Theory overview on amplitude analyses with charm decays

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Outline

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Impressive hadronic multibody decay data of D^0, D^+, D_s^+

- Dalitz plots $\to$ accumulations of events at different invariant masses: presence of meson resonances $\Leftrightarrow$ interferences
- Standard model (SM) : null CP asymmetries, if deviation: physics beyond SM
- D^0-$\bar{D}^0$ mixing: new physics?
- Multibody hadronic decays of $D_{(s)}$ mesons $\to$ microscopic flavor changing weak process: $c \to d$ and/or $c \to s$ via W meson interaction + hadronization + final state meson-meson strong interaction processes, FSI.
- Basic amplitude analyses $\to$ isobar model or sum of relativistic Breit-Wigner terms representing the different possible implied resonances + non resonant background: beyond?

$\Rightarrow$ Theoretical constraints on amplitudes: FSI - diagrammatic approach - QCD factorization.

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Different effective hadronic formalism approaches

⇒ Satoshi X. Nakamura, *A coupled-channel analysis of $D^+ \to K^-\pi^+\pi^+$ decay*, Phys. Rev. D **93**, 014005 (2016), arXiv:1504.02557: coupled-channel analysis of pseudo-data generated from the isobar model of the E791 Collaboration for Dalitz plot. Work to be presented by S.X. Nakamura on Wednesday afternoon.

- P. Magalhães, M. Robilotta, *A complete model for $D^+ \to K^-\pi^+\pi^+$ S wave*, arXiv:1312.2033, proceedings CHARM 2013.
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- P. C. Magalhães, M. R. Robilotta , *$D^+ \to K^-\pi^+\pi^+$ the weak vector current* , Phys. Rev. D **92**, 094005 (2015), arXiv:1504.06346, assumption of dominance weak vector current + chiral effective lagrangians + phenomenological form factors.

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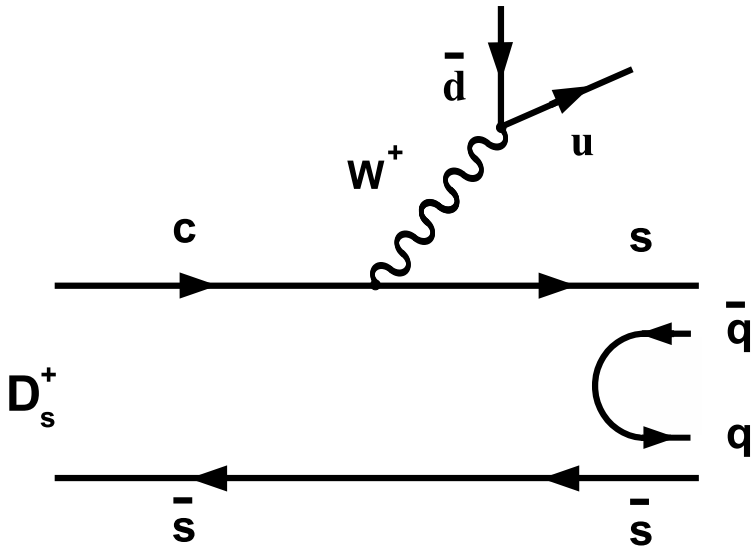
D_s^+ decays into π^+ and $q\bar{q} \to$ two pseudoscalar mesons:

We describe in detail the work [2] - same mechanism as in [1] where branching ratios for $a_0(980)$ and $f_0(980)$ production in good agreement with experiment.

[1] *The low lying scalar resonances in the D^0 decays into K_s^0 and $f_0(500)$, $f_0(980)$, $a_0(980)$*, Ju-Jun Xie, L.R. Dai, E. Oset, Physics Letters B **742**, 363 (2015), arXiv:1409.0401 (work included in *Weak decays of heavy hadrons into dynamically generated resonances*, Eulogio Oset *et al.*, arXiv:1601.03972).

[2] *$f_0(980)$ production in $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays*, J. M. Dias, F. S.Navarra, M. Nielsen, E. Oset, arXiv:1601.04635.



External π^+ emission + inserted $\bar{q}q \equiv \bar{u}u + \bar{d}d + \bar{s}s$

The $q\bar{q}$ M matrix:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s)$$

In the standard $\eta - \eta'$ mixing the matrix M is

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}.$$

Neglecting η' we have:
$s\bar{s}\,(\bar{u}u + \bar{d}d + \bar{s}s) \equiv (\Phi \cdot \Phi)_{33} = K^- K^+ + \bar{K}^0 K^0 + \frac{1}{3}\eta\eta$.
The K^+K^- pair after rescattering can produce $\pi^+\pi^-$ and K^+K^-. The D_s^+ decay width $\Gamma_{P^+P^-}$, $P^+P^- \equiv K^+K^-$ or $\pi^+\pi^-$ satisfies $\frac{d\Gamma_{P^+P^-}}{dM_{inv}} = \frac{1}{(2\pi)^3}\frac{p_\pi \tilde{p}_P}{4M_{D_s}^2}|T_{P^+P^-}|^2$, with,

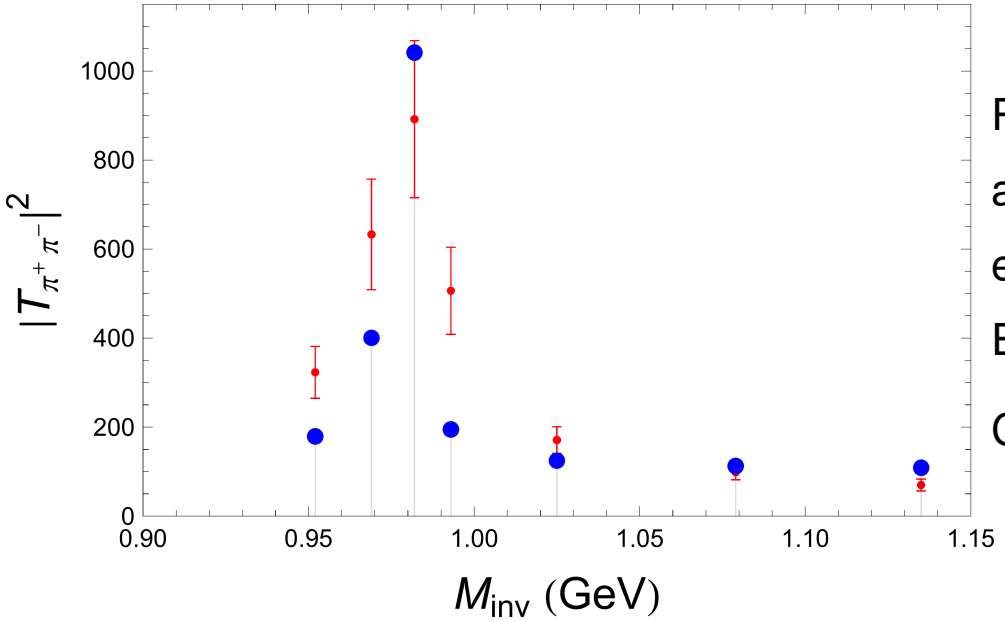
$$T_{K^+K^-} = V_0\,(1 + G_{K^+K^-}\,t_{K^+K^-\to K^+K^-} + G_{K^0\bar{K}^0}\,t_{K^0\bar{K}^0\to K^+K^-} + \frac{2}{3}\,\frac{1}{\sqrt{2}}\,G_{\eta\eta}\,t_{\eta\eta\to K^+K^-})\,,$$

$$T_{\pi^+\pi^-} = V_0\,(G_{K^+K^-}\,t_{K^+K^-\to\pi^+\pi^-} + G_{K^0\bar{K}^0}\,t_{K^0\bar{K}^0\to\pi^+\pi^-} + \frac{2}{3}\,\frac{1}{\sqrt{2}}\,G_{\eta\eta}\,t_{\eta\eta\to\pi^+\pi^-}).$$

Function G_l: loop function, $G_l(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(p-q)^2 - m_1^2 + i\varepsilon}\,\frac{1}{q^2 - m_2^2 + i\varepsilon}$, m_1, m_2: meson masses in loop l; integral on q^0 analytical; cut-off, $|\vec{q}_{max}| = 600\ \mathrm{MeV/c}$ introduced in integral on $\vec{q}$.

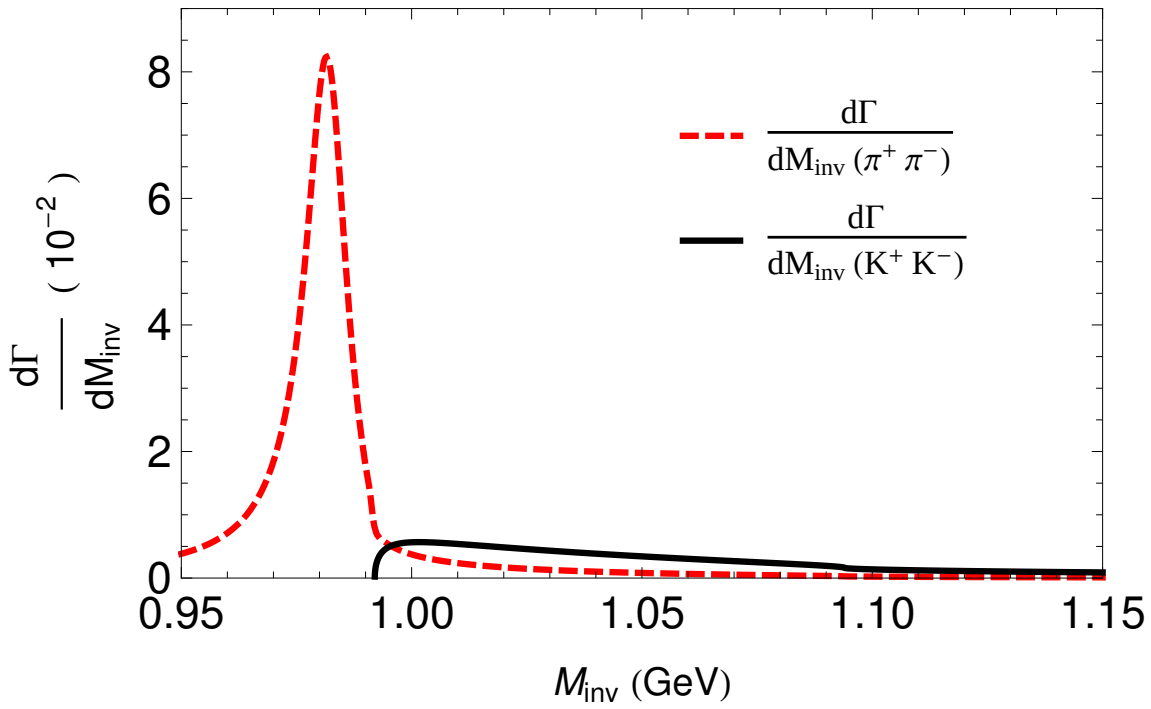
Comparison of the $T_{\pi\pi}$ amplitude with that of the experimental data

$t_{i\to j}$: coupled-channel Bethe-Salpeter $t_{i\to j}(s) = V_{ij}(s) + \sum_{l=1}^{5} V_{il}(s) G_l(s) t_{l\to j}(s)$; l channels $\to$ 1: $\pi^+\pi^-$, 2: $\pi^0\pi^0$, 3: K^+K^-, 4: $K^0\bar{K}^0$, 5: $\eta\eta$. Kernel $V \to$ tree-level transition amplitudes from phenomenological Lagrangians: J. A. Oller and E. Oset, *Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the sigma, $f_0(980)$, $a_0(980)$ scalar mesons*, NPA **620**, 438 (1997), NPA **652**, 407 (1999).



