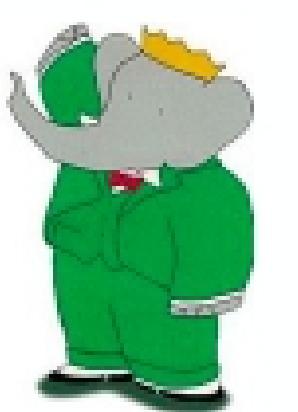




*CHARM Mixing, CP-Violation,
and non leptonic decays
at B-factories*

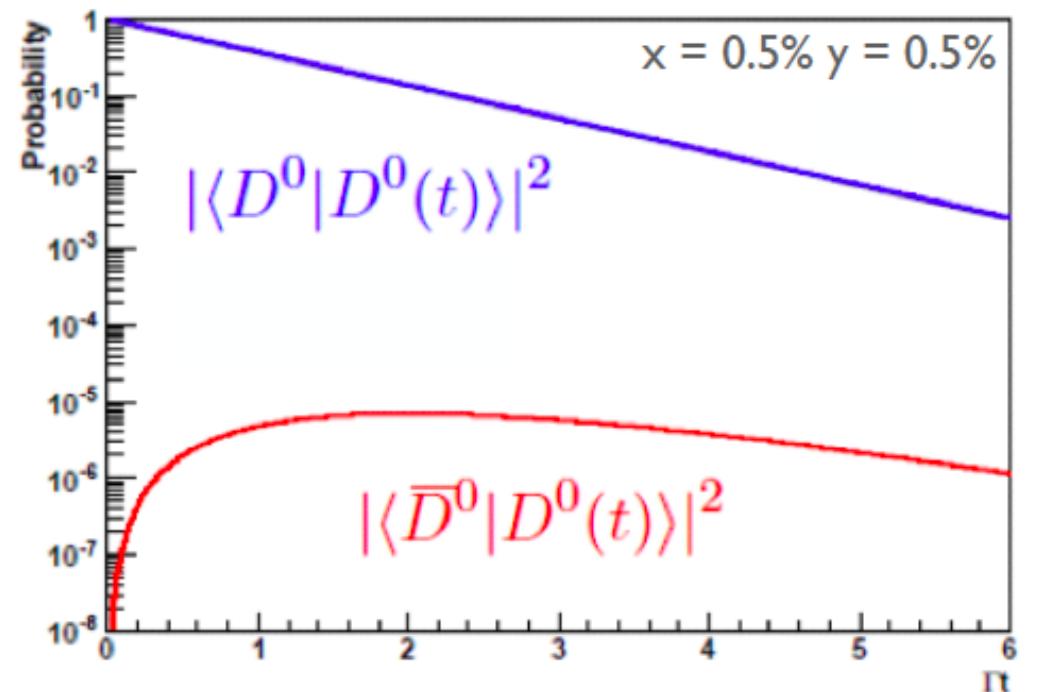
Franco Simonetto, INFN & UNIPD
 \mathcal{D}^o $\overline{\mathcal{D}}^o$
VIII CHARM, Bo



- Brief theory reminder
- Charm @ the B-factories
- Recent results
 - Mixing with time-dependent Dalitz analysis of $D^{\circ} \rightarrow \pi^+ \pi^- \pi^0$ @ BABAR
 - CPV and BR with $D^{\circ} \rightarrow \Phi/K^*/\rho \gamma$ @ Belle
 - CPV with $D^{\circ} \rightarrow K_s K_s$ @ Belle
- Conclusions

Theory reminder : mixing

- Production : flavor eigenstates $D^{\circ}/\bar{D}^{\circ}$
 - Time evolution : mass eigenstates D_1/D_2
 - Flavor oscillation :
- $$|\langle D^{\circ}|D^{\circ}(t)\rangle|^2 \propto [\cosh(yt/\tau) + \cos(xt/\tau)] \quad \text{"UNMIXED"}$$
- $$|\langle \bar{D}^{\circ}|D^{\circ}(t)\rangle|^2 \propto [\cosh(yt/\tau) - \cos(xt/\tau)] \quad \text{"MIXED"}$$
- Mixing parameters :
- $$\Gamma = (\Gamma_1 + \Gamma_2)/2 = \tau / \tau$$
- $$x = (m_1 - m_2)/\Gamma$$
- $$y = (\Gamma_1 - \Gamma_2)/(\Gamma_1 + \Gamma_2)$$



Theory reminder : CPV

- Direct :

$$|\mathcal{A}_f| = |\mathcal{A}(D \rightarrow f)| \neq |\mathcal{A}(\bar{D} \rightarrow \bar{f})| = |\overline{\mathcal{A}}_{\bar{f}}|$$

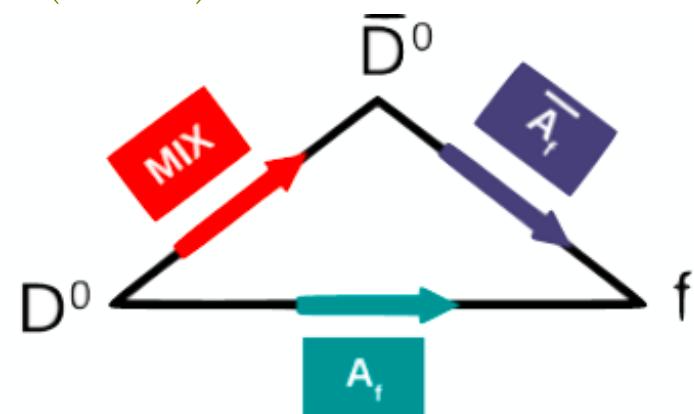
$$\mathcal{A}_{CP} = \frac{|\mathcal{A}_f|^2 - |\overline{\mathcal{A}}_{\bar{f}}|^2}{|\mathcal{A}_f|^2 + |\overline{\mathcal{A}}_{\bar{f}}|^2}$$

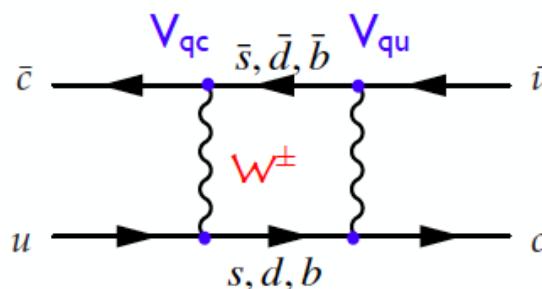
- Mixing induced :

$$\text{Prob}(D^0 \rightarrow \bar{D}^0) \neq \text{Prob}(\bar{D}^0 \rightarrow D^0) \quad \Leftrightarrow \quad |q/p| \neq 1$$

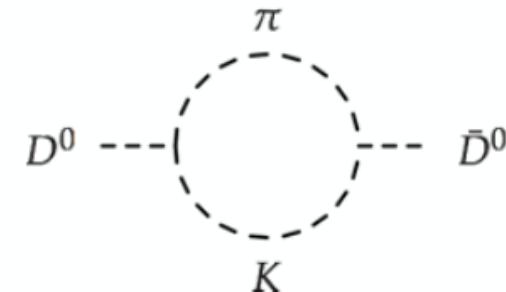
- Interference of the two in CP eigenstates ($f \equiv \bar{f}$) :

$$\lambda_f = \frac{q}{p} \frac{\overline{\mathcal{A}}_f}{\mathcal{A}_f} = \left| \frac{q}{p} \frac{\overline{\mathcal{A}}_f}{\mathcal{A}_f} \right| e^{i(\delta_f + \phi_f)}$$





+



- Short distance:
 - light quarks in the loop
 - GIM & CKM suppressed
- Long distance:
 - real particles in the loop
 - dominant contribution
- Large uncertainties in the calculation, expect
 - $x, y \sim 0.5\%$, $\mathcal{A}_{CP} \sim 10^{-3}$
- Values in excess of $\sim 1\%$:
 - may be sign of NEW PHYSICS

