Weak Charm Decays with lattice QCD: status and prospects

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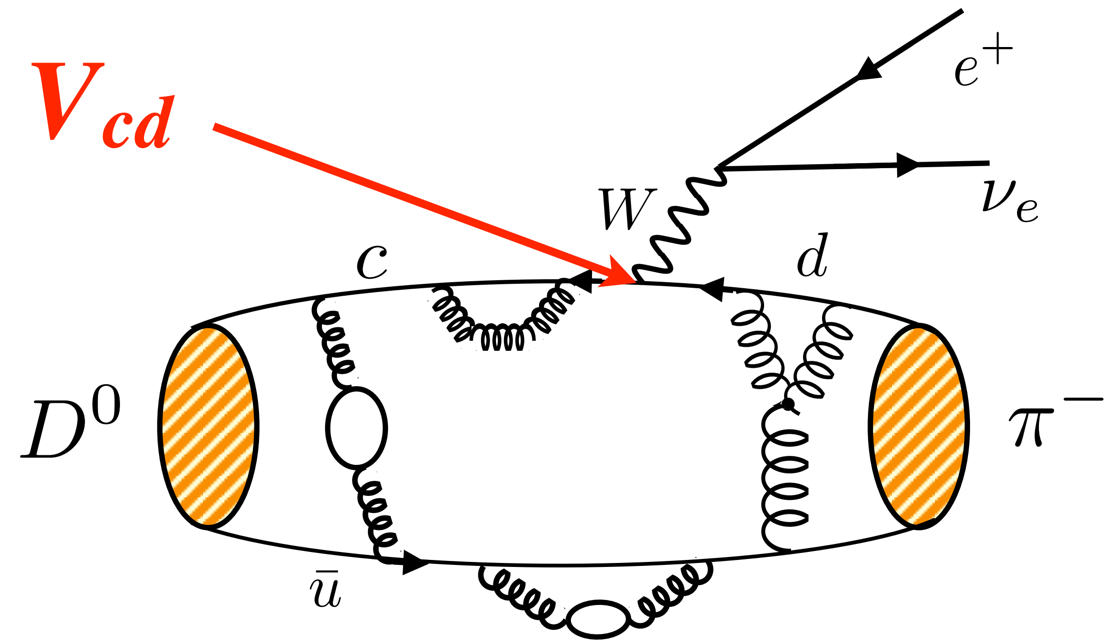
Outline

- Motivation and Introduction
  - lattice QCD
- Results
  - leptonic D-meson decays
  - semileptonic D-meson decays
  - neutral D-meson mixing
- Phenomenology
  - CKM determinations
  - second row unitarity test
  - NP scale
- Summary and Outlook

Introduction

example: $D \to \pi \ell \nu$



generic weak process involving hadrons:

(experiment) = (known) x (**CKM element**) x (had. matrix element)

$\Gamma_{K\ell 3}, \Gamma_{K\ell 2}, \ldots$

$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}, \frac{d\Gamma(D \to K \ell \nu)}{dq^2}, \ldots$

$\Delta m_{d(s)}$

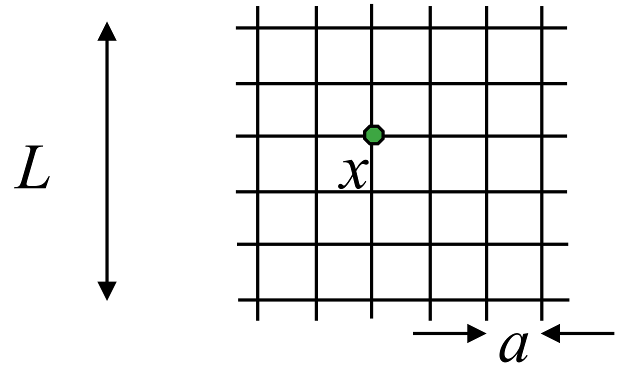
$\vdots$

Lattice QCD

parameterize the ME in terms of form factors, decay constants, bag parameters, ...

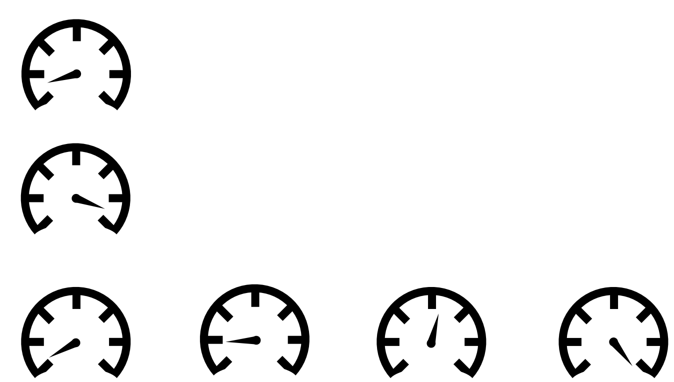
Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$



- discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

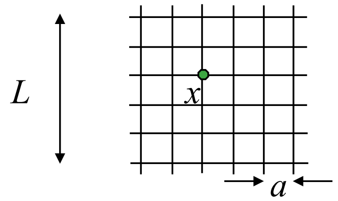
adjustable parameters



- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty,\, T > L$
- quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses $m_f \to m_{f,\rm phys}$
  extrapolations/interpolations

m_{ud} m_s m_c m_b

- also: n_f = number of sea quarks: 3 (2+1), 4 (2+1+1)



Lattice QCD Introduction

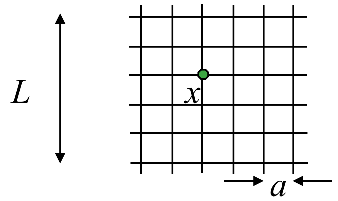
$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis



Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD

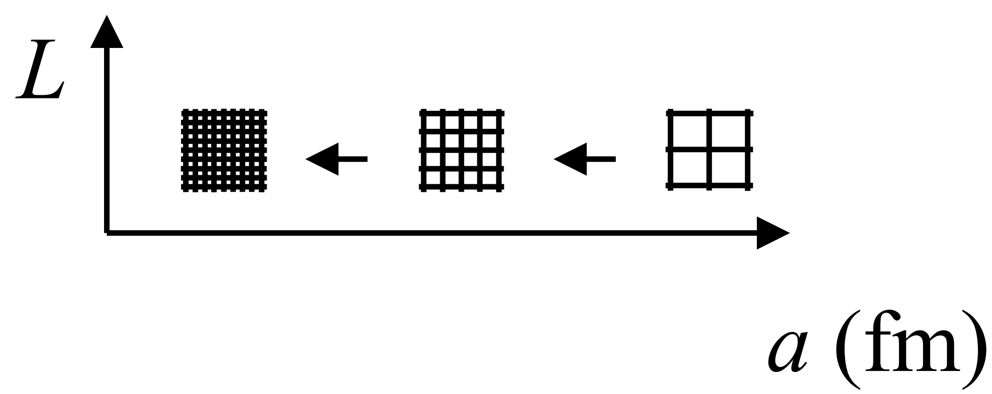
→ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

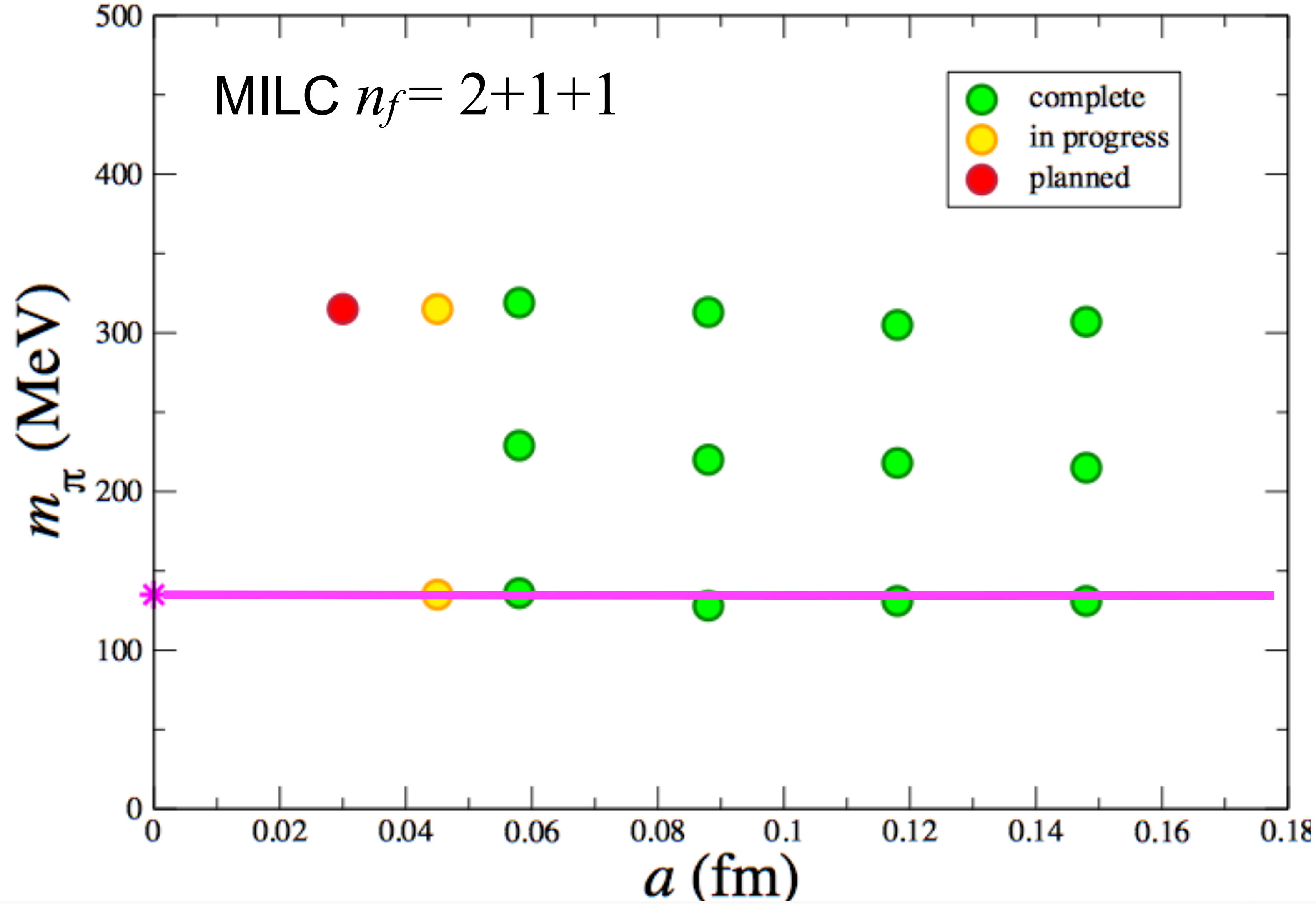
To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...



chiral-continuum extrapolation (interpolation)

Example: Set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM

Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{\rm QCD}$), discretization errors ~ $\alpha_s^k (a\Lambda_{\rm QCD})^n$
- For heavy quarks, discretization errors ~ $\alpha_s^k (am_h)^n$ with currently available lattice spacings

  for *b* quarks $am_b > 1$

  for charm $am_c \sim 0.15\text{-}0.6$

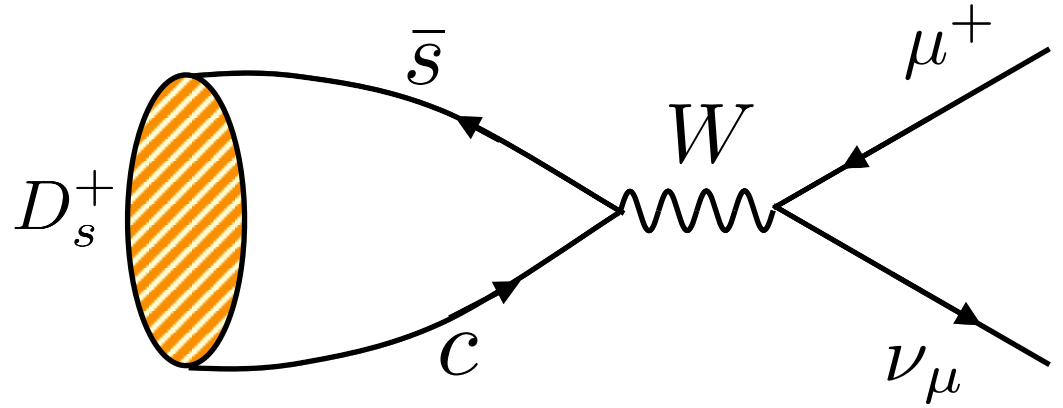
⇨ for charm can use light quark methods, if action is sufficiently improved (HISQ, tmWilson, NP imp. Wilson,…)

need effective field theory methods for *b* quarks

- avoid errors of $(am_c)^n$ in the action by matching to continuum HQET:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba) can be used for charm and bottom

Leptonic D decay

example: $D_s^+ \to \mu^+ \nu_\mu$

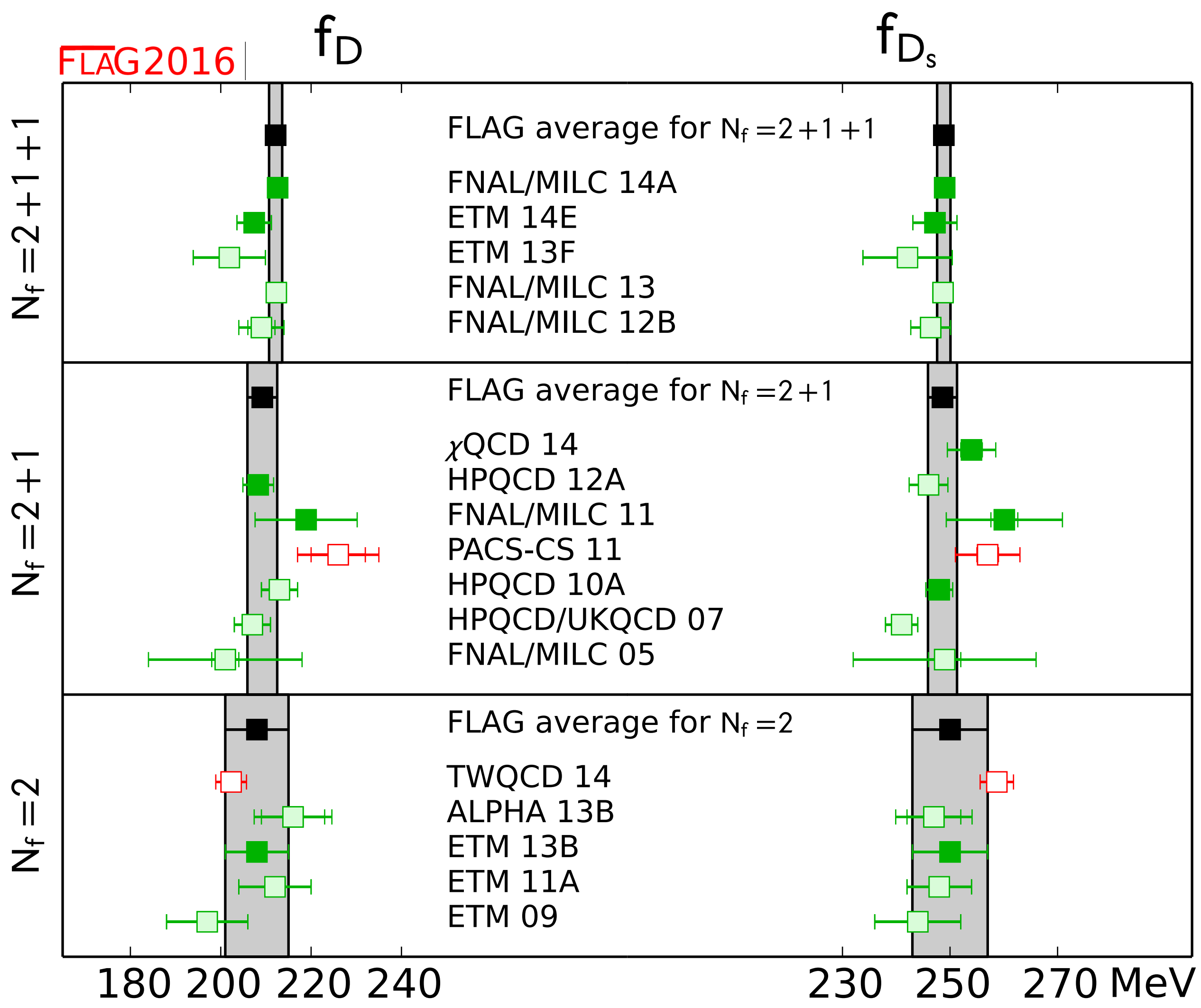


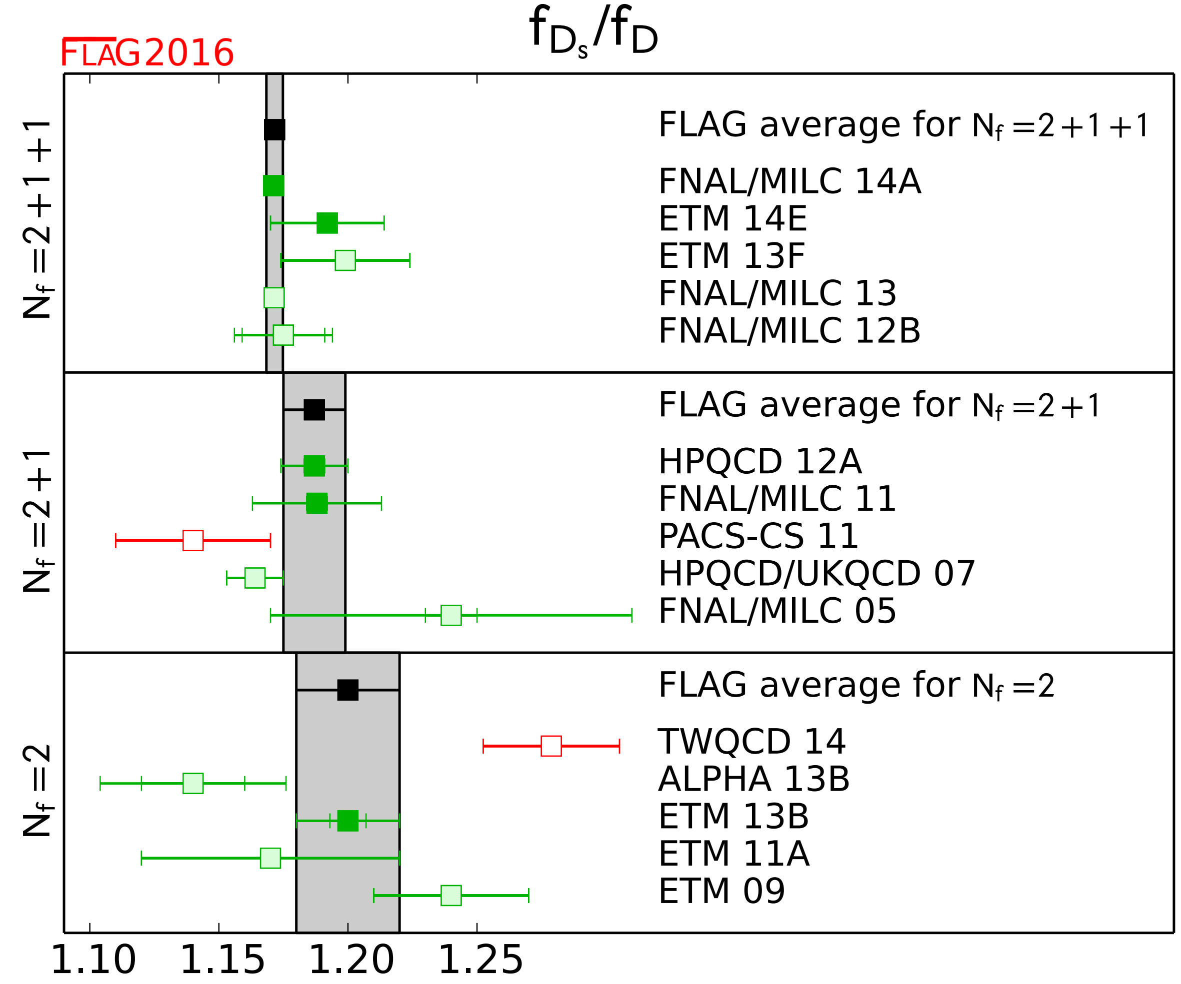
$$\Gamma(D_s^+ \to \ell^+ \nu_\ell(\gamma)) = (\text{known}) \times (1 + \delta_{\text{EM}}^\ell) \times |V_{cs}|^2 f_{D_s}^2$$

- use experiment + LQCD input for determination of CKM element
- similar for B ($|V_{ub}|$) and K ($|V_{us}|$) mesons
- SU(3) **ratio** f_{D_s}/f_{D^+} : statistical and systematic errors tend to cancel.
- δ_{EM}^ℓ includes structure dependent EM corrections. It is needed to relate the "pure QCD" decay constant to experiment and is currently estimated phenomenologically.

