

Mixing and CP violation: General Introduction and Lattice Perspectives

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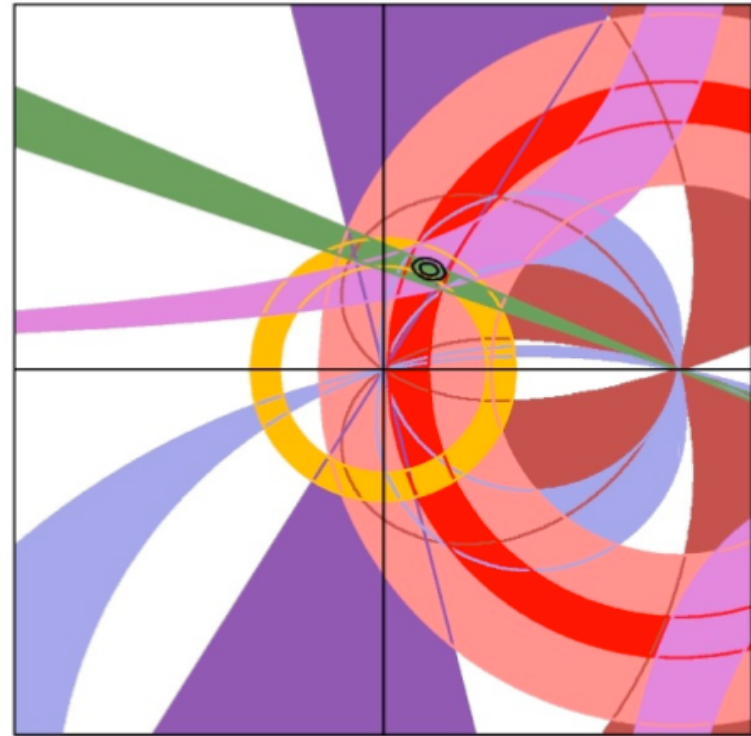
International School for Advanced Studies



PLAN OF THE TALK

- *General introduction to the Unitary Triangle Fit*
- *SM Analysis*
- *Tensions and unknown*
- *New/old ideas from Lattice QCD vs Charm Physics*
- *Conclusion*

- *New Physics -> M. Ciuchini*
- *More on Charm D. Derkach*



*Thanks to Bona,
Lubicz and
Silvestrini,*

Flavor physics in the Standard Model

In the SM, the quark mass matrix, from which the CKM matrix and CP violation originate, is determined by the coupling of the Higgs boson to fermions.

$$\mathcal{L}_{\text{quarks}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Yukawa}}$$

CP invariant

CP and symmetry breaking are strictly correlated

$$\mathcal{L}(\Lambda_{\text{Fermi}}) = \mathcal{L}(\Lambda, H, H^\dagger) + \mathcal{L}^{\text{kin}} + \mathcal{L}_{\text{SM}}^{\text{gauge}} + \mathcal{L}_{\text{SM}}^{\text{Yukawa}} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \dots$$

EWSB

has many accidental symmetries

may violate accidental symmetries

Absence of FCNC at tree level (& GIM suppression of FCNC @loop level)

Almost no CP violation at tree level

Flavour Physics is extremely sensitive to New Physics (NP)

In competition with Electroweak Precision Measurements

WHY RARE DECAYS ?

Rare decays are a manifestation of broken (accidental) symmetries e.g. of physics beyond the Standard Model

Proton decay

baryon and lepton
number conservation

$$\mu \rightarrow e + \gamma$$

lepton flavor number

$$\nu_i \rightarrow \nu_k \text{ found !}$$

RARE DECAYS WHICH ARE ALLOWED IN THE STANDARD MODEL

FCNC:

$$q_i \rightarrow q_k + \nu \bar{\nu}$$

$$q_i \rightarrow q_k + l^+ l^-$$

$$q_i \rightarrow q_k + \gamma$$

these decays occur
only via loops because
of *GIM* and are
suppressed by *CKM*

THUS THEY ARE SENSITIVE TO
NEW PHYSICS

Flavour and New Physics

Flavour phenomenology plays a fundamental role in indirect searches of New Physics:

- *looks for deviation from the SM whatever the origin*
- *needs good theoretical control of the SM contribution only*
- *in general cannot provide precise information on the NP scale, but a positive result would be a strong evidence that NP is not too far (i.e. in the multi-TeV region)*

the path leading to TeV NP is narrower after the results of the LHC

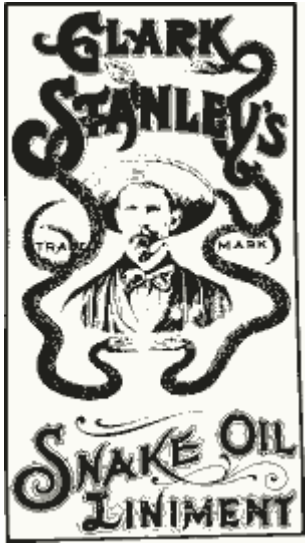
this will be further explored in the present run



- 1) A fundamental issue is **to find signatures of new physics** and to unravel the underlying theoretical structure;
- 2) Precision Flavor physics is a key tool, complementary to the large energy searches at the LHC, in this endeavour;
- 3) If the LHC discovers new elementary particles BSM, then precision flavor physics will be necessary to understand the underlying framework;
- 4) **The discovery potential of precision flavor physics should also not be underestimate;**
- 5) Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

$$Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$



www.utfit.org

C. Alpigiani, A. Bevan, M.B., M. Ciuchini,
D. Derkach, E. Franco, V. Lubicz, G. Martinelli,
F. Parodi, M. Pierini, C. Schiavi, L. Silvestrini,
A. Stocchi, V. Sordini, C. Tarantino and V. Vagnoni

Other UT analyses exist, by:

CKMfitter (<http://ckmfitter.in2p3.fr/>),

Laiho&Lunghi&Van de Water (<http://latticeaverages.org/>)

Lunghi&Soni (1010.6069)

CP Violation in the Standard Model

$N(N-1)/2$ angles and $(N-1)(N-2)/2$ phases

$N=3$ 3 angles + 1 phase KM
 the phase generates complex couplings i.e. CP
violation;

6 masses +3 angles +1 phase = 10 parameters

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

**NO Flavour Changing Neutral Currents (FCNC)
at Tree Level
(FCNC processes are good candidates for observing
NEW PHYSICS)**

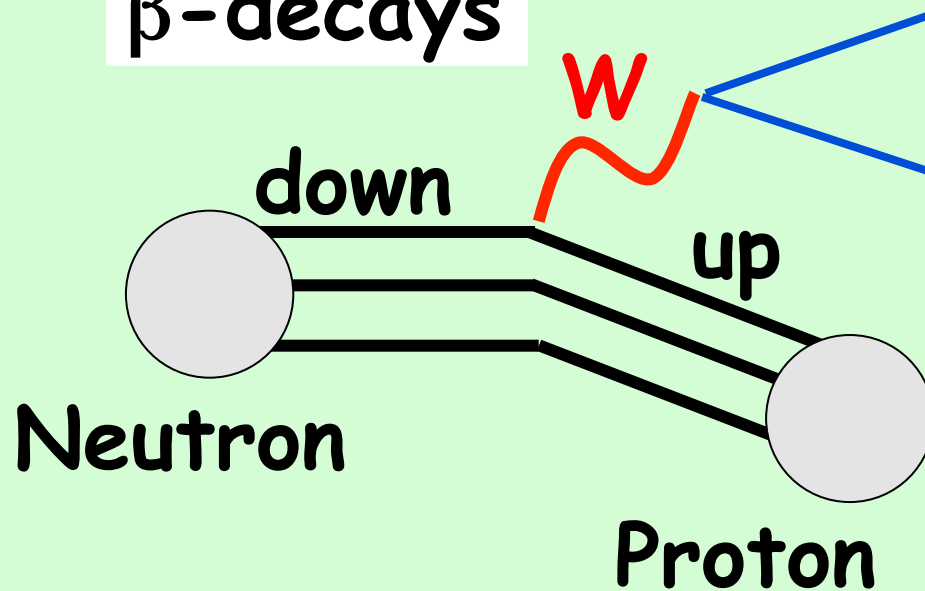
**CP Violation is natural with three quark
generations (Kobayashi-Maskawa)**

**With three generations all CP
phenomena are related to the same
unique parameter (δ)**

Quark masses & Generation Mixing

V_{ud}	V_{us}	V_{ub}
V_{cd}	V_{cs}	V_{cb}
V_{td}	V_{ts}	V_{tb}

β -decays



$$|V_{ud}|$$

e^-
 $\bar{\nu}_e$

$$|V_{ud}| = 0.9735(8)$$

$$|V_{us}| = 0.2196(23)$$

$$|V_{cd}| = 0.224(16)$$

$$|V_{cs}| = 0.970(9)(70)$$

$$|V_{cb}| = 0.0406(8)$$

$$|V_{ub}| = 0.00409(25)$$

$$|V_{tb}| = 0.99(29)$$

$$(0.999)$$

The Wolfenstein Parametrization

$1 - 1/2 \lambda^2$	λ	$A \lambda^3(\rho - i \eta)$
$-\lambda$	$1 - 1/2 \lambda^2$	$A \lambda^2$
$A \lambda^3 \times$ $(1 - \rho - i \eta)$	$-A \lambda^2$	1

V_{ub}

$+ O(\lambda^4)$

V_{td}

$$\lambda \sim 0.2 \quad A \sim 0.8$$

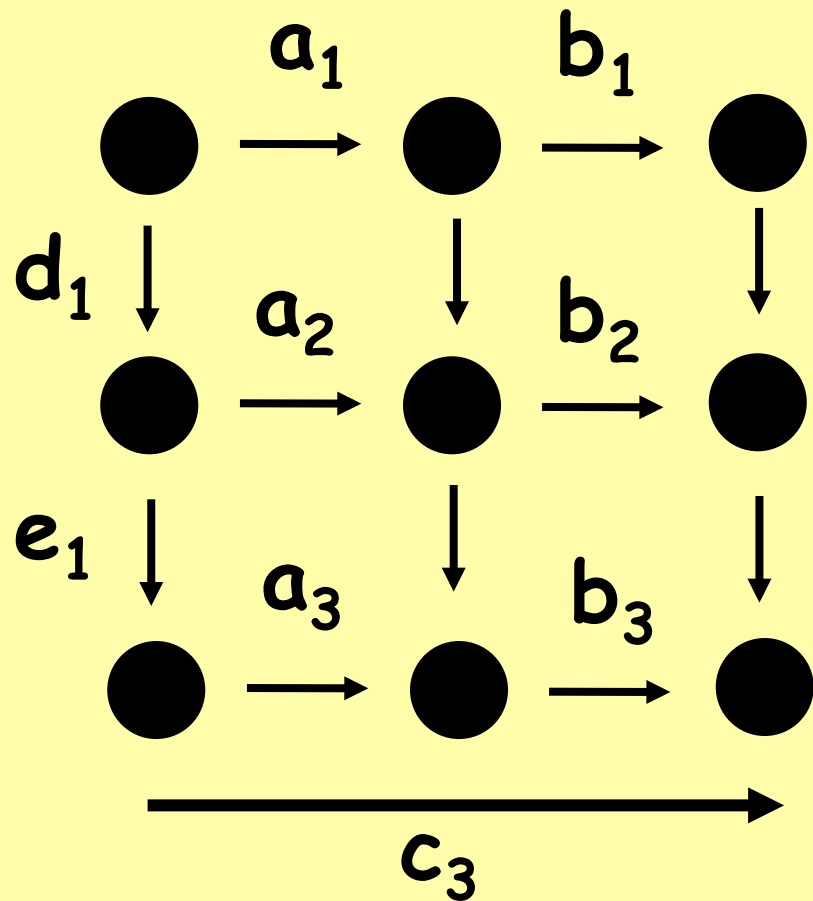
$$\eta \sim 0.2 \quad \rho \sim 0.3$$

$$\sin \theta_{12} = \lambda$$

$$\sin \theta_{23} = A \lambda^2$$

$$\sin \theta_{13} = A \lambda^3(\rho - i \eta)$$

The Bjorken-Jarlskog Unitarity Triangle



$|V_{ij}|$ is invariant under phase rotations

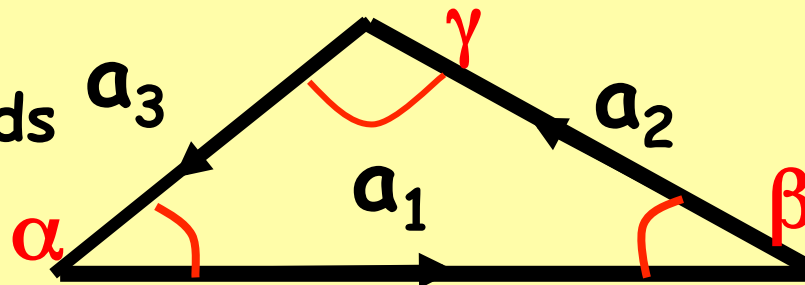
$$a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^*$$

$$a_2 = V_{21} V_{22}^* \quad a_3 = V_{31} V_{32}^*$$

$$a_1 + a_2 + a_3 = 0$$

$$(b_1 + b_2 + b_3 = 0 \text{ etc.})$$

Only the orientation depends on the phase convention



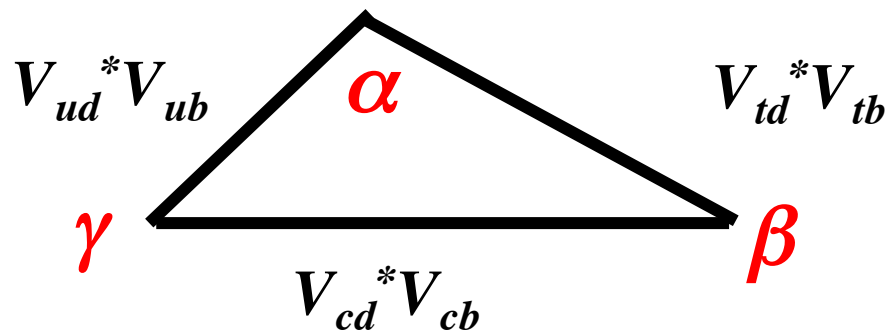
Physical quantities correspond to invariants under phase reparametrization i.e. $|a_1|, |a_2|, \dots, |e_3|$ and the area of the Unitary Triangles

$$J = \text{Im} (a_1 a_2^*) = |a_1 a_2| \sin \beta$$

a precise knowledge of the moduli (angles) would fix J

$$\mathcal{CP} \propto J$$

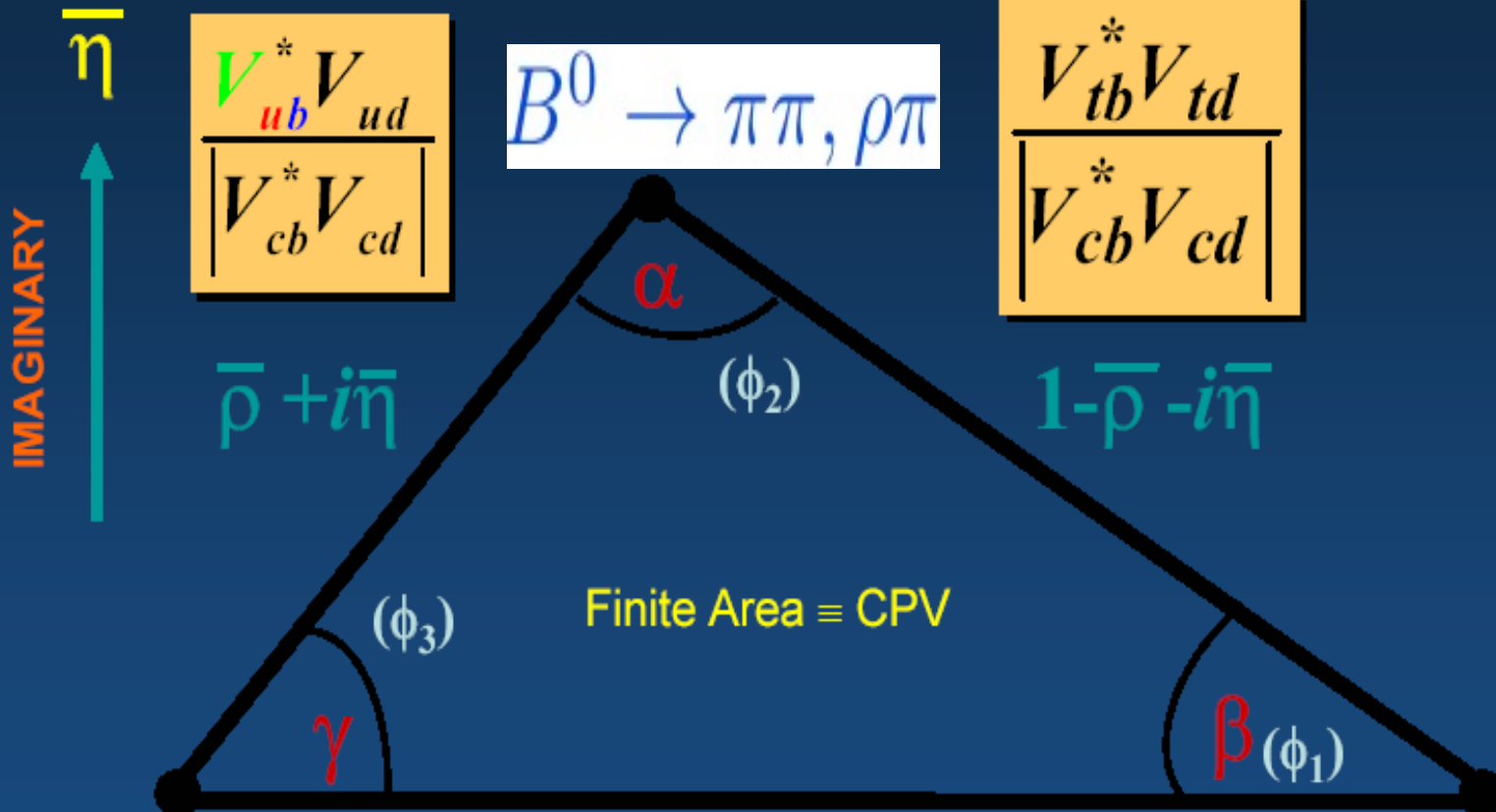
$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$



