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Charmonium and XYZ states: progress and prospects in Lattice QCD

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Plan

- Introduction and background
- A consumers guide to Lattice QCD
  - brief discussion of discretisation and calculating correlators
  - practicalities (aka compromises) and consequences
- Discussion and selected results
  - parallel tracks for progress
  - **precision spectroscopy of single hadron states including excited and exotic states**
  - **spectroscopy of scattering states - progress and challenges**
- Summary

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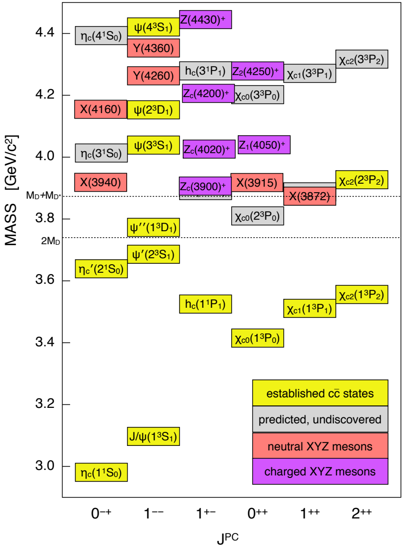
Many details and topics omitted for time constraints - APOLOGIES!

Motivation - a charm revolution

X,Y,Z states: narrow charmonium-like structures above the open charm threshold.
Not all in $^{2S+1}L_J$ (quark model) pattern: what is the nature of these states?

- Below open charm threshold:
  $\eta_c(1S, 2S)$, $JPsi(1S, 2S)$,
  $\chi_{c(0,1,2)}(1P)$, $h_c(1P)$.
- Above:
  - X(3872): close to $D\bar{D}^*$ threshold - a molecular meson?
  - X(4260): 1^{--} hybrid meson?
  - X(4430)$^{\pm}$: charged - not $c\bar{c}$: tetraquark?
  - No clear picture has emerged.

S. Olsen Bormio2015, 1511.01589 ⟶

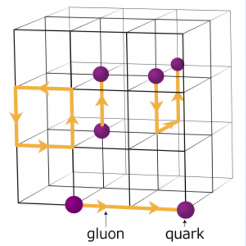


A lattice QCD primer

Start from the QCD Lagrangian:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\psi - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

- Gluon fields are SU(3) matrices - links of a hypercube.
  $A_{\mu}(x) \rightarrow U(x,\mu) = e^{-iagA^{b}_{\mu}(x)t^{b}}$
- Quark fields $\psi(x)$ on sites with color, flavor, Dirac indices.
  Fermion discretisation - Wilson, Staggered, Overlap.
- Derivatives $\rightarrow$ finite differences:
  $\nabla^{\mathrm{fwd}}_{\mu}\psi(x) = \frac{1}{a}\left[U_{\mu}(x)\psi(x+a\hat{\mu}) - \psi(x)\right]$



Solve the QCD path integral on a finite lattice with spacing $a \neq 0$ estimated stochastically by Monte Carlo. Can only be done effectively in a Euclidean space-time metric (no useful importance sampling weight for the theory in Minkowski space).

Observables determined from (Euclidean) path integrals of the QCD action

$$\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \; \mathcal{O}[U,\bar{\psi},\psi] e^{-S[U,\bar{\psi},\psi]}$$

Correlation functions for spectroscopy

- $\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U \det M^2 \mathcal{O} e^{-S_G}$ after integrating the quark (Grassman) fields.
- Calculated by importance sampling of gauge fields and averaging over ensembles: $\langle \mathcal{O} \rangle = \lim_{N_{\text{cfg}} \to \infty} \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \mathcal{O}_i[U_i]$
- Interested in two-point correlation functions built from interpolating operators
  - Eg the local meson operator $\phi(x) = \bar{\psi}_a(x) \Gamma \psi_b(x)$ with Γ an element of the Dirac algebra with possible displacements; a and b flavour indices

Euclidean-time correlation function

$$
\begin{aligned}
C(t) &= \langle 0 | \phi(t) \quad \phi^\dagger(0) | 0 \rangle \\
&= \sum_{k,k'} \langle 0 | \phi | k \rangle \langle k | e^{-\hat{H}t} | k' \rangle \langle k' | \phi^\dagger | 0 \rangle \\
&= \sum_{k} | \langle 0 | \phi | k \rangle |^2 e^{-E_k t}
\end{aligned}
$$

So $\lim_{t \to \infty} C(t) = Z e^{-E_0 t}$ ← the ground state energy.

