$
\begin{aligned}
& \Gamma_{3}\left(s_{1}, s_{2}, s_{3}\right)=\frac{1}{i(2 \pi)^{4}} \int \frac{d^{4} q_{1}}{\left(q_{1}^{2}-m_{1}^{2}+i \epsilon\right)\left(q_{2}^{2}-m_{2}^{2}+i \epsilon\right)\left(q_{3}^{2}-m_{3}^{2}+i \epsilon\right)} \\
&=\frac{-1}{16 \pi^{2}} \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} d a_{1} d a_{2} d a_{3} \frac{\delta\left(1-a_{1}-a_{2}-a_{3}\right)}{D-i \epsilon} \\
& D \equiv \sum_{i, j=1}^{3} a_{i} a_{j} Y_{i j}, \quad Y_{i j}=\frac{1}{2}\left[m_{i}^{2}+m_{j}^{2}-\left(q_{i}-q_{j}\right)^{2}\right]
\end{aligned}
$$



The ATS occurs when all the three internal particles can be simultaneously on shell. It corresponds to

$$
\partial D / \partial a_{j}=0 \quad \text { for all } \mathrm{j}=1,2,3 . \quad \square \quad \operatorname{det}\left[Y_{i j}\right]=0
$$

L. D. Landau, Nucl. Phys. 13, 181 (1959).
G. Bonnevay, I. J. R. Aitchison and J. S. Dowker, Nuovo Cim. 21, 3569 (1961).
R.F. Peierls, Phys. Rev. Lett. 6 (1961) 641.
S. Coleman, R.E. Norton, Nuovo Cimento 38 (1965) 438.
P. Landshoff, S. Treiman, Phys. Rev. 127 (1962) 649.
C. Schmid, Phys. Rev. 154 (1967) 1363.

The ATS condition for fixed $s_{1}, m_{\mathrm{j}}$, and $s_{3}$ is:

$$
s_{2}^{ \pm}=\left(m_{1}+m_{3}\right)^{2}+\frac{1}{2 m_{2}^{2}}\left[\left(m_{1}^{2}+m_{2}^{2}-s_{3}\right)\left(s_{1}-m_{2}^{2}-m_{3}^{2}\right)-4 m_{2}^{2} m_{1} m_{3}\right.
$$

$$
\left.\pm \lambda^{1 / 2}\left(s_{1}, m_{2}^{2}, m_{3}^{2}\right) \lambda^{1 / 2}\left(s_{3}, m_{1}^{2}, m_{2}^{2}\right)\right]
$$

Or for fixed $s_{2}, m_{j}$, and $s_{3}$ :

$$
\begin{aligned}
s_{1}^{ \pm} & =\left(m_{2}+m_{3}\right)^{2}+\frac{1}{2 m_{1}^{2}}\left[\left(m_{1}^{2}+m_{2}^{2}-s_{3}\right)\left(s_{2}-m_{1}^{2}-m_{3}^{2}\right)-4 m_{1}^{2} m_{2} m_{3}\right. \\
& \left. \pm \lambda^{1 / 2}\left(s_{2}, m_{1}^{2}, m_{3}^{2}\right) \lambda^{1 / 2}\left(s_{3}, m_{1}^{2}, m_{2}^{2}\right)\right] \\
& \text { with } \lambda(x, y, z) \equiv(x-y-z)^{2}-4 y z .
\end{aligned}
$$

X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016); arXiv:1507.01674 [hep-ph]

Single dispersion relation in $s_{2}$ in the complex plane of $s_{2}{ }^{\prime}$ :

$$
\Gamma_{3}\left(s_{1}, s_{2}, s_{3}\right)=\frac{1}{\pi} \int_{\left(m_{1}+m_{3}\right)^{2}}^{\infty} \frac{d s_{2}^{\prime}}{s_{2}^{\prime}-s_{2}-i \epsilon} \sigma\left(s_{1}, s_{2}^{\prime}, s_{3}\right)
$$

The spectral function $\sigma\left(s_{1}, s_{2}, s_{3}\right)$ can be obtained by means of the Cutkosky's rules (absorptive part of the loop amplitude):

$$
\sigma\left(s_{1}, s_{2}, s_{3}\right)=\frac{-1}{16 \pi} \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} d a_{1} d a_{2} d a_{3} \delta\left(1-a_{1}-a_{2}-a_{3}\right) \delta(D) .
$$

which reads

$$
\begin{aligned}
\sigma\left(s_{1}, s_{2}, s_{3}\right) & =\sigma_{+}-\sigma_{-}, \\
\sigma_{ \pm}\left(s_{1}, s_{2}, s_{3}\right) & =\frac{-1}{16 \pi \lambda^{1 / 2}\left(s_{1}, s_{2}, s_{3}\right)} \log \left[-s_{2}\left(s_{1}+s_{3}-s_{2}+m_{1}^{2}+m_{3}^{2}-2 m_{2}^{2}\right)\right. \\
& \left.-\left(s_{1}-s_{3}\right)\left(m_{1}^{2}-m_{3}^{2}\right) \pm \lambda^{1 / 2}\left(s_{1}, s_{2}, s_{3}\right) \lambda^{1 / 2}\left(s_{2}, m_{1}^{2}, m_{3}^{2}\right)\right] .
\end{aligned}
$$

For fixed $s_{1}, s_{3}$ and $m_{\mathrm{i}}$, the spectral function $\sigma\left(s_{1}, s_{2}, s_{3}\right)$ has logarithmic branch points $s^{ \pm}{ }_{2}$, which correspond to the anomalous thresholds by solving the Landau equation.

