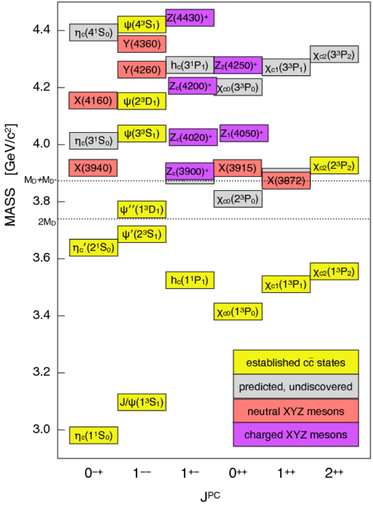
Hybrid and Exotic Spectroscopy with Charm Quarks in Lattice QCD

Gavin Cheung
for the Hadron Spectrum Collaboration

DAMTP, University of Cambridge

7 September 2016

Introduction



S. Olsen, arxiv:1511.01589

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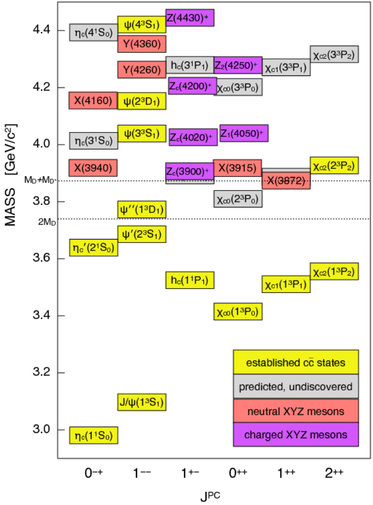

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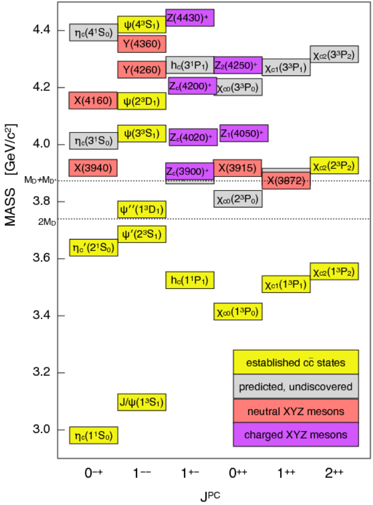

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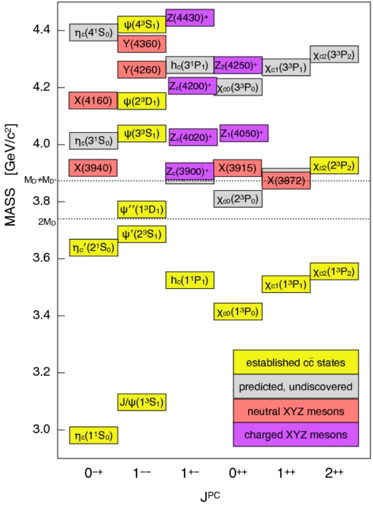

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- Tetraquarks? Molecules? Cusps? Hybrids?
- First principles calculations using lattice QCD to understand these states.

Calculation Details

- Determine spectrum of hidden and open charmed states including excitations and states with intrinsic gluonic component on the lattice at pion mass $M_\pi \sim 240$ MeV.

  [G.C., C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, D. Tims, to appear]

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$24^3 \times 128$	391	553
$32^3 \times 256$	236	484

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- Finite volume errors $\sim e^{-M_\pi L}$.

Extracting Observables

- Construct a creation operator $O_i^\dagger(t)$ that has the same quantum numbers as the states we're interested in. E.g. for the $\eta_c(0^{-+})$, $O^\dagger = \bar{c}\gamma^5 c$.