Beyond ground state spectroscopy

- Construct a basis of local and non-local operators $\bar{\psi}(x)\Gamma D_i D_j \ldots \psi(x)$
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\phi_i \phi_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

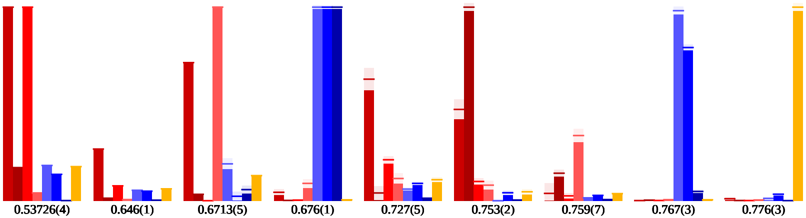
- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem
  $C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

- operators of definite J^{PC} constructed in step 1 are subduced into the relevant irrep(s)
- a subduced operator carries a "memory" of continuum spin J from which it was subduced - it **overlaps** predominantly with states of this J.

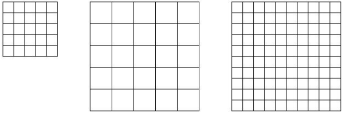
J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins at finite a
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is **Spin 1**, **Spin 3** and **Spin 4**.



Practicalities of numerical simulation

1. Working in a finite box at finite grid spacing



- Recover continuum **QCD** at small grid spacing, a and large volume, V.

A costly procedure but a regular feature in lattice calculation s now

2. Simulating at physical quark masses

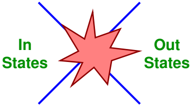
- Computational cost grows rapidly with decreasing quark mass $\rightarrow m_q = m_{u,d}$ costly. Care needed vis location of decay thresholds and identification of resonances.
- Heavy quark discretisation errors grow as $(\mathcal{O}(am_q)^n)$ and can be large for charm and bottom quarks.
  - effective field theory methods for b - FNAL, NRQCD, ...
  - relativistic (improved) actions suitable for charm. The HadSpec anisotropic approach helps here keeping $a_t m_q$ small.

Simulations with physical light quark masses already available. Heavy quark methods now robustly tested.

3. Breaking symmetry

- Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
  Classify states by irreps of O_h and relate by subduction to J values of O_3. Lots of degeneracies in subduction for $J \geq 2$ and physical near-degeneracies. Complicates spin identification.

Spin identification at finite lattice spacing: Dudek et al 0707.4162, 1204.5425



4. Working in Euclidean time.

- Scattering matrix elements not directly accessible from Euclidean QFT [*Maiani-Testa theorem*].
  Scattering matrix elements: asymptotic $|\text{in}\rangle$, $|\text{out}\rangle$ states: $\langle\text{out}|e^{i\hat{H}t}|\text{in}\rangle \rightarrow \langle\text{out}|e^{-\hat{H}t}|\text{in}\rangle$.
  Recall euclidean metric projects onto ground state at $t \rightarrow \infty$
- Enables $C(t) \sim e^{-E_n t}$.

Lüscher and generalisations of: method for indirect access.

5. Calculating all elements of correlation functions.

- Traditional point-to-all algorithms for quark propagation don't allow for all Wick contractions.
- Disconnected diagrams require all-to-all techniques - often noisy and expensive.

Distillation provides efficient precise evaluation of disconnected diagrams and also for very large bases of operators. Enabling progress in precision spectroscopy and scattering. [PRD80 (2009) 054506].

6. Quenching

No longer an issue: Simulations done with $N_f = 2, 2+1, 2+1+1$.

Lattice Hadron Spectroscopy
precision & pioneering results

Spectroscopy: two strategies for progress

Below thresholds: "gold-plated" quantities characterised by

- Using well-tried and tested and validated methods
- High statistics and improved actions for precise results
- Different actions in agreement
- Most/All systematic errors accounted for e.g. continuum extrapolation, finite volume, physical light quark masses, Nf, EM effects

Above thresholds: calculation of high spin & exotic states relatively new

- Physics of molecular/multi hadron states needs relevant operators.
- No continuum extrapolation
- Relatively heavy pions ← already changing
- Improvements underway now that methods are proven

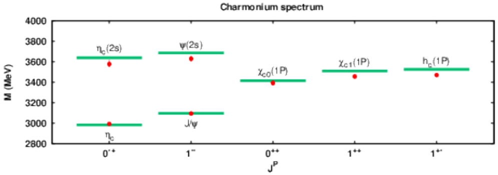
new ideas crucial

- Distillation - quark propagation enabling isoscalars, precision spectrosco py ...
- Framework for scattering & coupled channels