Results: adjusting V_0 and comparing theoretical amplitudes with those available from the experimental data leads to a fair agreement. Experimental data: B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **79** (2009) 032003.

Invariant mass distribution for $D_s^+ \to \pi^+\pi^-\pi^+$ (dashed line) and $D_s^+ \to \pi^+K^-K^+$ (solid line).



Results for the $f_0(980)$ signal in the spectra are (up to a global common normalization factor) predictions of the Chiral Unitary approach with no free parameters.
$\Rightarrow$ Interesting issue: the $\pi^+\pi^0\eta$ decay mode which generates the $a_0(980)$ will lead to informations on $f_0(980)$ and $a_0(980)$ mixing.

Model-independent analysis based on flavor SU(3) symmetry.

- The *c* quark mass too high to apply chiral perturbation theory and too light to use heavy quark expansion approaches
- $\Rightarrow$ Diagrammatic approach: flavor-flow diagrams with all strong interaction effects included.
- Model-independent analysis based on flavor SU(3) symmetry $\Rightarrow$ topological amplitudes $\leftrightarrow$ underlying decay mechanisms.
- Introduced by L. L. Chau, *Quark Mixing in Weak Interactions*, Phys. Rep. **95**, 1 (1983).
- Possible topologies of weak interactions: $\Rightarrow$ Fig. 1 [Hai-Yang Cheng, Cheng-Wei Chiang, *Direct CP violation in two-body hadronic charmed meson decays*, Phys. Rev. D **85**, 034036 (2012), arXiv:1201.0785]

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Flavor-flow diagrams with all strong interaction effects included.

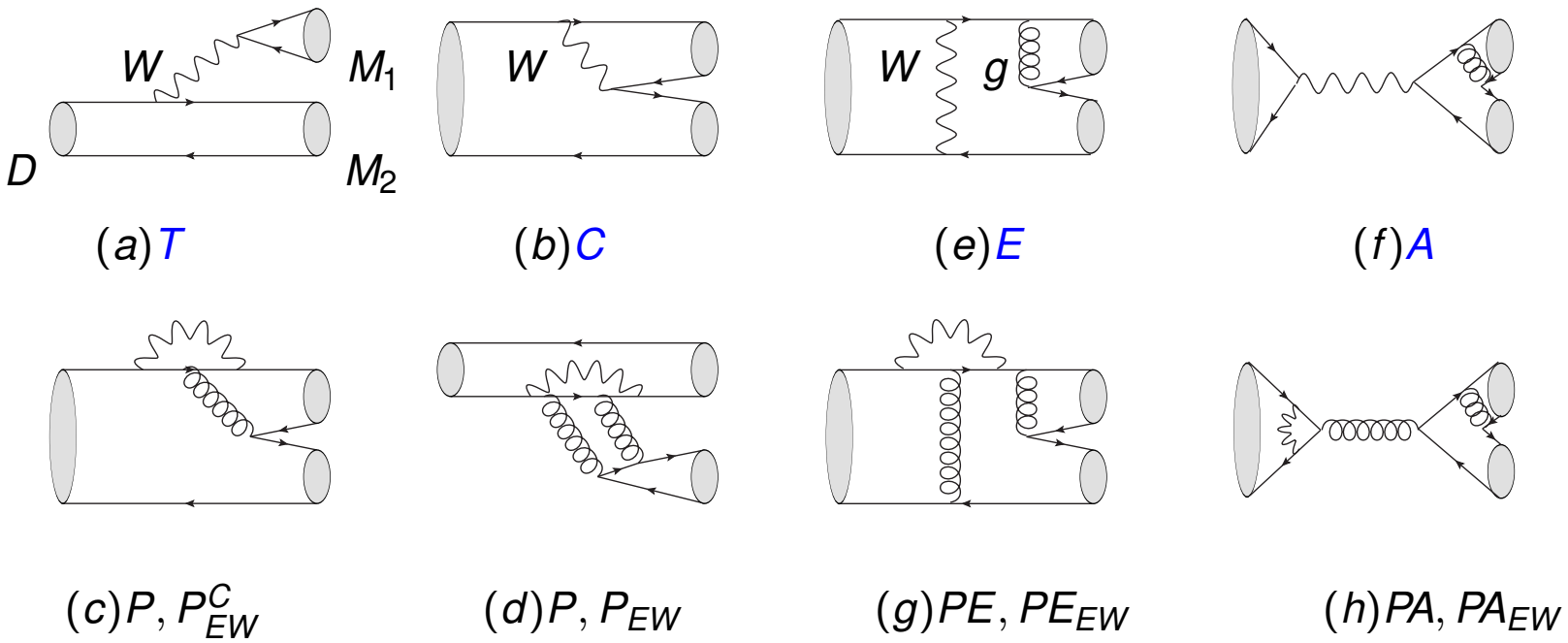

Figure: (a) T: color-allowed tree, (b) C: color-suppressed tree, (e)E: W-exchange , (f) A: W-annihilation, (c)P: QCD-penguin, (d)S: singlet QCD-penguin with 2 (3) gluon lines for M_2 being a pseudoscalar meson P (a vector meson V), (g)PE: QCD-penguin exchange, (h)PA: QCD-penguin annihilation.

Global analysis of two-body decays.

We describe here some results of the work of Hai-Yang Cheng, Cheng-Wei Chiang, An-Li Kuo, *A global analysis of two-body D to VP decays within the framework of flavor symmetry*, Phys. Rev. D **93**, 114010 (2016), arXiv:1604.03761.

- Within SU(3) flavor symmetry only four types of amplitudes for all $D \to VP$ decays: color-allowed T, color-suppressed C, W-exchange E, W-annihilation A.
- Subscript P or V to each amplitude, *e.g.*, $T_{P(V)}$, denote the amplitude in which the spectator quark goes to the pseudoscalar or vector meson in the final state.
- The 8 complex amplitudes, $T_{P(V)}, C_{P(V)}, E_{P(V)}, A_{P(V)}$ (15 real parameters, T_V chosen to be real) determined by performing a χ^2 fit of 16 experimental branching fractions for Cabibbo-favored decays $\propto$ CKM factors $Y_{sd} \equiv V_{cs}^* V_{ud} \sim \mathcal{O}(1)$. Several solutions: favored one (A1).
- Comparison made with pole model: genralized factorization + pole in annihilation diagrams [Fu-Sheng Yu, Xiao-Xia Wang, Cai-Dian Lü, [YWL], *Nonleptonic Two Body Decays of Charmed Mesons*, Phys. Rev. D **84**, 074019 (2011), arXiv:1101.4714; see also: Qin Qin, Hsiang-nan Li, Cai-Dian Lü, Fu-Sheng Yu, *Branching ratios and direct CP asymmetries in $D \to PV$ decays*, Phys. Rev. D **89**, 054006 (2014), arXiv:1305.7021].

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Fitted branching fractions for Cabibbo-favored (CF) $D^0 \to VP$ decays.

Table: Units: %, data: PDG, $Y_{sd} \equiv V_{cs}^* V_{ud}$, $s_\phi \equiv \sin\phi$, $c_\phi \equiv \cos\phi$ (mixing angle η-η'). For comparison $\mathcal{B}$(pole): pole model of [YWL].

Mode	Amplitudes	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}$(pole)
$K^{*-}\pi^+$	$Y_{sd}(T_V + E_P)$	5.43 ± 0.44	5.45 ± 0.64	3.1 ± 1.0
$K^-\rho^+$	$Y_{sd}(T_P + E_V)$	11.1 ± 0.9	11.3 ± 2.70	8.8 ± 2.2
$\overline{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}} Y_{sd}(C_P - E_P)$	3.75 ± 0.29	3.72 ± 0.49	2.9 ± 1.0
$\overline{K}^0\rho^0$	$\frac{1}{\sqrt{2}} Y_{sd}(C_V - E_V)$	$1.28^{+0.14}_{-0.16}$	1.30 ± 0.78	1.7 ± 0.7
$\overline{K}^{*0}\eta$	$Y_{sd}((C_P + E_P) \times c_\phi/\sqrt{2} - E_V s_\phi)$	0.96 ± 0.30	0.92 ± 0.36	0.7 ± 0.2
$\overline{K}^{*0}\eta'$	$-Y_{sd}((C_P + E_P) \times s_\phi/\sqrt{2} + E_V c_\phi)$	< 0.11	0.003 ± 0.002	0.016 ± 0.005
$\overline{K}^0\omega$	$-\frac{1}{\sqrt{2}} Y_{sd}(C_V + E_V)$	2.22 ± 0.12	2.24 ± 0.84	2.5 ± 0.7
$\overline{K}^0\phi$	$-Y_{sd} E_P$	$0.847^{+0.066}_{-0.034}$	0.848 ± 0.050	0.80 ± 0.2

Fitted branching fractions for Cabibbo-favored $D^+_{(s)} \to VP$ decays .

Table: Recent measurement of $D_s^+ \to \pi^+ \rho^0 \Rightarrow A_{P,V}$. Units: %, data: PDG but $\mathcal{B}_{\rho^+ \eta'}$ from BESIII Coll., Phys. Lett. B **750**, 466 (2015).

Mode	Amplitude	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}$(pole)
		$D^+ \to VP$		
$\overline{K}^{*0}\pi^+$	$Y_{sd}(T_V + C_P)$	1.57 ± 0.13	1.57 ± 0.25	1.4 ± 1.3
$\overline{K}^0 \rho^+$	$Y_{sd}(T_P + C_V)$	$12.08^{+1.20}_{-0.68}$	12.15 ± 11.69	15.1 ± 3.8
		$D_s^+ \to VP$		
$\overline{K}^{*0} K^+$	$Y_{sd}(C_P + A_V)$	3.92 ± 0.14	3.92 ± 1.13	4.2 ± 1.7
$\overline{K}^0 K^{*+}$	$Y_{sd}(C_V + A_P)$	5.4 ± 1.2	4.38 ± 1.19	1.0 ± 0.6
$\rho^+ \pi^0$	$\frac{1}{\sqrt{2}} Y_{sd}(A_P - A_V)$	—	0.021 ± 0.087	0.4 ± 0.4
$\rho^+ \eta$	$-Y_{sd}((A_P + A_V) \times \frac{1}{\sqrt{2}} c_\phi - T_P s_\phi)$	8.9 ± 0.8	8.85 ± 1.69	8.3 ± 1.3
$\rho^+ \eta'$	$Y_{sd}((A_P + A_V) \times \frac{1}{\sqrt{2}} s_\phi + T_P c_\phi)$	5.80 ± 1.46	2.75 ± 0.46	3.0 ± 0.5
$\pi^+ \rho^0$	$\frac{1}{\sqrt{2}} Y_{sd}(A_V - A_P)$	0.020 ± 0.012	0.021 ± 0.087	0.4 ± 0.4
$\pi^+ \omega$	$\frac{1}{\sqrt{2}} Y_{sd}(A_V + A_P)$	0.24 ± 0.06	0.24 ± 0.15	0
$\pi^+ \phi$	$Y_{sd} T_V$	4.5 ± 0.4	4.49 ± 0.40	4.3 ± 0.6

Cabibbo-favored amplitudes resulting from the branching fraction fit.

