Theory overview on amplitude analyses with charm decays

B. Loiseau

LPNHE, Groupe Phénoménologie (Paris, France)

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- Basic weak interaction + chiral unitary approach

DIAGRAMMATIC APPROACH: $D \rightarrow M_1 M_2$
- $D \rightarrow VP$ decays within SU(3) flavor symmetry
- Fit on branching fractions of $D \rightarrow VP$ decays
- Concluding remarks on this $D \rightarrow VP$ study

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Impressive hadronic multibody decay data of $D^0$, $D^+$, $D_s^+$

- **Dalitz plots** → accumulations of events at different invariant masses: presence of meson resonances ⇔ interferences
- Standard model (SM): null $CP$ asymmetries, if deviation: physics beyond SM
- $D^0$-$\bar{D}^0$ mixing: new physics?
- Multibody hadronic decays of $D_{(s)}$ mesons → microscopic flavor changing weak process: $c \to d$ and/or $c \to s$ via $W$ meson interaction + hadronization + final state meson-meson strong interaction processes, FSI.
- Basic amplitude analyses → isobar model or sum of relativistic Breit-Wigner terms representing the different possible implied resonances + non resonant background: beyond?
  ⇒ **Theoretical constraints** on amplitudes: FSI - diagrammatic approach - QCD factorization.
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Different effective hadronic formalism approaches


⇒ R. T. Aoude, P. C. Magalhães, A. C. dos Reis, M. R. Robilotta, *Multi-Meson Model applied to $D^+ \to K^+K^-K^+$*, arXiv:1604.02904, model as an alternative to isobar model, with free parameters predicted by the theory to be fine-tuned by a fit to data, to be presented by P. C. Magalhães on Wednesday afternoon.
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Basic weak interaction + chiral unitary approach

$D_s^+$ decays into $\pi^+$ and $q\bar{q} \rightarrow$ two pseudoscalar mesons:

We describe in detail the work [2] - same mechanism as in [1] where branching ratios for $a_0(980)$ and $f_0(980)$ production in good agreement with experiment.


![Diagram of $D_s^+$ decay](image)

The $q\bar{q}$ $M$ matrix:

$$M = \begin{pmatrix}
  u\bar{u} & u\bar{d} & u\bar{s} \\
  d\bar{u} & d\bar{d} & d\bar{s} \\
  s\bar{u} & s\bar{d} & s\bar{s}
\end{pmatrix}$$

$$M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s)$$
In the standard $\eta - \eta'$ mixing the matrix $M$ is

$$
\Phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{3}} \eta + \frac{1}{\sqrt{6}} \eta' \\
\pi^- \\
K^-
\end{pmatrix}
\begin{pmatrix}
\pi^+ \\
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{3}} \eta + \frac{1}{\sqrt{6}} \eta' \\
\bar{K}^0 \\
\frac{1}{\sqrt{3}} \eta + \sqrt{\frac{2}{3}} \eta'
\end{pmatrix}
\begin{pmatrix}
K^+ \\
K^0 \\
\bar{K}^0 \\
\eta\eta
\end{pmatrix}.
$$

Neglecting $\eta'$ we have:

$$
s\bar{s} (\bar{u}u + \bar{d}d + \bar{s}s) \equiv (\Phi \cdot \Phi)_{33} = K^- K^+ + \bar{K}^0 K^0 + \frac{1}{3} \eta\eta.
$$

The $K^+ K^-$ pair after rescattering can produce $\pi^+ \pi^-$ and $K^+ K^-$. The $D_s^+$ decay width

$$
\Gamma_{P^+ P^-}, \quad P^+ P^- \equiv K^+ K^- \quad \text{or} \quad \pi^+ \pi^- \quad \text{satisfies} \quad \frac{d\Gamma_{P^+ P^-}}{dM_{inv}} = \frac{1}{(2\pi)^3} \frac{p_\pi \bar{p}_P}{4M_{Ds}^2} |T_{P^+ P^-}|^2, \quad \text{with},
$$

$$
T_{K^+ K^-} = V_0 \left(1 + G_{K^+ K^-} t_{K^+ K^-} \rightarrow K^+ K^- + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0} \rightarrow K^+ K^- + \frac{2}{3} \frac{1}{\sqrt{2}} G_{\eta\eta} t_{\eta\eta} \rightarrow K^+ K^- \right),
$$

$$
T_{\pi^+ \pi^-} = V_0 \left(G_{K^+ K^-} t_{K^+ K^-} \rightarrow \pi^+ \pi^- + G_{K^0 \bar{K}^0} t_{K^0 \bar{K}^0} \rightarrow \pi^+ \pi^- + \frac{2}{3} \frac{1}{\sqrt{2}} G_{\eta\eta} t_{\eta\eta} \rightarrow \pi^+ \pi^- \right).
$$

Function $G_l$: loop function, $G_l(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p-q)^2 - m_1^2 + i\epsilon} \frac{1}{q^2 - m_2^2 + i\epsilon}, m_1, m_2$: meson masses in loop $l$; integral on $q^0$ analytical; cut-off, $|\bar{q}_{\text{max}}| = 600$ MeV/c introduced in integral on $\bar{q}$. 

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Comparison of the $T_{\pi\pi}$ amplitude with that of the experimental data

$t_{i\rightarrow j}$: coupled-channel Bethe-Salpeter $t_{i\rightarrow j}(s) = V_{ij}(s) + \sum_{l=1}^{5} V_{il}(s) G_l(s) t_{l\rightarrow j}(s)$;

$l$ channels $\rightarrow$ 1: $\pi^+\pi^-$, 2: $\pi^0\pi^0$, 3: $K^+K^-$, 4: $K^0\bar{K}^0$, 5: $\eta\eta$. Kernel $V \rightarrow$ tree-level transition amplitudes from phenomenological Lagrangians: J. A. Oller and E. Oset, *Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the sigma, $f_0(980)$, $a_0(980)$ scalar mesons*, NPA 620, 438 (1997), NPA 652, 407 (1999).

Results: adjusting $V_0$ and comparing theoretical amplitudes with those available from the experimental data leads to a fair agreement. Experimental data: B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D 79 (2009) 032003.
Basic weak interaction + chiral unitary approach

Invariant mass distribution for $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ (dashed line) and $D_s^+ \rightarrow \pi^+K^-K^+$ (solid line).

Results for the $f_0(980)$ signal in the spectra are (up to a global common normalization factor) predictions of the Chiral Unitary approach with no free parameters.

$\Rightarrow$ Interesting issue: the $\pi^+\pi^0\eta$ decay mode which generates the $a_0(980)$ will lead to informations on $f_0(980)$ and $a_0(980)$ mixing.
Model-independent analysis based on flavor SU(3) symmetry.

- The *c quark mass* too high to apply chiral perturbation theory and too light to use heavy quark expansion approaches.
  - \( \Rightarrow \) Diagrammatic approach: flavor-flow diagrams with all strong interaction effects included.
  - Model-independent analysis based on flavor SU(3) symmetry \( \Rightarrow \) topological amplitudes \( \leftrightarrow \) underlying decay mechanisms.
  - Possible topologies of weak interactions: \( \Rightarrow \) Fig. 1 [Hai-Yang Cheng, Cheng-Wei Chiang, *Direct CP violation in two-body hadronic charmed meson decays*, Phys. Rev. D 85, 034036 (2012), arXiv:1201.0785]
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Flavor-flow diagrams with all strong interaction effects included.

Figure: (a) $T$: color-allowed tree, (b) $C$: color-suppressed tree, (e) $E$: $W$-exchange, (f) $A$: $W$-annihilation, (c) $P$: QCD-penguin, (d) $S$: singlet QCD-penguin with 2 (3) gluon lines for $M_2$ being a pseudoscalar meson $P$ (a vector meson $V$), (g) $PE$: QCD-penguin exchange, (h) $PA$: QCD-penguin annihilation.
Global analysis of two-body decays.


- Within SU(3) flavor symmetry only four types of amplitudes for all \( D \rightarrow VP \) decays: color-allowed \( T \), color-suppressed \( C \), \( W \)-exchange \( E \), \( W \)-annihilation \( A \).

- Subscript \( P \) or \( V \) to each amplitude, e.g., \( T_P(V) \), denote the amplitude in which the spectator quark goes to the pseudoscalar or vector meson in the final state.

- The 8 complex amplitudes, \( T_P(V), C_P(V), E_P(V), A_P(V) \) (15 real parameters, \( T_V \) chosen to be real) determined by performing a \( \chi^2 \) fit of 16 experimental branching fractions for Cabibbo-favored decays \( \propto \) CKM factors \( Y_{sd} \equiv V_{cs}^* V_{ud} \sim O(1) \). Several solutions: favored one (A1).

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Fitted branching fractions for Cabibbo-favored (CF) $D^0 \to VP$ decays.

