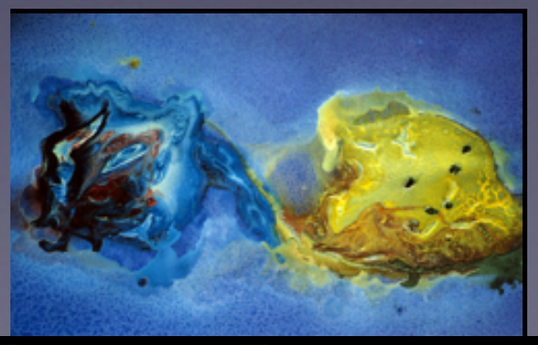
TECHNISCHE UNIVERSITÄT MÜNCHEN

Charmonium EXOTICS with Effective Field Theories



Physik Department TUM
T30f

Nora Brambilla

- X, Y, Z the new revolution
  - Charmonium a golden system
    - Our present understanding of charmonium below threshold: EFTs and lattice
- Charmonium at and above threshold
  - EFT for charmonium hybrids: results and comparison to the data

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutr members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment ($\#\sigma$)	Year	Statu
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\,...$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi)\,...$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma\, J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5) LHCb [1003] (> 10)	2005	Ok
				$B \to K(\gamma\, \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2) LHCb [1003] (4.4)	2008	NC!
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2) T. Xiao *et al.* [CLEO data] [1009] (>5)	2013	Ok
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

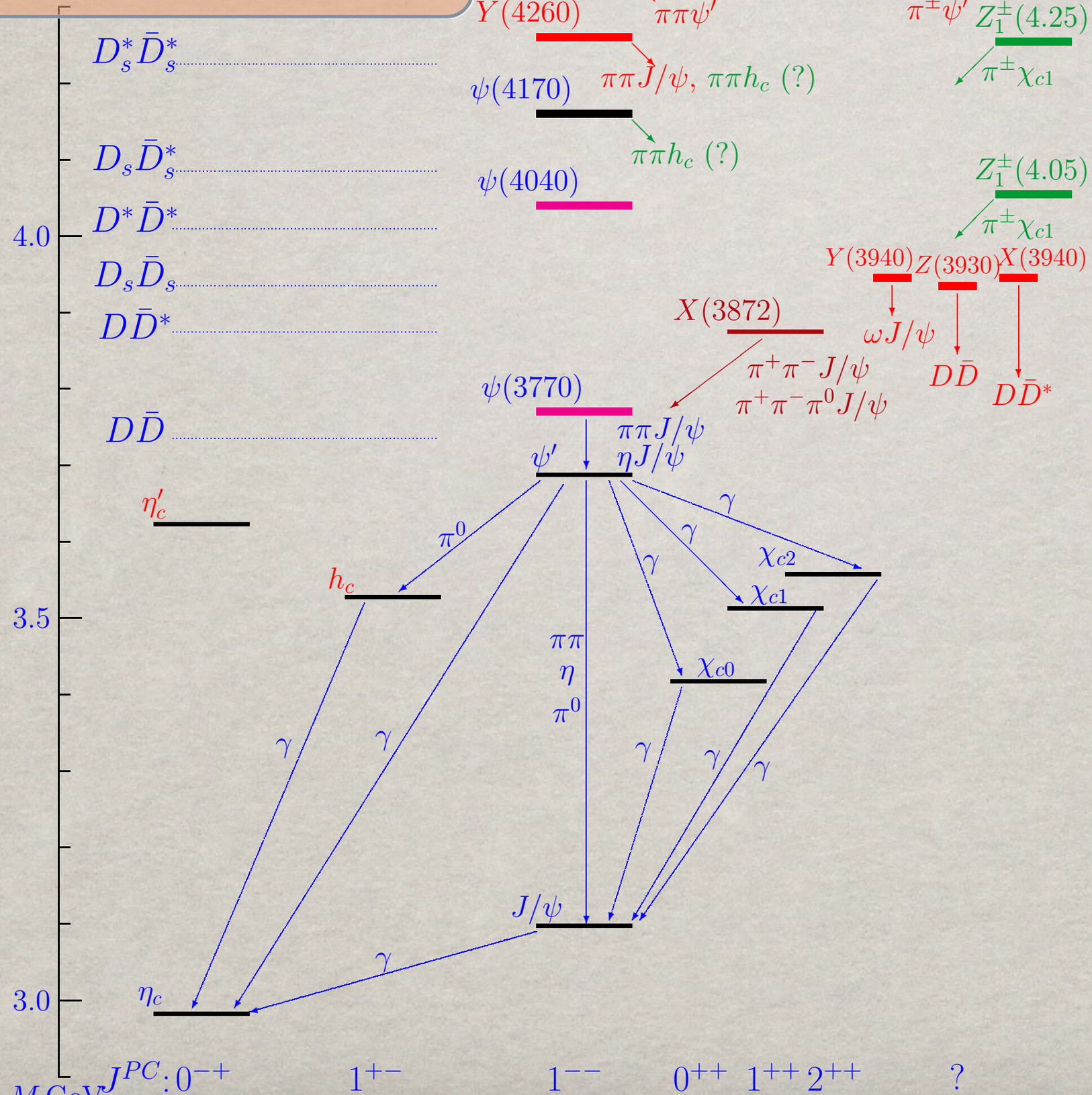
arXiv:1404.3723v1

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment ($\#\sigma$)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5) D0 [1064] (3.1)	2009	NC!
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11) Belle [1008, 1056] (15), BES III [1007] (np)	2005	Ok
				$e^+e^- \to (f_0(980)J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4) LHCb [1077] (13.9)	2007	Ok
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

arXiv:1404.3723v1

Charmonium
the present revolution



$Y(4660)$ — $\pi\pi\psi'$

$4320 \div 4360$ — $\pi\pi\psi'$

$Y(4260)$ — $\pi\pi J/\psi$, $\pi\pi h_c$ (?)

$\psi(4170)$ — $\pi\pi h_c$ (?)

$Z^{\pm}(4430)$ — $\pi^{\pm}\psi'$

$Z_1^{\pm}(4.25)$ — $\pi^{\pm}\chi_{c1}$

$Z_1^{\pm}(4.05)$ — $\pi^{\pm}\chi_{c1}$

$Y(3940)$ — $\omega J/\psi$

$Z(3930)$ — $D\bar{D}$

$X(3940)$ — $D\bar{D}^*$

$X(3872)$ — $\pi^+\pi^- J/\psi$, $\pi^+\pi^-\pi^0 J/\psi$

THE COLLIDER DETECTOR AT FERMILAB

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BELLE

BABAR Collaboration Home Page

BES

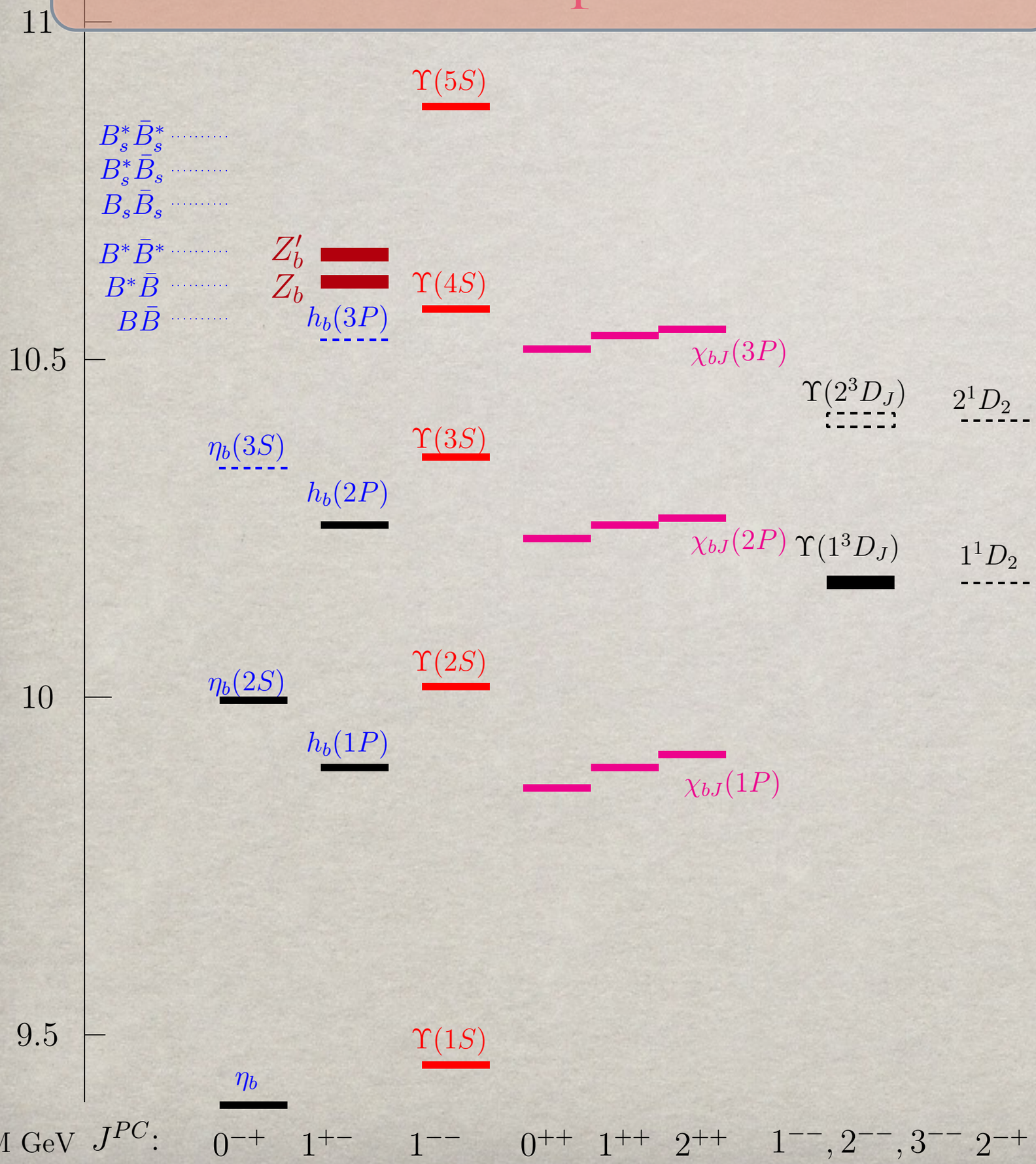
CLEO

ATLAS

CMS

LHCb

bottomonium: the present revolution



THE COLLIDER DETECTOR AT FERMILAB

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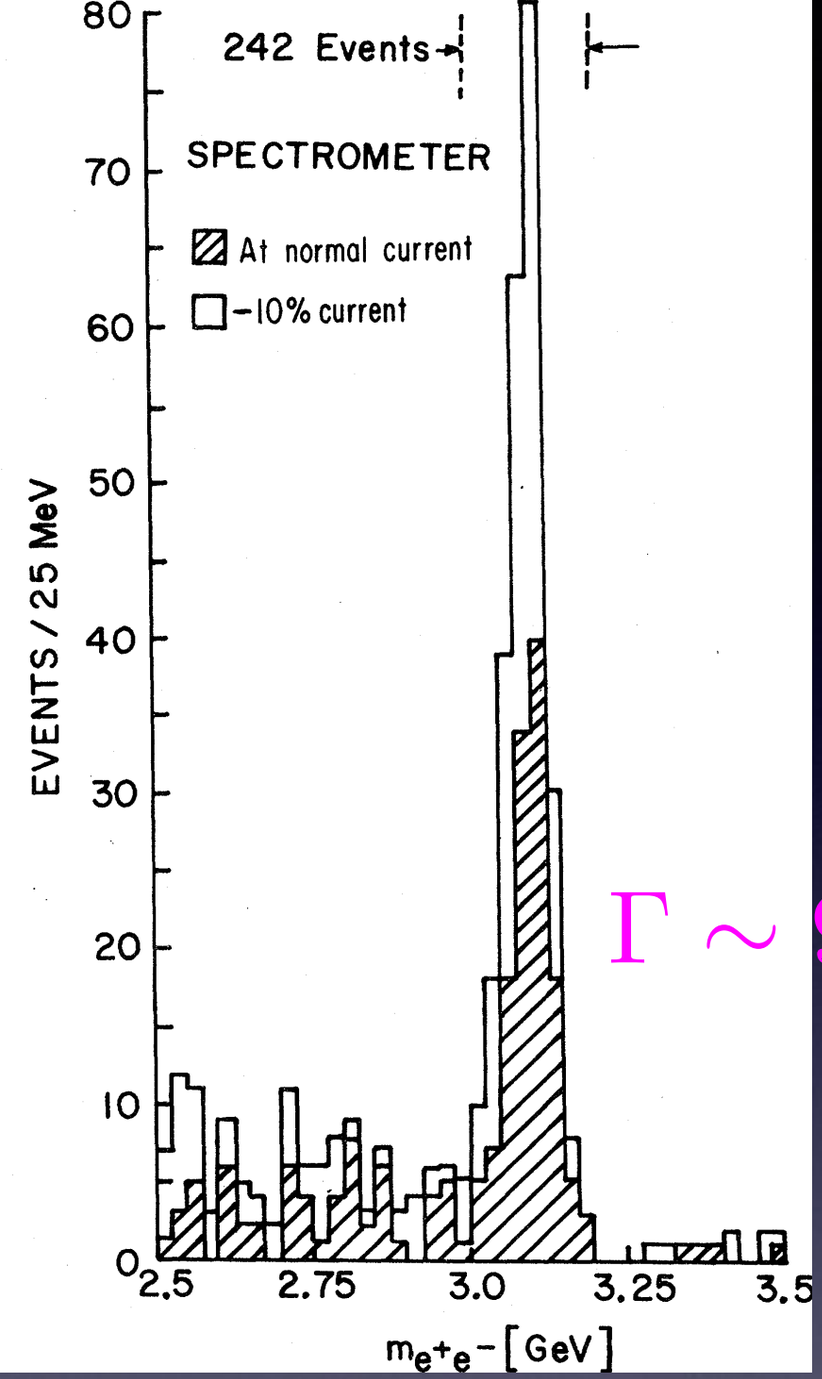
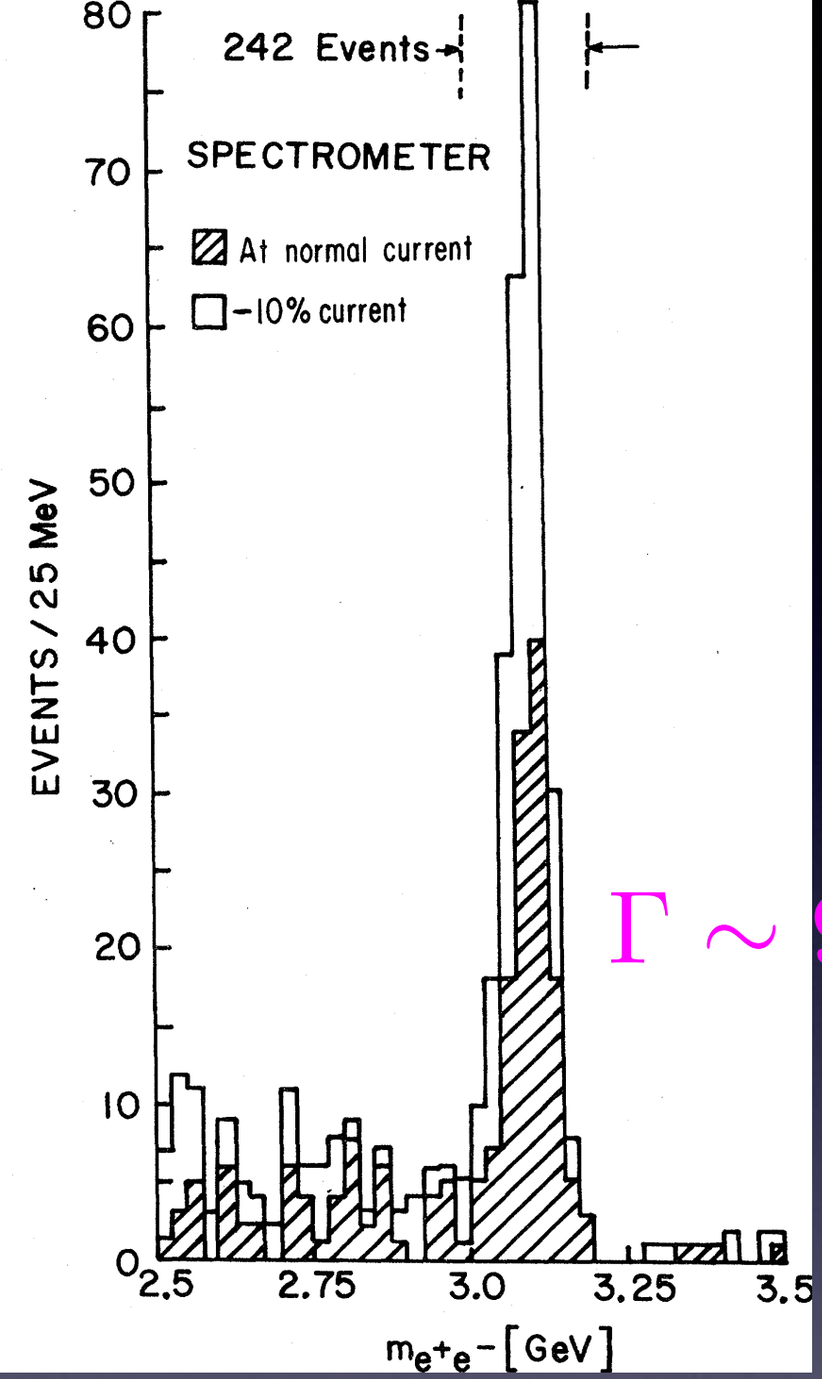
The November revolution in 1974: the J/ψ discovery



$\Gamma \sim 90\,\mathrm{KeV}$

Aubert et al. BNL 74

The November revolution in 1974: the J/ψ discovery

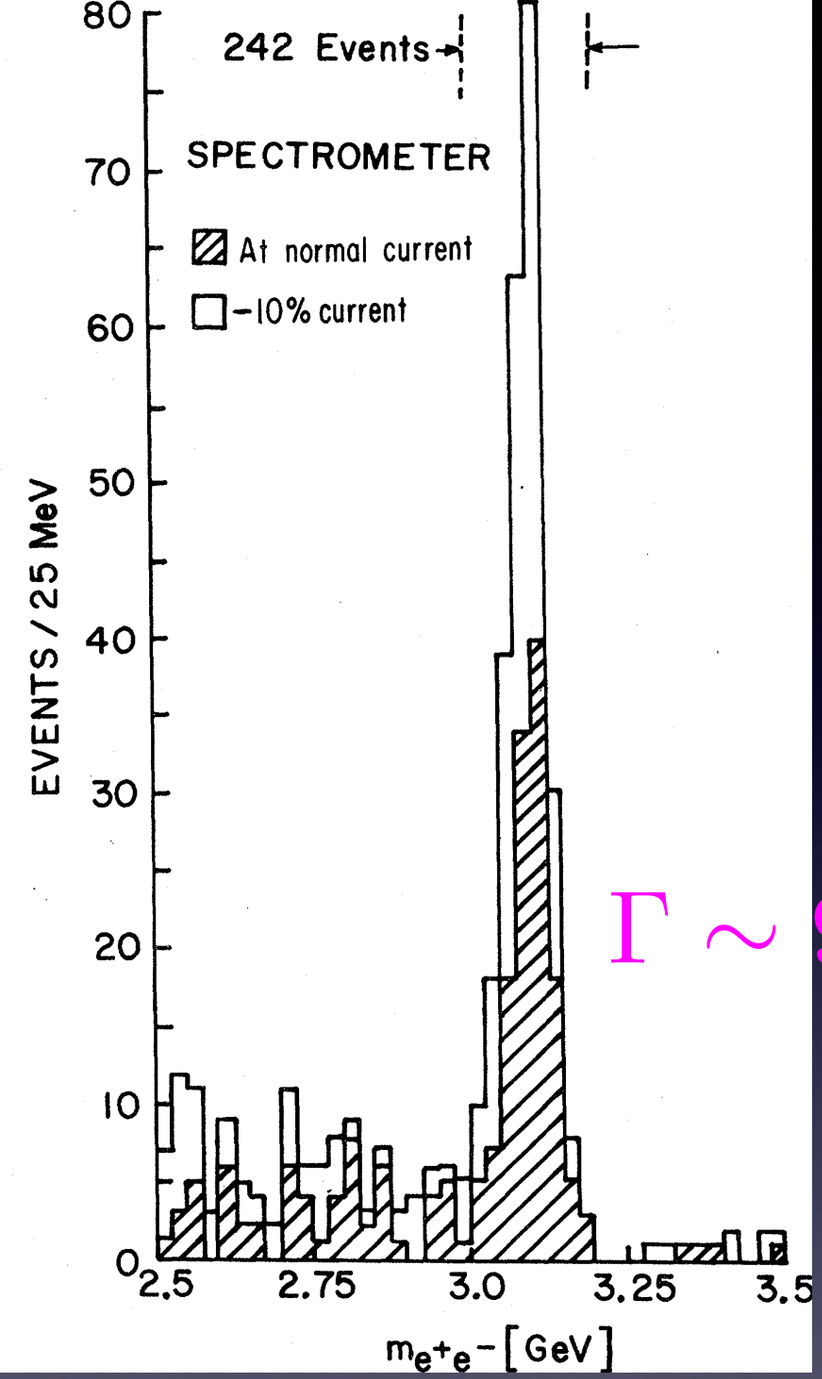


Aubert et al. BNL 74

Samuel Ting: "It is like to stumble on a village where people live 70000 years"

$\Gamma \sim 90\,\mathrm{KeV}$

The November revolution in 1974: the J/ψ discovery



$\Gamma \sim 90\,\mathrm{KeV}$

Samuel Ting: "It is like to stumble on a village where people live 70000 years"

it has been the confirmation of the charm quark prediction and of QCD (strong int theory) foundations

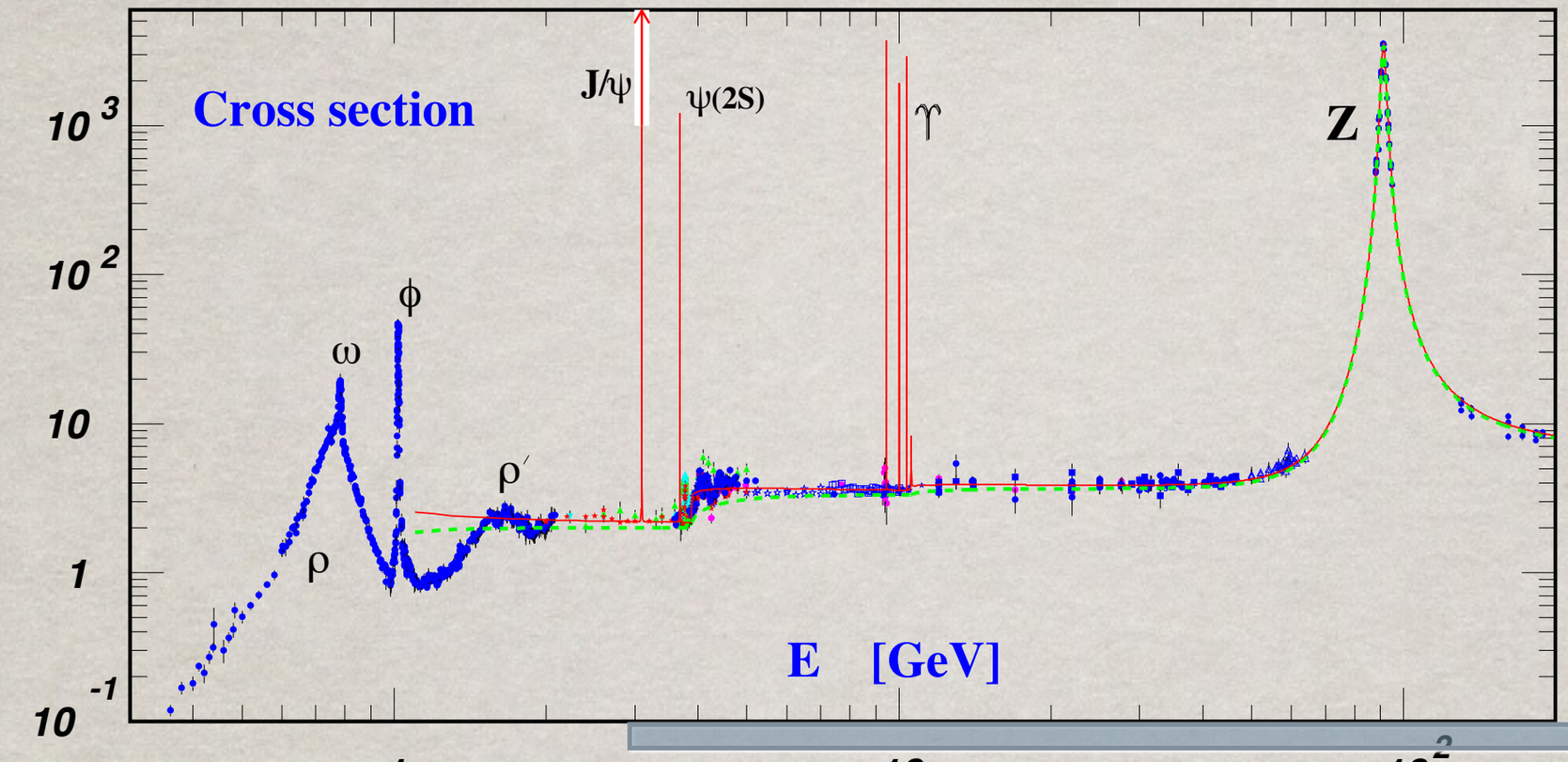
Aubert et al. BNL 74

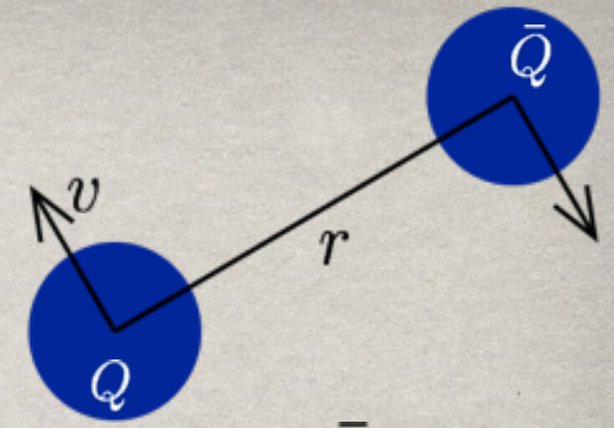
narrow width and asymptotic freedom

annihilation at large scale controlled by small α_s

first discovery of a quark of large mass moving "slowly"