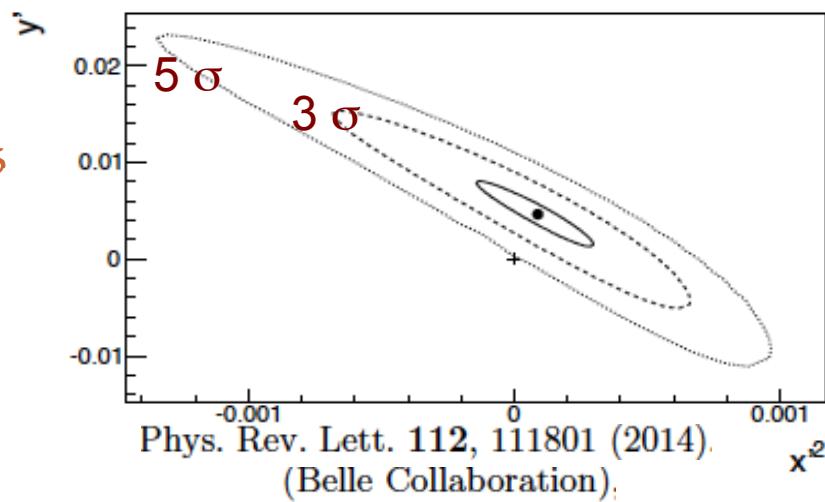
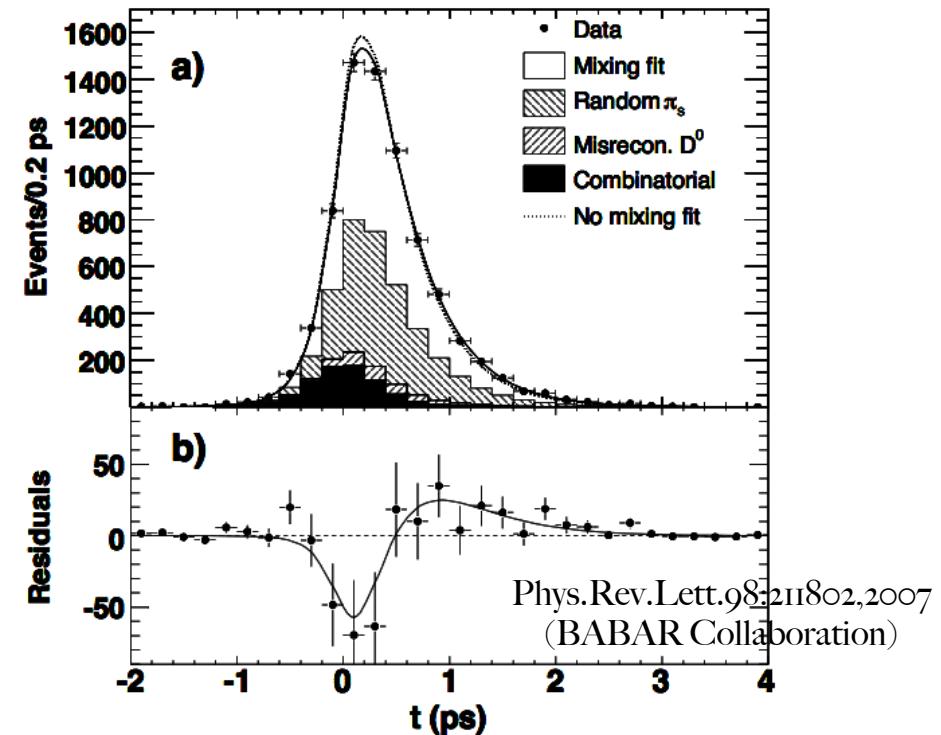
Status : mixing

- Mixing assessed at the B-factories in $D^0 \rightarrow K\pi$ transitions
- Strong phases : measure effective parameters

$$x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi}$$

$$y' = y \cos \delta_{K\pi} - x \sin \delta_{K\pi}$$

- Then :
 - focus on phase-free approaches
 - determine x and y



- No positive observation – yet
- Where to look ?
- Nierste :
 - SCS decays in two pseudoscalar mesons ($D^0 \rightarrow K_s K_s$)
- Bigj :
 - “3 & 4-body final states not backup for information from two-body – landscapes are very different”

“Charm and Strange, recent results and future perspectives”
(Gagan Mohanty, summary talk at ICHEP 2016)

Measurements at B-factories

- Extensive study of mixing and CPV in several final states

BABAR Contributions

(Giulia Casarosa, XIII Int. Conf. on Heavy Quarks and Leptons)

- mixing*
- $D^0 \rightarrow \pi^+\pi^-, K^+K^-$ y_{CP}
 - $D^0 \rightarrow K^+\pi^-$ x'^2, y'
 - $D^0 \rightarrow K_S\pi^+\pi^-, K_SK^+K^-$ x, y
 - $D^0 \rightarrow K^+\pi^-\pi^0$ y'', x''^2
 - $D^0 \rightarrow \pi^+\pi^-\pi^0$ x, y
- NEW**
- PRELIMINARY RESULT**

- direct CPV*
- $D^0 \rightarrow \pi^+\pi^-, K^+K^-$ $A_{CP}, \Delta A_{CP}$
 - $D^0 \rightarrow K^+K^-\pi^0$
 - $D^0 \rightarrow \pi^+\pi^-\pi^0$
 - $D_s^+ \rightarrow K_S\pi^+$ $A_{CP} = \frac{(\Gamma_D - \Gamma_{\bar{D}})}{(\Gamma_D + \Gamma_{\bar{D}})}$
 - $D_s^+ \rightarrow K_SK^+$
 - $D^+ \rightarrow K_S\pi^+$
 - $D^+ \rightarrow K^+K^-\pi^+$
 - ★ $D^+ \rightarrow \pi^+\pi^0$ **ALMOST READY...**

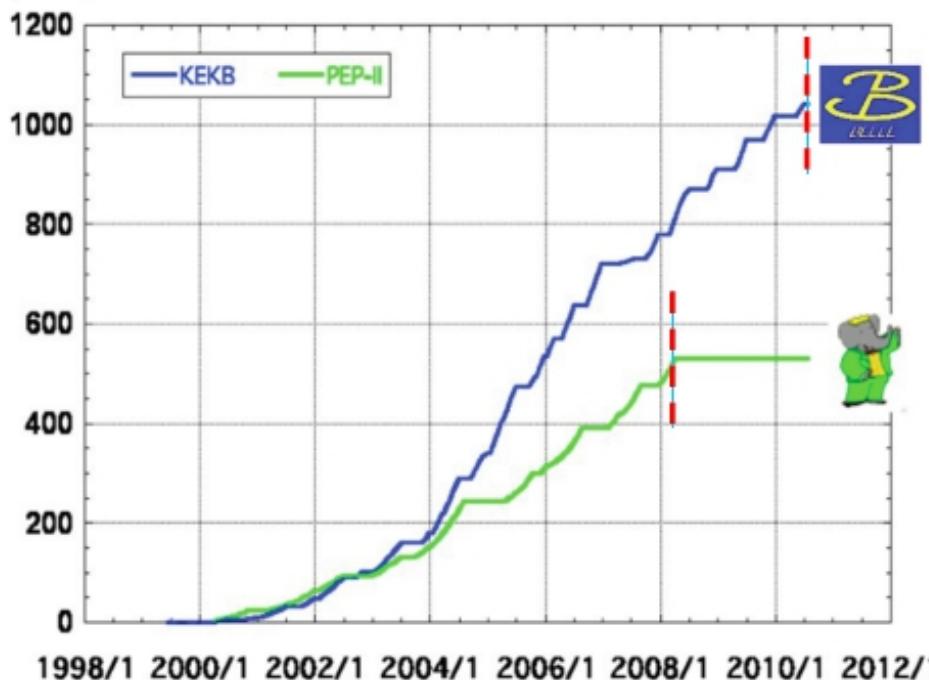
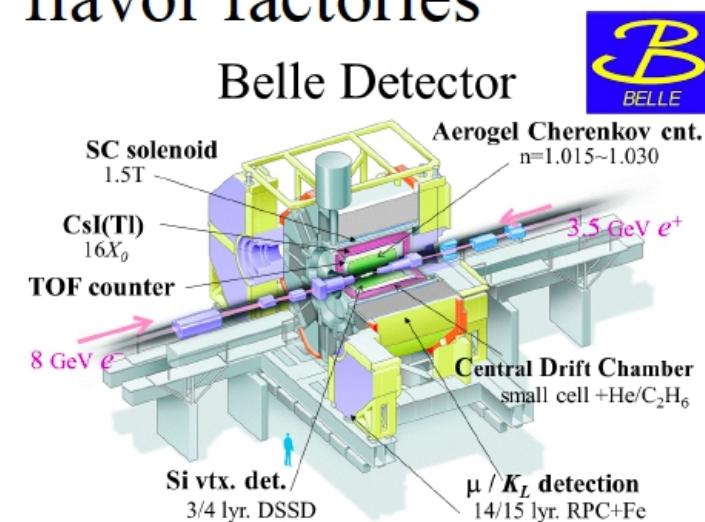
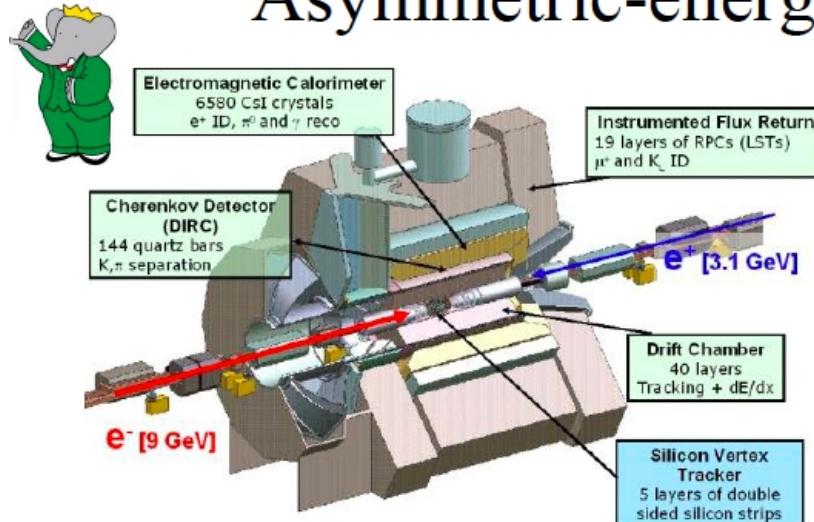
- direct vs indirect CPV*
- $A_{CP}, \Delta Y$ (aka A_T)
- $D^0 \rightarrow \pi^+\pi^-, K^+K^-$

- CP violation with triple-product asymmetries*
- $$C_T = (\vec{p}_1 \times \vec{p}_2) \cdot \vec{p}_3$$
- $D^0 \rightarrow K^+K^-\pi^+\pi^-$
 - $D^+ \rightarrow K_SK^+\pi^+\pi^-$
 - $D_s^+ \rightarrow K_SK^+\pi^+\pi^-$