How the logarithmic branch points $s^{ \pm}{ }_{2}$ move as $s_{1}$ increases from the threshold of $\left(m_{2}+m_{3}\right)^{2}$, with $s_{3}$ and $m_{i}$ fixed?

Substituting $s_{1} \rightarrow s_{1}+i \varepsilon, s^{ \pm}{ }_{2}$ in the $s^{\prime}$-plane are then located at

$$
s_{2}^{ \pm}\left(s_{1}+i \epsilon\right)=s_{2}^{ \pm}\left(s_{1}\right)+i \epsilon \frac{\partial s_{2}^{ \pm}}{\partial s_{1}},
$$

With $\partial s_{2}^{ \pm} / \partial s_{1}=0\left(\partial s_{1}^{ \pm} / \partial s_{2}=0\right)$
the normal and critical thresholds for $s_{1}$ and $s_{2}$ can be determined:

$$
\begin{aligned}
& s_{1 N}=\left(m_{2}+m_{3}\right)^{2}, s_{1 C}=\left(m_{2}+m_{3}\right)^{2}+\frac{m_{3}}{m_{1}}\left[\left(m_{2}-m_{1}\right)^{2}-s_{3}\right], \\
& s_{2 N}=\left(m_{1}+m_{3}\right)^{2}, s_{2 C}=\left(m_{1}+m_{3}\right)^{2}+\frac{m_{3}}{m_{2}}\left[\left(m_{2}-m_{1}\right)^{2}-s_{3}\right],
\end{aligned}
$$

Trajectories of $s^{ \pm}{ }_{2}$ in the complex $s_{2}^{\prime}$-plane with $s_{1}$ increasing from $s_{1 N} \rightarrow \infty$ :

$\mathrm{A}^{+}:\left(s_{1}=\mathrm{s}_{1 N^{\prime}}, s_{2}{ }^{+}=s_{2 C}+i \varepsilon\right) \rightarrow \mathrm{B}^{+}:\left(s_{1}=s_{1 C}, s_{2}{ }^{+}=s_{2 N}+\mathrm{m}_{3} \lambda\left(s_{3}, \mathrm{~m}_{1}{ }^{2}, \mathrm{~m}_{2}{ }^{2}\right) /\left(\mathrm{m}_{1} \mathrm{~m}_{2}\right)+i \varepsilon\right)$
$A^{-}:\left(s_{1}=s_{1 N}, s_{2}^{-}=s_{2 C}-i \varepsilon\right) \rightarrow B^{-}:\left(s_{1}=s_{1 C}, s_{2}^{-}=s_{2 N}\right)$
$\mathbf{P}: s_{2}+i \epsilon$.


$$
\Gamma_{3}\left(s_{1}, s_{2}, s_{3}\right)=\frac{1}{\pi} \int_{\left(m_{1}+m_{3}\right)^{2}}^{\infty} \frac{d s_{2}^{\prime}}{s_{2}^{\prime}-s_{2}-i \epsilon} \sigma\left(s_{1}, s_{2}^{\prime}, s_{3}\right)
$$

The difference between the normal and anomalous thresholds decides the kinematic range of the ATS effects:

$$
\begin{aligned}
\Delta_{s_{1}} & =\sqrt{s_{1}^{-}}-\sqrt{s_{1 N}} \\
\Delta_{s_{2}} & =\sqrt{s_{2}^{-}}-\sqrt{s_{2 N}} .
\end{aligned}
$$



When $\mathrm{s}_{2}=\mathrm{s}_{2 \mathrm{~N}}\left(\mathrm{~s}_{1}=\mathrm{s}_{1 \mathrm{~N}}\right)$, we will obtain the maximum value of $\Delta \mathrm{s}_{1}\left(\Delta \mathrm{~s}_{2}\right)$,

$$
\begin{aligned}
& \Delta_{s_{1}}^{\max }=\sqrt{s_{1 C}}-\sqrt{s_{1 N}} \approx \frac{m_{3}}{2 m_{1}\left(m_{2}+m_{3}\right)}\left[\left(m_{2}-m_{1}\right)^{2}-s_{3}\right], \\
& \Delta_{s_{2}}^{\max }=\sqrt{s_{2 C}}-\sqrt{s_{2 N}} \approx \frac{m_{3}}{2 m_{2}\left(m_{1}+m_{3}\right)}\left[\left(m_{2}-m_{1}\right)^{2}-s_{3}\right] .
\end{aligned}
$$

Larger values of $\Delta_{\mathrm{s}}{ }^{\text {max }}$ means more significant effects from the ATS mechanism!