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

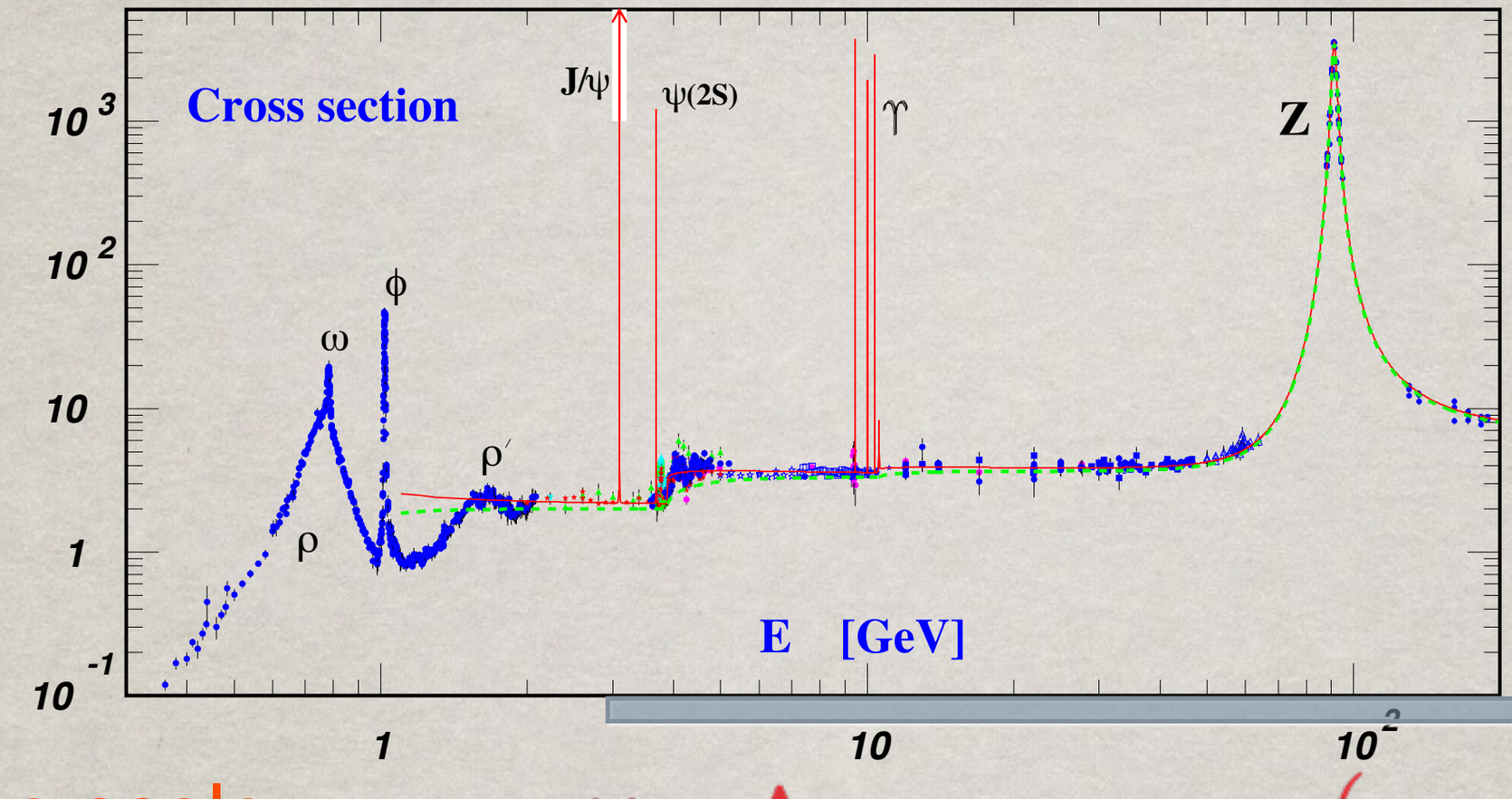
$m_c \sim 1.5\text{GeV}$

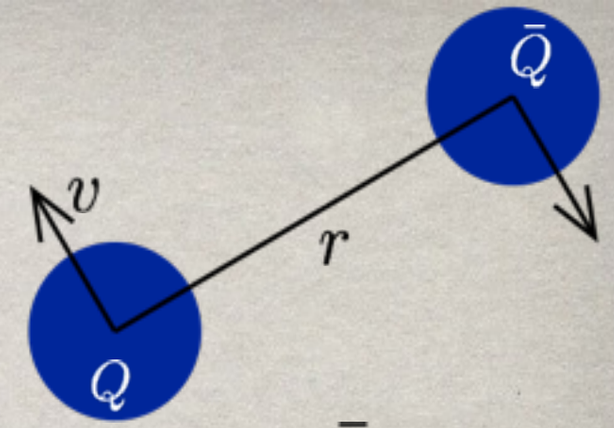
$m_b \sim 5\text{GeV}$

$m_t \sim 170\text{GeV}$

A large scale $m_Q \gg \Lambda_{\rm QCD}$ $\alpha_s(m_Q) \ll 1$

Heavy quarks offer a privileged access





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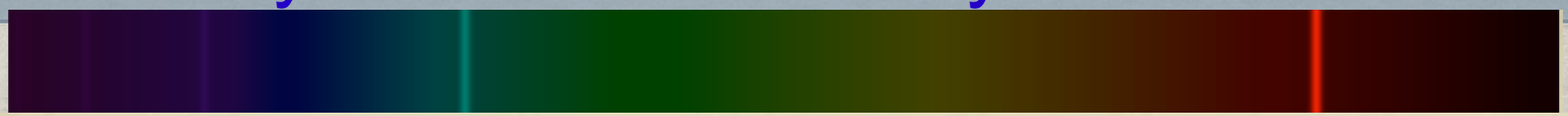
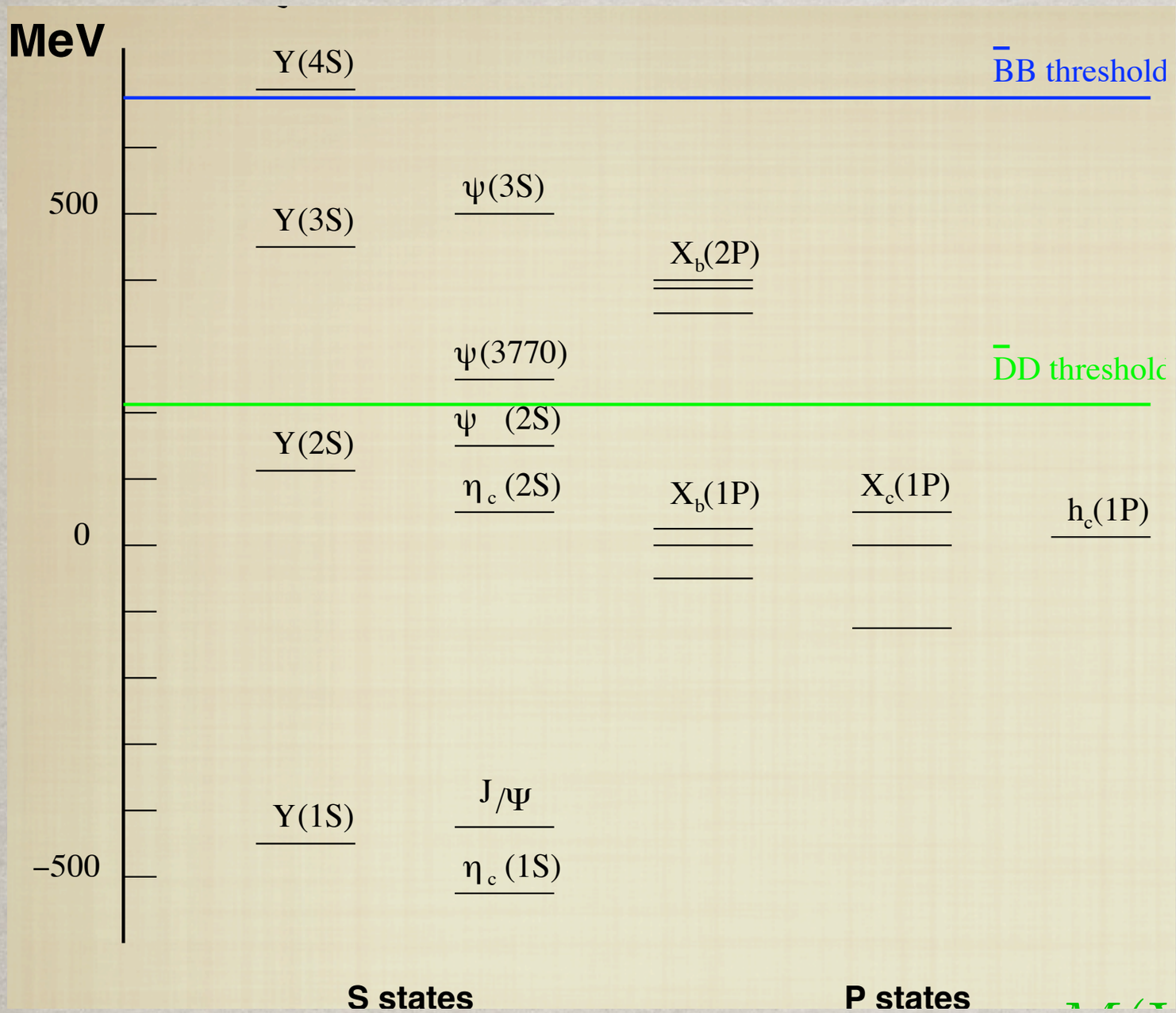
Heavy quarkonia are nonrelativistic bound systems: multiscale systems



many scales: a challenge and an opportunity

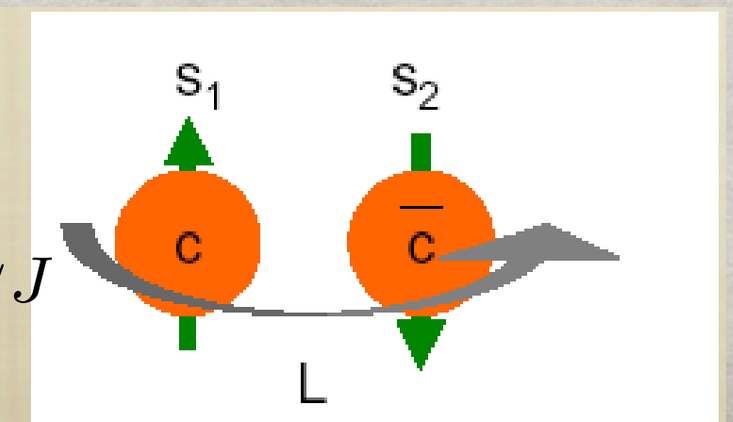
Electromagnetic bound states: atoms, molecules,

Quarkonium scales

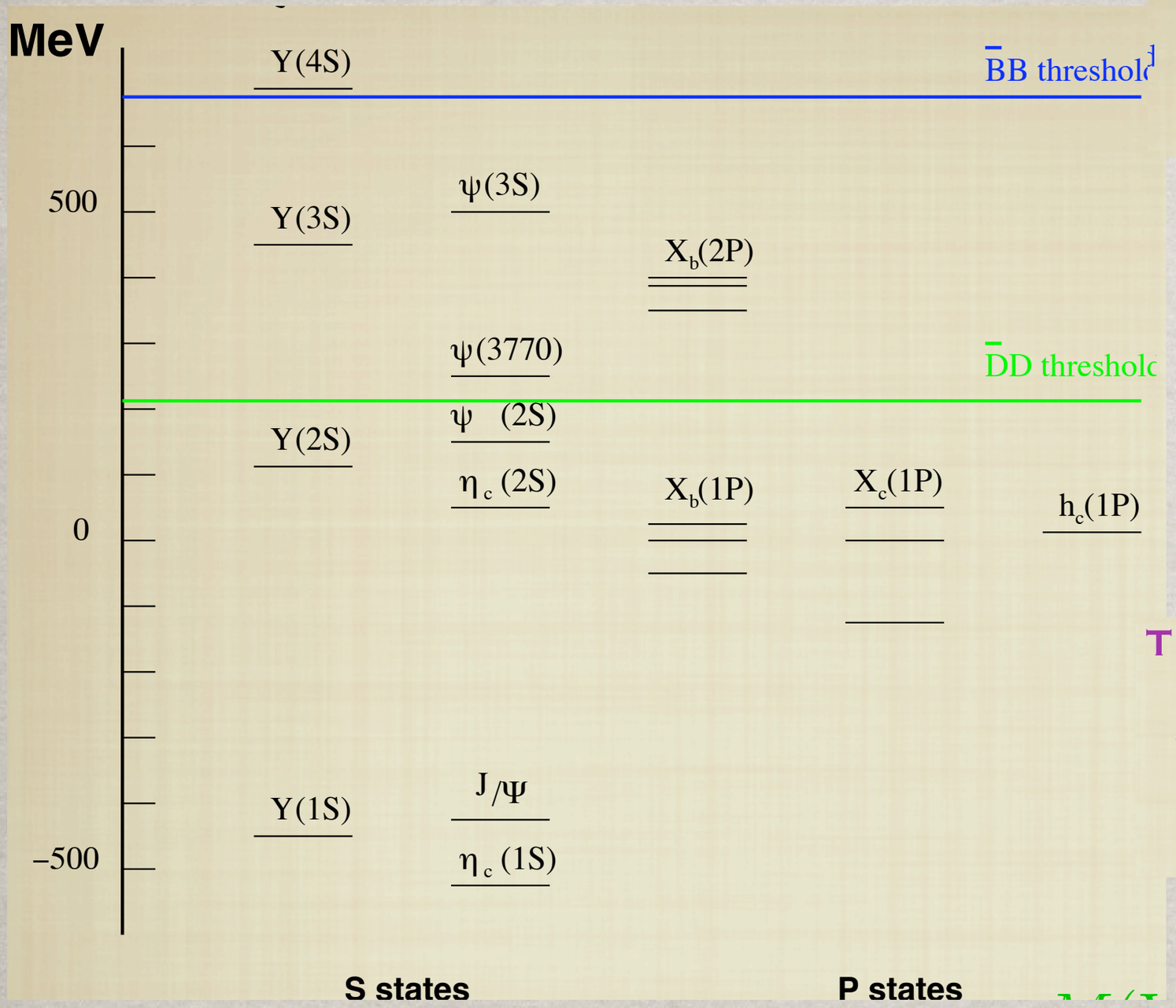


Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

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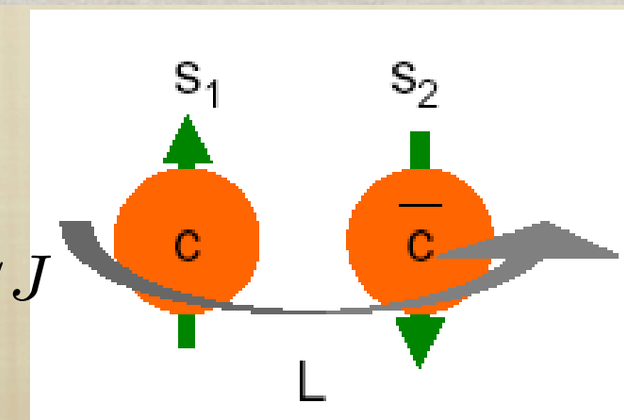


NR bound states have at least 3 scales

$$m \gg mv \gg mv^2 \quad v \ll 1$$

$$mv \sim r^{-1}$$

and $\Lambda_{\rm QCD}$

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

The mass scale is perturbative

$$m_Q \gg \Lambda_{\rm QCD}$$

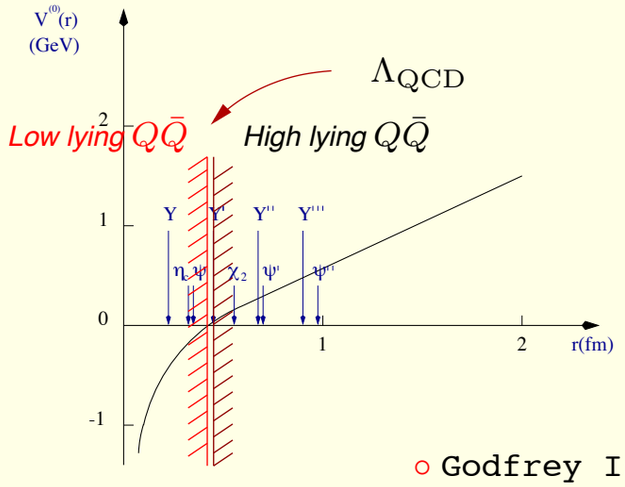
$$m_b \simeq 5\,{\rm GeV}; m_c \simeq 1.5\,{\rm GeV}$$

Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

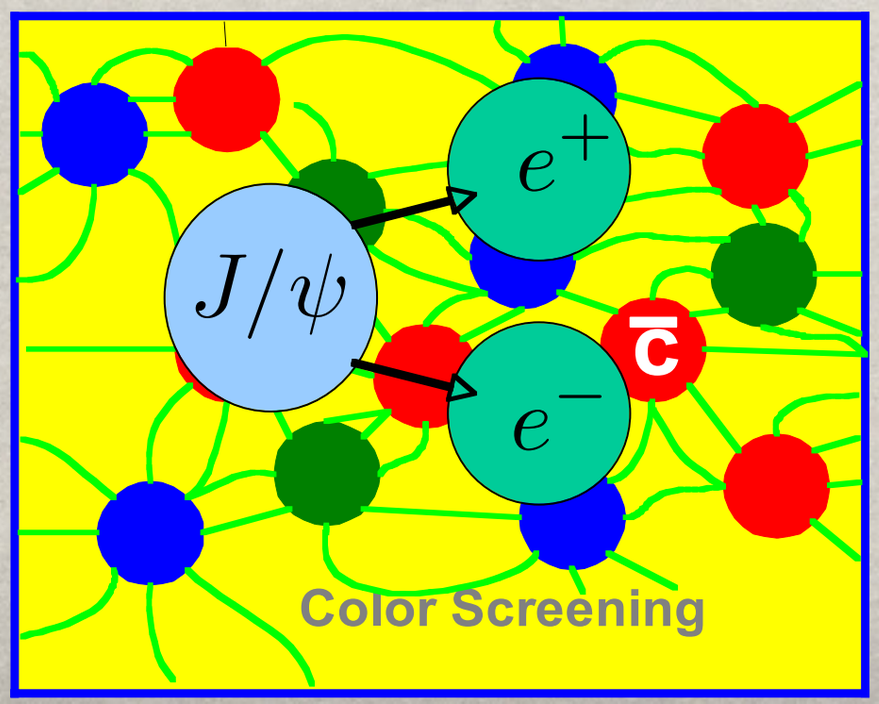
- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



○ Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

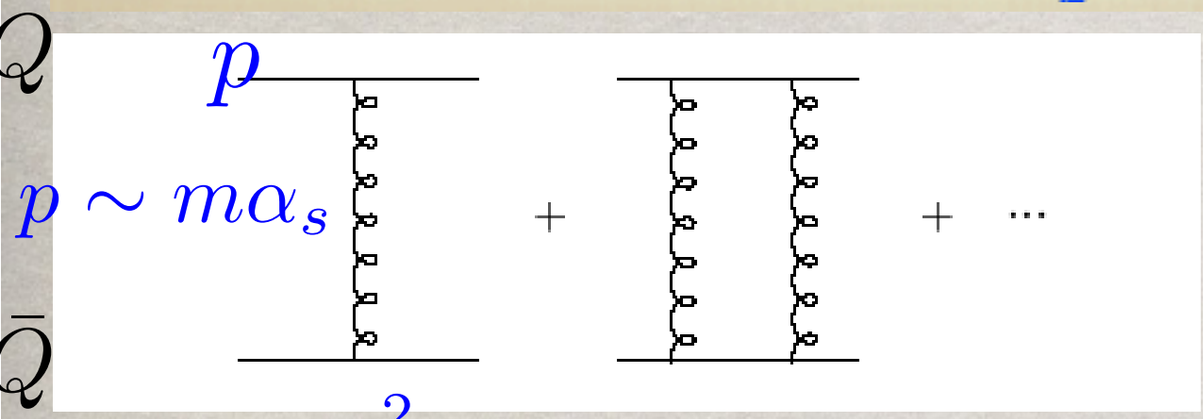
At finite temperature



quarkonia dissociate at different temperatures in dependences of their radius: they are a QGP thermometer

QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$

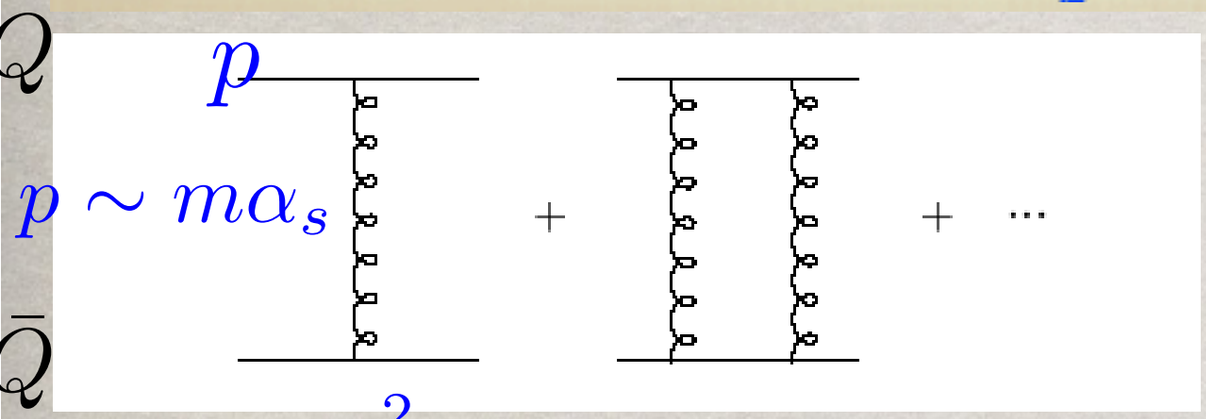


$$\frac{g^2}{p^2}\left(1+\frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E-\left(\frac{p^2}{m}+V\right)}$$

QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$



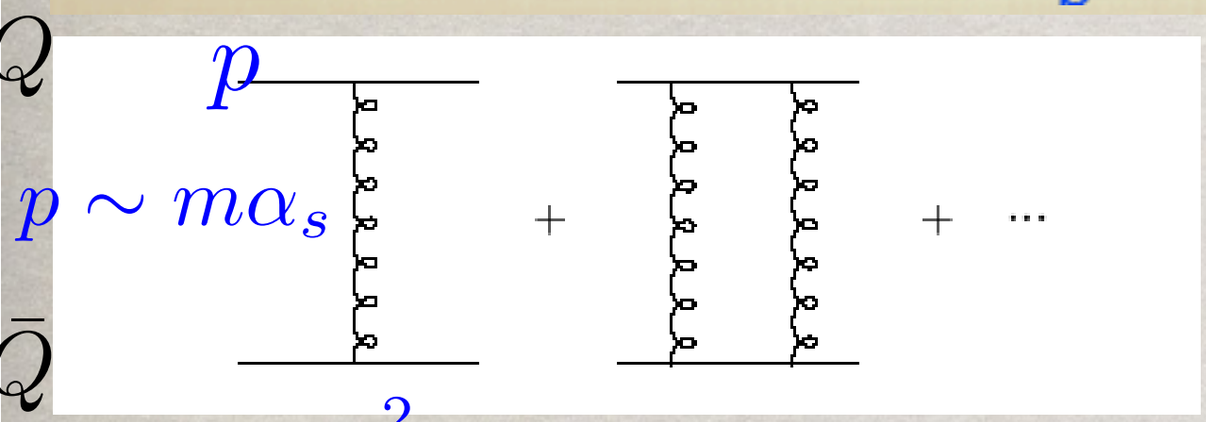
$$\frac{g^2}{p^2}\left(1+\frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E-\left(\frac{p^2}{m}+V\right)}$$

- From $\left(\frac{p^2}{m}+V\right)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m}+V \sim mv^2$.

QCD theory of Quarkonium: a very hard problem

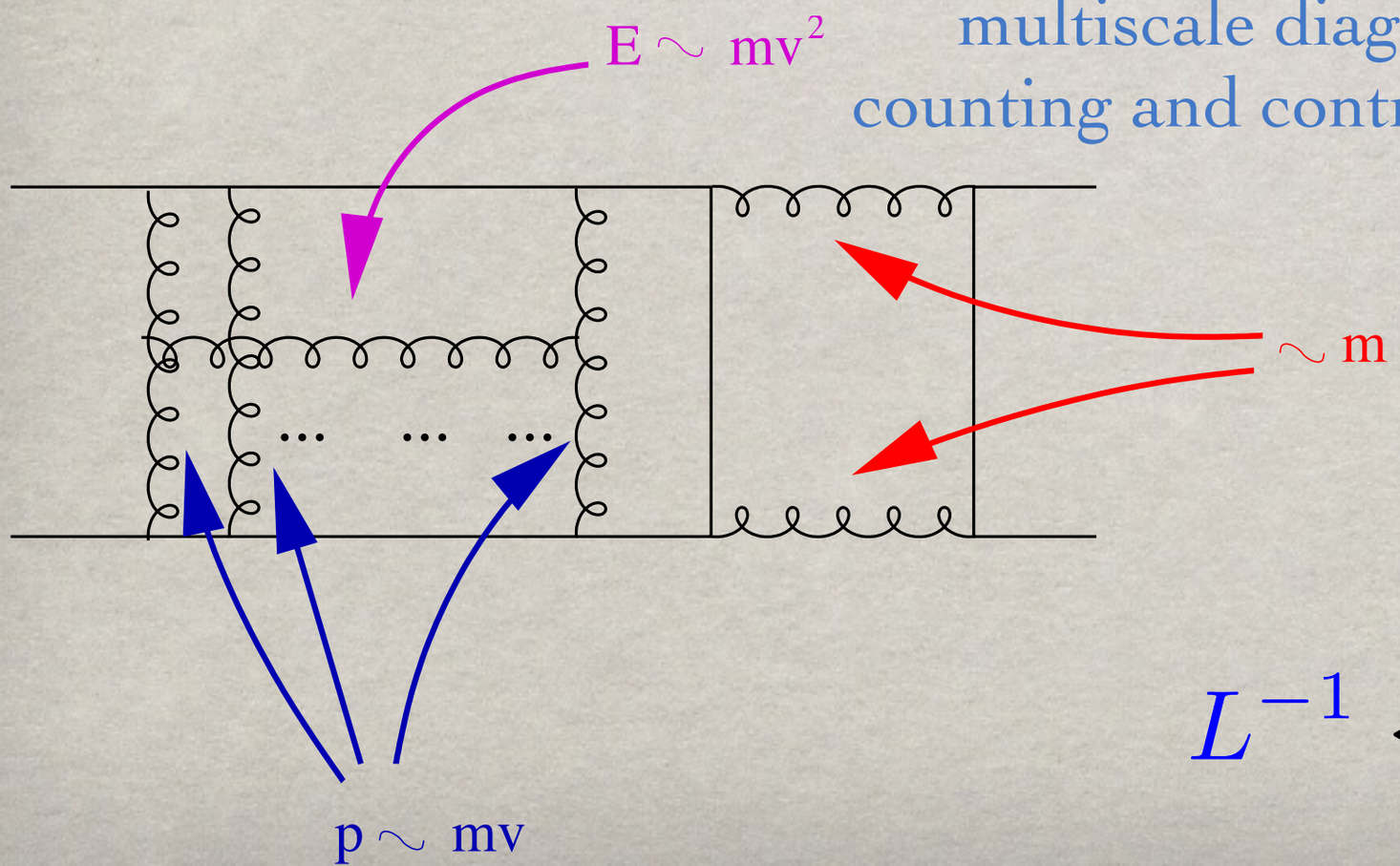
Close to the bound state $\alpha_s \sim v$



$$\frac{g^2}{p^2}\left(1+\frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E-\left(\frac{p^2}{m}+V\right)}$$

- From $\left(\frac{p^2}{m}+V\right)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m}+V \sim mv^2$.