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- Computing the two-point correlation function,

$$
\begin{aligned}
C_{ij}(t) &= \langle 0|O_i(t)O_j^\dagger(0)|0\rangle \\
&= \sum_n \langle 0|e^{Ht}O_i(0)e^{-Ht}\frac{|n\rangle\langle n|}{2E_n}O_j^\dagger(0)|0\rangle \\
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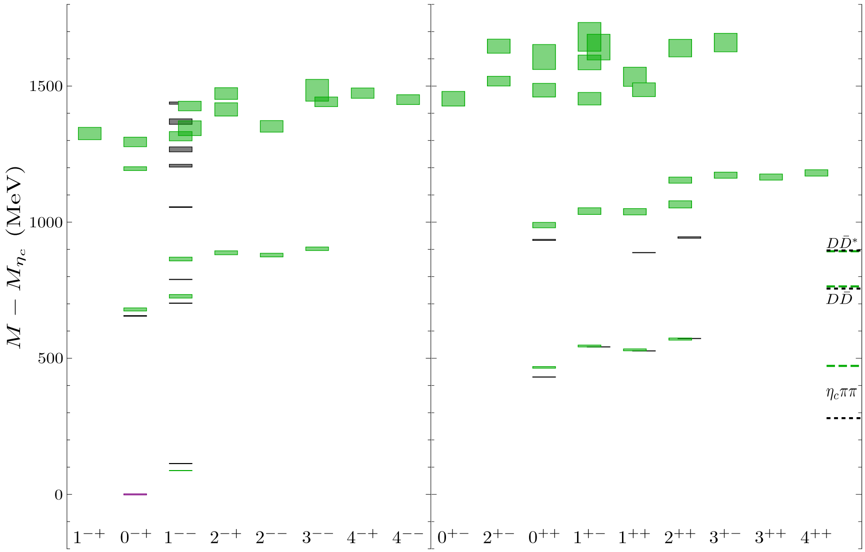
- The spectrum is contained in $C_{ij}(t)$. In practice, we use many $O_i^\dagger \sim \bar{c}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} c$.

Charmonium Spectrum at $M_\pi \sim 240$ MeV



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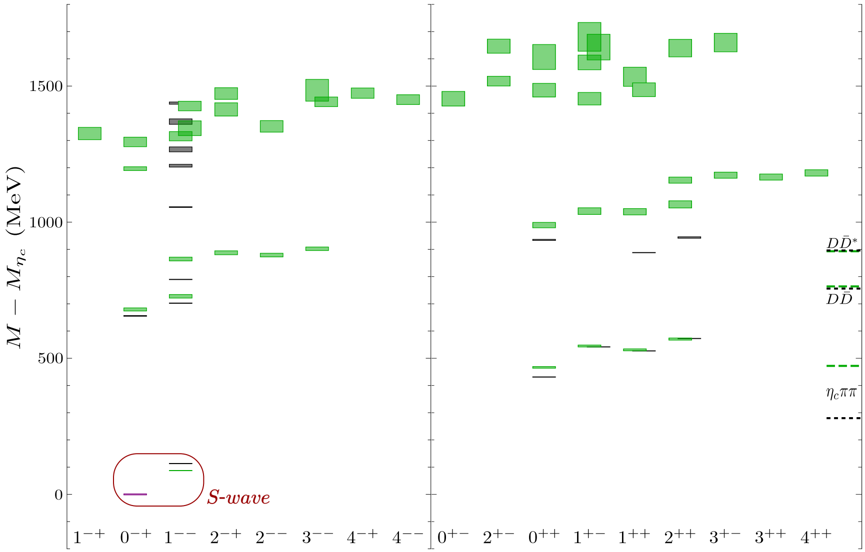

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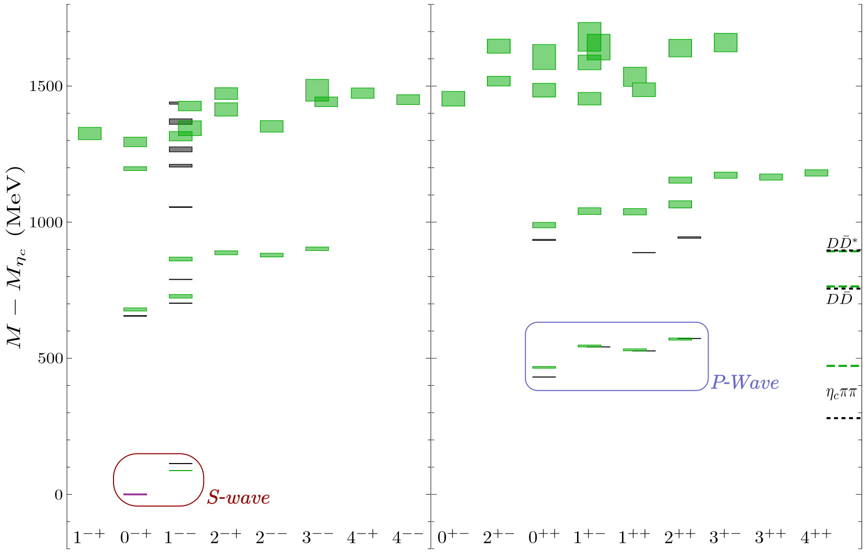

