Weak Charm Decays with lattice QCD: status and prospects

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CHARM 2016 workshop        Bologna, Italy, 5-9 Sep 2016
Outline

- Motivation and Introduction
  - lattice QCD

- Results
  - leptonic $D$-meson decays
  - semileptonic $D$-meson decays
  - neutral $D$-meson mixing

- Phenomenology
  - CKM determinations
  - second row unitarity test
  - NP scale

- Summary and Outlook
Introduction

example: \( D \rightarrow \pi \ell \nu \)

generic weak process involving hadrons:

\[
\text{(experiment)} = (\text{known}) \times (\text{CKM element}) \times (\text{had. matrix element})
\]

\[
\Gamma_{K\ell 3}, \Gamma_{K\ell 2}, \ldots
\]

\[
\frac{d\Gamma(B\rightarrow\pi\ell\nu)}{dq^2}, \quad \frac{d\Gamma(D\rightarrow K\ell\nu)}{dq^2}, \ldots
\]

\[
\Delta m_{d(s)}
\]

\[
\vdots
\]

parameterize the ME in terms of form factors, decay constants, bag parameters, ...
**Lattice QCD Introduction**

\[ \mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu} \]

- discrete Euclidean space-time (spacing \( a \)) derivatives \( \rightarrow \) difference operators, etc…
- finite spatial volume (\( L \))
- finite time extent (\( T \))

**Adjustable parameters**

- lattice spacing: \( a \rightarrow 0 \)
- finite volume, time: \( L \rightarrow \infty, T > L \)
- quark masses \( (m_f) \): \( M_{H,\text{lat}} = M_{H,\text{exp}} \)
  - tune using hadron masses extrapolations/interpolations
  - \( m_f \rightarrow m_{f,\text{phys}} \)
  - \( m_{ud}, m_s, m_c, m_b \)

- also: \( n_f = \) number of sea quarks: 3 (2+1), 4 (2+1+1)
Lattice QCD Introduction

\[ \langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \mathcal{O}(\psi, \bar{\psi}, A) e^{-S} \]
\[ S = \int d^4x \left[ \bar{\psi} (\mathcal{D} + m) \psi + \frac{1}{4} (F_{\mu\nu}^c)^2 \right] \]

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves \( \det(\mathcal{D} + m) \) in the integrand. The correlation functions, \( \mathcal{O} \), are then written in terms of \( (\mathcal{D} + m)^{-1} \) and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to \( \det(\mathcal{D} + m) \ e^{-S} \)
2. calculate quark propagators, \( (\mathcal{D} + m_q)^{-1} \), for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis
systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD ➔ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...

\[ L \]
\[ a \text{ (fm)} \]
chiral-continuum extrapolation (interpolation)

Example: Set of ensembles by MILC collaboration

MILC $n_f = 2+1+1$

Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM
Heavy Quark Treatment

• For light quarks \( m_\ell < \Lambda_{\text{QCD}} \), discretization errors \( \sim \alpha_s^k (a\Lambda_{\text{QCD}})^n \)

• For heavy quarks, discretization errors \( \sim \alpha_s^k (am_h)^n \)
  with currently available lattice spacings
  for \( b \) quarks \( am_b > 1 \)
  for charm \( am_c \sim 0.15-0.6 \)

\[ \text{for charm can use light quark methods, if action is sufficiently improved (HISQ, tmWilson, NP imp. Wilson,...)} \]

\[ \text{need effective field theory methods for } b \text{ quarks} \]

• avoid errors of \( (am_c)^n \) in the action by matching to continuum HQET:
  ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  can be used for charm and bottom
Leptonic $D$ decay

**example:** $D_s^+ \rightarrow \mu^+ \nu_\mu$

\[
\Gamma(D_s^+ \rightarrow \ell^+ \nu_\ell(\gamma)) = (\text{known}) \times (1 + \delta^\ell_{\text{EM}}) \times |V_{cs}|^2 \frac{f_{D_s}^2}{f_{D^+}}
\]