$$\gamma = \delta_{CKM}$$

Unitarity:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



$$\gamma = \tan^{-1} \left(\frac{\bar{\eta}}{\bar{\rho}} \right)$$

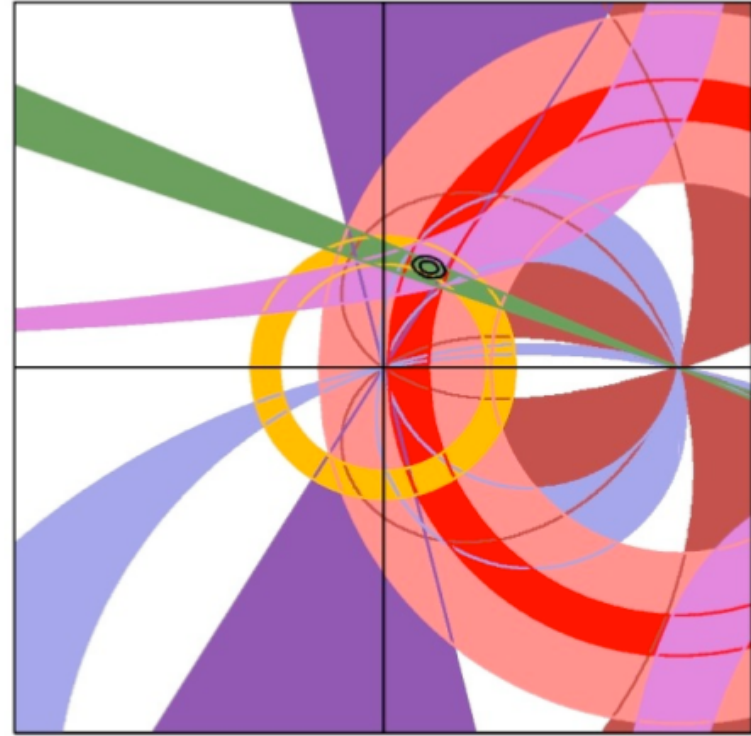
$B^0 \rightarrow DK^{(*)}$

REAL \rightarrow $\bar{\rho}$

$$\beta = \tan^{-1} \left(\frac{\bar{\eta}}{(1-\bar{\rho})} \right)$$

$B^0 \rightarrow J/\psi K_s$

***STANDARD
MODEL
UNITARITY
TRIANGLE
ANALYSIS
(Flavor Physics)***



- *Provides the best determination of the CKM parameters;*
- *Tests the consistency of the SM (“direct” vs “indirect” determinations) @ the quantum level;*
- *Provides predictions for SM observables (in the past for example $\sin 2\beta$ and Δm_s)*
- *It could lead to new discoveries (CP violation, Charm, !?)*

Measure	V_{CKM}	Other NP parameters
$\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$\bar{\Lambda}, \lambda_1, F(1), \dots$
ε_K	$\eta [(1 - \bar{\rho}) + \dots]$	B_K
Δm_d	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d/\Delta m_1$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	ξ
$A_{CP}(B_d \rightarrow J/\psi K_s)$	$\sin 2\beta$	—

$$Q^{EXP} = V_{CKM} \times \langle H_F | \hat{O} | H_I \rangle$$

For details see:
 UTfit Collaboration
<http://www.utfit.org>

classical UT analysis

DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible theor. uncertainties

$$A_{CP}(B \rightarrow J/\psi K_s) \quad \gamma \text{ from } B \rightarrow DK$$

$$K^0 \rightarrow \pi^0 \nu \bar{\nu}$$

2) Second class quantities, with theoretical errors of O(10%) or less that can be reliably estimated

$$\varepsilon_K \quad \Delta M_{d,s}$$

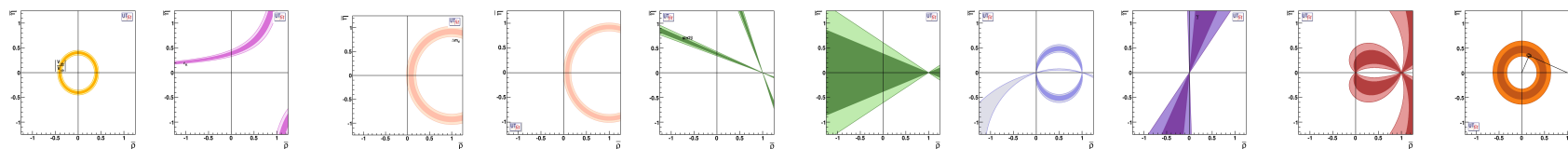
$$\Gamma(B \rightarrow c, u), \quad K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepancies we cannot tell whether is new physics or we must blame the model

$$B \rightarrow K \pi \quad B \rightarrow \pi^0 \pi^0$$

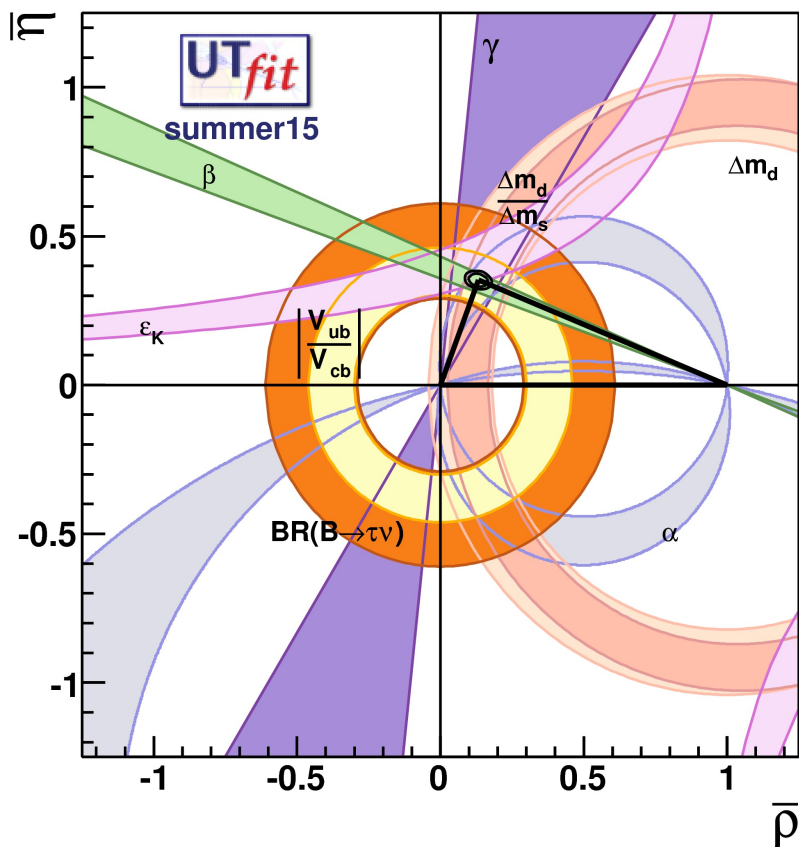
$$B \rightarrow \phi K_s$$



2016 results

$$\bar{\rho} = 0.153 \pm 0.013 \quad \bar{\eta} = 0.343 \pm 0.011$$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation



$$\alpha = (92.0 \pm 2.0)^\circ$$

$$\sin 2\beta = 0.696 \pm 0.018$$

$$\beta = (21.82 \pm 0.72)^\circ$$

$$\gamma = (65.8 \pm 1.9)^\circ$$

$$A = 0.833 \pm 0.012$$

$$\lambda = 0.22497 \pm 0.00069$$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

CKM Matrix in the SM 2016

CKM matrix thus looks like

$$V_{CKM} = \begin{pmatrix} (0.97431 \pm 0.00015) & (0.22512 \pm 0.00067) & (0.00365 \pm 0.00012)e^{i(-65.88 \pm 1.88)^\circ} \\ (-0.22497 \pm 0.00067)e^{i(0.0352 \pm 0.0010)^\circ} & (0.97344 \pm 0.00015)e^{i(-0.001877 \pm 0.000055)^\circ} & (0.04255 \pm 0.00069) \\ (0.00869 \pm 0.00014)e^{i(-22.00 \pm 0.73)^\circ} & (-0.04156 \pm 0.00056)e^{i(1.040 \pm 0.035)^\circ} & (0.999097 \pm 0.000024) \end{pmatrix}$$

Standard Parametrization (PDG)

$$\text{Sin } \theta_{12} = 0.22497 \pm 0.00069$$

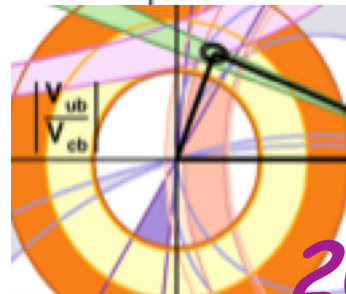
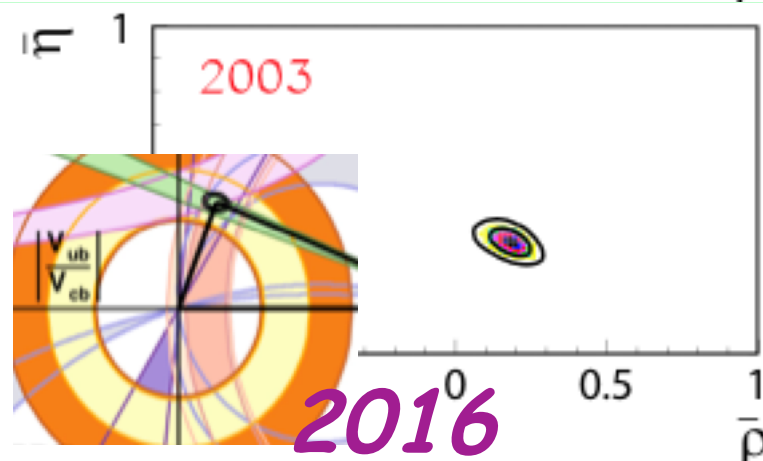
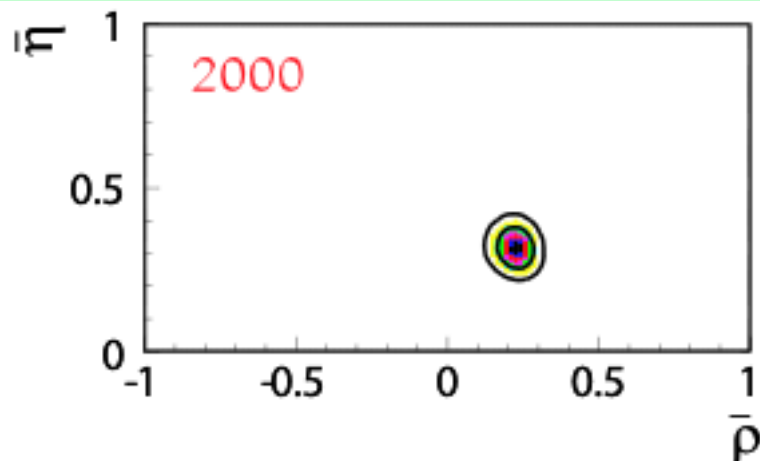
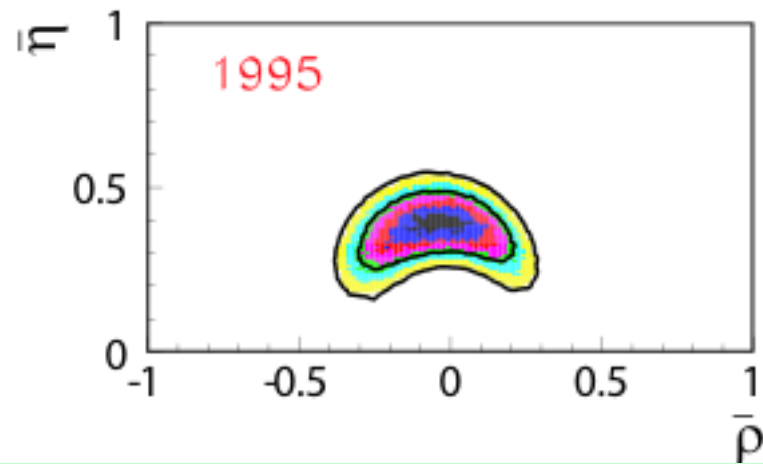
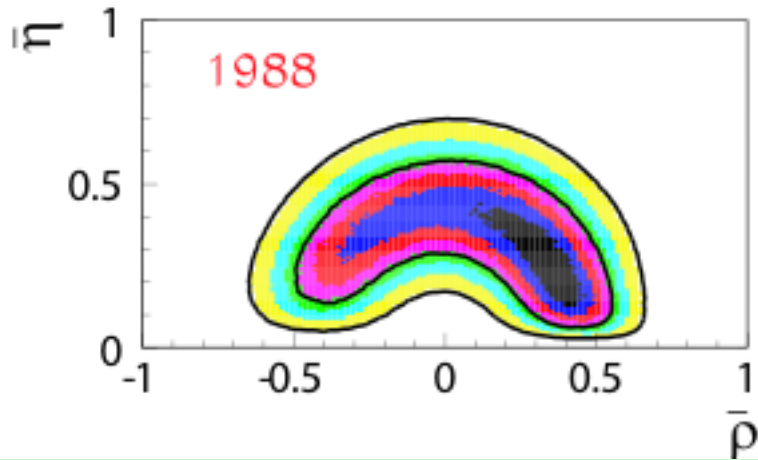
$$\text{Sin } \theta_{23} = 0.04229 \pm 0.00057$$

$$\text{Sin } \theta_{13} = 0.00368 \pm 0.00002 \quad \delta = 65.9 \pm 2.0$$

Wolfenstein Parametrization (PDG)

$$\lambda = 0.22497 \pm 0.00069 \quad A = 0.833 \pm 0.012$$

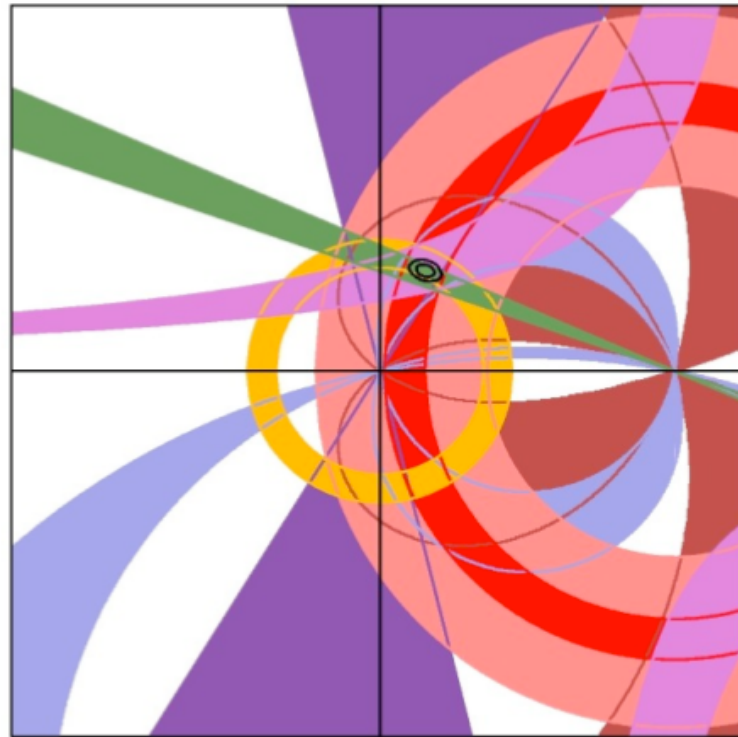
PROGRESS SINCE 1988



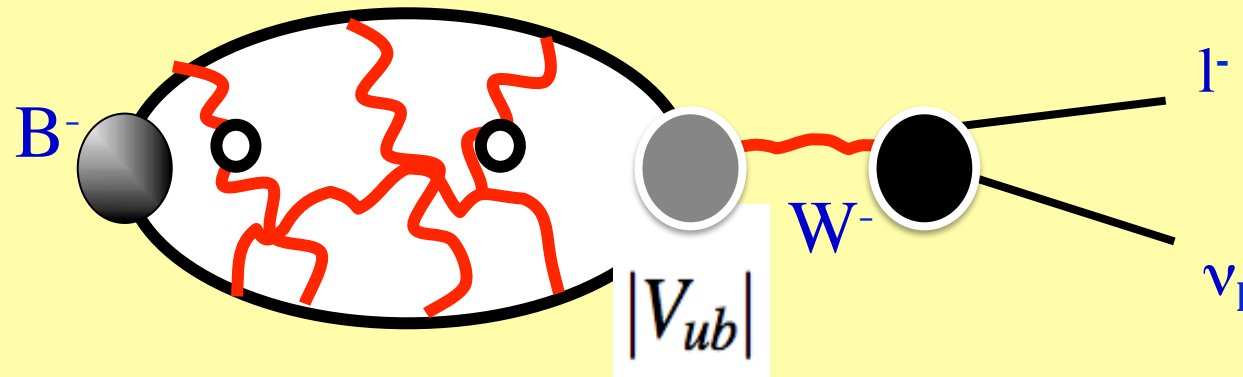
Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing
- 5) $R(D)$ and $R(D^*)$
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