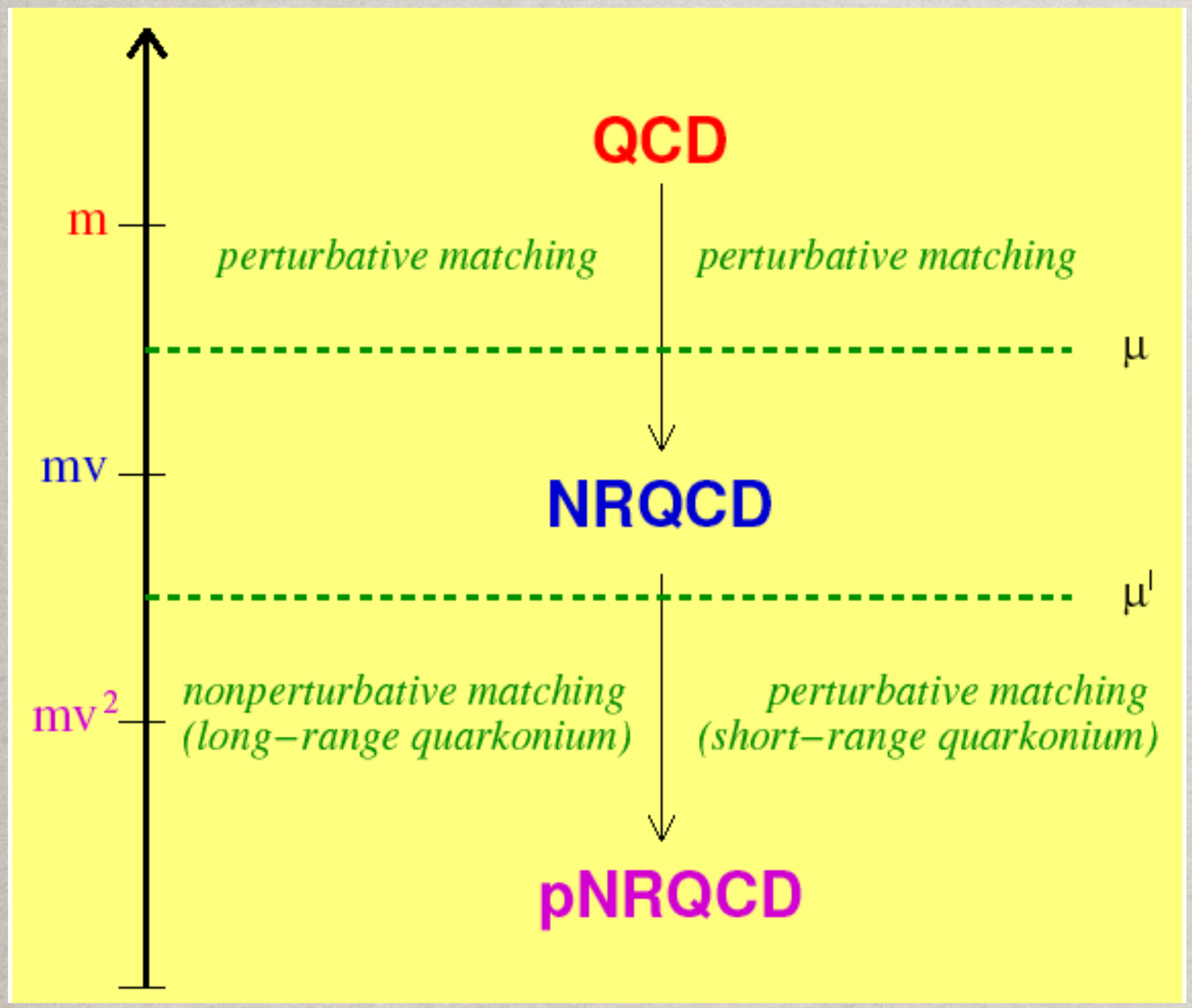


multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Quarkonium with Non relativistic Effective Field Theories



Color degrees of freedom
3X3=1+8
singlet and octet QQbar

Hard

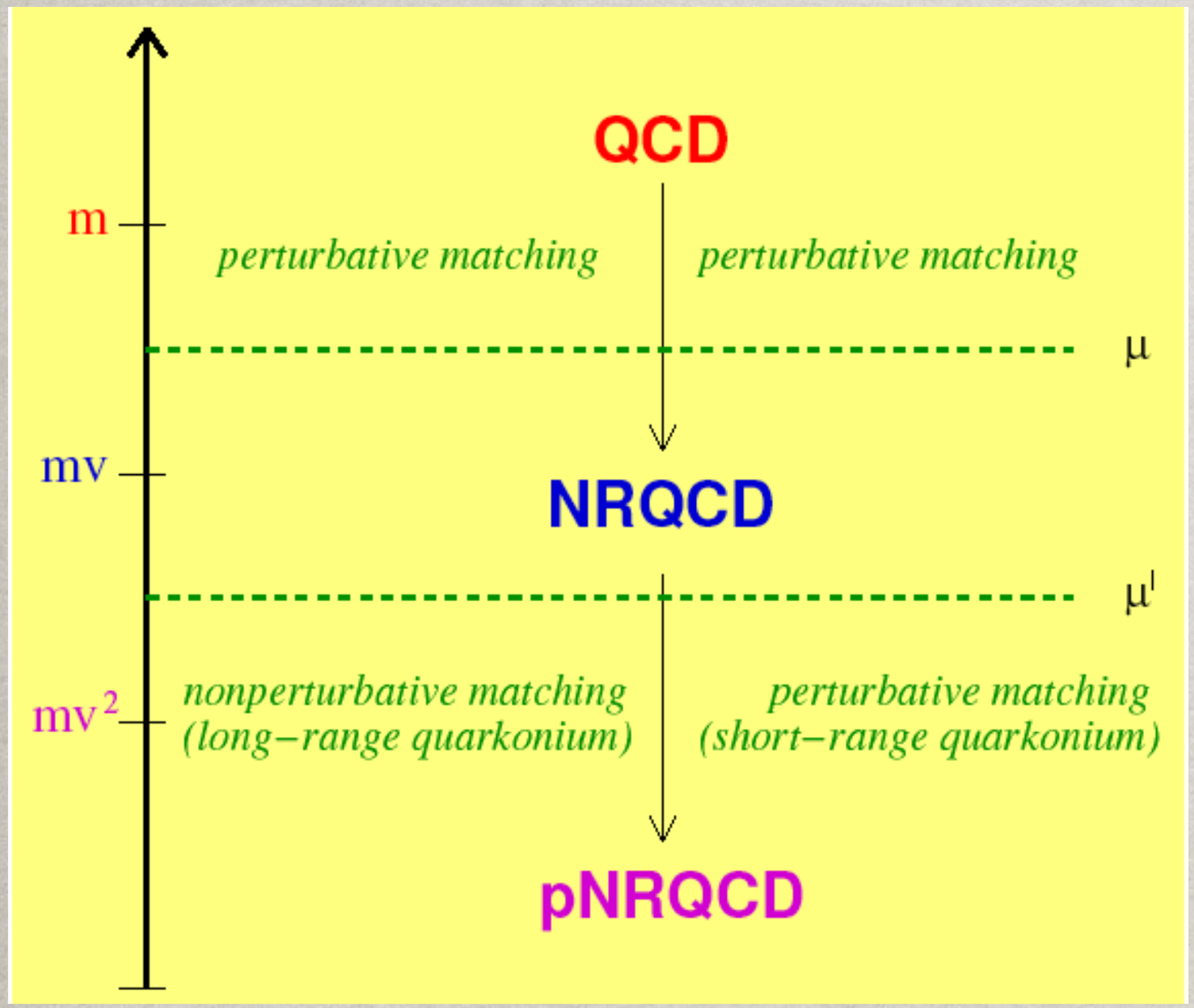
Soft
(relative
momentum)

Ultrasoft
(binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda} \qquad \langle O_n\rangle \sim E_\lambda^n$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
momentum)

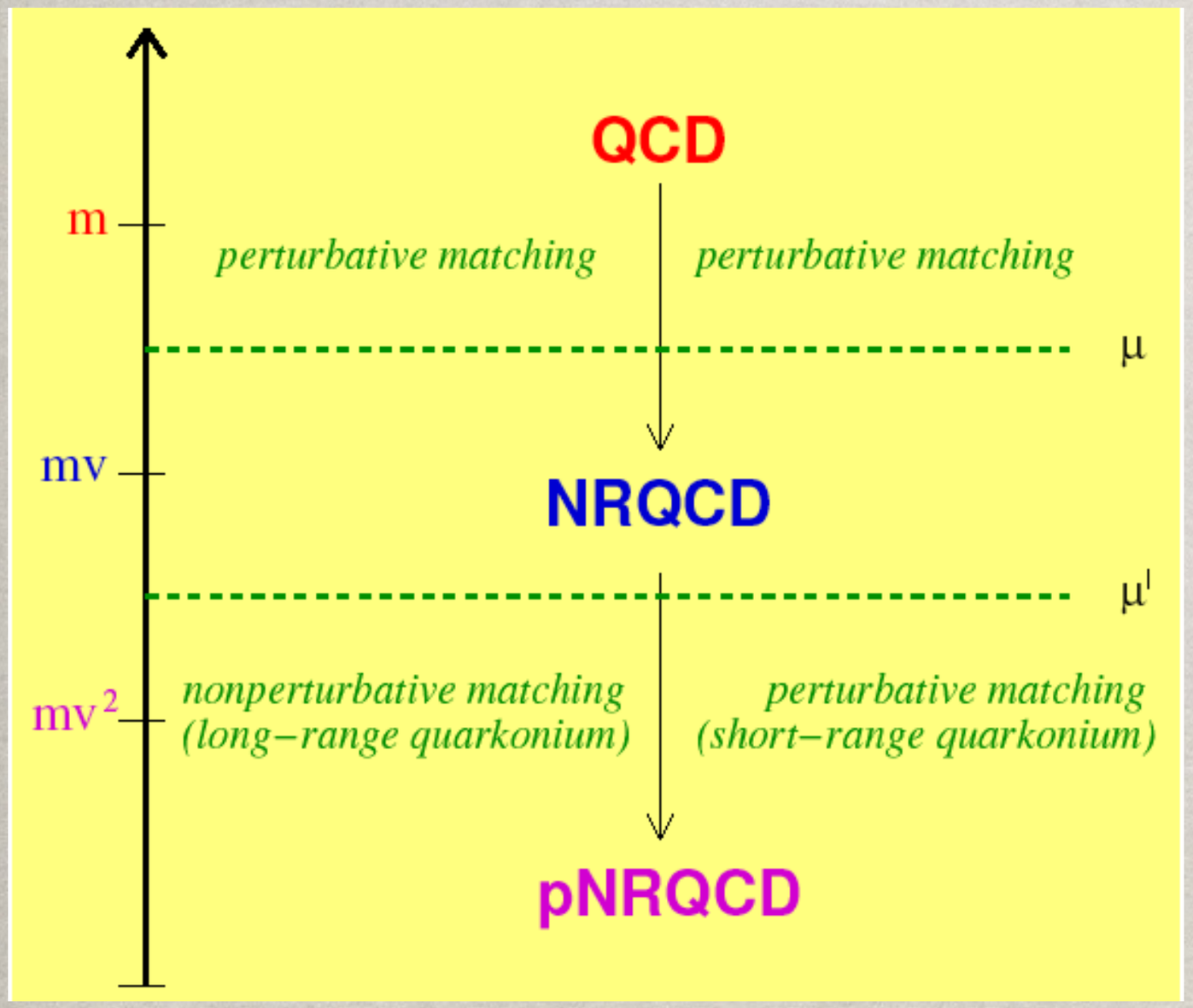
Ultrasoft
(binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
momentum)

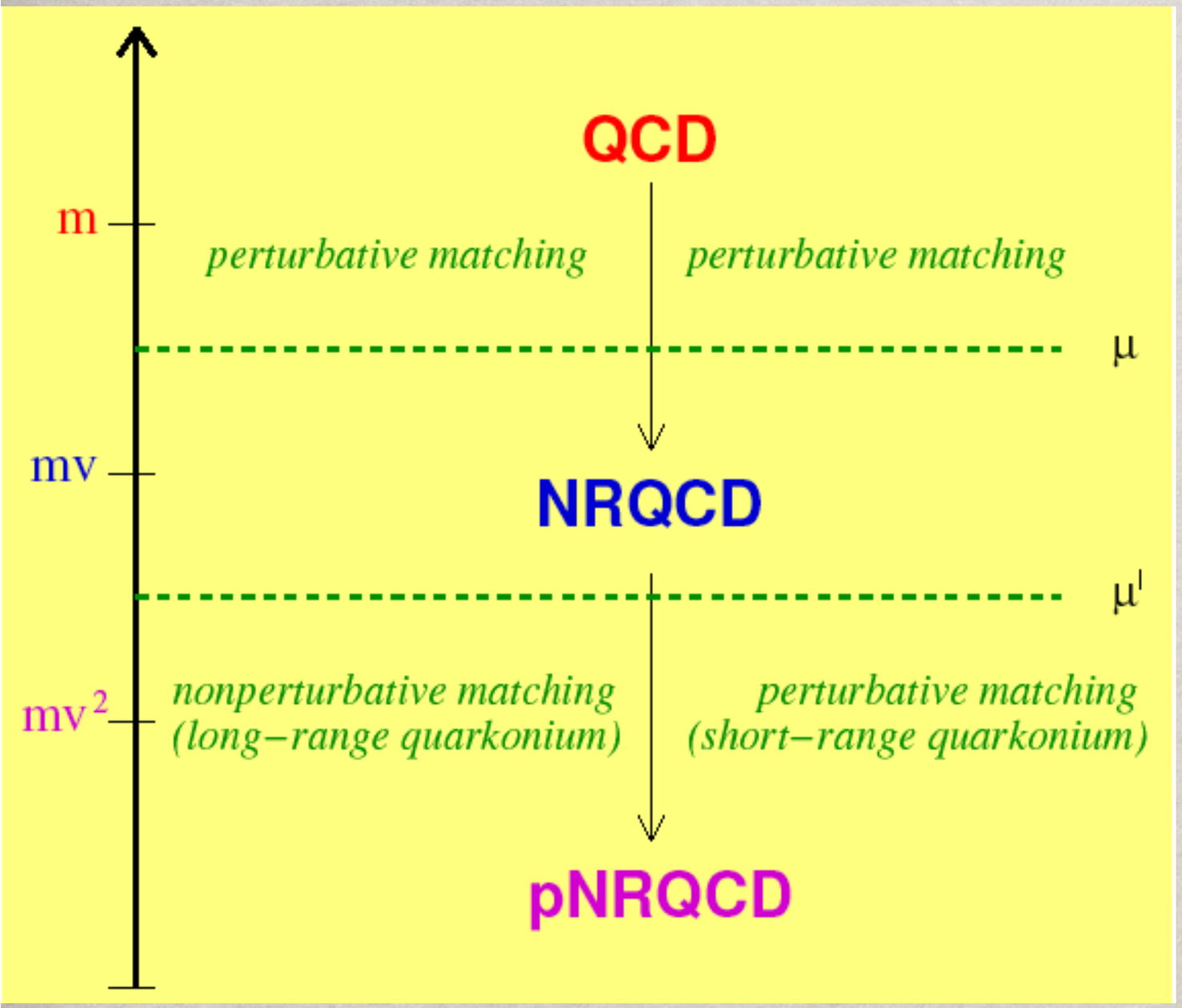
$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

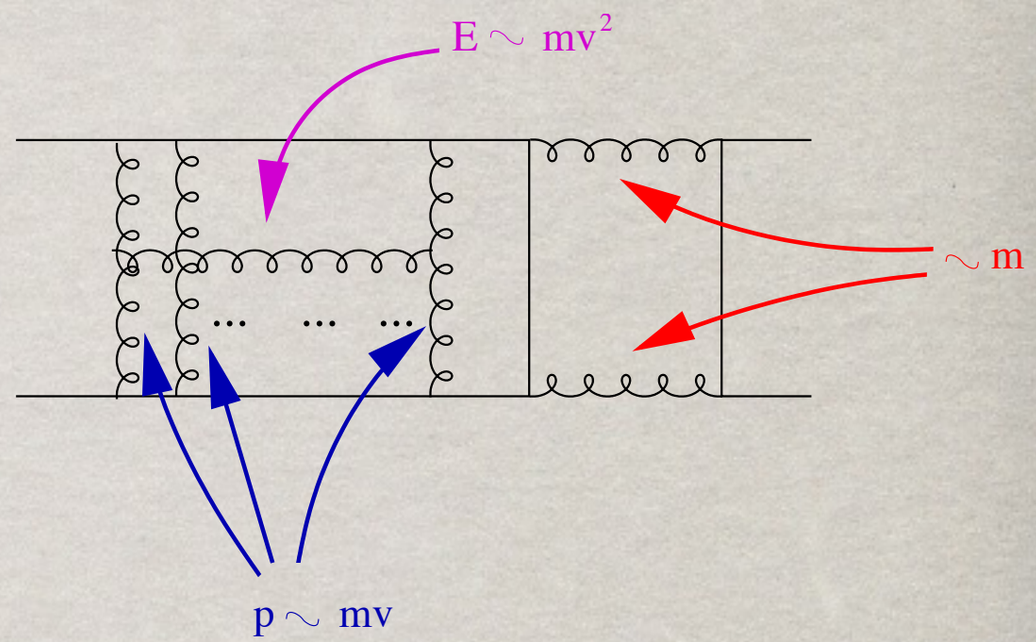
Ultrasoft
(binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

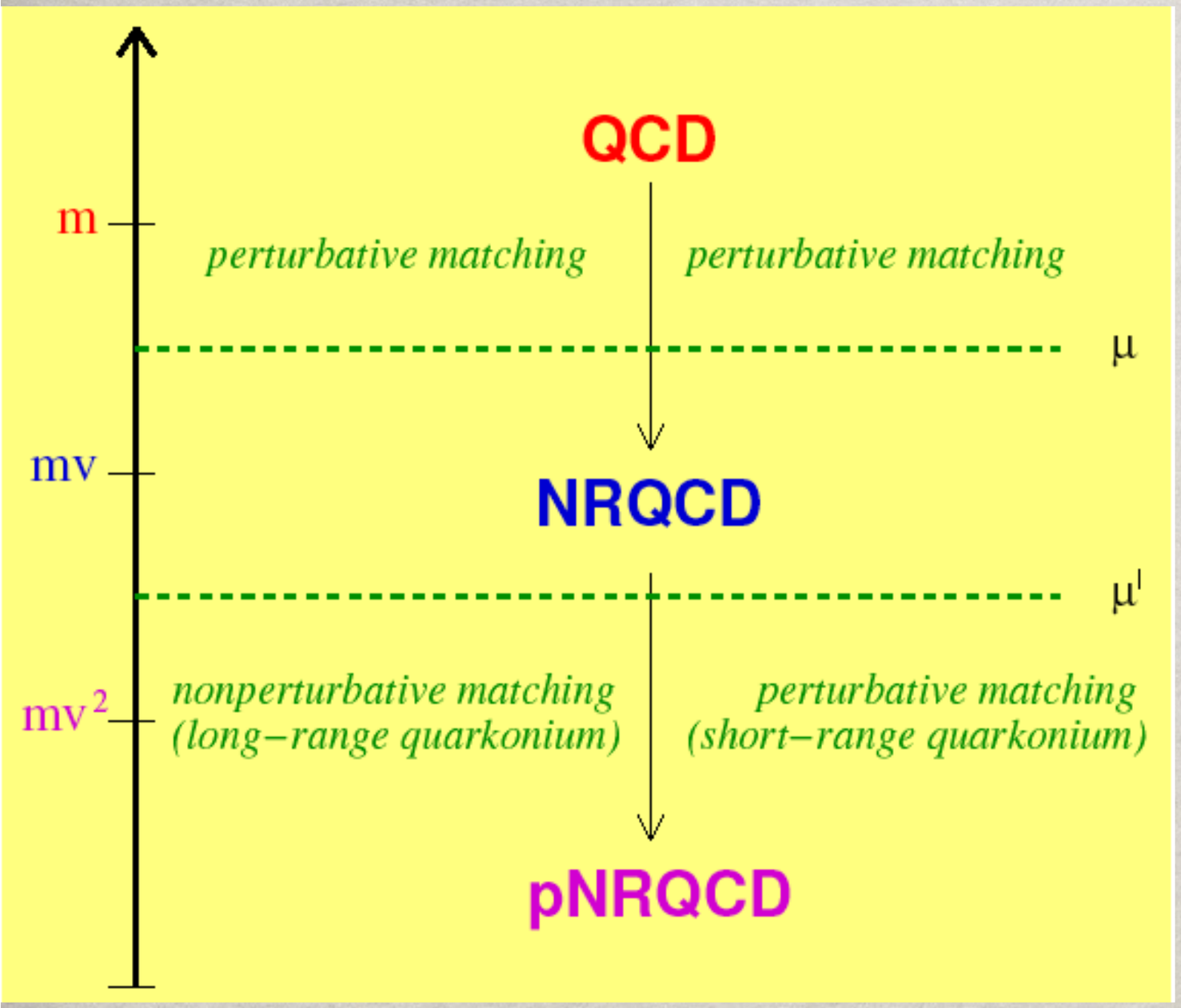
$$\langle O_n \rangle \sim E_\lambda^n$$

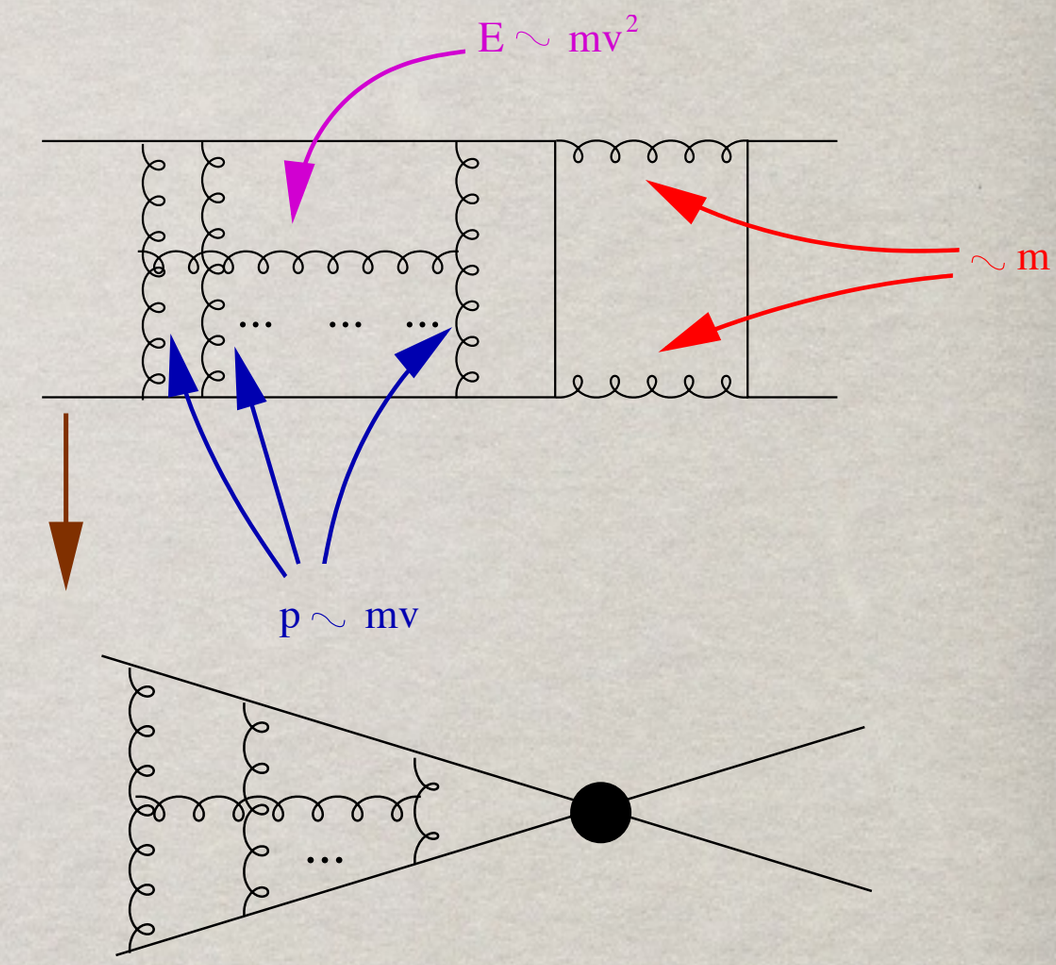
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