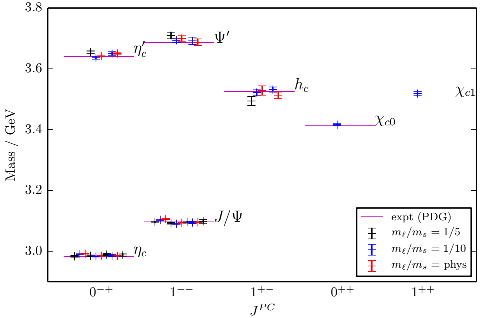
Charmonium below threshold - "gold-plated"

- Methods: tested, validated; Different actions in agreement.
- High statistics and improved actions for precise results.
- Simulation at m_q^{phys} or extrapolation $m_q \to m_q^{phys}$.
- Discretisation errors $\mathcal{O}(am_c)$ and $\mathcal{O}(am_b)$ under control.

Perez-Rubio, Collins, Bali 1503.08440



Charmonium, HPQCD 1411.1318



No disconnected diagrams in $c\bar{c}$ spectrum: OZI suppressed - assumed to be small
$\Rightarrow$ mixing with lighter states not included.

Precision Spectroscopy:
single hadron states near/above thresholds

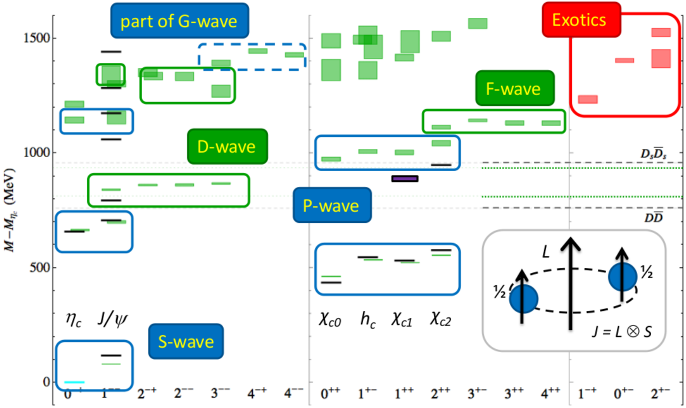
CHARMONIUM: SINGLE HADRON STATES ABOVE THRESHOLD

Precision calculation of high spin ($J \geq 2$) and exotic states is relatively new

Caveat Emptor

- Only single-hadron operators
- Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- Relatively heavy pions ← already changing

Charmonium

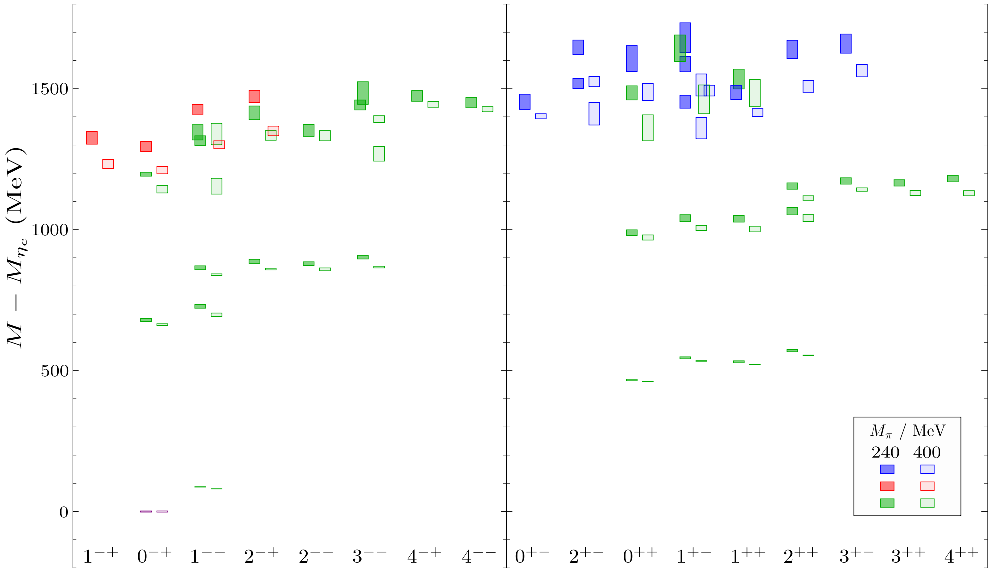


from HSC 2012

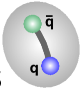
→ Expect improvements now methods established