**Table:** Units: %, data: PDG, $Y_{sd} \equiv V_{cs}^* V_{ud}$, $s_\phi \equiv \sin \phi$, $c_\phi \equiv \cos \phi$
(mixing angle $\eta-\eta'$). For comparison $B_{(pole)}$: pole model of [YWL].

<table>
<thead>
<tr>
<th>Mode</th>
<th>Amplitudes</th>
<th>$B_{exp}$</th>
<th>$B_{theory}(A1)$</th>
<th>$B_{(pole)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{*-} \pi^+$</td>
<td>$Y_{sd}(T_V + E_P)$</td>
<td>5.43 ± 0.44</td>
<td>5.45 ± 0.64</td>
<td>3.1 ± 1.0</td>
</tr>
<tr>
<td>$K^- \rho^+$</td>
<td>$Y_{sd}(T_P + E_V)$</td>
<td>11.1 ± 0.9</td>
<td>11.3 ± 2.70</td>
<td>8.8 ± 2.2</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \pi^0$</td>
<td>$\frac{1}{\sqrt{2}} Y_{sd}(C_P - E_P)$</td>
<td>3.75 ± 0.29</td>
<td>3.72 ± 0.49</td>
<td>2.9 ± 1.0</td>
</tr>
<tr>
<td>$\overline{K}^0 \rho^0$</td>
<td>$\frac{1}{\sqrt{2}} Y_{sd}(C_V - E_V)$</td>
<td>$1.28^{+0.14}_{-0.16}$</td>
<td>1.30 ± 0.78</td>
<td>1.7 ± 0.7</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \eta$</td>
<td>$Y_{sd}((C_P + E_P)$ \times $c_\phi/\sqrt{2} - E_V s_\phi)$</td>
<td>0.96 ± 0.30</td>
<td>0.92 ± 0.36</td>
<td>0.7 ± 0.2</td>
</tr>
<tr>
<td>$\overline{K}^{*0} \eta'$</td>
<td>$-Y_{sd}((C_P + E_P)$ \times $s_\phi/\sqrt{2} + E_V c_\phi)$</td>
<td>$&lt; 0.11$</td>
<td>$0.003 \pm 0.002$</td>
<td>$0.016 \pm 0.005$</td>
</tr>
<tr>
<td>$\overline{K}^0 \omega$</td>
<td>$-\frac{1}{\sqrt{2}} Y_{sd}(C_V + E_V)$</td>
<td>2.22 ± 0.12</td>
<td>2.24 ± 0.84</td>
<td>2.5 ± 0.7</td>
</tr>
<tr>
<td>$\overline{K}^0 \phi$</td>
<td>$-Y_{sd}E_P$</td>
<td>$0.847^{+0.066}_{-0.034}$</td>
<td>$0.848 \pm 0.050$</td>
<td>$0.80 \pm 0.2$</td>
</tr>
</tbody>
</table>
**Fitted branching fractions for Cabibbo-favored** $D_s^{+} \rightarrow VP$ **decays**.

**Table:** Recent measurement of $D_s^{+} \rightarrow \pi^+ \rho^0 \Rightarrow A_{P,V}$. Units: %, data: PDG but $B_{\rho^+ \eta'}$, from BESIII Coll., Phys. Lett. B 750, 466 (2015).

<table>
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<tr>
<th>Mode</th>
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<th>$B_{exp}$</th>
<th>$B_{theory}(A1)$</th>
<th>$B$(pole)</th>
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</thead>
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<tr>
<td>$\bar{K}^{*0} \pi^+$</td>
<td>$Y_{sd}(T_V + C_P)$</td>
<td>$1.57 \pm 0.13$</td>
<td>$1.57 \pm 0.25$</td>
<td>$1.4 \pm 1.3$</td>
</tr>
<tr>
<td>$\bar{K}^0 \rho^+$</td>
<td>$Y_{sd}(T_P + C_V)$</td>
<td>$12.08^{+1.20}_{-0.68}$</td>
<td>$12.15 \pm 11.69$</td>
<td>$15.1 \pm 3.8$</td>
</tr>
<tr>
<td>$D_s^{+} \rightarrow VP$</td>
<td></td>
<td></td>
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<td>$\bar{K}^{*0} K^+$</td>
<td>$Y_{sd}(C_P + A_V)$</td>
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<td>$\bar{K}^0 K^{*+}$</td>
<td>$Y_{sd}(C_V + A_P)$</td>
<td>$5.4 \pm 1.2$</td>
<td>$4.38 \pm 1.19$</td>
<td>$1.0 \pm 0.6$</td>
</tr>
<tr>
<td>$\rho^+ \pi^0$</td>
<td>$\frac{1}{\sqrt{2}} Y_{sd}(A_P - A_V)$</td>
<td>—</td>
<td>$0.021 \pm 0.087$</td>
<td>$0.4 \pm 0.4$</td>
</tr>
<tr>
<td>$\rho^+ \eta$</td>
<td>$- Y_{sd}((A_P + A_V)$</td>
<td>$8.9 \pm 0.8$</td>
<td>$8.85 \pm 1.69$</td>
<td>$8.3 \pm 1.3$</td>
</tr>
<tr>
<td></td>
<td>$\times \frac{1}{\sqrt{2}}c_\phi - T_P s_\phi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho^+ \eta'$</td>
<td>$Y_{sd}((A_P + A_V)$</td>
<td>$5.80 \pm 1.46$</td>
<td>$2.75 \pm 0.46$</td>
<td>$3.0 \pm 0.5$</td>
</tr>
<tr>
<td></td>
<td>$\times \frac{1}{\sqrt{2}}s_\phi + T_P c_\phi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^+ \rho^0$</td>
<td>$\frac{1}{\sqrt{2}} Y_{sd}(A_V - A_P)$</td>
<td>$0.020 \pm 0.012$</td>
<td>$0.021 \pm 0.087$</td>
<td>$0.4 \pm 0.4$</td>
</tr>
<tr>
<td>$\pi^+ \omega$</td>
<td>$\frac{1}{\sqrt{2}} Y_{sd}(A_V + A_P)$</td>
<td>$0.24 \pm 0.06$</td>
<td>$0.24 \pm 0.15$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\pi^+ \phi$</td>
<td>$Y_{sd} T_V$</td>
<td>$4.5 \pm 0.4$</td>
<td>$4.49 \pm 0.40$</td>
<td>$4.3 \pm 0.6$</td>
</tr>
</tbody>
</table>
Cabibbo-favored amplitudes resulting from the branching fraction fit.

Table: Fit results (solution A1) \( \phi = 43.5^\circ \). Units: \( 10^{-6} \), strong phases in degrees. \( |T_V| = 4.21^{+0.18}_{-0.19} \).

| \( |T_P| \) | \( \delta_{T_P} \) | \( |C_V| \) | \( \delta_{C_V} \) | \( |C_P| \) | \( \delta_{C_P} \) | \( |E_V| \) |
|----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 8.46^{+0.22}_{-0.25} | 57^{+35}_{-41} | 4.09^{+0.16}_{-0.25} | -145^{+29}_{-39} | 4.08^{+0.37}_{-0.36} | -157 \pm 2 | 1.19^{+0.64}_{-0.46} |

| \( \delta_{E_V} \) | \( |E_P| \) | \( \delta_{E_P} \) | \( |A_P| \) | \( \delta_{A_P} \) | \( |A_V| \) | \( \delta_{A_V} \) |
|----------------|--------------|--------------|--------------|--------------|--------------|--------------|
| -85^{+42}_{-39} | 3.06 \pm 0.09 | 98 \pm 5 | 0.64^{+0.14}_{-0.27} | 152^{+48}_{-50} | 0.52^{+0.24}_{-0.19} | 122^{+70}_{-42} |

- **Modulus** of color-allowed tree \( T_P \) amplitude is the **largest**.
- **Moduli** of the \( W \)-annihilation \( A_P(V) \) amplitudes are the **smallest**.
INTRODUCTION
FSI CONSTRAINTS
DIAGRAMMATIC APPROACH: $D \rightarrow M_1 M_2$
FACTORIZATION APPROACH: $D \rightarrow m_1 m_2 m_3$
CONCLUDING REMARKS

$D \rightarrow VP$ decays within SU(3) flavor symmetry

Fit on branching fractions of $D \rightarrow VP$ decays

Concluding remarks on this $D \rightarrow VP$ study

Predictions for some singly Cabibbo-suppressed decays,

**Table:** Units of $10^{-3}$. $Y_d \equiv V_{cd}^* V_{ud} \sim \mathcal{O}(\lambda)$, $Y_s \equiv V_{cs}^* V_{us} \sim \mathcal{O}(\lambda)$, $\lambda = 0.22543$ (CKMfitter). No SU(3) breaking: $T' \equiv T$ etc ....