- *What can be computed and what cannot be computed*



The Simplest Example

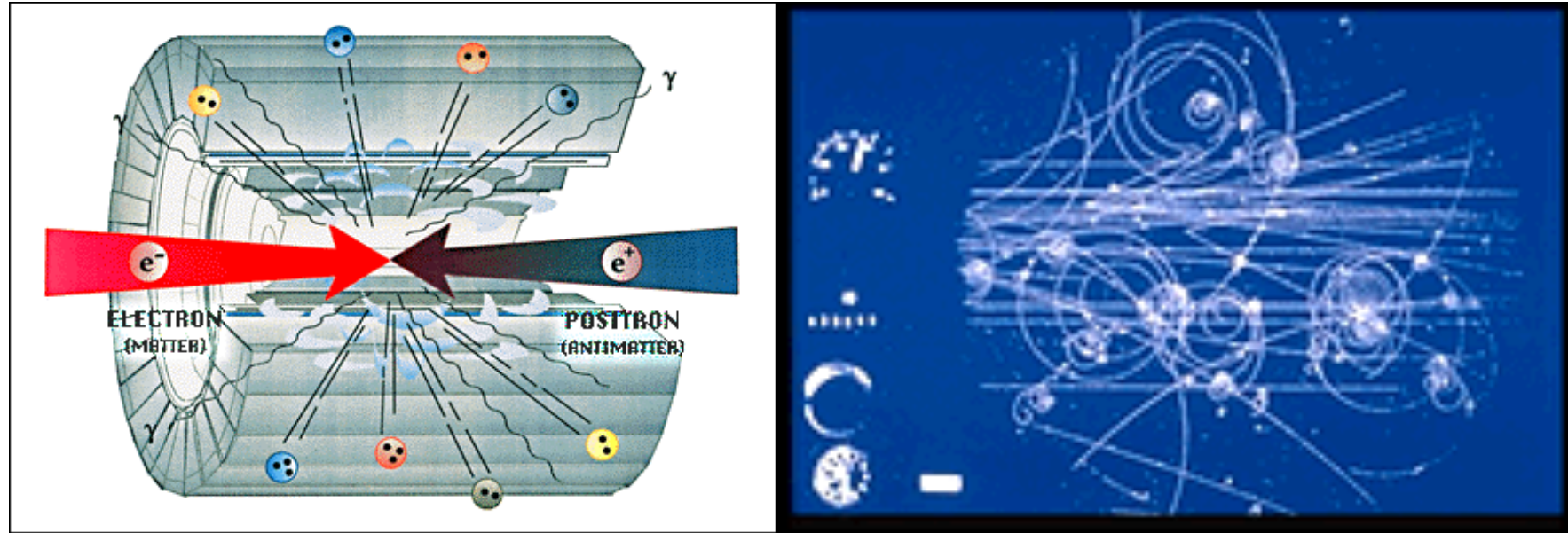


$$BR(B^- \rightarrow \tau^- \bar{\nu}_\tau) = f_B^2 |V_{ub}|^2 \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_B$$

$$f_B^2 |V_{ub}|^2$$

$$\langle 0 | \bar{b} \gamma_\mu \gamma_5 d | B^0(p) \rangle = i f_B p_\mu$$

COULD WE COMPUTE THIS PROCESS WITH
SUFFICIENT COMPUTER POWER ?



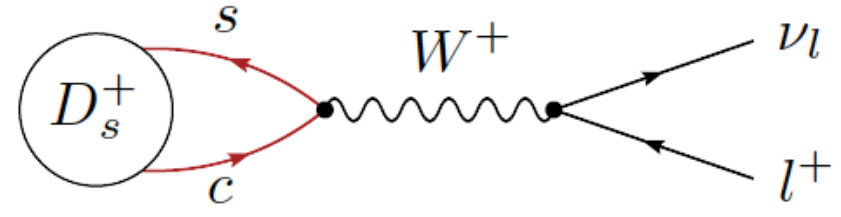
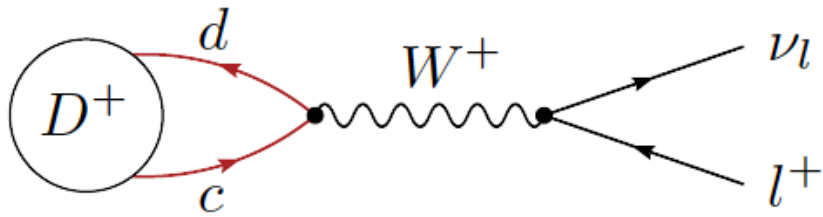
THE ANSWER IS: NO

IT IS NOT ONLY A QUESTION OF COMPUTER POWER
BECAUSE THERE ARE COMPLICATED
FIELD THEORETICAL PROBLEMS

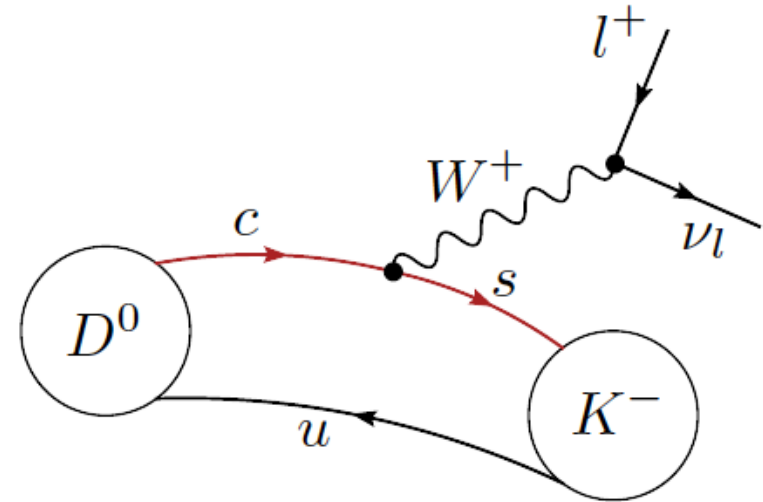
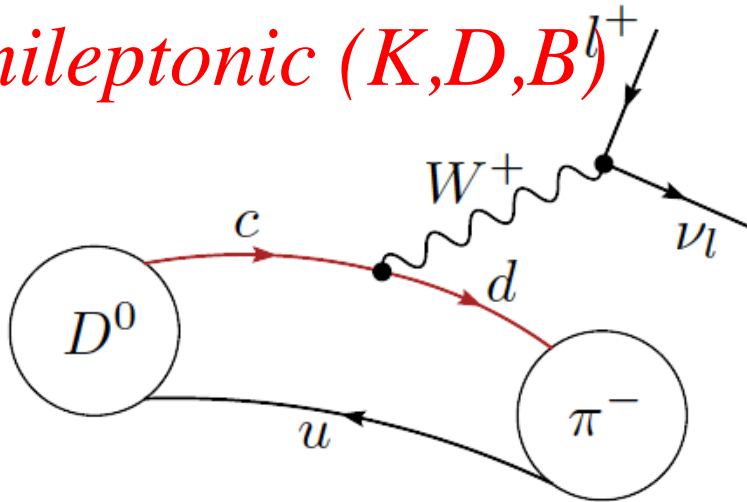
Euclidean vs Minkowski



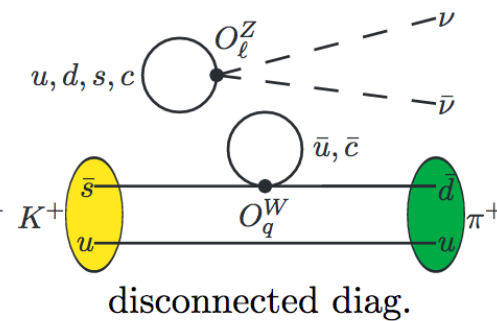
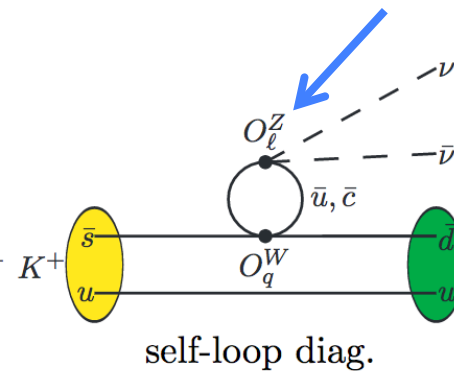
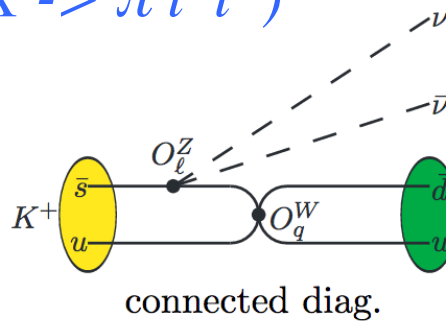
Leptonic (π, K, D, B)



Semileptonic (K, D, B)



(some) Radiative and Rare long distance effects
(also $K \rightarrow \pi l^+ l^-$)

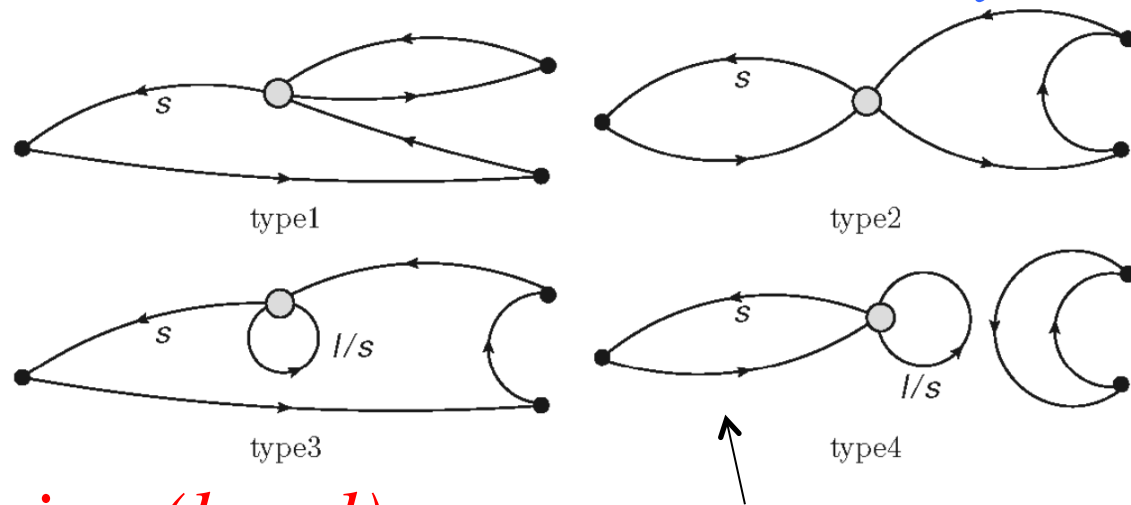


Non-leptonic

but only below the inelastic threshold

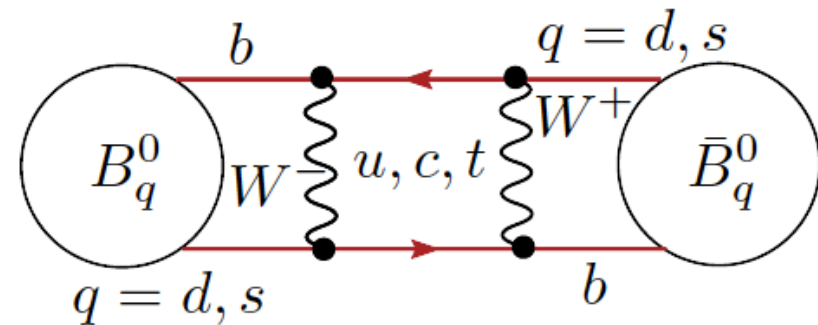
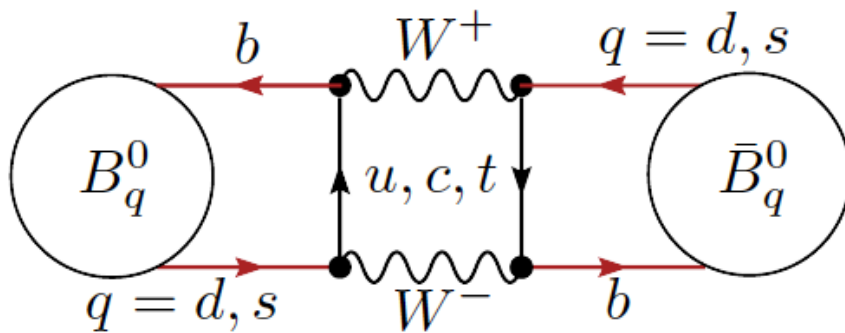
(may be also 3 body decays)

B -> ππ, Kπ, etc. No ! (Not yet)



Neutral meson mixing (local)

D -> Kπ probably yes



+ some long distance contributions to K and D neutral meson mixing + short distance contributions to B -> K* l+l-

Radiative corrections to weak amplitudes

important for hadron masses, leptonic and semileptonic decays, $|V_{us}|$, but also for D and B decays

13

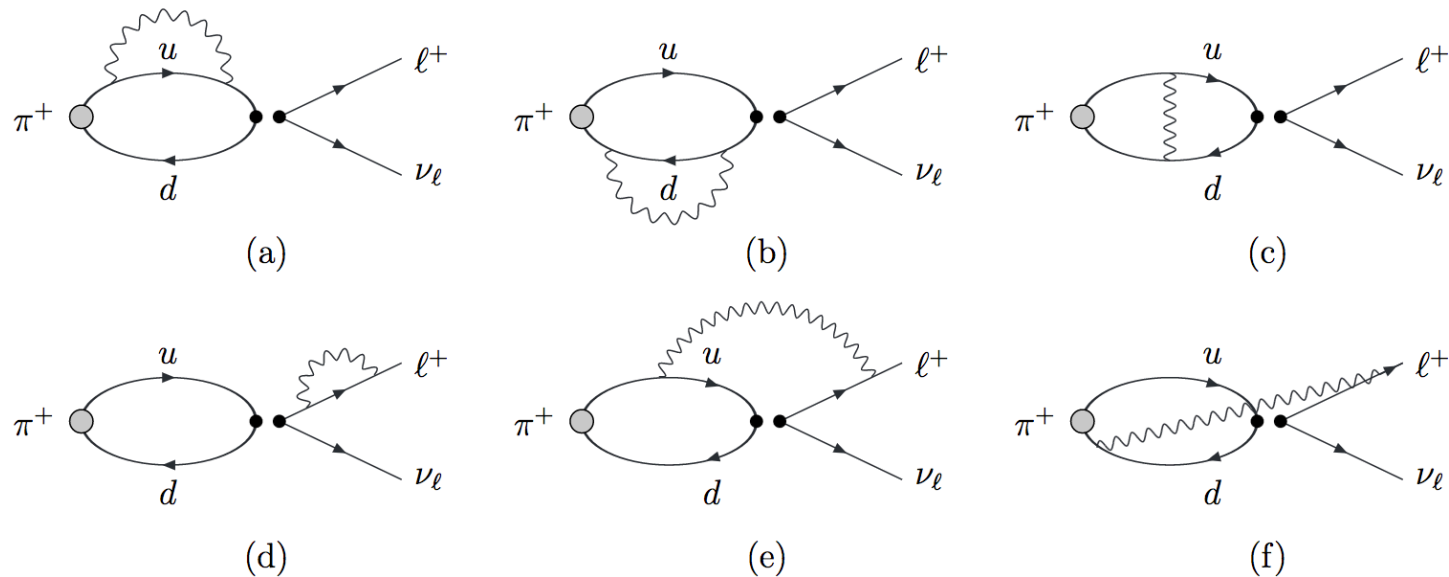
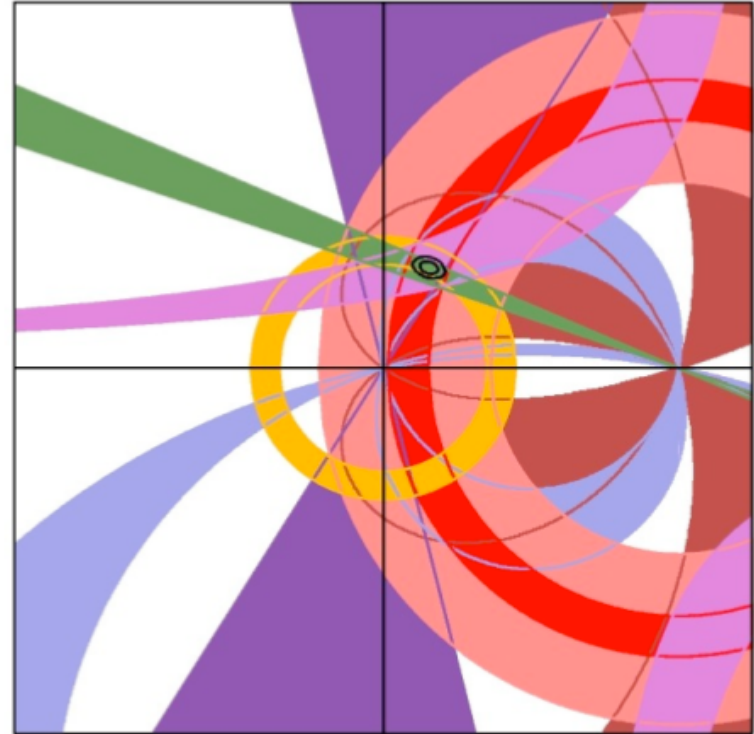


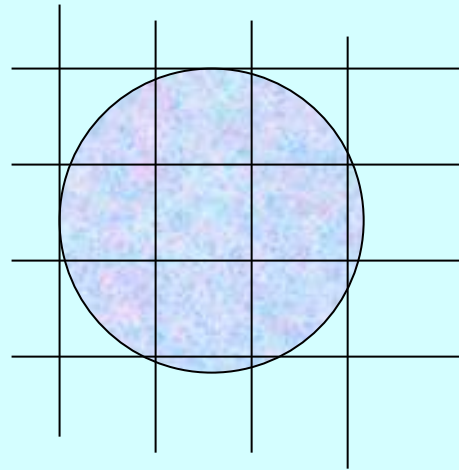
FIG. 5: Connected diagrams contributing at $O(\alpha)$ contribution to the amplitude for the decay $\pi^+ \rightarrow \ell^+ \nu_\ell$.