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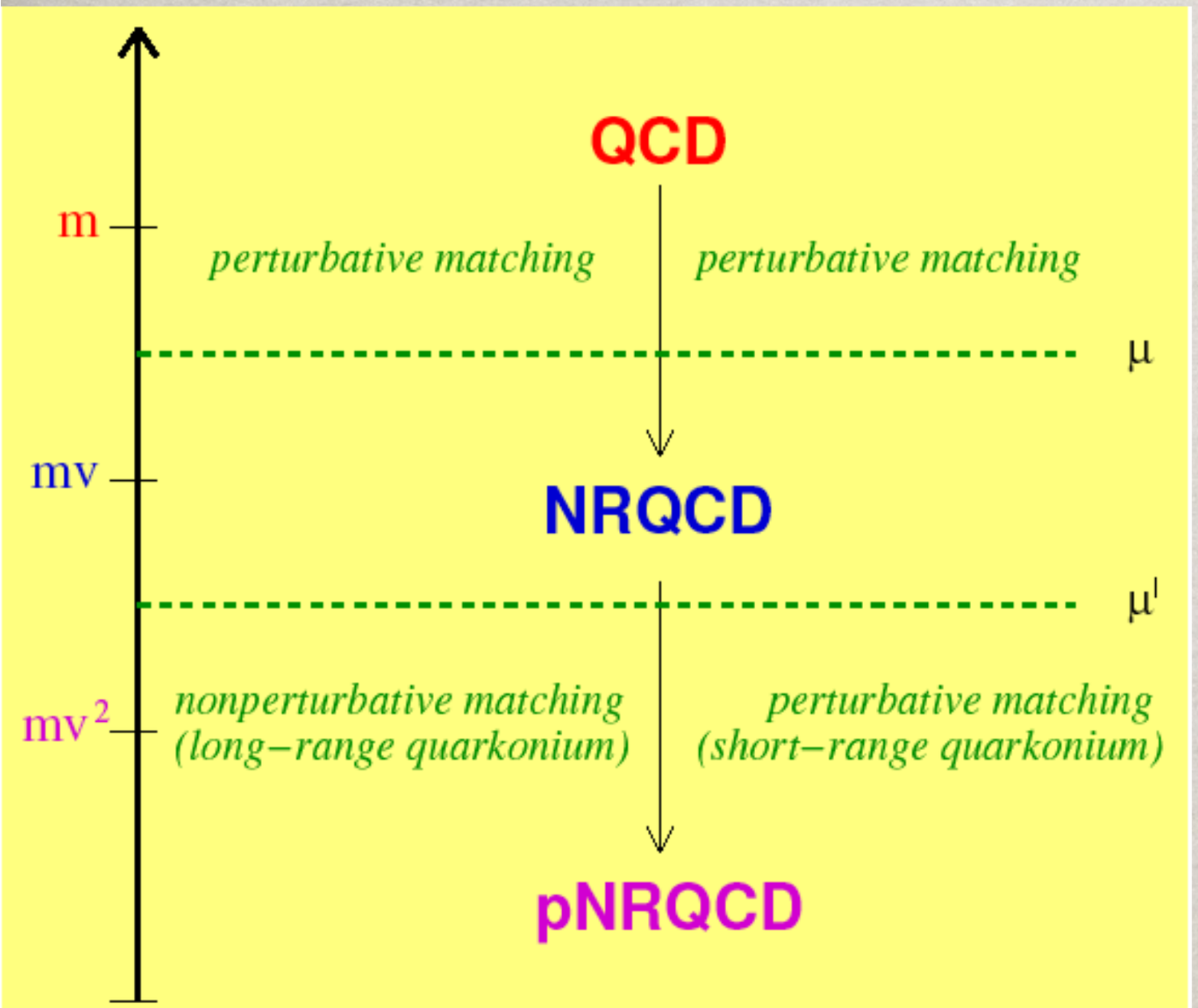
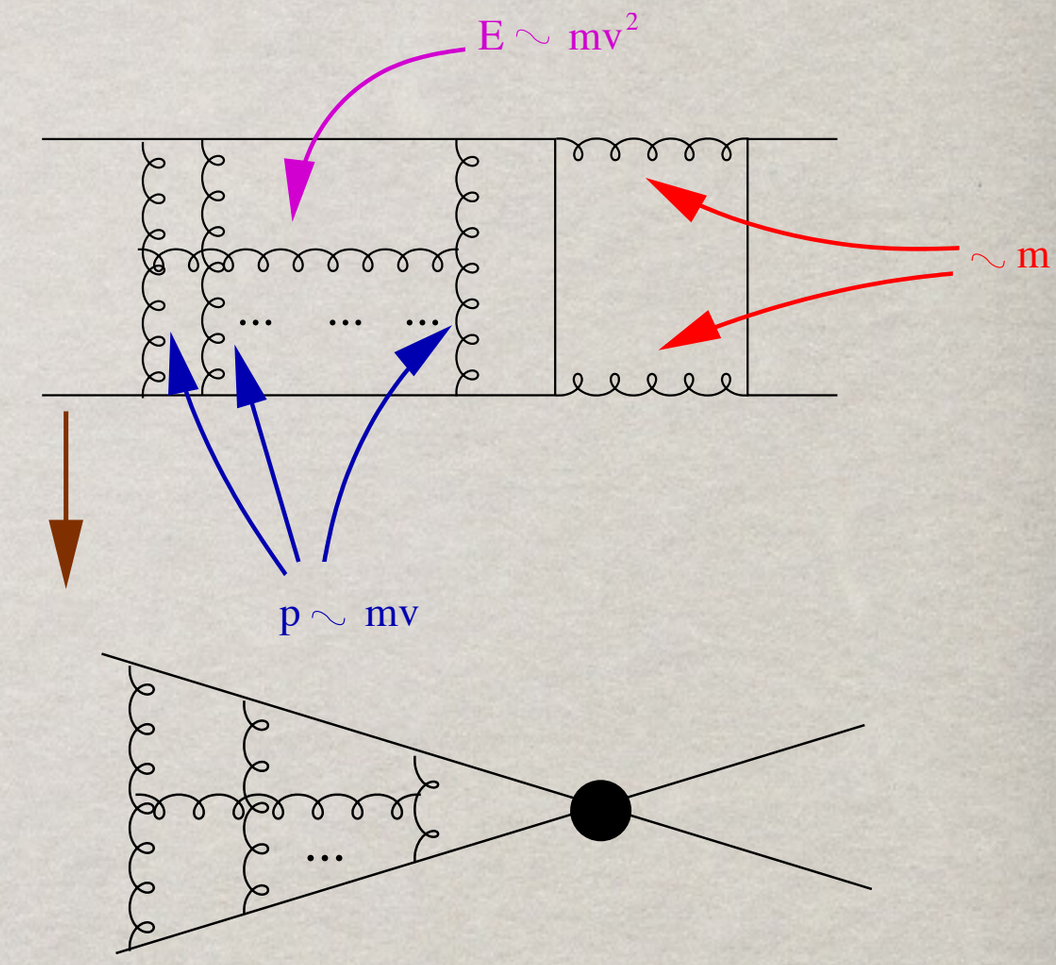
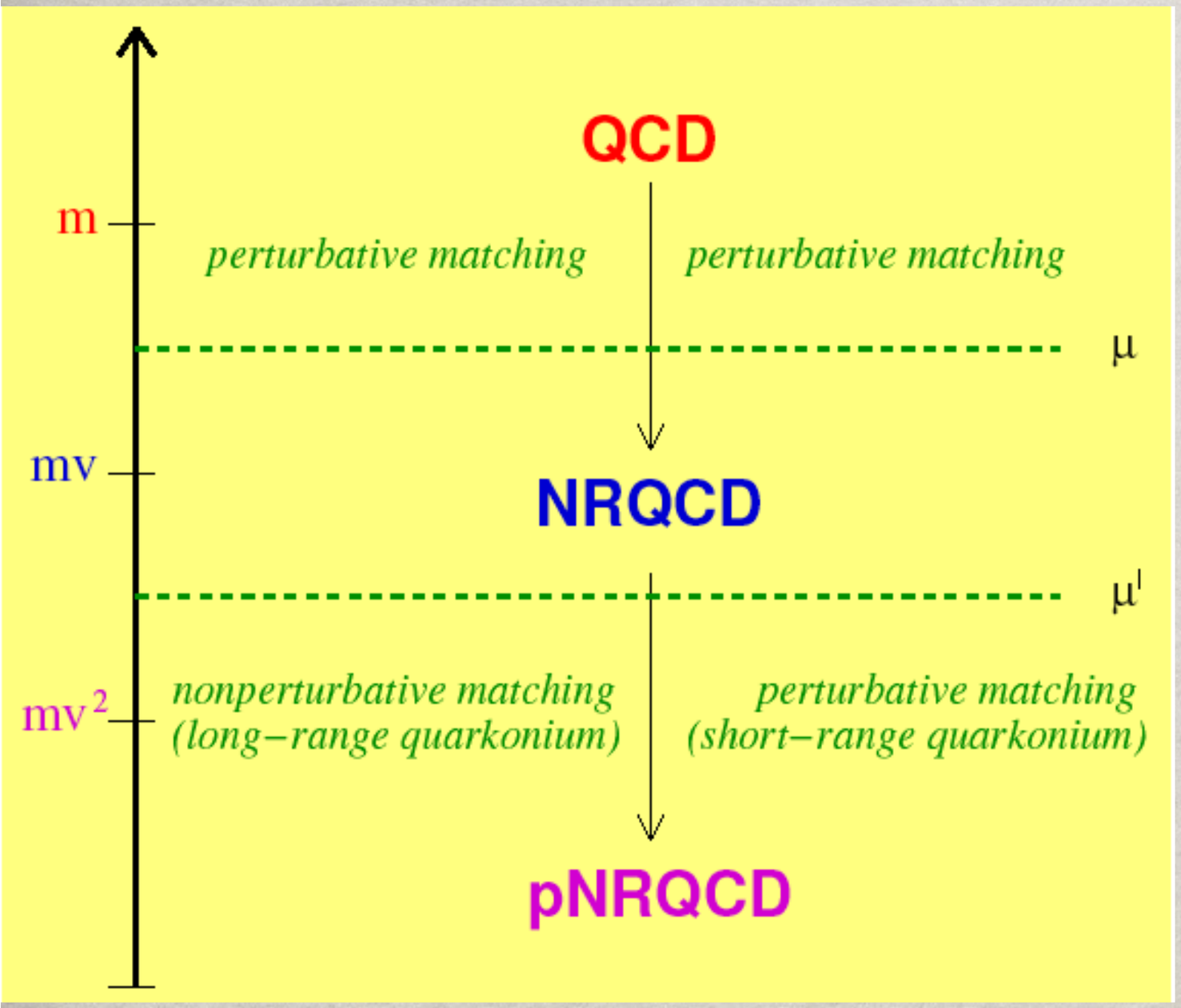
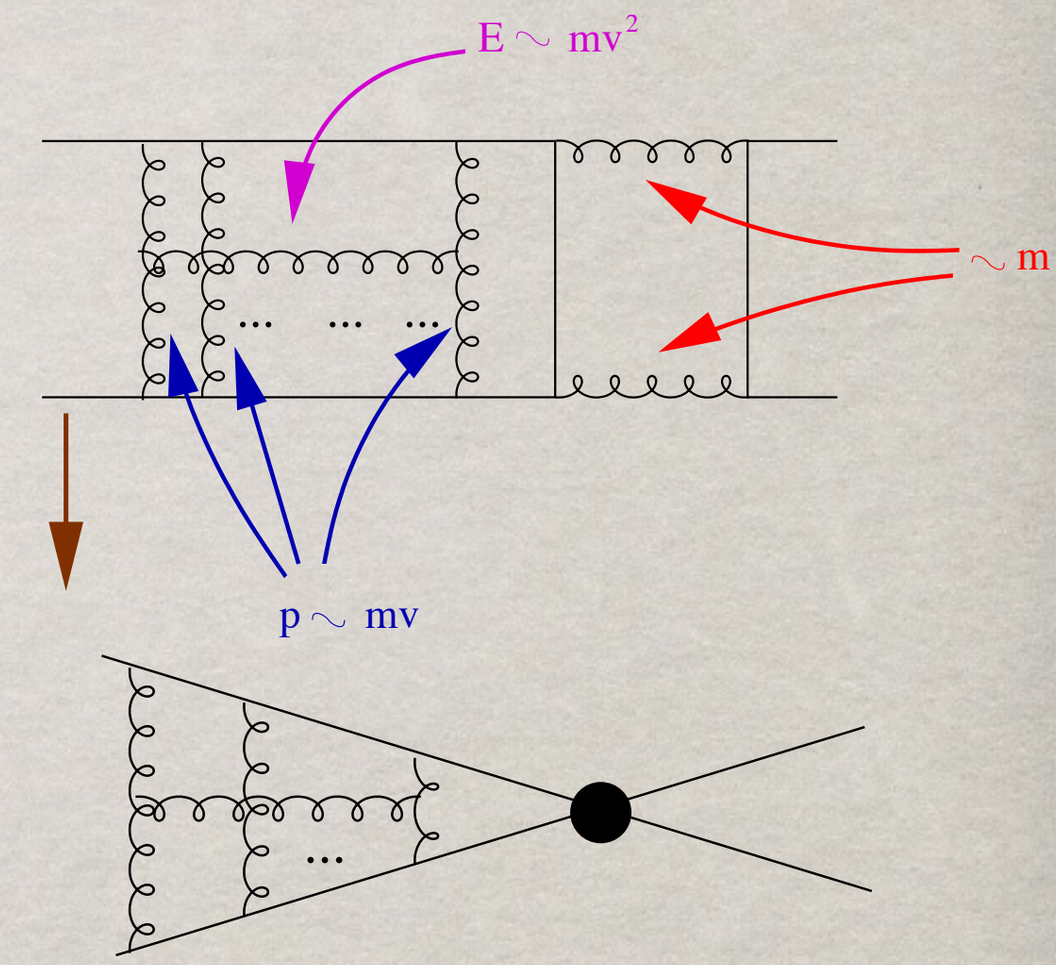




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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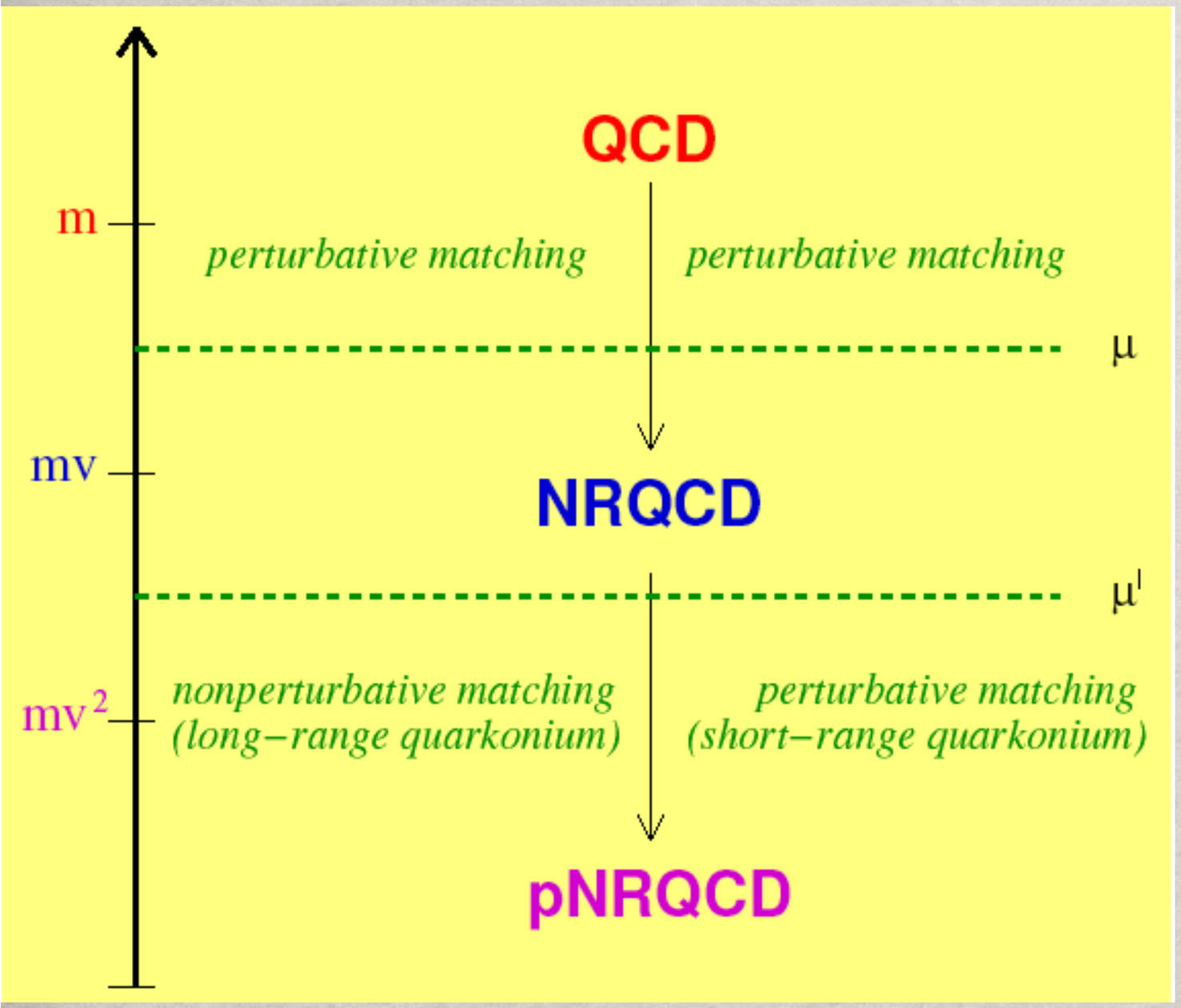

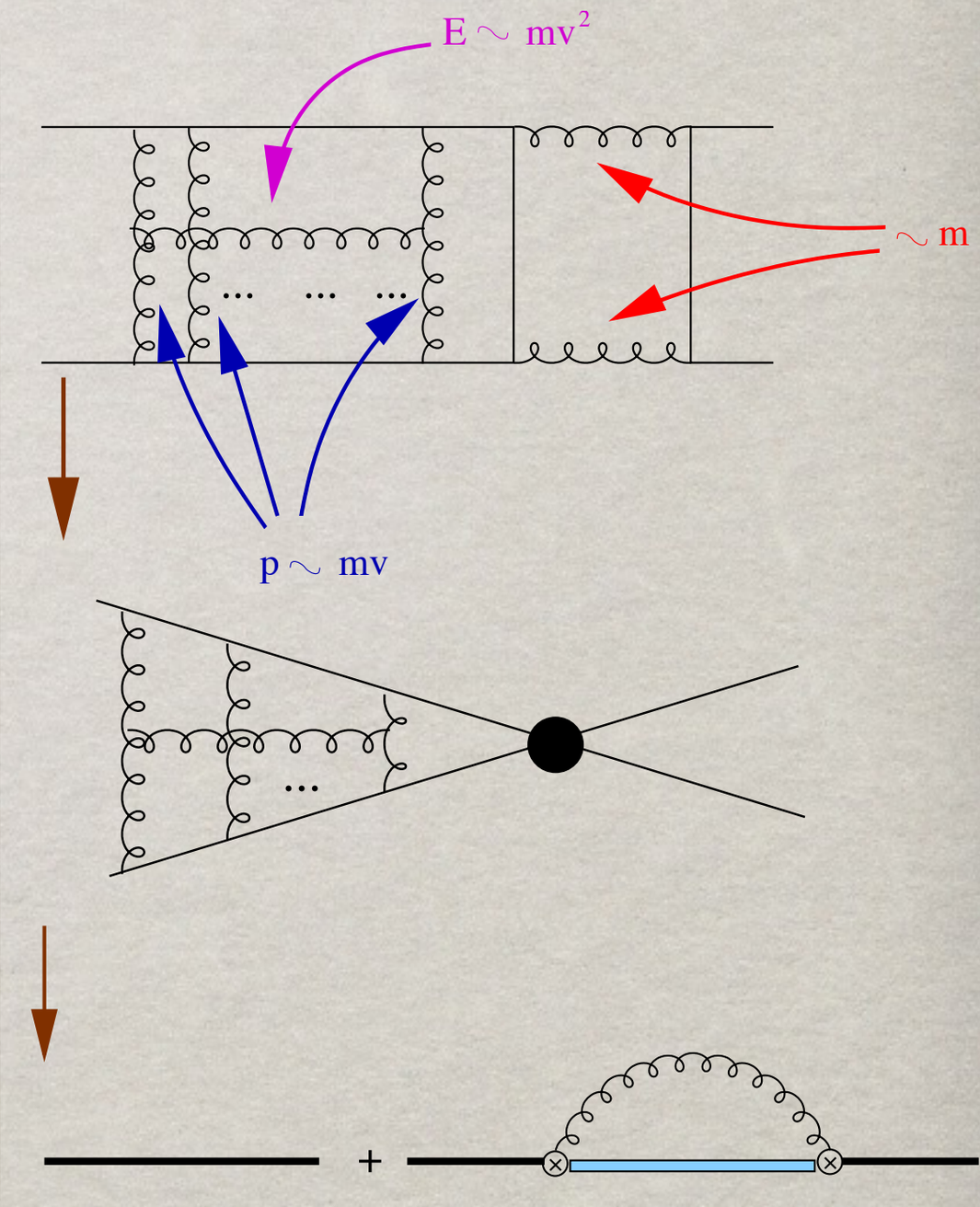

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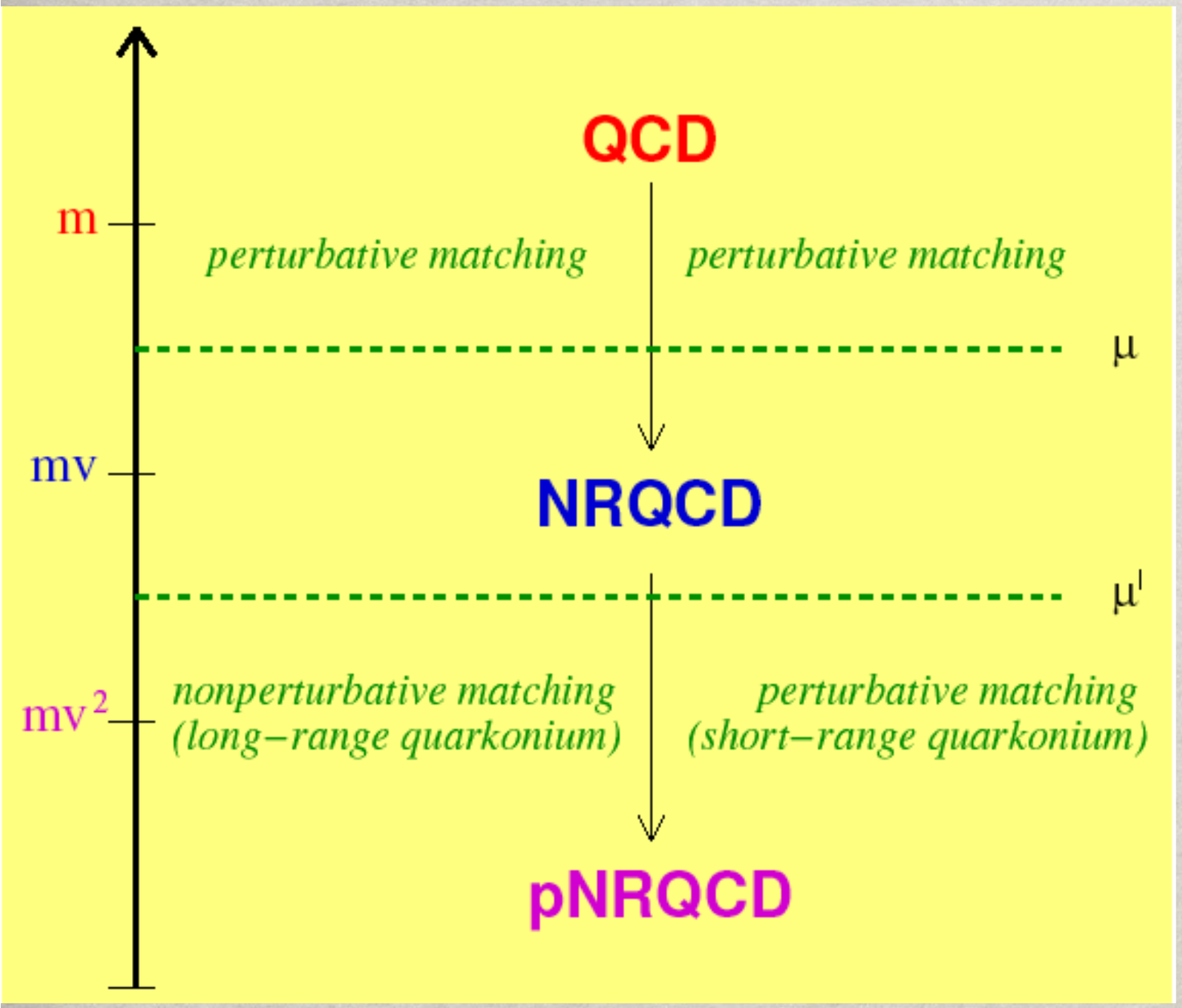
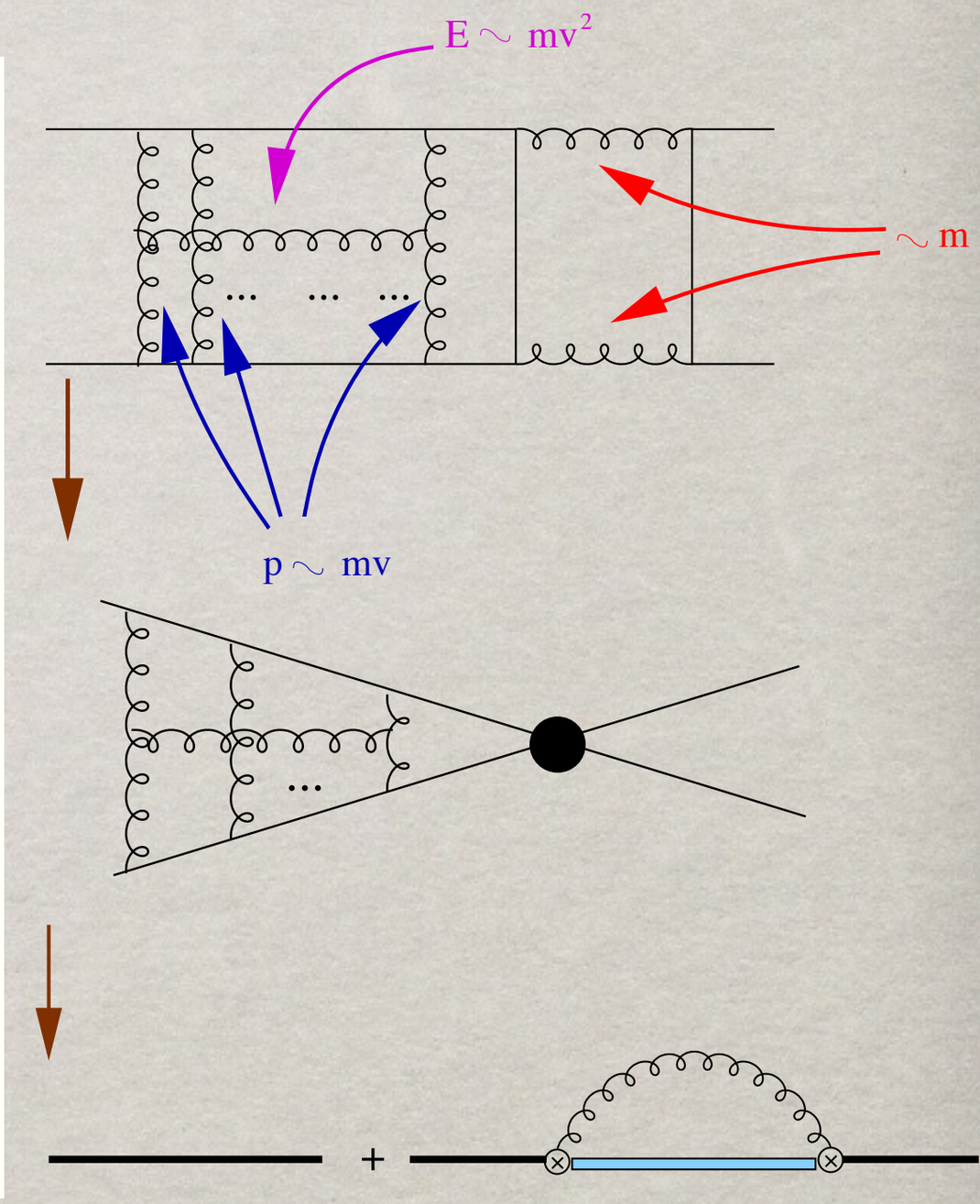
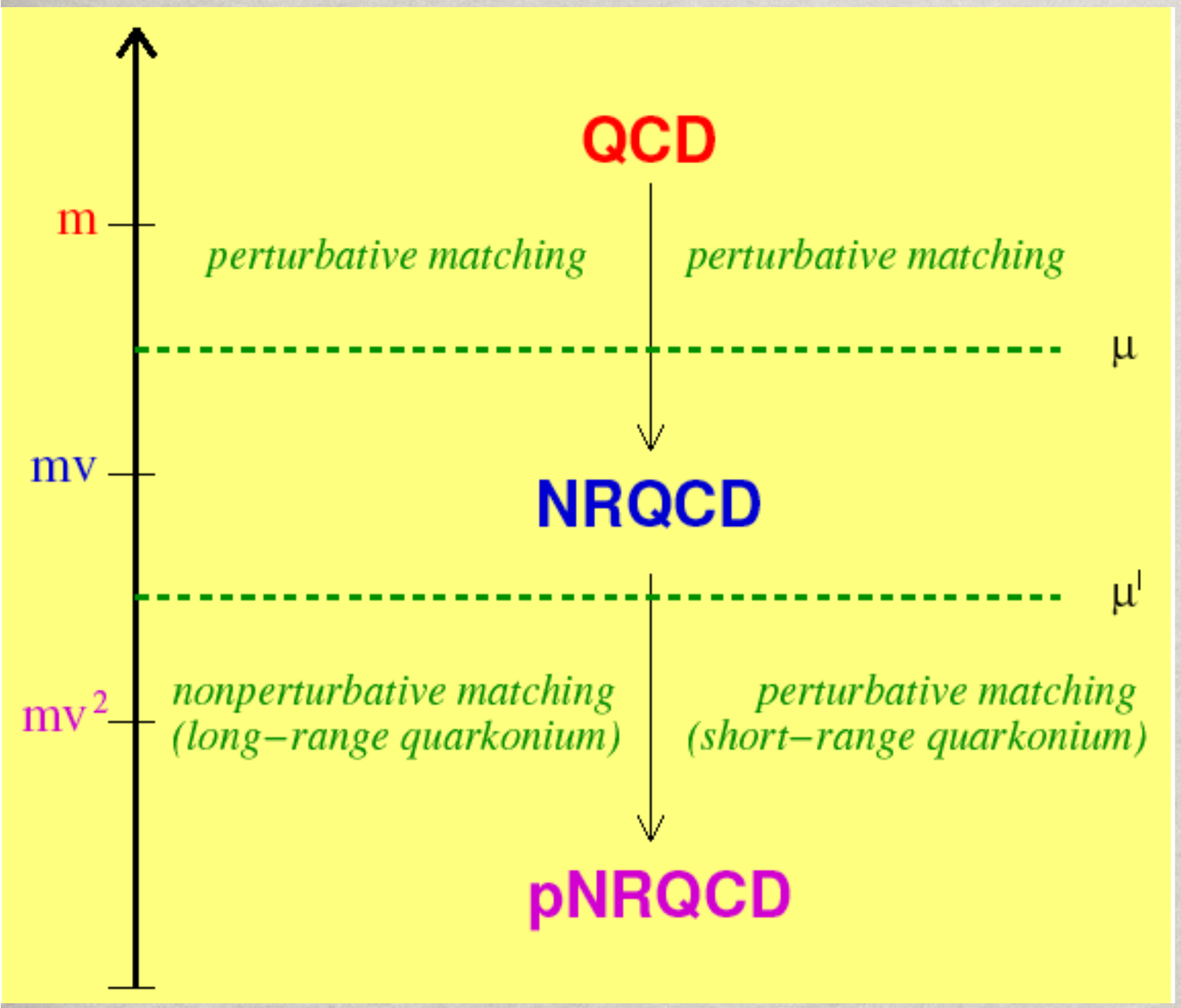
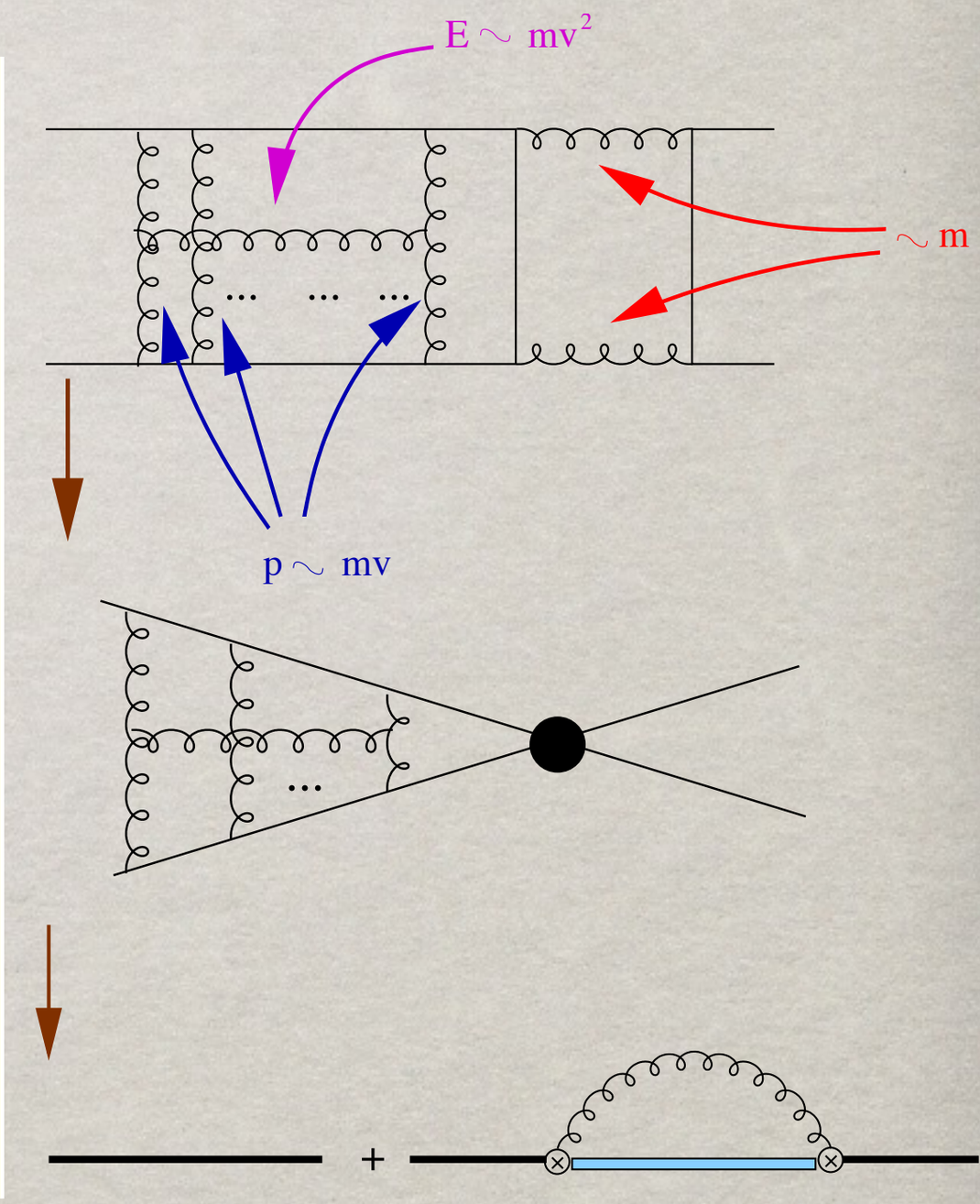