<table>
<thead>
<tr>
<th>Mode</th>
<th>Amplitude</th>
<th>$\mathcal{B}_{\text{exp}}$</th>
<th>$\mathcal{B}_{\text{theory}}(A1)$</th>
<th>$\mathcal{B}$(pole)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow VP$</td>
<td>$\pi^+ \rho^-$: $Y_d(T'_V + E'_P)$</td>
<td>$5.09 \pm 0.34$</td>
<td>$3.61 \pm 0.43$</td>
<td>$3.5 \pm 0.6$</td>
</tr>
<tr>
<td></td>
<td>$\pi^- \rho^+$: $Y_d(T'_P + E'_V)$</td>
<td>$10.0 \pm 0.6$</td>
<td>$8.73 \pm 2.09$</td>
<td>$10.2 \pm 1.5$</td>
</tr>
<tr>
<td></td>
<td>$\pi^0 \phi$: $\frac{1}{\sqrt{2}} Y_s C'_P$</td>
<td>$1.35 \pm 0.10$</td>
<td>$0.77 \pm 0.14$</td>
<td>$1.0 \pm 0.3$</td>
</tr>
<tr>
<td>$D^+ \rightarrow VP$</td>
<td>$\pi^+ \rho^0$: $\frac{1}{\sqrt{2}} Y_d(T'_V + C'_P - A'_P + A'_V)$</td>
<td>$0.84 \pm 0.15$</td>
<td>$0.51 \pm 0.28$</td>
<td>$0.8 \pm 0.7$</td>
</tr>
<tr>
<td></td>
<td>$K^+\bar{K}^{*0}$: $Y_d A'_V + Y_s T'_V$</td>
<td>$3.84^{+0.14}_{-0.23}$</td>
<td>$4.00 \pm 0.82$</td>
<td>$4.1 \pm 1.0$</td>
</tr>
<tr>
<td></td>
<td>$\bar{K}^0 K^{*+}$: $Y_d A'_P + Y_s T'_P$</td>
<td>$34 \pm 16$</td>
<td>$14.45 \pm 2.45$</td>
<td>$12.4 \pm 2.4$</td>
</tr>
<tr>
<td>$D_s^+ \rightarrow VP$</td>
<td>$\pi^+ K^{*0}$: $Y_d T'_V + Y_s A'_V$</td>
<td>$2.13 \pm 0.36$</td>
<td>$3.51 \pm 0.72$</td>
<td>$1.5 \pm 0.7$</td>
</tr>
<tr>
<td></td>
<td>$K^+ \rho^0$: $\frac{1}{\sqrt{2}} (Y_d C'_P - Y_s A'_P)$</td>
<td>$2.5 \pm 0.4$</td>
<td>$1.58 \pm 0.38$</td>
<td>$1.0 \pm 0.6$</td>
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<tr>
<td></td>
<td>$K^+ \phi$: $Y_s(T'_V + C'_P + A'_V)$</td>
<td>$0.164 \pm 0.041$</td>
<td>$0.111 \pm 0.060$</td>
<td>$0.3 \pm 0.3$</td>
</tr>
</tbody>
</table>
INTRODUCTION

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DIAGRAMMATIC APPROACH: \( D \rightarrow M_1 M_2 \)

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CONCLUDING REMARKS

Predictions for some doubly Cabibbo-suppressed decays. No SU(3) breaking: \( T'' \equiv T \) etc....

<table>
<thead>
<tr>
<th>Table: Units: ( 10^{-4} ). ( Y_{ds} \equiv V_{cd}^* V_{us} \sim O(\lambda^2) ).</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Meson</strong></td>
</tr>
<tr>
<td>( D^0 )</td>
</tr>
<tr>
<td>( \phi K^0 )</td>
</tr>
<tr>
<td>( D^+ )</td>
</tr>
<tr>
<td>( K^{*+} \pi^0 )</td>
</tr>
<tr>
<td>( \rho^0 K^+ )</td>
</tr>
<tr>
<td>( D_s^+ )</td>
</tr>
<tr>
<td>( K^{*0} K^+ )</td>
</tr>
</tbody>
</table>

- Doubly Cabibbo-suppressed channels: good agreement with data.
- Singly Cabibbo-suppressed \( \propto Y_d \equiv V_{cd}^* V_{ud} \) and \( \propto Y_s \equiv V_{cs}^* V_{us} \) have flavor SU(3) symmetry breaking effects.
Exact flavor SU(3) describes reasonably well the available data.

- If $T$ and $C$ amplitudes factorizable $\Rightarrow$ effective Wilson coefficients $a_{1,2}$, $|a_2/a_1|$ and $\arg(a_2/a_1)$ (see next Section) from Cabibbo-favored $D^+ \rightarrow K^*\pi^+$ and $K^0\rho^+$ [solutions (A1)]

<table>
<thead>
<tr>
<th></th>
<th>$K^*\pi^+$</th>
<th>$K^0\rho^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>a_1</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>a_2</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>a_2/a_1</td>
<td>$</td>
</tr>
<tr>
<td>$\arg(a_2/a_1)$</td>
<td>$-(157 \pm 2)^\circ$</td>
<td>$(158 \pm 51)^\circ$</td>
</tr>
</tbody>
</table>


- SU(3) symmetry breaking in color-allowed $T$ and color-suppressed $C$ tree amplitudes needed in general to have a better agreement with experiment.

$\rightarrow$ Nevertheless, the exact flavor SU(3)-symmetric approach alone is adequate to provide an overall explanation for the current data.
Factorization in $D$ decays: phenomenological approach

- QCD factorization beyond naive factorization $\rightarrow$ expansion in $\alpha_s$ and $1/m_b$

- In $D$ decays, $m_c \sim m_b/3$ $\rightarrow$ significant corrections to the factorized results. $\Rightarrow$ factorization phenomenological approach, based on the seminal work by Bauer, Stech and Wirbel [Exclusive Non-Leptonic Decays of $D$-, $D_s$- and $B$-Mesons], Z. Phys. C 34, 103 (1987).

- Applied successfully to $D$ decays, treating Wilson coefficients as phenomenological parameters to account for non-factorizable corrections.

- No factorization theorem for three-body decays but important contributions from intermediate resonances as $\rho(770)$, $K^*(892)$ and $\phi(1020)$ $\Rightarrow$ three-body decays quasi-two-body decays.

- Two of the three final-state mesons form a single state originating from a quark-antiquark pair. This hypothesis leads to a quasi-two-body final state to which the factorization procedure is applied.

  $\Rightarrow$ Dalitz plot studies of $D^0 \to K^0_S \pi^+ \pi^-$ [J.-P. Dedonder, R. R Kamiński, L. Leśniak, B. L., Phys. Rev. D 89, 094018 (2014), arXiv:1403.2971].
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Factorization in $D$ decays: phenomenological approach

- QCD factorization beyond naive factorization → expansion in $\alpha_s$ and $1/m_b$
  applied with success to charmless nonleptonic two-body $B$ decays
  [M. Beneke, M. Neubert, QCD factorization for $B \to PP$ and $B \to PV$ decays

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Weak effective Hamiltonian

- No penguin ($W$-loop diagram) in $D^0 \to K_S^0 \pi^+ \pi^-$, then

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_{i=1,2} C_i(\mu) O_i(\mu) + h.c.
\]

- $V_{\text{CKM}}$: quark mixing couplings, $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$: Fermi coupling, $C_i(\mu)$: QCD Wilson coefficients ($W$ exchange), $\mu$: renormalization scale, $\mu \sim m_c = 1.3$ GeV (charm quark mass).

- $O_{1,2}$ are left-handed quark current-current operators, e.g. ($\alpha$ and $\beta$ color indices)

\[
O_1 = j_1 \otimes j_2, \quad j_1 = \bar{s}_\alpha \gamma^\nu (1 - \gamma^5) c_\alpha \equiv (\bar{s}c)_{V-A}, \quad j_2 = \bar{u}_\beta \gamma^\nu (1 - \gamma^5) d_\beta \equiv (\bar{u}d)_{V-A}.
\]

- In the amplitude and at leading order in $\alpha_s$, the following real effective QCD coefficients $a_1(m_c)$ and $a_2(m_c)$ will appear,

\[
a_1(m_c) = C_1(m_c) + \frac{C_2(m_c)}{N_C}, \quad a_2(m_c) = C_2(m_c) + \frac{C_1(m_c)}{N_C}, \quad N_C = 3
\]

being the number of colors (from now on $a_i(m_c) \equiv a_i, i = 1, 2$).
Weak effective Hamiltonian

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$N_C = 3$ being the number of colors (from now on $a_i(m_c) \equiv a_i, i = 1, 2$).
Operator Product Expansion + large W mass \Rightarrow two-body factorization approximation

\[ \text{Factorization: } \langle M_1 M_2 | O_i(\mu) | D^0 \rangle = \langle M_1 | j_1 | 0 \rangle \langle M_2 | j_2 | D^0 \rangle + \text{higher order corrections} \]