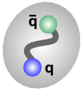
Charmonium with lighter pions - HadSpec

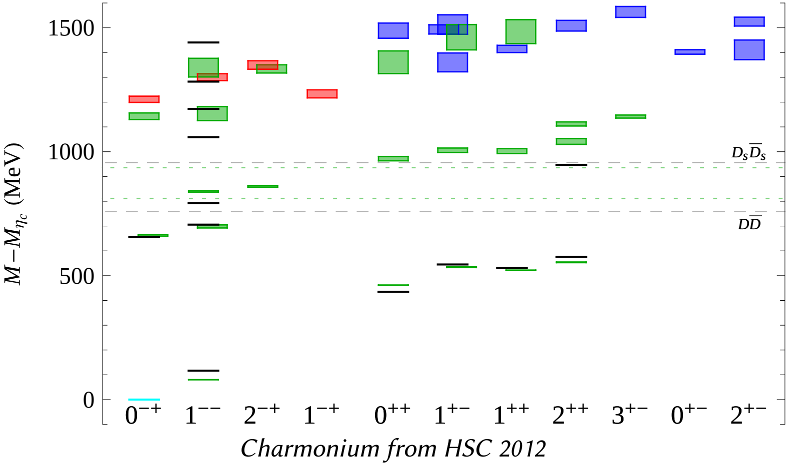
See Gavin Cheung's talk (Wed) for details



Hybrids







Lightest hybrid supermultiplet and excited hybrid supermultiplet same pattern and scale as in open charm and light[*HadSpec*:1106.5515] sectors.

Scattering and resonances:

in charmonium and elsewhere

Scattering in a Euclidean theory

Recall, lose direct access to scattering in (Euclidean) lattice calculations

- The large Euclidean time limit dominated by threshold or off-shell states. Maiani-Testa, PLB (1990).
- Analytic continuation of numerical (euclidean) correlators is and ill-posed problem.

Lüscher found a way to extract $\pi\pi \to \pi\pi$ scattering information in the elastic region from LQCD . [NPB354, 531-578 (1991)]

- Use the finite volume as a tool
- Related lattice energy levels in a finite volume to a decomposition of the scattering amplitude in partial waves in infinite volume

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$

and $\cot\phi$ a known function (containing a generalised zeta function).

- Also known from perturbative NR-QM. Huang-Yang PRD105 (1957)
- To use this idea many technical improvements were needed. Eg need disconnected diagrams and energy levels at extraordinary precision. This is why it has taken a while ...

The method has been generalised for

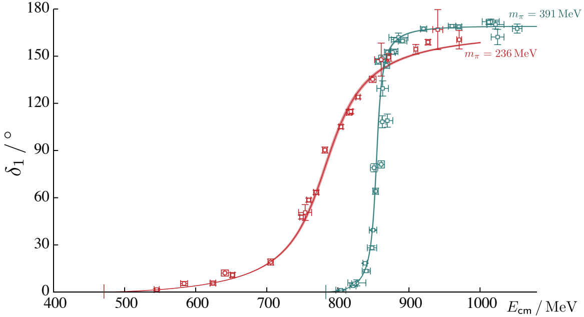
- **moving frames; non-identical particles; multiple two-particle channels, particles with spin**

  Rummukainen and Gottlieb, Nucl. Phys. B450, 397 (1995)
  Beane, Bedaque, Parreno, and Savage, Nucl. Phys. A747, 55 (2005)
  Kim, Sachrajda, and Sharpe, Nucl. Phys. B727, 218 (2005)
  Christ, Kim, Yamazaki, Phys. Rev. D72, 114506 (2005)
  Bernard, Lage, Meissner, and Rusetsky, JHEP, 1101, 019 (2011)
  Hansen and Sharpe, Phys.Rev. D86 (2012) 016007
  Briceño and Davoudi, Phys.Rev. D88 (2013) 094507
  Li and Liu, Phys. Rev. D87, 014502 (2013)
  Briceño, Phys. Rev. D 89, 074507 (2014)

- The precision and robustness of some numerical implementations is now very impressive. *[See talks at Lattice 2015/6]*
- No practical implementation of three-coupled channels but formalism exists. No generalisation beyond ...