$D_{(s)}$ decay constant results

S. Aoki et al (FLAG review, arXiv:1607.00299)





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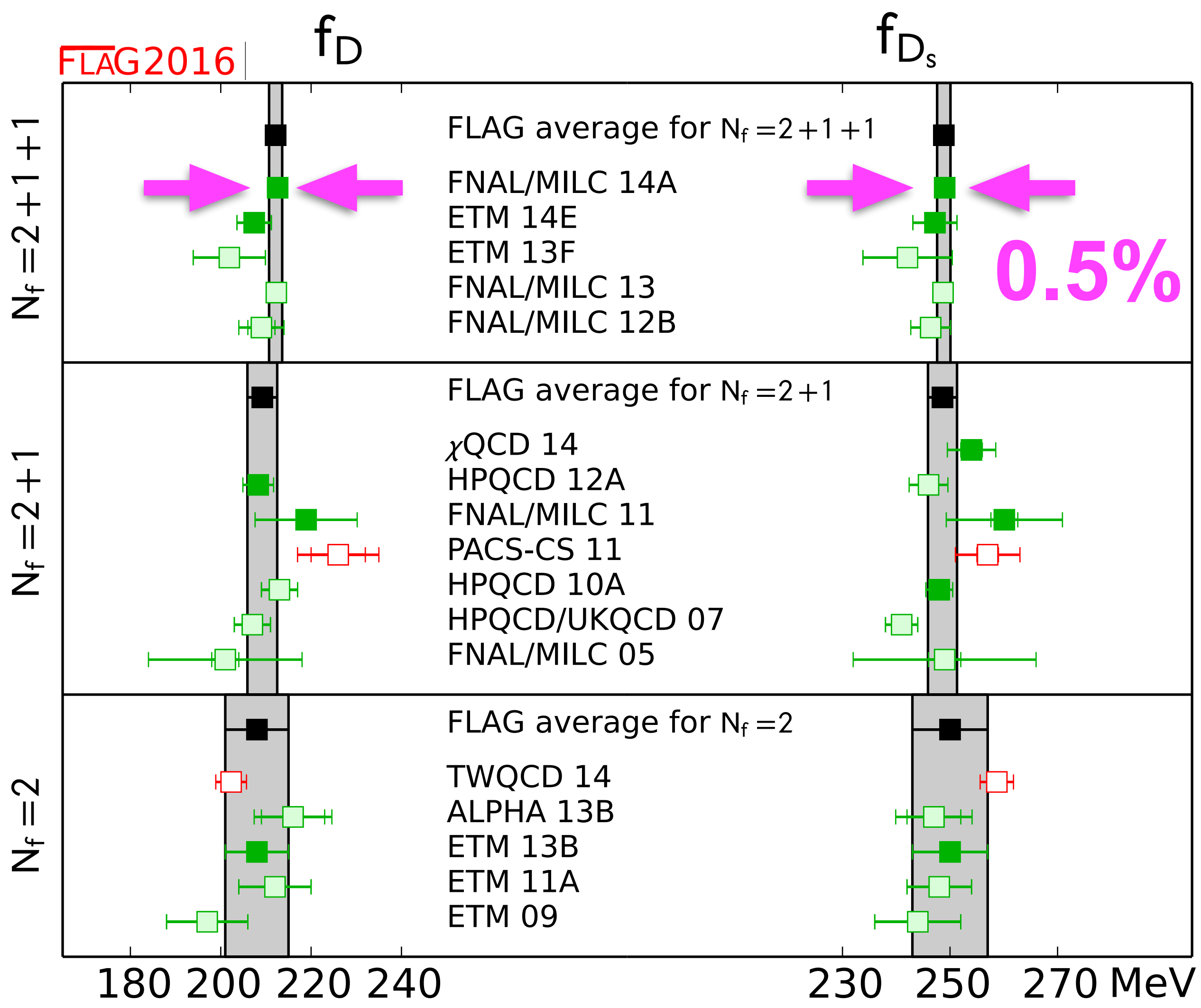
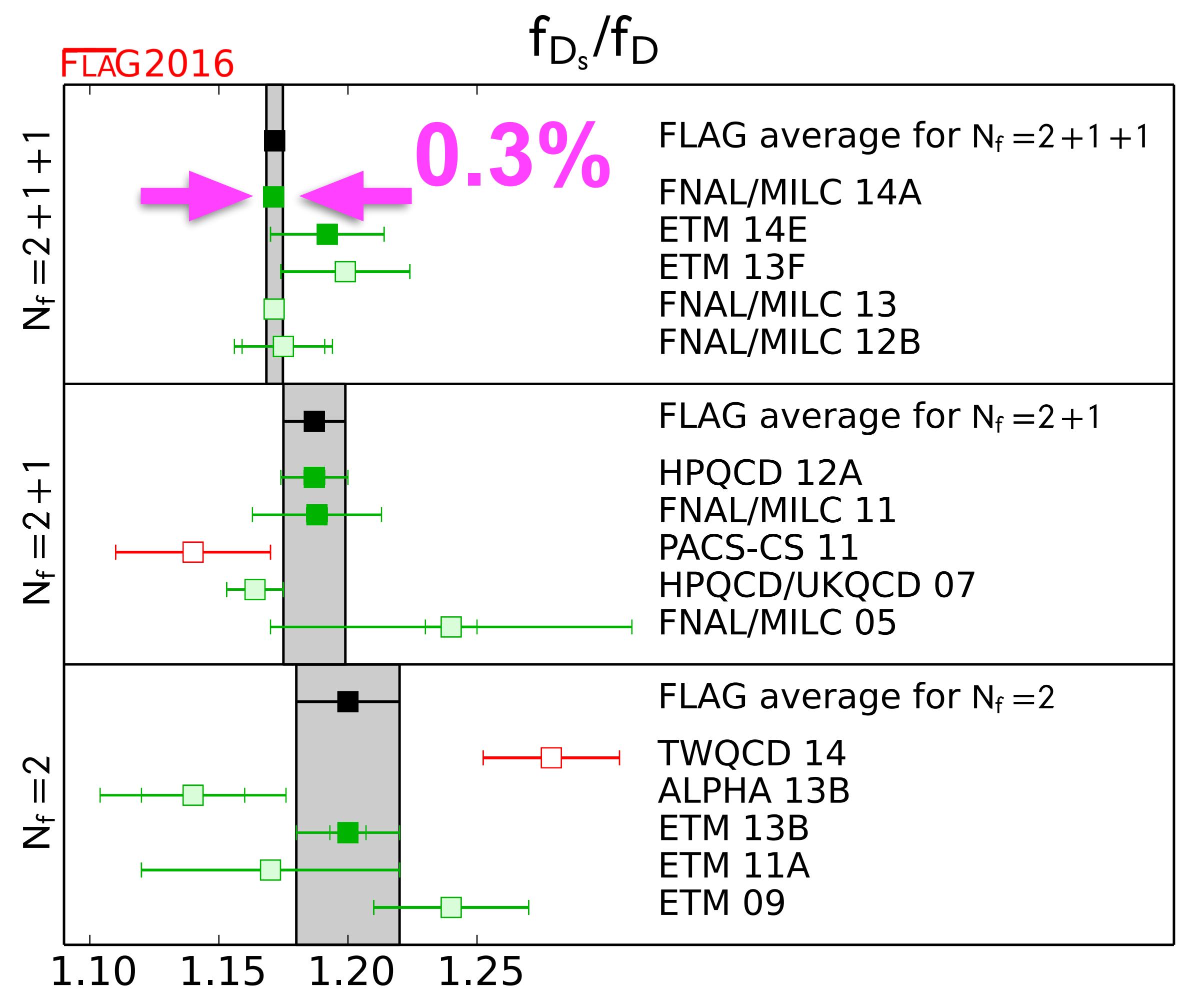




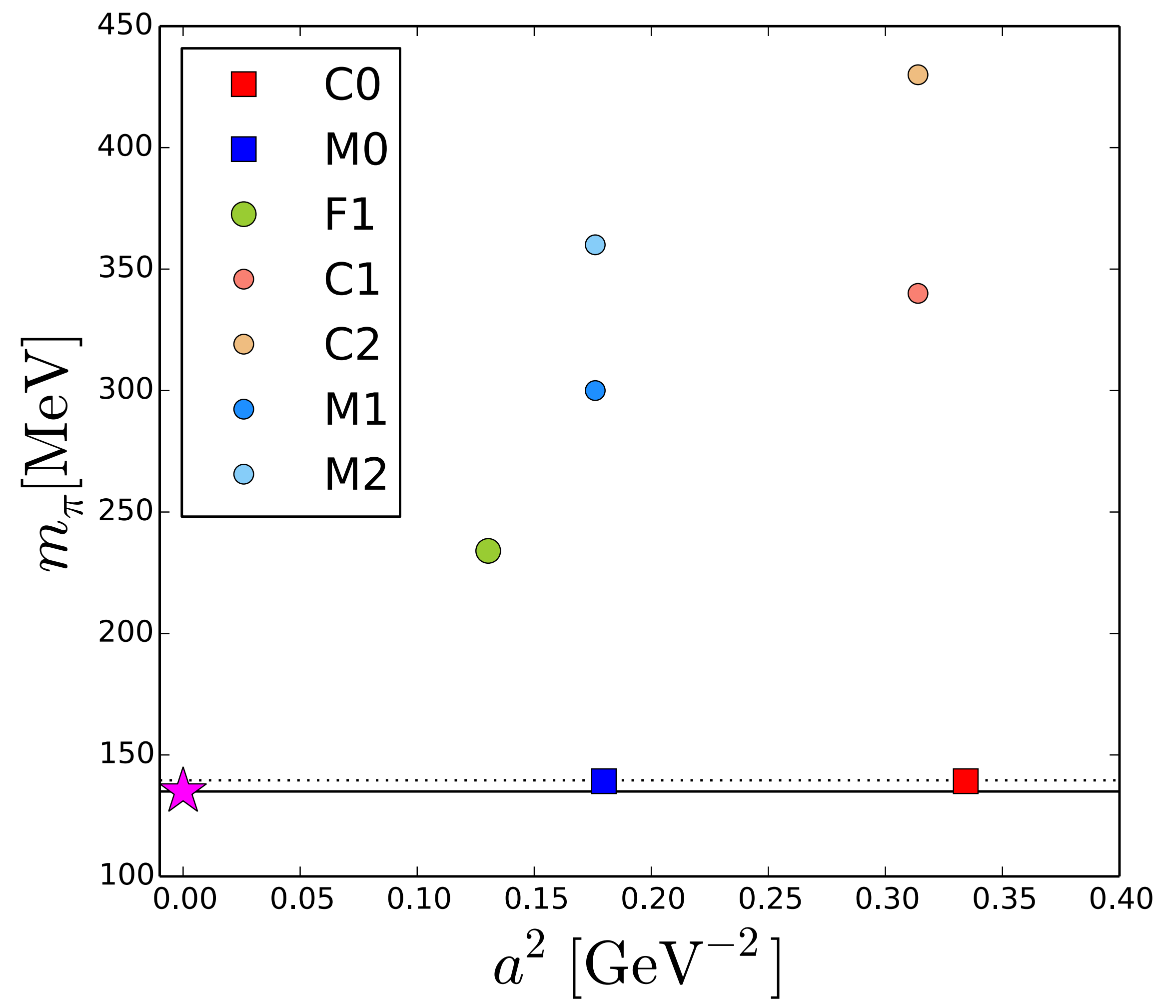
small errors in FNAL/MILC 14A (arXiv:1407.3772, 2014 PRD) due to

- physical mass ensembles
- improved action (small discretization errors)
- small lattice spacings
- PCAC (no renormalization)

$D_{(s)}$ decay constant results

RBC/UKQCD (J.T. Tsang @ Lattice 2016):

- 2+1 flavors of DW fermions
- physical mass ensembles
- PCAC (no renormalization)



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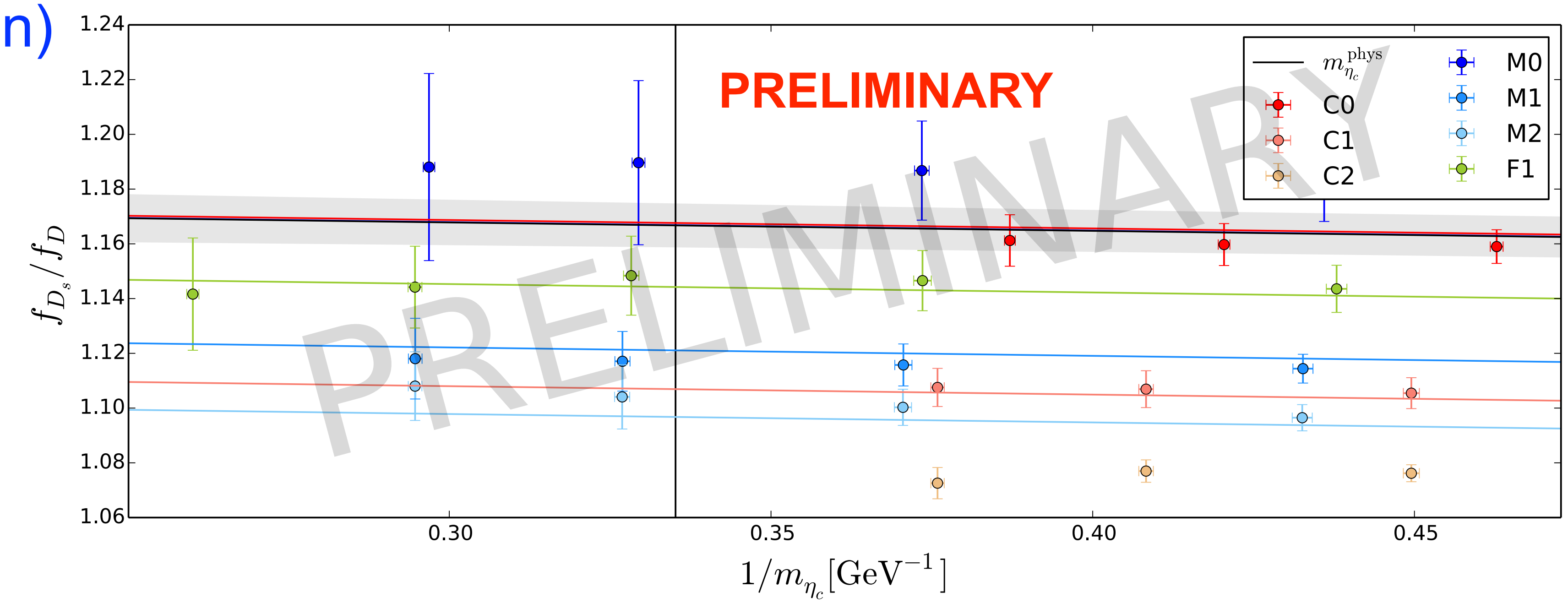