- Similar list for Belle

Charm Physics at the B-factories

Asymmetric-energy flavor factories



Belle: $\sim 1.2 \cdot 10^9$ ($e^+ e^- \rightarrow c\bar{c}$) events

Babar: $\sim 0.7 \cdot 10^9$ ($e^+ e^- \rightarrow c\bar{c}$) events

Results from B-factories

- This talk, focus on new results :

$D^\circ - \bar{D}^\circ$ mixing with $D^\circ \rightarrow \pi^+ \pi^- \pi^\circ$ decays at BaBar

arXiv:1604.00857

\mathcal{A}_{CP} with $D^\circ \rightarrow K_s K_s$ decays at Belle

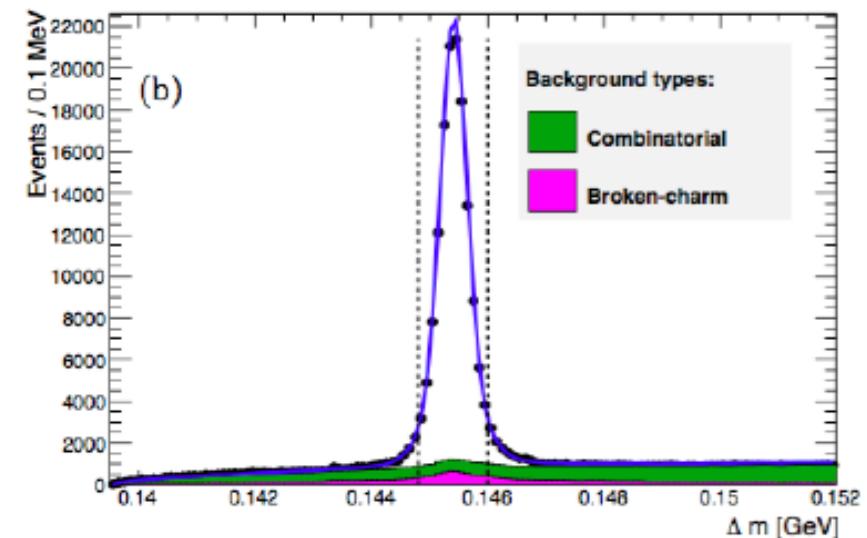
arXiv:1603.03257

\mathcal{A}_{CP} with $D^\circ \rightarrow \phi/\rho/K^*\gamma$ at Belle

Belle Conf 1609

Common Features

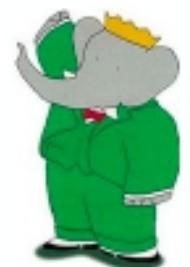
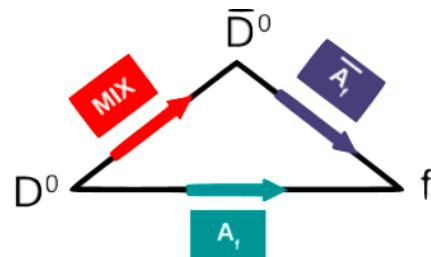
- Use $D^{*+} \rightarrow \pi^+ D^0$ decays (and cc)
 - very good signal / noise
 - soft pion charge tags D^0 flavor at production time ($\sim 99\%$ efficient)
- Focus on prompt charm
 - cut on $p_T > p_{\text{threshold}}$ (~ 2.5 GeV)
- apply beam spot constraint in kinematic fits
 - improve S/N
 - improve proper time resolution



- Common final state to D^0 and \bar{D}^0

- CP :

- exchange D^0 and \bar{D}^0
- exchange π^+ and π^-
- invert all momenta



$$\begin{aligned} \text{CP } | \pi^+ (\vec{p}_1) \pi^- (\vec{p}_2) \pi^0 (\vec{p}_3) \rangle &= \\ &= | \pi^- (-\vec{p}_1) \pi^+ (-\vec{p}_2) \pi^0 (-\vec{p}_3) \rangle \end{aligned}$$

- Assuming CP conservation, \bar{D}^0 and D^0 belong to the same Dalitz plot with:

$$\bar{\mathcal{A}}_f(s_-, s_+) = \mathcal{A}_f(s_+, s_-)$$

$$s_+ = m^2(\pi^+, \pi^0)$$

$$s_- = m^2(\pi^-, \pi^0)$$

- Strong phases not an issue anymore

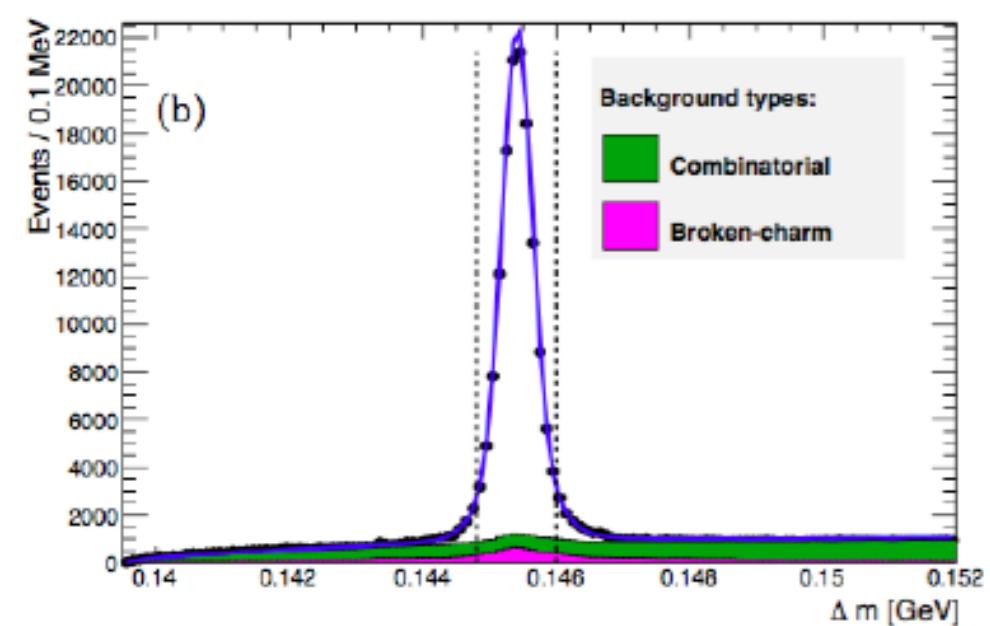
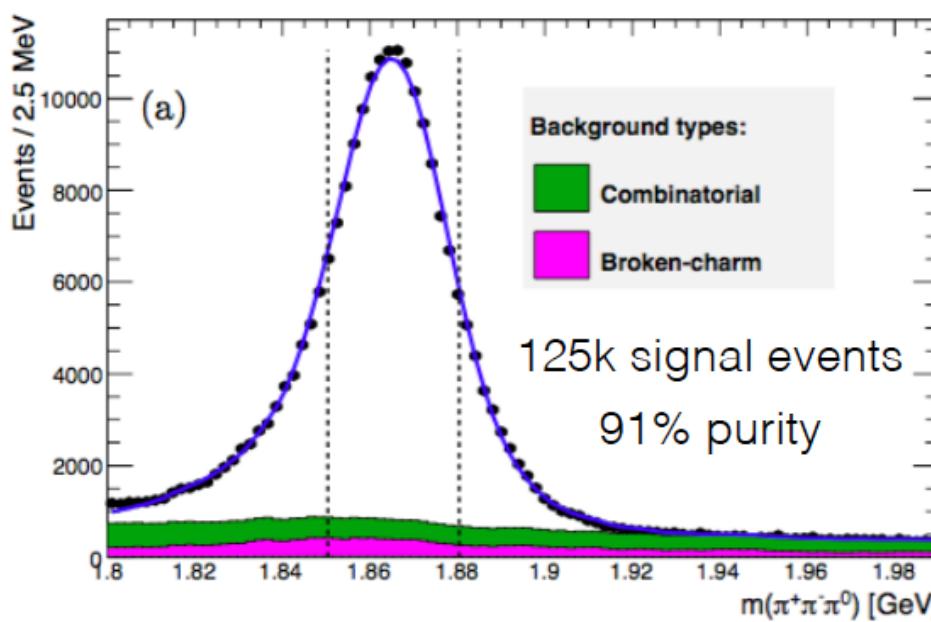
$$\bar{\mathcal{A}}_f = \mathcal{A}(\bar{D}^0 \rightarrow \pi^+ \pi^- \pi^0)$$

$$\mathcal{A}_f = \mathcal{A}(D^0 \rightarrow \pi^+ \pi^- \pi^0)$$

Mixing with $D^0 \rightarrow \pi\pi\pi$

- Tight cuts on $m(D^0)$, Δm
 - 125 k evts
 - 91% purity
- Time-dependent analysis across the Dalitz plot
- Unbinned maximum likelihood fit to (t, s_+, s_-)
- Blind analysis

Veto on $D^0 \rightarrow K^-\pi^+$, $D^0 \rightarrow K^-\pi^+\pi^0$,
 $D^0 \rightarrow K_S \pi^+\pi^0$, $D^0 \rightarrow K_S \pi^0$
 $E_{\text{lab}}(\pi^0) > 350$ MeV
 $p_{\text{cms}}(D^0) > 2.8$ GeV to remove $B \rightarrow D$ events
 $-2 < t(D^0) < 3$ ps, $\sigma_t < 0.8$ ps
 $P(\chi^2) > 0.1\%$ for the D^* candidates



Time function

$$|\mathcal{M}(D^0)|^2 \propto \frac{1}{2} e^{-\Gamma_D t} \left\{ |A_f|^2 [\cosh(y\Gamma_D t) + \cos(x\Gamma_D t)] \right.$$