## Physical cases for recognizing the ATS

(I) S-wave open flavor threshold in vector channel

- $\mathrm{Zc}(3900)$ production in $\mathrm{Y}(4260)$ decays



## $Y(4260)$ could be a hadronic molecule made of $D_{1}(2420)$

"threshold state"


- Zc(3900) can be dynamically generated.
- The peak can also obtain contributions from the ATS mechanism.
- Test of Schmid theorem?
Q. Wang, C. Hanhart, Q.Z., PRL111, 132003 (2013); PLB(2013)
W. Qin, S.R. Xue, Q.Z., arXiv:1605.02407[hep-ph]


## Singularity kinematics in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{Y}(4260) / \mathrm{Y}(4360) \rightarrow$ $\mathrm{J} / \psi \pi \pi$


Q. Wang, C. Hanhart, Q. Zhao, PLB2013; arXiv: 1305.1997[hep-ph]; Liu, Oka, and Q. Zhao, PLB2016; arXiv: 1507.01674[hep-ph]


## Singularity kinematics in $\mathrm{e}^{+} \mathrm{e}^{-}-\mathrm{Y}(\mathrm{nS}) \pi \pi$



-Two singularity regions by the $\mathrm{BB}^{*}$ and $\mathrm{B}^{*} \mathrm{~B}^{*}$ thresholds can appear in the same c.m. energy in the $Y(6 S)$ decays. Two peaks are expected!
Q. Wang, C. Hanhart, Q. Zhao, PLB2013; arXiv: 1305.1997[hep-ph]

(II) The $\eta(1405) / \eta(1475)$ puzzle, and $\mathrm{f}_{1}(1420) / \mathrm{a}_{1}(1420)$ problem

The nonet structure repeats itself in the radial excitations in the $\bar{q} q$ meson scenario.


- The abundance of $0^{-+}$ (I=0) states implies a glueball candidate?

Positive: Flux tube model favors $\mathrm{M}_{\mathrm{G}} \cong 1.4 \mathrm{GeV}$

Caveat: LQCD (quenched) favors $\mathrm{M}_{\mathrm{G}} \cong 2.4 \mathrm{GeV}$

More problems arising from $3 \eta$ scenario!

## Isospin-violating decay of $\mathrm{J} / \psi \rightarrow \gamma \pi \pi \pi$



BES-III Collaboration, Phys. Rev. Lett. 108, 182001 (2012)
See also plenary talk by S.S. Fang



- $\mathrm{f}_{0}(980)$ is extremely narrow: $\Gamma \cong 10 \mathrm{MeV}$ ! PDG: $\Gamma \cong 40 \sim 100 \mathrm{MeV}$.
- Anomalously large isospin violation!

$$
\frac{\operatorname{Br}\left(\eta(1405) \rightarrow f_{0}(980) \pi^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}\right)}{\operatorname{Br}\left(\eta(1405) \rightarrow a_{0}^{0}(980) \pi^{0} \rightarrow \eta \pi^{0} \pi^{0}\right)}
$$



$$
\begin{aligned}
&\left.g\left(a_{0} K^{+} K^{-}\right)\right) \\
&=-g\left(\mathrm{a}_{0} \mathrm{~K}^{0} \mathrm{~K}^{0} \mathrm{~K}^{+} \mathrm{K}^{-}\right) \\
& g\left(\mathrm{f}_{0} \mathrm{~K}^{0}\right. \\
& M\left(\mathrm{~K}^{0}\right)-M\left(\mathrm{~K}^{ \pm}\right)= \mathrm{m}_{\mathrm{d}}-\mathrm{m}_{\mathrm{u}}
\end{aligned}
$$

"Triangle singularity"
Internal $\bar{K} K *(K)$ approach the on-shell condition simultaneously!


A novel isospin breaking mechanism!

Triangle loop amplitudes:


Absorptive amplitudes


Dispersive amplitudes


## $\eta(1440) \rightarrow K \bar{K} \pi$ decay mechanism:



Data from Mark III, BES-I, and DM2


$$
\begin{aligned}
& \frac{d \Gamma_{J / \psi \rightarrow \gamma \eta(1440) \rightarrow \gamma A B C}}{d \sqrt{s_{0}}} \\
& \quad=\frac{2 s_{0}}{\pi} \frac{\Gamma_{J / \psi \rightarrow \gamma \eta(1440)}\left(s_{0}\right) \Gamma_{\eta(1440) \rightarrow A B C}\left(s_{0}\right)}{\left(s_{0}-m_{\eta(1440)}^{2}\right)^{2}+\Gamma_{\eta(1440)}^{2} m_{\eta(1440)}^{2}},
\end{aligned}
$$

J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012)

- So far, BESIII do not see two peak signals in any exclusive process.
- The same state has different peak positions and lineshapes in different decay channels.
- $\eta(1405)$ and $\eta(1475)$ are likely to be the same state.