Table: Fit results (solution A1) $\phi = 43.5°$. Units: 10^{-6}, strong phases in degrees. $|T_V| = 4.21^{+0.18}_{-0.19}$.

$\lvert T_P\rvert$	δ_{T_P}	$\lvert C_V\rvert$	δ_{C_V}	$\lvert C_P\rvert$	δ_{C_P}	$\lvert E_V\rvert$
$8.46^{+0.22}_{-0.25}$	57^{+35}_{-41}	$4.09^{+0.16}_{-0.25}$	-145^{+29}_{-39}	$4.08^{+0.37}_{-0.36}$	-157 ± 2	$1.19^{+0.64}_{-0.46}$
δ_{E_V}	$\lvert E_P\rvert$	δ_{E_P}	$\lvert A_P\rvert$	δ_{A_P}	$\lvert A_V\rvert$	δ_{A_V}
-85^{+42}_{-39}	3.06 ± 0.09	98 ± 5	$0.64^{+0.14}_{-0.27}$	152^{+48}_{-50}	$0.52^{+0.24}_{-0.19}$	122^{+70}_{-42}

- Modulus of color-allowed tree T_P amplitude is the largest.
- Moduli of the W-annihilation $A_P(V)$ amplitudes are the smallest.

Predictions for some singly Cabibbo-suppressed decays,

Table: Units of 10^{-3}. $Y_d \equiv V_{cd}^* V_{ud} \sim \mathcal{O}(\lambda)$, $Y_s \equiv V_{cs}^* V_{us} \sim \mathcal{O}(\lambda)$, $\lambda = 0.22543$ (CKMfitter). No SU(3) breaking: $T' \equiv T$ etc ....

Mode	Amplitude	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}$(pole)
		$D^0 \to VP$		
$\pi^+ \rho^-$	$Y_d(T'_V + E'_P)$	5.09 ± 0.34	3.61 ± 0.43	3.5 ± 0.6
$\pi^- \rho^+$	$Y_d(T'_P + E'_V)$	10.0 ± 0.6	8.73 ± 2.09	10.2 ± 1.5
$\pi^0 \phi$	$\frac{1}{\sqrt{2}} Y_s C'_P$	1.35 ± 0.10	0.77 ± 0.14	1.0 ± 0.3
		$D^+ \to VP$		
$\pi^+ \rho^0$	$\frac{1}{\sqrt{2}} Y_d(T'_V + C'_P - A'_P + A'_V)$	0.84 ± 0.15	0.51 ± 0.28	0.8 ± 0.7
$K^+ \overline{K}^{*0}$	$Y_d A'_V + Y_s T'_V$	$3.84^{+0.14}_{-0.23}$	4.00 ± 0.82	4.1 ± 1.0
$\overline{K}^0 K^{*+}$	$Y_d A'_P + Y_s T'_P$	34 ± 16	14.45 ± 2.45	12.4 ± 2.4
		$D_s^+ \to VP$		
$\pi^+ K^{*0}$	$Y_d T'_V + Y_s A'_V$	2.13 ± 0.36	3.51 ± 0.72	1.5 ± 0.7
$K^+ \rho^0$	$\frac{1}{\sqrt{2}}(Y_d C'_P - Y_s A'_P)$	2.5 ± 0.4	1.58 ± 0.38	1.0 ± 0.6
$K^+ \phi$	$Y_s(T'_V + C'_P + A'_V)$	0.164 ± 0.041	0.111 ± 0.060	0.3 ± 0.3

Predictions for some doubly Cabibbo-suppressed decays. No SU(3) breaking: $T'' \equiv T$ etc....

Table: Units: 10^{-4}. $Y_{ds} \equiv V_{cd}^* V_{us} \sim \mathcal{O}(\lambda^2)$.

Meson	Mode	Amplitude	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}$(pole)
D^0	$K^{*+}\pi^-$	$Y_{ds}(T''_P + E''_V)$	$3.45^{+1.80}_{-1.02}$	3.77 ± 0.90	2.7 ± 0.6
	$K^{*0}\pi^0$	$\frac{1}{\sqrt{2}} Y_{ds}\left(C''_P - E''_V\right)$	—	0.49 ± 0.23	0.8 ± 0.3
	ϕK^0	$-Y_{ds} E''_V$	—	0.04 ± 0.03	0.2 ± 0.06
D^+	$K^{*0}\pi^+$	$Y_{ds}(C''_P + A''_V)$	3.9 ± 0.6	2.94 ± 0.85	2.2 ± 0.9
	$K^{*+}\pi^0$	$\frac{1}{\sqrt{2}} Y_{ds}\left(T''_P - A''_V\right)$	—	5.76 ± 0.85	4.0 ± 0.9
	$\rho^0 K^+$	$\frac{1}{\sqrt{2}} Y_{ds}(T''_V - A''_P)$	2.1 ± 0.5	1.66 ± 0.24	0.5 ± 0.4
D_s^+	$K^{*+}K^0$	$Y_{ds}(T''_P + C''_V)$	—	1.55 ± 1.49	2.3 ± 0.06
	$K^{*0}K^+$	$Y_{ds}(T''_V + C''_P)$	0.90 ± 0.51	0.17 ± 0.03	0.2 ± 0.02

- Doubly Cabibbo-suppressed channels: good agreement with data.
- Singly Cabibbo-suppressed $\propto Y_d \equiv V_{cd}^* V_{ud}$ and $\propto Y_s \equiv V_{cs}^* V_{us}$ have flavor SU(3) symmetry breaking effects.

Exact flavor SU(3) describes reasonably well the available data.

- If T and C amplitudes factorizable $\Rightarrow$ effective Wilson coefficients $a_{1,2}$, $|a_2/a_1|$ and $\arg(a_2/a_1)$ (see next Section) from Cabibbo-favored $D^+ \to \overline{K}^{*0}\pi^+$ and $\overline{K}^0\rho^+$ [solutions (A1)]

	$\overline{K}^{*0}\pi^+$	$\overline{K}^0\rho^+$
$\lvert a_1\rvert$	1.34 ± 0.06	1.43 ± 0.04
$\lvert a_2\rvert$	0.69 ± 0.06	1.05 ± 0.05
$\lvert a_2/a_1\rvert$	0.52 ± 0.05	0.73 ± 0.04
$\arg(a_2/a_1)$	$-(157 \pm 2)^\circ$	$(158 \pm 51)^\circ$

- Within the diagrammatic approach we quote the study of validity of flavor SU(3) by B. Bhattacharya and J. L. Rosner, *Decays of Charmed Mesons to PV Final States*, Phys. Rev. D **79**, 034016 (2009); Erratum Phys. Rev. D **81**, 099903 (2010), arXiv:0812.3167.
- SU(3) symmetry breaking in color-allowed T and color-suppressed C tree amplitudes needed in general to have a better agreement with experiment.

$\to$ Nevertheless, the exact flavor SU(3)-symmetric approach alone is adequate to provide an overall explanation for the current data.

Factorization in D decays: phenomenological approach

- QCD factorization beyond naive factorization $\to$ expansion in α_s and $1/m_b$ applied with success to charmless nonleptonic two-body B decays [M. Beneke, M. Neubert, *QCD factorization for $B \to PP$ and $B \to PV$ decays* Nuc. Phys. B **675**, 333 (2003), arXiv:hep-ph/0308039].
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- Two of the three final-state mesons form a single state originating from a quark-antiquark pair. This hypothesis leads to a quasi-two-body final state to which the factorization procedure is applied.

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Weak effective Hamiltonian

- No penguin (W-loop diagram) in $D^0 \to K_S^0 \pi^+ \pi^-$, then

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_{i=1,2} C_i(\mu) O_i(\mu) + h.c.$$

V_{CKM}: quark mixing couplings, $G_F = 1.166 \times 10^{-5}$ GeV^{-2}: Fermi coupling, $C_i(\mu)$: QCD Wilson coefficients (W exchange), μ: renormalization scale, $\mu \sim m_c = 1.3$ GeV (charm quark mass).

- $O_{1,2}$ are left-handed quark current-current operators, e.g. (α and β color indices)

$$O_1 = j_1 \otimes j_2, \quad j_1 = \bar{s}_\alpha \gamma^\nu (1-\gamma^5) c_\alpha \equiv (\bar{s}c)_{V-A}, \quad j_2 = \bar{u}_\beta \gamma_\nu (1-\gamma_5) d_\beta \equiv (\bar{u}d)_{V-A}.$$

- In the amplitude and at leading order in α_s, the following real effective QCD coefficients $a_1(m_c)$ and $a_2(m_c)$ will appear,

$$a_1(m_c) = C_1(m_c) + \frac{C_2(m_c)}{N_C}, \qquad a_2(m_c) = C_2(m_c) + \frac{C_1(m_c)}{N_C},$$

$N_C = 3$ being the number of colors (from now on $a_i(m_c) \equiv a_i, i = 1, 2$).