- No three-body factorization scheme \Rightarrow \text{quasi-two-body approximation:}

\[ \bar{K}^0 \pi^+ \pi^- \simeq [\bar{K}^0 \pi^\pm]_L \pi^\mp \text{ or } \bar{K}^0 [\pi^+ \pi^-]_L \]

- e.g. \( M_1 = [\pi^+ K^0]_L, M_2 = \pi^- \) or \( M_1 = \bar{K}^0, M_2 = [\pi^+ \pi^-]_L \)

\rightarrow \text{the state } [m_1 m_2]_L \text{ in } L = S, P \text{ or } D \text{ wave originates from a } q' \bar{q} \text{ state.}

- For instance, with \( V_{CKM} = V_{CS}^* V_{ud} \equiv \Lambda_1: \langle \bar{K}^0 h^- h^+ | H_{\text{eff}} | D^0 \rangle \Rightarrow \)

\[ G_F \Lambda_1 a_2 \left\langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \right\rangle \left\langle [\pi^+ \pi^-]_L | (\bar{u}c)_{V-A} | D^0 \right\rangle \propto \text{if}_{K^0} p_{K^0} \cdot \left\langle \bar{D}^0 [\pi^+ \pi^-]_L | (\bar{u}c)_{V-A} | 0 \right\rangle \]

- Decay constant ↑

- Transition form factor ↑

- If \( M_1 = [\pi^+ K^0]_L \leftrightarrow \text{Form Factor } \langle [\pi^+ K^0]_L | (\bar{s}u)_{V-A} | 0 \rangle : [\pi^+ K^0]_L \text{ interaction} \)
Operator Product Expansion + large W mass ⇒ two-body factorization approximation

No three-body factorization scheme ⇒ quasi-two-body approximation:

\[ \bar{K}^0 \pi^+ \pi^- \simeq [\bar{K}^0 \pi^\pm]_L \pi^\mp \text{ or } \bar{K}^0 [\pi^+ \pi^-]_L \]

e.g. \( M_1 = [\pi^+ K^0]_L, M_2 = \pi^- \) or \( M_1 = \bar{K}^0, M_2 = [\pi^+ \pi^-]_L \)

→ the state \([m_1 m_2]_L \) in \( L = S, P \) or \( D \) wave originates from a \( q' \bar{q} \) state.

For instance, with \( V_{CKM} = V_{cs}^* V_{ud} \equiv \Lambda_1 \):

\[ G_F \Lambda_1 a_2 \left\langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \right\rangle \left\langle [\pi^+ \pi^-]_L | (\bar{u}c)_{V-A} | D^0 \right\rangle \propto i f_{K^0} p_{K^0} \cdot \left\langle D^0 [\pi^+ \pi^-]_L | (\bar{u}c)_{V-A} | 0 \right\rangle \]

Decay constant ↑ Transition form factor ↑

If \( M_1 = [\pi^+ K^0]_L \) ↔ Form Factor \( \left\langle [\pi^+ K^0]_L | (\bar{s}u)_{V-A} | 0 \right\rangle : [\pi^+ K^0]_L \) interaction
Quasi-two-body factorization for $D^0 \to K_S^0 \pi^+ \pi^-$

**Form factors**

**Dalitz-plot fit**

**Operator Product Expansion + large W mass ⇒ two-body factorization approximation**

Factorization: $\langle M_1 M_2 | O_i(\mu) | D^0 \rangle = \langle M_1 | j_1 | 0 \rangle \langle M_2 | j_2 | D^0 \rangle + \text{higher order corrections}$

- No three-body factorization scheme ⇒ **quasi-two-body approximation**: 
  
  $\bar{K}^0 \pi^+ \pi^- \simeq [\bar{K}^0 \pi^\pm]_L \pi^\mp$ or $\bar{K}^0 [\pi^+ \pi^-]_L$

  e.g. $M_1 = [\pi^+ K^0]_L$, $M_2 = \pi^-$ or $M_1 = \bar{K}^0$, $M_2 = [\pi^+ \pi^-]_L$

  $\to$ the state $[m_1 m_2]_L$ in $L = S, P$ or $D$ wave originates from a $q' \bar{q}$ state.

- For instance, with $V_{CKM} = V_{cs}^* V_{ud} \equiv \Lambda_1$: 
  
  $G_F \Lambda_1 a_2 \left\langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \right\rangle \left\langle [\pi^+ \pi^-]_L | (\bar{u}c)_{V-A} | D^0 \right\rangle \propto i f_{K^0 p_{K^0}} \cdot \left\langle \bar{D}^0 [\pi^+ \pi^-]_L | (\bar{u}c)_{V-A} | 0 \right\rangle$

  Decay constant $\uparrow$ Transition form factor $\uparrow$

- If $M_1 = [\pi^+ K^0]_L \leftrightarrow$ Form Factor $\langle [\pi^+ K^0]_L | (\bar{s}u)_{V-A} | 0 \rangle : [\pi^+ K^0]_L$ interaction
Quasi-two-body factorization for $D^0 \to K_S^0 \pi^+ \pi^-$

Form factors

Dalitz-plot fit

Resonance contributions [note: $s = (p_1 + p_2)^2$, $s_{\pm} = (p_{\pi \pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$]

Contribution of resonances, e.g. from $q \bar{u}$, in a given $[m_1 m_2]_L$ channel:

$$\langle [m_1 m_2]_L | (\bar{q} c)_{V-A} | D^0 \rangle \simeq \sum_{R_L} F^{D^0 R_L} (m_{m_3}^2) G_{R_L} (s) \langle R_L^{[m_1 m_2]} | q \bar{u} \rangle$$

$$\simeq c_{R_L} \chi \sum_{R_L} F^{D^0 R_L} (m_{m_3}^2) \langle [m_1 m_2]_L | (q \bar{u})_{V-A} | 0 \rangle \simeq c_{R_L} \chi F^{D^0 \bar{R}_L} (m_{m_3}^2) F^{m_1 m_2} (s).$$

$$G_{R_L} (s) = \chi \langle [m_1 m_2]_L | (q \bar{u})_{V-A} | 0 \rangle$$

$$c_{R_L} = \langle R_L^{[m_1 m_2]} | q \bar{u} \rangle,$$ $\bar{R}_L$: L-wave dominant resonance

With $L = S$, $q = u$, $m_3 = \bar{K}^0$, $m_1 = \pi^+$, $m_2 = \pi^-$, $\bar{R}_S = f_0 (980)$, $c_{f_0} = 1/\sqrt{2}$:

$$F^{[\text{Cabibbo Favored}]}_{\bar{K}^0 [\pi^+ \pi^-]} (s_0, s_-, s_+) = -\frac{G_F}{2} a_2 \Lambda_1 \chi (m_{D^0}^2 - s_0) f_{K^0} F^{D^0 f_0 (980)} (m_{K^0}^2) F_{\pi^+ \pi^-} (s_0)$$

$\to$ Form factor $F_{\pi^+ \pi^-} (s_0)$ includes contribution of $f_0 (500), f_0 (980), f_0 (1400)$

$\to$ If a resonance is largely dominant, like the $\rho (770)^0$ in $[\pi^+ \pi^-]$ then $\chi \propto 1/f_{\rho}$
Resonance contributions [note: $s = (p_1 + p_2)^2$, $s_\pm = (p_{\pi\pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$]

- Contribution of resonances, e.g. from $q\bar{u}$, in a given $[m_1 m_2]_L$ channel:

$$
\langle [m_1 m_2]_L |(\bar{q}c)_V\ A| D^0 \rangle \simeq \sum_{R_L} F^{D^0 R_L}(m_{m_3}^2) G_{R_L}(s) \langle R_L^{[m_1 m_2]} | q\bar{u} \rangle
$$

$$
\simeq c_{R_L} \chi \sum_{R_L} F^{D^0 R_L}(m_{m_3}^2) \langle [m_1 m_2]_L |(\bar{q}u)_V - A| 0 \rangle \simeq c_{R_L} \chi F^{D^0 \bar{R}_L}(m_{m_3}^2) F^{m_1 m_2}_L(s).
$$

$$
G_{R_L}(s) = \chi \langle [m_1 m_2]_L |(\bar{q}u)_V - A| 0 \rangle
$$

$$
c_{R_L} = \langle R_L^{[m_1 m_2]} | q\bar{u} \rangle, \ \bar{R}_L: \text{L-wave dominant resonance}
$$

- With $L = S$, $q = u$, $m_3 = K^0$, $m_1 = \pi^+$, $m_2 = \pi^-$, $\bar{R}_S = f_0(980)$, $c_{f_0} = 1/\sqrt{2}$:

$$
T^{[\text{Cabibbo Favored}]}_{K^0 [\pi^+ \pi^-]}(s_0, s_-, s_+) = -\frac{G_F}{2} a_2 \Lambda_1 \chi (m_{D^0}^2 - s_0) f_{K^0} F^{D^0 f_0(980)}(m_{K^0}^2) F_{0}^{\pi^+ \pi^-}(s_0)
$$