- use experiment + LQCD input for determination of CKM element
- similar for $B$ ($|V_{ub}|$) and $K$ ($|V_{us}|$) mesons
- SU(3) ratio $f_{D_s}/f_{D^+}$: statistical and systematic errors tend to cancel.
- $\delta^\ell_{\text{EM}}$ includes structure dependent EM corrections. It is needed to relate the “pure QCD” decay constant to experiment and is currently estimated phenomenologically.
$D_s$ decay constant results

**$D_{s}$ decay constant results**


<table>
<thead>
<tr>
<th>(N_f=2+1+1)</th>
<th>(f_D)</th>
<th>(f_{D_s})</th>
<th>(f_{D_s}/f_D)</th>
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<tbody>
<tr>
<td>FNAL/MILC 14A</td>
<td>o</td>
<td>x</td>
<td>0.3%</td>
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<tr>
<td>ETM 14E</td>
<td>x</td>
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<td>ETM 13F</td>
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<td>FNAL/MILC 13</td>
<td>o</td>
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<tr>
<td>FNAL/MILC 12B</td>
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<th>(f_D)</th>
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<td>FNAL/MILC 05</td>
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<th>(f_{D_s})</th>
<th>(f_{D_s}/f_D)</th>
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<td>TWQCD 14</td>
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Small errors in FNAL/MILC 14A (arXiv:1407.3772, 2014 PRD) due to
- physical mass ensembles
- improved action (small discretization errors)
- small lattice spacings
- PCAC (no renormalization)
$D_{(s)}$ decay constant results

RBC/UKQCD (J.T. Tsang @ Lattice 2016):
- 2+1 flavors of DW fermions
- physical mass ensembles
- PCAC (no renormalization)
$D_{(s)}$ decay constant results

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J. T. Tsang (RBC/UKQCD) @ Lattice 2016:

RBC/UKQCD (J.T. Tsang @ Lattice 2016):
- 2+1 flavors of DW fermions
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also ongoing work by:
- ALPHA/RQCD (imp. Wilson)
- FNAL/MILC (with Fermilab charm)
+ new results from ETM on $f_{D^*(s)}$
  (Melis @ Lattice 2016)
Semileptonic $D$-meson decay

Example: $D \rightarrow \pi \ell \nu$

\[
\frac{d\Gamma(D \rightarrow \pi \ell \nu)}{dq^2} = (\text{known}) \times |V_{cd}|^2 f_+(q^2)
\]

\[
\ell = e, \mu
\]

★ can calculate the form factors for the entire recoil energy range
★ can use $z$-expansion* for model-independent parameterization of $q^2$ dependence
★ calculate both form factors $f_+(q^2), f_0(q^2)$
★ can compare shape between experiment and lattice
★ extension to rare SL decay form factors ($f_T$) straightforward

*see backup slides
**D SL form factor results**

$D$ SL form factor results


**new preliminary results @ Lattice 2016:**

- **ETM (G. Salerno)**
  2+1+1 flavors of tmWilson calculate all form factors over whole $q^2$ range modified $z$-expansion preliminary sys. errors

- **FNAL/MILC (S. Gottlieb, T. Primer)**
  no central values (yet)
  2+1+1 flavors of HISQ physical mass ensembles calculate directly at zero $q^2$

- **JLQCD (T. Kaneko)**
  2+1 flavors of DW fermions extrapolate to zero $q^2$ with $z$-expansion chiral-continuum extrapolation still adding ensembles to analysis
**D SL form factor results**

adapted from S. Aoki et al (arXiv:1607.00299)

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$D$ SL form factor results

ETM (G. Salerno) @ Lattice 2016

- 2+1+1 flavors of tmWilson
- calculate $f_+, f_0$ over whole $q^2$ range
- modified $z$-expansion
- correct for hypercubic discretization effects
- preliminary sys. errors

### Details of the ensembles used in this Nf = 2+1+1 analysis

The valence light quark mass is put equal to the sea quark mass

Range of the simulated pion masses

Three different values of the lattice spacing: 0.06 $fm \div 0.09$ $fm$

Different volumes: 2 $fm \div 3$ $fm$

Pion masses in range $210 \div 440$ MeV

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![Graph showing $f_0(q^2)$ and $f_+(q^2)$](image_url)