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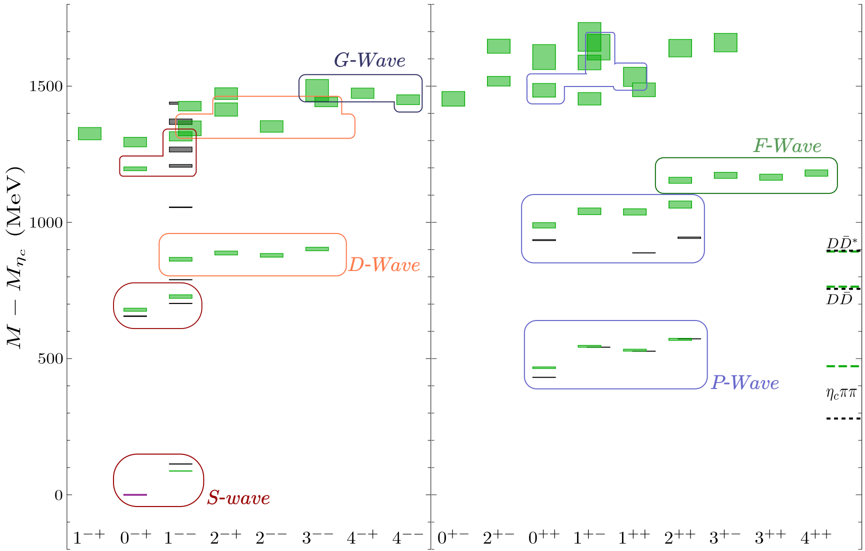

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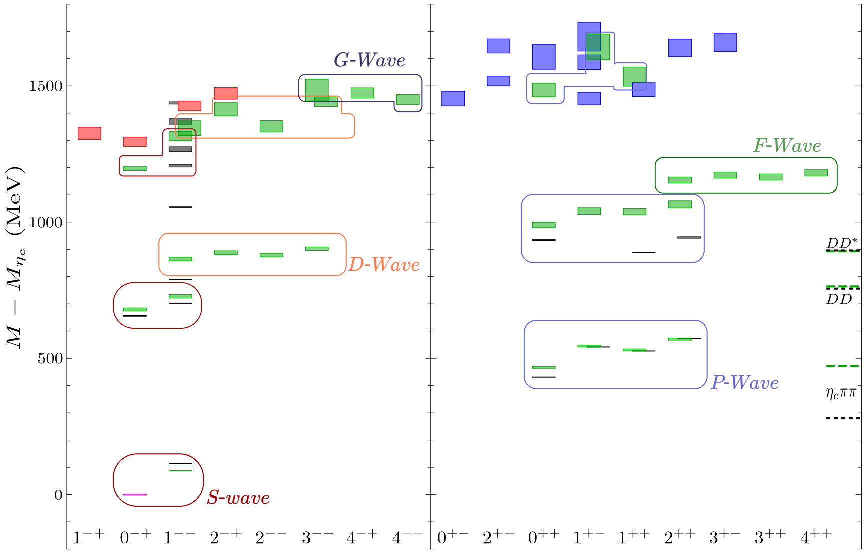
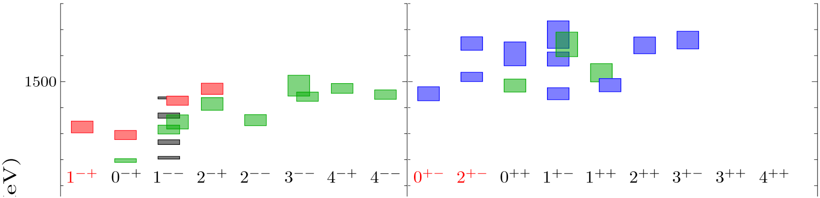


