Hybrid and Exotic Spectroscopy with Charm Quarks in Lattice QCD

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Plethora of unexpected charmonium-like ($X$, $Y$, $Z$) states discovered experimentally.

- Masses and widths of some $D_s$ states significantly lower than those expected from quark model.
- Tetraquarks? Molecules? Cusps? Hybrids?
- First principles calculations using lattice QCD to understand these states.

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- Finite volume errors $\sim e^{-M_\pi L}$. 
Extracting Observables

- Construct a creation operator $O_i^\dagger(t)$ that has the same quantum numbers as the states we’re interested in. E.g. for the $\eta_c(0^{-+})$, $O^\dagger = \bar{c}\gamma^5 c$. 

Computing the two-point correlation function, 

$$C_{ij}(t) = \langle 0 | O_i(t) O_{\dagger j}(0) | 0 \rangle = \sum_n \langle 0 | e^{iHt} O_i(0) e^{-iHt} | n \rangle \langle n | \frac{1}{2}E_n \quad O_{\dagger j}(0) | 0 \rangle.$$ 

The spectrum is contained in $C_{ij}(t)$. In practice, we use many $O_i^\dagger \sim \bar{c}\Gamma \rightarrow D \cdots \rightarrow D c$. 

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Charmonium Spectrum at $M_\pi \sim 240$ MeV
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$S$-wave
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$P$-Wave

$M - M_{\eta_c}$ (MeV)

1++ 0++ 1-- 2-- 2++ 2++ 3-- 4-- 4++ 0-- 1-- 1++ 2++ 3-- 3++ 4++
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Hybrid Mesons

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$$\{0^{-+}; 1^{--}\} \rightarrow \{1^{--}; 0^{-+}, 1^{-+}, 2^{-+}\}$$

$q\bar{q} \ L = 1$

$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \rightarrow \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{--}(3), 2^{+-}(2), 3^{+-}\}$$
$D_s$ Spectrum at $M_\pi \sim 240$ MeV

- S-Wave
- P-Wave
- F-Wave
- D-Wave
- G-Wave
$D$ Spectrum at $M_\pi \sim 240$ MeV
Charmonium Comparison

![Graph showing the comparison of Charmonium states. The graph plots the mass difference $M - M_{\eta_c}$ (MeV) against various quantum numbers. Different colors and markers represent different states such as $D\bar{D}$, $D^0\bar{D}$, and $\eta_c\pi\pi$. The mass resolution is indicated for two values: 240 and 400 MeV.](image-url)
$D_s$ Comparison

\[ M - M_{\eta_c}/2 \text{ (MeV)} \]

- $M_{\pi}$ / MeV
  - 240
  - 400

- States:
  - $0^-$
  - $1^-$
  - $2^-$
  - $3^-$
  - $4^-$
  - $0^+$
  - $1^+$
  - $2^+$
  - $3^+$
  - $4^+$

- $DK$
$D$ Comparison

$M - M_{\eta_c}/2$ (MeV)

$M_\pi / \text{MeV}$

240  400

$D^{*+}$

$D_{0^-}$
Summary

- At $M_{\pi} \sim 240$ MeV, we find states that are consistent with the $n^{2S+1}L_J$ pattern.

- Additionally, we also identify states that are consistent with a quark-antiquark combination coupled to a $1^{-+}$ gluonic excitation.

- Comparing with previous results at $M_{\pi} \sim 400$ MeV, we find minor differences and conclude that the overall qualitative structure is the same.

- This work has set the foundation for future scattering calculations.
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Next Steps?

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- Need to include multibody operators. Example of spectrum using meson-meson and tetraquark operators in $c\bar{c} 1^{++}$ isospin-1 channel relevant for charged $Z$ states.
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Many interesting channels, $DK, D\bar{D}, D\bar{D}^*, \ldots$ to understand $D_{s0}^*(2317), X(3872), \ldots$ etc.
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Lattice QCD will play a crucial role in understanding current and future exotic states.
Thank you for listening!
The Variational Method

\[ C_{ij}(t) = \langle O_i(t)O_j^\dagger(0) \rangle = \sum_n \frac{Z_i^n Z_j^{n\ast}}{2E_n} e^{-E_n t} \]

Solve the generalised eigenvalue problem

\[ C_{ij}(t)v^n_j = \lambda^n(t)C_{ij}(t_0)v^n_j. \]

Then the eigenvalue is related to the mass by

\[ \lambda^n(t) = e^{-(t-t_0)E_n} + O(e^{-(t-t_0)E_m}) \]

and the overlap is related to the eigenvector by

\[ Z_i^n = \sqrt{2E_n}e^{E_nt_0/2}v_j^{n\ast}C_{ji}(t_0). \]
Constructing the Charmonium Operator

1. Construct a fermion bilinear with a gamma matrix $\Gamma$ and a number of derivatives $D$ all with the same quantum numbers.

$$O(t) = \bar{c}\Gamma D_1 D_2 \ldots c.$$ 

2. Couple to a continuum angular momentum irrep using Clebsch-Gordan coefficients. For example, for one gamma matrix and one derivative,

$$O^{J, M}(t) = \sum_{m_1, m_2} \langle J_1, m_1; J_2, m_2 | J, M \rangle \bar{c}\Gamma_{m_1} D_{m_2} c.$$ 

3. Project onto a lattice irrep using ‘subduction’ coefficients.

$$O_{\Lambda, \lambda}^{[J]}(t) = \sum_M S_{\Lambda, \lambda}^{J, M} O^{J, M}(t).$$
Spin Identification

Overlaps for first 5 states in $T_{2}^{++}$.
$J=2$ (green), $J=3$ (blue), $J=4$ (yellow)
Hybrid Identification

Look for high overlap onto operators proportional to \([D_1, D_2]\).

Overlaps for first 4 states in 0^{−+}.

J=0 (black), Hybrid J=0 (grey), J=4 (yellow)