- Form factor $F_{0}^{\pi^+ \pi^-}(s_0)$ includes contribution of $f_0(500)$, $f_0(980)$, $f_0(1400)$

- If a resonance is largely dominant, like the $\rho(770)^0$ in $[\pi^+ \pi^-]_P$ then $\chi \propto 1/f_{\rho}$
Resonance contributions [note: \( s = (p_1 + p_2)^2, s_\pm = (p_\pi^\pm + p_{K^0})^2, s_0 = (p_{\pi^+} + p_{\pi^-})^2 \)]

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  \[
  \langle [m_1 m_2]_L | (\bar{q}c)_{V-A} | D^0 \rangle \approx \sum_{R_L} F^{D^0} R_L (m_{m_3}^2) G_{R_L} (s) \langle R_L^{[m_1 m_2]} | q\bar{u} \rangle
  \]

\[
\approx c_{R_L} \chi \sum_{R_L} F^{D^0} R_L (m_{m_3}^2) \langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \rangle \approx c_{R_L} \chi F^{D^0} \tilde{R}_L (m_{m_3}^2) F^{m_1 m_2}_L (s).
\]

\[
G_{R_L} (s) = \chi \langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \rangle
\]

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Resonance contributions [note: $s = (p_1 + p_2)^2$, $s_\pm = (p_{\pi \pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$]

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  \]

  \[
  \simeq c_{R_L} \chi \sum_{R_L} F_{D^0 R_L} (m_{m_3}^2) \langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \rangle \simeq c_{R_L} \chi F_{D^0 \bar{R}_L} (m_{m_3}^2) F^{m_1 m_2}_L (s).
  \]

  \[
  G_{R_L} (s) = \chi \langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \rangle
  \]

  \[
  c_{R_L} = \langle R_{L [m_1 m_2]} | q\bar{u} \rangle, \quad \bar{R}_L: \text{L-wave dominant resonance}
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  \[
  T_{[\text{Cabibbo Favored}]_{\bar{K}^0 \pi^+ \pi^-}}^{[\pi^+ \pi^-]} (s_0, s_+, s_-) = -\frac{G_F}{2} a_2 \Lambda_1 \chi (m_{D^0}^2 - s_0) f_{K^0} F_{D^0 f_0(980)}^{m_2_{K^0}} F_{0}^{\pi^+ \pi^-} (s_0)
  \]

\rightarrow Form factor $F_{0}^{\pi^+ \pi^-} (s_0)$ includes contribution of $f_0(500)$, $f_0(980)$, $f_0(1400)$

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### Table: CF, Cabibbo-favored ($\propto V_{cs}^* V_{ud}$) and DCS, doubly-Cabibbo-suppressed ($\propto V_{cd}^* V_{us}$) amplitudes.

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Channel</th>
<th>Dominant resonances</th>
<th>Form Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}_{1\pi}$: CF(Tr+An)</td>
<td>$[K_0^0 \pi^+]_S \pi^+$</td>
<td>$K^<em>_0(800)^-, K^</em>_0(1430)^-$</td>
<td>$F^K_{\pi}(s_-)$</td>
</tr>
<tr>
<td>$\mathcal{M}_{2\pi}$: CF+DCS(Tr+An)</td>
<td>$K_0^0 [\pi^+\pi^-]_S$</td>
<td>$f_0(500), f_0(980), f_0(1400)$</td>
<td>$F^K_{\pi\pi}(s_0)$</td>
</tr>
<tr>
<td>$\mathcal{M}_{3\pi}$: CF(Tr+An)</td>
<td>$[K_0^0 \pi^-]_P \pi^+$</td>
<td>$K^*(892)^-$</td>
<td>$F^{K\pi}(s_-)$</td>
</tr>
<tr>
<td>$\mathcal{M}_{4\pi}$: CF+DCS(Tr+An)</td>
<td>$K_0^0 [\pi^+\pi^-]_P$</td>
<td>$\rho(770)^0$</td>
<td>$F^{K\pi}(s_0)$</td>
</tr>
<tr>
<td>$\mathcal{M}_{5\pi}$: CF+DCS(Tr+An)</td>
<td>$K_0^0 [\pi^+\pi^-]_\omega$</td>
<td>$\omega(782)$</td>
<td>Breit-Wigner</td>
</tr>
<tr>
<td>$\mathcal{M}_{6\pi}$: CF(Tr+An)</td>
<td>$[K_0^0 \pi^-]_D \pi^+$</td>
<td>$K^*_2(1430)^-$</td>
<td>Breit-Wigner</td>
</tr>
<tr>
<td>$\mathcal{M}_{7\pi}$: CF+DCS(Tr+An)</td>
<td>$K_0^0 [\pi^+\pi^-]_D$</td>
<td>$f_2(1270)$</td>
<td>Breit-Wigner</td>
</tr>
<tr>
<td>$\mathcal{M}_{8\pi}$: DCS(Tr+An)</td>
<td>$[K_0^0 \pi^+]_S \pi^-$</td>
<td>$K^<em>_0(800)^+, K^</em>_0(1430)^+$</td>
<td>$F^K_{\pi}(s_+)$</td>
</tr>
<tr>
<td>$\mathcal{M}_{9\pi}$: DCS(Tr+An)</td>
<td>$[K_0^0 \pi^+]_P \pi^-$</td>
<td>$K^*(892)^+$</td>
<td>$F^{K\pi}(s_+)$</td>
</tr>
<tr>
<td>$\mathcal{M}_{10\pi}$: DCS(An)</td>
<td>$[K_0^0 \pi^+]_D \pi^-$</td>
<td>$K^*_2(1430)^+$</td>
<td>Breit-Wigner</td>
</tr>
</tbody>
</table>

Here: $s_{\pm} = (p_{\pi^\pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$
Unitary scalar $K\pi$ form factor used in our model for $D^0 \rightarrow K^0_S\pi^+\pi^-$

$K^*_0(800)$  $K^*_0(1430)$

$|F_{K^0\pi^-}(m)|$ (GeV)  phase of $F_{K^0\pi^-}^0(\phi)(m)$ (degrees)

$|F_{K^0\pi^-}(m)|$ scalar $K\pi$ form factor

$\Rightarrow$ Best fit with the $K\pi$ scalar form factor: Muskhelishvili-Omnès’s 2 coupled channel equations using experimental kaon-pion $T$ matrix + chiral symmetry + asymptotic QCD constraints + $f_K/f_\pi=1.175$ [B. Moussallam private communication, see also B. El-Bennich et al. Phys. Rev. D 79, 094005 (2009)]
Unitary scalar-isoscalar $\pi\pi$ form factor $F^{\pi\pi}_0(\propto \Gamma^\pi_1)$ used in our model for $D^0 \to K^0_S \pi^+ \pi^-$

Quasi-two-body factorization for $D^0 \rightarrow K^0_S \pi^+ \pi^-$

Form factors

Dalitz-plot fit

\[ K\pi \text{ and } \pi\pi \text{ vector form factor used in our model for } D^0 \rightarrow K^0_S \pi^+ \pi^- \]

- Mass and width of the $K^*(892)$ meson are free parameters entering also in the $K\pi$ vector form factor taken from the Belle Collaboration fit to the $\tau^- \rightarrow K^0_S \pi^- \nu_\tau$ decays

- Contributions of $K^*(892)$ and $K^*(1410)$ resonances taken but not that of the $K^*(1680)$

- Alternatively to this experimental parametrization we use the model of the $K\pi$ vector form factor of D. R. Boito et al. [JHEP 1009, 031 (2010)] in which some constraints from analyticity and elastic unitarity are incorporated

- Two types of the pion vector form factor tested:
  - the experimental parametrization used by Belle Collaboration [2008] in the data analysis of $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays
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Input and free parameters, here choice: $a_1 = 1.1 \quad a_2 = -0.5$

<table>
<thead>
<tr>
<th>fixed parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0^{D^0R_S[K^0\pi^-]}(m_\pi^2)$</td>
<td>0.48</td>
</tr>
<tr>
<td>$F_0^{D^0R_S[\pi^+\pi^-]}(m_{K^0}^2)$</td>
<td>0.18</td>
</tr>
<tr>
<td>$A_0^{D^0R_P[K^0\pi^-]}(m_\pi^2)$</td>
<td>0.76</td>
</tr>
<tr>
<td>$A_0^{D^0R_P[\pi^+\pi^-]}(m_{K^0}^2)$</td>
<td>0.7</td>
</tr>
<tr>
<td>$A_0^{D^0}\omega(m_{K^0}^2)$</td>
<td>0.669</td>
</tr>
<tr>
<td>$g_{\omega\pi\pi}$</td>
<td>0.3504</td>
</tr>
<tr>
<td>$g_{f_2\pi^+\pi^-}$</td>
<td>18.55 GeV$^{-1}$</td>
</tr>
<tr>
<td>$g_{K^*\pi^-K^0\pi^-}$</td>
<td>11.72 GeV$^{-1}$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ 33 free parameters $\rightarrow$ 14 complex: $\chi[\pi\pi]_S, \chi[\pi\pi]_S, F_0^{R_S[K^0\pi^-]\pi^+}(m_{D^0}^2)$,

$F_0^{R_S[\pi^+\pi^-]}(m_{D^0}^2)$, $A_0^{R_P[K^0\pi^-]\pi^+}(m_{D^0}^2)$, $A_0^{R_S\omega}(m_{D^0}^2)$ + 6 parameters for $D$-waves

+ 2 for possible charge independence violation in $M_{8\pi}$ and $M_{9\pi}$,

$\rightarrow$ 5 real parameters: $A_0^{R_P[\pi^+\pi^-]}(m_{D^0}^2)$ + 2 for pion-scalar form factor, $\kappa, c$

+ 2 for kaon-vector form factor, $m_{K^*\mp}, \Gamma_{K^*}$. 