## Where to look for pseudoscalar glueball candidate?


$\boldsymbol{J} / \boldsymbol{\psi} \rightarrow \boldsymbol{\omega} \boldsymbol{\eta} \boldsymbol{\pi}^{+} \boldsymbol{\pi}^{-}$

PRL 107, 182001 (2011)

Pure gauge LQCD: $\mathrm{M}_{\mathrm{G}} \cong 2.4 \mathrm{GeV}$

## The puzzle of $\mathrm{f}_{1}(1420)$ and $\mathrm{a}_{1}(1420)$



## Partial wave analysis of $\mathrm{J} / \psi \rightarrow \gamma \eta(1405) / \mathrm{f}_{1}(1420) \rightarrow \gamma \pi \pi \pi$



Dashed: eta(1440) Dotted: f1(1420) Solid: eta(1440) + f1

$$
\begin{gathered}
\chi^{2} / \text { d.o.f }=38.3 / 14 ; \quad b_{\gamma}=118.5 \pm 8.8, c=0.538 \pm 0.312 \\
\chi^{2} / \text { d.o. } f=19.8 / 12 ; \quad b_{f_{0}}=145.7 \pm 10.7, c_{1}=0.314 \pm 0.128, c_{2}=0.141 \pm 0.317
\end{gathered}
$$


BESIII results:

| immediate states | $\chi^{2} /$ d.o. $f$ for $\cos \theta_{\gamma}$ | $\chi^{2} /$ d.o.f for $\cos \theta_{f_{0}}$ |
| :---: | :---: | :---: |
| $\eta(1440)$ | $40.2 / 15$ | $26.8 / 14$ |
| $f_{1}(1420)$ | $59.0 / 15$ | $26.4 / 13$ |
| $\eta(1440)$ and $f_{1}(1420)$ | $38.3 / 14$ | $19.8 / 12$ |


(a) $\mathrm{M}(\mathrm{K} \overline{\mathrm{K}} \pi)(\mathrm{GeV})$

(b) $\mathrm{M}\left(\pi^{+} \pi^{-} \pi^{0}\right)(\mathrm{GeV})$

(d) $\mathrm{M}\left(\pi^{+} \pi\right)(\mathrm{GeV})$

(c) $\mathrm{M}\left(\eta \pi^{0} \pi^{0}\right)(\mathrm{GeV})$

## Implication of existence of $a_{1}(1420)$ in isospin 1 channel



Due to the "triangle singularity", the same "state" produces different resonance-like lineshapes in different channels!




## Observation of a new state $\mathrm{a}_{1}(1420)$ at COMPASS



# (III) The heavy pentaquark $\operatorname{Pc}(4380)$ and $\mathrm{Pc}(4450)$ production 

Exp. evidence for heavy pentaquarks with hidden charm




$$
\begin{aligned}
& \mathrm{M}\left[\mathrm{Pc}^{+}(4380)\right]=(4380 \pm 8 \pm 29) \mathrm{MeV}, \Gamma=(205 \pm 18 \pm 86) \mathrm{MeV} \\
& \mathrm{M}\left[\mathrm{Pc}^{+}(4450)\right]=(4449.8 \pm 1.7 \pm 2.5) \mathrm{MeV}, \Gamma=(39 \pm 5 \pm 19) \mathrm{MeV}
\end{aligned}
$$

$$
J^{P}=\left(3 / 2^{-}, 5 / 2^{+}\right) \text {or }\left(3 / 2^{+}, 5 / 2^{-}\right)
$$

## Immediate theoretical studies:

1) Molecular states:
R. Chen, X. Liu, X.-Q. Li, S.-L. Zhu, PRL(2015); arXiv:1507.03704[hep-ph]
L. Roca, J. Nieves and E. Oset, arXiv:1507.04249 [hep-ph].
A. Feijoo, V. K. Magas, A. Ramos and E. Oset, arXiv:1507.04640 [hep-ph]
J. He, arXiv:1507.05200 [hep-ph]
U.-G. Meissner, J.A. Oller, arXiv:1507.07478v1 [hep-ph]
2) Multiquark state as an overall color singlet
L. Maiani, A.D. Polosa, and V. Riquer, arXiv:1507.04980 [hep-ph]
R.L. Lebed, arXiv:1507.05867 [hep-ph]
V.V. Anisovich et al., arXiv:1507.07652[hep-ph]
G.-N. Li, X.-G. He, M. He, arXiv:1507.08252 [hep-ph]
3) Soliton model
N.N. Scoccolaa, D.O. Riska, Mannque Rho, arXiv:1508.01172 [hep-ph]

## 4) Sum rules study

H. X. Chen, W. Chen, X. Liu, T.G. Steele and S. L. Zhu, PRL(2015); arXiv:1507.03717
Z.-G. Wang, arXiv:1508.01468.

## Some early studies:

J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010) [arXiv:1007.0573 [nucl-th]].
J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. C 84, 015202 (2011) [arXiv:1011.2399 [nucl-th]].
J. J. Wu, T.-S. H. Lee and B. S. Zou, Phys. Rev. C 85, 044002 (2012) [arXiv:1202.1036 [nucl-th]].
Z. C. Yang, Z. F. Sun, J. He, X. Liu and S. L. Zhu, Chin. Phys. C 36, 6 (2012) [arXiv:1105.2901 [hep-ph]].