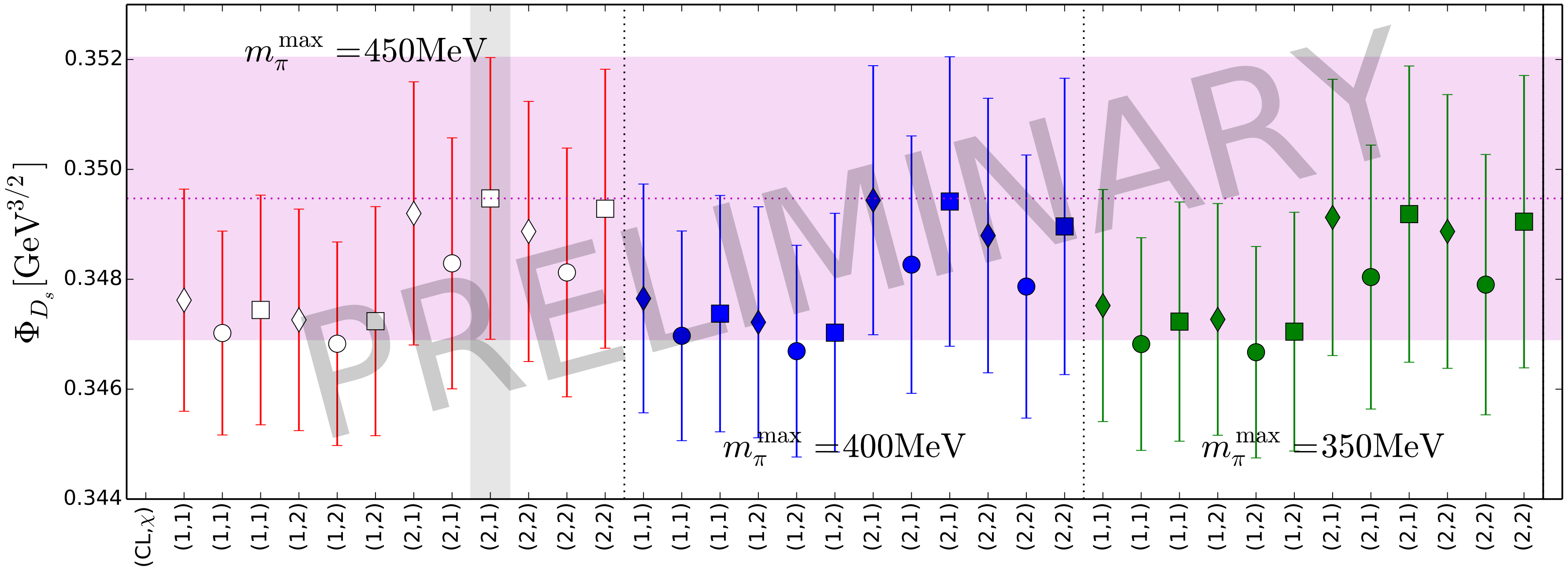

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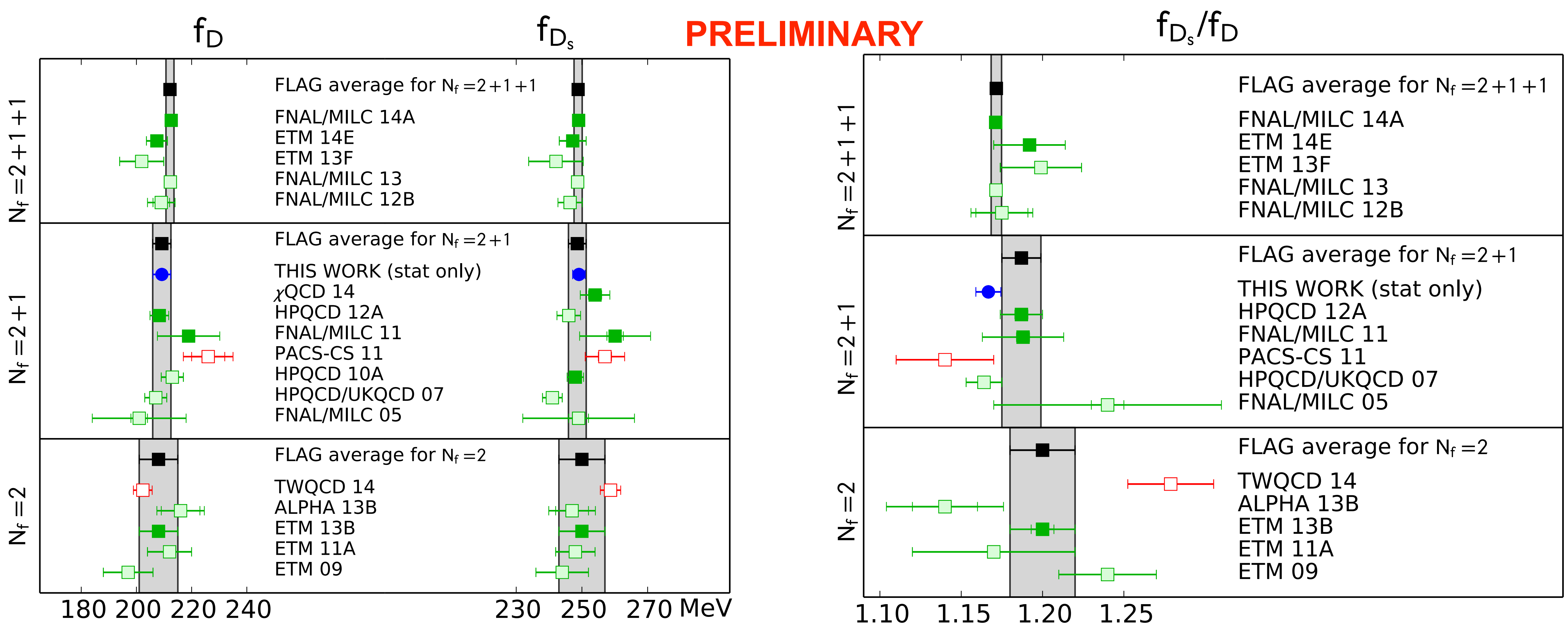
J. T. Tsang (RBC/UKQCD) @ Lattice 2016:



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- 2+1 flavors of DW fermions
- physical mass ensembles
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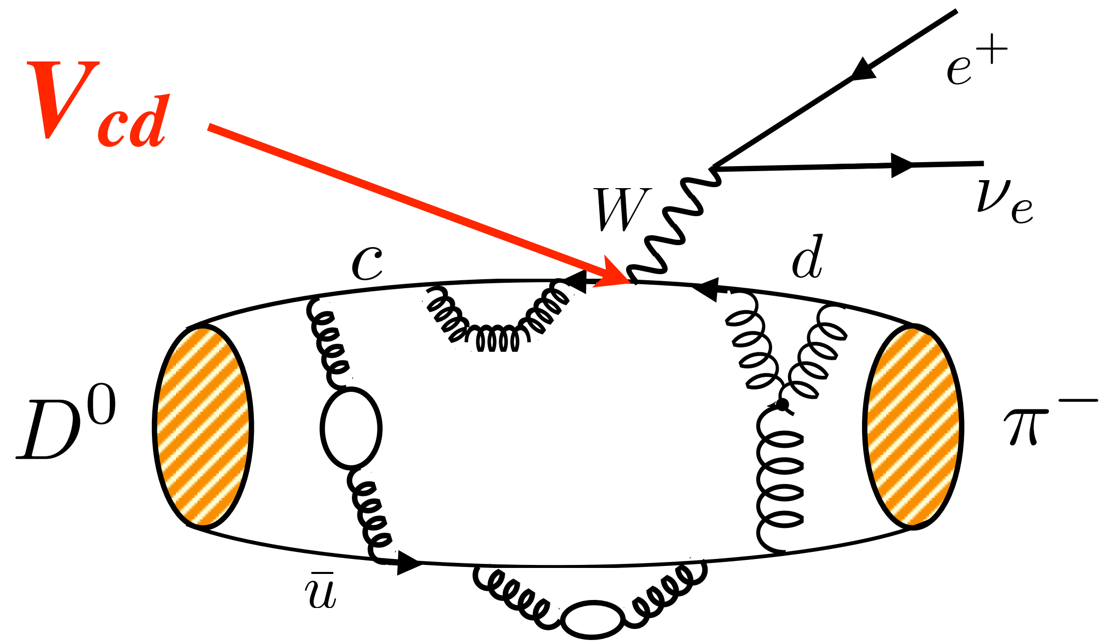
also ongoing work by:

- ALPHA/RQCD (imp. Wilson)
- FNAL/MILC (with Fermilab charm)

+ new results from ETM on $f_{D^*(s)}$ (Melis @ Lattice 2016)

Semileptonic D-meson decay

Example: $D \to \pi \ell \nu$



$$\frac{d\Gamma(D \to \pi \ell \nu)}{dq^2} = (\text{known}) \times |V_{cd}|^2 f_+^2(q^2)$$

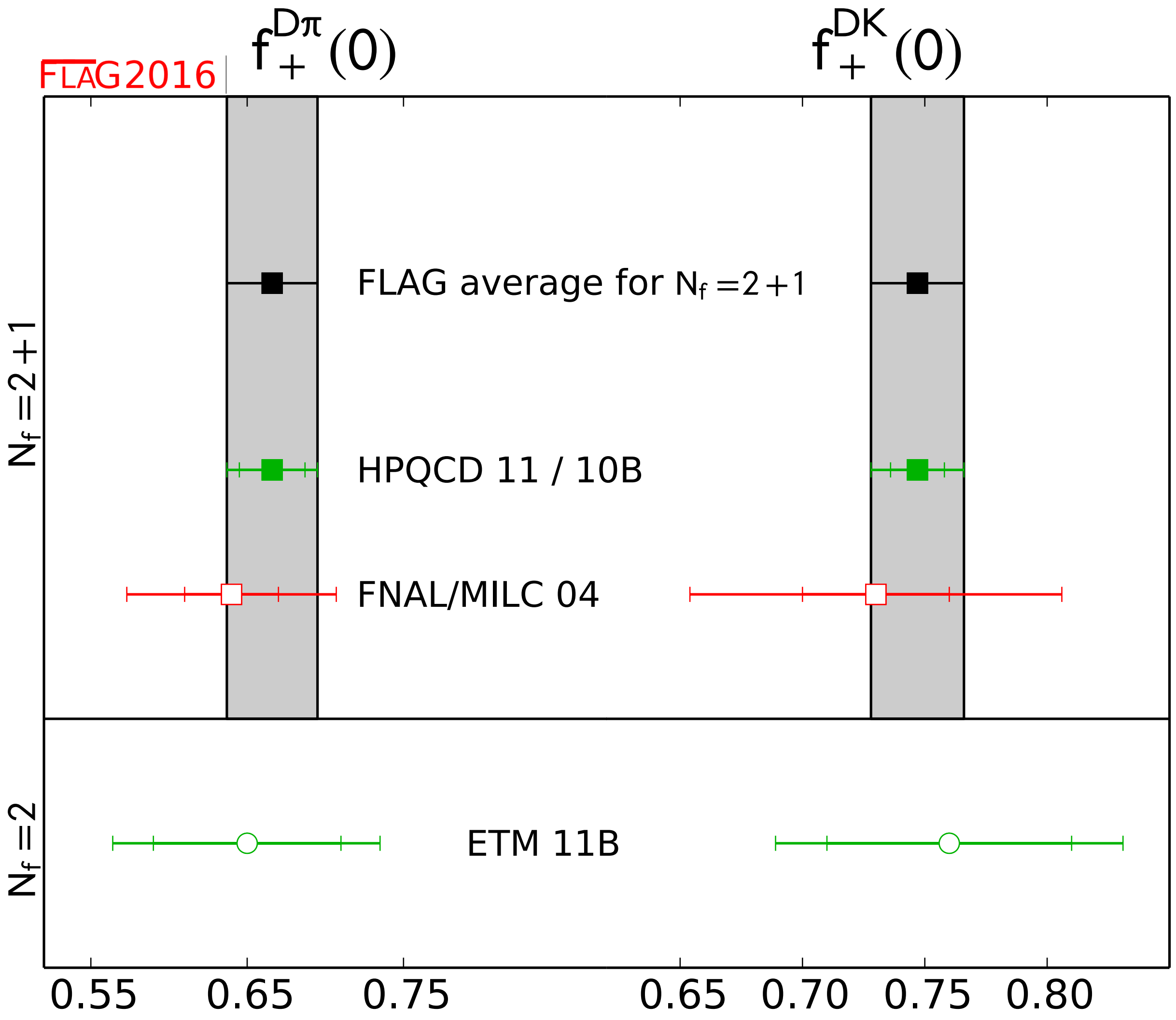
$\ell = e, \mu$

- ★ can calculate the form factors for the entire recoil energy range
- ★ can use z-expansion* for model-independent parameterization of q^2 dependence
- ★ calculate both form factors $f_+(q^2), f_0(q^2)$
- ★ can compare shape between experiment and lattice
- ★ extension to rare SL decay form factors (f_T) straightforward

*see backup slides

D SL form factor results

S. Aoki et al (FLAG review, arXiv:1607.00299)



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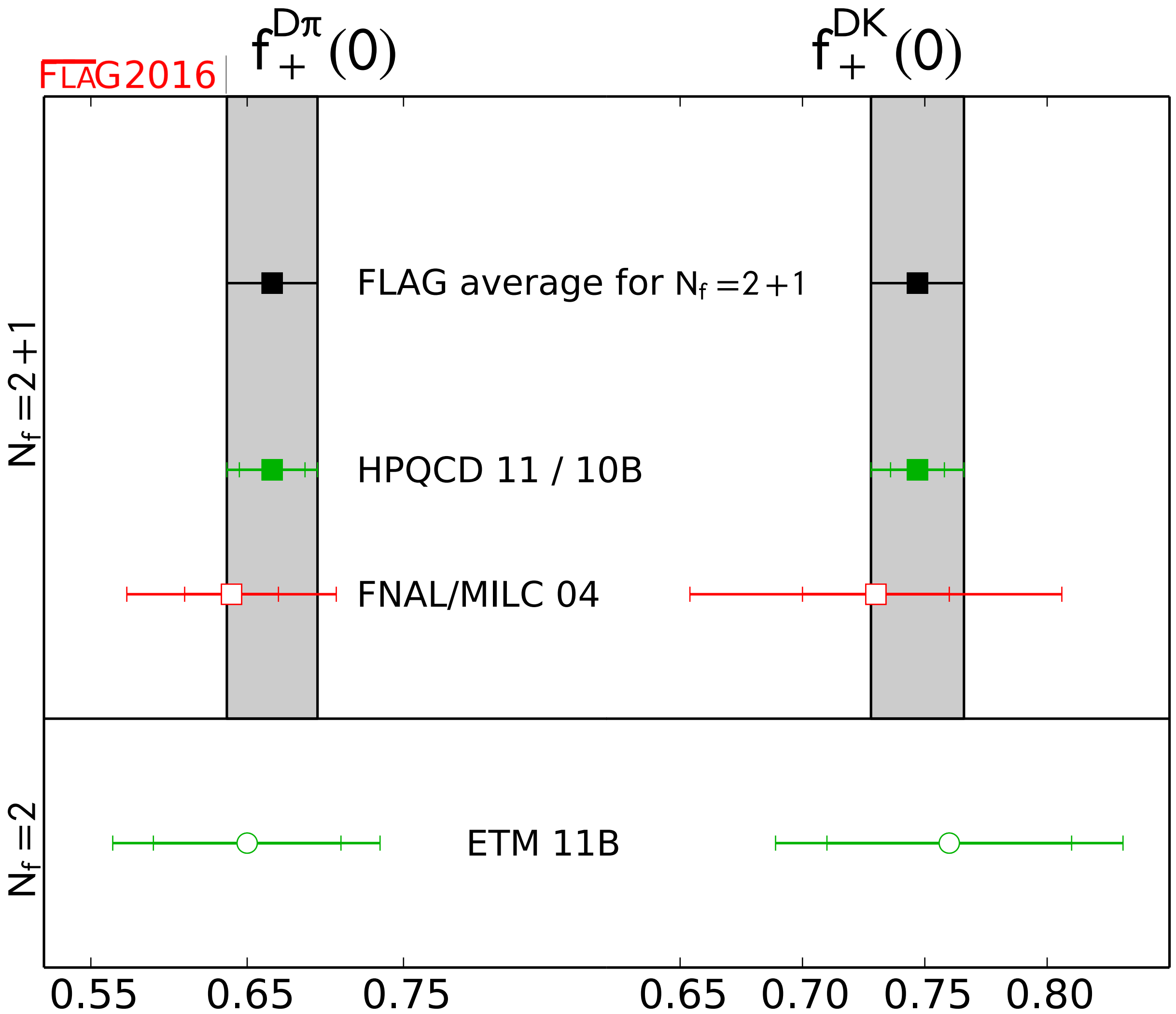
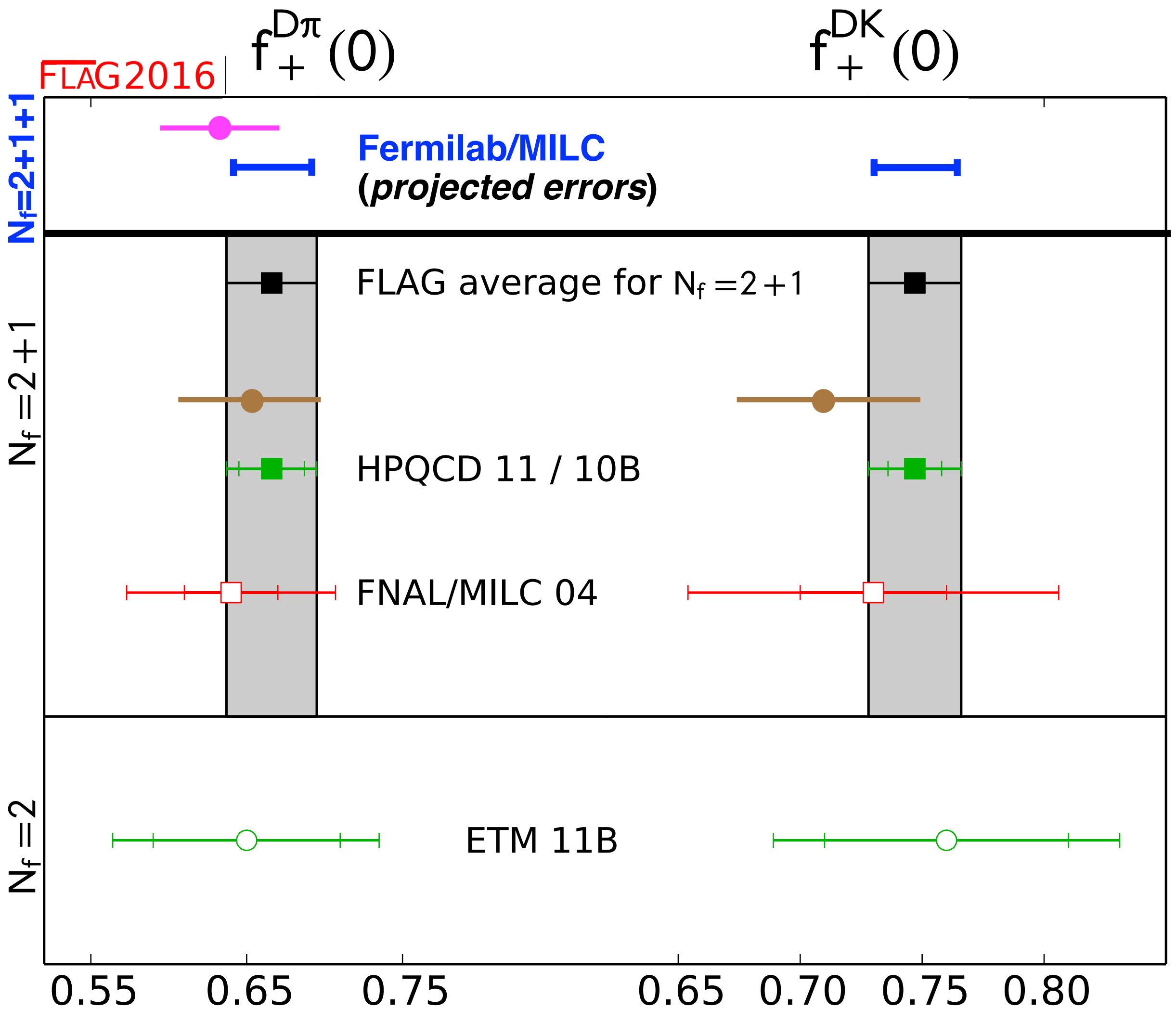


new preliminary results @ Lattice 2016:

- ETM (G. Salerno)
  2+1+1 flavors of tmWilson
  calculate all form factors over whole q^2 range
  modified z-expansion
  preliminary sys. errors
- FNAL/MILC (S. Gottlieb, T. Primer)
  no central values (yet)
  2+1+1 flavors of HISQ
  physical mass ensembles
  calculate directly at zero q^2
- JLQCD (T. Kaneko)
  2+1 flavors of DW fermions
  extrapolate to zero q^2 with z-expansion
  chiral-continuum extrapolaton
  still adding ensembles to analysis

D SL form factor results

adapted from S. Aoki et al (arXiv:1607.00299)



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ETM (G. Salerno) @ Lattice 2016