B. Loiseau
CHARM 2016 - Bologna, Italy - September 05, 2016 - 26
INTRODUCTION

FSI CONSTRAINTS

DIAGRAMMATIC APPROACH: $D \rightarrow M_1 M_2$

FACTORIZATION APPROACH: $D \rightarrow m_1 m_2 m_3$

CONCLUDING REMARKS

Quasi-two-body factorization for $D^0 \rightarrow K^0_S \pi^+ \pi^-$

Form factors

Dalitz-plot fit

Fit to Belle Dalitz plot and distribution of $\chi^2$ values larger than 4
Branching fractions (Br) for all quasi two-body channels. $\sum \text{Br}=132.81 \% \leftrightarrow \text{interferences.}$

Table: $\mathcal{M}_{1\pi} : K_0^*(800)^-, K_0^*(1430)^-; \mathcal{M}_{2\pi} : f_0(500), f_0(980), f_0(1400); \mathcal{M}_{3\pi} : K^*(892)^-; \mathcal{M}_{4\pi} : \rho(770)^0$

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>channel</th>
<th>Br</th>
<th>tree</th>
<th>ann. low</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{M}_{1\pi}$</td>
<td>$[K_S^0 \pi^-]_S \pi^+$</td>
<td>$25.03 \pm 3.61 \pm 0.18$</td>
<td>$8.24 \pm 0.10$</td>
<td>$7.88 \pm 0.11$</td>
</tr>
<tr>
<td>$\mathcal{M}_{2\pi}$</td>
<td>$K_S^0[\pi^- \pi^+]_S$</td>
<td>$16.92 \pm 1.27 \pm 0.02$</td>
<td>$14.70 \pm 0.17$</td>
<td>$2.92 \pm 0.09$</td>
</tr>
<tr>
<td>$\mathcal{M}_{3\pi}$</td>
<td>$[K_S^0 \pi^-]_P \pi^+$</td>
<td>$62.72 \pm 4.45 \pm 0.15$</td>
<td>$24.69 \pm 5.65$</td>
<td>$8.74 \pm 2.97$</td>
</tr>
<tr>
<td>$\mathcal{M}_{4\pi}$</td>
<td>$K_S^0[\pi^- \pi^+]_P$</td>
<td>$21.96 \pm 1.55 \pm 0.06$</td>
<td>$4.36 \pm 0.06$</td>
<td>$6.74 \pm 0.04$</td>
</tr>
<tr>
<td>$\mathcal{M}_{5\pi}$</td>
<td>$K_S^0 \omega$</td>
<td>$0.79 \pm 0.07 \pm 0.04$</td>
<td>$0.24 \pm 0.01$</td>
<td>$0.16 \pm 0.02$</td>
</tr>
<tr>
<td>$\mathcal{M}_{6\pi}$</td>
<td>$[K_S^0 \pi^-]_D \pi^+$</td>
<td>$1.41 \pm 0.11 \pm 0.04$</td>
<td>$2.15 \pm 0.19 \pm 0.10$</td>
<td>$2.15 \pm 0.19 \pm 0.10$</td>
</tr>
<tr>
<td>$\mathcal{M}_{7\pi}$</td>
<td>$K_S^0[\pi^- \pi^+]_D$</td>
<td>$0.56 \pm 0.07 \pm 0.03$</td>
<td>$0.07 \pm 0.00$</td>
<td>$0.29 \pm 0.02$</td>
</tr>
<tr>
<td>$\mathcal{M}_{8\pi}$</td>
<td>$[K_S^0 \pi^+]_S \pi^-$</td>
<td>$0.64 \pm 0.06 \pm 0.02$</td>
<td>$0.77 \pm 0.15$</td>
<td>$0.01 \pm 0.01$</td>
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<td>$0.01 \pm 0.01$</td>
</tr>
<tr>
<td>$\mathcal{M}_{10\pi}$</td>
<td>$[K_S^0 \pi^+]_D \pi^-$</td>
<td>$0.63 \pm 0.07 \pm 0.11$</td>
<td>$0$</td>
<td>$0.63 \pm 0.11$</td>
</tr>
</tbody>
</table>

- Branching fractions compare well with those of Belle 's analysis
- **Belle** $\text{Br}_{K_S^0 \sigma_1} + \text{Br}_{K_S^0 f_0(980)} + \text{Br}_{K_S^0 \sigma_2} + \text{Br}_{K_S^0 f_0(1370)} = 18.6\% \sim 16.9\%$ value
- **Annihilation** contributions can be important
FSI described in term of meson-meson form factors

**Quasi two body-factorization** approaches have also been performed by:


- From these phenomenologically successful studies ⇒ amplitude parametrizations to be readily implemented in experimental analyses.
- Two-body hadronic final state interactions taken into account in terms of unitary \( S \)- and \( P \)-wave \( \pi \pi \), \( \pi K \) and \( K\bar{K} \) form factors.
- Sound alternative to the simplistic and widely used isobar model.

⇒ Explicit amplitude expressions [4,5] for the decays \( D^+ \rightarrow \pi^- \pi^+ \pi^+ \), \( D^+ \rightarrow K^- \pi^+ \pi^+ \), \( D^0 \rightarrow K_S^0 \pi^+ \pi^- \), \( D^0 \rightarrow K_S^0 K^+ K^- \).


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Amplitude parametrizations
Outlook
Backup slides

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  \[ D^+ \rightarrow K^-\pi^+\pi^+, \]
  \[ D^0 \rightarrow K^0_S\pi^+\pi^-, \]
  \[ D^0 \rightarrow K^0_S K^+K^- . \]


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**Quasi two body-factorization** approaches have also been performed by:


- From these **phenomenologically successful** studies \( \Rightarrow \) **amplitude parametrizations** to be readily implemented in experimental analyses.
- Two-body hadronic final state interactions taken into account in terms of **unitary S- and P-wave \( \pi \pi \), \( \pi K \) and \( K\bar{K} \) form factors**.
- Sound **alternative** to the simplistic and widely used **isobar model**.

\( \Rightarrow \) Explicit amplitude expressions [4,5] for the decays \( D^+ \rightarrow \pi^- \pi^+ \pi^+ \), \( D^+ \rightarrow K^- \pi^+ \pi^+ \), \( D^0 \rightarrow K_S^0 \pi^+ \pi^- \), \( D^0 \rightarrow K_S^0 K^+ K^- \).


FSI described in term of meson-meson form factors

Quasi two body-factorization approaches have also been performed by:


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Improved models needed for getting out the most information from new data

Impressive amount of high quality hadronic multibody decay data of $D^0$, $D^+$, $D_s^+$ and in this review I described some available potentialities for constraining amplitude analyses in these charm decays:

- Final state constraints: effective-hadronic formalism approach with combination of basic elements of the weak interaction with the framework of the chiral unitary approach in coupled channel.

- Diagrammatic approach: model-independent analysis based on flavor SU(3) symmetry topological amplitudes allowing to understand the relative importance of different underlying decay mechanisms.

- Three-body decays analyzed in a phenomenological quasi-two-body factorization approach in which two-body hadronic final state interactions are fully taken into account in terms of unitary form factors.
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Some recent works on hadronic $D$ decays

  → Explore consequences of constraint from CPT symmetry on three-body $D$ decays.
  → Simulate $D^\pm \rightarrow \pi^\mp K^+ K^-$ and discuss correlations with measured $D^\pm \rightarrow \pi^\mp \pi^+ \pi^-$. 

  → Based on the $k_T$ factorization, results agree with the existing experiment data for most channels.
Some recent works on hadronic $D$ decays

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  → Based on the $k_T$ factorization, results agree with the existing experiment data for most channels.
Comparison of the $T_{K^+ K^-}$ amplitude with that of the experimental data

→ FSI in [2] $f_0(980)$ production in $D_s \rightarrow \pi^+ \pi^+ \pi^-$ and $D_s \rightarrow \pi^+ K^+ K^-$ decays, J. M. Dias, F. S. Navarra, M. Nielsen, E. Oset, arXiv:1601.04635.