- $f_0(q^2)$
- $f_+(q^2)$
- Preliminary

Data from:
- Cleo D+
- Cleo D0
- Belle
- BaBar

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see talk by G. Salerno on Thursday
\( D \) SL form factor results

FNAL/MILC (S. Gottlieb, T. Primer) @ Lattice 2016

- 2+1+1 HISQ ensembles
  - physical light quark masses
- HISQ valence charm, strange, light
- calculate directly at zero \( q^2 \)
  - chiral-continuum extrapolation
- preliminary systematic error analysis
- next step:
  - vector and scalar form factors
  - + range of recoil momenta
  - \( \Rightarrow \) whole \( q^2 \) range
- will yield better precision
  - and shape comparison with experiment
FNAL/MILC (S. Gottlieb, T. Primer) @ Lattice 2016

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$d$ SL form factor results
$D$ SL form factor results

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JLQCD (T. Kaneko) @ Lattice 2016

- 2+1 flavors of DW fermions
- extrapolate to zero $q^2$ with $z$-expansion
- chiral-continuum extrapolation
- still adding ensembles to analysis

\[ M_\pi^2 \text{ [GeV}^2\text{]} \]

\[ a^2 \times 100 \text{ [fm}^2\text{]} \]
$D$ SL form factor results

JLQCD (T. Kaneko) @ Lattice 2016

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Neutral $D$ meson mixing

$$M_{12} - \frac{i}{2} \Gamma_{12} \propto \langle D^0 | H_W^{\Delta c=2} | \bar{D}^0 \rangle + \sum_n \frac{\langle D^0 | H_W^{\Delta c=1} | n \rangle \langle n | H_W^{\Delta c=1} | \bar{D}^0 \rangle}{M_D - E_n + i\epsilon}$$

short distance

long distance
Neutral $D$ meson mixing

\[ M_{12} - \frac{i}{2} \Gamma_{12} \propto \langle D^0 | H_W^{\Delta c=2} | \bar{D}^0 \rangle + \sum_n \frac{\langle D^0 | H_W^{\Delta c=1} | n \rangle \langle n | H_W^{\Delta c=1} | \bar{D}^0 \rangle}{M_D - E_n + i\epsilon} \]

**“Simple”**
- can use the same methods as for $B$ mixing (and decay constants, form factors)
- BSMs with heavy new particles can contribute here

**“Hard”**
- large contribution
- intermediate state can include multiple (>2) hadrons:
  - not a problem for Kaon mixing
Neutral $D$ meson mixing

$$M_{12} - \frac{i}{2} \Gamma_{12} \propto \langle D^0 | H^\Delta W_{c=2} | \overline{D}^0 \rangle + \sum_n \frac{\langle D^0 | H^\Delta W_{c=1} | n \rangle \langle n | H^\Delta W_{c=1} | \overline{D}^0 \rangle}{M_D - E_n + i\epsilon}$$

"Simple"
- can use the same methods as for $B$ mixing (and decay constants, form factors)
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Neutral $D$ meson mixing

In the SM and beyond:

$$H_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu) O_i(\mu)$$

$$O_1 = \bar{c}\gamma^\mu Lu \bar{\gamma}^\mu Lu$$
$$O_2 = \bar{c}Lu\bar{c}Lu$$
$$O_3 = \bar{c}^\alpha Lu^\beta \bar{\gamma}^\beta Lu^\alpha$$
$$O_4 = \bar{c}Lu\bar{c}Ru$$
$$O_5 = \bar{c}^\alpha Lu^\beta \bar{\gamma}^\beta Ru^\alpha c$$

$$\langle O_i \rangle \equiv \langle D^0|O_i|\bar{D}^0 \rangle(\mu) = e_i M_D^2 f_D^2 B_D^{(i)}(\mu)$$

- choose $\mu = 3 \text{ GeV}$

- calculate the matrix elements of all five local operators.
$D$ mixing results in comparison