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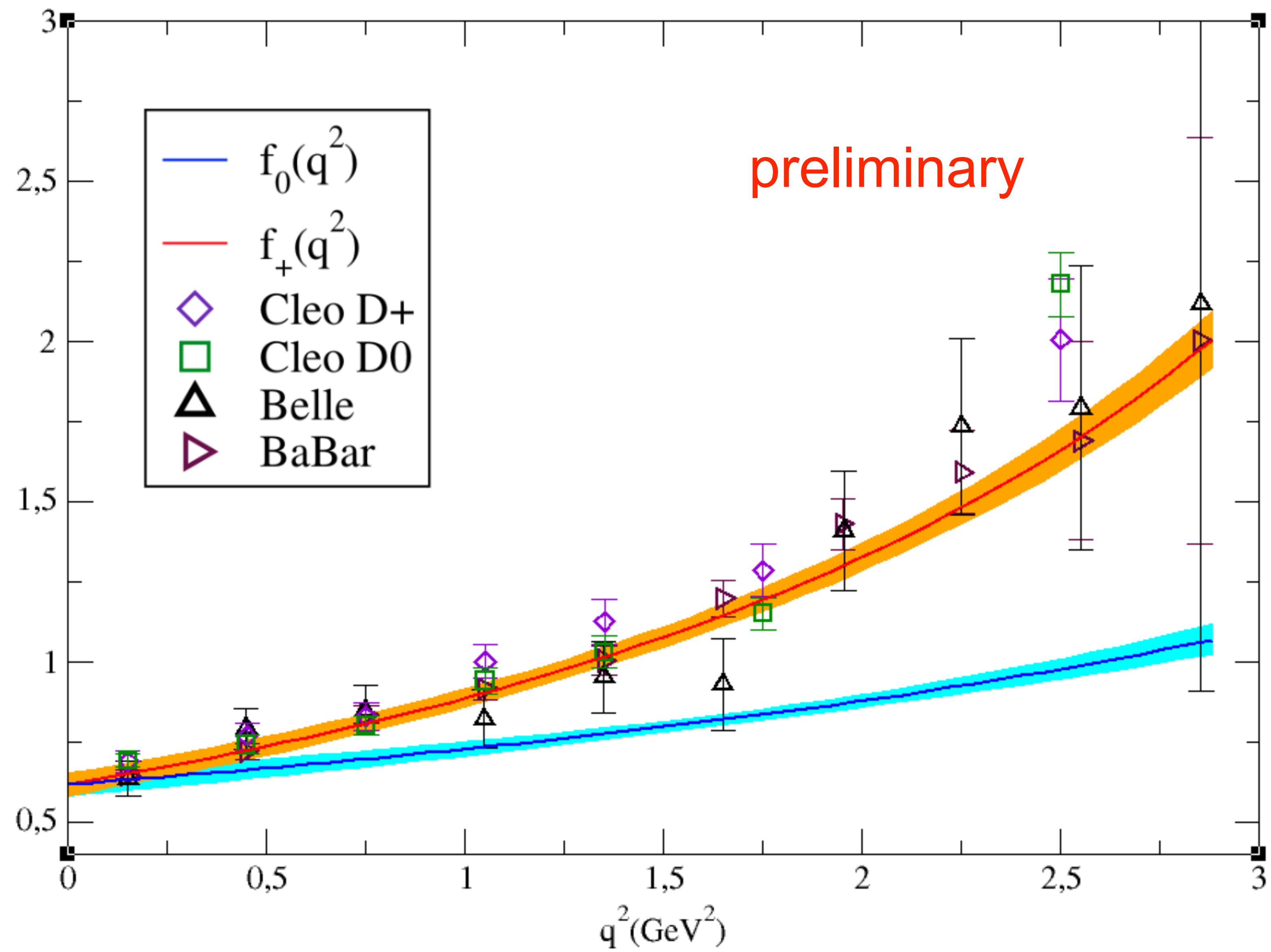
- calculate f_+, f_0 over whole q^2 range
- modified z-expansion
- correct for hypercubic discretization effects
- preliminary sys. errors

see talk by G. Salerno on Thursday

Three different values of the lattice spacing: 0.06 *fm* ÷ 0.09 *fm*

Different volumes: 2 *fm* ÷ 3 *fm*

Pion masses in range 210 ÷ 440 MeV



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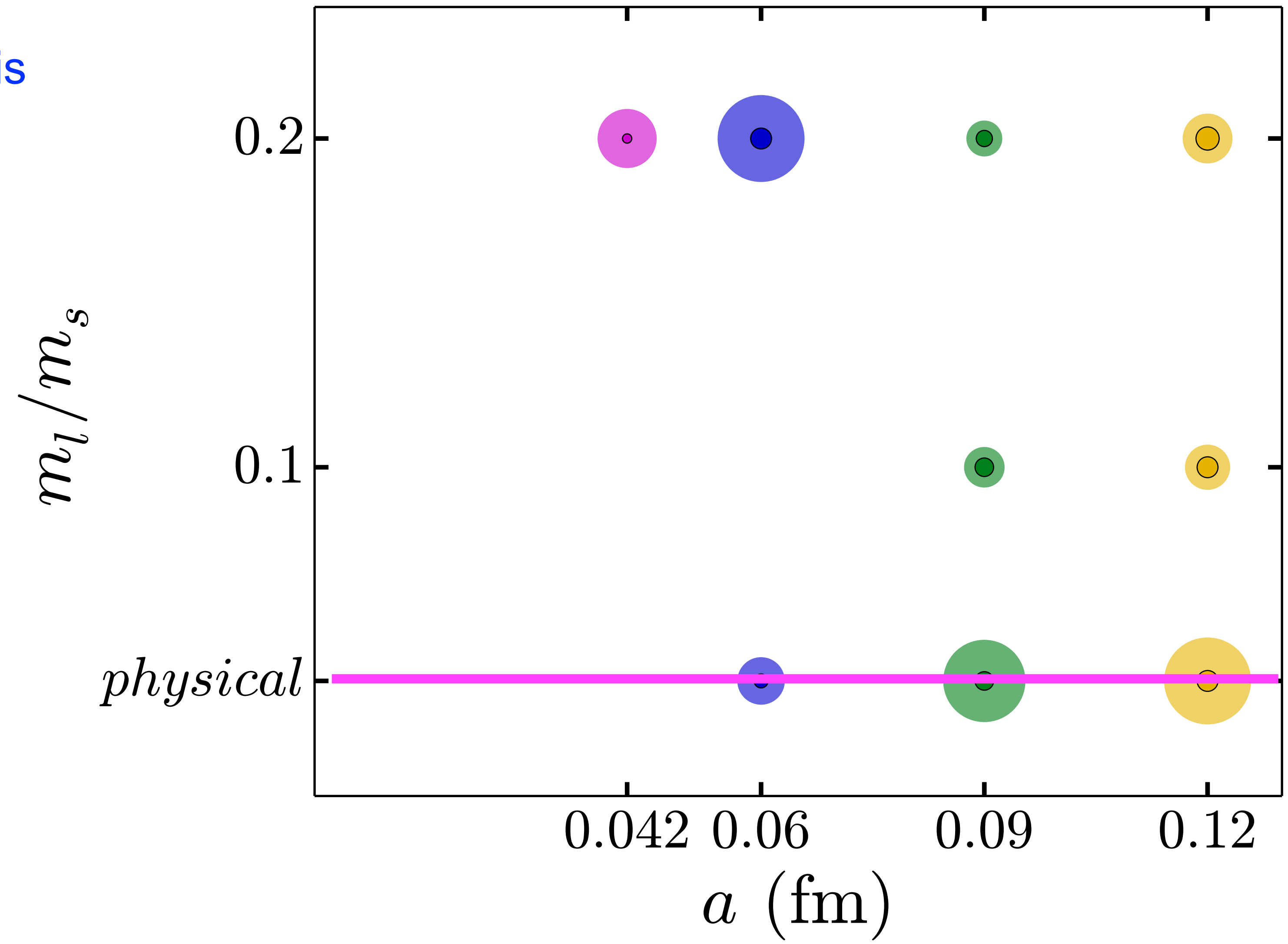
- next step:
  vector and scalar form factors
  + range of recoil momenta
  $\Rightarrow$ whole q^2 range
- will yield better precision
  and shape comparison with
  experiment



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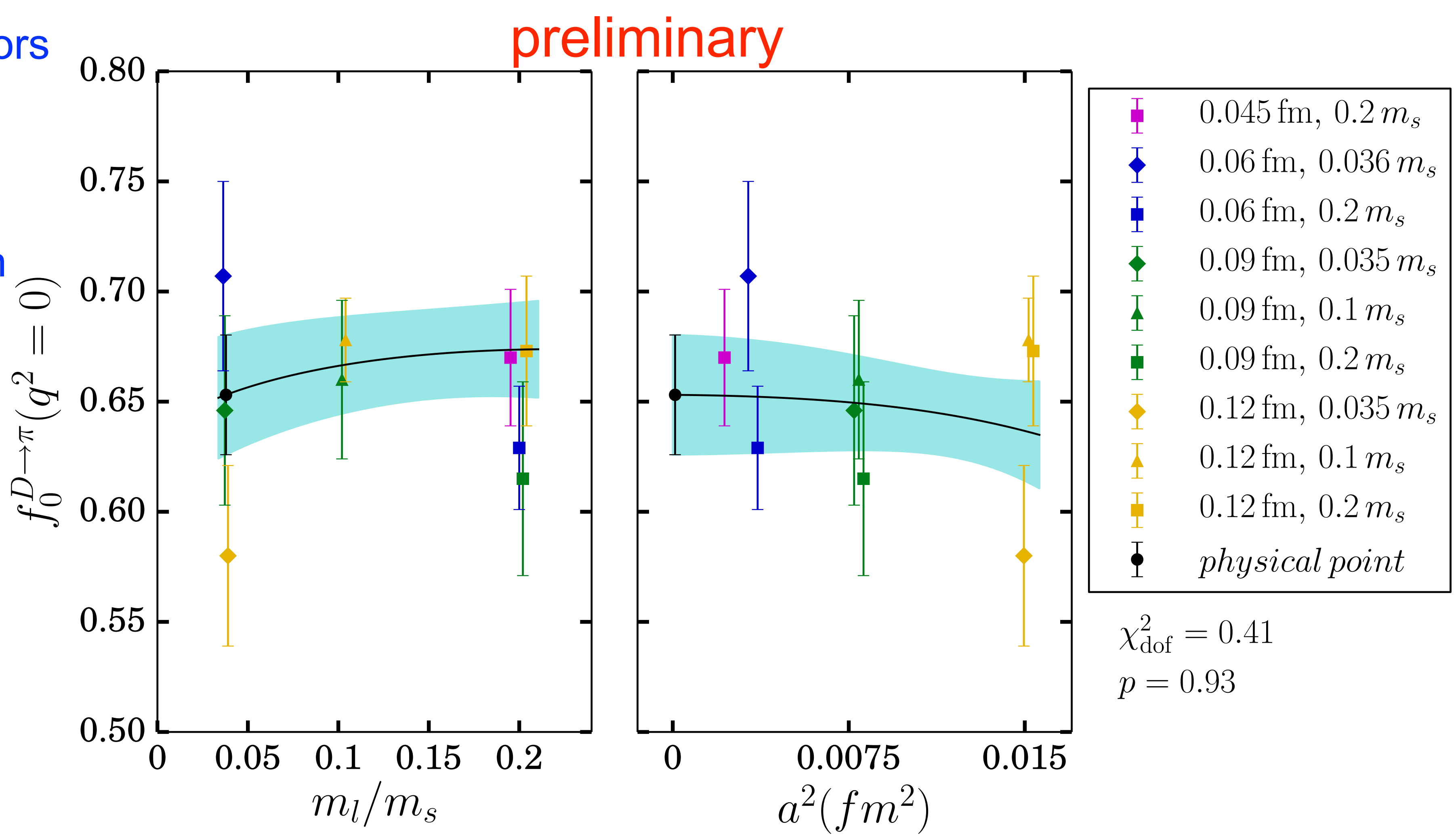

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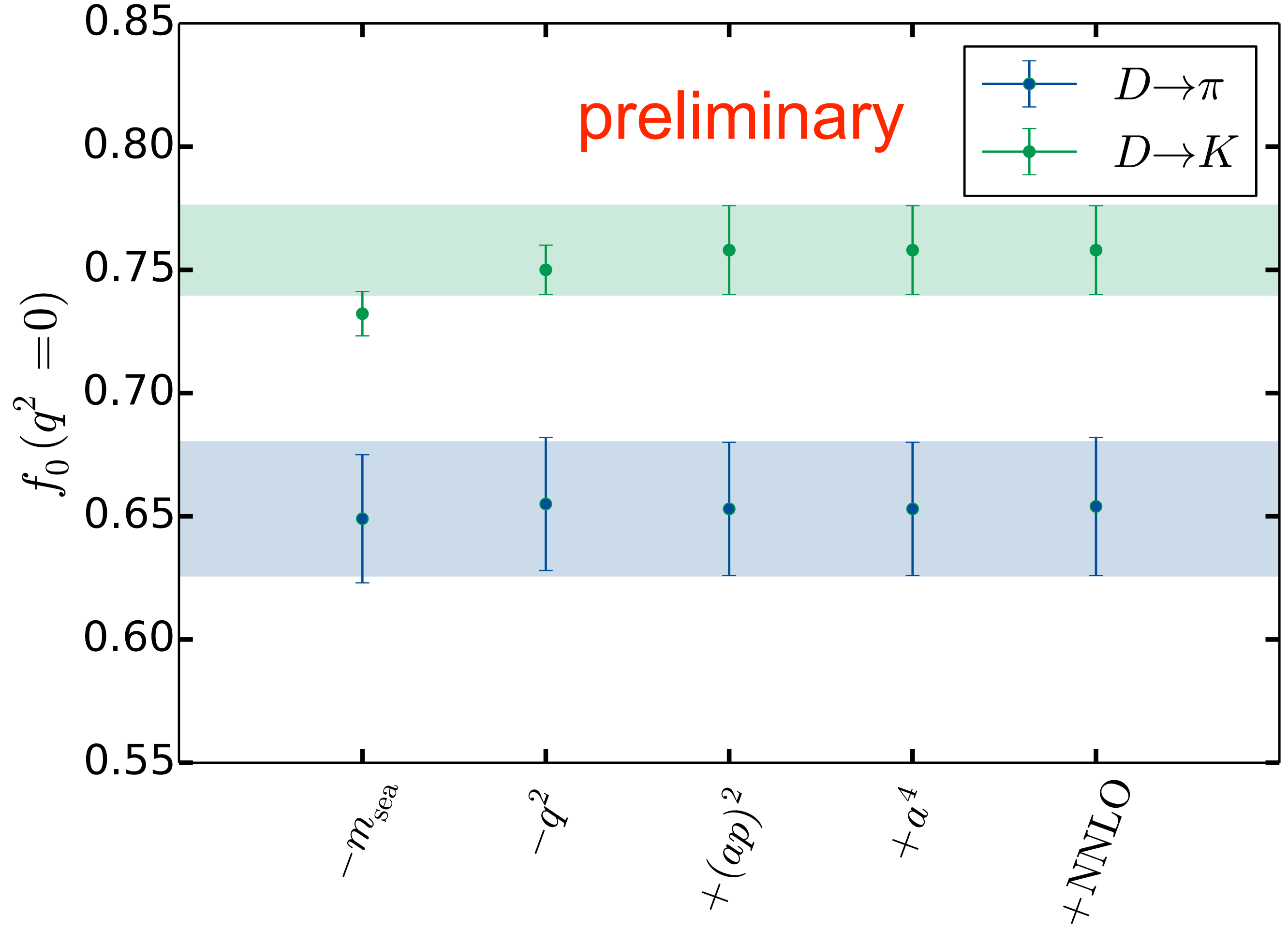

D SL form factor results

JLQCD (T. Kaneko) @ Lattice 2016

- 2+1 flavors of DW fermions
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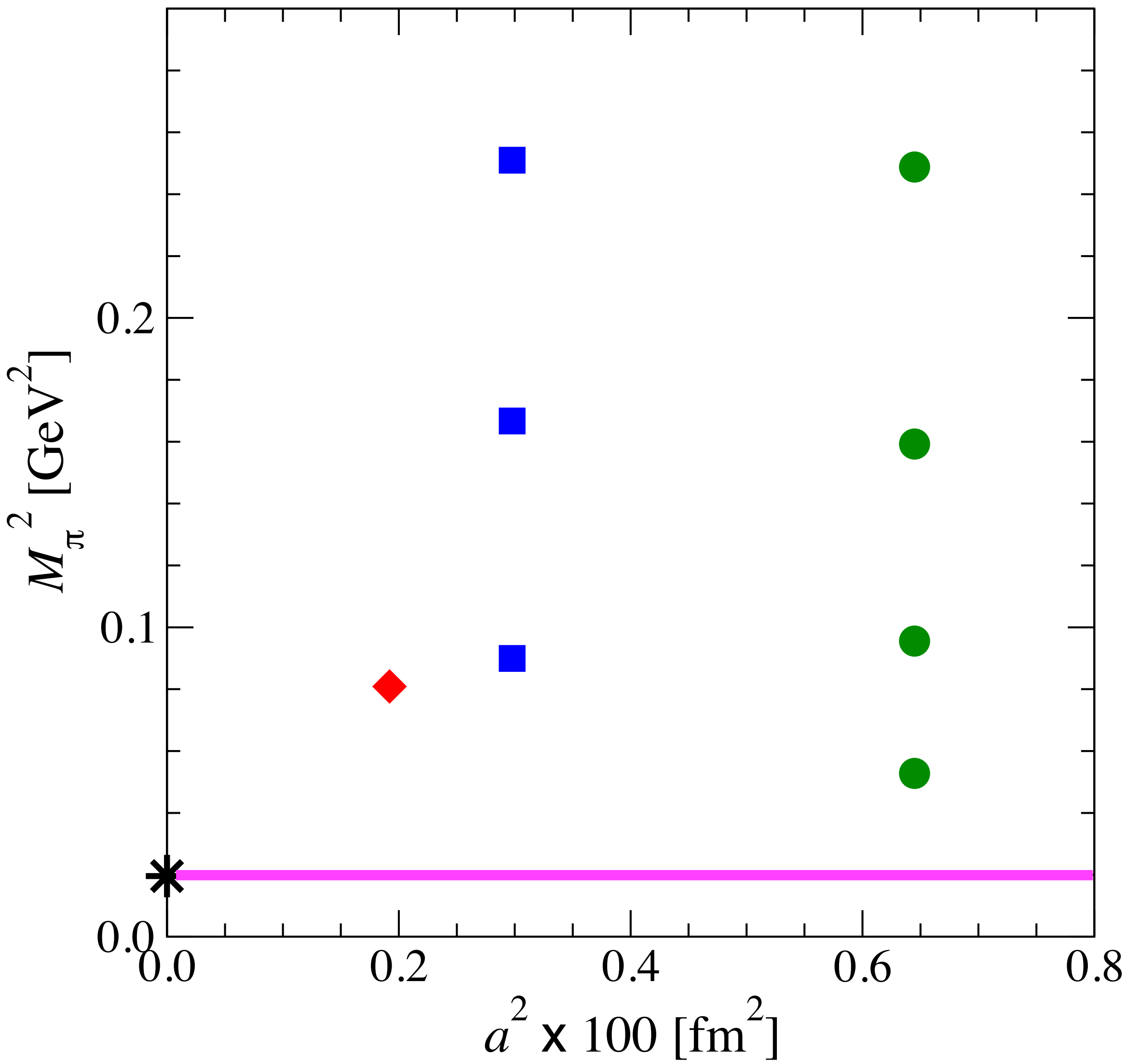
- chiral-continuum extrapolaton
- still adding ensembles to analysis