Direct decay



$$+ \left| \frac{q}{p} \bar{A}_f \right|^2 [\cosh(y\Gamma_D t) - \cos(x\Gamma_D t)]$$

Through mixing

$$- 2 \left[\text{Re} \left(\frac{q}{p} A_f^* \bar{A}_f \right) \sinh(y\Gamma_D t) - \text{Im} \left(\frac{q}{p} A_f^* \bar{A}_f \right) \sin(x\Gamma_D t) \right] \right\}$$

Interference

Resolution function : sum of three Gaussian

Account for correlations between $\sigma(t)$ and s_+, s_-

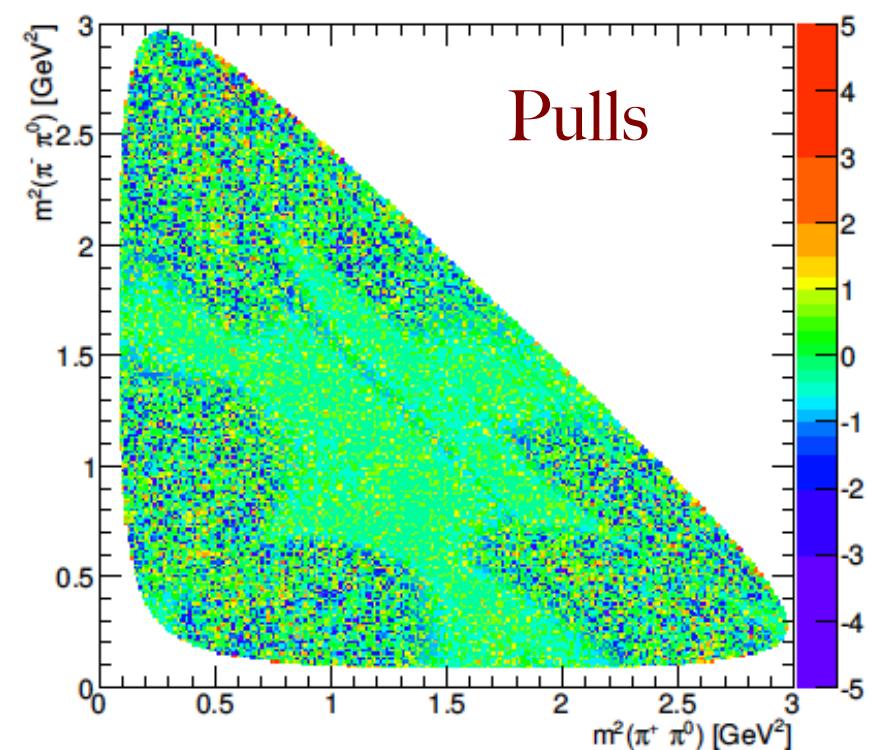
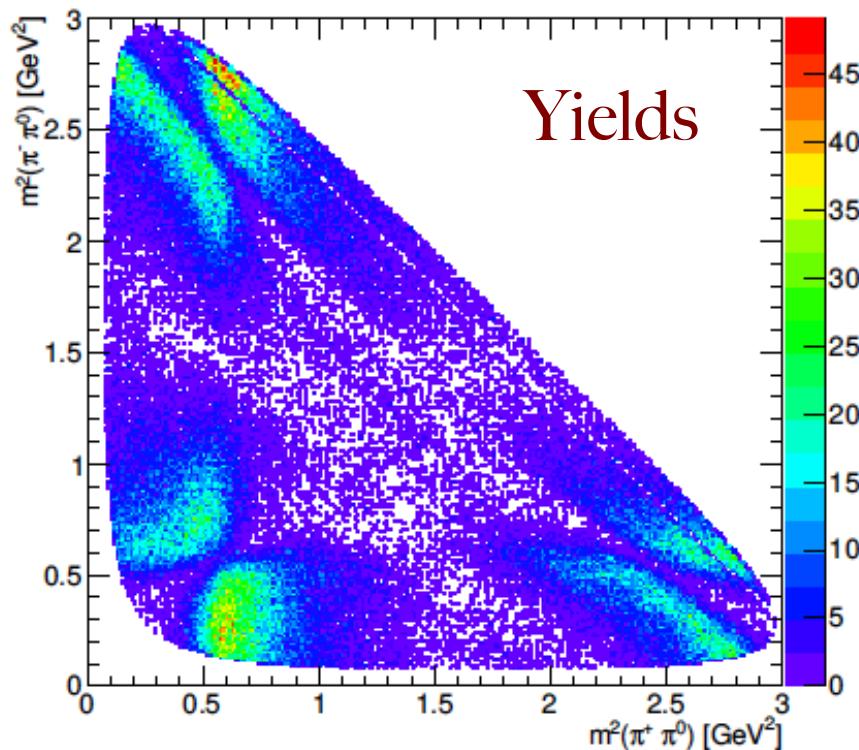
Dalitz Analysis

- Isobar model $\bar{A}_f(s_-, s_+) = A_f(s_+, s_-) = \sum_k c_k \mathcal{W}_k(s_+, s_-)$



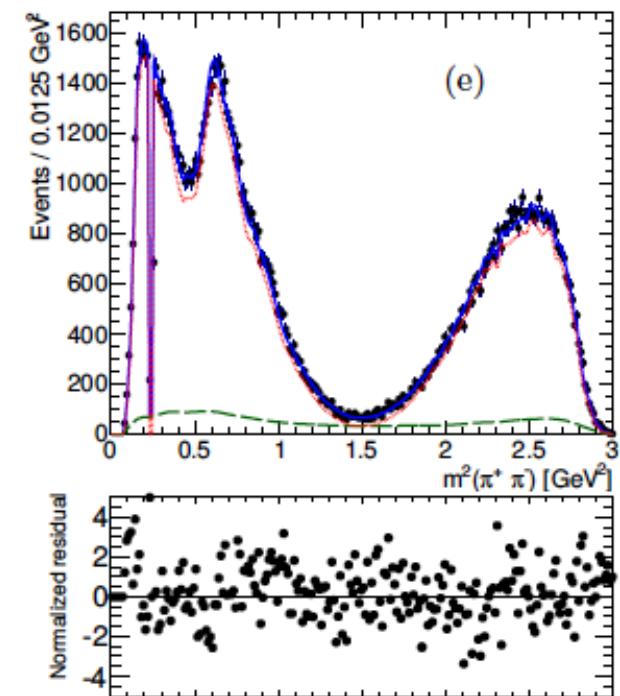
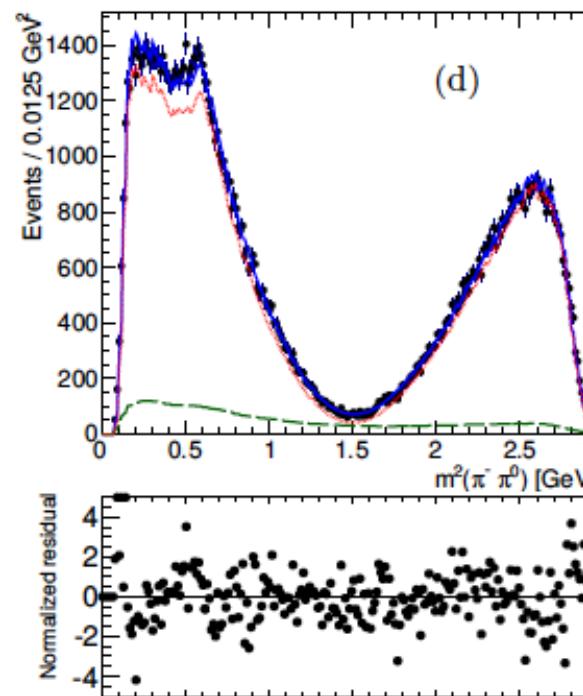
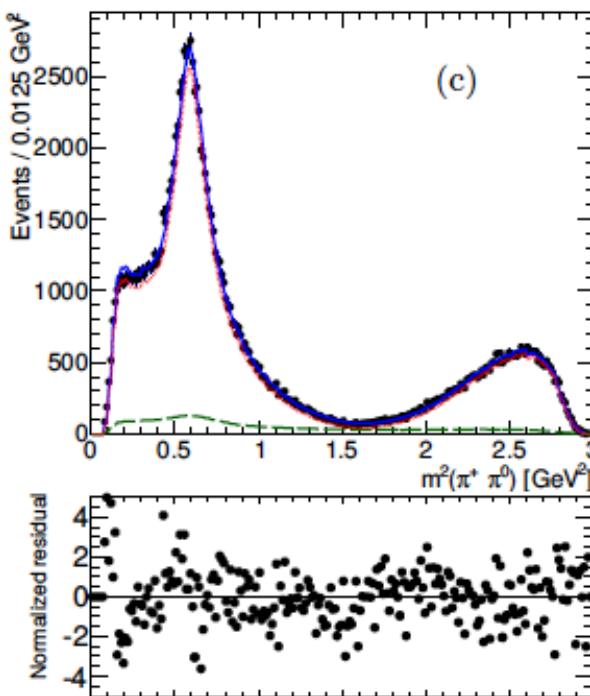
\mathcal{W}_k : relativistic Breit Wigner

- × spin dependent angular factor
- × Blatt Weisskopf form factors



Dalitz Analysis

- Large pull values near phase space boundaries
 - observed in MC as well
 - finite resolution effect causing migration from the edges of the Dalitz plot
- Bias from test on MC samples : $\Delta x = +0.58\%$
 $\Delta y = -0.05\%$