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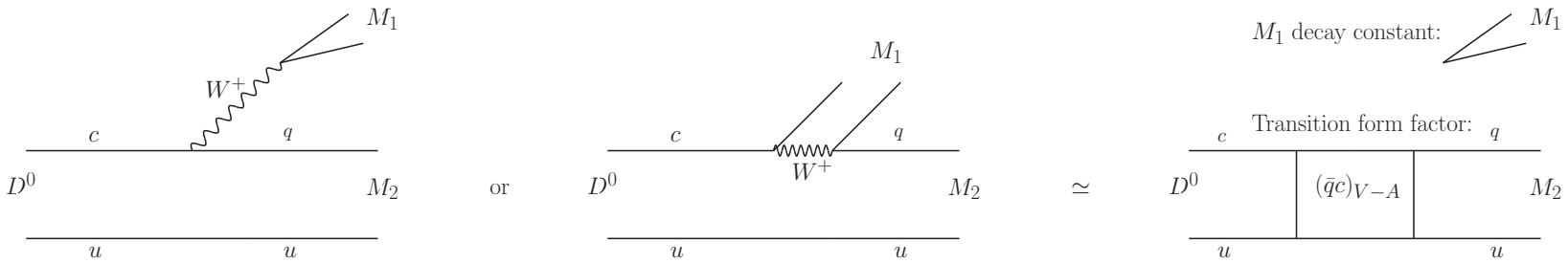
Operator Product Expansion + large W mass ⇒ two-body factorization approximation

Factorization: $\langle M_1 M_2 | O_i(\mu) | D^0 \rangle = \langle M_1 | j_1 | 0 \rangle \langle M_2 | j_2 | D^0 \rangle$ + higher order corrections



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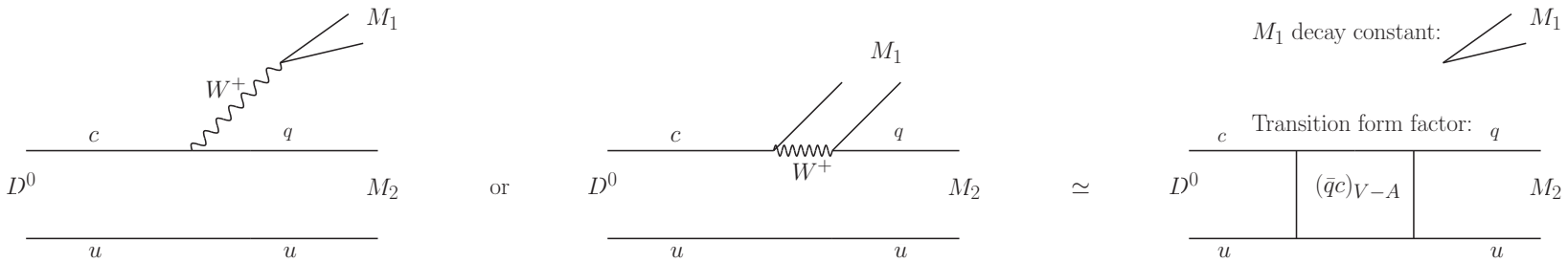

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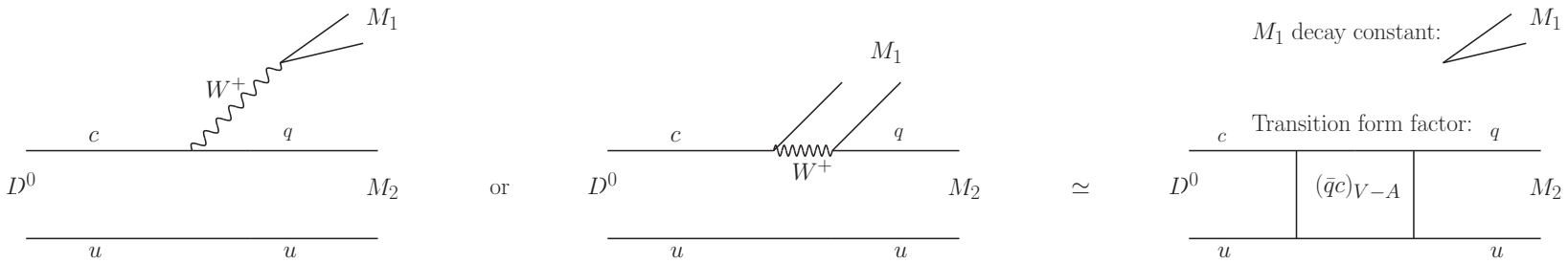

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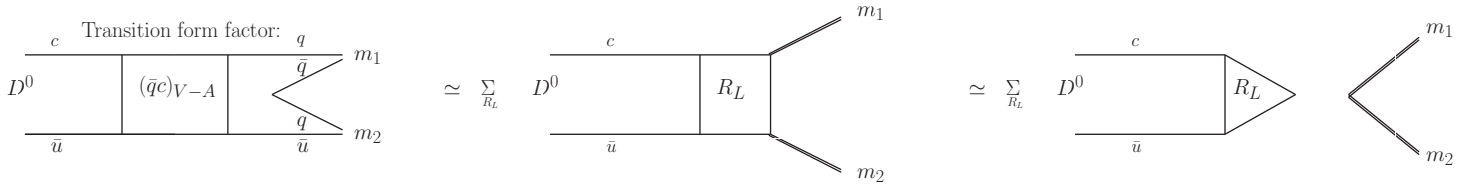
Decay constant ↑ Transition form factor ↑

- If $M_1 = [\pi^+ K^0]_L \leftrightarrow$ Form Factor $\langle [\pi^+ K^0]_L | (\bar{s}u)_{V-A} | 0 \rangle$: $[\pi^+ K^0]_L$ interaction

Resonance contributions [note: $s = (p_1 + p_2)^2$, $s_\pm = (p_{\pi^\pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$]

- Contribution of resonances, e.g. from $q\bar{u}$, in a given $[m_1 m_2]_L$ channel:

$$\left\langle [m_1 m_2]_L | (\bar{q}c)_{V-A} | D^0 \right\rangle \simeq \sum_{R_L} F^{D^0 R_L}(m^2_{m_3}) G_{R_L}(s) \left\langle R_L^{[m_1 m_2]} | q\bar{u} \right\rangle$$



$$\simeq c_{R_L} \chi \sum_{R_L} F^{D^0 R_L}(m^2_{m_3}) \langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \rangle \simeq c_{R_L} \chi F^{D^0 \tilde{R}_L}(m^2_{m_3}) F_L^{m_1 m_2}(s).$$

$$G_{R_L}(s) = \chi \langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \rangle$$

$c_{R_L} = \left\langle R_L^{[m_1 m_2]} | q\bar{u} \right\rangle$, $\tilde{R}_L$: L-wave dominant resonance

- With $L = S$, $q = u$, $m_3 = \bar{K}^0$, $m_1 = \pi^+$, $m_2 = \pi^-$, $\tilde{R}_S = f_0(980)$, $c_{f_0} = 1/\sqrt{2}$:

$$T^{[\text{Cabibbo Favored}]}_{\bar{K}^0\ [\pi^+\pi^-]_S}(s_0, s_-, s_+) = -\frac{G_F}{2}\, a_2\, \Lambda_1\, \chi\, (m^2_{D^0} - s_0)\, f_{K^0}\, F_0^{D^0 f_0(980)}(m^2_{K^0})\, F_0^{\pi^+\pi^-}(s_0)$$

$\to$ Form factor $F_0^{\pi^+\pi^-}(s_0)$ includes contribution of $f_0(500)$, $f_0(980)$, $f_0(1400)$

$\to$ If a resonance is largely dominant, like the $\rho(770)^0$ in $[\pi^+\pi^-]_P$ then $\chi \propto 1/f_\rho$

Resonance contributions [note: $s = (p_1 + p_2)^2$, $s_\pm = (p_{\pi^\pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$]

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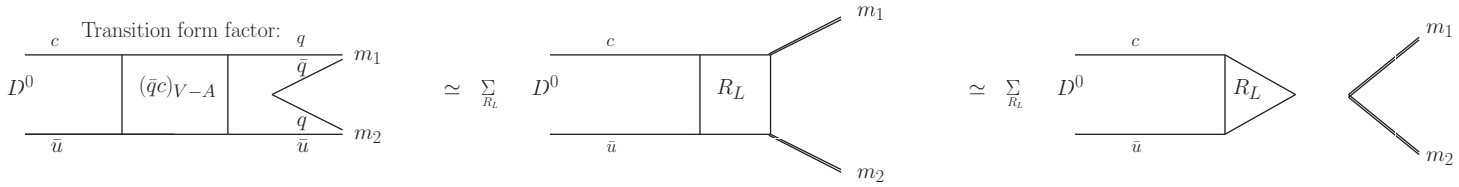

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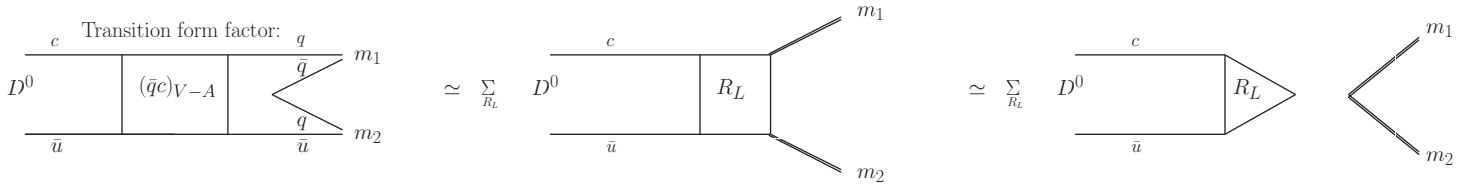

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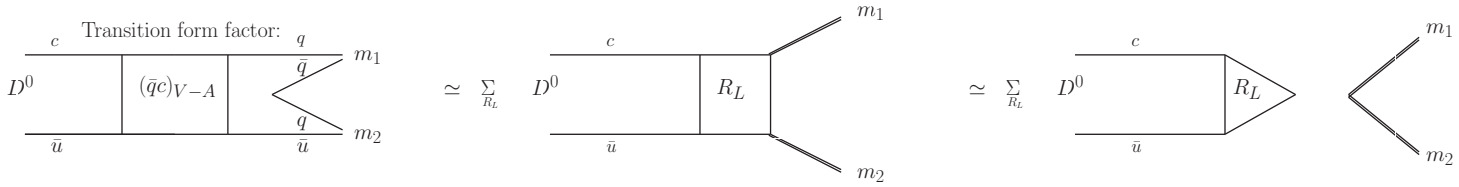