Hybrid Mesons



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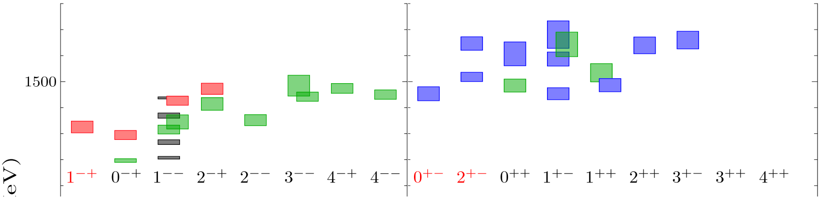

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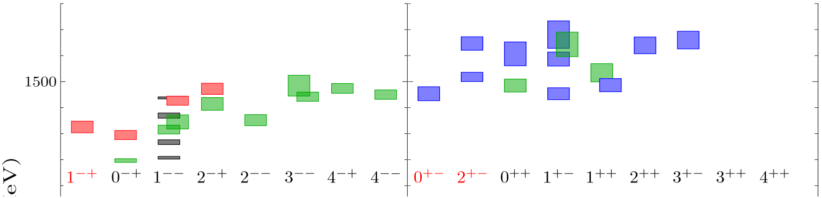

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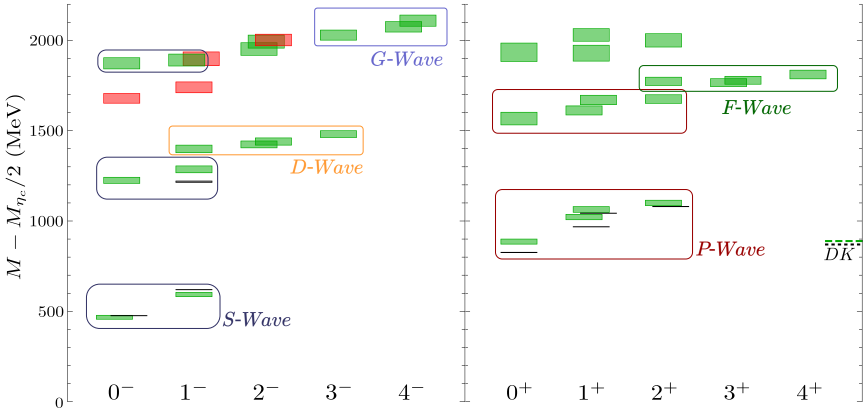
$q\bar{q}$ $L = 1$

$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \rightarrow \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{+-}(3), 2^{+-}(2), 3^{+-}\}$$

D_s Spectrum at $M_\pi \sim 240$ MeV



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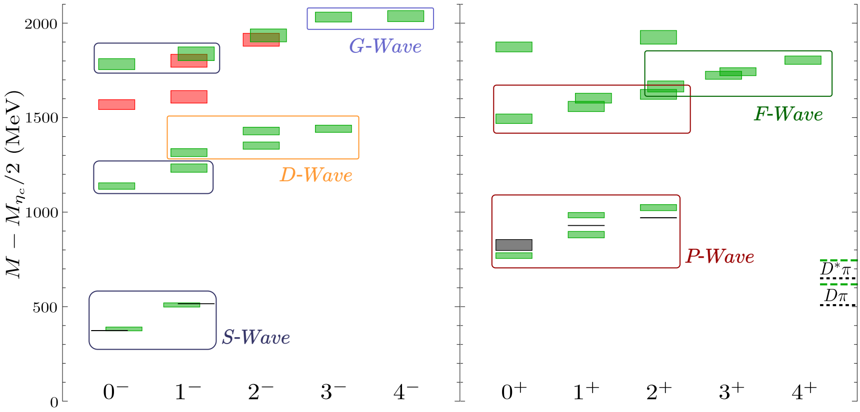
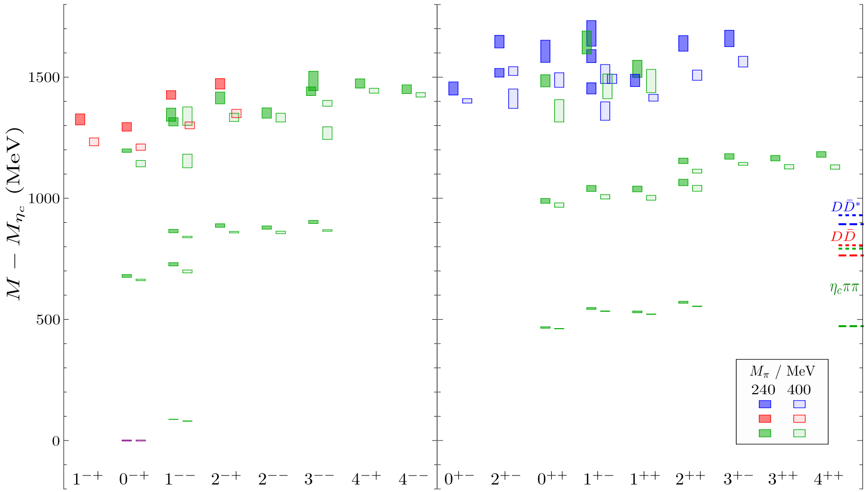