$x = (1.5 \pm 1.2 \pm 0.6)\%$
$y = (0.2 \pm 0.9 \pm 0.5)\%$
$\tau(D^0) = 410.2 \pm 3.8 \text{ fs}$



Output of amplitude analysis

State	Fit to data results		
	Magnitude	Phase (°)	Fraction f_r (%)
$\rho(770)^+$	1	0	66.4 ± 0.5
$\rho(770)^0$	0.55 ± 0.00	16.1 ± 0.4	23.9 ± 0.3
$\rho(770)^-$	0.73 ± 0.01	-1.6 ± 0.5	35.6 ± 0.4
$\rho(1450)^+$	0.55 ± 0.07	-7.7 ± 8.2	1.1 ± 0.3
$\rho(1450)^0$	0.19 ± 0.07	-70.4 ± 15.9	0.1 ± 0.1
$\rho(1450)^-$	0.53 ± 0.06	8.2 ± 6.7	1.0 ± 0.2
$\rho(1700)^+$	0.91 ± 0.15	-23.3 ± 10.3	1.5 ± 0.5
$\rho(1700)^0$	0.60 ± 0.13	-56.3 ± 16.0	0.7 ± 0.3
$\rho(1700)^-$	0.98 ± 0.17	78.9 ± 8.5	1.7 ± 0.6
$f_0(980)$	0.06 ± 0.00	-58.8 ± 2.9	0.3 ± 0.0
$f_0(1370)$	0.20 ± 0.03	-19.6 ± 9.5	0.3 ± 0.1
$f_0(1500)$	0.18 ± 0.02	7.4 ± 7.4	0.3 ± 0.1
$f_0(1710)$	0.40 ± 0.08	42.9 ± 8.8	0.3 ± 0.1
$f_2(1270)$	0.25 ± 0.01	8.8 ± 2.6	0.9 ± 0.0
$f_0(500)$	0.26 ± 0.01	-4.1 ± 3.7	0.9 ± 0.1
NR	0.43 ± 0.07	-22.1 ± 11.7	0.4 ± 0.1

Systematic uncertainties

Source		x [%]	y [%]
“Lucky” false slow pion fraction		0.01	0.01
Time resolution dependence on reconstructed D^0 mass		0.03	0.02
Amplitude-model variations		0.31	0.12
Resonance radius		0.02	0.10
DP efficiency parametrization		0.03	0.03
DP normalization granularity		0.03	0.04
Background DP distribution		0.21	0.11
Decay time window		0.18	0.19
σ_t cutoff		0.01	0.01
Number of σ_t ranges		0.11	0.26
σ_t parametrization		0.05	0.03
Background-model MC time distribution parameters		0.06	0.11
Fit bias correction		0.29	0.02
SVT misalignment		0.20	0.23
Total		0.56	0.46



CPV

- SM extensions may enhance \mathcal{A}_{CP} to few %

G. Isidori and J. F. Kamenik, PRL 109 (2012) 171801

- Decay $D^0 \rightarrow \rho^0\gamma$ not observed before
- BR and \mathcal{A}_{CP} measured w.r.t. normalization samples

$$\mathcal{B}_{sig} = \mathcal{B}_{norm} \times \frac{\mathcal{N}_{sig}}{\mathcal{N}_{norm}} \times \frac{\epsilon_{sig}}{\epsilon_{norm}}$$

$$\mathcal{A}_{CP}^{sig} = \mathcal{A}_{raw}^{sig} - \mathcal{A}_{raw}^{norm} + \mathcal{A}_{CP}^{norm}$$

$$\mathcal{A}_{raw} = \frac{\mathcal{N}(\pi^+ V\gamma) - \mathcal{N}(\pi^- V\gamma)}{\mathcal{N}(\pi^+ V\gamma) + \mathcal{N}(\pi^- V\gamma)}$$

Signal	Norm.	Signal Yield
--------	-------	--------------

$D^0 \rightarrow \phi\gamma \rightarrow K^+K^-\gamma$	$D^0 \rightarrow K^+K^-$	524 ± 35
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$D^0 \rightarrow \bar{K}^{*0}\gamma \rightarrow K^-\pi^+\gamma$	$D^0 \rightarrow K^-\pi^-$	9104 ± 396
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$D^0 \rightarrow \rho^0\gamma \rightarrow \pi^+\pi^-\gamma$	$D^0 \rightarrow \pi^+\pi^-$	500 ± 85
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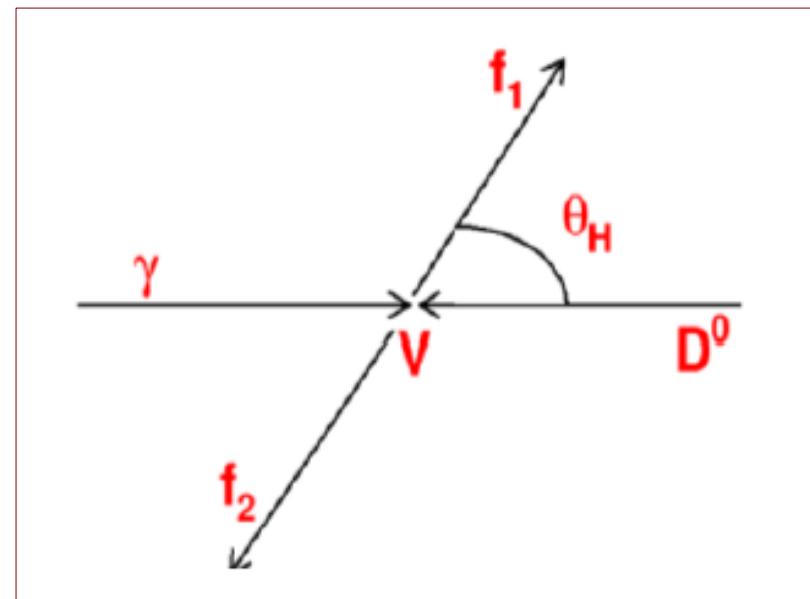


Belle : CPV in $D^0 \rightarrow V\gamma$

- Main background : $D^0 \rightarrow V\pi^0$, with a missed γ
 - Neural Network to optimize π^0 rejection by removing $\gamma\gamma$ pairs
- Tags :
 - $m(V\gamma)$
 - helicity angle θ_H

$$\text{signal} \propto (1 - \cos \theta_H)^2$$

$$\pi^0 \text{ backg} \simeq (\cos \theta_H)^2$$



Belle : CPV in $D^{\circ} \rightarrow V\gamma$

γ signal red

π° background green

$\phi\gamma$

$K^*\gamma$

$\rho\gamma$

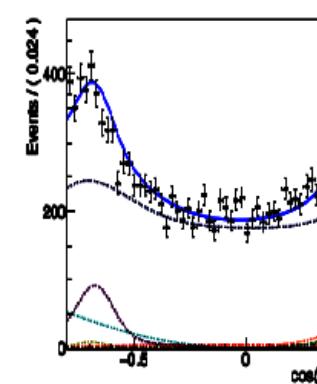
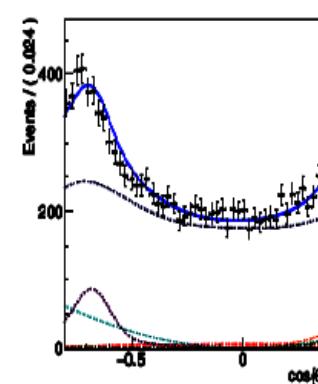
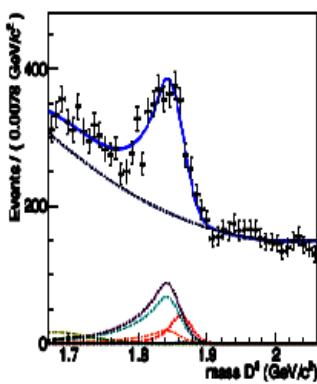
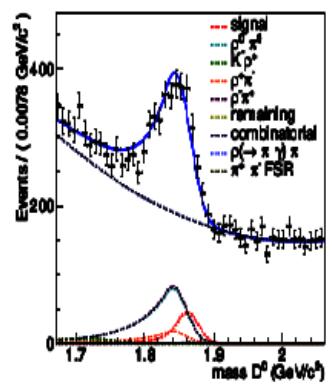
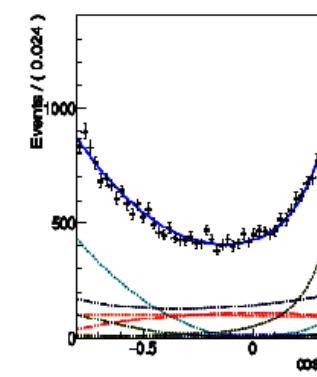
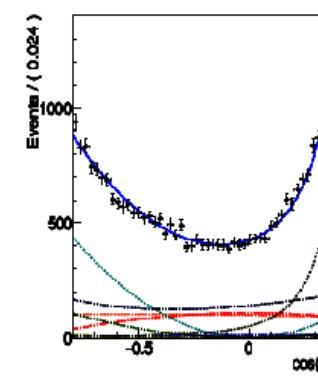
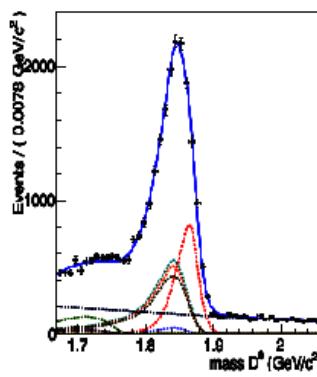
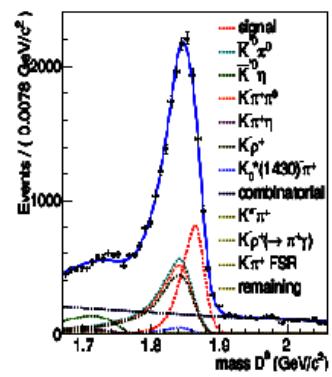
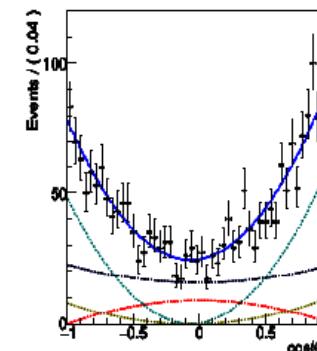
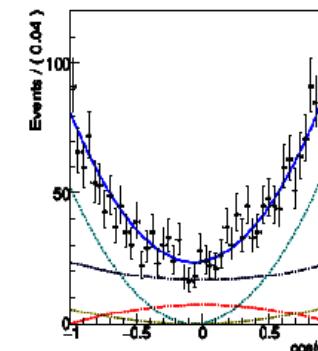
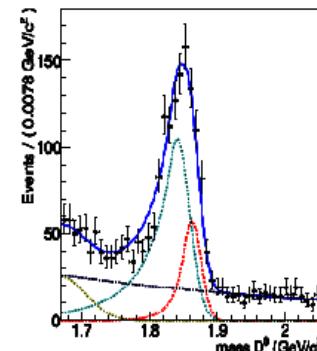
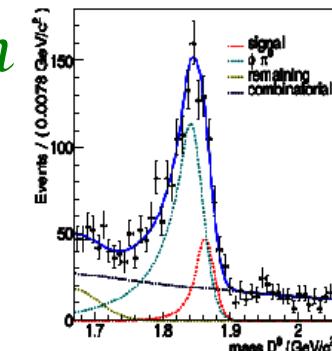