preliminary



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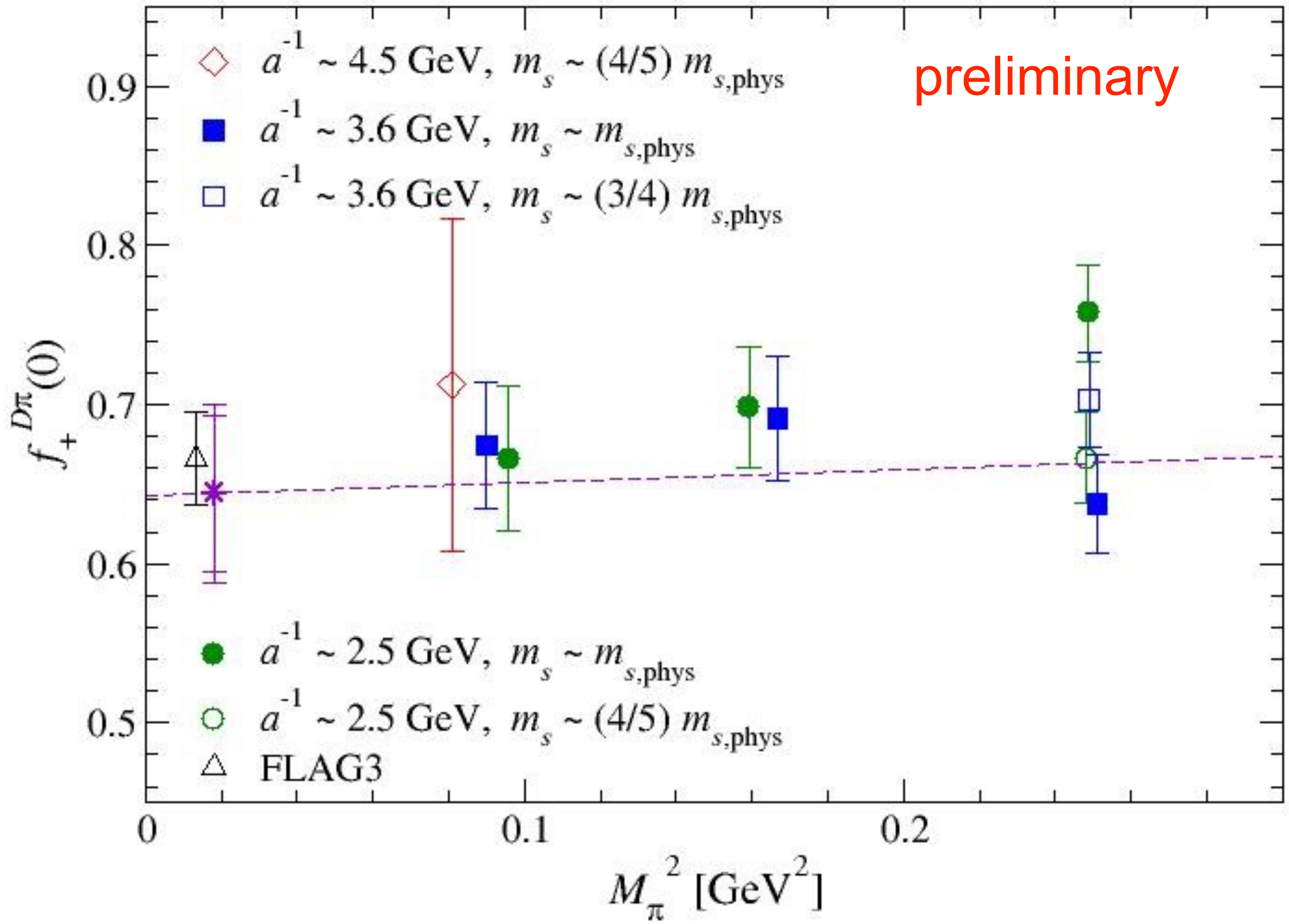
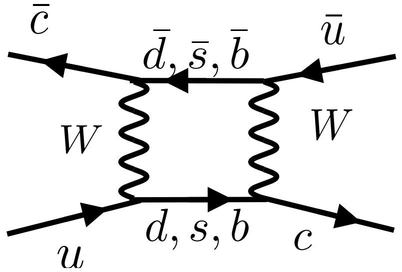
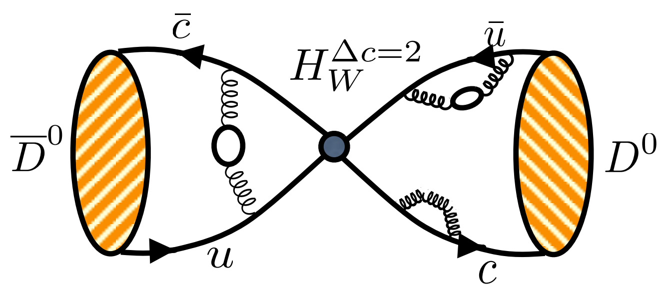
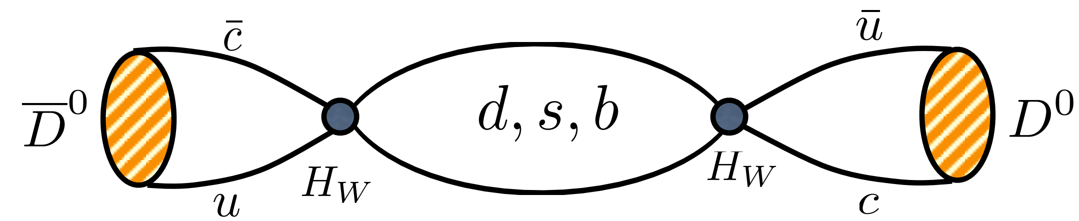


Neutral D meson mixing

$$M_{12} - \frac{i}{2}\Gamma_{12} \;\propto\; \langle D^0 | H_W^{\Delta c=2} | \overline{D}^0 \rangle \;+\; \sum_n \frac{\langle D^0 | H_W^{\Delta c=1} | n \rangle \langle n | H_W^{\Delta c=1} | \overline{D}^0 \rangle}{M_D - E_n + i\epsilon}$$







short distance

long distance

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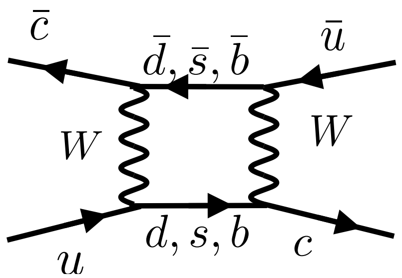

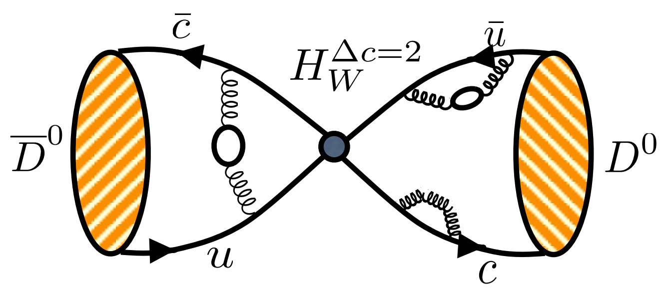

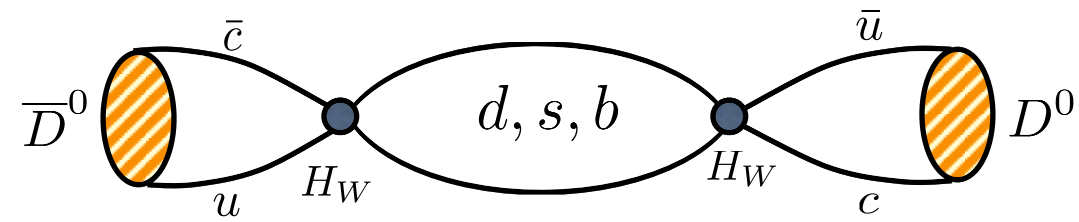

short distance

long distance

“Simple”

- can use the same methods as for B mixing (and decay constants, form factors)
- BSMs with heavy new particles can contribute here

“Hard”

- large contribution
- intermediate state can include multiple (>2) hadrons:
  - ✦ formalism for multi-hadron states still under development (Hansen & Sharpe, arXiv:1602.00324, 2016 PRD)
  - ✦ not a problem for Kaon mixing
    - ➠ first calculation of long-distance contribution already exists (RBC/UKCD, arXiv:1406.0916, 2014 PRL)

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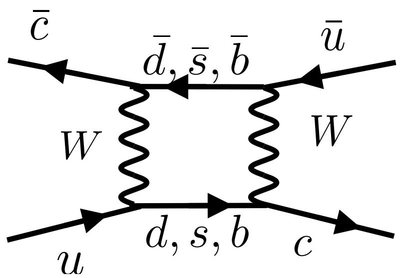

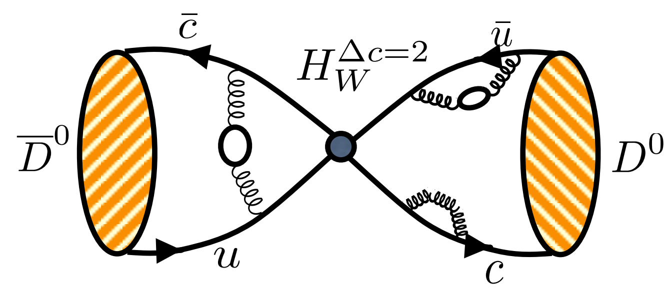

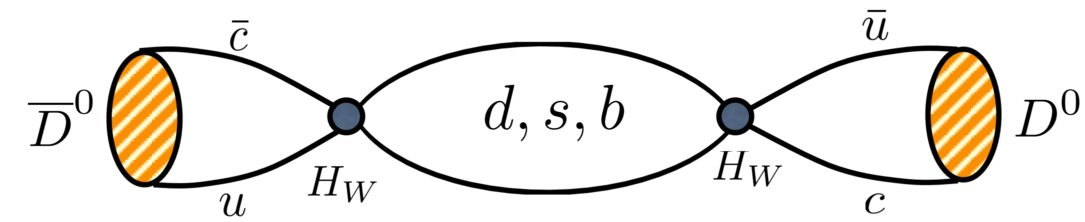

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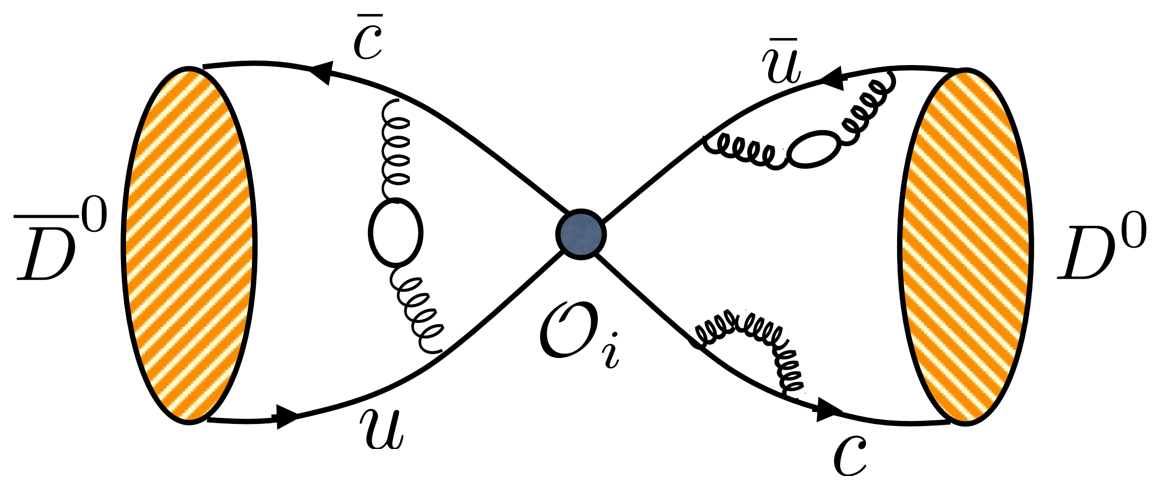


In the SM and beyond:

$$\mathcal{H}_{\rm eff} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_1 = \bar{c}\gamma^\mu L u\,\bar{c}\gamma^\mu L u$$
$$\mathcal{O}_2 = \bar{c}Lu\,\bar{c}Lu$$
$$\mathcal{O}_3 = \bar{c}^\alpha L u^\beta\,\bar{c}^\beta L u^\alpha$$
$$\mathcal{O}_4 = \bar{c}Lu\,\bar{c}Ru$$
$$\mathcal{O}_5 = \bar{c}^\alpha L u^\beta\,\bar{c}^\beta R u^\alpha c$$

$$\langle \mathcal{O}_i \rangle \equiv \langle D^0 | \mathcal{O}_i | \overline{D}^0 \rangle(\mu) = e_i M_D^2 f_D^2 B_D^{(i)}(\mu)$$

choose $\mu = 3$ GeV

- calculate the matrix elements of all five local operators.

D mixing results in comparison

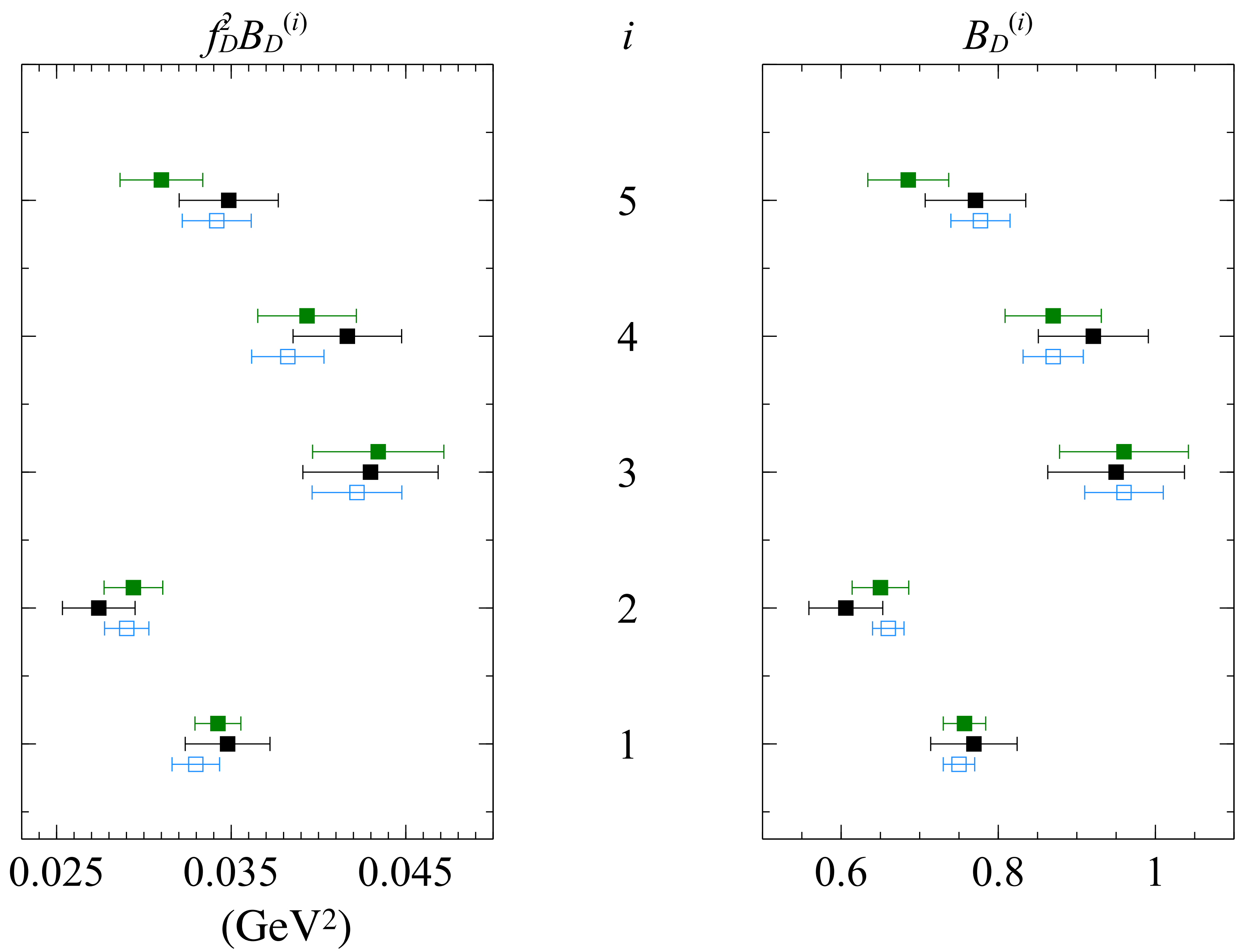
$\mu = 3$ GeV

A. Kronfeld @ Lattice 2016 (plot by C.C. Chang)

- ETM:
  $n_f = 2+1+1$
  arXiv:1505.06639

- Fermilab/MILC:
  $n_f = 2+1$

- ETM:
  $n_f = 2$
  arXiv:1403.7302

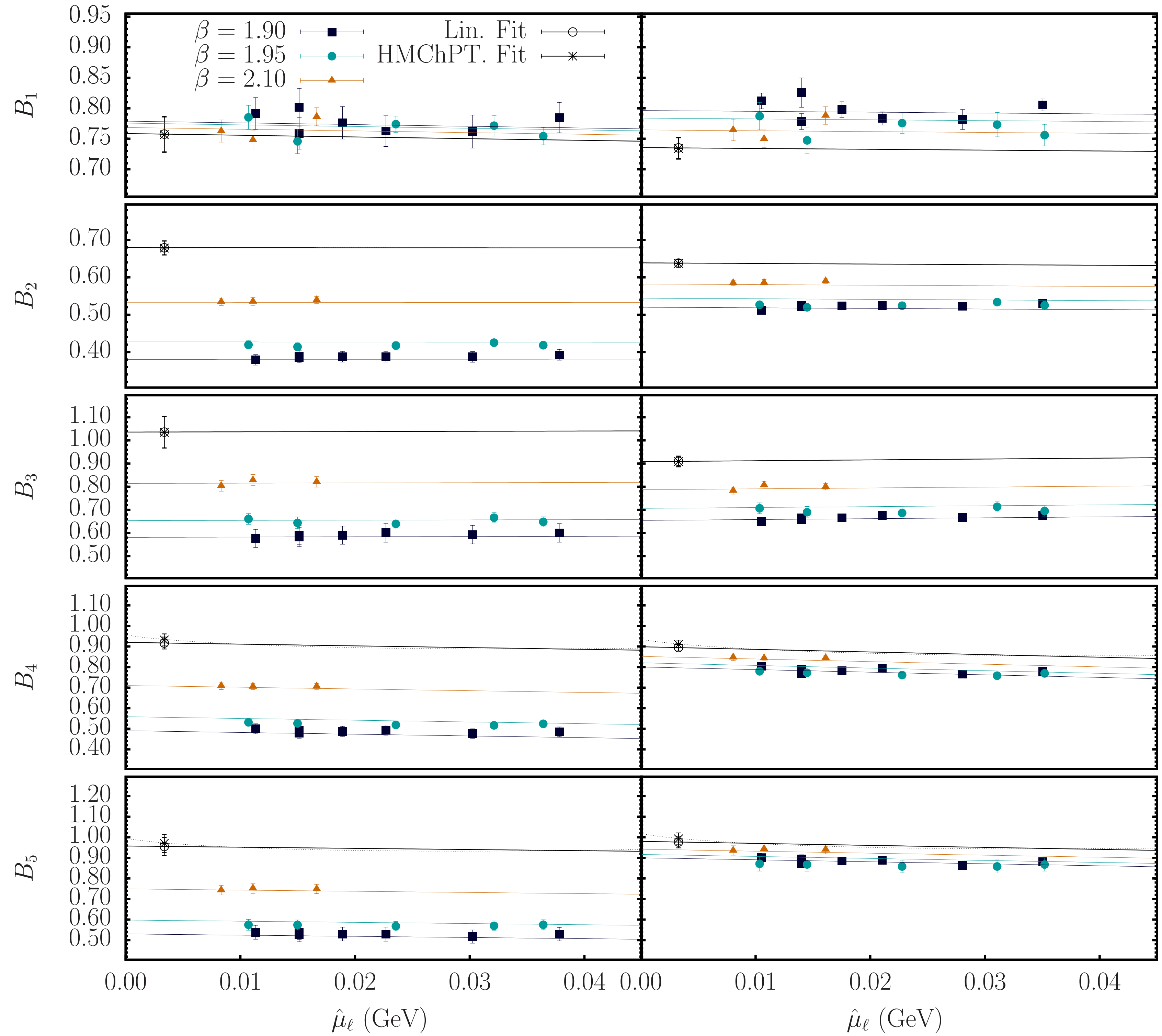


D mixing results

ETM

$n_f = 2+1+1$ tmWilson

(arXiv:1505.06639, 2015 PRD)