- *Uncertainties in lattice QCD calculations*

Not in this talk



Continuum limit, discretization and finite volume errors



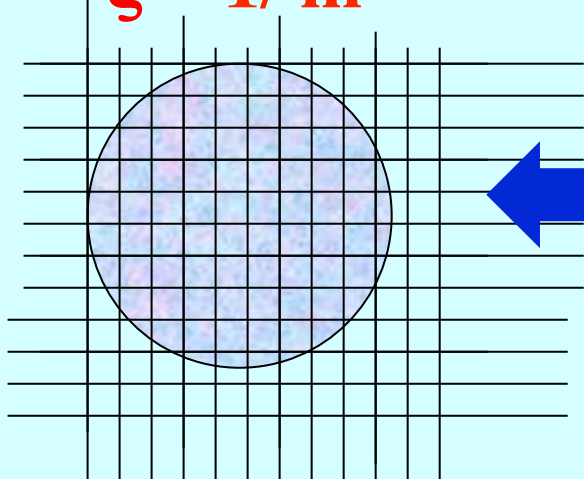
a

Formal $\lim_{a \rightarrow 0} S_{\text{Lattice}}(\phi) \rightarrow S_{\text{Continuum}}(\phi)$

$a/\xi = m a \sim 1$ The size of the object is comparable to the lattice spacing



$\xi = 1/m$



$a/\xi \ll 1$ i.e. $ma \rightarrow 0$ The size of the object is much larger than the lattice spacing

Similar to $a \sum_n \rightarrow \int dx$

Physics Reach (Mainly Heavy Flavor Physics)

many slides from lattice 2015 particularly from C. Pena

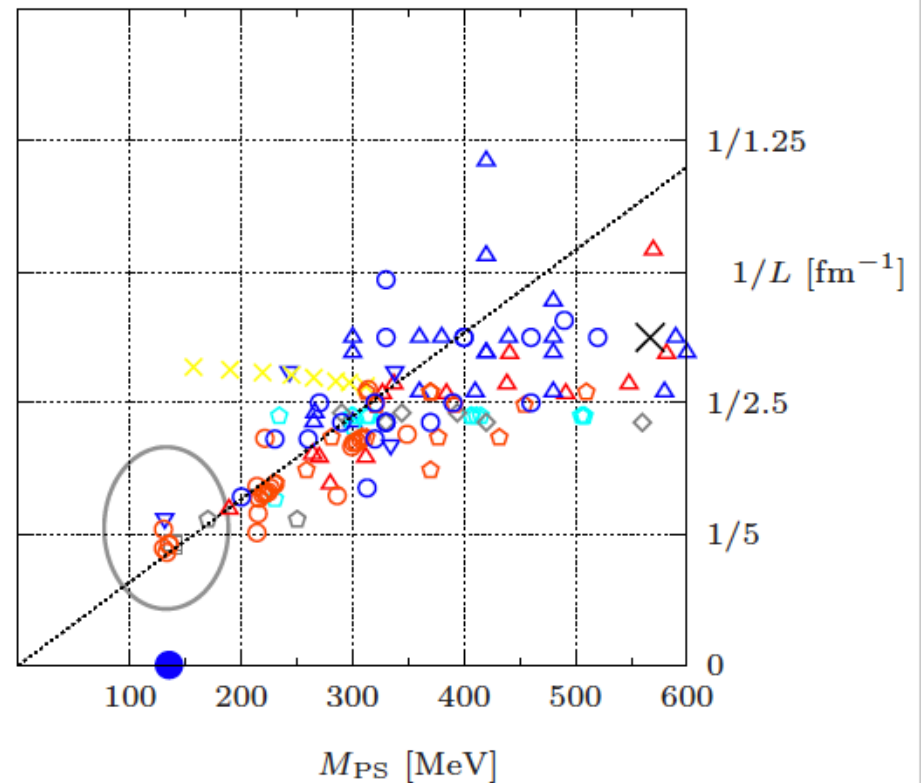
- charm physics directly accessible for some time now
- fraction of available ensembles used for HQ physics still limited

CLS	$N_f = 2$	\triangle
ETMC	$N_f = 2$	\triangle
(clover) ETMC	$N_f = 2$	∇
(Iwa) TWQCD	$N_f = 2$	\times
(Möbius) JLQCD	$N_f = 2 + 1$	\circ
RBC-UKQCD	$N_f = 2 + 1$	\diamond
(DSDR) RBC-UKQCD	$N_f = 2 + 1$	\circ
(Möbius) RBC-UKQCD	$N_f = 2 + 1$	\square
MILC	$N_f = 2 + 1$	\circ
MILC	$N_f = 2 + 1 + 1$	\circ
ETMC	$N_f = 2 + 1 + 1$	\circ
JLQCD/CP-PACS (2001)	$N_f = 2$	\times
	M_π (experiment)	\bullet

————— $\Lambda_{UV} \sim 1/a$

————— m_Q

————— Λ_{QCD}

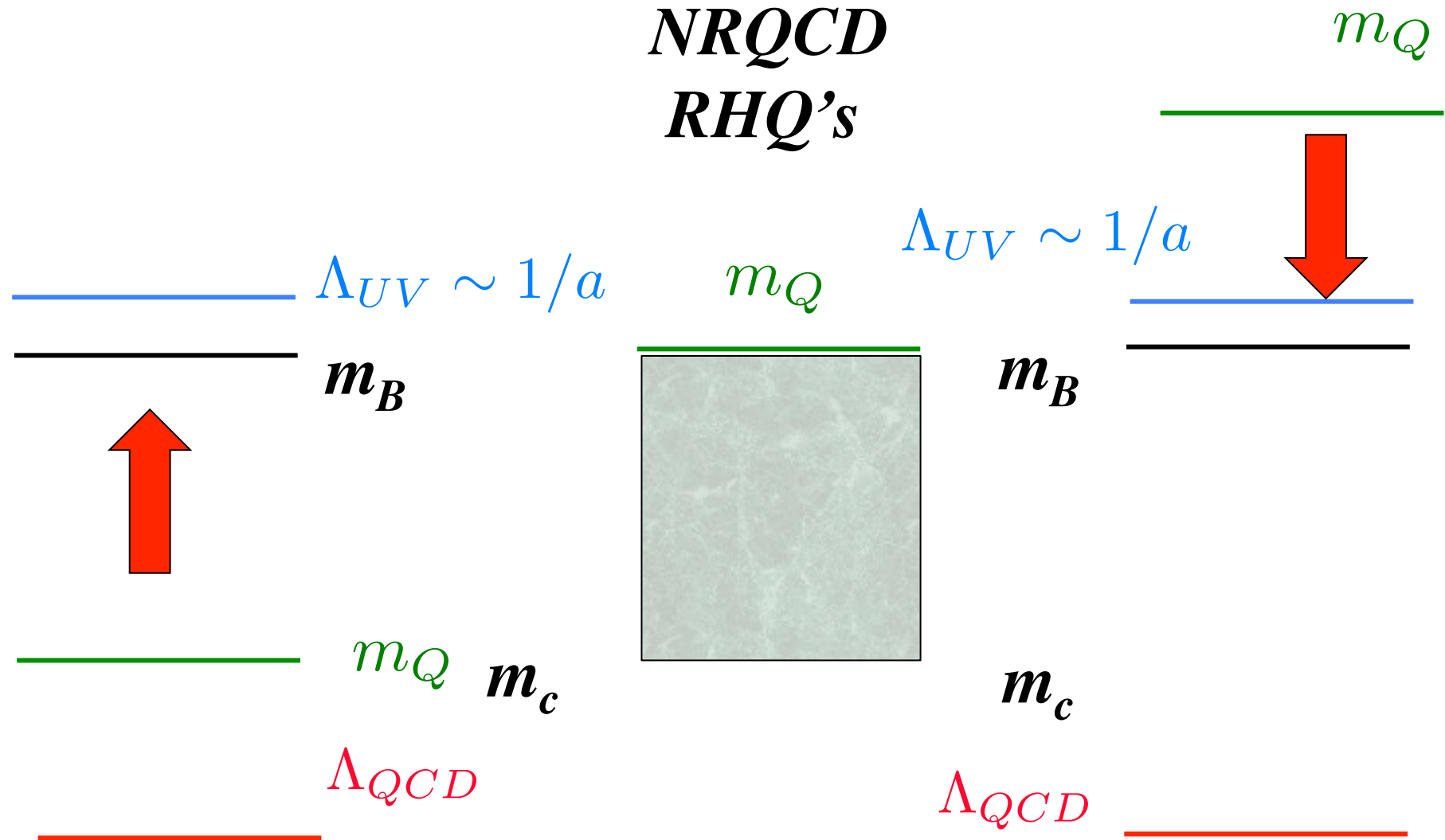


a crosscheck of different approaches is fundamental

*Extrapolation in $1/m_Q$
Ratio Method*

npHQET

*NRQCD
RHQ's*



ATTENTION TO THE QUOTED ERRORS

significant differences in estimates of fit and systematic uncertainties
in otherwise very similar computations

well-known example from light-quark physics (both computations use MILC ensembles, relatively minor differences)

MILC 13

$$f_{K^\pm} / f_{\pi^\pm} |_{N_f=2+1+1} = 1.1947 \overset{\text{stat}}{\downarrow} \overset{\text{CL}}{\downarrow} \overset{\text{FV}}{\downarrow} \overset{\text{e.m.}}{\downarrow} (26)(33)(17)(2)$$

HPQCD 13

$$f_{K^\pm} / f_{\pi^\pm} |_{N_f=2+1+1} = 1.1916 \overset{\text{stat}}{\uparrow} \overset{\text{CL}}{\uparrow} \overset{\text{FV}}{\uparrow} \overset{\text{(misc.)}}{\uparrow} (15)(12)(1)(10)$$

+ perturbative renormalization
courtesy of C. Pena

$\Lambda_b \rightarrow p \ell^- \bar{\nu}_\ell$ and $\Lambda_b \rightarrow \Lambda_c \ell^- \bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks

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¹Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

²Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

³Department of Physics, University of Arizona, Tucson, AZ 85721, USA

⁴RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

Very nice paper – interesting for LHCb

Parameter	coarse	fine
$am_Q^{(b)}$	8.45	3.99
$\xi^{(b)}$	3.1	1.93
$c_{E,B}^{(b)}$	5.8	3.57
$am_Q^{(c)}$	0.1214	-0.0045
$\xi^{(c)}$	1.2362	1.1281
$c_E^{(c)}$	1.6650	1.5311
$c_B^{(c)}$	1.8409	1.6232

TABLE II. Parameters of the bottom and charm quark actions [51, 52].

the parameters ν , c_E , c_B as functions of am_Q , heavy-quark discretization errors proportional to powers of am_Q can be removed to all orders. The remaining discretization errors are of order $a^2|\mathbf{p}|^2$, where $|\mathbf{p}|$ is the typical magnitude of the spatial momentum of the heavy quark inside the hadron. As the continuum limit $a \rightarrow 0$ is approached, the

FLAG-2 on B mixing

$BB_s = 1.32(5)$ $N_f=2$, ETMC

$BB_s = 1.33(6)$ $N_f=2+1$

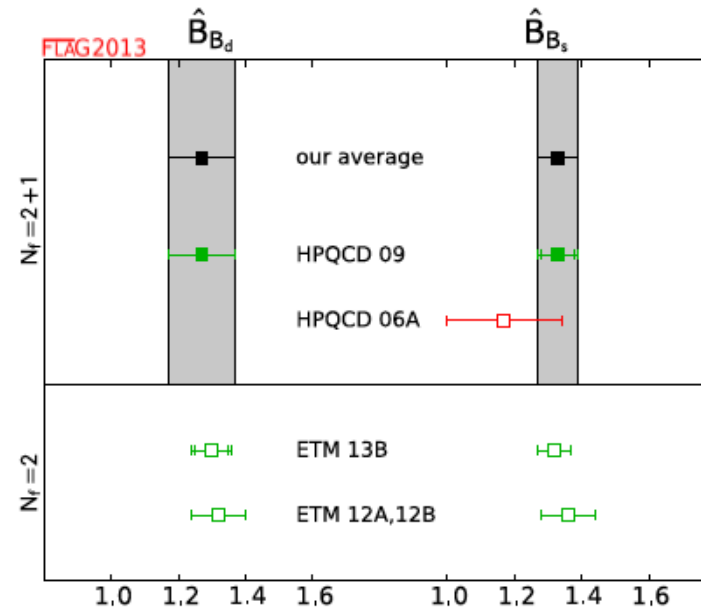
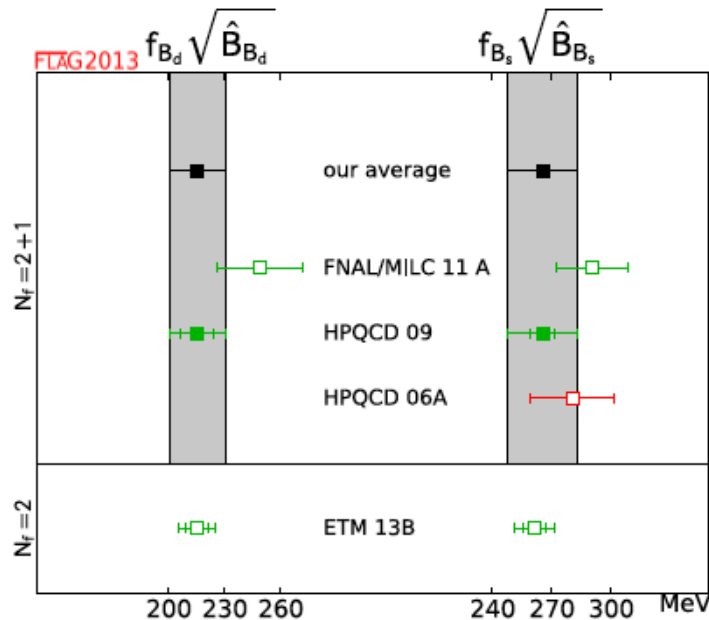
HPQCD

$BB_s = 1.492(92)$ $N_f=2+1$, **NEW**

FNAL/MILC

UTFIT AV. $BB_s = 1.38(11)$

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	$f_{B_d} \sqrt{\hat{B}_{B_d}}$	$f_{B_s} \sqrt{\hat{B}_{B_s}}$	\hat{B}_{B_d}	\hat{B}_{B_s}
FNAL/MILC 11A	[411]	2+1	C	★	○	★	○	✓	250(23) [†]	291(18) [†]	–	–
HPQCD 09	[402]	2+1	A	○	○ [▽]	★	○	✓	216(15)*	266(18)*	1.27(10)*	1.33(6)*
HPQCD 06A	[412]	2+1	A	■	■	★	○	✓	–	281(21)	–	1.17(17)
now published												
ETM 13B	[334]	2	P	★	○	★	★	✓	216(6)(8)	262(6)(8)	1.30(5)(3)	1.32(5)(2)
ETM 12A, 12B	[392, 413]	2	C	★	○	★	★	✓	–	–	1.32(8) [°]	1.36(8) [°]