## Alternative solutions? Or some further concerns?

Threshold enhancement produced by anomalous triangle singularity:
F.-K. Guo, U.-G. Meissner, W. Wang, and Z. Yang, arXiv:1507.04950 [hep-ph]
X.-H. Liu, Q. Wang, and Q. Zhao, arXiv:1507.05359 [hep-ph]
M. Mikhasenko, arXiv:1507.06552v1 [hep-ph]

## Production mechanism in $\Lambda_{b}$ decay



Rescattering via triangle diagrams


## A new leading order mechanism


(c)

$$
\begin{aligned}
& \left\langle Y_{c} \bar{K} \bar{D}\right| \hat{H}_{w}\left|\Lambda_{b}\right\rangle_{(c)} \\
= & \frac{1}{2 \sqrt{2}}\left[-\Sigma_{c}^{++} K^{-} D^{-}+\frac{1}{2} \Sigma_{c}^{+} \bar{K}^{0} D^{-}-\frac{1}{2} \Sigma_{c}^{+} K^{-} \bar{D}^{0}+\Sigma_{c}^{0} \bar{K}^{0} \bar{D}^{0}+\frac{1}{2} \Lambda_{c}^{+} K^{-} \bar{D}^{0}-\frac{1}{2} \Lambda_{c}^{+} \bar{K}^{0} D^{-}\right]
\end{aligned}
$$

Rescattering to generate a pole?


Favored by the molecular picture
(c)

Thresholds for $\chi_{c \jmath} p$

| Threshold masses $[\mathrm{MeV}]$ | $\chi_{c 0}(1 P) 0^{+}$ | $\chi_{c 1}(1 P) 1^{+}$ | $\chi_{c 2}(1 P) 2^{+}$ |
| :---: | :---: | :---: | :---: |
| $p 1 / 2^{+}$ | 4353 | 4449 | 4494 |



(a)


| Threshold masses $[\mathrm{MeV}]$ | $\Lambda_{c}(2286)$ | $1 / 2^{+}$ | $\Lambda_{c}(2595) 1 / 2^{-}$ | $\Lambda_{c}(2625) 3 / 2^{-}$ | $\Lambda_{c}(2880) 5 / 2^{+}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |


| $\bar{D}_{s}(1968) 0^{-}$ | 4254 | 4563 | 4593 | 4848 |
| :---: | :---: | :---: | :---: | :---: |
| $D_{s}^{*}(2112) 1^{-}$ | 4398 | 4707 | 4737 | 4994 |
| $D_{s 0}(2317) 0^{+}$ | 4585 | 4912 | 4942 | 5197 |
| $D_{s 1}(2460) 1^{+}$ | 4728 | 5055 | 5085 | 5340 |
| $\bar{D}_{s 1}(2536) 1^{+}$ | 4822 | 5131 | 5161 | 5416 |
| $\bar{D}_{s 2}(2573) 2^{+}$ | 4859 | 5168 | 5198 | 5453 |
| $\bar{D}_{s 1}(2700) 1^{-}$ | 4986 | 5295 | 5325 | 5580 |
| $D_{s J}(2860) ?^{?}$ | 5146 | 5455 | 5485 | $[5740]$ |
| $D_{s, J}(3040) ?^{?}$ | 5331 | $[5636]$ | $[5672]$ | $[5926]$ |


| Threshold masses [GeV] | $\Sigma_{c}(2455) 1 / 2^{+}$ | $\Sigma_{c}(2520) 3 / 2^{+}$ | $\Sigma_{c}(2625) ? ?$ |
| :---: | :---: | :---: | :---: |
| $\bar{D}(1865) 0^{-}$ | 4.321 | 4.385 | 4.668 |
| $\bar{D}^{*}(2007) 1^{-}$ | 4.463 | 4.527 | 4.810 |
| $\bar{D}_{1}(2420) 1^{+}$ | 4.875 | 4.939 | 5.222 |
| $\bar{D}_{2}(2460) 2^{+}$ | 4.917 | 4.981 | 5.264 |




## Invariant mass distribution of $\mathrm{J} / \Psi \mathrm{p}$ with different $\mathrm{K}^{-} \mathrm{p}$ momentum cuts

(a) $m_{K_{p}}<1.55 \mathrm{GeV}$,(b) $1.55 \mathrm{GeV}<m_{K_{p}}<1.07 \mathrm{GeV}$,
(c) $1.07 \mathrm{GeV}<m_{K_{p}}<12.0 \mathrm{GeV}$, (d) $m_{K_{p}}>2.0 \mathrm{GeV}$.




The ATS can mimic a resonance behavior in certain cases

F.-K. Guo, U.-G. Meissner, W. Wang, and Z. Yang, arXiv:1507.04950 [hep-ph] See also talk by F.-K. Guo

## Kinematic features of the production mechanism



Scattering angle

1) Forward angle peaking is predominant due to the diffractive process, i.e. Pomeron exchanges.
2) S-channel resonance excitations contribute to the cross sections at middle and backward angles.
3) U-channel contributes to backward

Interferences from different transition mechanisms

## $s$ and $u$-channel pentaquark production



Coupling vertices for $\gamma N P_{c}$ :

$$
\begin{aligned}
\mathcal{L}_{\gamma N P_{c}}^{3 / 2 P^{ \pm}}= & \frac{i e h_{1}}{2 M_{N}} \bar{N} \Gamma_{\nu}^{( \pm)} F^{\mu \nu} P_{c \mu}-\frac{e h_{2}}{\left(2 M_{N}\right)^{2}} \partial_{\nu} \bar{N} \Gamma^{( \pm)} F^{\mu \nu} P_{c \mu} \\
& + \text { H.c. }, \\
\mathcal{L}_{\gamma N P_{c}}^{5 / 2^{ \pm}}= & \frac{e h_{1}}{\left(2 M_{N}\right)^{2}} \bar{N} \Gamma_{\nu}^{(\mp)} \partial^{\alpha} F^{\mu \nu} P_{c \mu \alpha} \\
& -\frac{i e h_{2}}{\left(2 M_{N}\right)^{3}} \partial_{\nu} \bar{N} \Gamma_{\nu}^{(\mp)} \partial^{\alpha} F^{\mu \nu} P_{c \mu \alpha}+\text { H.c., } \quad \Gamma_{\mu}^{( \pm)} \equiv\binom{\gamma_{\mu} \gamma_{5}}{\gamma_{\mu}}, \quad \Gamma^{( \pm)} \equiv\binom{\gamma_{5}}{1},
\end{aligned}
$$