Using the method

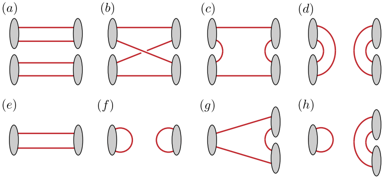
isovector $\pi\pi$ scattering

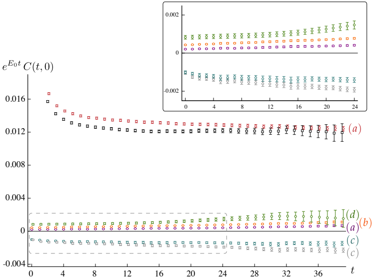


Hadron Spectrum Collaboration

- coupled channel analysis $\pi\pi$, $K\bar{K}$ at $m_\pi = 236$MeV.
- increasingly complex systems studied mainly for light mesons including $\pi\pi$, $\pi K - \eta K$ and $\pi\eta - K\bar{K}$.

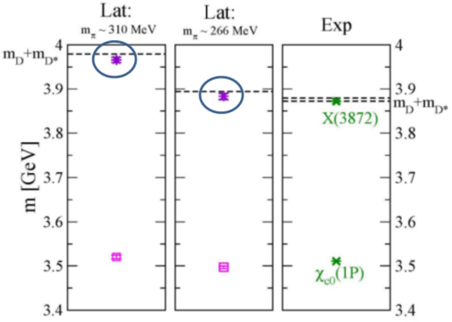
- includes all Wick contractions [enabled by distillation]



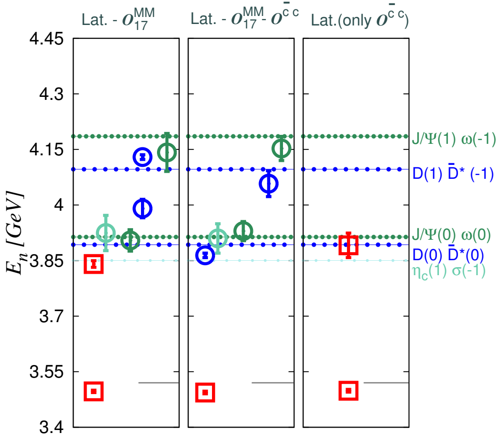


X(3872) - a first look (no coupled channels)

Prelovsek & Leskovec 1307.5172



Padmanath, Lang, Prelovsek 1503.03257



ground state: $\chi_{c1}(1P)$
$D\bar{D}^*$ scattering mx: pole just below thr.
Location of thr., finite vol effects controlled?

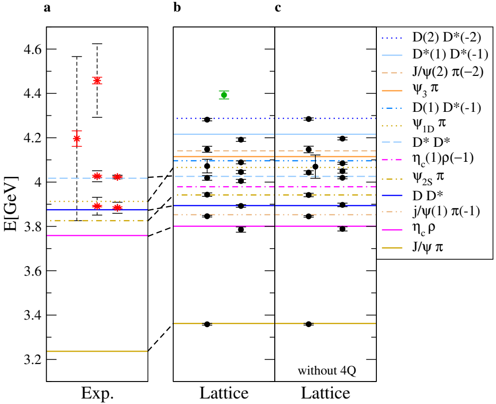
X(3872) not found if $c\bar{c}$ not in basis.

Also results from Lee et al 1411.1389

Within 1MeV of $D^0\bar{D}^{0*}$ and 8MeV of D^+D^* thresholds: isospin breaking effects important?

Z_c^+ - First look on the lattice (no coupled channels)

Manifestly exotic hadron i.e. does not fit in the quark model picture.



Prelovsek, Lang, Leskovec, Mohler: 1405.7615

- 13 expected 2-meson e'states found (black)
- no additional state below 4.2GeV
- no Z_c^+ candidate below 4.2GeV

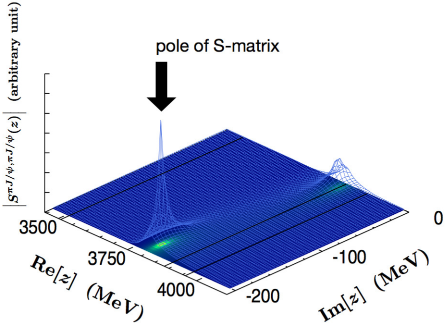
Similar conclusion from Lee et al [1411.1389] and Chen et al [1403.1318]

Why no eigenstate for Z_c? Is Z_c^+ a coupled channel effect? What can other groups say? Work needed!

Recent work: Z_c^+

HAL QCD recently did a first coupled-channel analysis [1602.03465].