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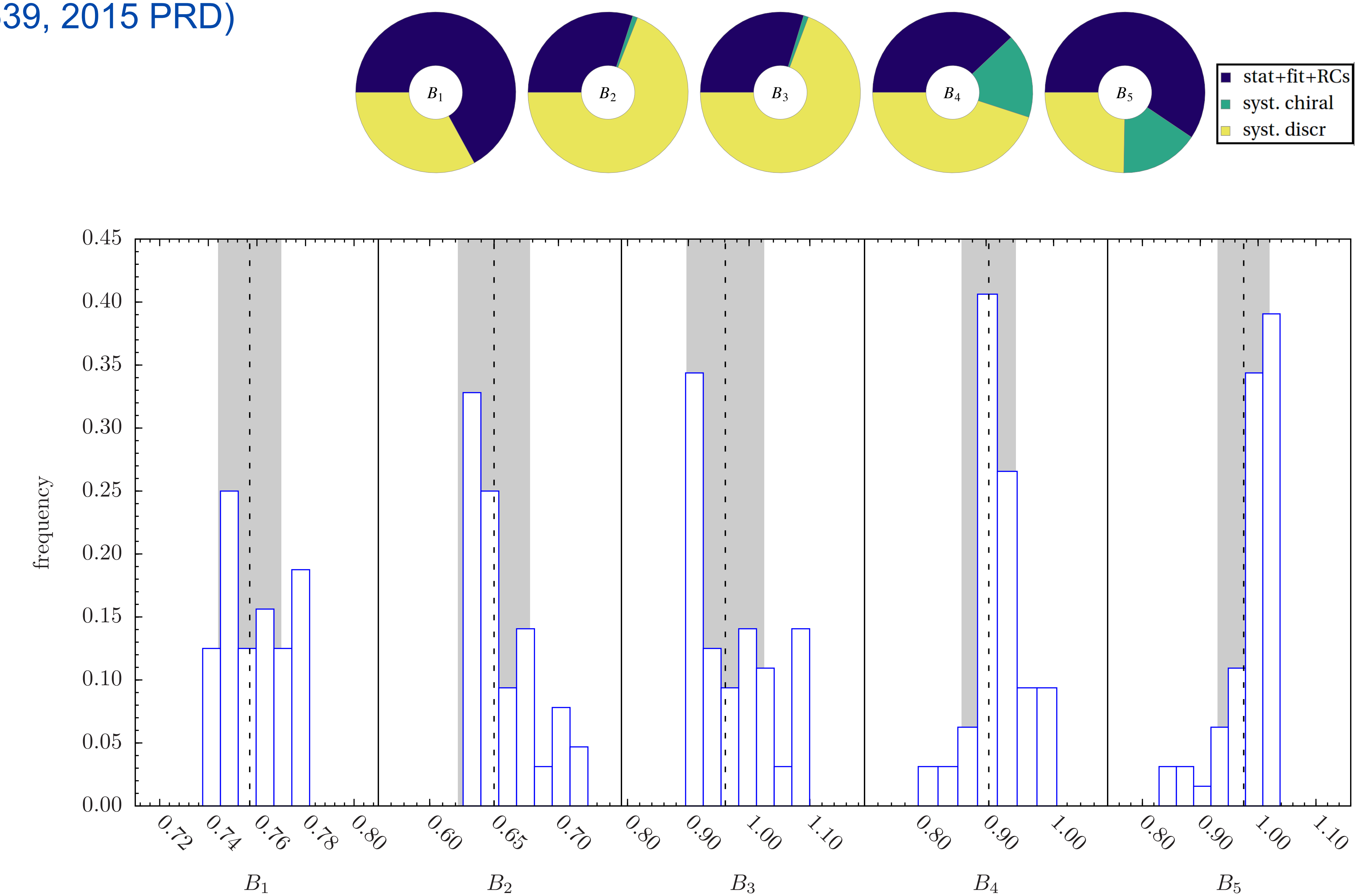
$n_f = 2+1+1$ tmWilson

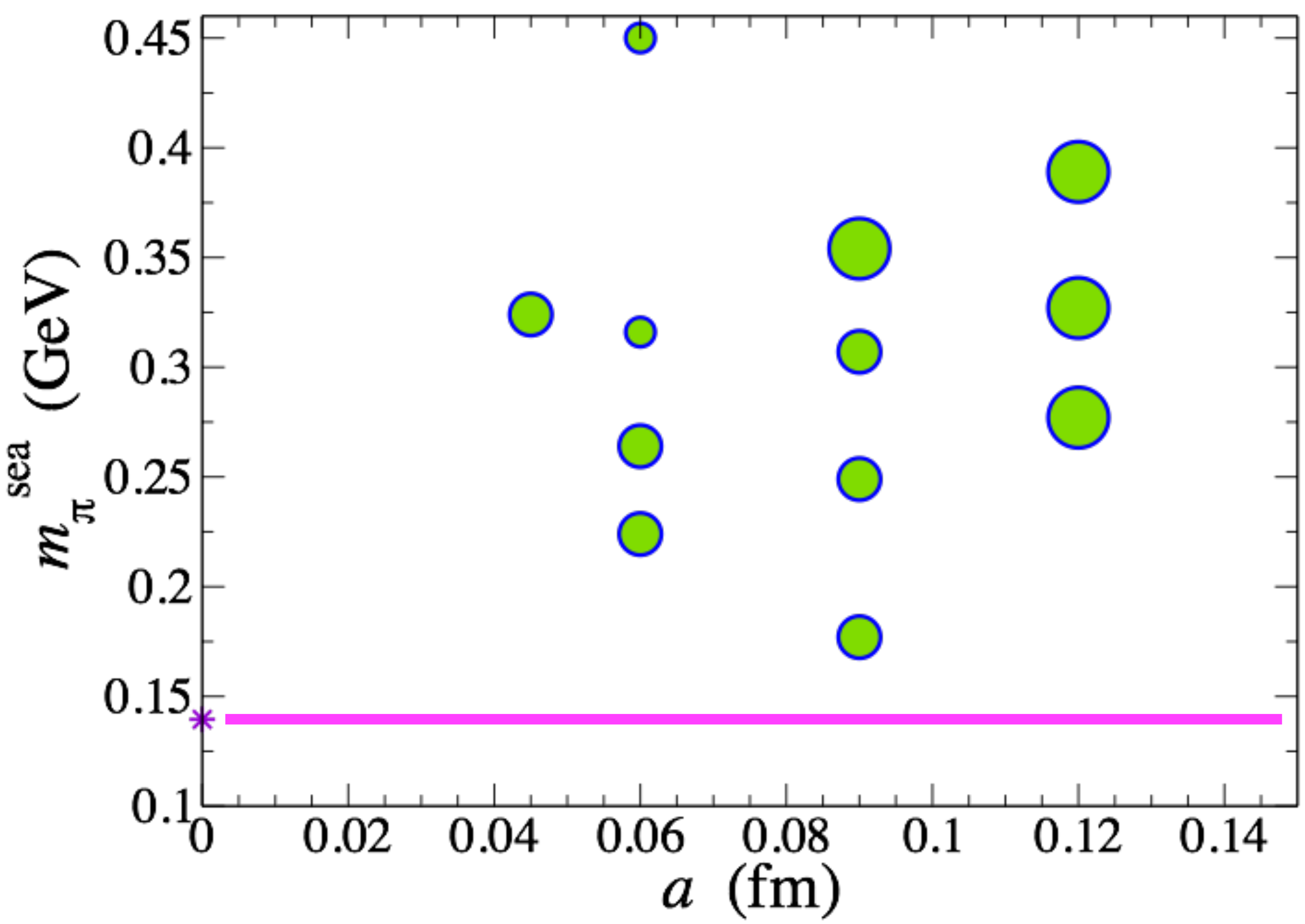
(arXiv:1505.06639, 2015 PRD)



D mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

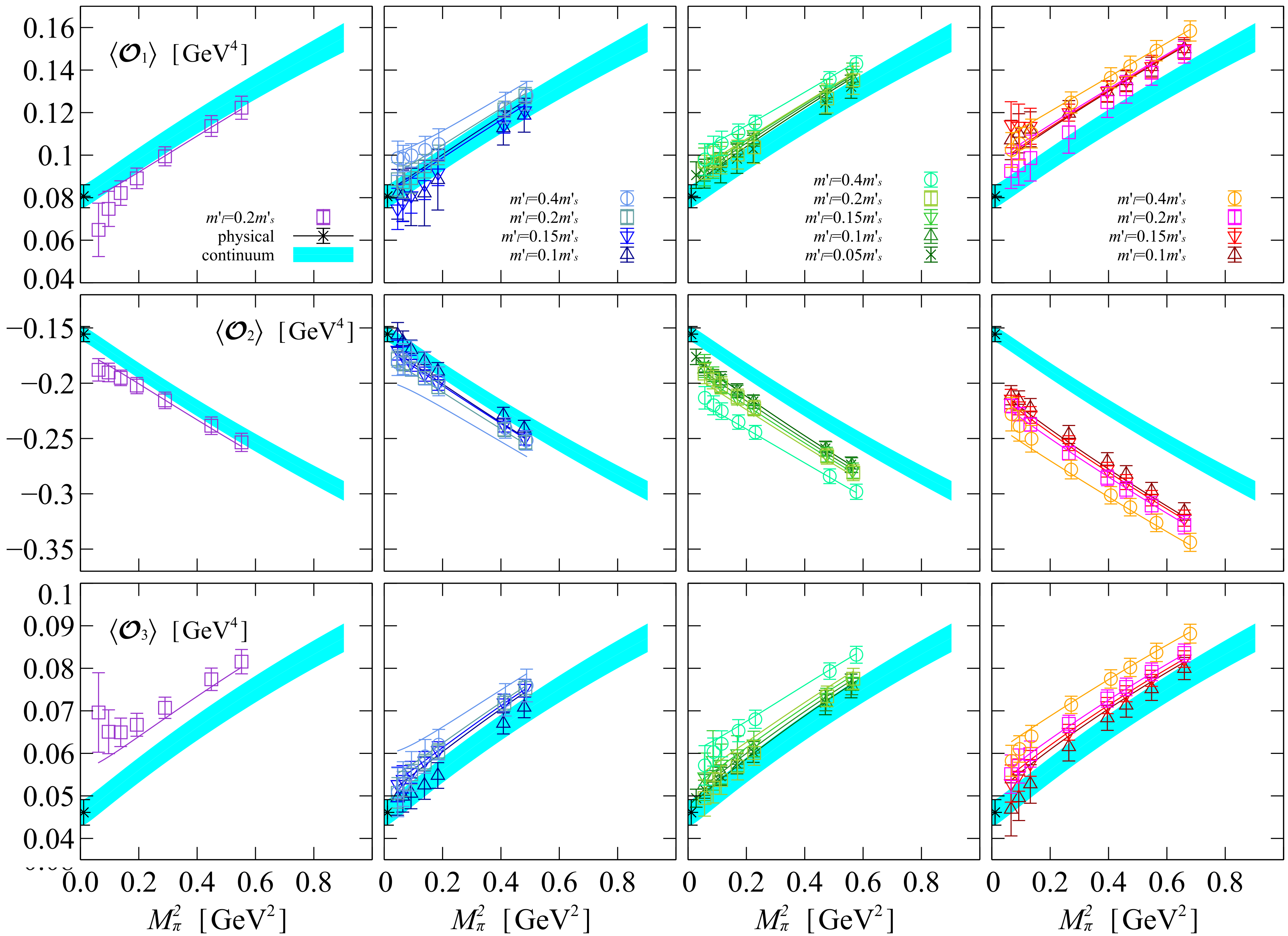
- 14 MILC asqtad ensembles
  4 lattice spacings
  ~ 4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  $\times\, 4$ time-sources per configuration
- Fermilab c quarks
- mNPR renormalization



D mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

Operators 1,2,3

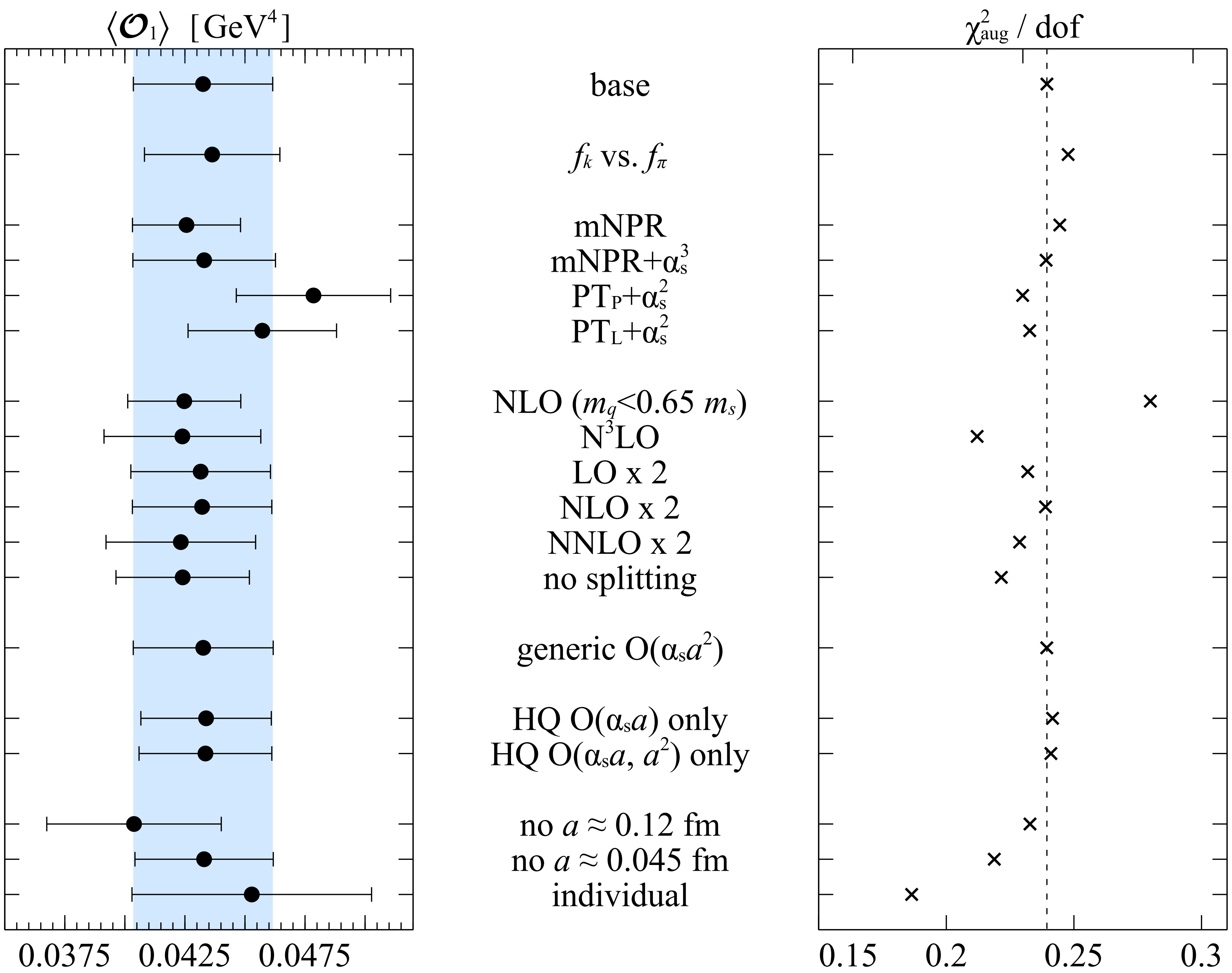


D mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

systematic error study

- remove or add higher order terms in fit function:
  - ✦ chiral expansion
  - ✦ heavy meson expansion
  - ✦ light quark discretization effects
  - ✦ HQ discretization effects
  - ✦ renormalization (perturbative expansion)
- change data included
- change inputs



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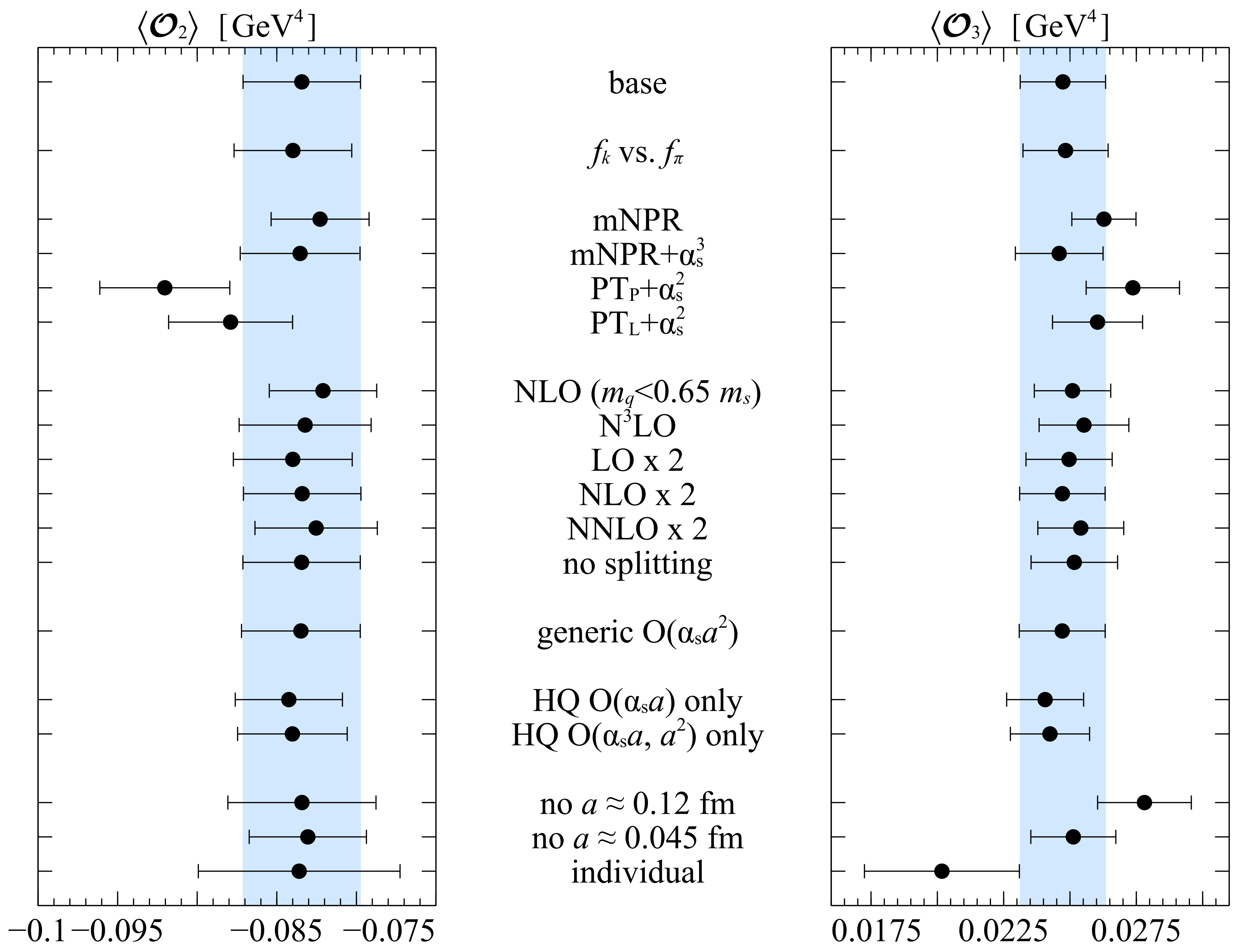