S. H. Kim, S. i. Nam, Y. Oh and H. C. Kim, PRD 84, 114023 (2011)
Q. Wang, X.-H. Liu, and Q. Zhao, arXiv:1508.00339 [hep-ph]

## Coupling vertices for $J / \psi N P_{c}$ :

$$
\begin{aligned}
\mathcal{L}_{P_{c} N \psi}^{3 / 2} & =-\frac{i g_{1}}{2 M_{N}} \bar{N} \Gamma_{\nu}^{( \pm)} \psi^{\mu \nu} P_{c \mu}-\frac{g_{2}}{\left(2 M_{N}\right)^{2}} \partial_{\nu} \bar{N} \Gamma^{( \pm)} \psi^{\mu \nu} P_{c \mu}+\frac{g_{3}}{\left(2 M_{N}\right)^{2}} \bar{N} \Gamma^{( \pm)} \partial_{\nu} \psi^{\mu \nu} P_{c \mu}+H . c ., \\
\mathcal{L}_{P_{c} N \psi}^{5 / 2^{ \pm}} & =\frac{g_{1}}{\left(2 M_{N}\right)^{2}} \bar{N} \Gamma_{\nu}^{(\mp)} \partial^{\alpha} \psi^{\mu \nu} P_{c \mu \alpha}-\frac{i g_{2}}{\left(2 M_{N}\right)^{3}} \partial_{\nu} \bar{N} \Gamma^{(\mp)} \partial^{\alpha} \psi^{\mu \nu} P_{c \mu \alpha}+\frac{i g_{3}}{\left(2 M_{N}\right)^{3}} \bar{N} \Gamma^{(\mp)} \partial^{\alpha} \partial_{\nu} \psi^{\mu \nu} P_{c \mu \alpha}+H . c .
\end{aligned}
$$

## Leading transition matrix elements:

$$
\begin{aligned}
\mathcal{M}^{3 / 2^{ \pm}} & =\frac{1}{s-M_{P_{c}}^{2}} \frac{e h_{1} g_{1}}{\left(2 M_{N}\right)} \epsilon_{\psi \nu}^{*} \bar{u}_{N} \Gamma_{\sigma}^{( \pm)} \Delta_{\beta \alpha}\left(P_{c}, k+p\right) \Gamma_{\delta}^{( \pm)}\left(k^{\alpha} g^{\mu \delta}-k^{\delta} g^{\alpha \mu}\right) u_{N} \epsilon_{\gamma \mu} \\
\mathcal{M}^{5 / 2^{ \pm}} & =\frac{1}{s-M_{P_{c}}^{2}} \frac{e h_{1} g_{1}}{\left(2 M_{N}\right) 4} \epsilon_{\psi \nu}^{*} \bar{u}_{N} q^{\sigma}\left(q^{\rho} g^{\nu \delta}-q^{\delta} g^{\nu \rho}\right) \Delta_{\rho \sigma ; \alpha \beta}\left(P_{c}, k+p\right) \Gamma_{\lambda}^{(\mp)} k^{\beta}\left(k^{\alpha} g^{\mu \lambda}-k^{\lambda} g^{\alpha \mu}\right) u_{N} \epsilon_{\gamma \mu}
\end{aligned}
$$

## Rarita-Schwinger spin projections:

$$
\begin{aligned}
\Delta_{\beta \alpha}(B, p) & =\left(\not p+M_{B}\right)\left[-g_{\beta \alpha}+\frac{1}{3} \gamma_{\beta} \gamma_{\alpha}+\frac{1}{3 M_{B}}\left(\gamma_{\beta} p_{\alpha}-\gamma_{\alpha} p_{\beta}\right)+\frac{2}{3 M_{B}^{2}} p_{\beta} p_{\alpha}\right] \\
\Delta_{\rho \sigma ; \alpha \beta}(B, p) & =\left(\not p+M_{B}\right)\left[\frac{1}{2}\left(\bar{g}_{\rho \alpha} \bar{g}_{\sigma \beta}+\bar{g}_{\rho \beta} \bar{g}_{\sigma \alpha}\right)-\frac{1}{5} \bar{g}_{\rho \sigma} \bar{g}_{\alpha \beta}-\frac{1}{10}\left(\bar{\gamma}_{\rho} \bar{\gamma}_{\alpha} \bar{g}_{\sigma \beta}+\bar{\gamma}_{\rho} \bar{\gamma}_{\beta} \bar{g}_{\sigma \alpha}+\bar{\gamma}_{\sigma} \bar{\gamma}_{\alpha} \bar{g}_{\rho \beta}+\bar{\gamma}_{\sigma} \bar{\gamma}_{\beta} \bar{g}_{\rho \alpha}\right)\right] \\
\text { with } & \left\{\begin{array}{l}
\bar{g}_{\alpha \beta}=g_{\alpha \beta}-\frac{p_{\alpha} p_{\beta}}{M_{B}^{2}}, \\
\bar{\gamma}_{\alpha}=\gamma_{\alpha}-\frac{p_{\alpha}}{M_{B}^{2} \not p}
\end{array}\right.
\end{aligned}
$$