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CPV in $D^{\circ} \rightarrow V\gamma$: results

$$\mathcal{B}(D^0 \rightarrow \phi\gamma) = (2.76 \pm 0.20 \pm 0.08) \times 10^{-5} \quad \text{consistent with BABAR}$$
$$\mathcal{B}(D^0 \rightarrow \bar{K}^{*0}\gamma) = (4.66 \pm 0.21 \pm 0.18) \times 10^{-4} \quad 3.3\sigma \text{ wrt BABAR}$$
$$\mathcal{B}(D^0 \rightarrow \rho^0\gamma) = (1.77 \pm 0.30 \pm 0.08) \times 10^{-5} \quad \text{First observation}$$

$$\mathcal{A}_{CP}(D^0 \rightarrow \phi\gamma) = -(0.094 \pm 0.066 \pm 0.001)$$
$$\mathcal{A}_{CP}(D^0 \rightarrow \bar{K}^{*0}\gamma) = -(0.003 \pm 0.020 \pm 0.000)$$
$$\mathcal{A}_{CP}(D^0 \rightarrow \rho^0\gamma) = \quad 0.056 \pm 0.151 \pm 0.006$$

- Results consistent with no CPV
- Statistical errors dominate



- S.M. 95% Upper Limit. for direct CPV : 1.1 % PRD 92, 054036 (2015)
- (Possible) interference with NP : sizable enhancement PRD 87, 014024 (2013)
- Normalization : $D^{\circ} \rightarrow K_s \pi^{\circ}$

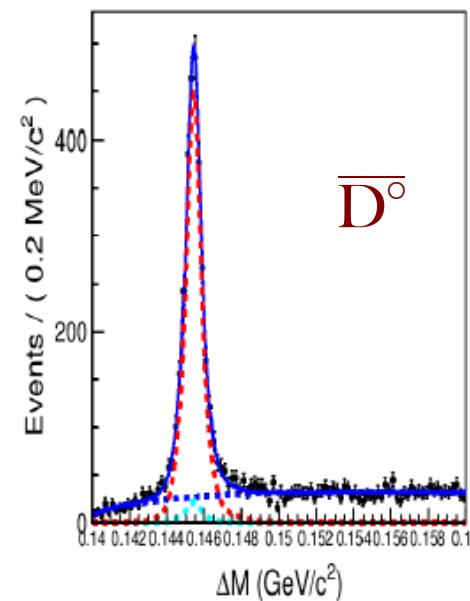
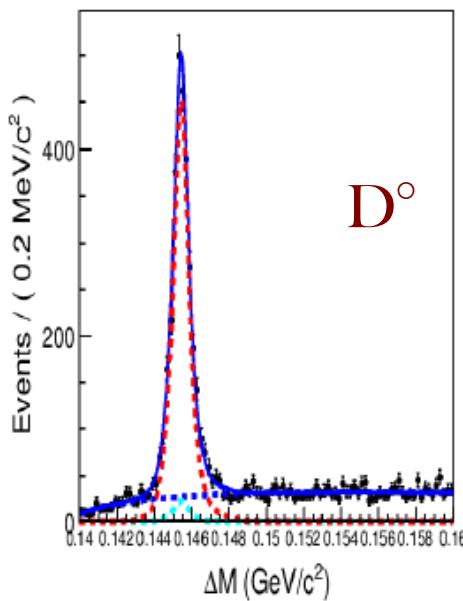
$$\mathcal{A}_{CP}^{D^{\circ} \rightarrow K_s K_s} = \mathcal{A}_{RAW}^{D^{\circ} \rightarrow K_s K_s} - \mathcal{A}_{RAW}^{D^{\circ} \rightarrow K_s \pi^{\circ}} + \mathcal{A}_{CP}^{D^{\circ} \rightarrow K_s \pi^{\circ}} + \mathcal{A}_{\epsilon}^{K^{\circ}/\bar{K}^{\circ}}$$

$$A_{\epsilon}^{K^{\circ}/\bar{K}^{\circ}} = (-0.11 \pm 0.01)\% \quad K^{\circ}/\bar{K}^{\circ} \text{ interaction with the detector material}$$
$$A_{CP}^{D^{\circ} \rightarrow K_s^0 \pi^0} = (-0.20 \pm 0.17)\% \quad \text{PDG}$$

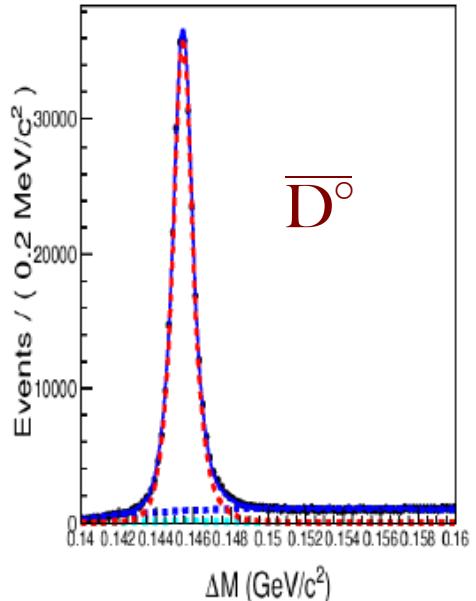
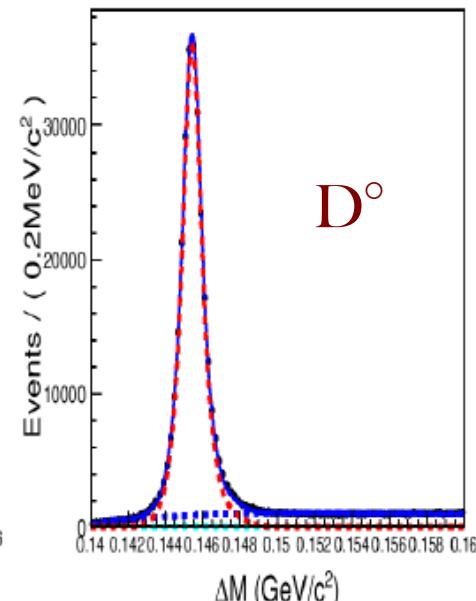
PRD 84, 111501 (2011)



$D \rightarrow K_S^0 K_S^0$: Signal mode



$D \rightarrow K_S^0 \pi^0$: Normalization mode



- Efficiency = $(11.04 \pm 0.02)\%$
- Signal Yield = 5399 ± 87
- $A^{raw} = (0.45 \pm 1.53)\%$

- Efficiency = $(12.60 \pm 0.02)\%$
- Signal Yield = 531807 ± 796
- $A^{raw} = (0.16 \pm 0.14)\%$



$$\mathcal{A}_{CP}(D^\circ \rightarrow K_s K_s) = (-0.002 \pm 1.53 \pm 0.17)\%$$

[Belle CONF-1609]

CLEO $(-23 \pm 19)\%$ 13.7 fb^{-1} PRD 63 (2001) 071101
LHCb $(-2.9 \pm 5.2 \pm 2.2)\%$ 3 fb^{-1} JHEP 10 (2015) 055

Source	Systematic uncertainty, in %
Signal shape	± 0.01
Peaking background	± 0.01
K^0/\bar{K}^0 material effects	± 0.01
A_{CP} measurement of $K_S^0\pi^0$	± 0.17
Total	± 0.17