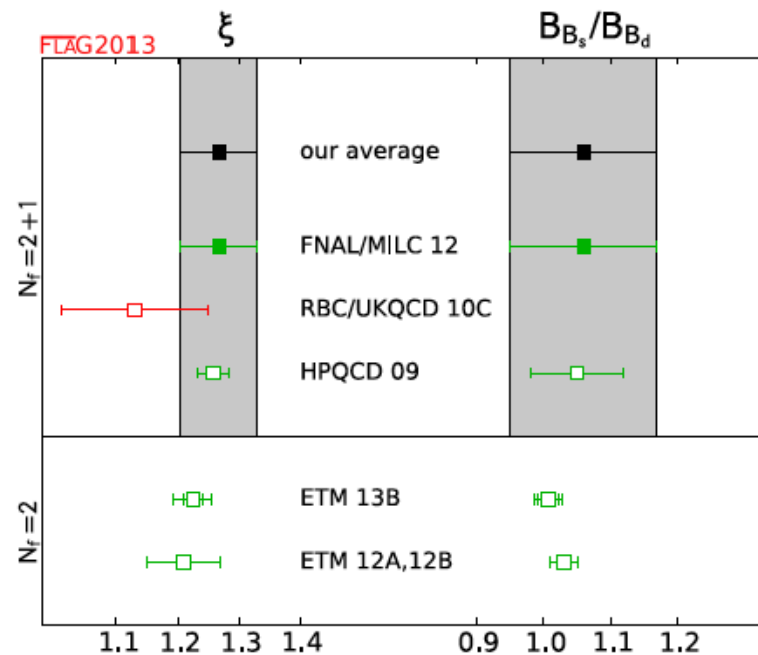


FLAG-2 on B mixing

$FLAG2\ BB_s/BB_d = 1.06(11)$
 $UTFIT\ BB_s/BB_d = 1.012(27)$

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	ξ	B_{B_s}/B_{B_d}
FNAL/MILC 12	[414]	2+1	A	○	○	★	○	✓	1.268(63)	1.06(11)
RBC/UKQCD 10C	[405]	2+1	A	■	■	★	○	✓	1.13(12)	–
HPQCD 09	[402]	2+1	A	○	○ [▽]	★	○	✓	1.258(33)	1.05(7)
ETM 13B	[334]	2	P	★	○	★	★	✓	1.225(16)(14)(22)	1.007(15)(14)
ETM 12A, 12B	[392, 413]	2	C	★	○	★	★	✓	1.21(6)	1.03(2)

now published



Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing
- 5) $R(D)$ and $R(D^*)$
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

CKM-TRIANGLE ANALYSIS

State of The Art 2015

	Measurement	Fit	Prediction	Pull
α	$(92.7 \pm 6.2)^{\circ}$ 6.7 %	$(90.1 \pm 2.7)^{\circ}$ 2.9 %	$(88.3 \pm 3.4)^{\circ}$ 3.8 %	0.6
$\sin 2\beta$	0.680 ± 0.024 3.5 %	0.696 ± 0.022 2.6 %	0.747 ± 0.039 5.2 %	1.8
γ	$(71.4 \pm 6.5)^{\circ}$ 9.1 %	$(67.4 \pm 2.8)^{\circ}$ 4.2 %	$(66.7 \pm 3.0)^{\circ}$ 4.5 %	0.7
$ V_{ub} \times 10^3$	3.81 ± 0.40 10 %	3.66 ± 0.12 3.3 %	3.64 ± 0.12 3.3 %	0.5
$ V_{cb} \times 10^2$	4.09 ± 0.11 2.6 %	4.206 ± 0.053 1.2 %	4.240 ± 0.062 1.4 %	0.9
$\varepsilon_K \times 10^3$	2.228 ± 0.011 0.5 %	2.227 ± 0.011 0.5 %	2.08 ± 0.18 8.7 %	0.8
Δm_s (ps ⁻¹)	17.761 ± 0.022 0.1 %	17.755 ± 0.022 0.1 %	17.3 ± 1.0 5.7 %	0.2
$BR(B \rightarrow \tau \nu) \times 10^4$	1.06 ± 0.20 18.9 %	0.83 ± 0.07 7.9 %	0.81 ± 0.7 8.2 %	1.3
$BR(B_s \rightarrow \mu\mu) \times 10^9$	2.9 ± 0.7 24.1 %	3.99 ± 0.15 3.8 %	3.94 ± 0.16 4.0 %	1.5 ew corrections not included
$BR(B_d \rightarrow \mu\mu) \times 10^9$	0.39 ± 0.15 38.5 %	0.1098 ± 0.0057 5.2 %	0.1103 ± 0.0058 5.2 %	1.9 ew corrections not included
β_s	$(0.97 \pm 0.95)^{\circ}$ 98 %	$(1.056 \pm 0.039)^{\circ}$ 4.4 %	$(1.056 \pm 0.039)^{\circ}$ 4.1 %	0.1 not included in the fit

$$B(B \rightarrow \tau \nu)_{\text{Old}} = (1.67 \pm 0.30) 10^{-4}$$

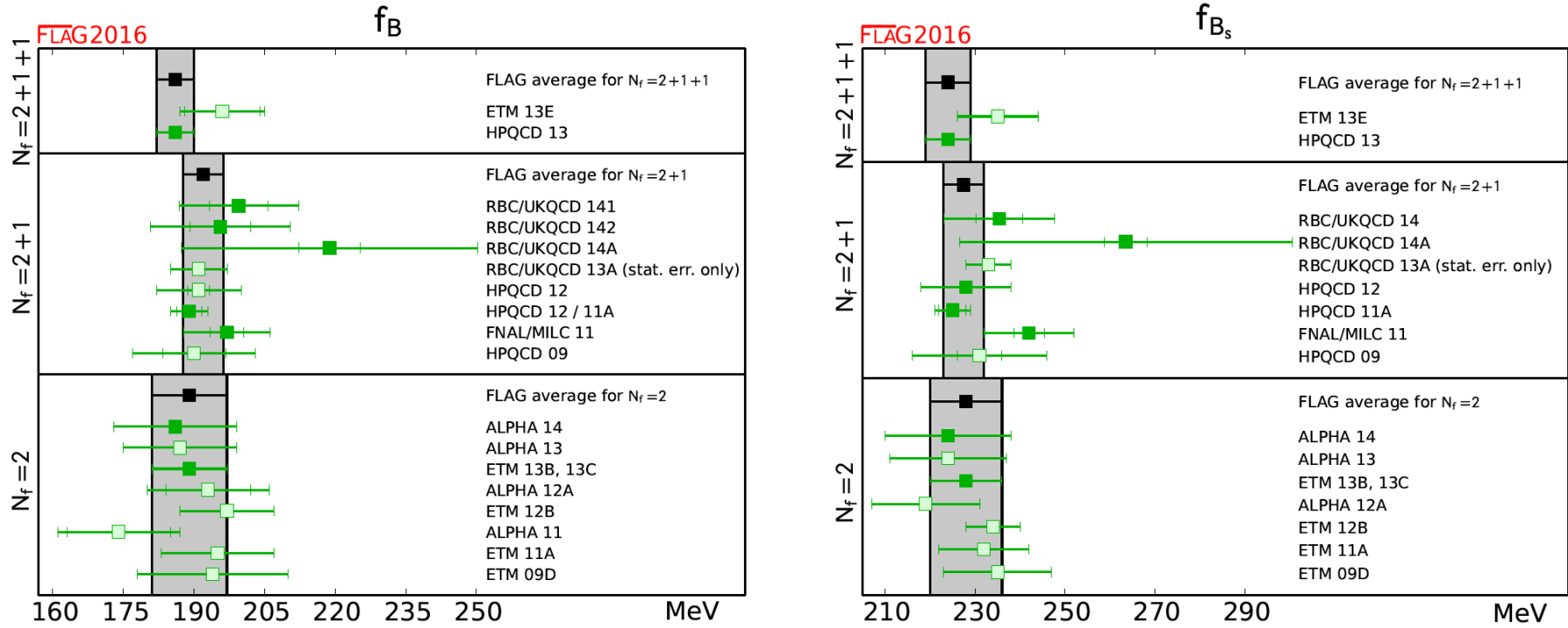


Figure 20: Decay constants of the B and B_s mesons. The values are taken from Tab. 32 (the

$$f_B = 192.0(4.3) \text{ MeV } \mathbf{(186)} \text{ Refs. [48, 53–56],}$$

$$N_f = 2 + 1 : \quad f_{B_s} = 228.4(3.7) \text{ MeV } \mathbf{(224)} \text{ Refs. [48, 53–56],}$$

$$\mathbf{N_f=2+1+1} \quad f_{B_s}/f_B = 1.201(16) \quad \mathbf{(1.205)} \quad \text{Refs. [48, 53–56].}$$

LATTICE PARAMETERS (2016)

It does not make sense to improve the precision on B_K if we do not control long distance effects; Similarly for f_π or f_K without radiative corrections

obtained excluding the given constraint from the fit

			Pull ($\# \sigma$)
B_K	0.740 ± 0.029	0.81 ± 0.07	< 1
f_{B_s}	0.226 ± 0.005	0.220 ± 0.007	< 1
f_{B_s}/f_{B_d}	1.203 ± 0.013	1.210 ± 0.030	< 1
B_{B_s}/B_{B_d}	1.032 ± 0.036	1.07 ± 0.05	< 1
B_{B_s}	1.35 ± 0.08	1.30 ± 0.07	< 1

in general: average the Nf=2+1+1 and Nf=2+1 FLAG averages, through eq.(28) in arXiv:1403.4504

for B_K , f_{B_s} , f_{B_s}/f_{B_d} :

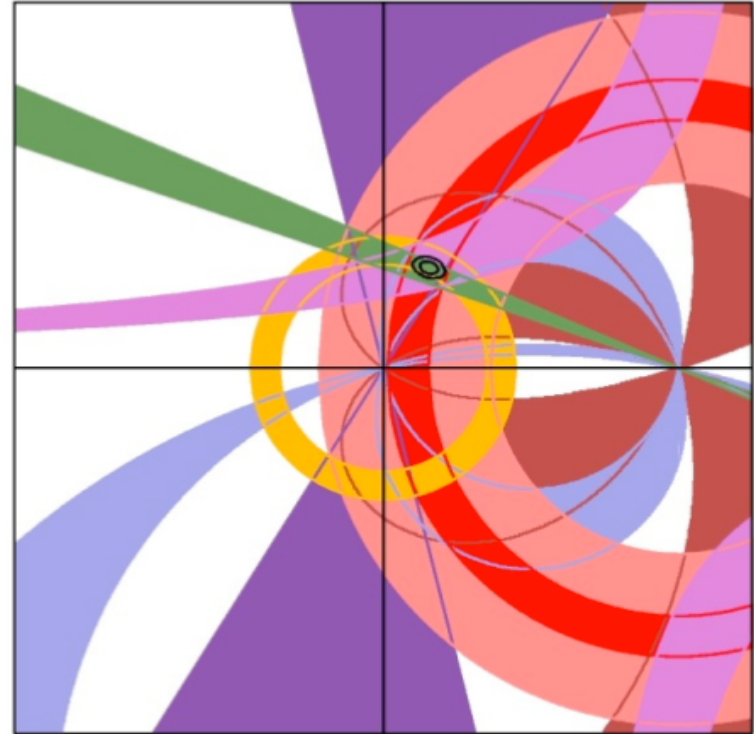
FLAG Nf=2+1+1 (single result) and Nf=2+1 average

for B_{B_s} , B_{B_s}/B_{B_d} :

update w.r.t. the Nf=2+1 FLAG average (no Nf=2+1+1 results yet)

updating the FNAL/MILC result to FNAL/MILC 2016 (1602.13560)

- *Future directions*



Long Distance Effects in Neutral Meson Mixing

N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1212.5931

Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1406.0916

Z.Bai (RBC-UKQCD), arXiv:1411.3210

exp

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12} \text{ MeV.}$$

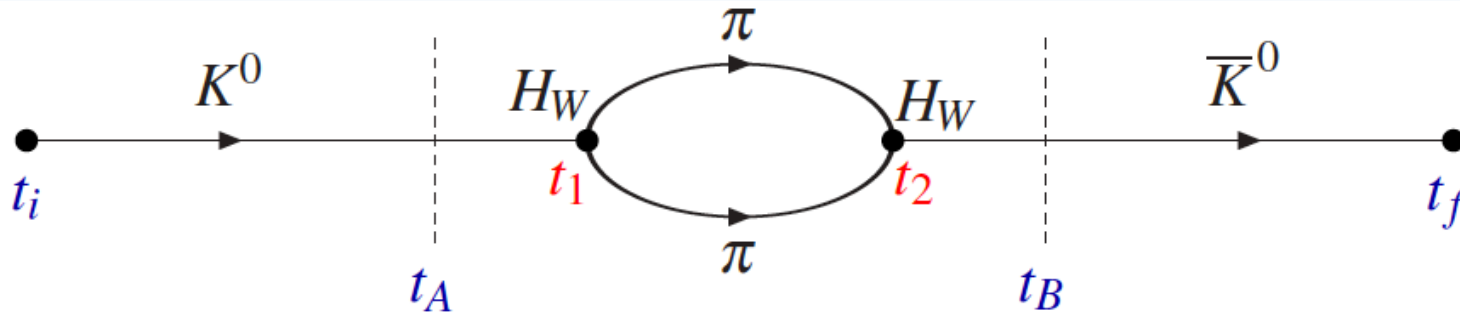
3.19(41)(96)
lattice unphysical
masses

- Historically led to the prediction of the energy scale of the charm quark.
Mohapatra, Rao & Marshak (1968); GIM (1970); Gaillard & Lee (1974)
- Tiny quantity \Rightarrow places strong constraints on BSM Physics.
- Within the standard model, Δm_K arises from $K^0 - \bar{K}^0$ mixing at second order in the weak interactions:

$$\Delta M_K = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | H_W | \alpha \rangle \langle \alpha | H_W | K^0 \rangle}{m_K - E_{\alpha}},$$

where the sum over $|\alpha\rangle$ includes an energy-momentum integral.

Long Distance Effects in Neutral Meson Mixing



- Δm_K is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | \mathcal{H}_W | \alpha \rangle \langle \alpha | \mathcal{H}_W | K^0 \rangle}{m_K - E_{\alpha}} = 3.483(6) \times 10^{-12} \text{ MeV}.$$

- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A, t_B; t_i, t_f) = |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)^2} \times \left\{ e^{(M_K - E_n)T} - (m_K - E_n)T - 1 \right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\text{FV}} \equiv 2 \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)}.$$

Long Distance Effects in Neutral Meson Mixing

- The general formula can be written:

N.H.Christ, G.Martinelli & CTS, arXiv:1401.1362

N.H.Christ, X.Feng, G.Martinelli & CTS, arXiv:1504.01170

$$\Delta m_K = \Delta m_K^{\text{FV}} - 2\pi v \langle \bar{K}^0 | H | n_0 \rangle_V v \langle n_0 | H | K^0 \rangle_V \left[\cot \pi h \frac{dh}{dE} \right]_{m_K},$$

where $h(E, L)\pi \equiv \phi(q) + \delta(k)$.

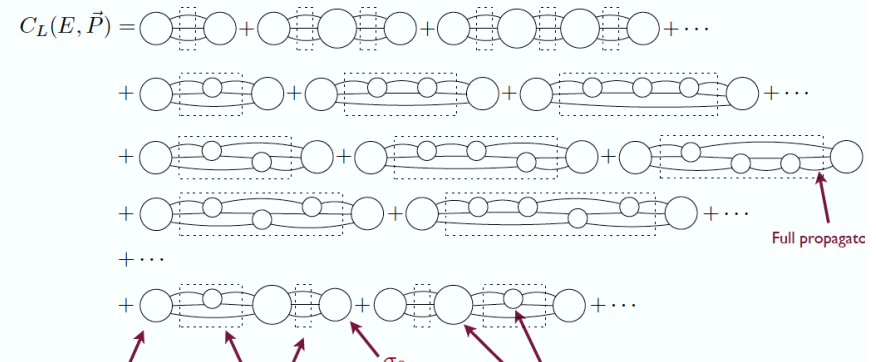
- This formula reproduces the result for the special case when the volume is such that there is a two-pion state with energy $= m_K$. N.H.Christ, arXiv:1012.6034
- Increasing the volumes keeping $h = n/2$ and thus avoiding the power corrections is an intriguing possibility.