$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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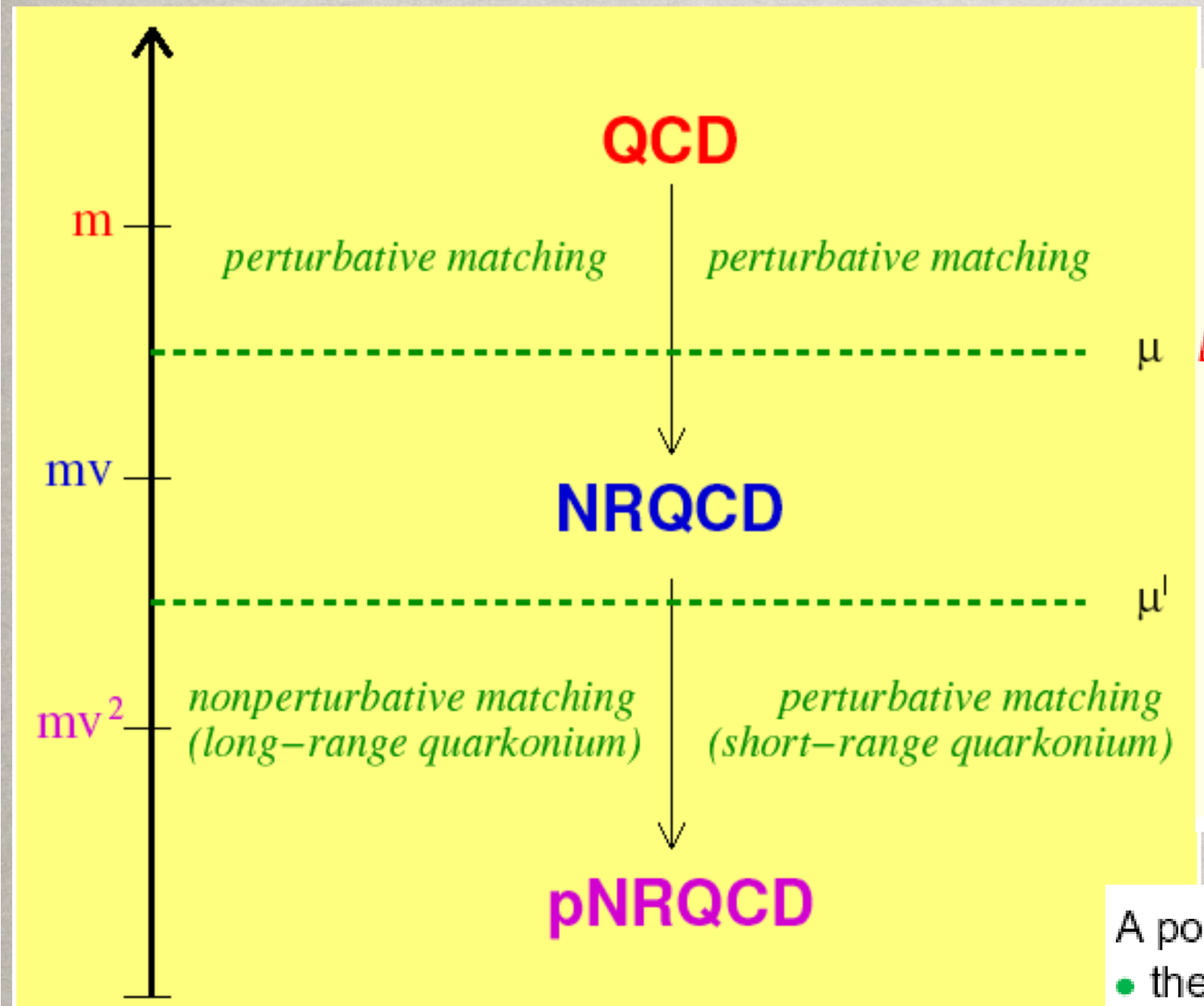
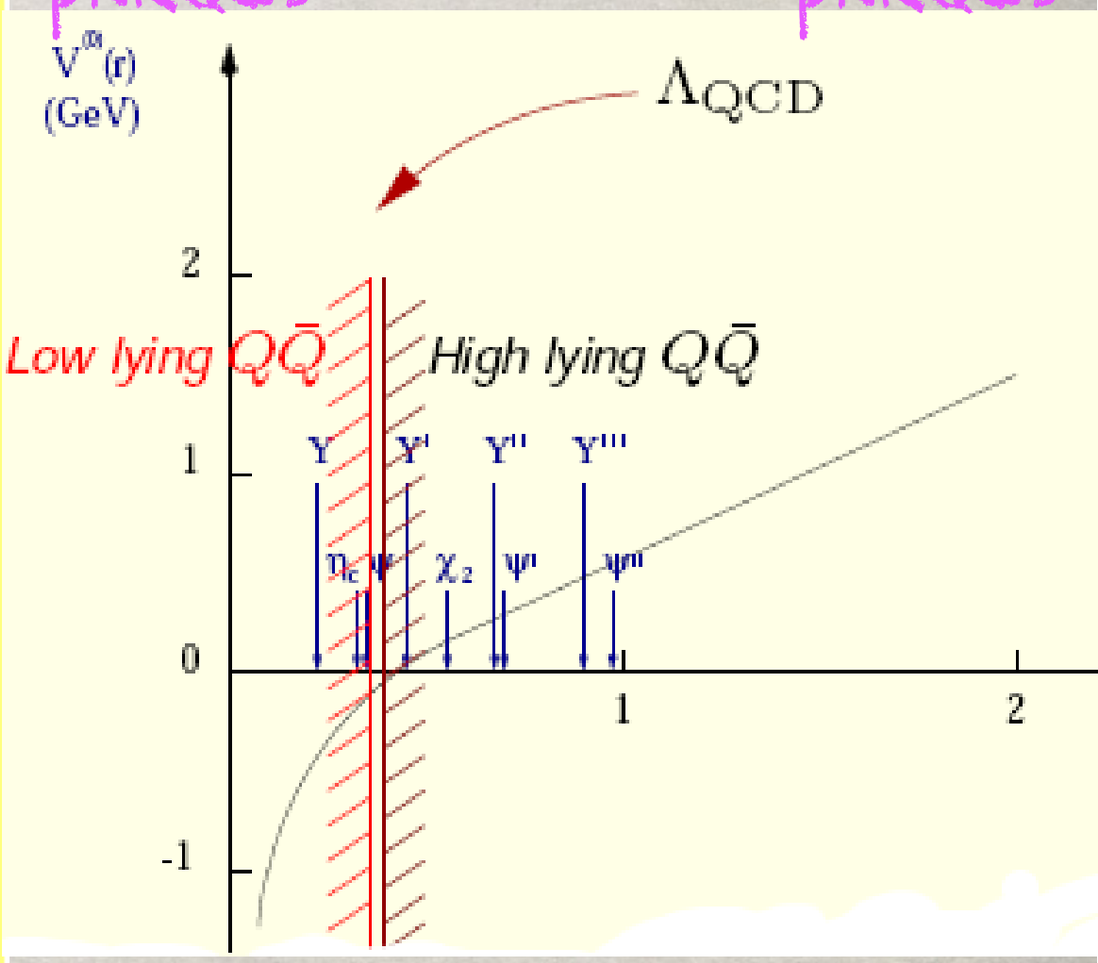
Quarkonium with NR EFT: pNRQCD



weakly coupled pNRQCD

strongly coupled pNRQCD



A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{\rm QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{\rm QCD}$

In QCD another scale is relevant $\Lambda_{\rm QCD}$

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

today we have a complete theory description for quarkonia states away from the strong decay threshold

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The EFT has been constructed pNRQCD

*Work at calculating higher order perturbative corrections in v and alpha_s

*Resumming the log

*Calculating/extracting nonperturbatively the low energy quantities

*Extending the theory (electromagnetic effect, 3 bodies)

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The issue here is precision physics and the study of confinement

- Precise and systematic high order calculations allow the extraction of precise determinations of standard model parameters like the quark masses and alpha_s

- The eft has allowed to systematically factorize and to study the low energy nonperturbative contributions

weakly coupled pNRQCD

$$r \ll \Lambda_{\rm QCD}^{-1}$$

Singlet static potential

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right.$$
$$\left. + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

Octet static potential

$$+ V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot g\mathbf{E}\, \mathrm{O} \right\}$$
$$+ \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r}\cdot g\mathbf{E} \right\}$$
$$+ \cdots$$

NLO in r

S singlet field

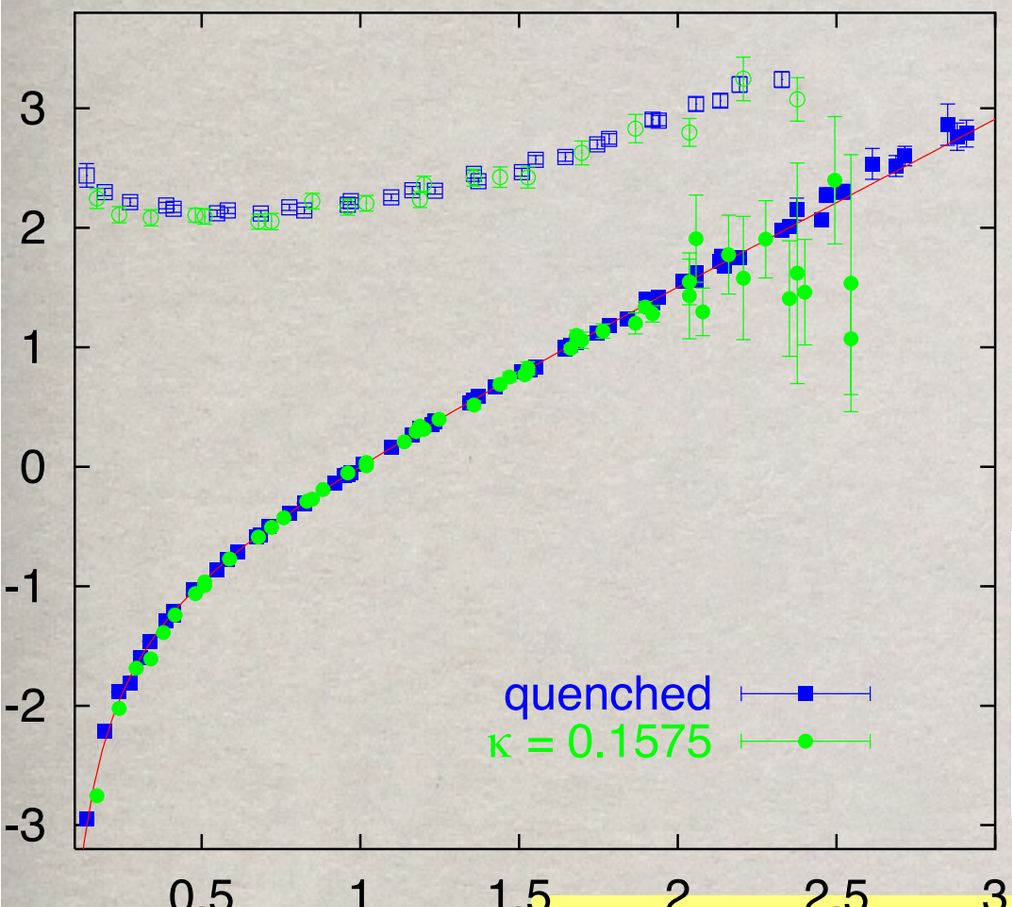
O octet field

——— singlet propagator

═══ octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



- integrate out all scales above mv^2
- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

Brambilla Pineda Soto V

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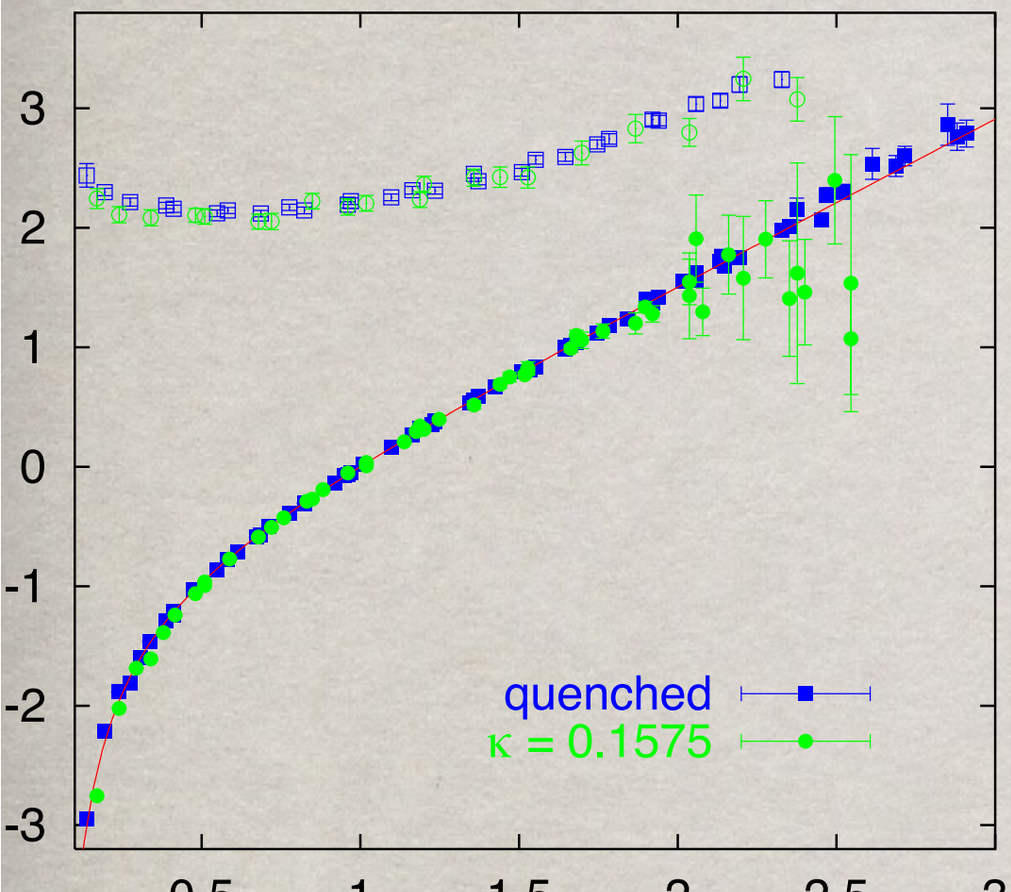

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- A potential description emerges from the EFT

Brambilla Pineda Soto V

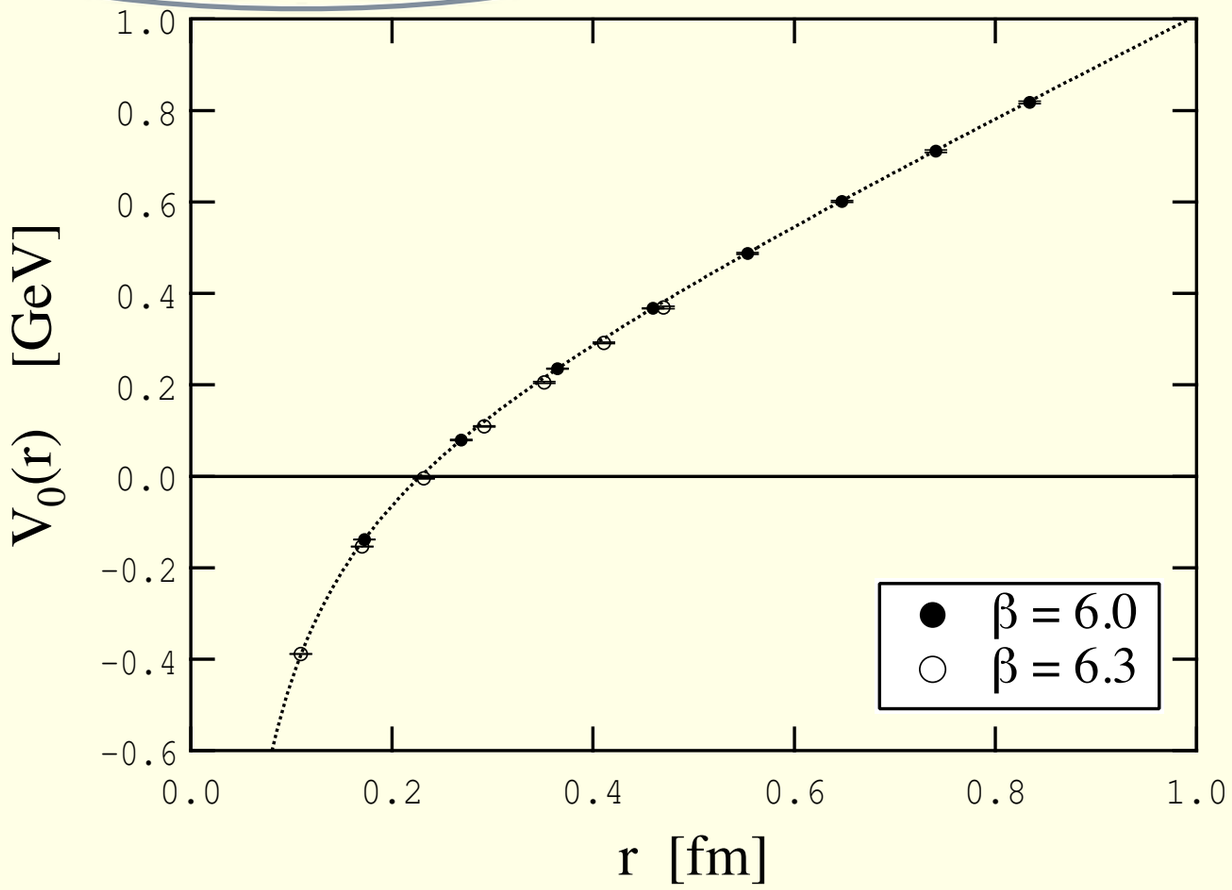
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

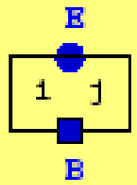
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r \times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$

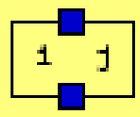


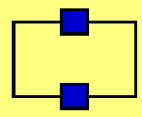
○ Koma Koma NPB 769(07)79

QCD Spin dependent potentials

$$V_{\rm SD}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \, \langle$$ 

$$\rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}$$

$$- {c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle$$ 

$$\rangle - \frac{\delta_{ij}}{3} \langle$$ 
$$\rangle \right)$$

$$\times \left(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \right)$$

$$+ \left(\frac{2}{3} {c_F}^2 i \int_0^\infty dt \langle$$
$$\rangle - 4(d_2 + C_F d_4) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S_2}$$

Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

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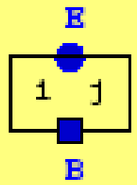
$$V_{\rm SD}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \, \langle$$ 

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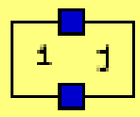
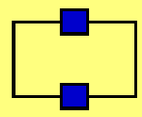
$$- {c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle$$ 

$$\rangle - \frac{\delta_{ij}}{3} \langle$$ 
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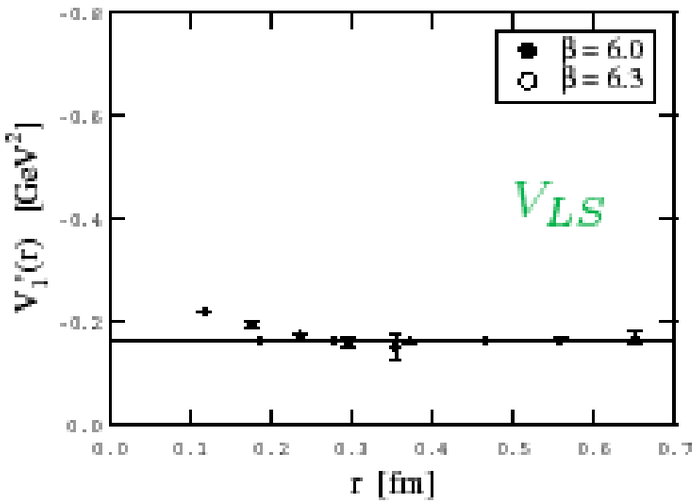
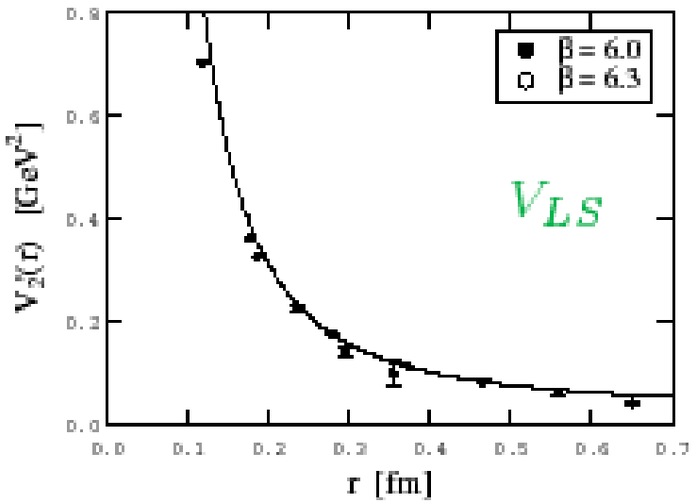
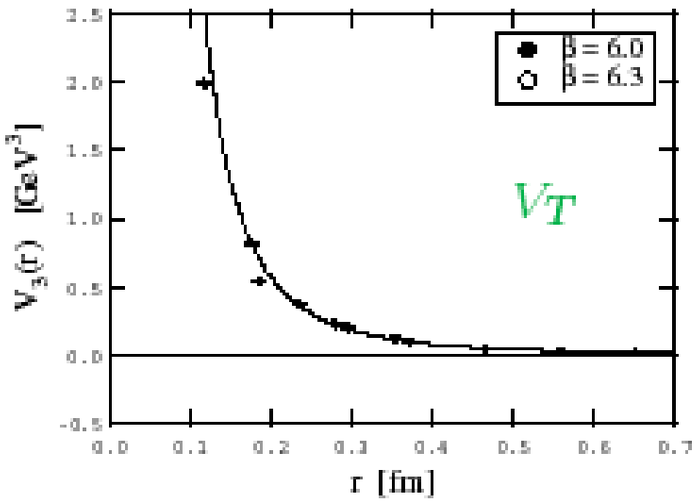
Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