- Years after shutdown, B-factories still provide competitive results on charm-Physics.
- I have discussed few brand new results on
 - Mixing in $D^0 \rightarrow \pi\pi\pi$ (BABAR)

$$x = (1.5 \pm 1.2 \pm 0.6)\%$$

$$y = (0.2 \pm 0.9 \pm 0.5)\%$$

- CPV (Belle)

$$\mathcal{A}_{CP}(D^0 \rightarrow K_s K_s) = (-0.002 \pm 1.53 \pm 0.17)\%$$

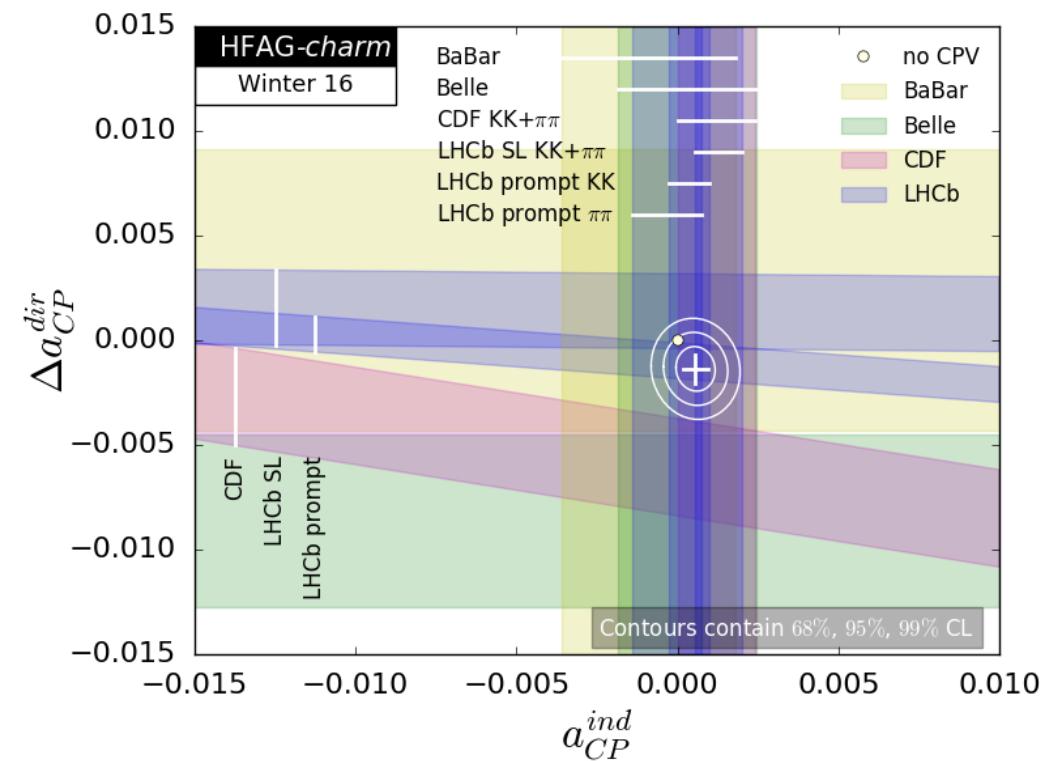
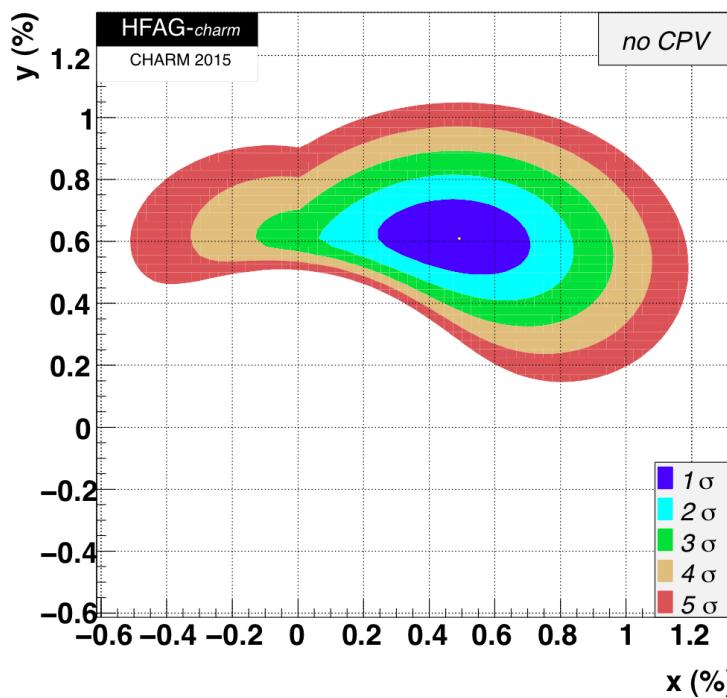
$$\mathcal{A}_{CP}(D^0 \rightarrow \phi\gamma) = -(0.094 \pm 0.066 \pm 0.001)$$

$$\mathcal{A}_{CP}(D^0 \rightarrow \bar{K}^{*0}\gamma) = -(0.003 \pm 0.020 \pm 0.000)$$

$$\mathcal{A}_{CP}(D^0 \rightarrow \rho^0\gamma) = 0.056 \pm 0.151 \pm 0.006$$

Summary (II)

- Mixing is well established, and being studied in several new channels
- ... whereas CPV has not yet been observed
- However all results are limited by $\sigma(\text{STAT})$
- ... paving the way for the super-B factory at BelleTwo



Backup



RARE non leptonic DECAY:

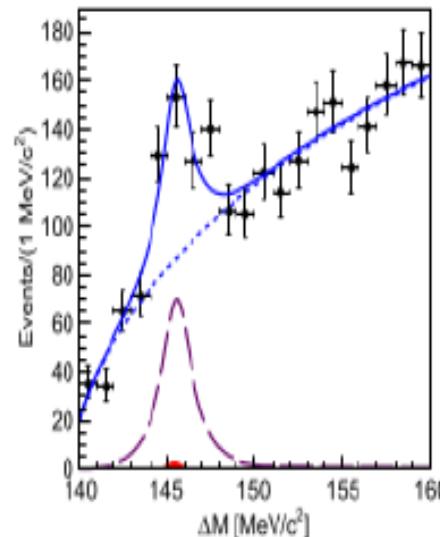
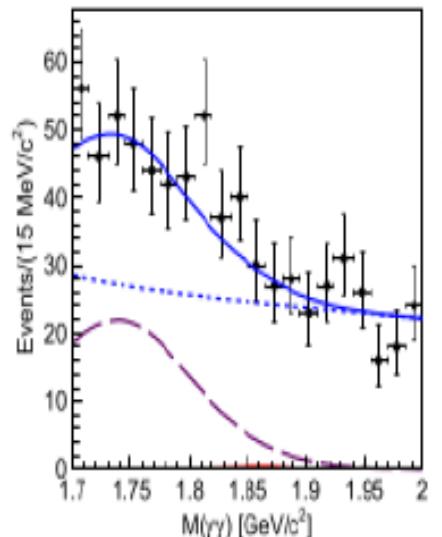
D $\rightarrow \gamma\gamma$ at Belle

[PRD 93 (2016) 051102]

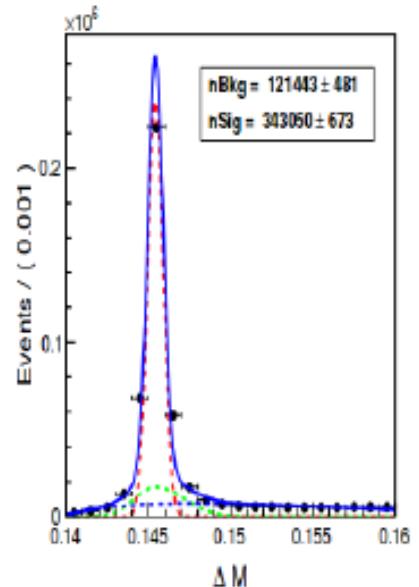
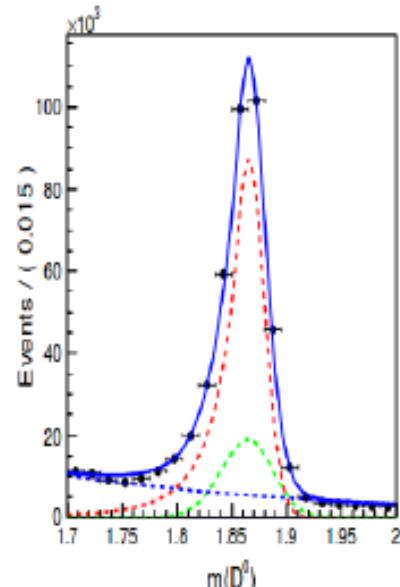


- Standard Model : $\mathcal{B}(D^0 \rightarrow \gamma\gamma) \sim 10^{-8}$
- MSSM with gluino exchange : $\mathcal{B}(D^0 \rightarrow \gamma\gamma) \sim 10^{-6}$
- Peaking backgrounds :
 - $D^0 \rightarrow hh'$ ($h,h' = \pi^0, \eta, K_{S,L}$) with unresolved $\gamma\gamma$
 - Use π^0 veto and e.m. cluster shape in calorimeter (E_9/E_{25})
- Signal extraction :
 - 2D fit to $m(\gamma\gamma)$, Δm