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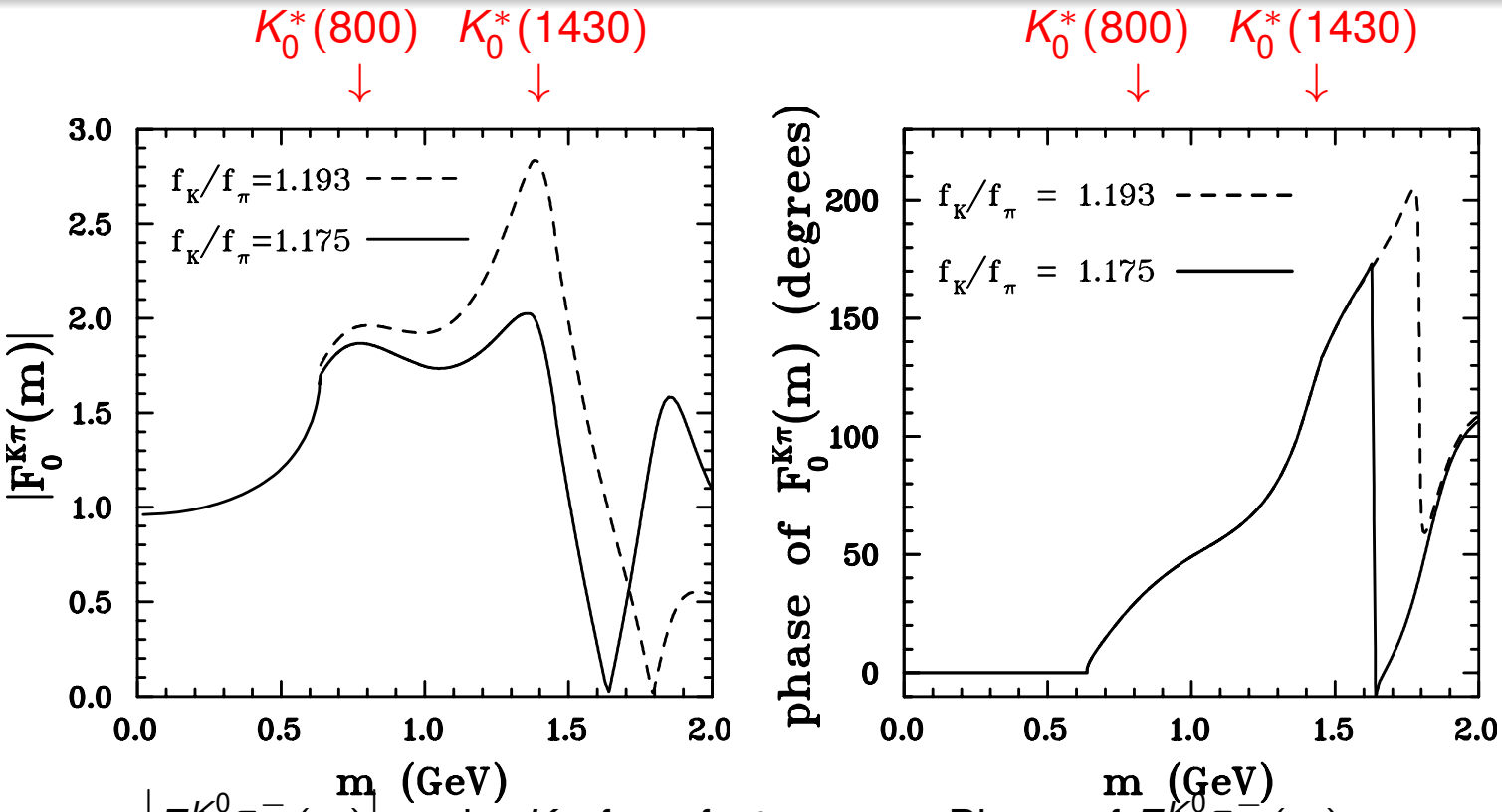
Dominant resonances in the quasi-two-body CF and DCS channel amplitudes

Table: CF, Cabibbo-favored ($\propto V_{cs}^* V_{ud}$) and DCS, doubly-Cabibbo-suppressed ($\propto V_{cd}^* V_{us}$) amplitudes. 13 tree-amplitudes (Tr), 14 W-exchange amplitudes (An) recast into 10 amplitudes.

Amplitude	Channel	Dominant resonances	Form Factor
$\mathcal{M}_{1\pi}$: CF(Tr+An)	$[K_S^0 \pi^-]_S \pi^+$	$K_0^*(800)^-$, $K_0^*(1430)^-$	$F_0^{K\pi}(s_-)$
$\mathcal{M}_{2\pi}$: CF+DCS(Tr+An)	$K_S^0 [\pi^+\pi^-]_S$	$f_0(500)$, $f_0(980)$, $f_0(1400)$	$F_0^{\pi\pi}(s_0)$
$\mathcal{M}_{3\pi}$: CF(Tr+An)	$[K_S^0 \pi^-]_P \pi^+$	$K^*(892)^-$	$F_1^{K\pi}(s_-)$
$\mathcal{M}_{4\pi}$: CF+DCS(Tr+An)	$K_S^0 [\pi^+\pi^-]_P$	$\rho(770)^0$	$F_1^{\pi\pi}(s_0)$
$\mathcal{M}_{5\pi}$: CF+DCS(Tr+An)	$K_S^0 [\pi^+\pi^-]_\omega$	$\omega(782)$	Breit-Wigner
$\mathcal{M}_{6\pi}$: CF(Tr+An)	$[K_S^0 \pi^-]_D \pi^+$	$K_2^*(1430)^-$	Breit-Wigner
$\mathcal{M}_{7\pi}$: CF+DCS(Tr+An)	$K_S^0 [\pi^+\pi^-]_D$	$f_2(1270)$	Breit-Wigner
$\mathcal{M}_{8\pi}$: DCS(Tr+An)	$[K_S^0 \pi^+]_S \pi^-$	$K_0^*(800)^+$, $K_0^*(1430)^+$	$F_0^{K\pi}(s_+)$
$\mathcal{M}_{9\pi}$: DCS(Tr+An)	$[K_S^0 \pi^+]_P \pi^-$	$K^*(892)^+$	$F_1^{K\pi}(s_+)$
$\mathcal{M}_{10\pi}$: DCS(An)	$[K_S^0 \pi^+]_D \pi^-$	$K_2^*(1430)^+$	Breit-Wigner

Here: $s_\pm = (p_{\pi^\pm} + p_{K^0})^2, s_0 = (p_{\pi^+} + p_{\pi^-})^2$

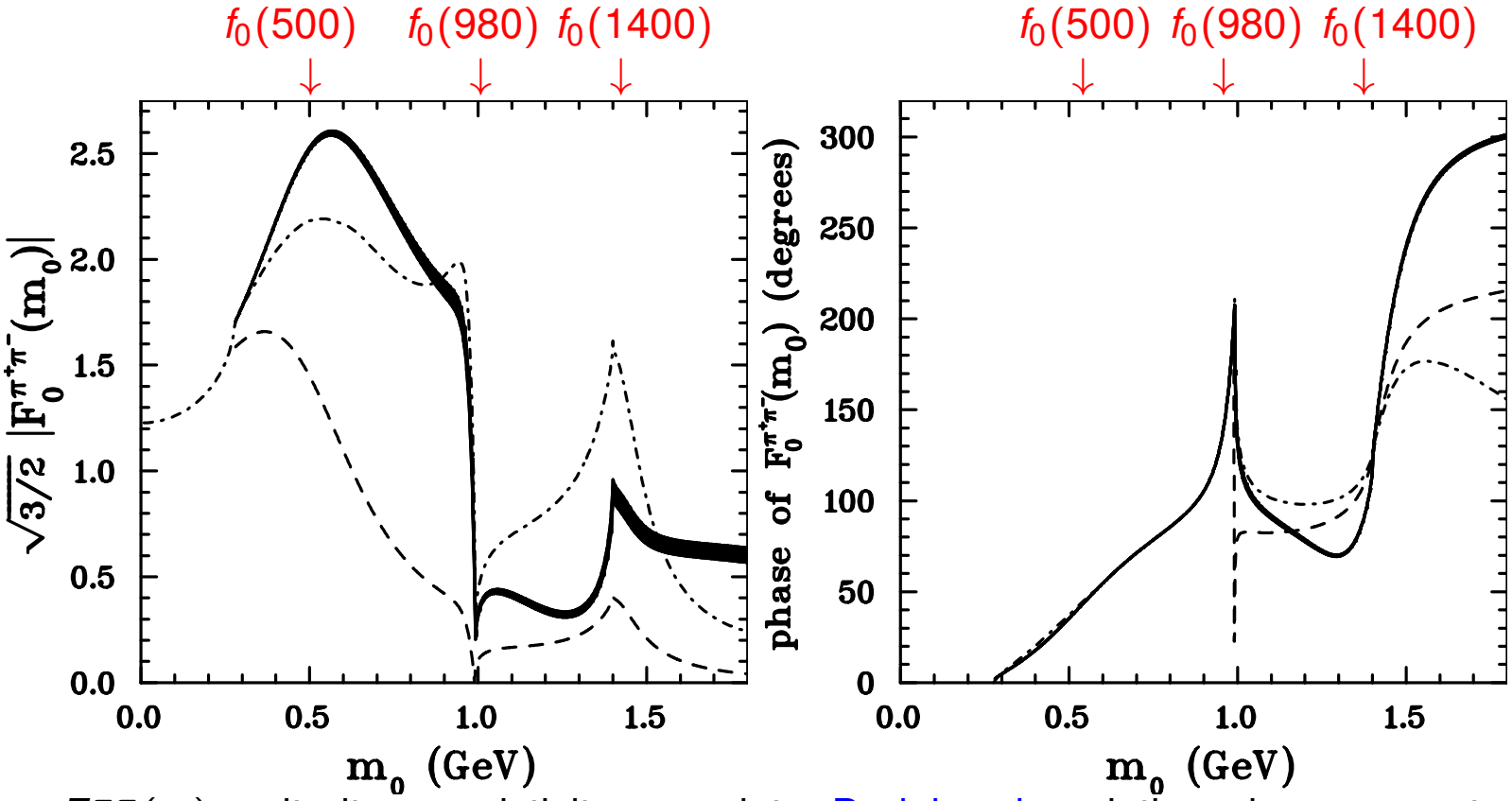
Unitary scalar $K\pi$ form factor used in our model for $D^0 \to K_S^0 \pi^+ \pi^-$



$\left|F_0^{K^0\pi^-}(m)\right|$ scalar $K\pi$ form factor Phase of $F_0^{K^0\pi^-}(m)$

$\Rightarrow$ Best fit with the $K\pi$ scalar form factor: Muskhelishvili-Omnès's 2 coupled channel equations using experimental kaon-pion T matrix + chiral symmetry + asymptotic QCD constraints + f_K/f_π =1.175 [B. Moussallam private communication, see also B. El-Bennich *et al.* Phys. Rev. D **79**, 094005 (2009)]

Unitary scalar-isoscalar $\pi\pi$ form factor $F_0^{\pi\pi}(\propto \Gamma_1^n)$ used in our model for $D^0 \to K_S^0 \pi^+ \pi^-$



$\Rightarrow$ $F_0^{\pi\pi}(m)$: unitarity + analyticity + $\pi\pi$ data. Dark band: variation when parameters vary within their errors. Dashed line: [$B \to 3\pi$, J.-P. Dedonder *et al.* Acta Phys. Pol. B **42**, 2013 (2011)]. Dotted-dashed line: Moussallam calculation [Eur. Phys. J. C. **14**, 111 (2000)] using Muskhelishvili-Omnès equations

$K\pi$ and $\pi\pi$ vector form factor used in our model for $D^0 \to K_S^0 \pi^+ \pi^-$

- Mass and width of the $K^*(892)$ meson are free parameters entering also in the $K\pi$ vector form factor taken from the Belle Collaboration fit to the $\tau^- \to K_S^0 \pi^- \nu_\tau$ decays
  Contributions of $K^*(892)$ and $K^*(1410)$ resonances taken but not that of the $K^*(1680)$
- Alternatively to this experimental parametrization we use the model of the $K\pi$ vector form factor of D. R. Boito *et al.* [JHEP **1009**, 031 (2010)] in which some constraints from analyticity and elastic unitarity are incorporated
- Two types of the pion vector form factor tested:
  - the experimental parametrization used by Belle Collaboration [2008] in the data analysis of $\tau^- \to \pi^- \pi^0 \nu_\tau$ decays
  - the unitary parametrization of Hanhart [Phys. Lett. B **170**, 710 (2012)] which also fit the $\tau^- \to \pi^- \pi^0 \nu_\tau$ data