Vector meson dominance


$$
\mathcal{L}_{V \gamma}=\sum_{V} \frac{e M_{V}^{2}}{f_{V}} V_{\mu} A^{\mu}
$$



$$
e h_{1}=-\frac{e M_{J / \psi}^{2}}{f_{J / \psi}} \frac{i g_{1}}{k^{2}-M_{J / \psi}^{2}}=i \frac{e}{f_{J / \psi}} g_{1}
$$

By assuming that the $\mathrm{J} / \psi \mathrm{p}$ saturate the decay widths of the Pc states, we have

$$
g_{\frac{3}{2}+}=1.07, \quad g_{\frac{3}{2}-}=1.40, \quad g_{\frac{5}{2}^{+}}=2.56, \quad g_{\frac{5}{2}}=5.58
$$

A form factor is included:

$$
\mathcal{F}\left(p^{2}\right)=\frac{\Lambda^{4}}{\Lambda^{4}+\left(p^{2}-M_{P_{c}}^{2}\right)^{2}}
$$

## Total cross sections predicted:

Full width prediction



Prediction with $5 \%$ of b.r. to $\mathrm{J} / \psi \mathrm{P}$ :



## Challenge:

J/psi p CANNOT be the dominant decay channel of these pentaquark candidates!

## Predicted differential cross sections at different energies:




## Predicted differential cross sections at different energies:



## Summary

- The ATS is strongly correlated with threshold phenomena and can produce observable effects when the condition is fulfilled in the physical regime.
- The ATS may mix with the threshold pole structure if a genuine state does exist. So energy-dependence of the threshold peak should be studied.
- An enhancement in elastic channel "almost" implies the existence of a genuine state.
- More criteria for judging the ATS effects and genuine states should be pursued.


## Recent developments on this relevant issue：

－J．－J．Wu，X．－H．Liu，and Q．Zhao，B．－S．Zou，PRL108， 081003 （2012）
－X．－G．Wu，J．－J．Wu，Q．Zhao，B．－S．Zou，PRD 87， 014023 （2013）
－Q．Wang，C．Hanhart，Q．Zhao，PRL111， 132003 （2013）
－Q．Wang，C．Hanhart，Q．Zhao，PLB725， 106 （2013）
－X．－H．Liu，M．Oka，Q．Zhao，PLB753，297（2016）；arXiv：1507．01674［hep－ph］
－F．－K．Guo，U．－G．Meissner，W．Wang，and Z．Yang，PRD（2015）；arXiv：1507．04950 ［hep－ph］
－X．－H．Liu，Q．Wang，and Q．Zhao，PLB（2016）；arXiv：1507．05359［hep－ph］
－X．－H．Liu and M．Oka，NPA 954 （2016）352；arXiv：1512．05474［hep－ph］
－A．P．Szczepaniak，PLB747， 410 （2015）［arXiv：1501．01691［hep－ph］］

## Thanks for your attention！

(IV) $\operatorname{Ds1} 1(2460)$ or $\operatorname{Ds1(2536)}$ decays into Ds pipi can go through the ATS process.

$D_{s 1}(2460)$ or $D_{s 1}(2536)$ are mixture of $3 \mathrm{P}_{1}$ and $1 \mathrm{P}_{1}$.


The ATS peaks appear at different energies


## References:

-J.-J. Wu, X.-H. Liu, and Q. Zhao, B.-S. Zou, PRL108, 081003 (2012)
-X.-G. Wu, J.-J. Wu, Q. Zhao, B.-S. Zou, PRD 87, 014023 (2013)
-Q. Wang, C. Hanhart, Q. Zhao, PRL111, 132003 (2013)
-Q. Wang, C. Hanhart, Q. Zhao, PLB725, 106 (2013)
-X.-H. Liu, M. Oka, Q. Zhao, PLB753, 297(2016); arXiv:1507.01674
[hep-ph]
-F.-K. Guo, U.-G. Meissner, W. Wang, and Z. Yang, PRD(2015);
arXiv:1507.04950 [hep-ph]
-X.-H. Liu, Q. Wang, and Q. Zhao, PLB(2016); arXiv:1507.05359 [hep-ph]
-X.-H. Liu and M. Oka, arXiv:1512.05474[hep-ph]
-A. P. Szczepaniak, PLB747, 410 (2015) [arXiv:1501.01691 [hep-ph]]

## Lagrangians in the NREFT

- $\mathrm{Y}(4260) \mathrm{D}_{1} \mathrm{D}$ coupling:

$$
\begin{aligned}
& \quad \mathcal{L}_{Y}=i \frac{y}{\sqrt{2}}\left(\bar{D}_{a}^{\dagger} Y^{i} D_{1 a}^{i \dagger}-\bar{D}_{1 a}^{i \dagger} Y^{i} D_{a}^{\dagger}\right)+\text { H.c. }, \\
& |y|=\left(3.28_{-0.28}^{+0.25} \pm 1.39\right) \mathrm{GeV}^{-1 / 2}
\end{aligned}
$$