Charmonium Comparison



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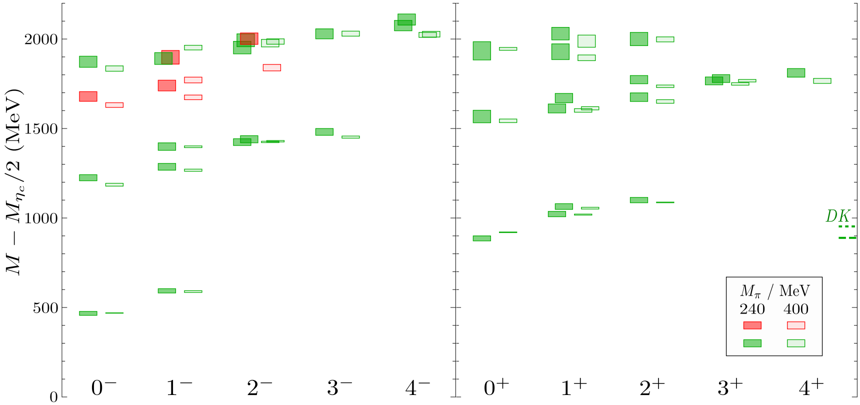

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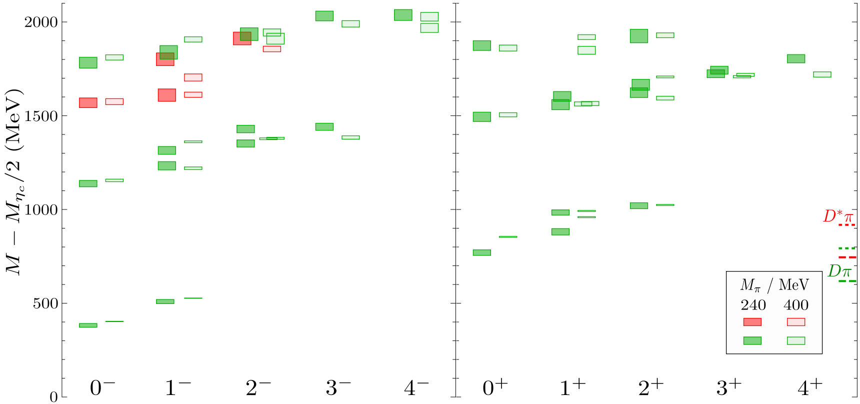

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- Additionally, we also identify states that are consistent with a quark-antiquark combination coupled to a 1^{+-} gluonic excitation.
- Comparing with previous results at $M_\pi \sim 400$ MeV, we find minor differences and conclude that the overall qualitative structure is the same.
- This work has set the foundation for future scattering calculations.

Next Steps?