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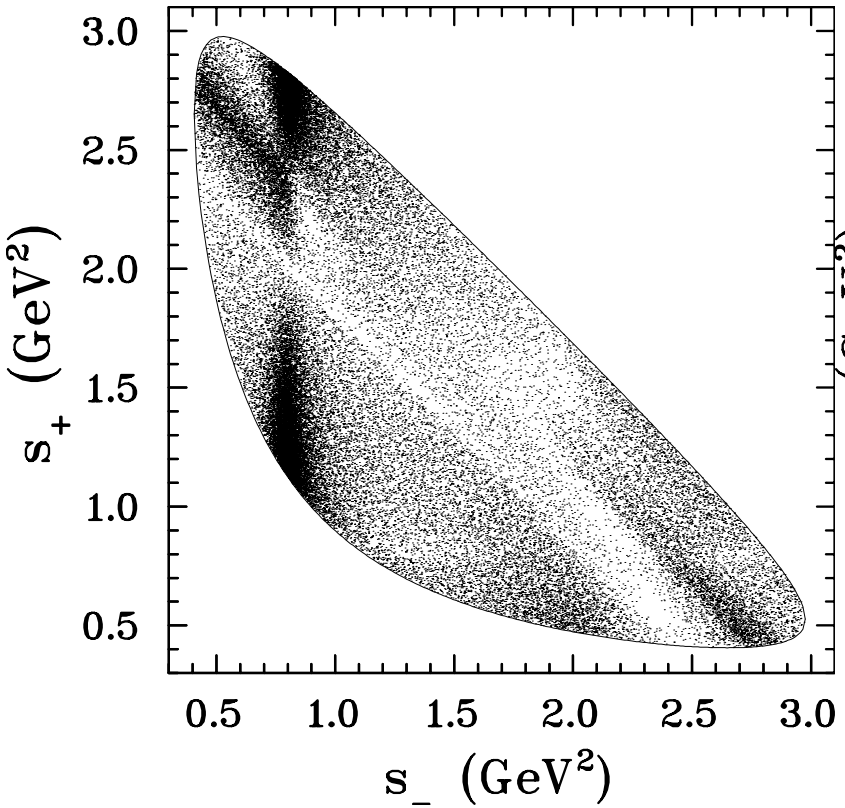
Input and free parameters, here choice: $a_1 = 1.1 \;\; a_2 = -0.5$

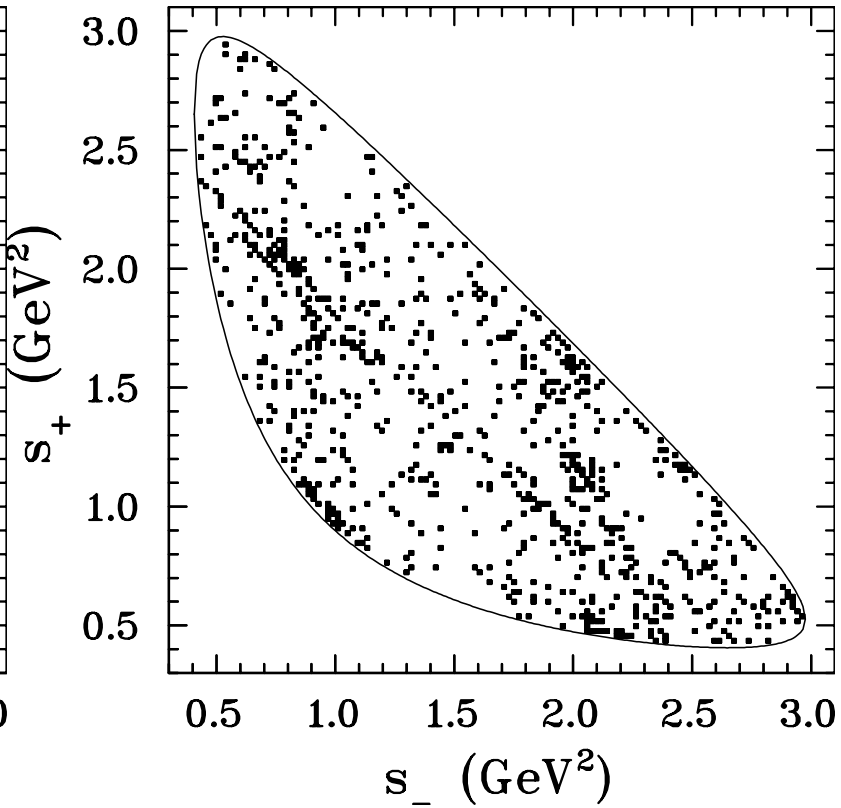
fixed parameters	value
$F_0^{D^0 R_S[\overline{K}^0 \pi^-]}(m_\pi^2)$	0.48
$F_0^{D^0 R_S[\pi^+\pi^-]}(m_{K^0}^2)$	0.18
$A_0^{D^0 R_P[\overline{K}^0 \pi^-]}(m_\pi^2)$	0.76
$A_0^{D^0 R_P[\pi^+\pi^-]}(m_{K^0}^2)$	0.7
$A_0^{D^0 \omega}(m_{K^0}^2)$	0.669
$g_{\omega\pi\pi}$	0.3504
$g_{f_2\pi^+\pi^-}$	18.55 GeV^{-1}
$g_{K_2^{*-}K^0\pi^-}$	11.72 GeV^{-1}

$\Rightarrow$ 33 free parameters $\to$ 14 complex: $\chi_{[\pi K]_S}$, $\chi_{[\pi\pi]_S}$, $F_0^{R_S[\bar{K}^0\pi^-]\pi^+}(m_{D^0}^2)$, $F_0^{\bar{K}^0 R_S[\pi^+\pi^-]}(m_{D^0}^2)$, $A_0^{R_P[\bar{K}^0\pi^-]\pi^+}(m_{D^0}^2)$, $A_0^{\bar{K}^0\omega}(m_{D^0}^2)$ + 6 parameters for D-waves + 2 for possible charge independence violation in $\mathcal{M}_{8\pi}$ and $\mathcal{M}_{9\pi}$,
$\to$ 5 real parameters: $A_0^{\bar{K}^0 R_P[\pi^+\pi^-]}(m_{D^0}^2)$ + 2 for pion-scalar form factor, κ, c + 2 for kaon-vector form factor, $m_{K^{*\mp}}$, Γ_{K^*}.

Fit to Belle Dalitz plot and distribution of χ^2 values larger than 4



$K^*(892)^-$ ↑. Fit to Belle Dalitz plot data



Black square: $\chi^2 > 4$

Branching fractions (Br) for all quasi two-body channels. $\sum$ Br=132.81 % $\leftrightarrow$ interferences.

Table: $\mathcal{M}_{1\pi}$: $K_0^*(800)^-$, $K_0^*(1430)^-$; $\mathcal{M}_{2\pi}$:$f_0(500)$, $f_0(980)$, $f_0(1400)$; $\mathcal{M}_{3\pi}$:$K^*(892)^-$; $\mathcal{M}_{4\pi}$: $\rho(770)^0$

Amplitude	channel	Br	tree	ann. low
$\mathcal{M}_{1\pi}$	$[K_S^0\,\pi^-]_S\,\pi^+$	$25.03 \pm 3.61 \pm 0.18$	8.24 ± 0.10	7.88 ± 0.11
$\mathcal{M}_{2\pi}$	$K_S^0[\pi^-\pi^+]_S$	$16.92 \pm 1.27 \pm 0.02$	14.70 ± 0.17	2.92 ± 0.09
$\mathcal{M}_{3\pi}$	$[K_S^0\,\pi^-]_P\,\pi^+$	$62.72 \pm 4.45 \pm 0.15$	24.69 ± 5.65	8.74 ± 2.97
$\mathcal{M}_{4\pi}$	$K_S^0[\pi^-\pi^+]_P$	$21.96 \pm 1.55 \pm 0.06$	4.36 ± 0.06	6.74 ± 0.04
$\mathcal{M}_{5\pi}$	$K_S^0\omega$	$0.79 \pm 0.07 \pm 0.04$	0.24 ± 0.01	0.16 ± 0.02
$\mathcal{M}_{6\pi}$	$[K_S^0\,\pi^-]_D\,\pi^+$	$1.41 \pm 0.11 \pm 0.04$		
$\mathcal{M}_{7\pi}$	$K_S^0[\pi^-\pi^+]_D$	$2.15 \pm 0.19 \pm 0.10$		
$\mathcal{M}_{8\pi}$	$[K_S^0\,\pi^+]_S\,\pi^-$	$0.56 \pm 0.07 \pm 0.03$	0.07 ± 0.00	0.29 ± 0.02
$\mathcal{M}_{9\pi}$	$[K_S^0\,\pi^+]_P\,\pi^-$	$0.64 \pm 0.06 \pm 0.02$	0.77 ± 0.15	0.01 ± 0.01
$\mathcal{M}_{10\pi}$	$[K_S^0\,\pi^+]_D\,\pi^-$	$0.63 \pm 0.07 \pm 0.11$	0	0.63 ± 0.11

- Branching fractions compare well with those of Belle 's analysis

$\to$ Belle $\mathrm{Br}_{K_S^0\sigma_1} + \mathrm{Br}_{K_S^0 f_0(980)} + \mathrm{Br}_{K_S^0\sigma_2} + \mathrm{Br}_{K_S^0 f_0(1370)}$ = 18.6% $\sim$ 16.9% value

- Annihilation contributions can be important

FSI described in term of meson-meson form factors

Quasi two body-factorization approaches have also been performed by:

[1] D. R. Boito, J.-P. Dedonder, B. El-Bennich, O. Leitner and B. Loiseau, *Scalar resonances in a unitary $\pi\pi$ S-wave model for $D^+ \to \pi^+\pi^-\pi^+$*, Phys. Rev. D **79**, 034020 (2009), arXiv:0812.3843.

[2] D. R. Boito and R. Escribano, *$K\pi$ form-factors and final state interactions in $D^+ \to K^-\pi^+\pi^+$ decays*, Phys. Rev. D **80**, 054007 (2009), arXiv:0907.0189.

[3] J.-P. Dedonder, R. R Kamiński, L. Leśniak and B. Loiseau, *Dalitz plot studies of $D^0 \to K_S^0 K^+ K^-$ decays in a factorization approach*, work in progress.

- From these phenomenologically successful studies $\Rightarrow$ amplitude parametrizations to be readily implemented in experimental analyses.
- Two-body hadronic final state interactions taken into account in terms of unitary S- and P-wave $\pi\pi$, πK and $K\bar{K}$ form factors.
- Sound alternative to the simplistic and widely used isobar model.

$\Rightarrow$ Explicit amplitude expressions [4,5] for the decays $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\pi^+$, $D^0 \to K_S^0\,\pi^+\pi^-$, $D^0 \to K_S^0\,K^+K^-$.

[4] D. Boito and B. Loiseau, on page 27. of J. H. Alvarenga Nogueira *et al.*, "Summary of the 2015 LHCb workshop on multi-body decays of D and B mesons," arXiv:1605.03889

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Improved models needed for getting out the most information from new data

Impressive amount of high quality hadronic multibody decay data of D^0, D^+, D_s^+ and in this review I described some available potentialities for constraining amplitude analyses in these charm decays:

- Final state constraints: effective-hadronic formalism approach with combination of basic elements of the weak interaction with the framework of the chiral unitary approach in coupled channel.
- Diagrammatic approach: model-independent analysis based on flavor SU(3) symmetry topological amplitudes allowing to understand the relative importance of different underlying decay mechanisms.
- Three-body decays analyzed in a phenomenological quasi-two-body factorization approach in which two-body hadronic final state interactions are fully taken into account in terms of unitary form factors.