- Zc(3900)DD* coupling:

$$
\begin{aligned}
\mathcal{L}_{Z} & =\frac{z}{\sqrt{2}}\left[\bar{V}^{\dagger i} Z^{i} P^{\dagger}-\bar{P}^{\dagger} Z^{i} V^{\dagger i}\right]+\text { H.c. } \\
Z_{b a}^{i} & =\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} Z^{0 i} & Z^{+i} \\
Z^{-i} & -\frac{1}{\sqrt{2}} Z^{0 i}
\end{array}\right)_{b a} \quad P(V)=\left(D^{(*) 0}, D^{(*)+}\right)
\end{aligned}
$$

- D1D*pi coupling:

$$
\begin{aligned}
& \mathcal{L}_{D_{1}}=i \frac{h^{\prime}}{f_{\pi}}\left[3 D_{1 a}^{i}\left(\partial^{i} \partial^{j} \phi_{a b}\right) D_{b}^{* \dagger j}-D_{1 a}^{i}\left(\partial^{j} \partial^{j} \phi_{a b}\right) D_{b}^{*+i}\right. \\
& \left.\quad-3 \bar{D}_{a}^{* \dagger i}\left(\partial^{i} \partial^{j} \phi_{a b}\right) \bar{D}_{1 b}^{j}+\bar{D}_{a}^{* i i}\left(\partial^{j} \partial^{j} \phi_{a b}\right) \bar{D}_{1 b}^{i}\right]+ \text { H.c., (2) }
\end{aligned}
$$

Q. Wang, C. Hanhart, QZ, PRL111, 132003 (2013); PLB(2013)

Q. Wang, C. Hanhart, Q.Z., PRL111, 132003 (2013); PLB(2013)
W. Qin, S.R. Xue, Q.Z., arXiv:1605.02407[hep-ph]

See talk by Si-Run Xue in Parallel Section: B-2/26-M-1, July 26, 2016

## Invariant mass spectra for $D \pi, D^{*} \pi$, and $D^{*}$



Signature for $D_{1}(2420)$ via the tree diagram.


The Zc(3900) could have a pole below the DD* threshold.


How a D wave can be present?

$$
\begin{aligned}
& \mathcal{M}=\epsilon_{Y}^{a} \epsilon_{Z_{c}}^{b}\left(C_{S} \delta^{a b}+C_{D}\left(\hat{q}^{a} \hat{q}^{b}-\frac{1}{3} \delta^{a b}\right)\right) \\
& \sum_{\text {polarizations }}|\mathcal{M}|^{2}=2 C_{S}^{2}-2 C_{S} C_{D} \cos ^{2} \theta_{\pi} \\
& +\frac{2 C_{S} C_{D}}{3}-\frac{C_{D}^{2} \cos ^{2} \theta_{\pi}}{3}+\frac{5 C_{D}^{2}}{9} \\
& \sum_{\lambda=1,2} \epsilon_{Y}^{\lambda a} \epsilon_{Y}^{* \lambda b}=\delta^{a b}-\delta^{a 3} \delta^{b 3}, \quad \sum_{\lambda=1,2,3} \epsilon_{Z_{c}}^{\lambda a} \epsilon_{Z_{c}}^{* \lambda b}=\delta^{a b}
\end{aligned}
$$




Non-BW lineshape around the $\mathrm{Y}(4260)$ mass region as an evidence for the molecular feature of $Y(4260)$ :


$$
e^{+} e^{-} \rightarrow J / \psi \pi^{+} \pi^{-}
$$



$$
e^{+} e^{-} \rightarrow h_{c} \pi^{+} \pi^{-}
$$

- Where is the peak position and pole position of $Y(4260)$ ?
- Is the J/psi pipi the dominant decay channel for $Y(4260)$ ?
- How to understand the non-trivial lineshape of the hc pipi channel?
- Could the exclusive channel cross section lineshape provide more information about the nature of $Y(4260)$ ?
- ......
M. Cleven, Q. Wang, C. Hanhart, U.-G. Meissner, and Q. Zhao, 1310.2190; PRD90, 074039 (2014)



## Angular distribution analysis





The asymmetry of events between $\left|\cos \left(\theta_{\pi D}\right)\right|>0.5$ and $\left|\cos \left(\theta_{\pi D}\right)\right|<0.5$,

$$
\mathcal{A}=\frac{n_{>0.5}-n_{<0.5}}{n_{>0.5}+n_{<0.5}}=(0.12 \pm 0.06) \quad A= \begin{cases}0.0 & \text { S wave } \\ 0.11 & \text { D wave } \\ 0.05 & \text { S+D }\end{cases}
$$




Forward-backward asymmetry for the DD* peak:

$$
\mathcal{A}_{f b}=\frac{n_{>0}-n_{<0}}{n_{>0}+n_{<0}} .= \begin{cases}0.0 & \text { S wave } \\ 0.37 & \mathrm{D} \text { wave } \\ 0.16 & \text { S+D }\end{cases}
$$