→ $f_0(980)$ in the 1 GeV region
Cabibbo favored and suppressed tree amplitudes in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$

$c \rightarrow su\bar{d}$ transition:

$$
\propto \frac{G_F}{\sqrt{2}} a_1(m_c) \Lambda_1 \{ \equiv V_{cs}^* V_{ud} \} (\bar{s}c) V_{-A} (\bar{u}d) V_{-A}
\rightarrow V_{cs} \approx V_{ud} \approx \cos \theta_C \approx 0.975,
$$

$\theta_C$ Cabibbo angle

$\Rightarrow$ 7 Cabibbo favored (CF) tree (Tr) amplitudes:

$[K\pi]_{S,P,D} \pi + K [\pi\pi]_{S,P,D} + K \omega \{ \omega \rightarrow [\pi\pi]_P \}$

by $G$-parity violation

$c \rightarrow du\bar{s}$ transition:

$$
\propto \frac{G_F}{\sqrt{2}} a_1(m_c) \Lambda_2 \{ \equiv V_{cd}^* V_{us} \} (\bar{d}c) V_{-A} (\bar{u}s) V_{-A}
\rightarrow V_{cd} \approx -\lambda, V_{us} \approx \lambda, \lambda = \sin \theta_C,
$$

$\Rightarrow$ 6 doubly Cabibbo suppressed (DCS) tree (Tr) amplitudes as no $W$ coupling to $[K\pi]_D$ state

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Annihilation - t-channel $W$-exchange amplitudes in $D^0 \rightarrow K_s^0 \pi^+ \pi^-$

$c \rightarrow s \bar{u} \bar{d}$ ($c \bar{u} \rightarrow s \bar{d}$):
\[
\propto \frac{G_F}{\sqrt{2}} a_2(m_c) \cos^2 \theta_C (\bar{s}c)_{V-A} (\bar{d}u)_{V-A}
\]

$\Rightarrow$ 7 Cabibbo favored annihilation (An) amplitudes

$c \rightarrow d \bar{u} \bar{s}$ ($c \bar{u} \rightarrow d \bar{s}$):
\[
\propto -\frac{G_F}{\sqrt{2}} a_2(m_c) \sin^2 \theta_C (\bar{d}c)_{V-A} (\bar{s}u)_{V-A}
\]

$(\sin \theta_C = 0.225)$

$\Rightarrow$ 7 doubly Cabibbo suppressed annihilation (An) amplitudes

Total of 27 non-zero amplitudes: 13 tree and 14 annihilation
Modified annihilation amplitudes

- Dalitz plot density distribution: \[ \frac{d^2 Br_{s-}}{ds_- ds_+} = \frac{|M|^2}{32(2\pi)^3 m^3 D^0 \Gamma D^0} \]

- The s_- one-dimensional density: \[ \frac{dB_{s-}}{ds_-} = \int \frac{(m_{D^0} - m_\pi)^2}{(m_\pi + m_{K^0})^2} \frac{d^2 Br_{s-}}{ds_- ds_+} ds_+ \]

- Out of the 10 phases, one phase (~770\[ρ(770)\]) is chosen to be real, e.g., \( \phi_4 \), cannot be determined \( \Rightarrow \) modified partial amplitudes \( \tilde{M}_i = e^{-i\phi_4} M_i \)

- Modified annihilation amplitudes \( A'_i: \tilde{\tilde{M}}_i = T_i + A'_i \) \( A'_i = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i \).

\[ \Rightarrow \]

- New coefficient for \( A'_4 \):
\[ \tilde{\tilde{A}}_0^{K^0[\pi^+\pi^-]} = (e^{-i\phi_4} - 1) \int f_{D^0}^{K^0} A_0^{D^0 R_P[\pi^+\pi^-]} (m_{K^0}) + e^{-i\phi_4} A_0^{K^0 R_P[\pi^+\pi^-]} (m_{D^0}) \]

- Similar for: \( \tilde{F}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{F}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \).
Dalitz plot density distribution: \[ \frac{d^2 Br}{ds_- ds_+} = \frac{|M|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \]

The \( s_- \) one-dimensional density: \[ \frac{d Br}{ds_-} = \int (m_{D_0} - m_{\pi})^2 \frac{d^2 Br}{ds_- ds_+} ds_+ \]

Out of the 10 phases, one phase \((M_4 \pi [\rho(770)^0])\) is chosen to be real, e. g., \( \phi_4 \), cannot be determined ⇒ modified partial amplitudes \( \tilde{M}_i = e^{-i\phi_4} M_i \)

\[ \frac{d^2 B_{i}^{\text{tree}}}{ds_- ds_+} = \frac{|T_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \quad \text{and} \quad \frac{d^2 B_{i}^{\text{ann}}}{ds_- ds_+} = \frac{|A_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} = \frac{|e^{i\phi_4} \tilde{M}_i - T_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \]

\[ |\tilde{M}_i|^2 + |T_i|^2 - 2|\tilde{M}_i||T_i| \leq |A_i|^2 \leq |\tilde{M}_i|^2 + |T_i|^2 + 2|\tilde{M}_i||T_i| \]

\[ Br_{i}^{\text{ann, low}} = Br_i + B_{i \text{tree}} - 2 \int \int ds_- ds_+ |\tilde{M}_i||T_i| \]

Modified annihilation amplitudes \( A'_i \): \( \tilde{M}_i = T_i + A'_i \quad A'_i = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i \)

⇒ New coefficient for \( A'_4 \):

\[ \tilde{A}_0^{K^0[\pi^+\pi^-]} = (e^{-i\phi_4} - 1) f_{K^0} f_{D_0} A_{D_0}^{D_0 R_P[\pi^+\pi^-]} (m_{K^0}) + e^{-i\phi_4} A_{0}^{K^0 R_P[\pi^+\pi^-]} (m_{D_0}^2) \]

Similar for: \( \tilde{F}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{F}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \), \( \tilde{A}_0^{K^0[\pi^+\pi^-]} \).
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**Modified annihilation amplitudes**

1. **Dalitz plot density distribution:**
   \[
   \frac{d^2 Br}{ds_- ds_+} = \frac{|M|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0}
   \]

2. **The $s_-$ one-dimensional density:**
   \[
   \frac{d Br}{ds_-} = \int (m_D^0 - m_\pi)^2 \frac{d^2 Br}{ds_- ds_+} ds_+
   \]

Out of the 10 phases, **one phase** $(\mathcal{M}_4 \pi [\rho(770)^0]$ is choosen to be real), e. g., $\phi_4$, **cannot be determined** ⇒ modified partial amplitudes $\tilde{M}_i = e^{-i\phi_4} M_i$

3. **Modified annihilation amplitudes** $A_i': \tilde{M}_i = T_i + A_i' \ A_i' = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i$.

⇒ **New coefficient for** $A_4'$:

4. **Similar for:** $\tilde{F}_0^{[\bar{K}^0 \pi -]_S \pi^+}, \ F_0^{[\bar{K}^0 \pi -]_S \pi^+}, \ \tilde{A}_0^{[\bar{K}^0 \pi -]_P \pi^+}, \ \tilde{A}_0^{\bar{K}^0 \omega} \leftrightarrow A'_1, A'_2, A'_3, A'_5$.
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Modified annihilation amplitudes

- Dalitz plot density distribution:
  \[ \frac{d^2 Br}{ds_- ds_+} = \frac{|M|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \]

- The $s_-$ one-dimensional density:
  \[ \frac{dB r}{ds_-} = \int (m_{D_0}^2 - m_{\pi}^2)^2 \frac{d^2 Br}{ds_- ds_+} \]

- Out of the 10 phases, one phase ($M_4 \pi [\rho(770)^0]$ is choosen to be real), e. g., $\phi_4$, cannot be determined $\Rightarrow$ modified partial amplitudes $\tilde{M}_i = e^{-i\phi_4} M_i$

- $\frac{d^2 B r_{i,\text{tree}}}{ds_- ds_+} = \frac{|T_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0}$ and $\frac{d^2 B r_{i,\text{ann}}}{ds_- ds_+} = \frac{|A_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0}$

\[ \Rightarrow |\tilde{M}_i|^2 + |T_i|^2 - 2|\tilde{M}_i||T_i| \leq |A_i|^2 \leq |\tilde{M}_i|^2 + |T_i|^2 + 2|\tilde{M}_i||T_i| \]

- Modified annihilation amplitudes $A_i'$: $\tilde{M}_i = T_i + A_i'$, $A_i' = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i$.

\[ \Rightarrow \text{New coefficient for } A_4': \tilde{A}_0^{K^0[\pi^+ \pi^-]} = (e^{-i\phi_4} - 1) f_{K^0} f_{D_0} D_0 R_{\rho[\pi^+ \pi^-]}(m_{K^0}^2) + e^{-i\phi_4} A_0^{K^0[\rho][\pi^+ \pi^-]}(m_{D_0}^2) \]