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Some recent works on hadronic D decays

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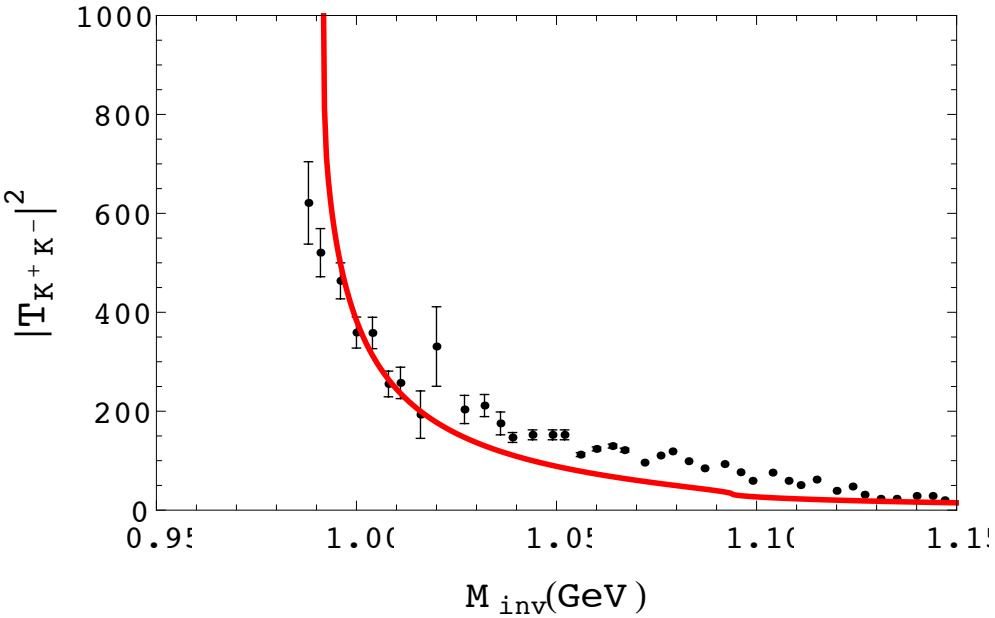
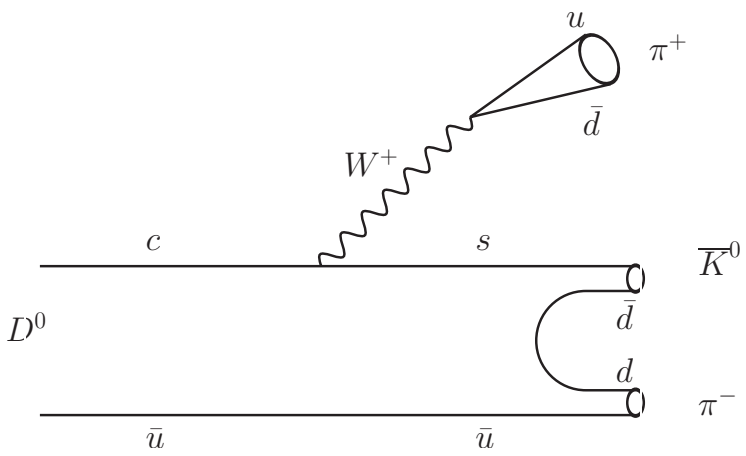
Comparison of the $T_{K^+K^-}$ amplitude with that of the experimental data

$\to$ FSI in [2] *$f_0(980)$ production in $D_s \to \pi^+\pi^+\pi^-$ and $D_s \to \pi^+K^+K^-$ decays*, J. M. Dias, F. S.Navarra, M. Nielsen, E. Oset, arXiv:1601.04635.



$\to$ Experimental data: P. del Amo Sanchez *et al.* [BABAR Collaboration], *Dalitz plot analysis of $D_s^+ \to K^+K^-\pi^+$*, Phys. Rev. D **83**, 052001 (2011), arXiv:1011.4190.
$\to$ $f_0(980)$ in the 1 GeV region

Cabibbo favored and suppressed tree amplitudes in $D^0 \to K_S^0 \pi^+ \pi^-$



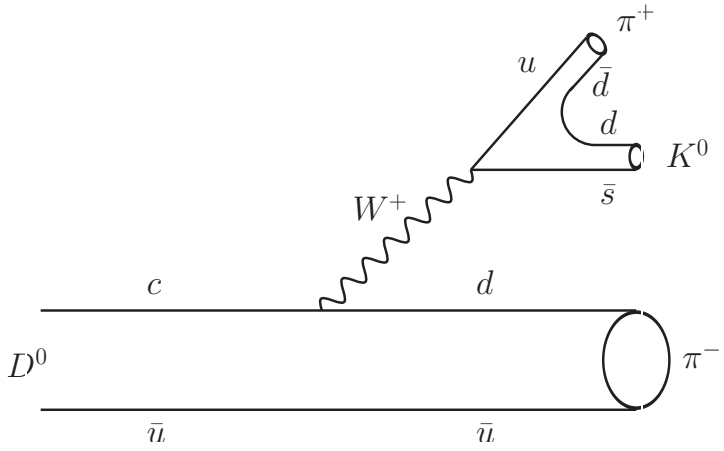
$c \to s u \bar{d}$ transition :

$$\propto \frac{G_F}{\sqrt{2}} a_1(m_c) \Lambda_1 \{\equiv V_{cs}^* V_{ud}\} \, (\bar{s}c)_{V-A} \, (\bar{u}d)_{V-A}$$

$\to V_{cs} \approx V_{ud} \approx \cos\theta_C \approx 0.975$,
θ_C Cabibbo angle

$\Rightarrow$ 7 Cabibbo favored (CF) tree (Tr) amplitudes:

$[K\pi]_{S,P,D}\, \pi + K\, [\pi\pi]_{S,P,D} + \; K\, \omega \, \{\omega \to [\pi\pi]_P$ by G-parity violation$\}$



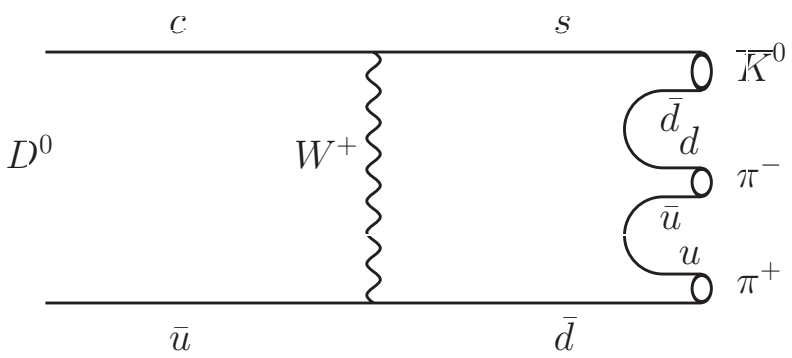
$c \to d u \bar{s}$ transition :

$$\propto \frac{G_F}{\sqrt{2}} a_1(m_c) \Lambda_2 \{\equiv V_{cd}^* V_{us}\} \, (\bar{d}c)_{V-A} \, (\bar{u}s)_{V-A}$$

$\to V_{cd} \approx -\lambda$, $V_{us} \approx \lambda$, $\lambda = \sin\theta_C$,

$\Rightarrow$ 6 doubly Cabibbo suppressed (DCS) tree (Tr) amplitudes as no W coupling to $[K\pi]_D$ state

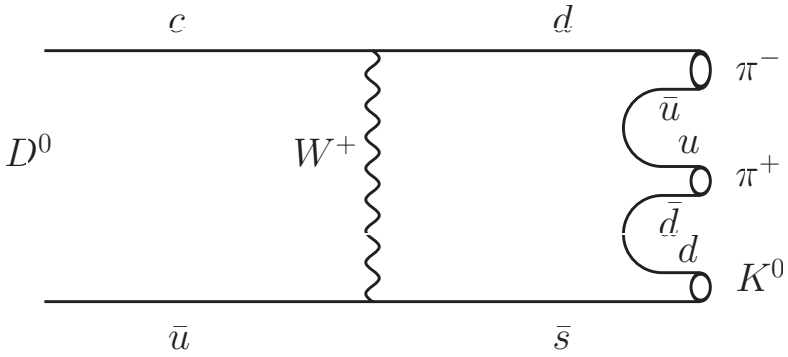
Annihilation - t-channel W-exchange amplitudes in $D^0 \to K_S^0 \pi^+ \pi^-$



$c \to s u \bar{d}$ $(c\bar{u} \to s\bar{d})$:

$$\propto \frac{G_F}{\sqrt{2}} a_2(m_c) \cos^2\theta_C \, (\bar{s}c)_{V-A} \, (\bar{d}u)_{V-A}$$

$\Rightarrow$ 7 Cabibbo favored annihilation (An) amplitudes



$c \to d u \bar{s}$ $(c\bar{u} \to d\bar{s})$:

$$\propto -\frac{G_F}{\sqrt{2}} a_2(m_c) \sin^2\theta_C \, (\bar{d}c)_{V-A} \, (\bar{s}u)_{V-A}$$

$(\sin\theta_C = 0.225)$

$\Rightarrow$ 7 doubly Cabibbo suppressed annihilation (An) amplitudes

Total of 27 non-zero amplitudes: 13 tree and 14 annihilation

Modified annihilation amplitudes

- Dalitz plot density distribution: $\frac{d^2 Br}{ds_- ds_+} = \frac{|\mathcal{M}|^2}{32(2\pi)^3 m_{D^0}^3 \Gamma_{D^0}}$
- The s_- one-dimensional density: $\frac{dBr}{ds_-} = \int_{(m_\pi + m_{K^0})^2}^{(m_{D^0} - m_\pi)^2} \frac{d^2 Br}{ds_- ds_+} ds_+$
- Out of the 10 phases, one phase ($\mathcal{M}_{4\pi}$ [$\rho(770)^0$] is choosen to be real), e. g., ϕ_4, cannot be determined $\Rightarrow$ modified partial amplitudes $\tilde{\mathcal{M}}_i = e^{-i\phi_4} \mathcal{M}_i$
- $\frac{d^2 Br_i^{tree}}{ds_- ds_+} = \frac{|\mathcal{T}_i|^2}{32(2\pi)^3 m_{D^0}^3 \Gamma_{D^0}}$ and $\frac{d^2 Br_i^{ann}}{ds_- ds_+} = \frac{|\mathcal{A}_i|^2}{32(2\pi)^3 m_{D^0}^3 \Gamma_{D^0}} = \frac{|\, e^{i\phi_4} \tilde{\mathcal{M}}_i - \mathcal{T}_i|^2}{32(2\pi)^3 m_{D^0}^3 \Gamma_{D^0}}$

$\Rightarrow$ $|\tilde{\mathcal{M}}_i|^2 + |\mathcal{T}_i|^2 - 2|\tilde{\mathcal{M}}_i|\,|\mathcal{T}_i| \leq |\mathcal{A}_i|^2 \leq |\tilde{\mathcal{M}}_i|^2 + |\mathcal{T}_i|^2 + 2|\tilde{\mathcal{M}}_i|\,|\mathcal{T}_i|$

$Br_i^{ann,\ low} = Br_i + Br_i^{tree} - 2 \int\int ds_- ds_+ |\tilde{\mathcal{M}}_i||T_i|$

- Modified annihilation amplitudes A_i': $\tilde{\mathcal{M}}_i = T_i + A_i'$ $A_i' = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i$.