**Within reasonable approximations
can be extended to D meson mixing**

**M. Ciuchini, V. Lubicz, L. Silvestrini, S. Simula
(progresses made by M. T. Hansen & S.
Sharpe, 1204.0826v4, 1409.7012v, 1504.04248v1)**

Also CPV in D $\rightarrow \pi\pi$ or KK

3-particle correlator



D MIXING

- D mixing is described by:
 - Dispersive $D \rightarrow \bar{D}$ amplitude M_{12}
 - SM: long-distance dominated, not calculable
 - NP: short distance, calculable w. lattice
 - Absorptive $D \rightarrow \bar{D}$ amplitude Γ_{12}
 - SM: long-distance, not calculable
 - NP: negligible
 - Observables: $|M_{12}|, |\Gamma_{12}|, \Phi_{12} = \arg(\Gamma_{12}/M_{12})$

Let us assume that the Standard Model contributions to M_{12} and Γ_{12} are real

"REAL SM" APPROXIMATION II

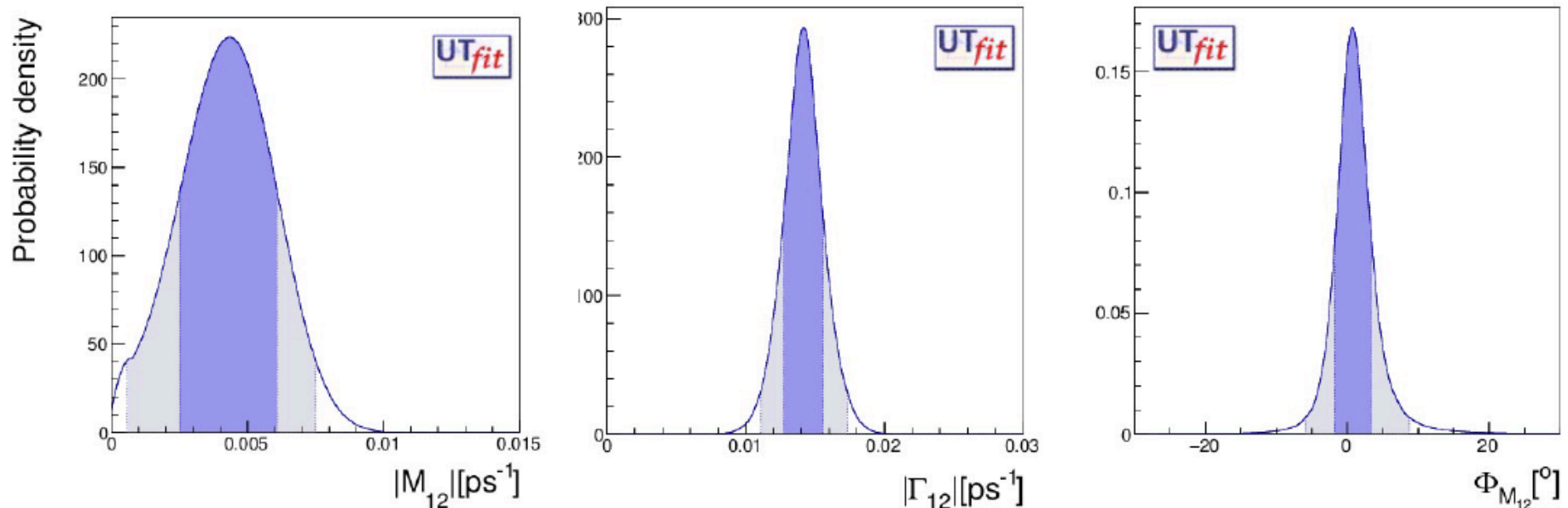
- Define $|D_{S,L}| = p|D^0| \pm q|D^0|$ and $\delta = (1 - |q/p|^2) / (1 + |q/p|^2)$. All observables can be written in terms of $x = \Delta m / \Gamma$, $y = \Delta \Gamma / 2\Gamma$ and δ , with

$$\begin{aligned} \sqrt{2} \Delta m &= \text{sign}(\cos \Phi_{12}) \sqrt{4|M_{12}|^2 - |\Gamma_{12}|^2 + \sqrt{(4|M_{12}|^2 + |\Gamma_{12}|^2)^2 - 16|M_{12}|^2|\Gamma_{12}|^2 \sin^2 \Phi_{12}}}, \\ \sqrt{2} \Delta \Gamma &= 2\sqrt{|\Gamma_{12}|^2 - 4|M_{12}|^2 + \sqrt{(4|M_{12}|^2 + |\Gamma_{12}|^2)^2 - 16|M_{12}|^2|\Gamma_{12}|^2 \sin^2 \Phi_{12}}}, \\ \delta &= \frac{2|M_{12}||\Gamma_{12}| \sin \Phi_{12}}{(\Delta m)^2 + |\Gamma_{12}|^2}, \end{aligned} \quad (7)$$

- Notice that $\phi = \arg(q/p) = \arg(y + i\delta x) - \arg \Gamma_{12}$
- $|q/p| \neq 1 \Leftrightarrow \phi \neq 0$ clear signals of NP

CPV IN CHARM MIXING

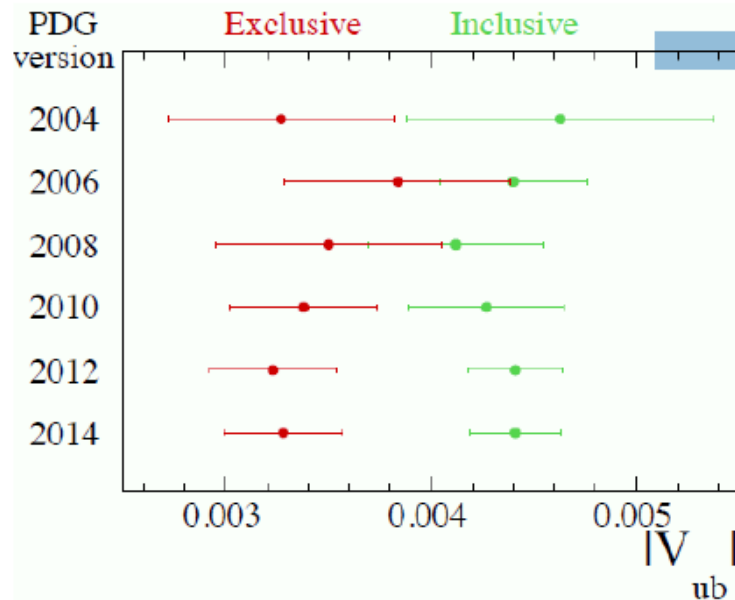
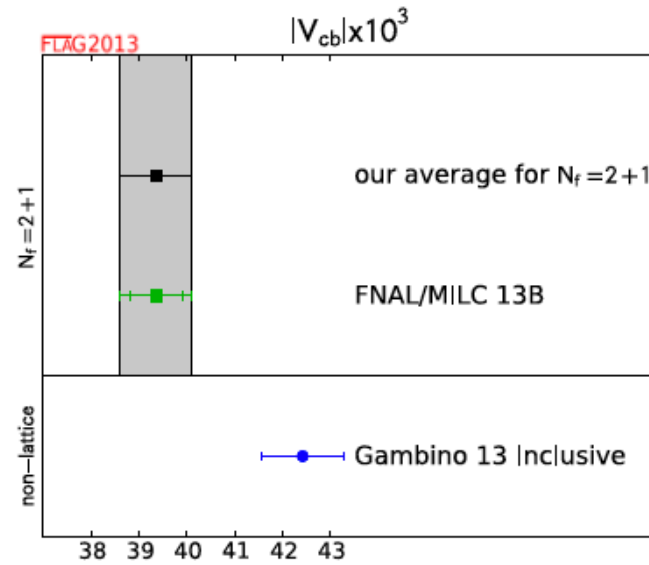
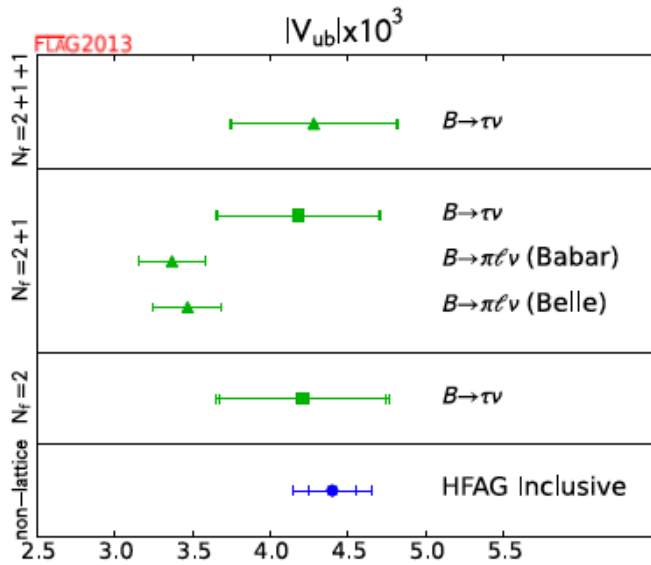
- Latest UTfit average (HFAG very similar):
 $x = (3.5 \pm 1.5) 10^{-3}$, $y = (5.8 \pm 0.6) 10^{-3}$,
 $|q/p|-1 = (0.7 \pm 1.8) 10^{-2}$, $\phi = \arg(q/p) = (0.20 \pm 0.56)^\circ$
 $|M_{12}| = (4 \pm 2)/fs$, $|\Gamma_{12}| = (14 \pm 1)/fs$, $\Phi_{12} = (0 \pm 3)^\circ$



Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing
- 5) $R(D)$ and $R(D^*)$
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

$|V_{ub}|$, $|V_{cb}|$



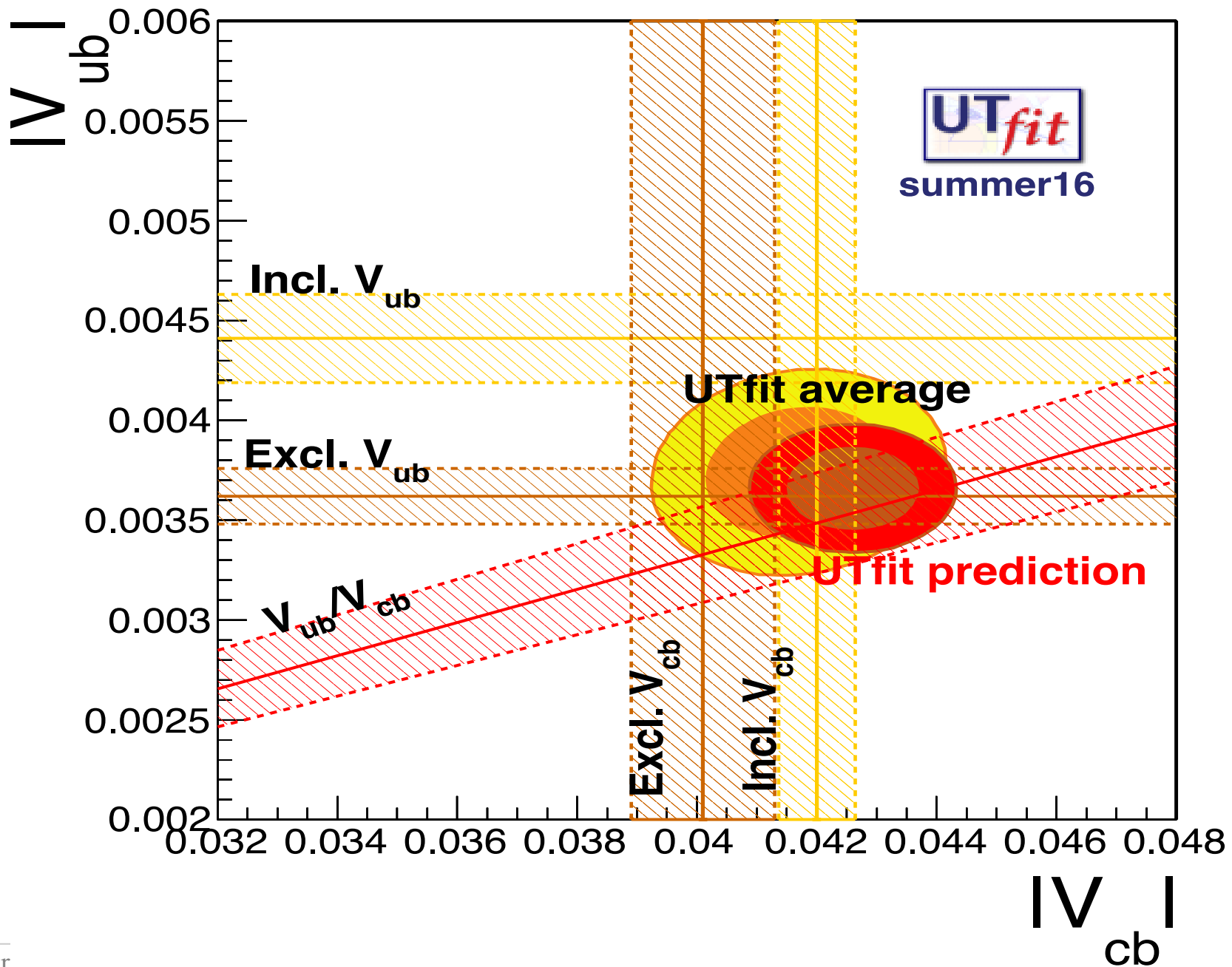
$V_{ub} \text{ Exclusive} = 0.00361 \pm 0.00013$

$V_{cb} \text{ Exclusive} = 0.0400 \pm 0.0011$

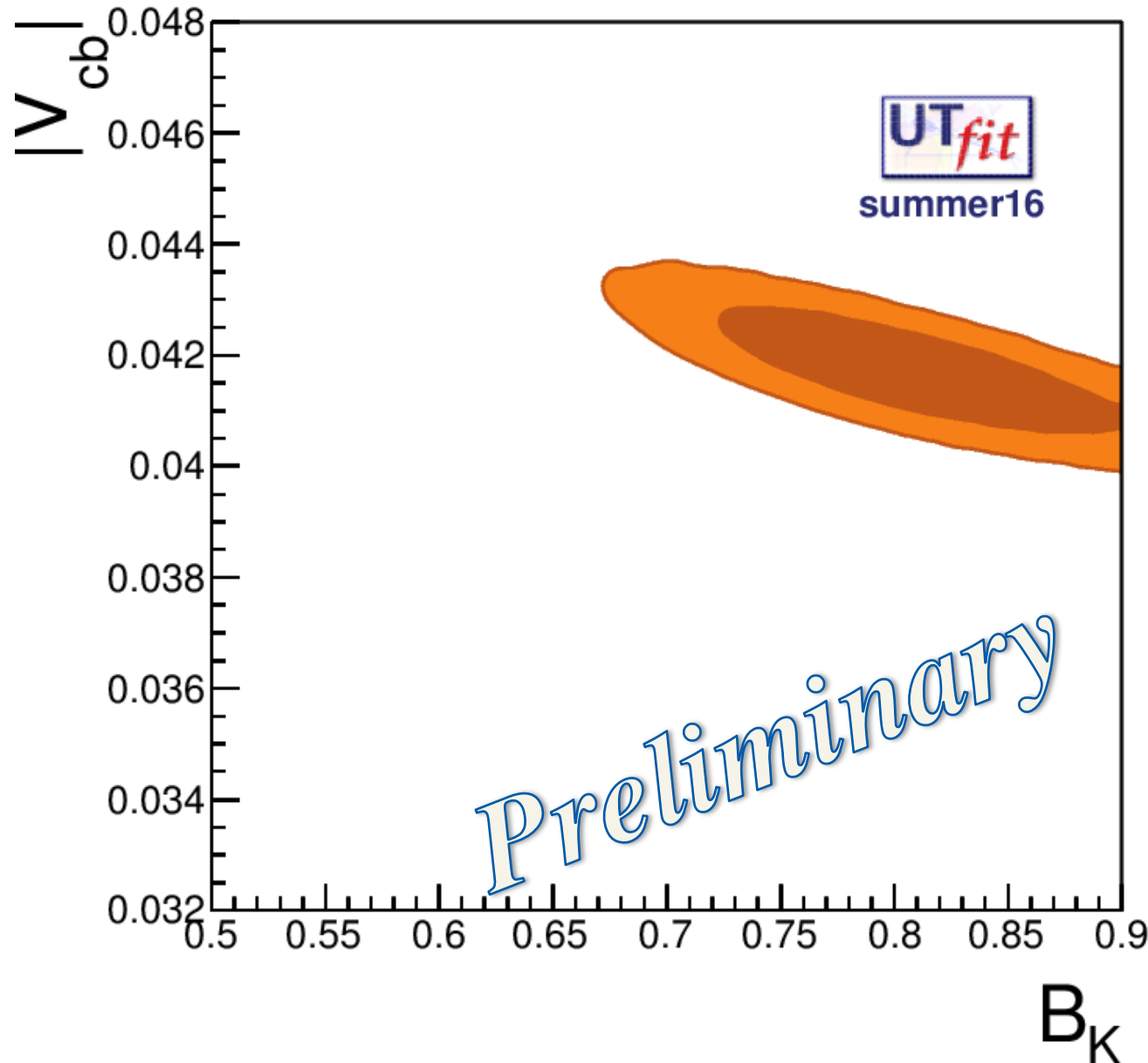
$V_{ub}/V_{cb} \text{ Exclusive} = 0.083 \pm 0.006$