Similar for: $\tilde{F}_0^{K^0[\rho][\pi^+ \pi^-]}$, $\tilde{F}_0^{K^0[\pi^+ \pi^-]}$, $\tilde{F}_0^{K^0[\pi^+ \pi^-]}$, $\tilde{A}_0^{K^0[\pi^+ \pi^-]}$, $\tilde{A}_0^{K^0[\omega]} \leftrightarrow A_1', A_2', A_3', A_5'$.
Modified annihilation amplitudes

- Dalitz plot density distribution: \[ \frac{d^2 Br}{ds_- ds_+} = \frac{|M|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \]

- The s_ one-dimensional density: \[ \frac{dB r}{ds_-} = \int (m_{D_0} - m_\pi)^2 \frac{d^2 Br}{ds_- ds_+} ds_+ \]

Out of the 10 phases, one phase \((M_{4\pi} \rho(770)^0)\) is chosen to be real, e. g., \(\phi_4\), cannot be determined \(\Rightarrow\) modified partial amplitudes \(\tilde{M}_i = e^{-i\phi_4} M_i\)

- Partial tree amplitudes: \[ \frac{d^2 Br_{i,\text{tree}}}{ds_- ds_+} = \frac{|T_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \]
  and \[ \frac{d^2 Br_{i,\text{ann}}}{ds_- ds_+} = \frac{|A_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} = \frac{|e^{i\phi_4}\tilde{M}_i - T_i|^2}{32(2\pi)^3 m^3 D_0 \Gamma D_0} \]

\[ \Rightarrow |\tilde{M}_i|^2 + |T_i|^2 - 2|\tilde{M}_i||T_i| \leq |A_i|^2 \leq |\tilde{M}_i|^2 + |T_i|^2 + 2|\tilde{M}_i||T_i| \]

- Modified annihilation amplitudes \(A_i': \tilde{M}_i = T_i + A_i'\)
  \[ A_i' = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i. \]

\[ \Rightarrow \] New coefficient for \(A_4': \]
\[ \tilde{A}^0_{K^0[\pi^+\pi^-]} = (e^{-i\phi_4} - 1)f_{K^0} f_{D_0}^0 R_p[\pi^+\pi^-](m_{K^0}) + e^{-i\phi_4} A^0_{K^0} R_p[\pi^+\pi^-](m_{D_0}^2) \]

Similar for: \(\tilde{f}_0^{K^0\pi^-}_S, \tilde{f}_0^{K^0\pi^-}_P, \tilde{A}^0_{K^0\pi^-}, \tilde{A}^0_{K^0\omega} \leftrightarrow A'_1, A'_2, A'_3, A'_5, \)
Modified annihilation amplitudes

- Dalitz plot density distribution: \( \frac{d^2 Br}{ds_- ds_+} = \frac{|M|^2}{32(2\pi)^3 m_D^0 \Gamma_D^0} \)

- The \( s_- \) one-dimensional density: \( \frac{dBr}{ds_-} = \int (m_D^0 - m_\pi^2) \frac{d^2 Br}{ds_- ds_+} ds_+ \)

Out of the 10 phases, one phase \((M_{4\pi} [\rho(770)^0] \) is choosen to be real), e. g., \( \phi_4 \), cannot be determined \( \Rightarrow \) modified partial amplitudes \( \tilde{M}_i = e^{-i\phi_4} M_i \)

- \( \frac{d^2 B_{i\text{tree}}}{ds_- ds_+} = \frac{|T_i|^2}{32(2\pi)^3 m_D^0 \Gamma_D^0} \)
- \( \frac{d^2 B_{i\text{ann}}}{ds_- ds_+} = \frac{|A_i|^2}{32(2\pi)^3 m_D^0 \Gamma_D^0} = \frac{|e^{i\phi_4} \tilde{M}_i - T_i|^2}{32(2\pi)^3 m_D^0 \Gamma_D^0} \)

\( \Rightarrow \) \( |\tilde{M}_i|^2 + |T_i|^2 - 2|\tilde{M}_i||T_i| \leq |A_i|^2 \leq |\tilde{M}_i|^2 + |T_i|^2 + 2|\tilde{M}_i||T_i| \)

- Modified annihilation amplitudes \( A'_i: \tilde{M}_i = T_i + A'_i \quad A'_i = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i \)

- New coefficient for \( A'_4 \):
  \[ \tilde{A}_0^{K^0[\pi^+\pi^-]}_{\rho} = (e^{-i\phi_4} - 1) \tilde{f}_D^{K^0} A_0^{D^0 R_{\rho}[\pi^+\pi^-]} (m_{K^0}^2) + e^{-i\phi_4} A_0^{K^0 R_{\rho}[\pi^+\pi^-]} (m_{D^0}^2) \]

Similar for: \( \tilde{F}_0^{K^0[\pi^+\pi^-]}_{\omega}, \tilde{F}_0^{K^0[\pi^+\pi^-]}_{\rho}, \tilde{A}_0^{K^0[\pi^+\pi^-]}_{\rho}, \tilde{A}_0^{K^0[\pi^+\pi^-]}_{\omega} \leftrightarrow A'_1, A'_2, A'_3, A'_5 \)
Modified annihilation amplitudes

- Dalitz plot density distribution: \[ \frac{d^2 Br}{ds_- ds_+} = \frac{|M_i|^2}{32(2\pi)^3 m_0^3 \Gamma_0 \Gamma_0} \]

- The \( s_- \) one-dimensional density: \[ \frac{d Br}{ds_-} = \int (m_0 - m_\pi)^2 \frac{d^2 Br}{ds_- ds_+} ds_+ \]

- Out of the 10 phases, one phase \( \phi_4 \) cannot be determined \( \Rightarrow \) modified partial amplitudes \( \tilde{M}_i = e^{-i\phi_4} M_i \)

- \[ \frac{d^2 B_{i,\text{tree}}}{ds_- ds_+} = \frac{|T_i|^2}{32(2\pi)^3 m_0^3 \Gamma_0 \Gamma_0} \quad \text{and} \quad \frac{d^2 B_{i,\text{ann}}}{ds_- ds_+} = \frac{|A_i|^2}{32(2\pi)^3 m_0^3 \Gamma_0 \Gamma_0} = \frac{|e^{i\phi_4} \tilde{M}_i - T_i|^2}{32(2\pi)^3 m_0^3 \Gamma_0 \Gamma_0} \]

- \( |\tilde{M}_i|^2 + |T_i|^2 - 2|\tilde{M}_i||T_i| \leq |A_i|^2 \leq |\tilde{M}_i|^2 + |T_i|^2 + 2|\tilde{M}_i||T_i| \)

- \( Br_{i,\text{ann, low}} = Br_i + B_{i,\text{tree}} - 2 \int \int ds_- ds_+ |\tilde{M}_i||T_i| \)

- Modified annihilation amplitudes \( A'_i: \tilde{M}_i = T_i + A'_i \), \( A'_i = T_i(e^{-i\phi_4} - 1) + e^{-i\phi_4} A_i \).

- New coefficient for \( A'_4 \):

\[ \tilde{A}_0^0[\pi^+\pi^-]_P = (e^{-i\phi_4} - 1) \frac{f_{K0}}{f_{D0}} A_0^{D0} R_P[\pi^+\pi^-] (m_{K0}^2) + e^{-i\phi_4} A_0^{K0} R_P[\pi^+\pi^-] (m_{D0}^2) \]

Similar for: \( \tilde{F}_0^0[\pi^+\pi^-]_P, \tilde{F}_0^{K0}[\pi^+\pi^-]_P, \tilde{A}_0^{K0} \omega \leftrightarrow A'_1, A'_2, A'_3, A'_5, \)
$K_S^0\pi^-$ effective mass squared distributions ($m_2^2 \equiv s_-$)

$K^*(892)^-$ \uparrow

⇒ Our model (solid curve) compared with Belle data (points with error bars).
Enlarged $K_S^0\pi^-$ effective mass squared distributions

⇒ Our model (solid curve) compared with Belle data (points with error bars). Vertical scale enlarged by a factor of 5 to enforce the differences at higher $K_S^0\pi^-$ masses
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**$K_S^0\pi^+$ and $\pi^+\pi^-$ effective mass squared distributions ($m_+^2 \equiv s_+$ and $m_0^2 \equiv s_0$)**

- **Left panel:** comparison of the $K_S^0\pi^+$ effective mass squared distributions for the **best fit** (solid curve) with the **Belle data** (points with error bars). **Right panel:** as in left panel but for the $\pi^+\pi^-$ effective mass squared.

- **The small shoulder** at $m_0^2 = 1.2 \text{ GeV}^2$ could correspond to the $\pi\pi \rightarrow \eta\eta$ contribution introduced in Belle’analysis but not included in our approach.