$\pi J/\Psi - \rho\eta_c - \bar{D}D^*$



Challenges:

- The Z_c^+ (and most of the XYZ states) lies above several thresholds and so decay to several two-meson final states
- requires a coupled-channel analysis for a rigorous treatment
- on a lattice the number of relevant coupled-channels is large for high energies.

- Potential method, rather different to Lüscher.
- Suggest Z_c^+ a threshold cusp. Pole found *below* $\bar{D}D^*$ - not a resonance.

Other lattice results are important to test this result

MANY OTHER STATES BEING INVESTIGATED

Tetraquarks:

- Double charm tetraquarks ($J^P = 1^+, I = 0$) by HALQCD [PLB712 (2012)]
  - attractive potential, no bound tetraquark state
- Charm tetraquarks: variational method with DD^*, D^*D^* and tetraquark operators finds no candidate.
- Preliminary results for meson-meson and tetraquarks in $c\bar{c}1^{++}, I = 1$ in talk by Cheung, for HadSpec, at this conference.

Y(4140)

- Ozaki and Sasaki [1211.5512] - no resonant Y(4140) structure found
- Padmanath, Lang, Prelovsek [1503.03257] considered operators: $c\bar{c}$, $(\bar{c}s)(\bar{s}c)$, $(\bar{c}c)(\bar{s}s)$, $[\bar{c}\bar{s}][cs]$ in $J^P = 1^+$. Expected 2-particle states found and χ_{c1}, $X(3872)$ **not** Y(4140).

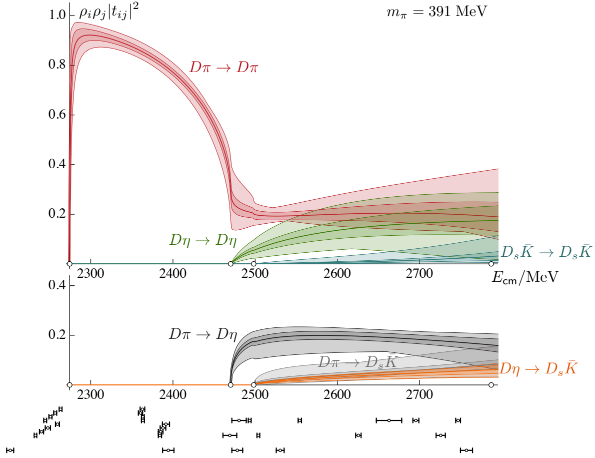
⋮

Prospects for charmonium physics

- There have been many “first looks” at exotic, XYZ charmonium states in 2012-2015.
- The existing framework for coupled channel analysis provides a path forward to study these states in more realistic scenarios.
- First coupled channel results with charm quarks by HAL QCD suggest Z_c^+ is not a resonance.
- The first coupled channel study with charm quarks, by HadSpec, was presented by Moir at this conference [1607.07093]. Similar studies for $c\bar{c}$ planned and a first look with tetraquark operators also presented.
- Charmonium radiative transitions also underway. Note resonant amplitudes also now studied: $\pi^+\gamma \to \rho \to \pi^+\pi^0$ in [Briceño et al, HadSpec 1604.03530].

$D\pi - D\eta - D_s\bar{K}$ scattering

S-wave, I=1/2



Moir, Peardon, SR, Thomas, Wilson

- Find a bound state just below threshold: $m = (2275.9 \pm 0.9)$ MeV.
- $D\pi$ threshold is (2276.4 ± 0.9) MeV and cf $D_0^*(2400)$
- In P-wave (1^-) a deeply bound pole - cf $D^*(2007)$.
- In D-wave (2^+) a narrow resonance - cf $D_2^*(2460)$.

Summary & Outlook

- Lattice QCD provides a robust formalism for calculating non-perturbative phenomena.
- New ideas are allowing rapid progress for the spectroscopy of resonances.
- Precision lattice calculations of excited and exotic hadron states available
  - includes hybrids and other exotics treated as "single-hadron" states.
- Studies of resonances, including multiple two-particle channels underway
- Expect significant progress in next few years in e.g X,Y,Z from lattice but there are particular challenges in charmonium
  - a proliferation of thresholds and coupled-channels
  - including disconnected contributions in $c\bar{c}$ still technically difficult.
- Numerically precision achievable but control of other "traditional" systematics is a challenge
- Many theoretical challenges remain e.g. no general framework for extracting scattering amplitudes involving more than two hadrons. Clever ideas needed!

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Thanks for listening!