$V_{ub} \text{ Inclusive} = 0.00440 \pm 0.00022$

$V_{cb} \text{ Inclusive} = 0.0420 \pm 0.0006$



UT-fit 2016 Correlation B_k vs V_{cb} in quest for theoretical improvement



- ϵ_K large V_{cb}
- B mixing with large lattice matrix elements small V_{cb}

2015

inclusives

vs

exclusives

$$V_{ub} \quad (4.40 \pm 0.22) \times 10^{-3}$$

$$(3.61 \pm 0.13) \times 10^{-3}$$

$$V_{cb} \quad (4.20 \pm 0.06) \times 10^{-2}$$

$$(4.00 \pm 0.11) \times 10^{-2}$$

$$V_{ub} \quad (3.73 \pm 0.21) \times 10^{-3}$$

$$V_{cb} \quad (4.17 \pm 0.10) \times 10^{-2}$$

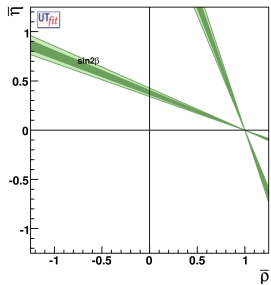
$$\sin 2\beta_{\text{exp}} =$$
$$0.680 \pm 0.023$$

$$\sin 2\beta_{\text{UTfit}} =$$
$$0.740 \pm 0.037$$
$$B_K = 0.81 \pm 0.07$$

$$\sin 2\beta_{\text{incl}} =$$
$$0.784 \pm 0.027$$
$$B_K = 0.74 \pm 0.05$$
$$(2015)$$

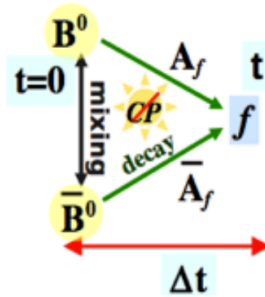
$$\sin 2\beta_{\text{excl}} =$$
$$0.703 \pm 0.021$$
$$B_K = 0.93 \pm 0.07$$
$$(2015)$$

Beta results



$B^0 \rightarrow J/\psi K^0$

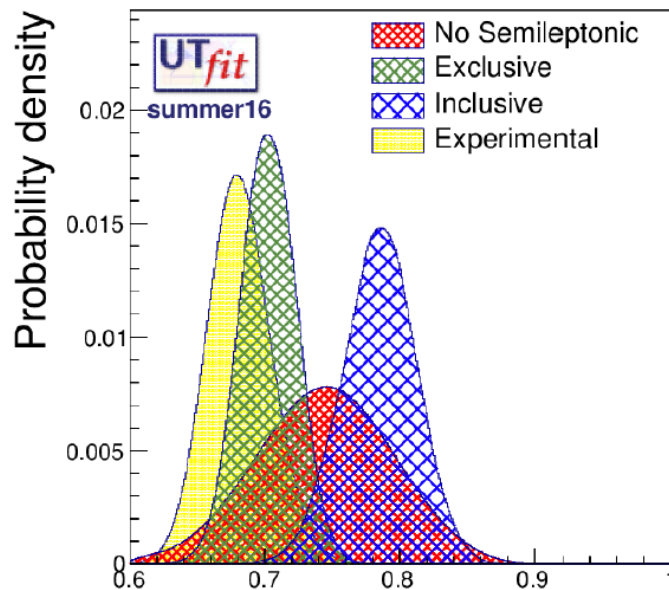
sin2β from time-dependent A_{CP} in $B \rightarrow J/\psi K$



$$a_{f_{CP}}(t) = \frac{\text{Prob}(B^0(t) \rightarrow f_{CP}) - \text{Prob}(\bar{B}^0(t) \rightarrow f_{CP})}{\text{Prob}(B^0(t) \rightarrow f_{CP}) + \text{Prob}(\bar{B}^0(t) \rightarrow f_{CP})} = C_f \cos \Delta m_d t + S_f \sin \Delta m_d t$$

The decay is dominated by a single (tree level) amplitude, thus a can be simplified:

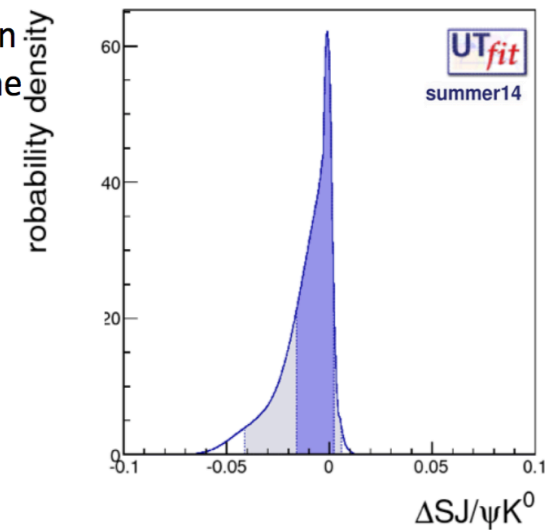
$$a_{f_{CP}}(t) = -\eta_{CP} \sin(\Delta m_d t) \sin 2\beta$$



We also analyse $\bar{B}^0 \rightarrow J/\psi \pi^0$ to obtain the theoretical uncertainty related to the penguin pollution in data-driven way. This gives us an additional correction:

data-driven theoretical uncertainty

$\Delta S \in [-0.02, 0.00]$ at 68% prob.



$$\sin(2\beta) = (0.680 \pm 0.023)$$

CKM Uncertainties

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.39 \pm 0.30) \cdot 10^{-11} \left[\frac{|V_{cb}|}{0.0407} \right]^{2.8} \left[\frac{\gamma}{73.2^\circ} \right]^{0.71}$$
$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.36 \pm 0.09) \cdot 10^{-11} \left[\frac{|V_{ub}|}{3.88 \cdot 10^{-3}} \right]^2 \left[\frac{|V_{cb}|}{0.0407} \right]^2 \left[\frac{\sin \gamma}{\sin(73.2)} \right]^2$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (65.3 \pm 3.1) \left[\bar{\text{Br}}(B_s \rightarrow \mu^+ \mu^-) \right]^{1.4} \left[\frac{\gamma}{70^\circ} \right]^{0.71} \left[\frac{227 \text{ MeV}}{F_{B_s}} \right]^{2.8}$$

A. Buras

AJB, Buttazzo,
Girrbach-Noe,
Knegjens
1503.02693

For $B_s \rightarrow \mu^+ \mu^-$ we use the formula from [56], slightly modified in [2]

$$\overline{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.65 \pm 0.06) \cdot 10^{-9} \left[\frac{m_t(m_t)}{163.5 \text{ GeV}} \right]^{3.02} \left[\frac{\alpha_s(M_Z)}{0.1184} \right]^{0.032} R_s$$

where

$$R_s = \left[\frac{F_{B_s}}{227.7 \text{ MeV}} \right]^2 \left[\frac{\tau_{B_s}}{1.516 \text{ ps}} \right] \left[\frac{0.938}{r(y_s)} \right] \left[\frac{|V_{ts}|}{41.5 \cdot 10^{-3}} \right]^2.$$

Now,

$$|V_{td}| = |V_{us}| |V_{cb}| R_t, \quad |V_{ts}| = \eta_R |V_{cb}|$$

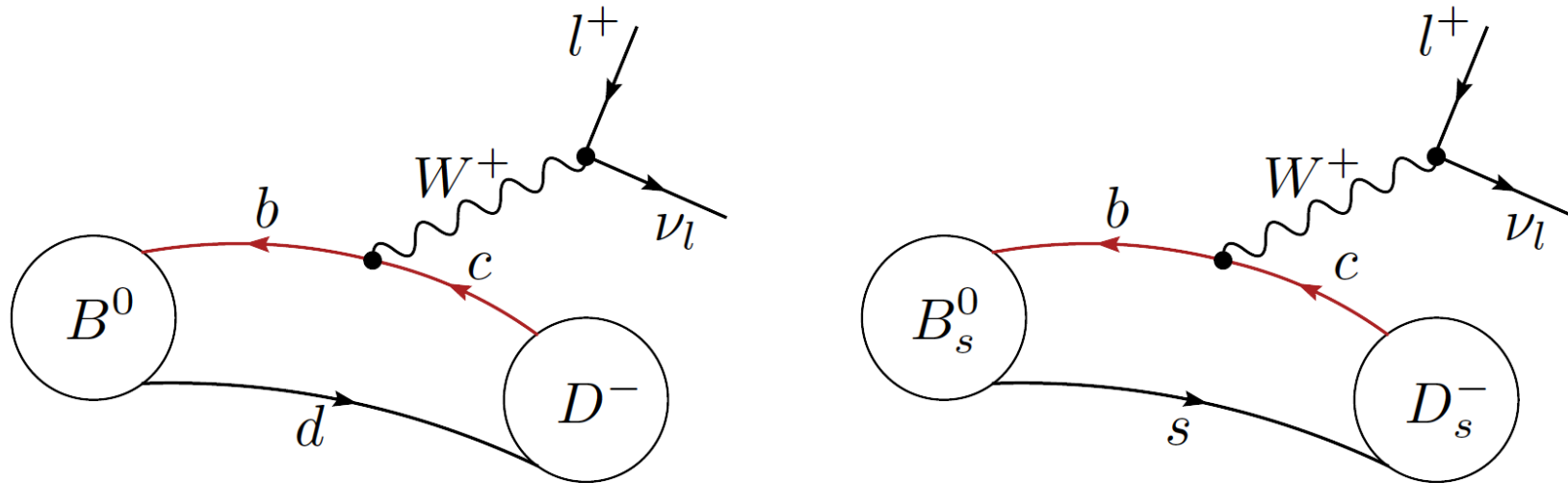
with R_t being one of the sides of the unitarity triangle (see Fig. 1) and

$$\eta_R = 1 - |V_{us}| \xi \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \cos \beta + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4) = 0.9825,$$

Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing (already discussed)
- 5) $R(D)$ and $R(D^*)$ (and V_{cb} of course)
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

B semileptonic decay: $|V_{cb}|$

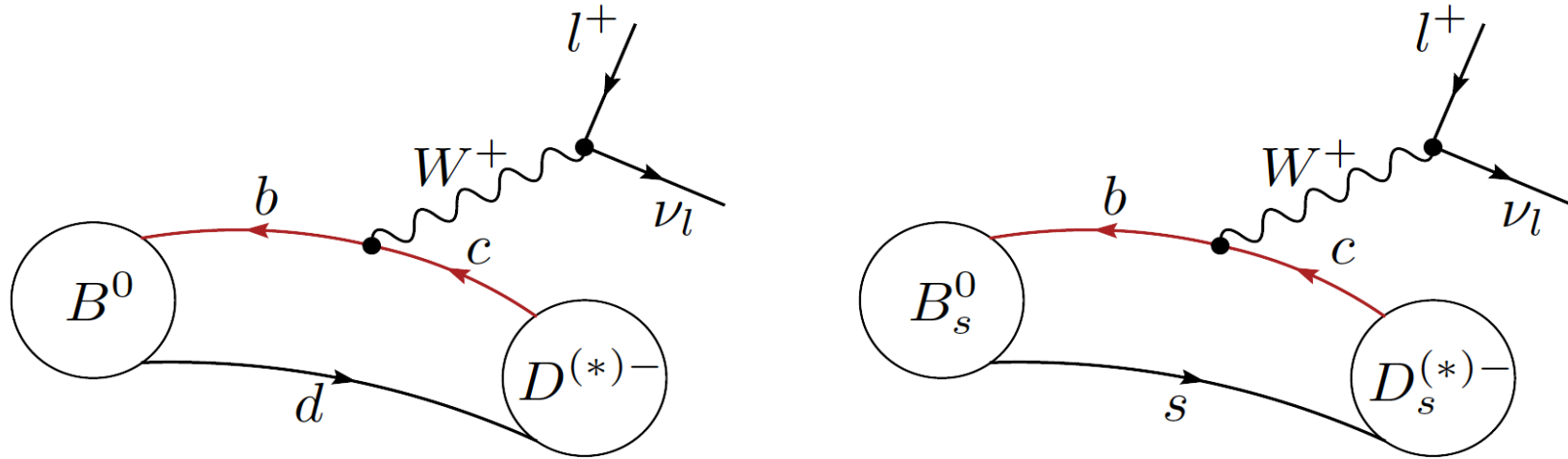


$$\frac{d\Gamma(B_{(s)} \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 \right. \\ \left. + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

e, μ suppressed ←

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

B semileptonic decay: $|V_{cb}|$



$$\frac{d\Gamma(B \rightarrow D l \nu_l)}{dw} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{d\Gamma(B \rightarrow D^* l \nu_l)}{dw} = \frac{G_F^2}{4\pi^3} (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{EW}|^2 \chi(w) |V_{cb}|^2 |\mathcal{F}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$w = \frac{p_B \cdot p_{D^*}}{m_B m_{D^*}} \qquad \mathcal{G}(w) = \frac{4 \frac{m_D}{m_B}}{1 + \frac{m_D}{m_B}} f_+(q^2) \quad \text{etc}$$

Low recoil region ($w=1$) accessible to lattice calculations

$B \rightarrow D-D^*$

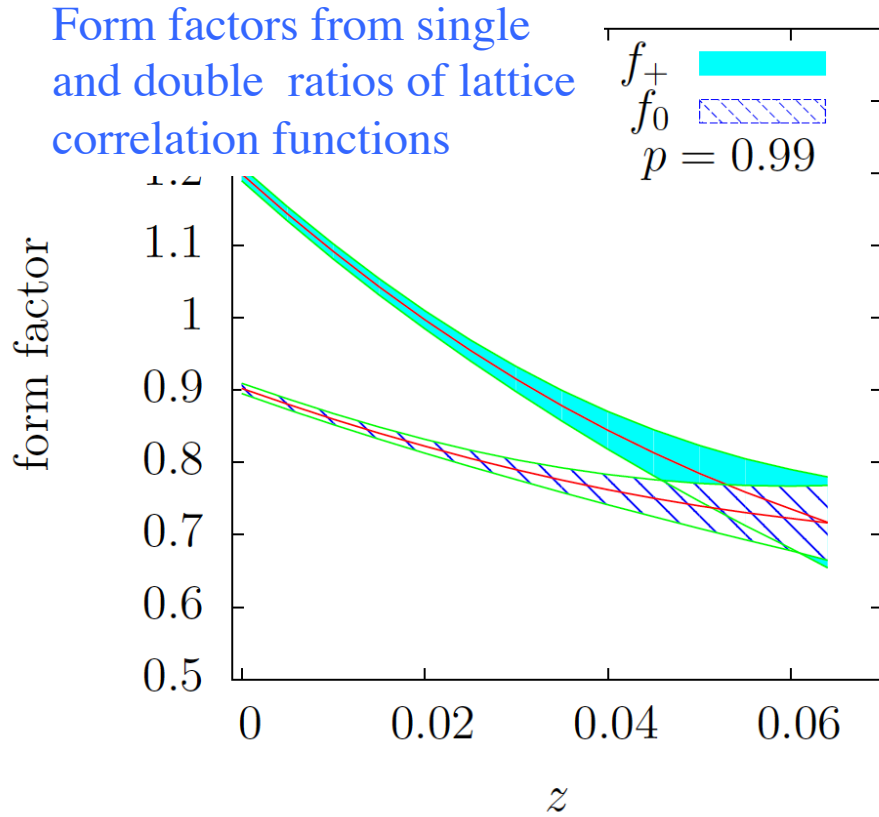
same lattice configurations used $m_b a \approx 1.1$ in the best case

	FNAL/MILC*	FNAL/MILC	HPQCD
process	$B \rightarrow D^* \ell \nu$	$B \rightarrow D \ell \nu$	$B \rightarrow D \ell \nu$
kinematics	$w = 1$	$w \geq 1$	$w \geq 1$
ensembles	MILC	MILC	MILC
N_f	2+1	2+1	2+1
a (fm)	5/0.045 – 0.15	4/0.045 – 0.12	2/0.09, 0.12
M_π^{\min} [MeV]	260	220	260
$M_\pi^{\min} L$	3.8	3.8	3.8
l quarks	asqtad	asqtad	asqtad
c quark	RHQ (Fermilab)	RHQ (Fermilab)	HISQ
b quark	RHQ (Fermilab)	RHQ (Fermilab)	NRQCD
reference	[1403.0635]	[1503.07237]	[1505.03925]

(* full publication of $B \rightarrow D^*$ results, no changes wrt proceedings value quoted in FLAG)