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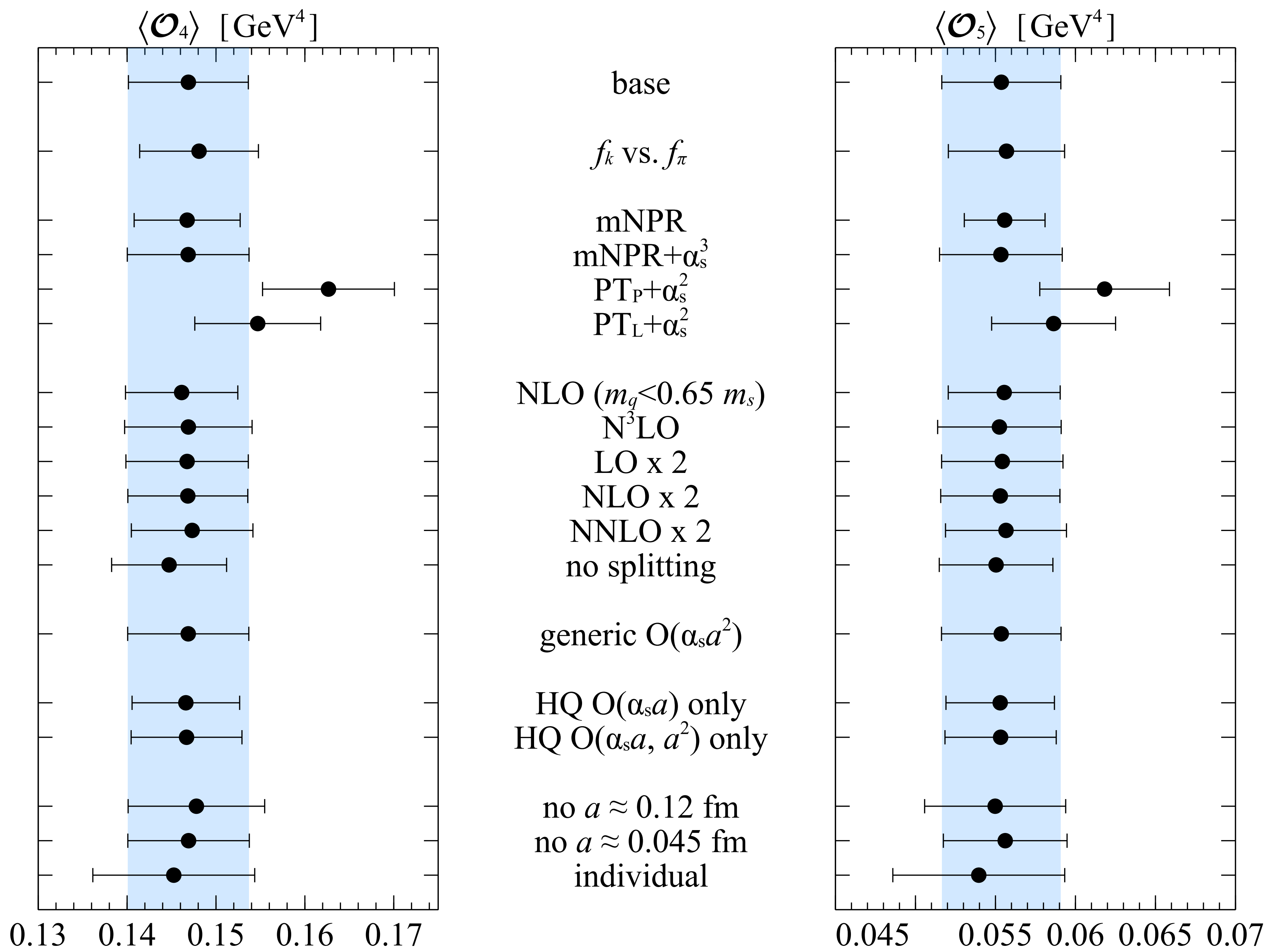
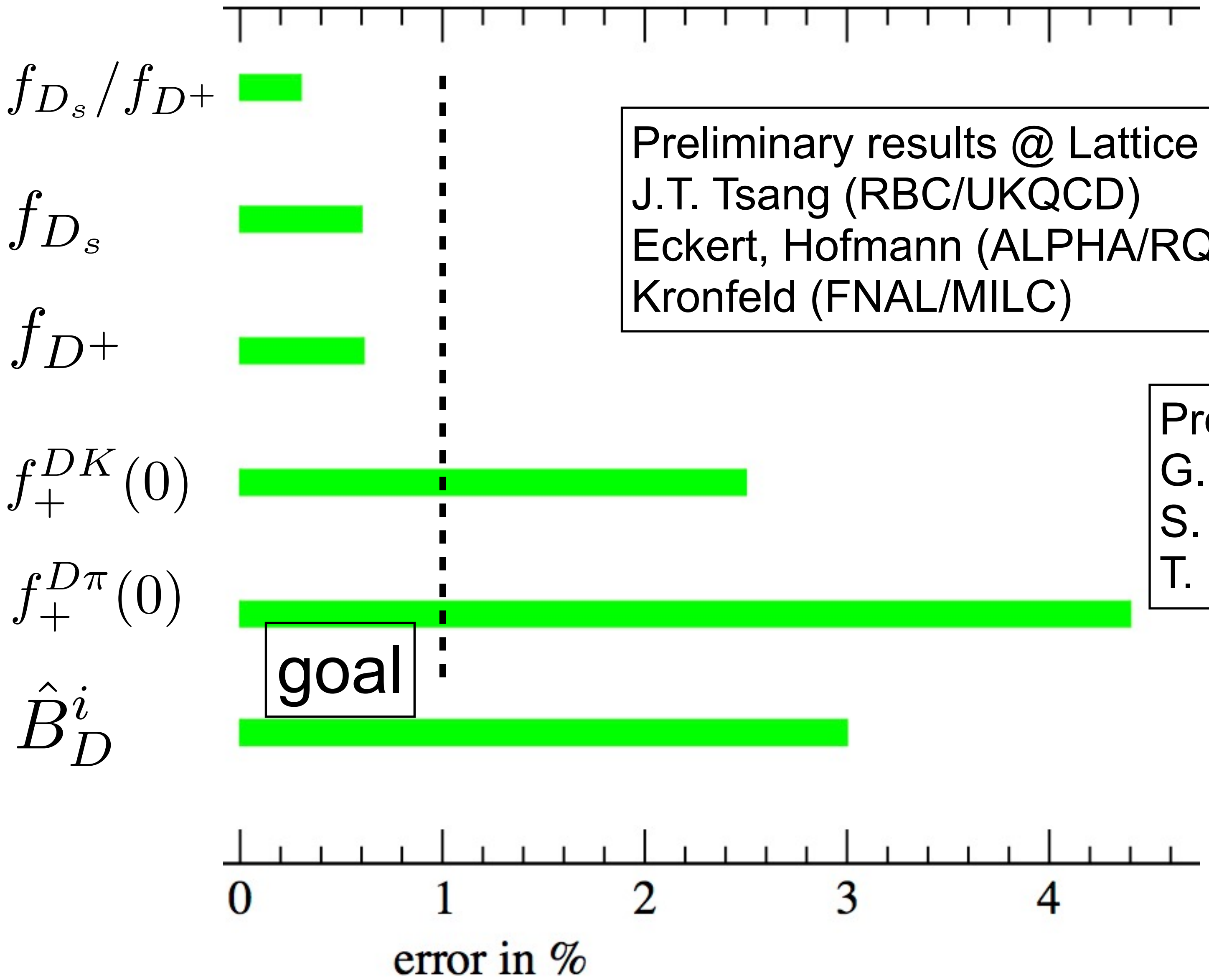


D meson summary

errors (in %) comparison:



Preliminary results @ Lattice 2016:
J.T. Tsang (RBC/UKQCD)
Eckert, Hofmann (ALPHA/RQCD)
Kronfeld (FNAL/MILC)

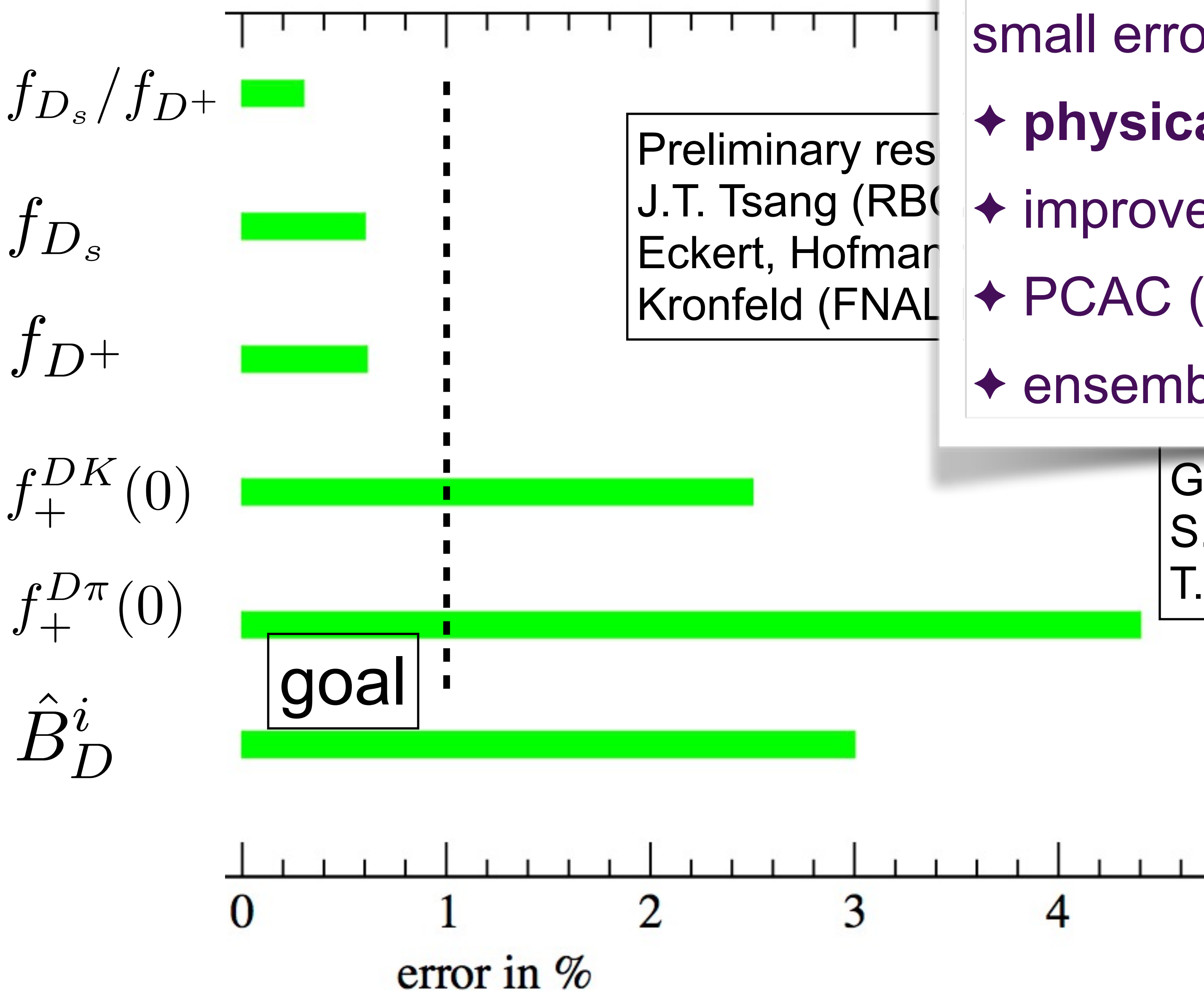
Preliminary results @ Lattice 2016:
G. Salerno (ETM)
S. Gottlieb, T. Primer (FNAL/MILC)
T. Kaneko (JLQCD)

D mixing MEs for all 5 local operators

- $n_f = 2$, 2+1+1 ETM (2013, 2014)
- $n_f = 2$+1 FNAL/MILC (Kronfeld, Chang, Lattice 2016)

D meson summary

errors (in %) comparison:



Preliminary res
J.T. Tsang (RB
Eckert, Hofmar
Kronfeld (FNAL

small errors due to

- ✦ **physical light quark masses**
- ✦ improved charm-quark action (HISQ)
- ✦ PCAC (no renormalization)
- ✦ ensembles with small lattice spacings

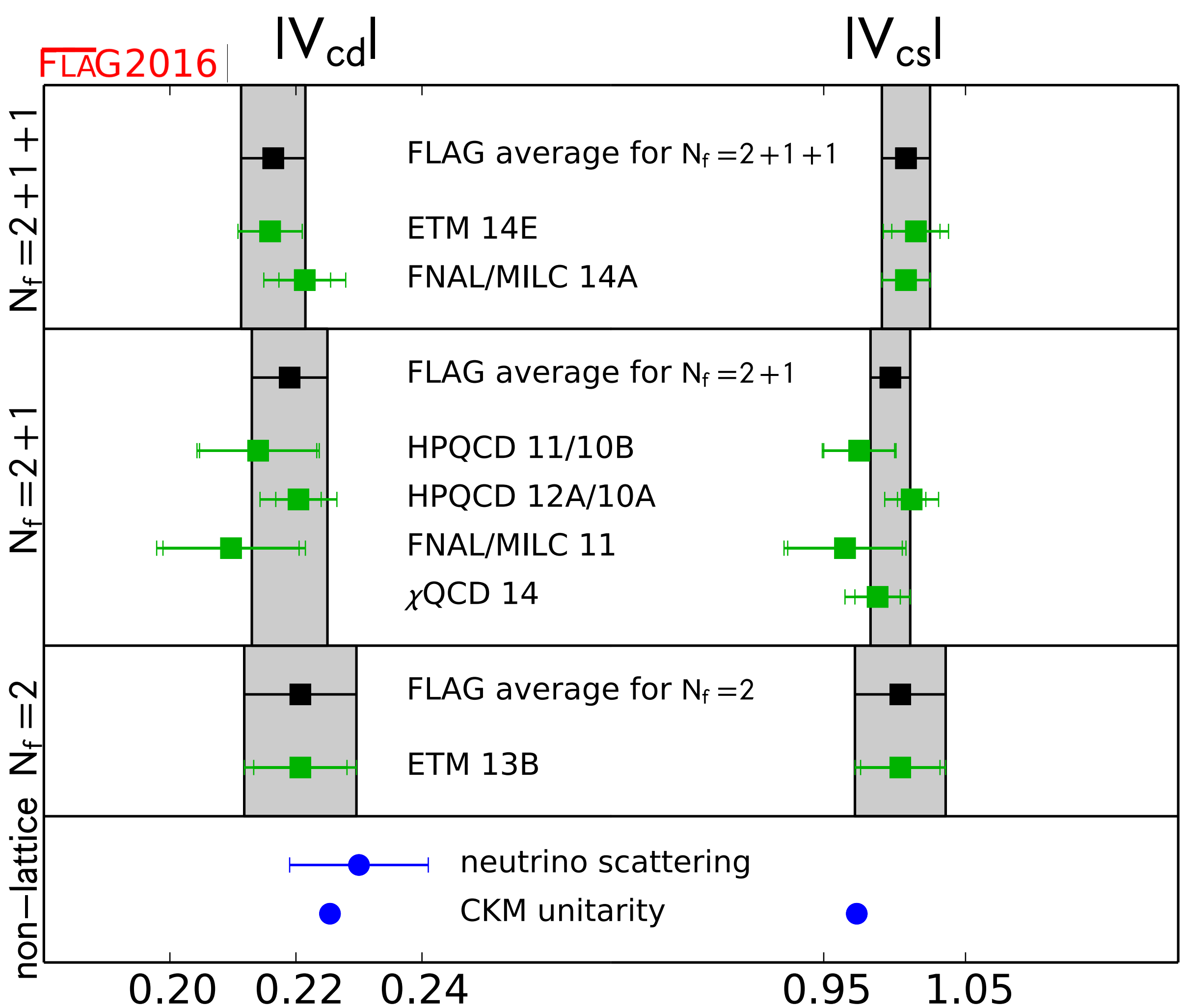
G. Salerno (ETM)
S. Gottlieb, T. Primer (FNAL/MILC)
T. Kaneko (JLQCD)

D **mixing MEs for all 5 local operators**

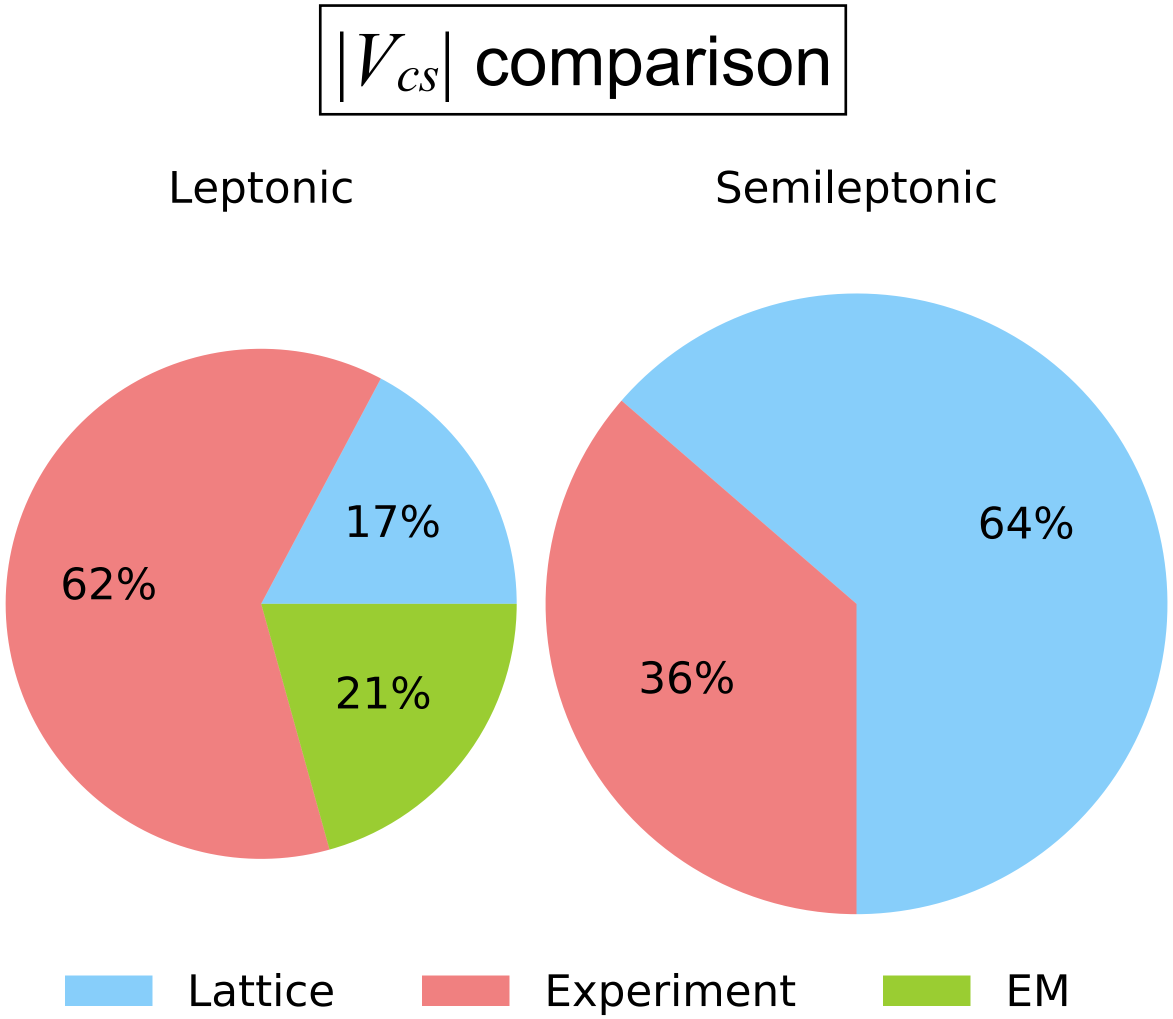
- $n_f = 2$, 2+1+1 ETM (2013, 2014)
- $n_f = 2+1$ FNAL/MILC (Kronfeld, Chang, Lattice 2016)