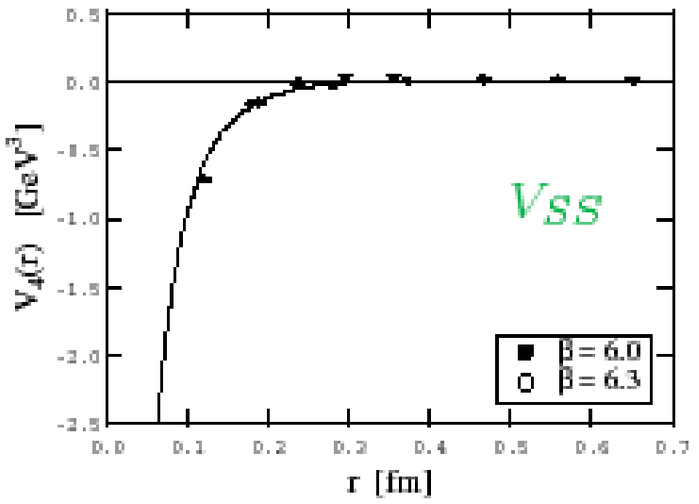
-factorization between high energy part in matching coefficient and low energy part in wilson loops; power counting;

Spin dependent potentials







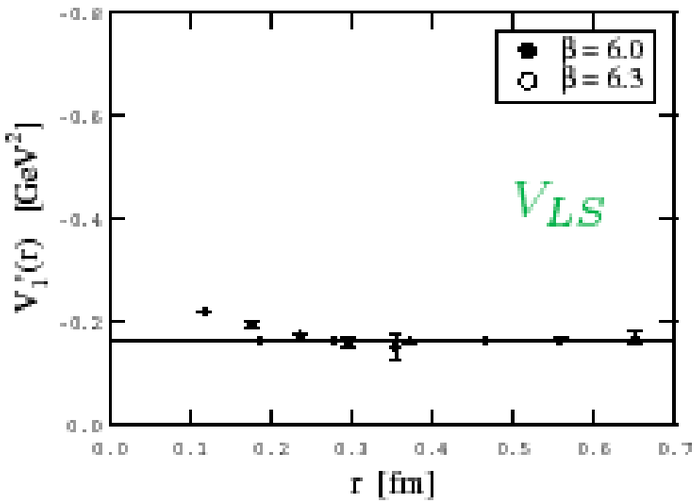


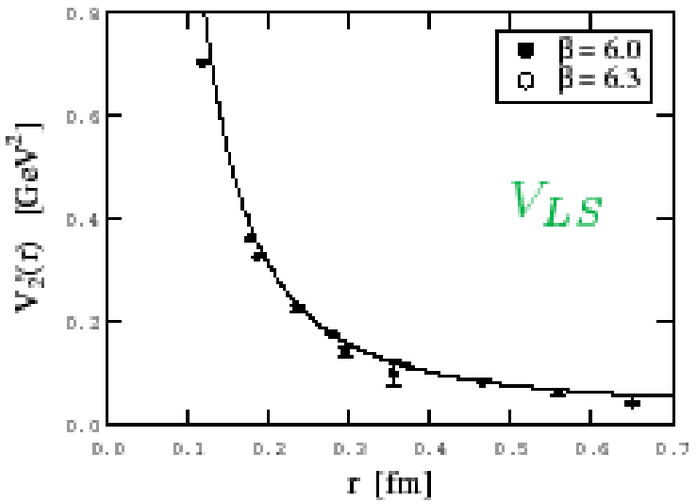
Koma Koma Wittig 05, Koma Koma 06

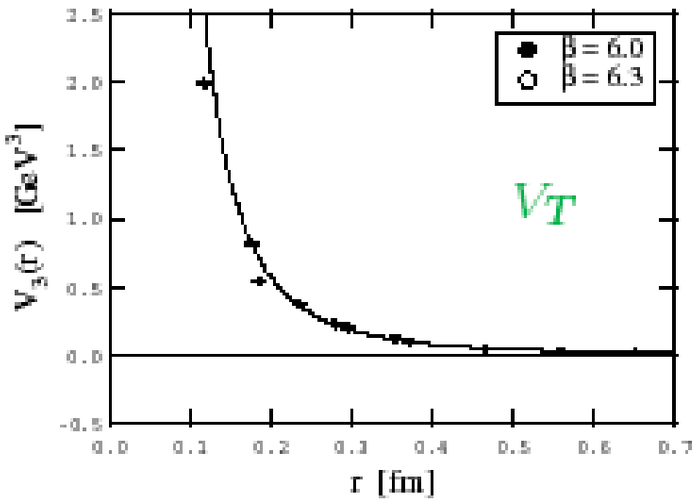
Terrific advance in the data precision with Lüscher multivel algorithm!

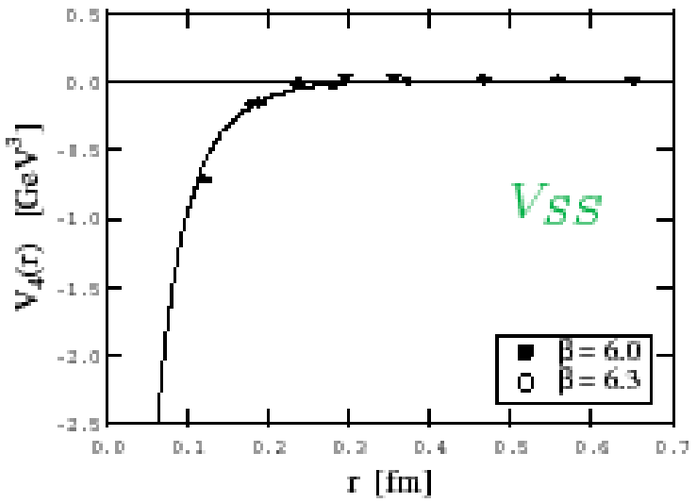
N. B., Martinez, Vairo 2014

Spin dependent potentials









Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, Vairo 2014

For states close or above the strong decay threshold the situation is much more complicated.

there is no mass gap between quarkonium and the creation of a heavy-light mesons couple

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Many phenomenological models exist

States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...

- Molecular states, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556

- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263

- The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).
  - Dubynskiy Voloshin PLB 666 (2008) 344

- Tetraquark states.

MAIANI, PICCININI, POLOSA ET AL. 2005--

  - Jaffe PRD 15(77)267 Vijande, Valcarce, Richard
  - Ebert Faustov Galkin PLB 634(06)214

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

  - Alexandrou et al. PRL 97(06)222002
  - Fodor et al. PoS LAT2005(06)310

States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...

- Molecular states, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556

- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263

- The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).
  - Dubynskiy Voloshin PLB 666 (2008) 344

- Tetraquark states.

MAIANI, PICCININI, POLOSA ET AL. 2005--

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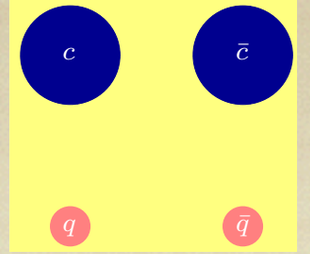
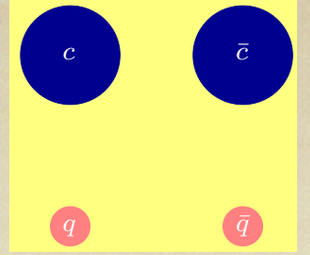
choosing one of these degrees of freedom and an interaction originates a model for exotics.

$X(3872)$: interpretations



4-quark state with $J^{PC} = 1^{++}$

$X(3872)$: interpretations



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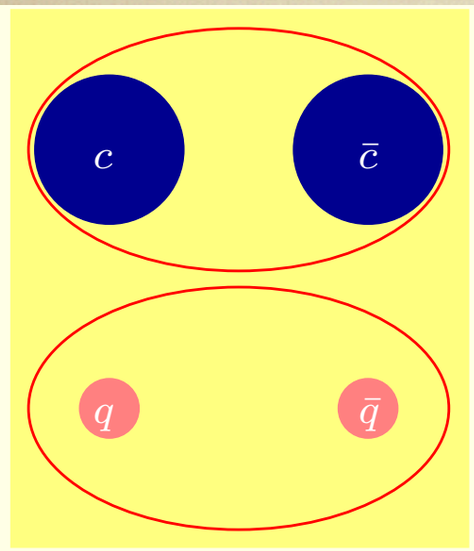
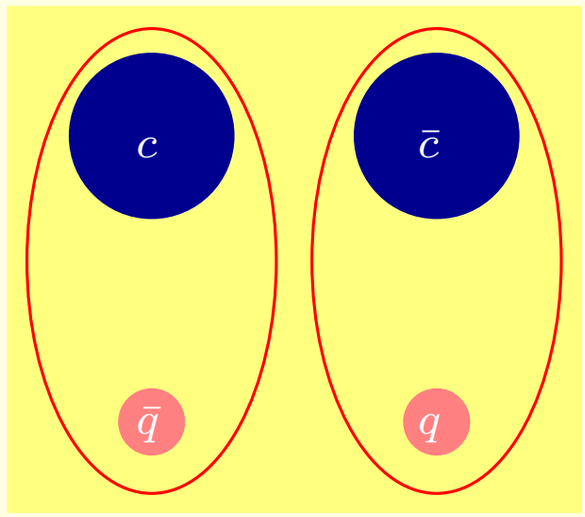


Høgassen et al 05

$$X \sim (c\bar{c})^8_{S=1} \otimes (q\bar{q})^8_{S=1}$$
$$\sim (c\bar{q})^1_{S=0} \otimes (q\bar{c})^1_{S=1} + (c\bar{q})^1_{S=1} \otimes (q\bar{c})^1_{S=0}$$

Molecular model

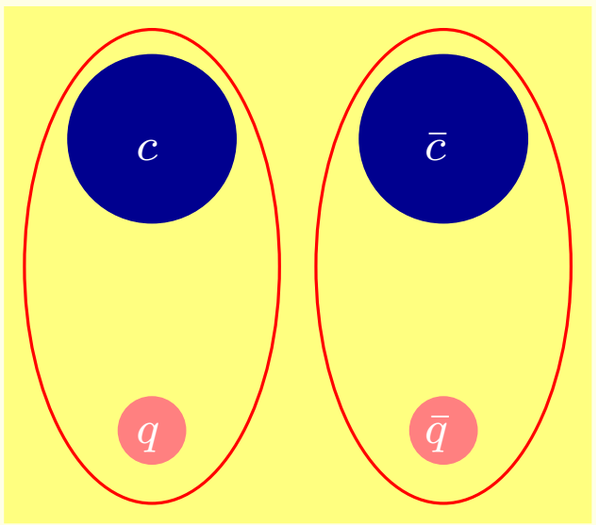
Predictions based on the phenomenological
$H = -\sum_{ij} C_{ij}\, T^a \otimes T^a\, \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$;



Törnqvist 93, Swanson 04

$$X \sim (c\bar{q})^1_{S=0} \otimes (q\bar{c})^1_{S=1} + (c\bar{q})^1_{S=1} \otimes (q\bar{c})^1_{S=0}$$
$$\sim D\,\bar{D}^* + D^*\,\bar{D}$$

This is assumed to be the dominant long-range Fock component; short-range components of the type $(c\bar{c})^1_{S=1} \otimes (q\bar{q})^1_{S=1}$ $\sim J/\psi\, \rho, \omega$ *are assumed as well.*



Maiani et al 04

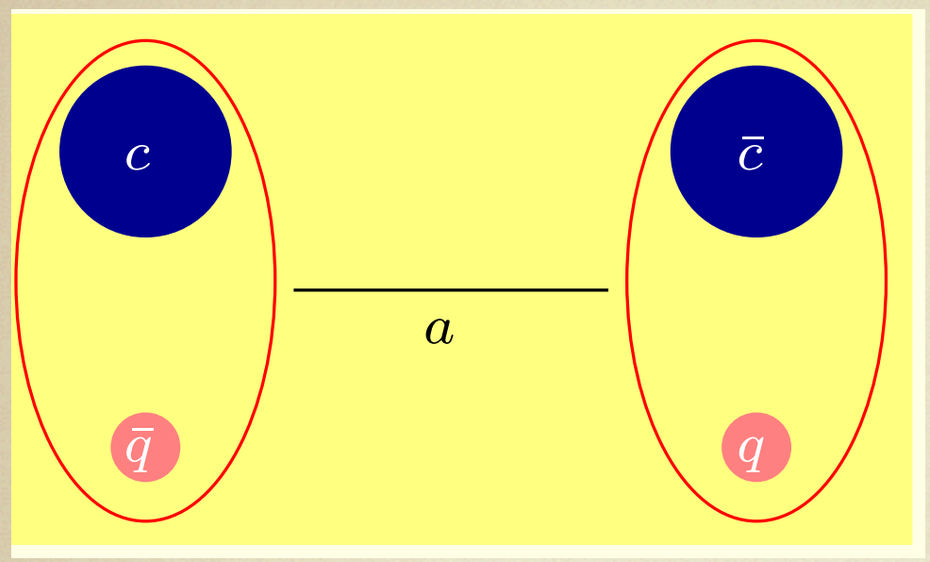
$$X \sim (cq)^{\bar{3}}_{S=1} \otimes (\bar{c}\bar{q})^3_{S=0} + (cq)^{\bar{3}}_{S=0} \otimes (\bar{c}\bar{q})^3_{S=1}$$

the dynamical assumption (there is no scale separation like in the doubly heavy baryons) is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

Tetraquark model

Predictions based on the phenomenological Hamiltonian: $H = \sum_{ij} \kappa_{ij}\, \boldsymbol{\sigma} \otimes \boldsymbol{\sigma}$; the

In some cases it is possible to develop an EFT owing to special dynamical condition



this happens if the state is sufficiently close to a threshold and if it has S-wave coupling to the threshold—> loosely bound molecule with universal properties

- An example is the $X(3872)$ intepreted as a $D^0\,\bar{D}^{*\,0}$ or $\bar{D}^0\,D^{*\,0}$ molecule.
  In this case, one may take advantage of the hierarchy of scales:

$$\Lambda_{\rm QCD} \gg m_\pi \gg m_\pi^2/M_{D^0} \approx 10\ \text{MeV} \gg E_{\rm binding}$$
$$\approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0)\ \text{MeV}$$

Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the $X(3872)$ *decaying into* $D^0\bar{D}^0\pi^0$ *is* $\mathcal{B}(X \to D^0\bar{D}^0\pi^0) \approx \mathcal{B}(D^{*\,0} \to D^0\pi^0) \approx 60\%$.

Pakvasa Suzuki 03, Voloshin 03, Braaten Kusunoki 03

Braaten Hammer 06

We need a description of states close or above threshold from QCD

Already the case of QCD without light quark is very interesting. The degrees of freedom are heavy quarkonium, heavy hybrids and glueballs

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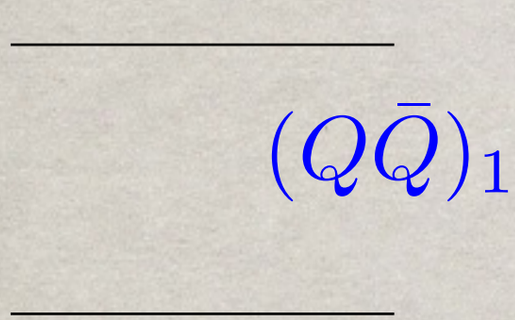

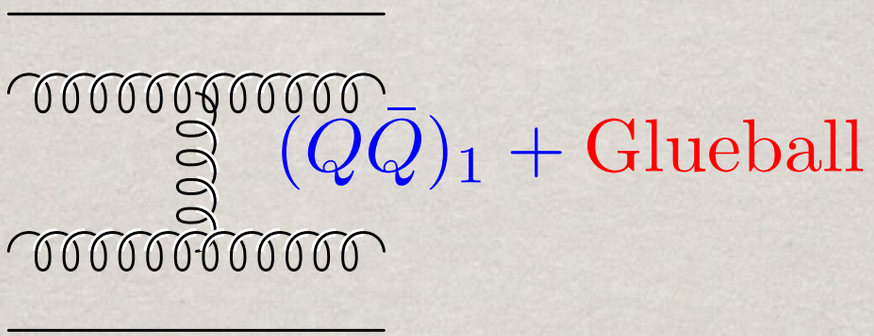

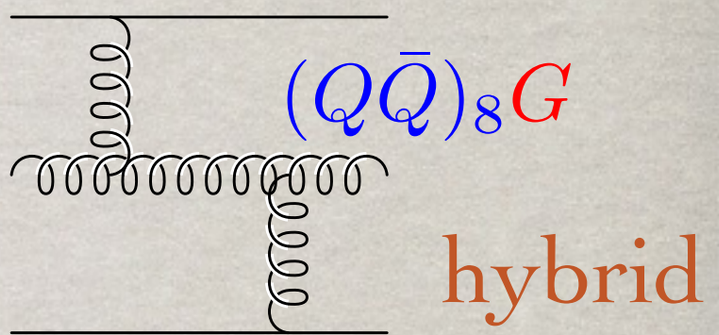

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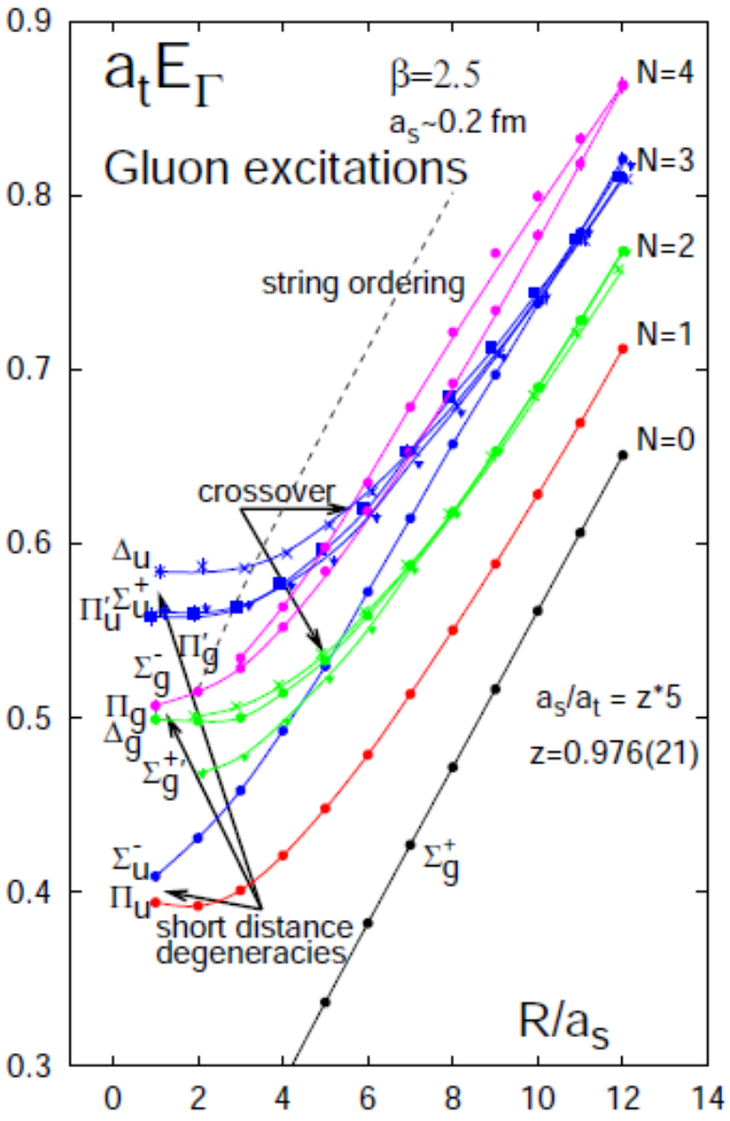
Static Lattice energies

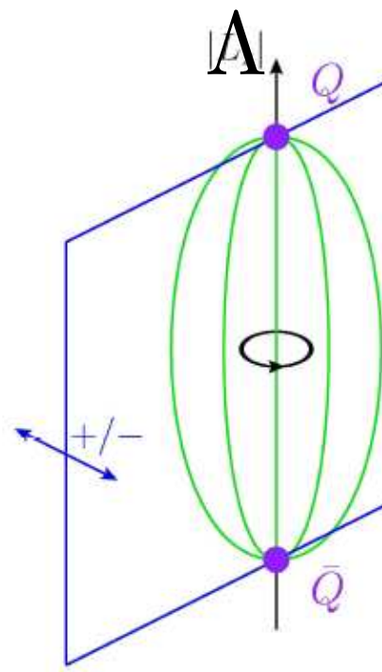
Juge Kuti Morningstar 2003



Symmetries

Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ^σ_η

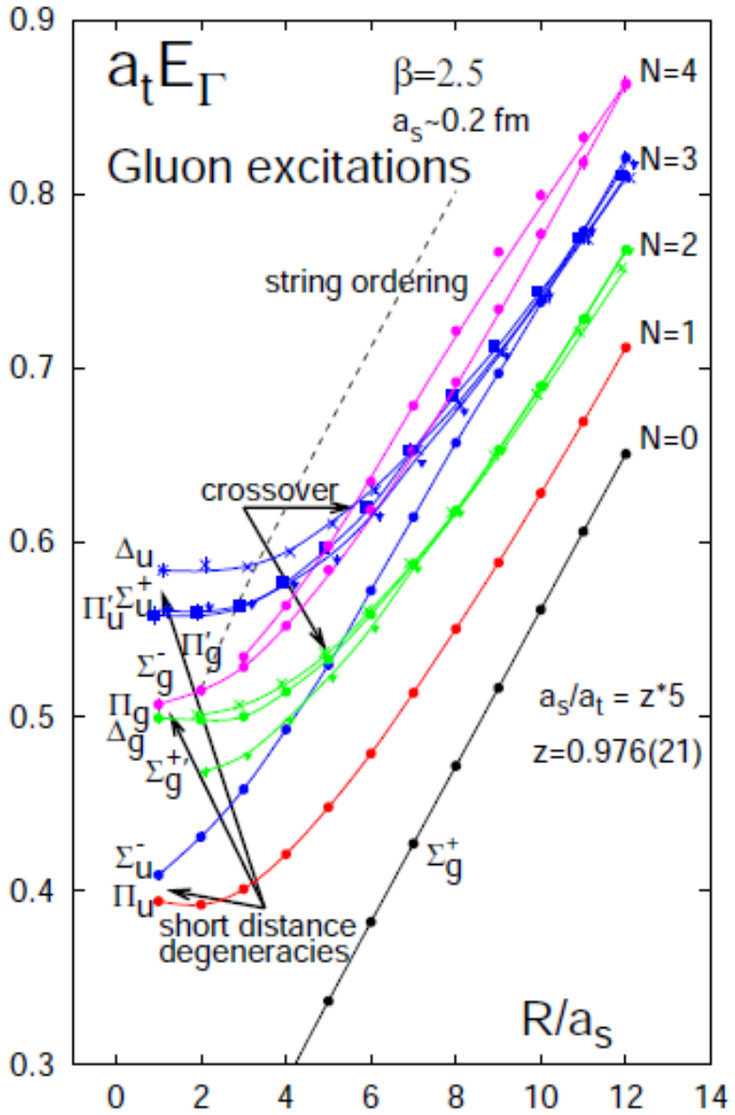
- Λ rotational quantum number
  $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to
  $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP:
  $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states
  (others are degenerate)



- The static energies correspond to the irreducible representations of D_∞
- In general it can be more than one state for each irreducible represent
  $D_{\infty h}$, usually denoted by primes, e.g. Π_u, Π'_u, Π''_u...

even the case without light quark is difficult

static Lattice energies



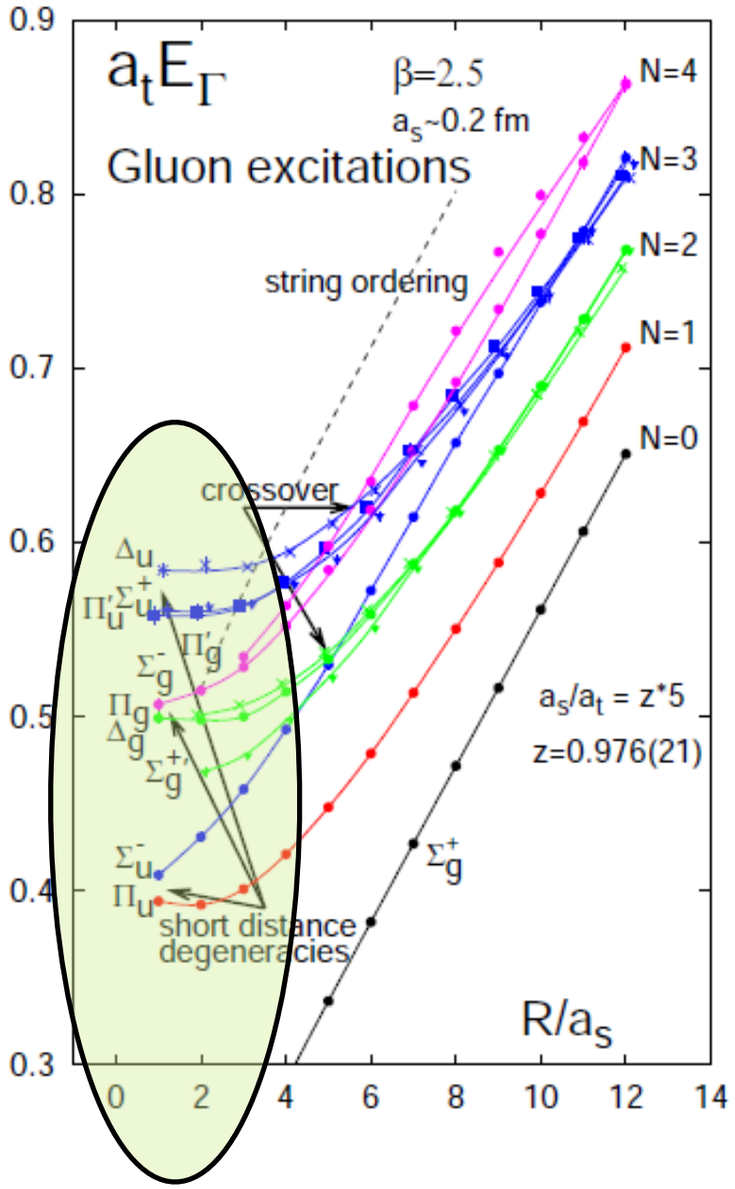
- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in Bali et al 2000 and good agreement was found below string breaking distance.