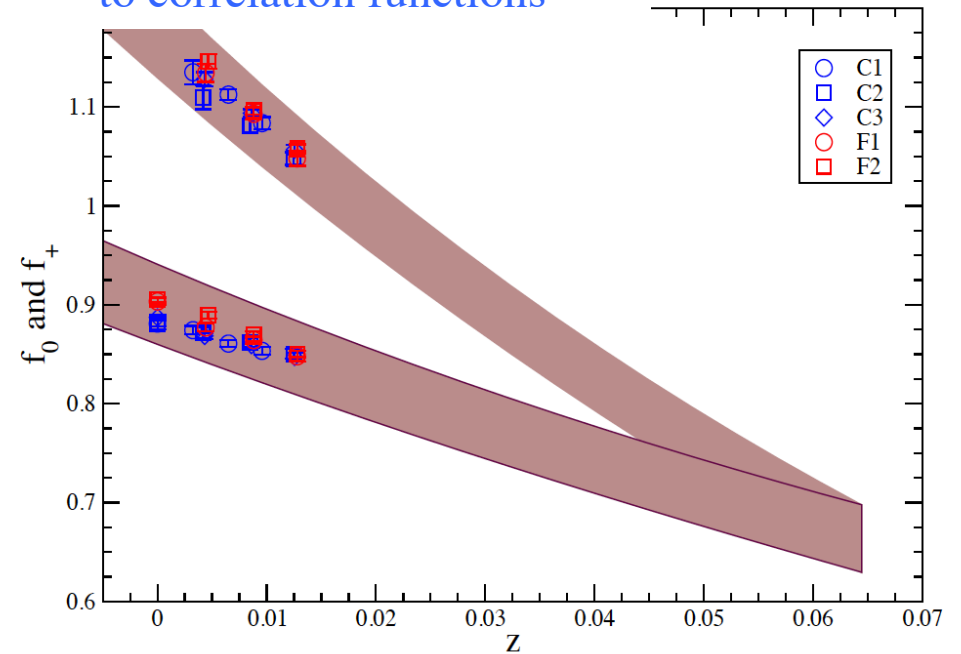
new results for $B \rightarrow D l \nu$

[FNAL/MILC]



[HPQCD]

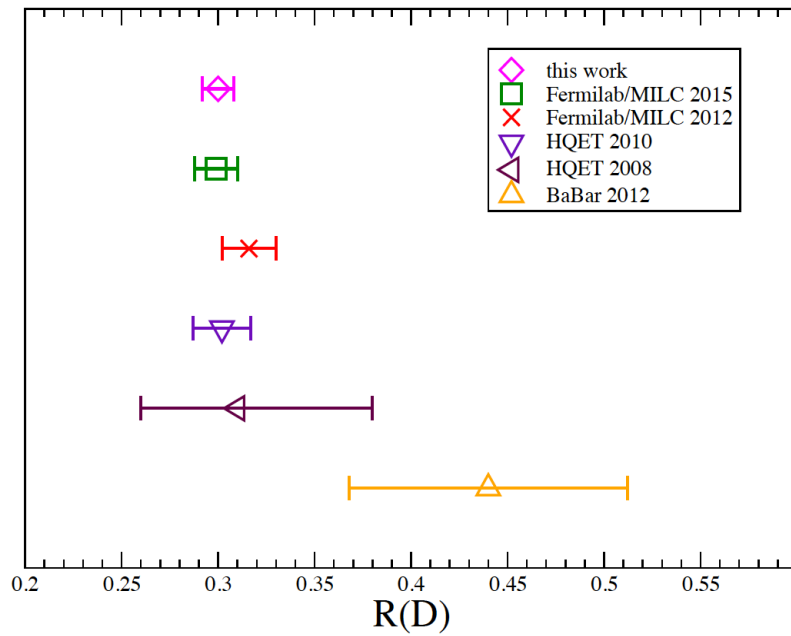
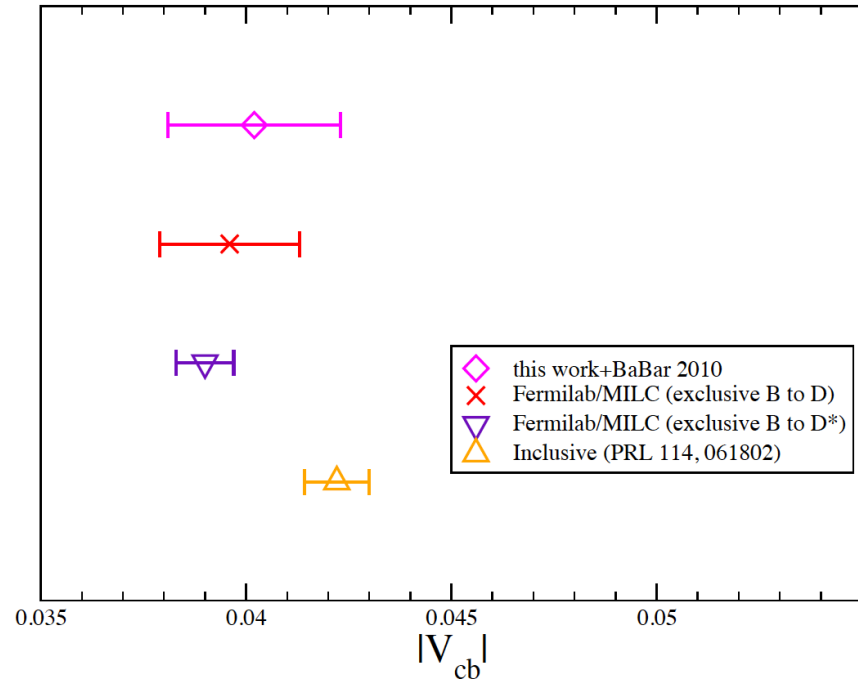
Form factors from direct fit to correlation functions



$$R(D) = \frac{\mathcal{B}(B \rightarrow D \tau \nu)}{\mathcal{B}(B \rightarrow D l \nu)} = 0.299(11)$$

$$0.300(8)$$

HPQCD June 13 2016

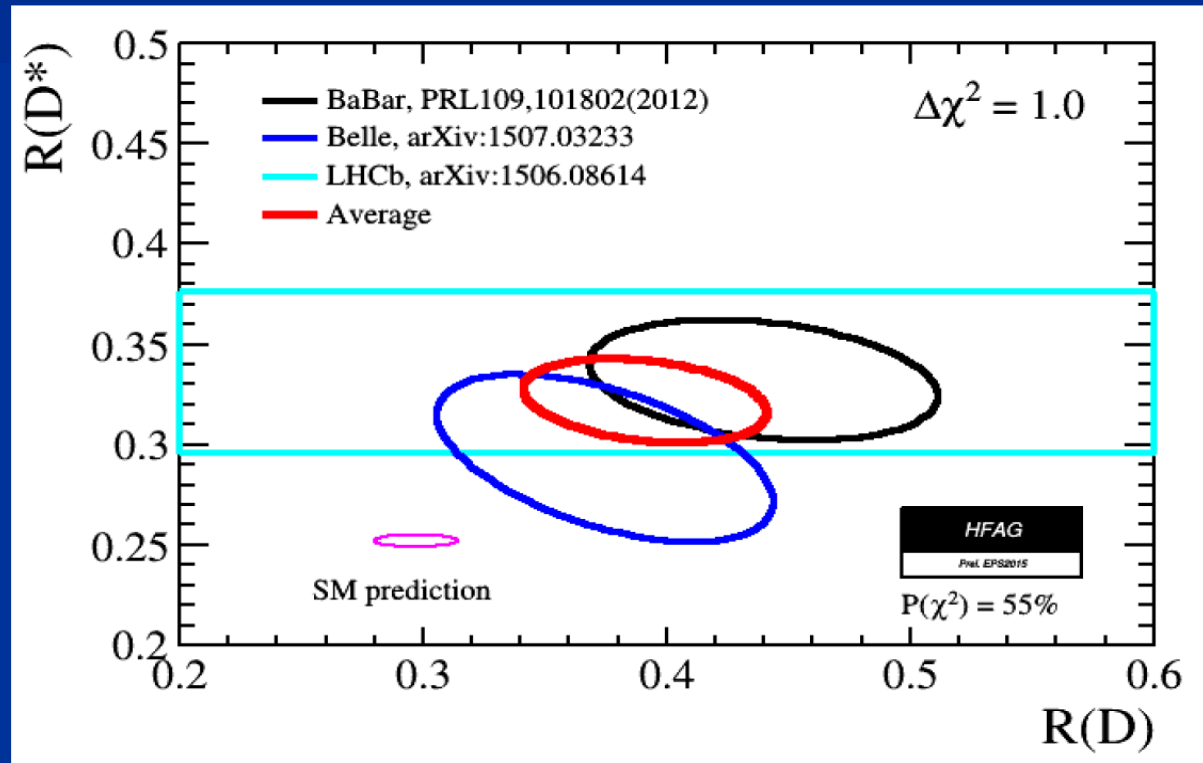


Tauonic B decays

Crivellin 2016

- Tree-level decays in the SM via W-boson

$$R(D^{(*)}) = B \rightarrow D^{(*)} \tau \nu / B \rightarrow D^{(*)} \ell \nu$$



Combined $\approx 4 \sigma$ deviation

$|V_{ub}|$ & $|V_{cb}|$ inclusive vs exclusive and all that

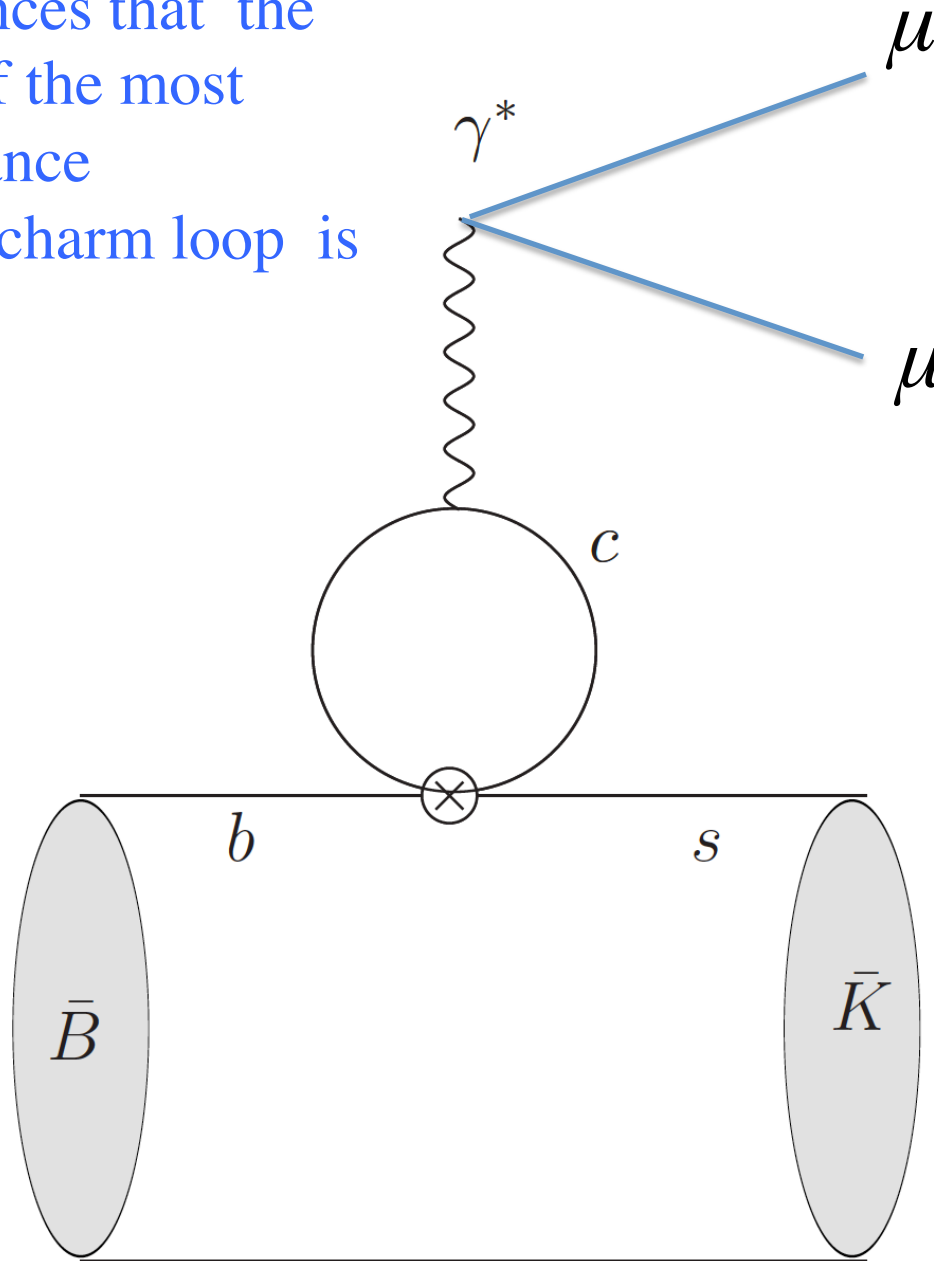
- 1) On the long run exclusive decays based on non-perturbative (lattice) determination of the relevant form factors will win;
- 2) The precision of the theoretical predictions for inclusive decays cannot be improved (are the present quoted errors reliable?);
- 3) Still (much) more work is needed, and different approaches to the physical B should be used and compared;
- 4) R(D) and R(D*) is an open problem; more lattice collaborations should work on these calculations;
- 5) Theoretical calculations and experimental analyses should not be biased by the HQFT - after all $\Lambda_{\text{QCD}}/m_c \approx O(1)$;
- 6) I hope to be wrong, but the possibility of new physics in tree level $b \rightarrow c$ decays looks to me quite remote.

Do we still care? Tensions and Unknowns

- 1) A “classical” example $B \rightarrow \tau \nu$
- 2) $|V_{ub}|$ and $|V_{cb}|$ inclusive vs exclusive
- 3) $|V_{cb}|$, B mixing and ϵ_K
- 4) D-mixing (already discussed)
- 5) $R(D)$ and $R(D^*)$ (and V_{cb} of course)
- 6) $B \rightarrow K^* \ell \ell$
- 7) Physics BSM ?

There are good chances that the lattice calculation of the most important long distance contributions via a charm loop is possible

M. Ciuchini,
V.Lubicz, G.M.,
L. Silvestrini,
S. Simula



RADIATIVE/RARE KAON DECAYS

*G. Isidori, G. M., and P. Turchetti, Phys.Lett. B633, 75 (2006),
arXiv:hep-lat/0506026*

N.H. Christ X. Feng A. Portelli and C.T. Sachrajda *Phys.Rev. D92*
(2015) no.9, 094512 [10.1103/PhysRevD.92.094512](https://arxiv.org/abs/10.1103/PhysRevD.92.094512) *

$$K \rightarrow \pi l^+ l^- \qquad K \rightarrow \pi \nu \bar{\nu}$$

Conserved currents and GIM important

2.1 $K \rightarrow \pi \ell^+ \ell^-$

G. Isidori, G. M., and P. Turchetti

The main non-perturbative correlators relevant for these decays are those with the electromagnetic current. In particular, the relevant T -product in Minkowski space is [7, 8]

$$(\mathcal{T}_i^j)_{\text{em}}^\mu(q^2) = -i \int d^4x e^{-iq \cdot x} \langle \pi^j(p) | T \{ J_{\text{em}}^\mu(x) [Q_i^u(0) - Q_i^c(0)] \} | K^j(k) \rangle , \quad (11)$$

$$J_{\text{em}}^\mu = \frac{2}{3} \sum_{q=u,c} \bar{q} \gamma^\mu q - \frac{1}{3} \sum_{q=d,s} \bar{q} \gamma^\mu q \quad (12)$$

for $i = 1, 2$ and $j = +, 0$. Thanks to gauge invariance we can write

$$(\mathcal{T}_i^j)_{\text{em}}^\mu(q^2) = \frac{w_i^j(q^2)}{(4\pi)^2} [q^2(k+p)^\mu - (m_k^2 - m_\pi^2)q^\mu] . \quad (13)$$

The normalization of (13) is such that the $O(1)$ scale-independent low-energy couplings $a_{+,0}$ defined in [8] can be expressed as

$$a_j = \frac{1}{\sqrt{2}} V_{us}^* V_{ud} \left[C_1 w_1^j(0) + C_2 w_2^j(0) + \frac{2N_j}{\sin^2 \theta_W} f_+(0) C_{7V} \right] . \quad (14)$$

A detailed analysis of the extraction of the amplitude from lattice correlators
by N.H. Christ X. Feng A. Portelli and C.T. Sachrajda

**Is the present picture showing a
Model Standardissimo ?**

An evidence, an evidence, my kingdom for an evidence

From Shakespeare's *Richard III*
and A. Stocchi

- 1) Fit of NP- $\Delta F=2$ parameters in a Model
“independent” way***
- 2) “Scale” analysis in $\Delta F=2$ ***

**Not today for lack of time see talk by M. Ciuchini*

CONCLUSIONS

- 1) The high precision of the SM UT Analysis allows to test the SM and to search for NP at a level which is competitive with direct searches
- 2) CKM matrix is the dominant source of flavour mixing and CP violation $\sigma(\rho) \sim 8\%$ & $\sigma(\eta) \sim 3\%$
- 3) SM analysis shows a **very** good overall consistency
- 4) The main tensions disappeared
- 5) Inclusive vs exclusive semileptonic decays still need theoretical improvement and BK/Kkbar mixing !!

Thus for the time being we have to remain with a **STANDARDISSIMO STANDARD MODEL** but ...



absence says more than presence

FRANK HERBERT
(Dune)

THANKS FOR YOUR ATTENTION



International School for Advanced Studies