Implications for $|V_{cs}|$, $|V_{cd}|$

S. Aoki et al (FLAG review, arXiv:1607.00299)

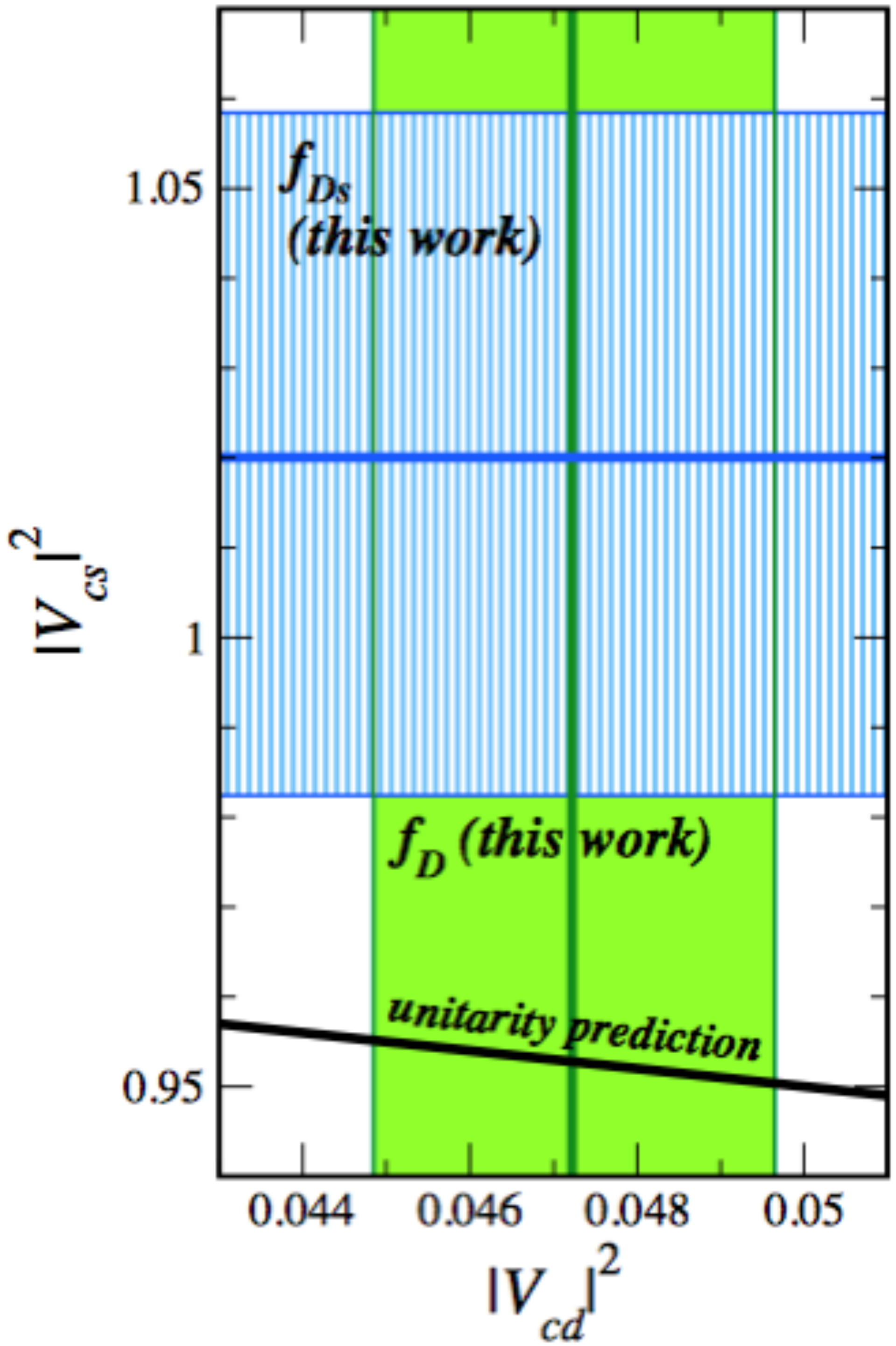


S. Gottlieb, T. Primer (FNAL/MILC) @ Lattice 2016



Implications for the 2nd row of the CKM Matrix

FNAL/MILC (arXiv:1407.3772, 2014 PRD)



errors on $|V_{cs}|$ and $|V_{cd}|$ are dominated by experiment (PDG 2015, arXiv:509.02220):

$$|V_{cd}| = 0.217\ (1)_{\rm LQCD}\ (5)_{\rm exp}$$
$$|V_{cs}| = 1.007\ (4)_{\rm LQCD}\ (16)_{\rm exp}$$

(based on the PDG average of 2+1 & 2+1+1 flavor LQCD results; average is dominated by FNAL/MILC)

2σ tension with unitarity:

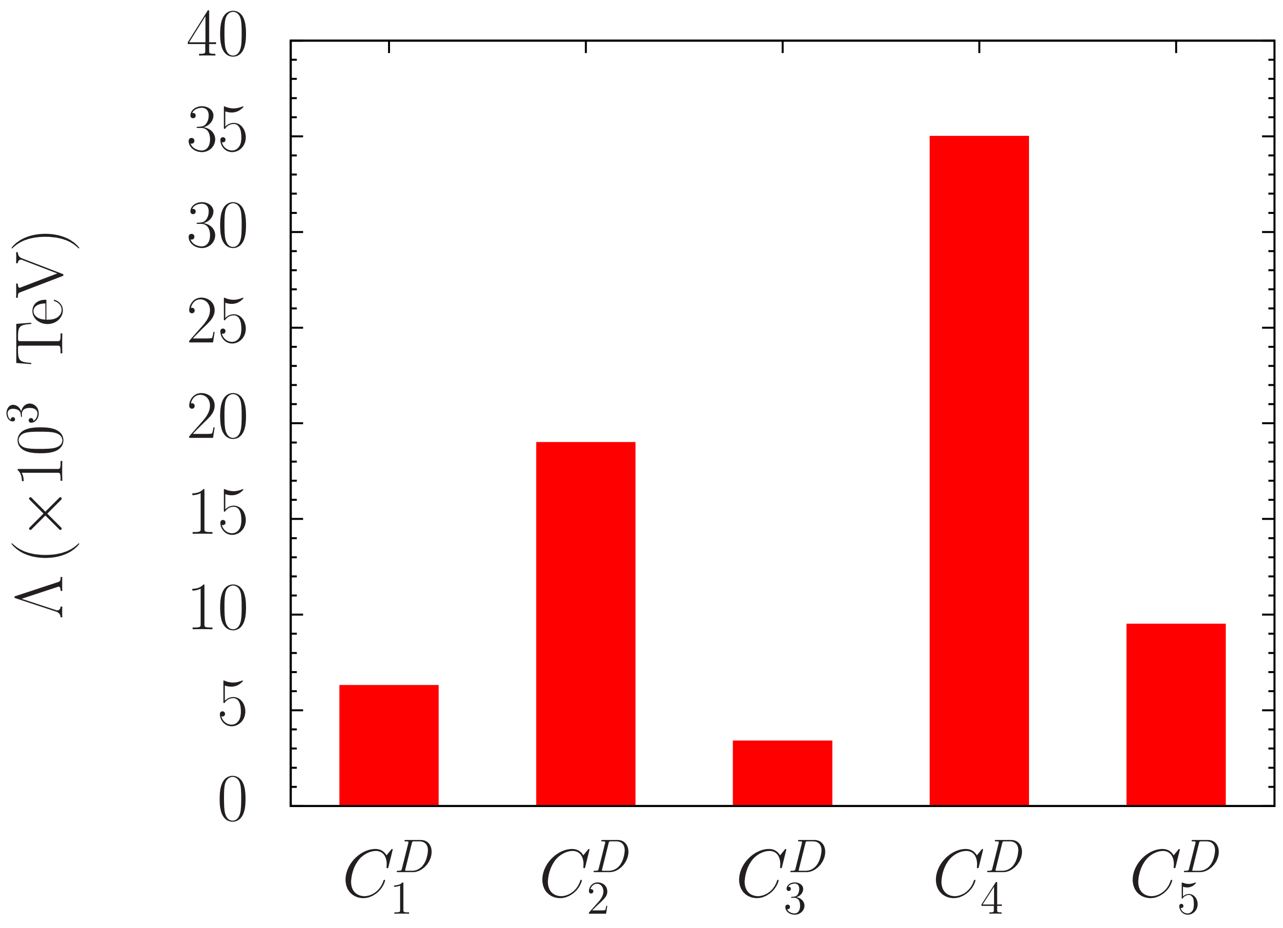
$$|V_{cs}|^2 + |V_{cd}|^2 + |V_{cb}|^2 - 1 = 0.064(32)$$

Lower bounds on New Physics

ETM (arXiv:1403.7302, 2014 PRD)

Wilson coefficients: $C_i(\Lambda) \sim \frac{F_i L_i}{\Lambda^2}$

Generic tree-level, strongly interacting: $L_i \sim F_i \sim 1$



Summary

- ☆ Gauge field ensembles with light sea quarks at their **physical masses** are being used in a growing number of LQCD calculations of D meson quantities
  ➠ removes chiral extrapolation errors ➠ better precision
- ☆ LQCD results for D, D_s decay constants are already very precise (~0.5% errors)
  ➠ uncertainties in CKM determinations are dominated by experimental contributions
  slight (2σ) tension with 2nd row unitarity
- ☆ For D semileptonic form factors, LQCD calculations still need better precision
  goal: ~1% errors
  ➠ need to calculate the form factors over the entire recoil range
  work in progress by several lattice groups
  extension to FCNC form factors (f_T) straightforward
- ☆ For neutral D meson mixing there are now two independent LQCD calculations of the matrix elements of the full set of five local operators.
  Further improvements are not needed until there are reliable predictions of the long-distance contributions.
- ☆ For semileptonic decays into vector meson final states, the finite volume formalism has recently been developed (Briceño et al, arXiv:1406.5965, 2015 PRD)
  Pilot studies for $B \to K^*$ are underway

Outlook

Amala Willenbrock

Outlook

How do/did we get to 1% total errors (or below)?

- ☆ physical mass ensembles are essential
- ☆ small lattice spacings
- ☆ calculate renormalizations nonperturbatively
- ☆ small statistical errors (straightforward, but expensive)

- ☆ will eventually need to include
  - ✦ strong isospin breaking ($m_u \neq m_d$) effects ✓
  - ✦ QED effects

    program being developed for kaon quantities, muon g-2

Extend the reach of LQCD to include

- SL decay to vector meson final states (in progress)
- hadronic D decays
- long-distance contributions to neutral D mixing

➠ formalism is being developed for multi-hadron states in finite volume

Already done for kaons

- ☆ excited state spectra, resonances, scattering states

  (see the talks by Sinead Ryan, Graham Moir, Gavin Cheung)

Thank you!

Farah Willenbrock

Backup slides

Heavy Quark Treatment

Relativistic Heavy Quarks - Fermilab formulation

- start with the relativistic Wilson action + $O(a)$ improvement
- with mass-dependent matching conditions, cut-off effects are

$\alpha_s^k f(m_h a)(a\Lambda)^n$ with

$$am_h \sim 1 : f(m_h a) \sim O(1)$$

FNAL/MILC implementation for action and currents:

tree-level tadpole $O(a)$ improved
mostly nonperturbative renormalization (mNPR)

Heavy Quark Treatment

HISQ action for charm:

- like asqtad, the HISQ action is a tree-level tadpole improved staggered action, with discretization errors for light quarks:

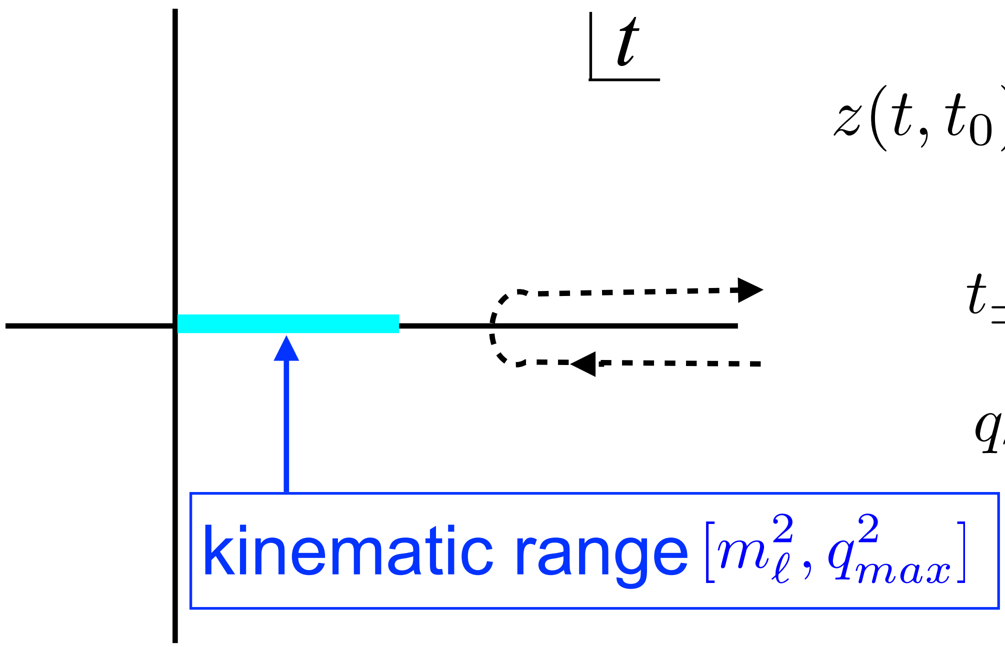
$$\alpha_s (a\Lambda)^2, (a\Lambda)^4$$

- HISQ action is highly improved for charm quarks:

$$\sim \alpha_s \Lambda/m_h (am_h)^2, (\Lambda/m_h)^2 (am_h)^4$$

- can also be used for heavier than charm quarks

The z-expansion

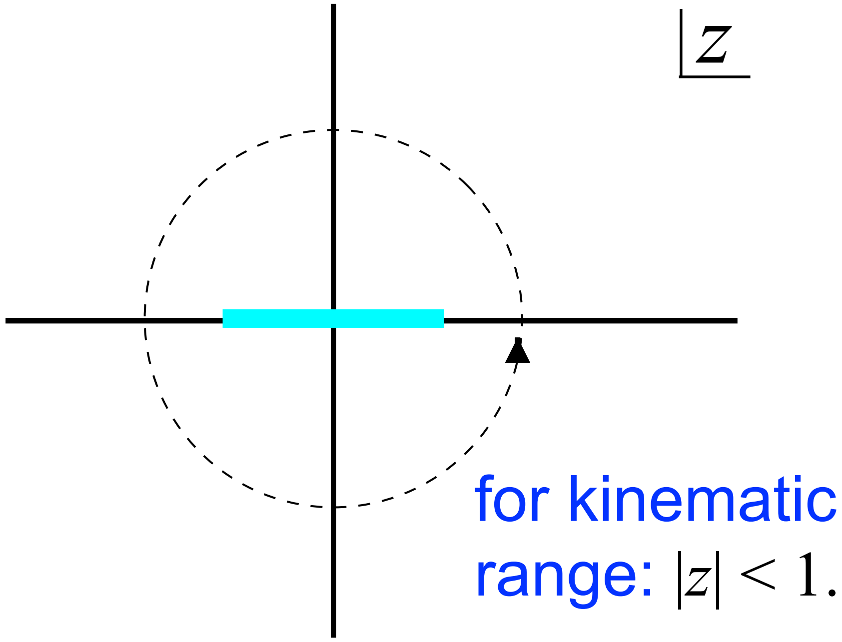


$$t = q^2$$

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$



Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd et al (hep-ph/9412324,PRL 95)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)

The form factor can be expanded as:

$$f(t) = \frac{1}{P(t)\phi(t, t_0)} \sum_{k=0} a_k(t_0) z(t, t_0)^k$$

- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.

chiral-continuum extrapolation

Some ensembles still have

$$m_{\text{light}} > 1/2\,(m_u + m_d)_{\text{phys}}$$

χPT guides the extrapolation/interpolation to the physical point.

- include (light quark) discretization effects (for example, staggered χPT)
- combined continuum-chiral extrapolation
- Heavy meson χPT: $\chi\text{PT} + 1/M$ expansion
- can also add HQ discretization terms to chiral-continuum fits

D mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

chiral-continuum extrapolation

$\mathrm{SU}(3)$ heavy-meson partially-quenched rooted staggered $\chi\mathrm{PT}$

- NLO chiral logs + staggered discretization corrections
- + analytic terms (up to $\mathrm{N^3LO}$)
- + leading $1/M$ terms in HM expansion
- + HQ discretization terms
- + higher order PT terms (up to $O(\alpha_s)^3$)

D mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

SU(3) heavy-meson partially-quenched rooted staggered χPT

- NLO chiral logs + taste-splittings + "wrong-spin" corrections
- + analytic terms (up to N^{3}LO)
- + B-meson hyperfine and flavor splittings
- + HQ discretization terms
- + higher order PT terms (up to $O(\alpha_s)^3$)

Schematically

$$\langle O_1^q \rangle = \beta_1 \left(1 + \text{NLO chiral logs + taste-splittings} + \text{wrong spin terms} \right) + (2\beta_2 + 2\beta_3)\,\text{w.s.} + (2\beta'_2 + 2\beta'_3)\,\text{w.s.}$$

+ analytic terms

LECs for $\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle, \langle \mathcal{O}_3 \rangle$

C. Bernard (Phys.Rev. D87 (2013) 114503, arXiv: 1303.0435)

- no new LECs with simultaneous fits to the operators that mix at NLO

  $[\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle, \langle \mathcal{O}_3 \rangle]$ and $[\langle \mathcal{O}_4 \rangle, \langle \mathcal{O}_5 \rangle]$