- Juge Kuti Morningstar PRL 90 (2003) 161601

even the case without light quark is difficult

static

Lattice energies



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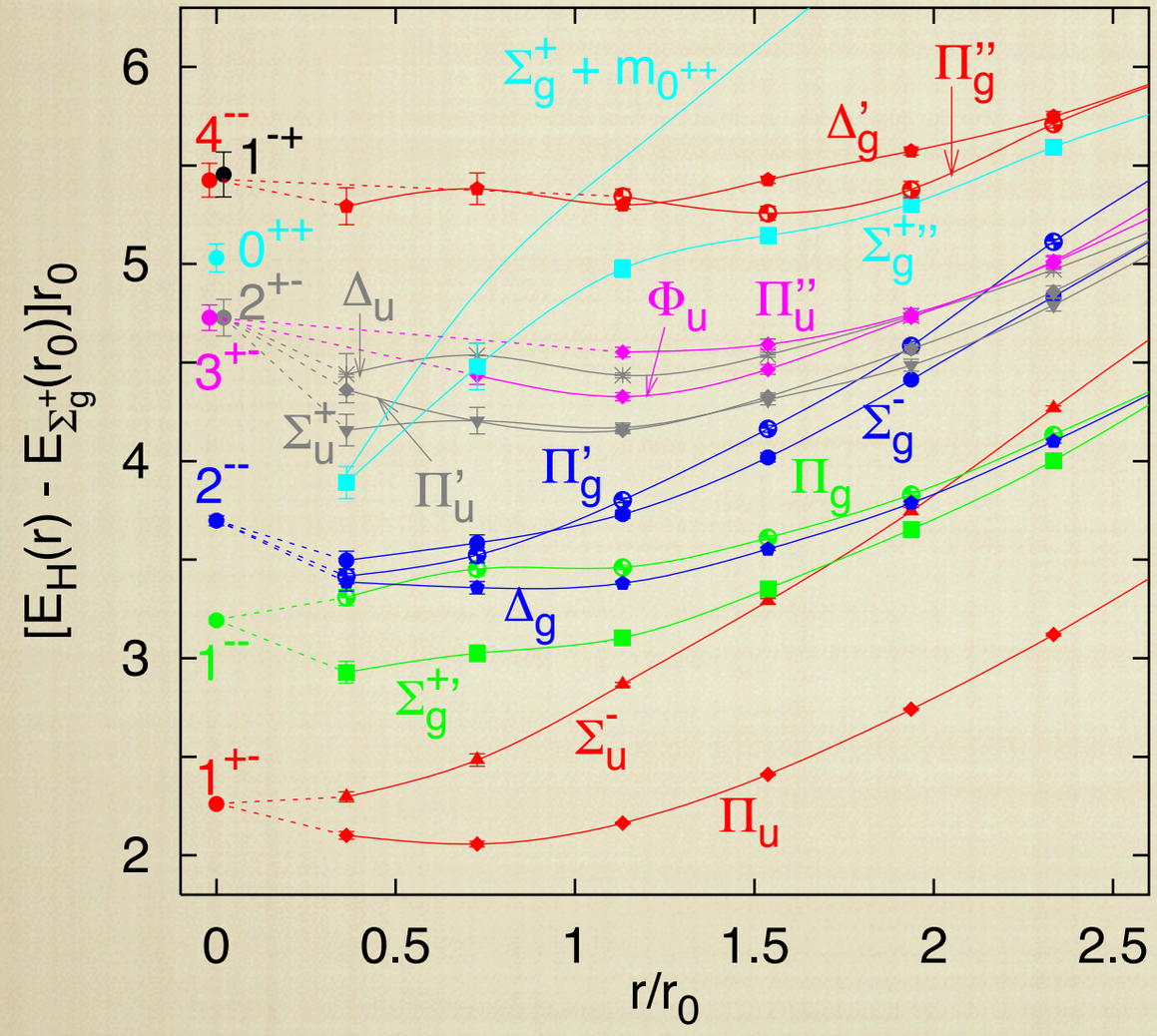
pNRQCD predicts the structure of multiplets at short distance and the ordering

In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.



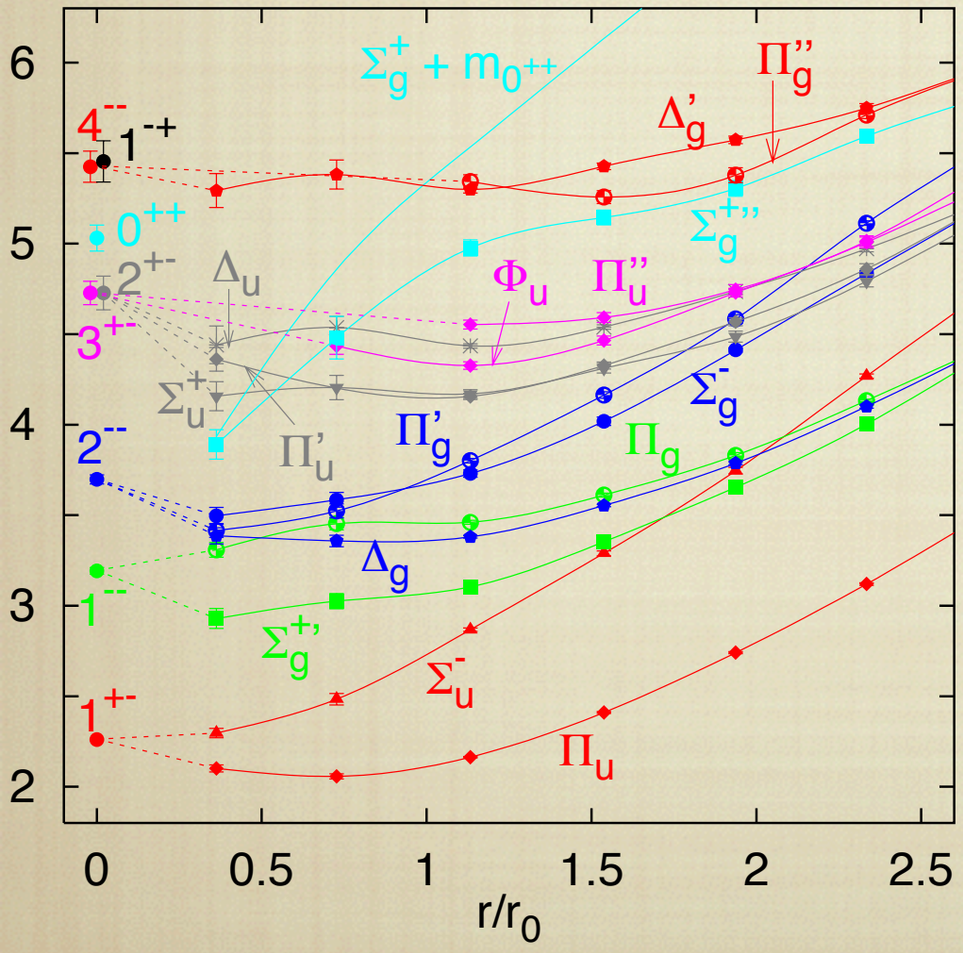
	$L=1$	$L=2$
$\Sigma_g^{+\prime}$	$\mathbf{r}\cdot\mathbf{E}$	
Σ_g^-		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times\mathbf{E}$	
Π_g'		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^-	$\mathbf{r}\cdot\mathbf{B}$	
Π_u	$\mathbf{r}\times\mathbf{B}$	
Π_u'		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

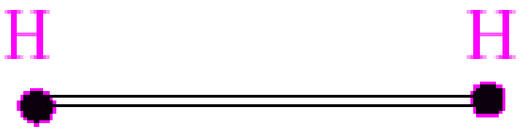
Brambilla Pineda Soto Vairo 0

Gluonic excitations in pNRQCD: one can determine the form of the potential

- *At lowest order in the multipole expansion, the singlet decouples while the octet is still coupled to gluons.*
- *Static hybrids at short distance are called gluelumps and are described by a static adjoint source (O) in the presence of a gluonic field (H):*

$$\mathrm{H}(R,r,t) = \mathrm{Tr}\{\mathrm{O}\,H\}$$





$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T}\ln\langle H^a(\frac{T}{2})\phi^{\mathrm{adj}}_{ab}H^b(-\frac{T}{2})\rangle$$

$$\langle H^a(\frac{T}{2})\phi^{\mathrm{adj}}_{ab}H^b(-\frac{T}{2})\rangle^{\mathrm{np}} \sim h\,e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H + O(r^2)$$

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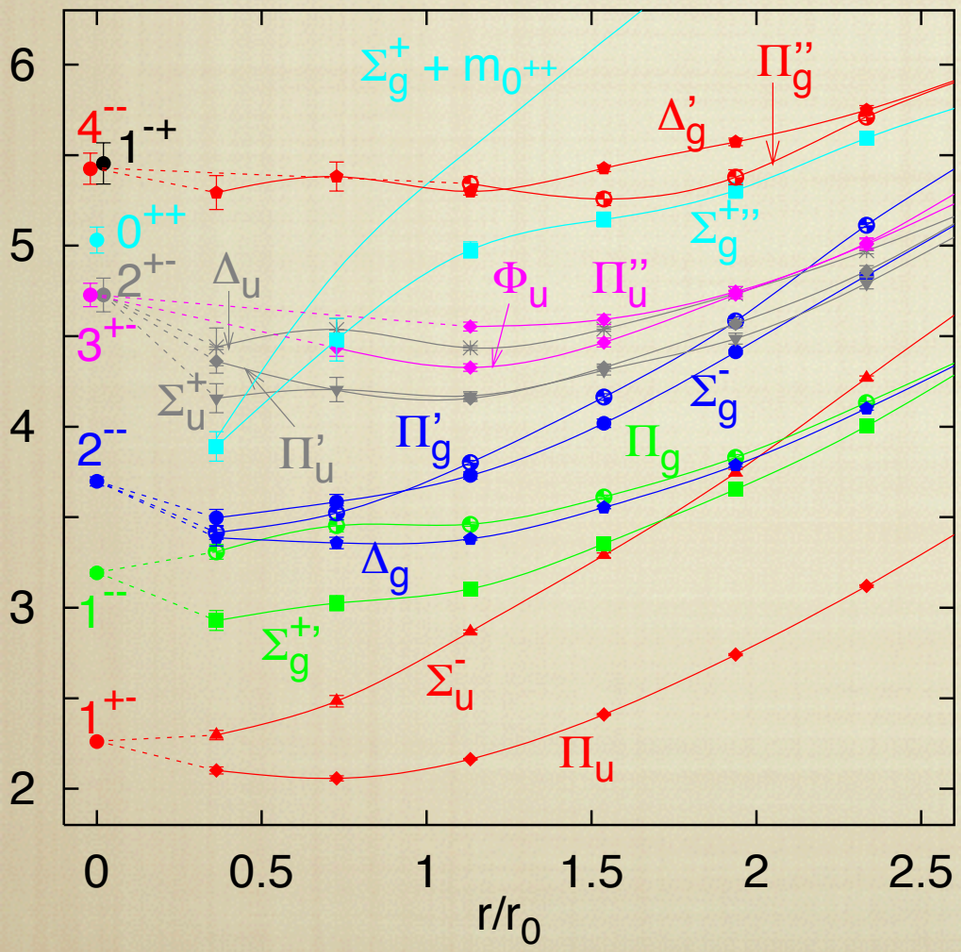

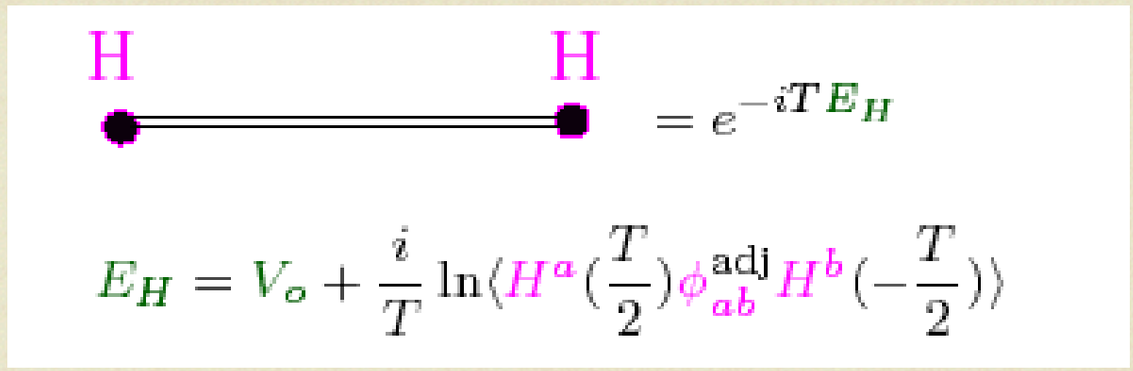

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octet potential

gluelump mass

correction softly bre the symmetry

We define symmetries and states in NRQCD

We match the energy and the states to pNRQCD at order 1/m in the expansion (but no spin for now) and identify coupled Schroedinger equations for Sigma_u and Pi_u

These are nonperturbative and would require lattice calculations of matrix elements

Lacking the lattice calculation, we identify the potentials with a multipole expansion in pNRQCD, solve the coupled equations and get the lowest ccbar, bbar and bcbar muliplets

Lowest energy multiplet $\Sigma_u^- - \Pi_u$

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

- The two lowest laying hybrid static energies are Π_u and Σ_u^-.
- They are generated by a gluelump with quantum numbers 1^{+-} and thus are degenerate at short distances.
- The kinetic operator mixes them but not with other multiplets.
- Well separated by a gap of ~ 1 GeV from the next multiplet with the same CP.

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Λ_H

- It is a non-perturbative quantity.
- It depends on the particular operator H^a, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster *et all* 1999; Bali, Pineda 2004; Marsh Lewis 2014
- At the subtraction scale $\nu_f = 1$ GeV: $\Lambda_{1^{+-}}^{RS} = 0.87(15)$ GeV.

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b_H

- It is a non-perturbative quantity.

- Proportional to r^2 due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.

Lowest energy multiplet $\Sigma_u^- - \Pi_u$

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

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Coupled radial Schrödinger equations

Projection vectors in matrix elements allow for two different solutions (coupled or uncoupled) for the Σ_u^- and Π_u radial wave functions:

1st solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)}\end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

- energy eigenvalue $\mathcal{E}$ gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

Lowest energy multiplet $\Sigma_u^- - \Pi_u$

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

The Lambda -doubling effect breaks the degeneracy between opposite parity spin-symmetry multiplets and lowers the mass of the multiplets that get mixed contributions of different static energies.

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2nd solution

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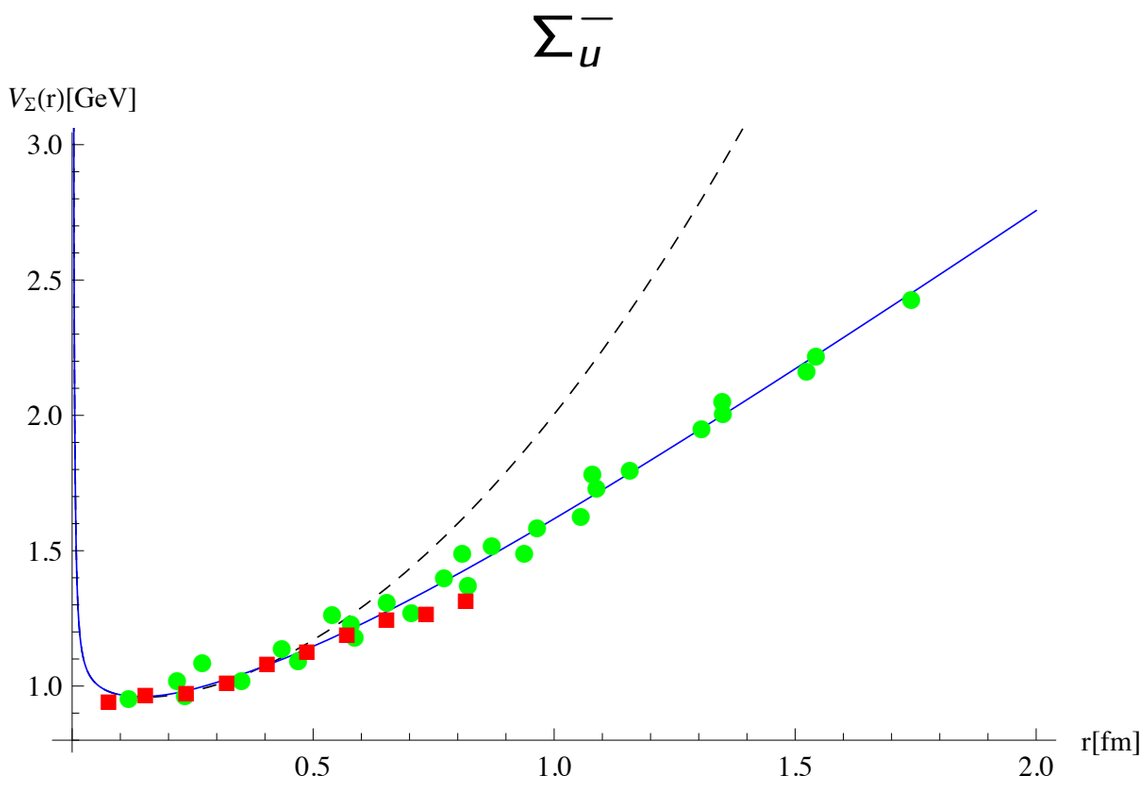
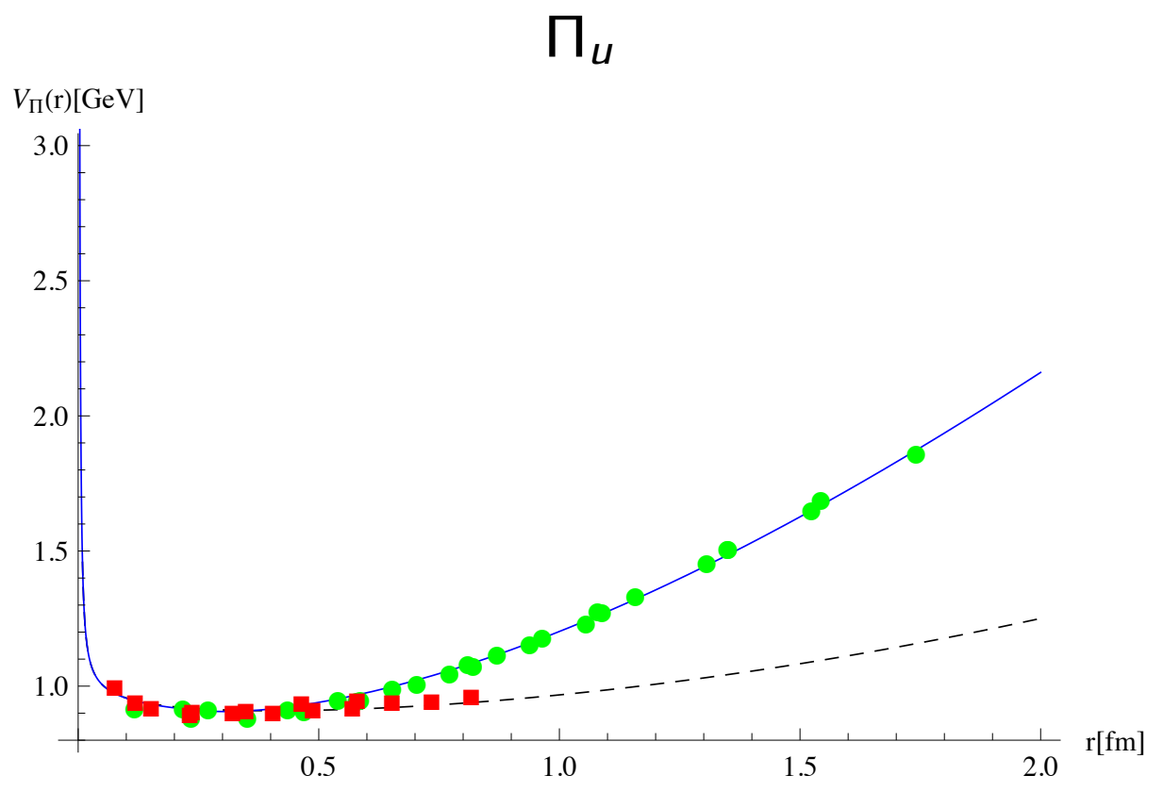




Lattice data:Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.

$$b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV/fm}^2, \quad b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV/fm}^2\,.$$

- $r > 0.25$ fm: phenomenological potential.
  - $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - *Constraint:* Continuity up to first derivatives.

Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H_1'	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H_2'	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

1. The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

- As expected the our approach works better in bottomonium than charmonium

- Spin symmetry multiplets

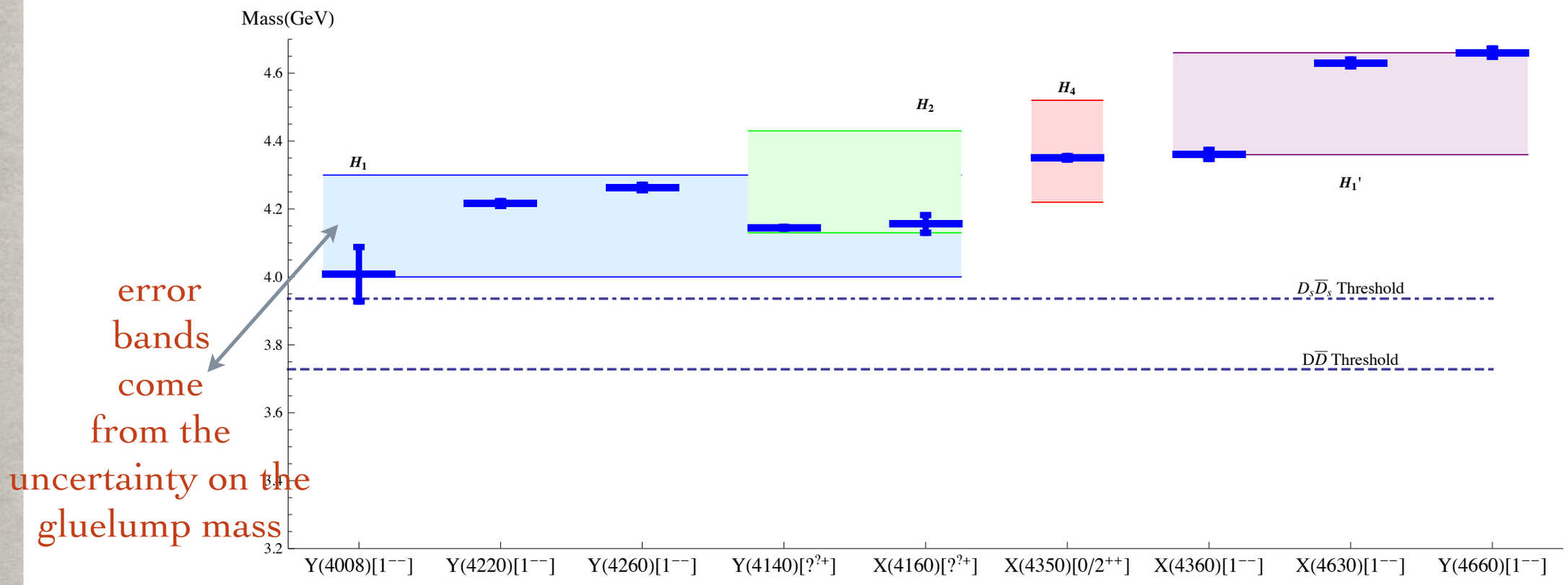
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^- , Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^- , Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2,3,4)^{-+}\}$	Σ_u^- , Π_u
H_7	$\{3^{++}, (2,3,4)^{+-}\}$	Π_u

Berwein,N.B. , Tarrus, Vairo arXiv:1510.04299

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively.

- Charmonium states (Belle, CDF, BESIII, Babar):



- Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, except for $Y(4220)$, all other candidates observed decay modes violate Heavy Quark Spin Symmetry.

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Comparison to direct lattice calculations

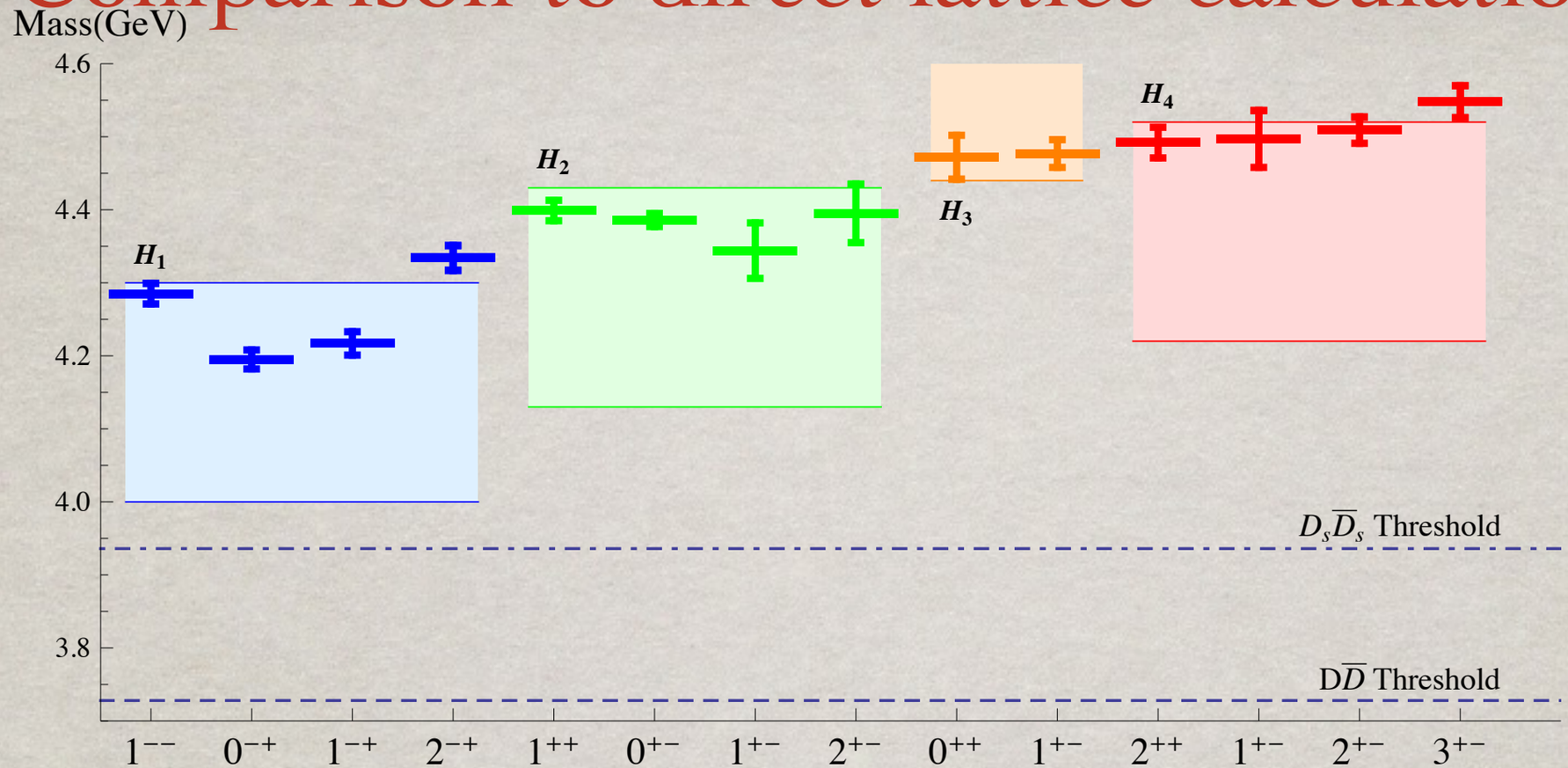
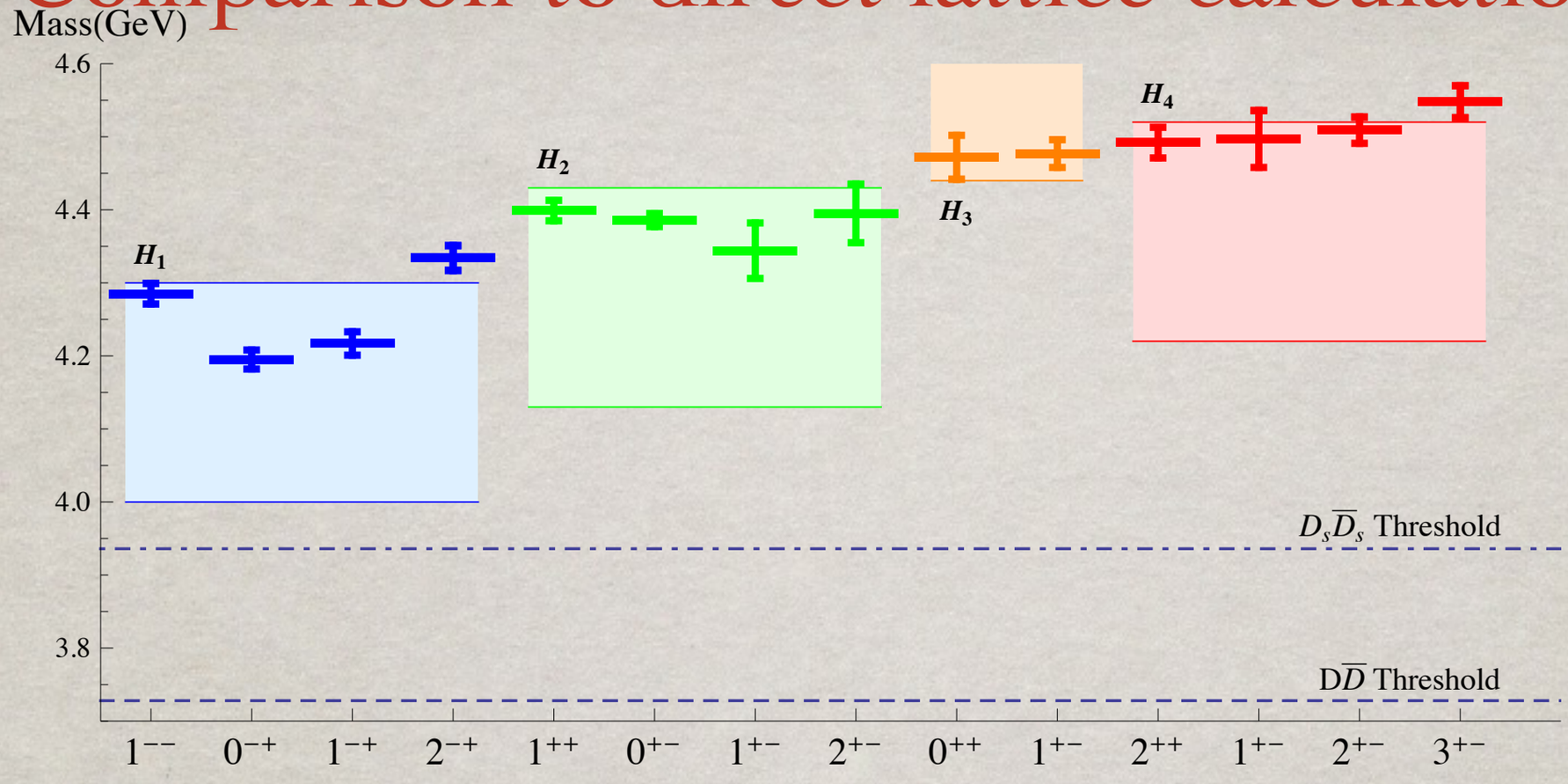


FIG. 5. Comparison of the results from direct lattice computations of the masses for charmonium hybrids [48] with our results using the $V^{(0.25)}$ potential. The direct lattice mass predictions are plotted in solid lines with error bars corresponding the mass uncertainties. Our results for the H_1, H_2, H_3, and H_4 multiplets have been plotted in error bands corresponding to the gluelump mass uncertainty of ± 0.15 GeV.