$\Rightarrow$ New coefficient for A_4':

$\tilde{A}_0^{\bar{K}^0[\pi^+\pi^-]_P} = (e^{-i\phi_4} - 1)\frac{f_{K^0}}{f_{D^0}} A_0^{D^0 R_P[\pi^+\pi^-]}(m_{K^0}^2) + e^{-i\phi_4} A_0^{\bar{K}^0 R_P[\pi^+\pi^-]}(m_{D^0}^2)$

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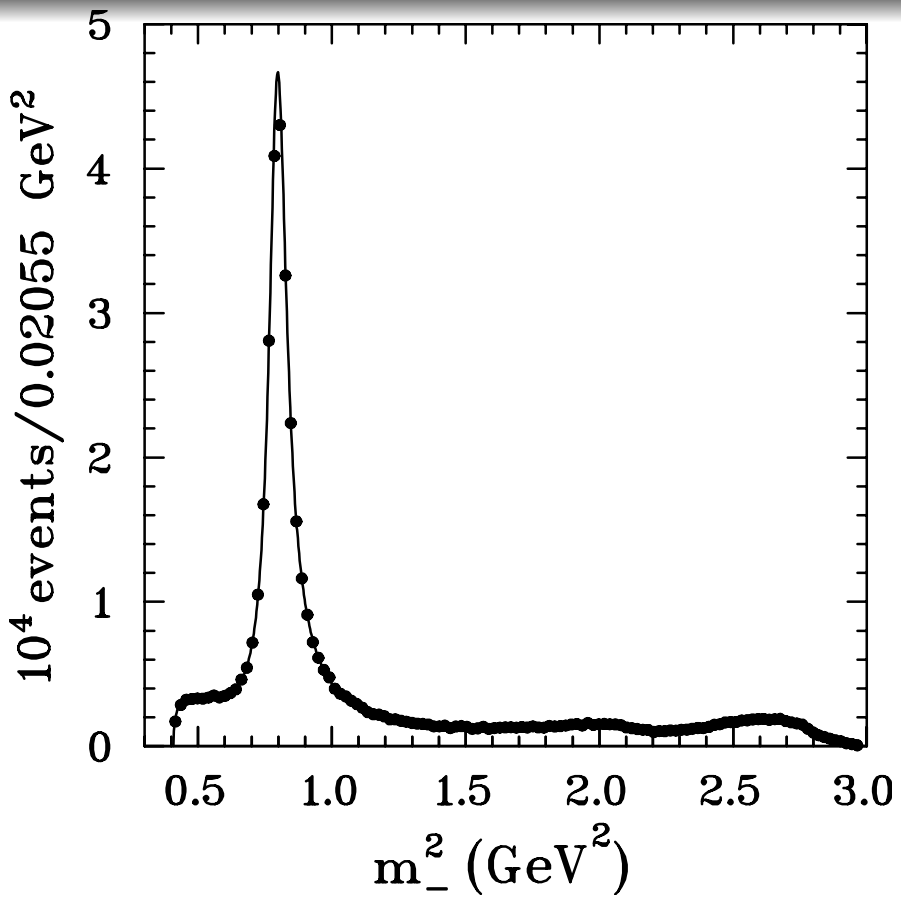
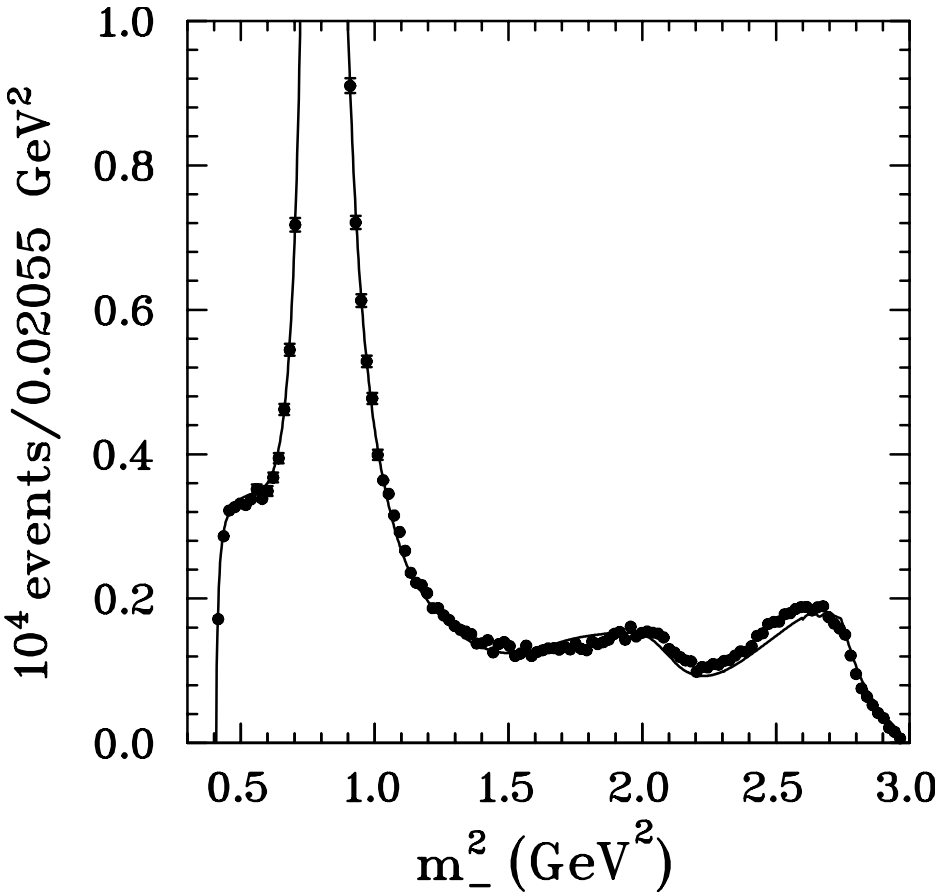
$K_S^0\pi^-$ effective mass squared distributions ($m_-^2 \equiv s_-$)



$K^*(892)^-$ ↑

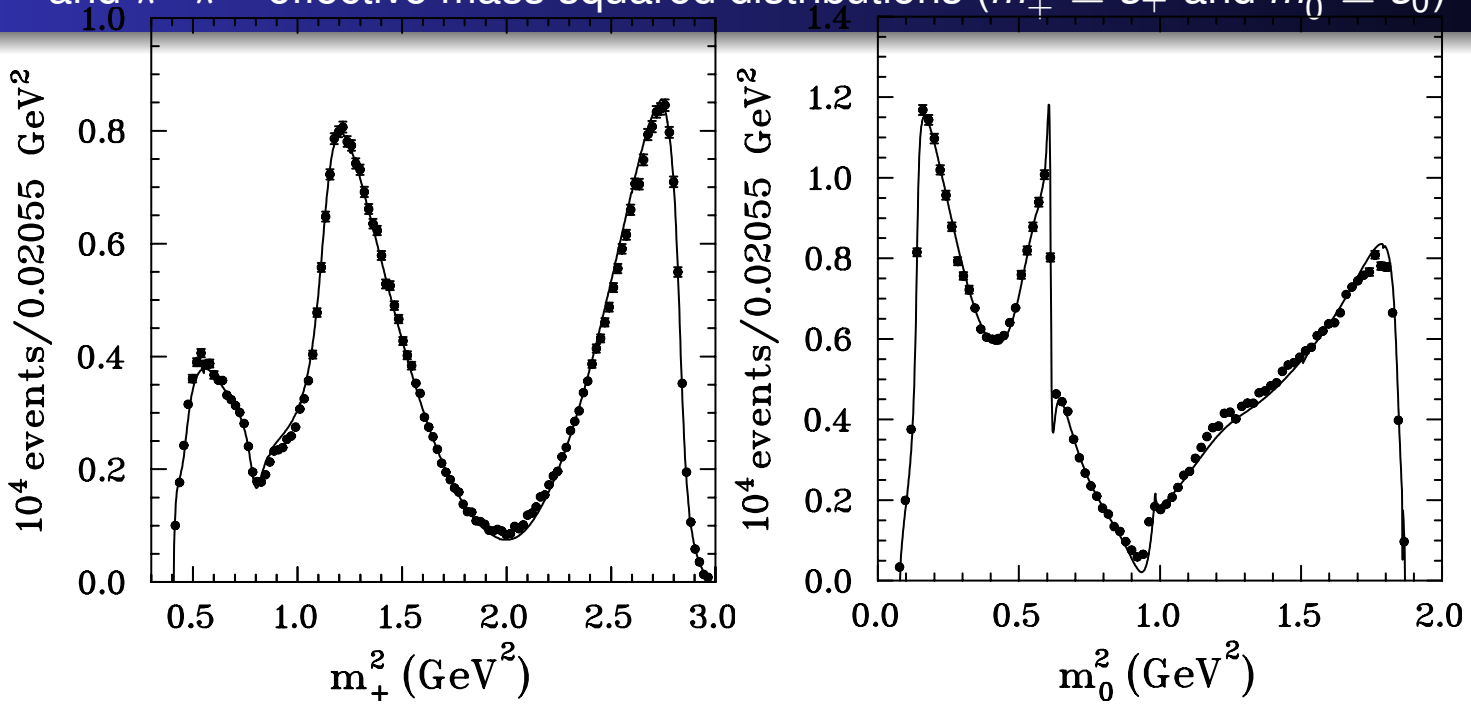
⇒ Our model (solid curve) compared with Belle data (points with error bars).

Enlarged $K_S^0\pi^-$ effective mass squared distributions



⇒ Our model (solid curve) compared with Belle data (points with error bars). Vertical scale enlarged by a factor of 5 to enforce the differences at higher $K_S^0\pi^-$ masses

$K_S^0\pi^+$ and $\pi^+\pi^-$ effective mass squared distributions ($m_+^2 \equiv s_+$ and $m_0^2 \equiv s_0$)



⇒Left panel: comparison of the $K_S^0\pi^+$ effective mass squared distributions for the best fit (solid curve) with the Belle data (points with error bars). Right panel: as in left panel but for the $\pi^+\pi^-$ effective mass squared.

- The small shoulder at $m_0^2 = 1.2$ GeV^2 could correspond to the $\pi\pi \to \eta\eta$ contribution introduced in Belle'analyzis but not included in our approach.