$\mu = 3$ GeV

- ETM: $n_f = 2+1+1$
  - arXiv:1505.06639

- Fermilab/MILC: $n_f = 2+1$

- ETM: $n_f = 2$
  - arXiv:1403.7302

A. Kronfeld @ Lattice 2016 (plot by C.C. Chang)
$D$ mixing results

ETM

$n_f = 2+1+1$ tmWilson

$D$ mixing results

ETM

$n_f = 2+1+1$ tmWilson

$D$ mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

- 14 MILC asqtad ensembles
  - 4 lattice spacings
  - ~4 sea quark masses per lattice spacing
  - ~600 - 2000 configurations
    - $\times$ 4 time-sources per configuration

- Fermilab $c$ quarks
- mNPR renormalization
$D$ mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)  
Operators 1,2,3

\[ \langle \mathcal{O}_i \rangle \text{ [GeV}^4] \]

\[ \langle \mathcal{O}_2 \rangle \text{ [GeV}^4] \]

\[ \langle \mathcal{O}_3 \rangle \text{ [GeV}^4] \]

\[ M_{\pi}^2 \text{ [GeV}^2] \]
$D$ mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

systematic error study

- remove or add higher order terms in fit function:
  - chiral expansion
  - heavy meson expansion
  - light quark discretization effects
  - HQ discretization effects
  - renormalization (perturbative expansion)
- change data included
- change inputs
$D$ mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

- remove or add higher order terms in fit function:
  - chiral expansion
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**systematic error study**

$$\langle O_3 \rangle [\text{GeV}^4]$$

- base
- $f_\pi$ vs. $f_\pi$
- $m_{NP}$
- $m_{NP} + \alpha_s^3$
- PT$_{P} + \alpha_s^2$
- PT$_{L} + \alpha_s^5$
- NLO ($m_q < 0.65 \ m_s$)
- N$^3$LO
- LO x 2
- NLO x 2
- NNLO x 2
- no splitting
- generic $O(\alpha_s a^2)$
- HQ $O(\alpha_s a)$ only
- HQ $O(\alpha_s a, a^2)$ only
- no $a \approx 0.12$ fm
- no $a \approx 0.045$ fm individual
**D mixing results**

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

- Remove or add higher order terms in fit function:
  - Chiral expansion
  - Heavy meson expansion
  - Light quark discretization effects
  - HQ discretization effects
  - Renormalization (perturbative expansion)

- Change data included

- Change inputs

---

**FNAL/MILC (Kronfeld, Chang @ Lattice 2016)**

- **systematic error study**

- **Base**
  - $f_s$ vs. $f_\pi$
  - mNPR
  - mNPR + $\alpha_s^3$
  - PT$_{F}$ + $\alpha_s^2$
  - PT$_{L}$ + $\alpha_s^3$

- **NLO** ($m_\pi < 0.65 m_s$)
  - N$^3$LO
  - LO x 2
  - NLO x 2
  - NNLO x 2
  - No splitting

- **Generic** $O(\alpha_s a^2)$

- **HQ** $O(\alpha_s a)$ only

- **HQ** $O(\alpha_s a, a^2)$ only

- No $a \approx 0.12$ fm

- No $a \approx 0.045$ fm

- Individual
**D meson summary**

errors (in %) comparison:

\[ \frac{f_{D_s}}{f_{D^+}} \]
\[ f_{D_s} \]
\[ f_{D^+} \]
\[ f^D_K(0) \]
\[ f^D_\pi(0) \]
\[ \hat{B}_D^i \]

Preliminary results @ Lattice 2016:
J.T. Tsang (RBC/UKQCD)  
Eckert, Hofmann (ALPHA/RQCD)  
Kronfeld (FNAL/MILC)

Preliminary results @ Lattice 2016:
G. Salerno (ETM)  
S. Gottlieb, T. Primer (FNAL/MILC)  
T. Kaneko (JLQCD)

\[ D \text{ mixing MEs for all 5 local operators} \]
- \( n_f = 2, 2+1+1 \) ETM (2013, 2014)
- \( n_f = 2+1 \) FNAL/MILC  
  (Kronfeld, Chang, Lattice 2016)
$D$ meson summary

errors (in %) comparison:

- $f_{D_s} / f_{D^+}$
- $f_{D_s}$
- $f_{D^+}$
- $f_{DK}^{(0)}$
- $f_{+}^{(0)}$
- $\hat{B}_D^i$

small errors due to:
- physical light quark masses
- improved charm-quark action (HISQ)
- PCAC (no renormalization)
- ensembles with small lattice spacings

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Preliminary results @ Lattice 2016:
- J.T. Tsang (RBC/UKQCD)
- Eckert, Hofmann (ALPHA/RQCD)
- Kronfeld (FNAL/MILC)

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$D$ mixing MEs for all 5 local operators:
- $n_f = 2, 2+1+1$ ETM (2013, 2014)
- $n_f = 2+1$ FNAL/MILC
  (Kronfeld, Chang, Lattice 2016)
Implications for $|V_{cs}|, |V_{cd}|$


S. Gottlieb, T. Primer (FNAL/MILC) @ Lattice 2016

|V_{cd}|
\[\begin{array}{c|c|c}
\text{FLAG2016} & \mid V_{cs} \mid & \\
\hline
N_r = 2 + 1 + 1 & \text{FLAG average for } N_r = 2 + 1 + 1 & \\
& \text{ETM 14E} & \\
& \text{FNAL/MILC 14A} & \\
\hline
N_r = 2 + 1 & \text{FLAG average for } N_r = 2 + 1 & \\
& \text{HPQCD 11/10B} & \\
& \text{HPQCD 12A/10A} & \\
& \text{FNAL/MILC 11} & \\
& \chiQCD 14 & \\
\hline
\text{non-lattice } N_r = 2 & \text{FLAG average for } N_r = 2 & \\
& \text{ETM 13B} & \\
\end{array}\]

|V_{cs}|

Leptonic
- Lattice: 62%
- Experiment: 21%
- EM: 17%

Semileptonic
- Lattice: 64%
- Experiment: 36%
- EM: 17%
Implications for the 2\textsuperscript{nd} row of the CKM Matrix

errors on $|V_{cs}|$ and $|V_{cd}|$ are dominated by experiment (PDG 2015, arXiv:509.02220):

$$
|V_{cd}| = 0.217 (1)_{\text{LQCD}} (5)_{\text{exp}}
$$
$$
|V_{cs}| = 1.007 (4)_{\text{LQCD}} (16)_{\text{exp}}
$$

(based on the PDG average of 2+1 & 2+1+1 flavor LQCD results; average is dominated by FNAL/MILC)

$2\sigma$ tension with unitarity:

$$
|V_{cs}|^2 + |V_{cd}|^2 + |V_{cb}|^2 - 1 = 0.064(32)
$$
Wilson coefficients: \[ C_i(\Lambda) \sim \frac{F_i L_i}{\Lambda^2} \]

Generic tree-level, strongly interacting: \[ L_i \sim F_i \sim 1 \]
Gauge field ensembles with light sea quarks at their **physical masses** are being used in a growing number of LQCD calculations of $D$ meson quantities ➤ removes chiral extrapolation errors ➤ better precision

LQCD results for $D$, $D_s$ decay constants are already very precise (~0.5% errors) ➤ uncertainties in CKM determinations are dominated by experimental contributions slight $(2\sigma)$ tension with 2nd row unitarity

For $D$ semileptonic form factors, LQCD calculations still need better precision goal: ~1% errors ➤ need to calculate the form factors over the entire recoil range work in progress by several lattice groups extension to FCNC form factors ($f_T$) straightforward

For neutral $D$ meson mixing there are now two independent LQCD calculations of the matrix elements of the full set of five local operators. Further improvements are not needed until there are reliable predictions of the long-distance contributions.

For semileptonic decays into vector meson final states, the finite volume formalism has recently been developed (Briceño et al, arXiv:1406.5965, 2015 PRD) Pilot studies for $B \rightarrow K^*$ are underway
Outlook

Amala Willenbrock
Outlook

How do/did we get to 1% total errors (or below)?