We observe the same Lambda-doubling pattern in lattice calculations, multiplets that receive mixed contributions from Sigma_u and Pi_u have lower masses then those that remain pure Pi_u states

L. Liu *et al.* [Hadron Spectrum Collaboration], JHEP **1207**, 126 (2012) [arXiv:1204.5425

Comparison to direct lattice calculations



support the result of the pNRQCD and BO approaches that the hybrid states appear in three distinct multiplets (H_2, H_3, and H_4) as compared to the constituent gluon picture, where they are assumed to form one supermultiplet together (cf. also the discussion in [34]).

uncertainty of ± 0.15 GeV.

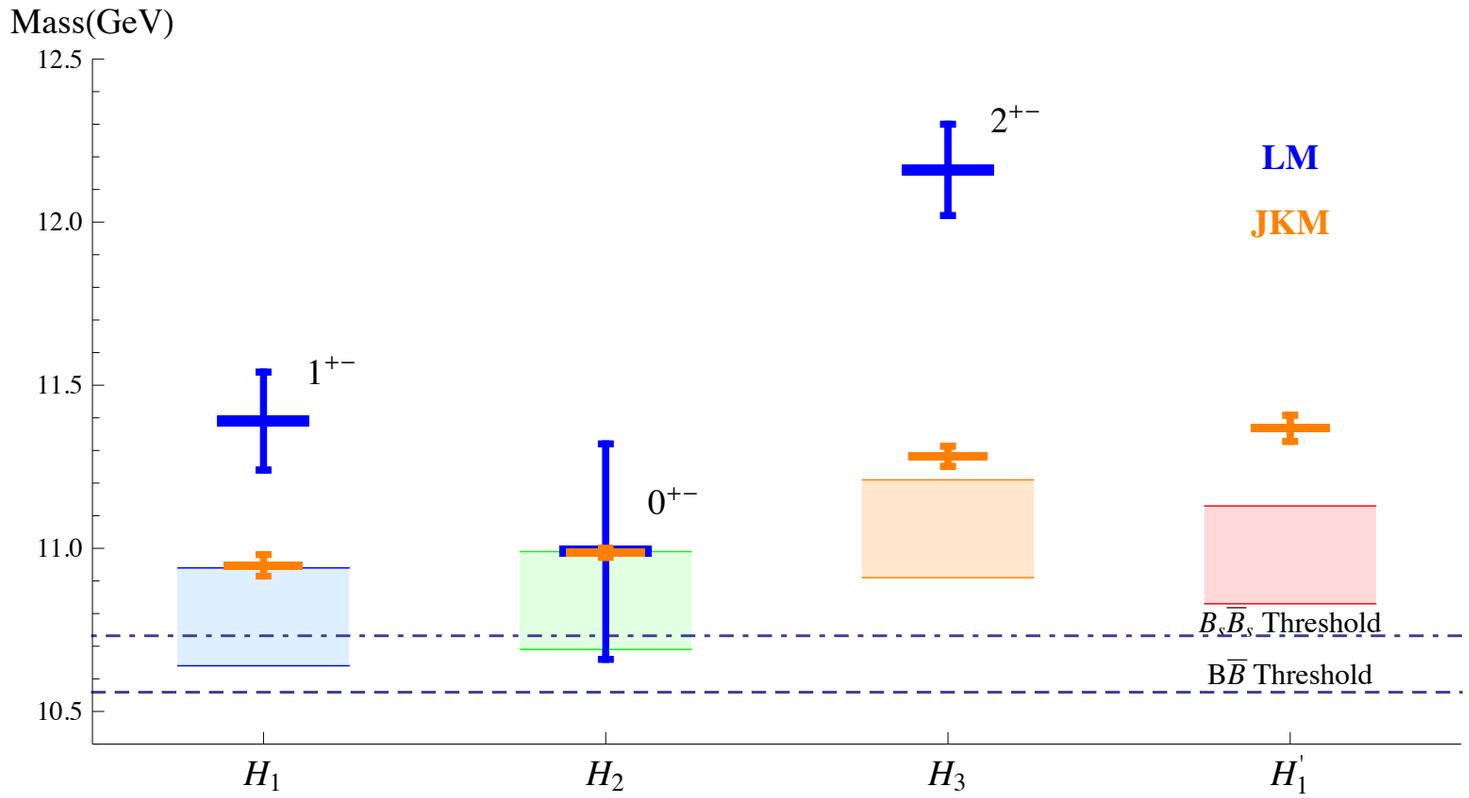
We observe the same Lambda-doubling pattern in lattice calculations, multiplets that receive mixed contributions from Sigma_u and Pi_u have lower masses then those that remain pure Pi_u states

L. Liu *et al.* [Hadron Spectrum Collaboration], JHEP **1207**, 126 (2012) [arXiv:1204.5425

Comparison with direct lattice computations

Bottomonium sector

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- Our masses are $0.15 - 0.25$ GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

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Comparison to sum rules

arXiv:1304.4522 [hep-ph]

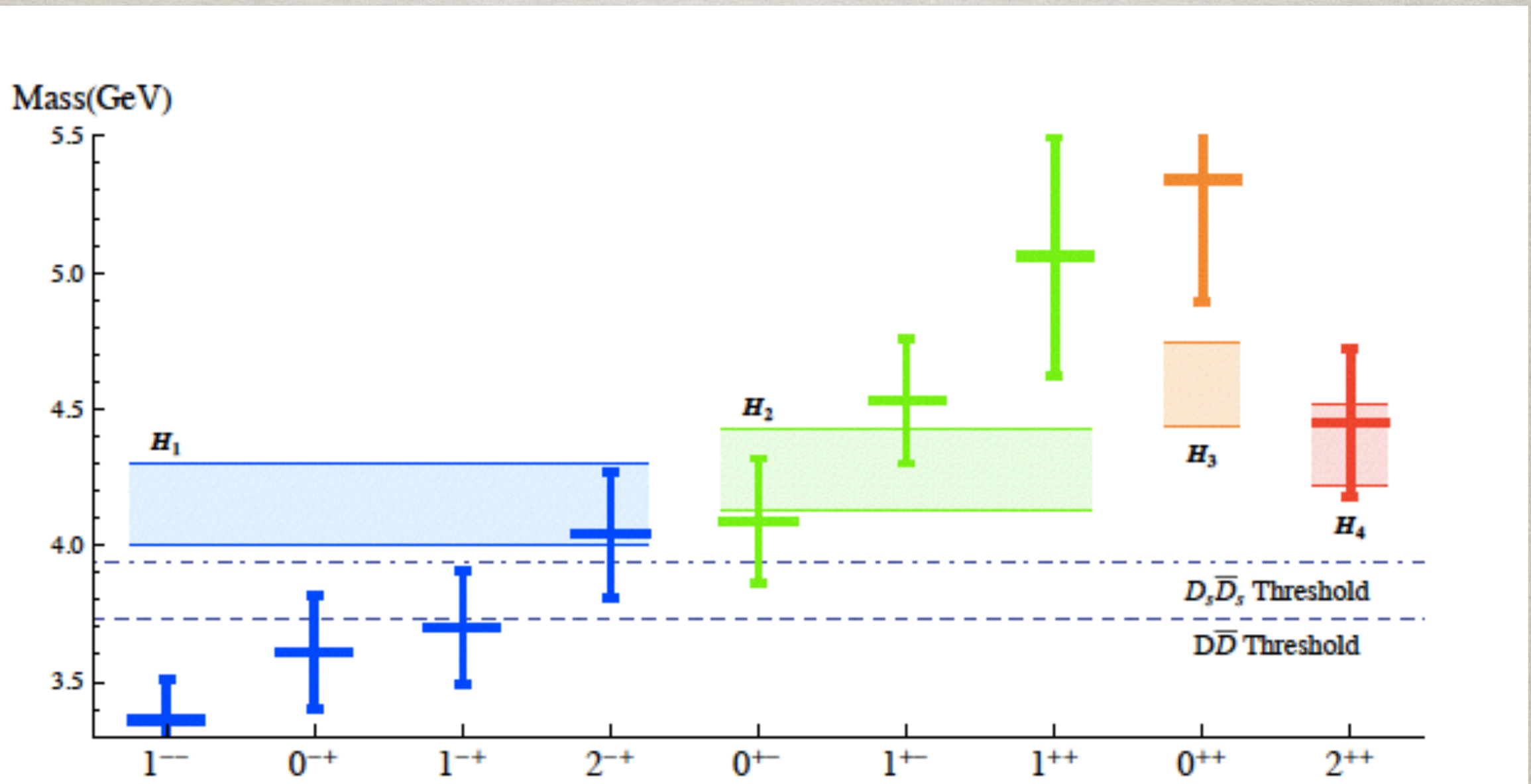


FIG. 7. Comparison of the mass predictions for charmonium hybrids in the upper figure and for bottomonium hybrids in the lower figure, obtained using QCD sum rules [68], with our results using the $V^{(0.25)}$ potential. The solid lines correspond to the QCD sum rules masses with error bars corresponding to their uncertainties. Our results for the H_1, H_2, H_3, and H_4 multiplets have been plotted in error bands corresponding to the gluelump mass uncertainty of ± 0.15 GeV.

Conclusions

Quarkonium is a golden system to study strong interactions

For states below threshold non relativistic EFTs provide a systematic tool to investigate a wide range of observables in the real of QCD

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

We are working to obtain decays and transitions, to obtain a version of strongly coupled pNRQCD including hybrids and to include light quark degrees of freedom

Lattice input needed: gluelump masses, tetra quark potentials

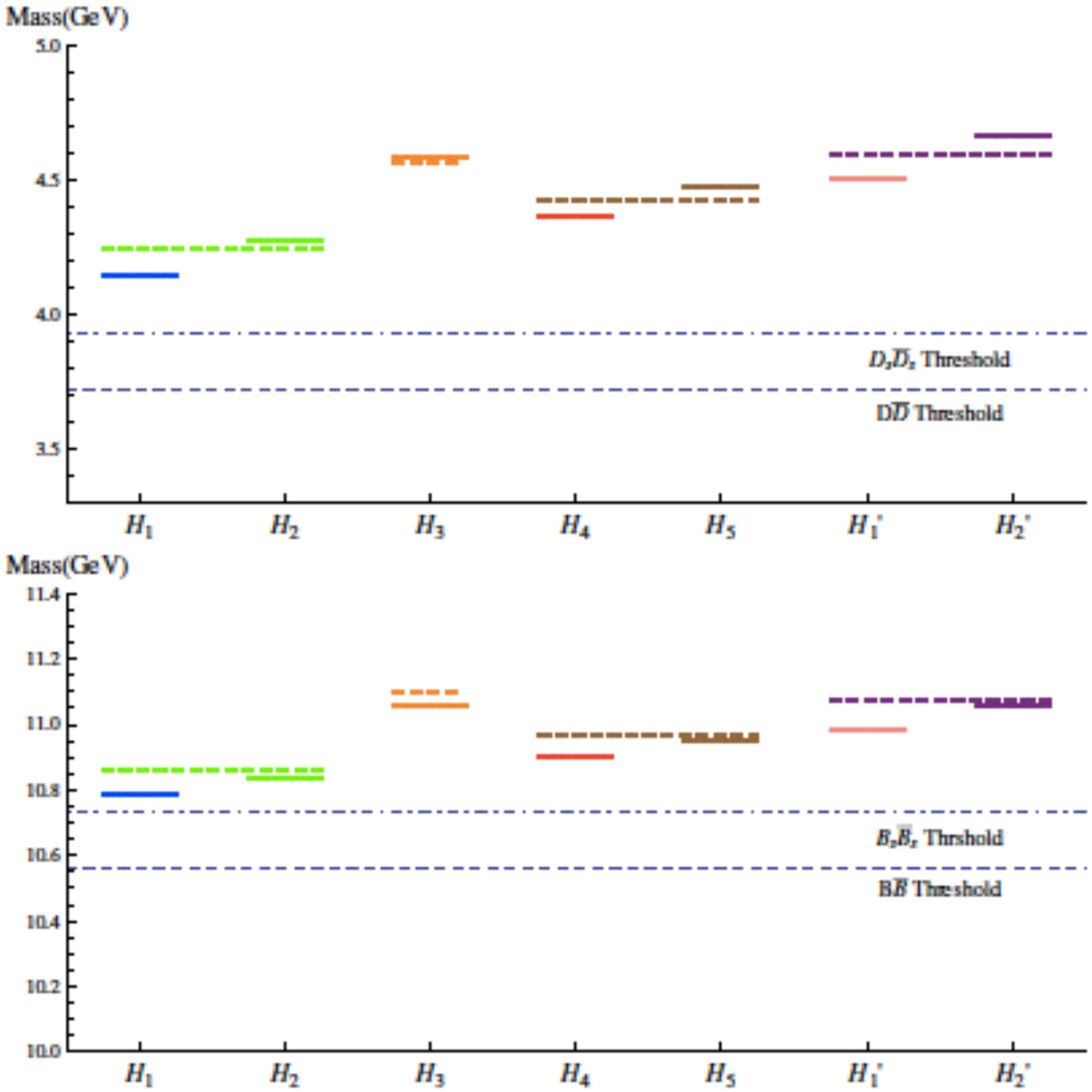
- Fundamental experimental input (like confirmation, quantum numbers, widths and masses) is still crucially missing for some of these states.

Experimental candidates for hybrids

State	M (MeV)	Γ (MeV)	J^{PC}	Decay modes	1st observation
$X(3823)$	3823.1 ± 1.9	< 24	$?^{?-}$	$\chi_{c1}\gamma$	Belle 2013
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$J/\psi\,\pi^+\pi^-,\ J/\psi\,\pi^+\pi^-\pi^0$	Belle 2003
				$D^0\bar{D}^0\pi^0,\ D^0\bar{D}^0\gamma$	
				$J/\psi\,\gamma,\ \psi(2S)\,\gamma$	
$X(3915)$	3917.5 ± 1.9	20 ± 5	0^{++}	$J/\psi\,\omega,$	Belle 2004
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$D\bar{D},$	Belle 2005
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$D^*\bar{D},\ D\bar{D}^*$	Belle 2007
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$D\bar{D},$	Babar 2007
$Y(4008)$	$4008^{+121}_{-\ 49}$	226 ± 97	1^{--}	$J/\psi\,\pi^+\pi^-,$	Belle 2007
$Y(4140)$	4144.5 ± 2.6	$15^{+11}_{-\ 7}$	$?^{?+}$	$J/\psi\,\phi$	CDF 2009
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$D^*\bar{D}^*$	Belle 2007
$Y(4220)$	4216 ± 7	39 ± 17	1^{--}	$h_c(1P)\,\pi^+\pi^-,$	BESIII 2013
$Y(4230)$	4230 ± 14	38 ± 14	1^{--}	$\chi_{c0}\,\omega,$	BESIII 2014
$Y(4260)$	4263^{+8}_{-9}	95 ± 14	1^{--}	$J/\psi\,\pi^+\pi^-,\ J/\psi\,\pi^0\pi^0$	Babar 2005
				$Z_c(3900)\,\pi,$	
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$J/\psi\,\phi$	CDF 2010
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0/2^{++}$	$J/\psi\,\phi,$	Belle 2009
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$\psi(2S)\,\pi^+\pi^-,$	Babar 2007
$X(4630)$	$4634^{+\ 9}_{-11}$	92^{+41}_{-32}	1^{--}	$\Lambda_c^+\Lambda_c^-,$	Belle 2007
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$\psi(2S)\,\pi^+\pi^-,$	Belle 2007
$Y_b(10890)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$\Upsilon(nS)\pi^+\pi^-$	Belle 2010

TABLE V: Neutral mesons above open flavor threshold excluding isospin partners of charged states.

Results I: Comparison to BO approximation



Braaten, Langmack and Smith 2014

For Hybrid multiplets:

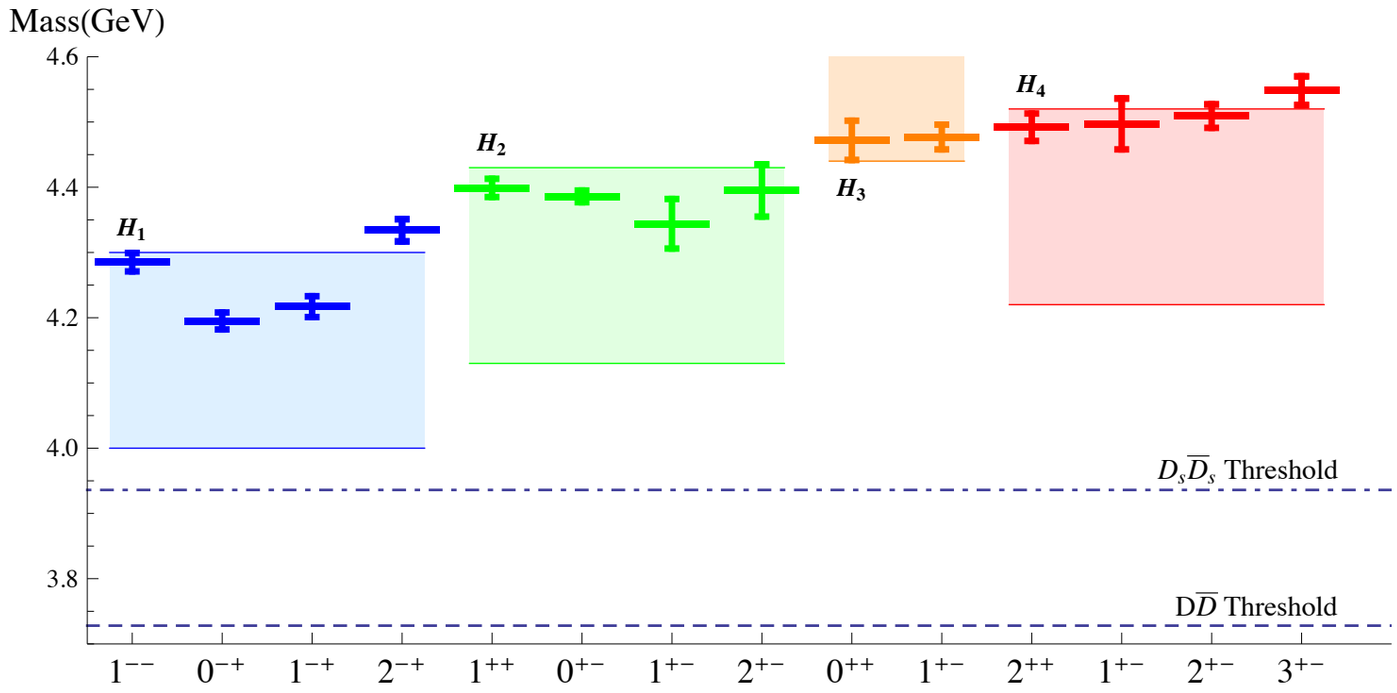
	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

- no distinction between opposite parity states in BO
- mixed states lie lower than pure
- discrepancy for H_2, H_3 and H_5 due to different potential fits

Comparison with direct lattice computations

Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu *et all* 2012
- They worked in the constituent gluon picture, which consider the multiplets H_2, H_3, H_4 as part of the same multiplet.
- Their results are given with the η_c mass subtracted.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- Our masses are $0.1 - 0.14$ GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^-.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

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Comparison to sum rules

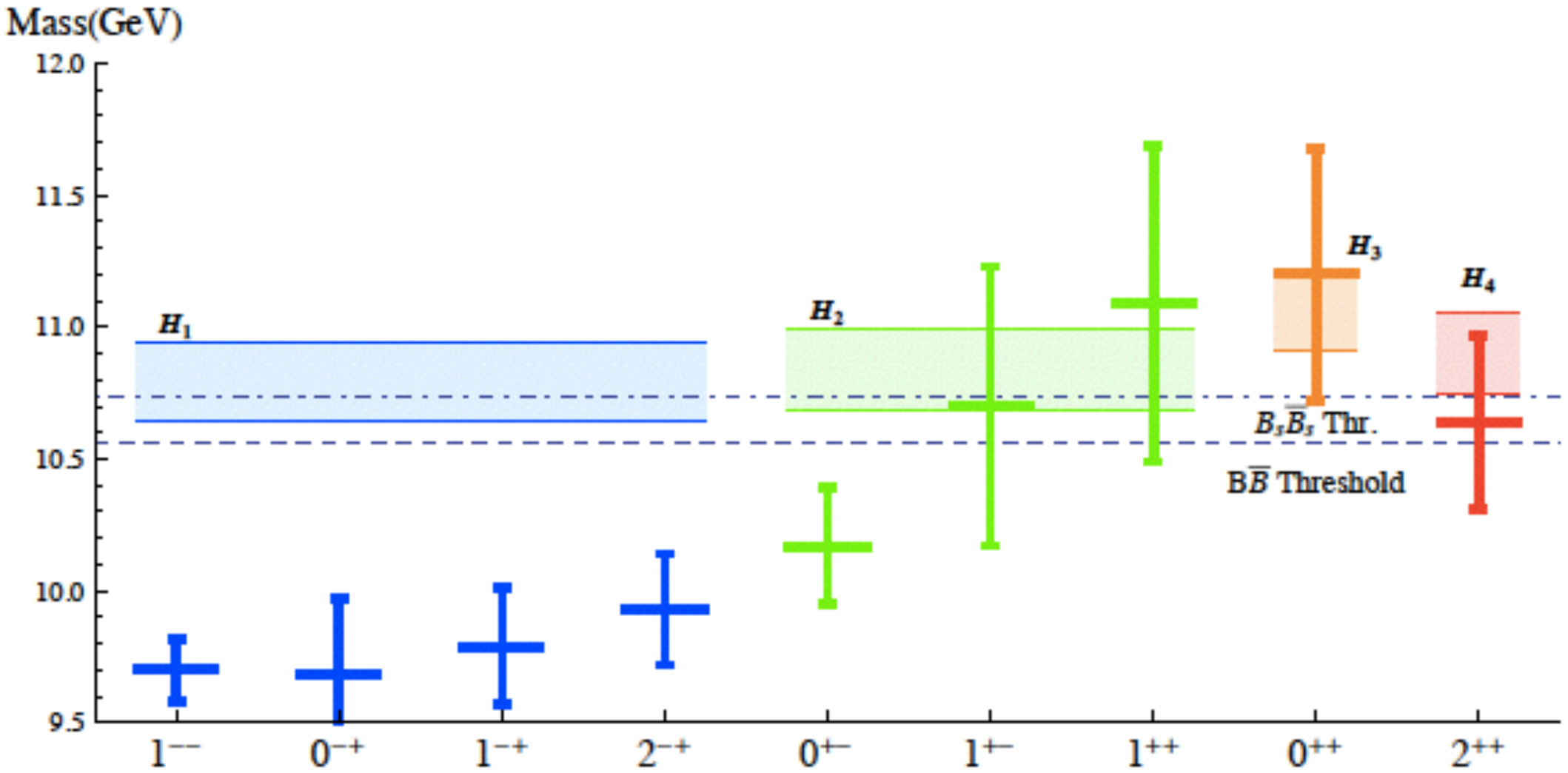

FIG. 7. Comparison of the mass predictions for charmonium hybrids in the upper figure and for bottomonium hybrids in the lower figure, obtained using QCD sum rules [68], with our results using the $V^{(0.25)}$ potential. The solid lines correspond to the QCD sum rules masses with error bars corresponding to their uncertainties. Our results for the H_1, H_2, H_3, and H_4 multiplets have been plotted in error bands corresponding to the gluelump mass uncertainty of ± 0.15 GeV.

- We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ–doubling terms by using coupled Schröringer equations.
- The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.
- Λ–doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- Several experimental candidates for Charmonium hybrids belonging to the H_1, H_2, H_4 and H_1' multiplets.
- One experimental candidate to the bottomonium H_1 multiplet.