$D^{\circ} \rightarrow \gamma\gamma$
 $D \rightarrow \gamma\gamma$: Signal mode


- Efficiency = 7.3%
- Signal Yield = 4 ± 15

 $D \rightarrow K_S^0 \pi^0$: Normalization mode


- Efficiency = 7.2%
- Signal Yield = 343050 ± 673

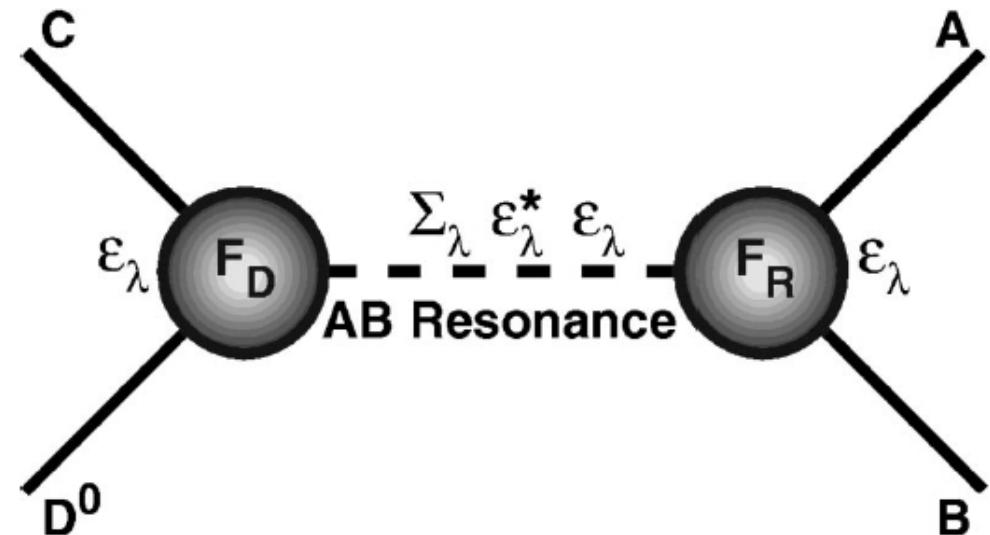
$$\mathcal{B}(D^{\circ} \rightarrow \gamma\gamma) < 8.5 \times 10^{-7} \text{ @ 90% CL}$$



Experiment	Luminosity	\mathcal{B} UL at 90% C.L.	References
CLEO	13.8 fb^{-1}	2.9×10^{-5}	PRL 90 (2003) 101801
BaBar	470.5 fb^{-1}	2.2×10^{-6}	PRD 85 (2012) 091107
BESIII	2.92 fb^{-1}	3.8×10^{-6}	PRD 91 (2015) 112015

The Dalitz Model

$$d\Gamma = \frac{|\mathcal{M}|^2}{256\pi^3 M_D^3} dM_{12}^2 dM_{23}^2$$



$$\sum_\lambda \epsilon_\lambda^{\mu*} \epsilon_\lambda^\nu$$

$$\mathcal{M} = F_D (P_{D^0} + P_C)_\mu \frac{1}{M_r^2 - M_{AB}^2 - i M_r \Gamma_{AB}} (P_A - P_B)_\nu F_r$$

$$\sum_\lambda \epsilon_\lambda^{\mu*} \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{P_{AB}^\mu P_{AB}^\nu}{M_{AB}^2}$$

$$\Gamma_{AB} = \Gamma_r \left(\frac{p_{AB}}{p_r} \right)^{2J+1} \left(\frac{M_r}{M_{AB}} \right) F_r^2 .$$

F_r, F_D : form factors, use Blatt-Weisskopf penetration factors

J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

Babar : systematic uncertainties

Time-Integrated Dalitz plot/3

$$A(s_+, s_-) = \sum_i c_i \frac{T(s)}{M_r - s - i M_r \Gamma(s)} F(s)$$

T is a tensor structure depending on spin

$$\Gamma(s) = \Gamma \left(\frac{q(s)}{q(M_r)} \right)^{2l+1} \left(\frac{M_r}{\sqrt{s}} \right) F^2$$

F is the Blatt-Weisskopf barrier factor

$$F_0 = 1, F_1 = \sqrt{\frac{1+R^2 q^2(M_r)}{1+R^2 q^2(s)}}, F_2 = \dots$$

Masses and widths fixed to the PDG value



State	J^{PC}	Resonance parameters		Fit to data results		
		Mass (MeV)	Width (MeV)	Magnitude	Phase (°)	Fraction f_r (%)
$\rho(770)^+$	1 ⁻⁻	775.8	150.3	1	0	66.4 ± 0.5
$\rho(770)^0$	1 ⁻⁻	775.8	150.3	0.55 ± 0.01	16.1 ± 0.4	23.9 ± 0.3
$\rho(770)^-$	1 ⁻⁻	775.8	150.3	0.73 ± 0.01	-1.6 ± 0.5	35.6 ± 0.4
$\rho(1450)^+$	1 ⁻⁻	1465	400	0.55 ± 0.07	-7.7 ± 8.2	1.1 ± 0.3
$\rho(1450)^0$	1 ⁻⁻	1465	400	0.19 ± 0.07	-70.4 ± 15.9	0.1 ± 0.1
$\rho(1450)^-$	1 ⁻⁻	1465	400	0.53 ± 0.06	8.2 ± 6.7	1.0 ± 0.2
$\rho(1700)^+$	1 ⁻⁻	1720	250	0.91 ± 0.15	-23.3 ± 10.3	1.5 ± 0.5
$\rho(1700)^0$	1 ⁻⁻	1720	250	0.60 ± 0.13	-56.3 ± 16.0	0.7 ± 0.3
$\rho(1700)^-$	1 ⁻⁻	1720	250	0.98 ± 0.17	78.9 ± 8.5	1.7 ± 0.6
$f_0(980)$	0 ⁺⁺	980	44	0.06 ± 0.01	-58.8 ± 2.9	0.3 ± 0.1
$f_0(1370)$	0 ⁺⁺	1434	173	0.20 ± 0.03	-19.6 ± 9.5	0.3 ± 0.1
$f_0(1500)$	0 ⁺⁺	1507	109	0.18 ± 0.02	7.4 ± 7.4	0.3 ± 0.1
$f_0(1710)$	0 ⁺⁺	1714	140	0.40 ± 0.08	42.9 ± 8.8	0.3 ± 0.1
$f_2(1270)$	2 ⁺⁺	1275.4	185.1	0.25 ± 0.01	8.8 ± 2.6	0.9 ± 0.1
$f_0(500)$	0 ⁺⁺	500	400	0.26 ± 0.01	-4.1 ± 3.7	0.9 ± 0.1
NR				0.43 ± 0.07	-22.1 ± 11.7	0.4 ± 0.1

To estimate systematics:

- We vary the radii R from 1.5 to 5 GeV $^{-1}$
- We remove a resonance from the fit, and if $\Delta\chi^2 < 100$, we estimate the variation in x, y
- We also allow the mass and width of $f_0(500)$ to float

To estimate any possible bias, the same fit is performed to MC samples with given

$$x = \pm 1\%, y = \pm 1\%$$

The mean bias is
 $\Delta x = 0.58\%, \Delta y = -0.05\%$

Dominant sources of systematics are:

- **Amplitude-model variations**, estimated removing the least relevant resonances
- **Combinatorial DP distribution**, when the MC is used instead of data
- **Different decay time windows**, and **number of σ_t ranges**
- **Fit bias correction**, taken as half of the bias measured from MC
- **Effect of SVT misalignment**, estimated creating MC signal samples with deliberately-wrong alignment files



Source	x [%]	y [%]
“Lucky” false slow pion fraction	0.01	0.01
Time resolution dependence on reconstructed D^0 mass	0.03	0.02
Amplitude-model variations	0.31	0.12
Resonance radius	0.02	0.10
DP efficiency parametrization	0.03	0.03
DP normalization granularity	0.03	0.04
Background DP distribution	0.21	0.11
Decay time window	0.18	0.19
σ_t cutoff	0.01	0.01
Number of σ_t ranges	0.11	0.26
σ_t parametrization	0.05	0.03
Background-model MC time distribution parameters	0.06	0.11
Fit bias correction	0.29	0.02
SVT misalignment	0.20	0.23
Total	0.56	0.46

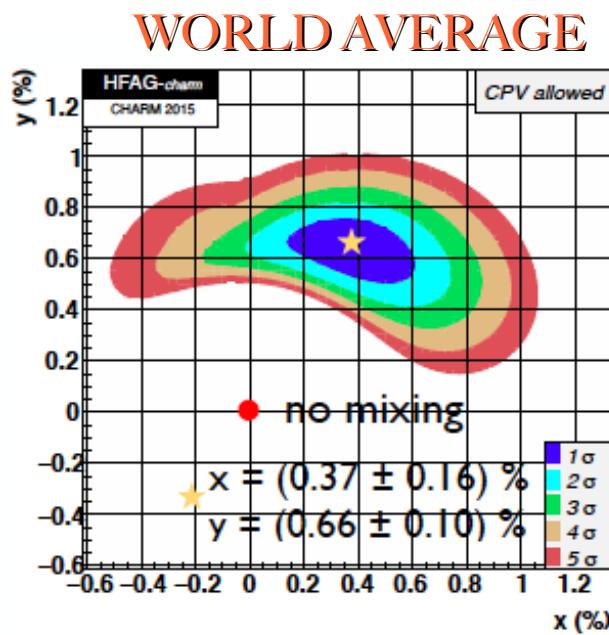
Results :

$$x = (1.5 \pm 1.2 \pm 0.6)\%$$

$$y = (0.2 \pm 0.9 \pm 0.5)\%$$

$$\tau(D^\circ) = 410.2 \pm 3.8 \text{ fs}$$

BABAR Collaboration arXiv 1604.00857



State	Fit to data results		
	Magnitude	Phase (°)	Fraction f_r (%)
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