🌟 physical mass ensembles are essential
🌟 small lattice spacings
🌟 calculate renormalizations nonperturbatively
🌟 small statistical errors (straightforward, but expensive)

🌟 will eventually need to include
  ✨ strong isospin breaking ($m_u \neq m_d$) effects ✓
  ✨ QED effects
    program being developed for kaon quantities, muon $g-2$

Extend the reach of LQCD to include

• SL decay to vector meson final states (in progress)
• hadronic $D$ decays
• long-distance contributions to neutral $D$ mixing
  ➠ formalism is being developed for multi-hadron states in finite volume

Already done for kaons

🌟 excited state spectra, resonances, scattering states
  (see the talks by Sinead Ryan, Graham Moir, Gavin Cheung)

Amala Willenbrock
Thank you!
Backup slides
Heavy Quark Treatment

Relativistic Heavy Quarks - Fermilab formulation

- start with the relativistic Wilson action + $O(a)$ improvement
- with mass-dependent matching conditions, cut-off effects are

$$\alpha_s^k f(m_ha)(a\Lambda)^n \text{ with}$$

$$am_h \sim 1 : f(m_ha) \sim O(1)$$

FNAL/MILC implementation for action and currents:

- tree-level tadpole $O(a)$ improved
- mostly nonperturbative renormalization (mNPR)
HISQ action for charm:

- like asqtad, the HISQ action is a tree-level tadpole improved staggered action, with discretization errors for light quarks:
  \[ \alpha_s (a\Lambda)^2, (a\Lambda)^4 \]

- HISQ action is highly improved for charm quarks:
  \[ \sim \alpha_s \Lambda/m_h (am_h)^2, (\Lambda/m_h)^2 (am_h)^4 \]

- can also be used for heavier than charm quarks
The $z$-expansion

\[ t = q^2 \]

\[ z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \]

\[ t_\pm = (m_B \pm m_\pi)^2 \]

\[ q_{max}^2 = t_- \]

The form factor can be expanded as:

\[ f(t) = \frac{1}{P(t)\phi(t, t_0)} \sum_{k=0} a_k(t_0) z(t, t_0)^k \]

- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function $\phi$ affects the unitarity bound on the $a_k$.
- In practice, only first few terms in expansion are needed.

Boyd et al (hep-ph/9412324, PRL 95)
Boyd & Savage (hep-ph/9702300, PRD 97)
Some ensembles still have

\[ m_{\text{light}} > \frac{1}{2} (m_u + m_d)_{\text{phys}} \]

\( \chiPT \) guides the extrapolation/interpolation to the physical point.

- include (light quark) discretization effects (for example, staggered \( \chiPT \))
- combined continuum-chiral extrapolation

- Heavy meson \( \chiPT \): \( \chiPT + 1/M \) expansion

- can also add HQ discretization terms to chiral-continuum fits
$D$ mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

| chiral-continuum extrapolation |

SU(3) heavy-meson partially-quenched rooted staggered $\chi$PT

- NLO chiral logs + staggered discretization corrections
- + analytic terms (up to $N^3$LO)
- + leading $1/M$ terms in HM expansion
- + HQ discretization terms
- + higher order PT terms (up to $O(\alpha_s)^3$)
\(D\) mixing results

FNAL/MILC (Kronfeld, Chang @ Lattice 2016)

SU(3) heavy-meson partially-quenched rooted staggered \(\chi\)PT

- NLO chiral logs + taste-splittings + “wrong-spin” corrections
- + analytic terms (up to \(N^3\)LO)
- + \(B\)-meson hyperfine and flavor splittings
- + HQ discretization terms
- + higher order PT terms (up to \(O(\alpha_s)^3\))

Schematically

\[
\langle O_1^q \rangle = \beta_1 \left(1 + \text{NLO chiral logs + taste-splittings} + \text{wrong spin terms} + (2\beta_2 + 2\beta_3) \text{w.s.} + (2\beta'_2 + 2\beta'_3) \text{w.s.} \right)
\]

\(+\) analytic terms

- no new LECs with simultaneous fits to the operators that mix at NLO
  
  \([\langle O_1 \rangle, \langle O_2 \rangle, \langle O_3 \rangle]\) and \([\langle O_4 \rangle, \langle O_5 \rangle]\)