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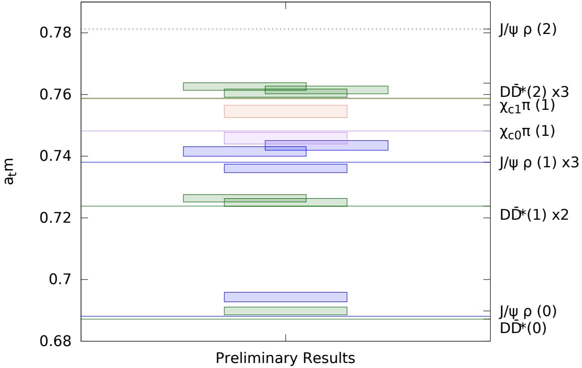
- Need to include multibody operators. Example of spectrum using meson-meson and tetraquark operators in $c\bar{c}$ 1^{++} isospin-1 channel relevant for charged Z states.



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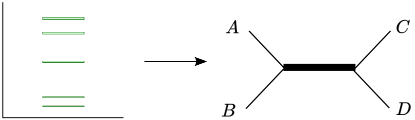

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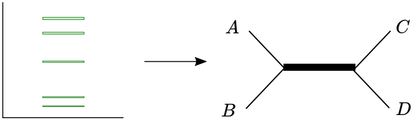

- Lüscher formalism relates discrete finite volume spectra to infinite volume S-matrix.
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  [Graham Moir 11:40am Weds. [arXiv:1607.07093]]

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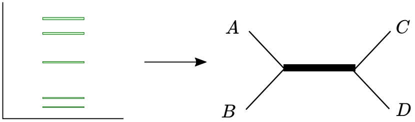

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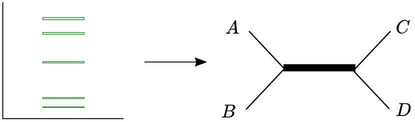

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- Radiative transitions between charmonium states can also be calculated on the lattice and are underway.
- Lattice QCD will play a crucial role in understanding current and future exotic states.

Thank you for listening!

The Variational Method

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n t}$$

Solve the generalised eigenvalue problem

$$C_{ij}(t) v_j^n = \lambda^n(t) C_{ij}(t_0) v_j^n.$$

Then the eigenvalue is related to the mass by

$$\lambda^n(t) = e^{-(t-t_0)E_n} + \mathcal{O}(e^{-(t-t_0)E_m})$$

and the overlap is related to the eigenvector by

$$Z_i^n = \sqrt{2E_n} e^{E_n t_0/2} v_j^{n*} C_{ji}(t_0).$$

Constructing the Charmonium Operator

1. Construct a fermion bilinear with a gamma matrix Γ and a number of derivatives D all with the same quantum numbers.

$$O(t) = \bar{c}\Gamma D_1 D_2 \ldots c.$$

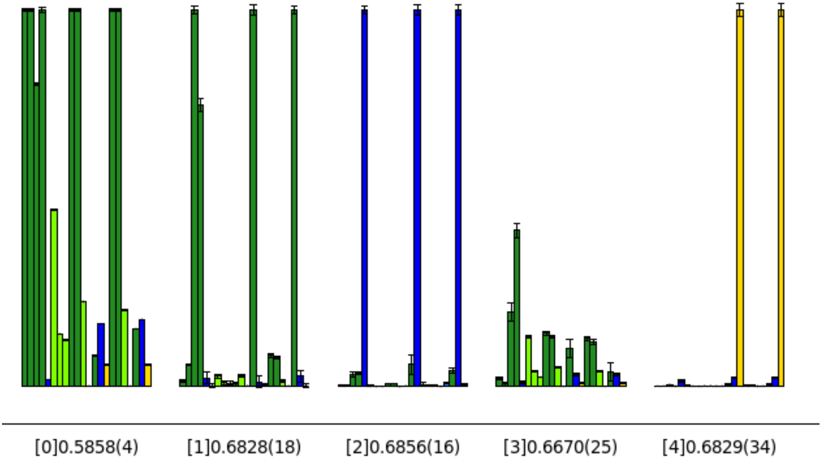
2. Couple to a continuum angular momentum irrep using Clebsch-Gordan coefficients. For example, for one gamma matrix and one derivative,

$$O^{J,M}(t) = \sum_{m_1,m_2} \langle J_1, m_1; J_2, m_2 | J, M \rangle \bar{c}\Gamma_{m_1} D_{m_2} c.$$

3. Project onto a lattice irrep using 'subduction' coefficients.

$$O^{[J]}_{\Lambda,\lambda}(t) = \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M}(t).$$

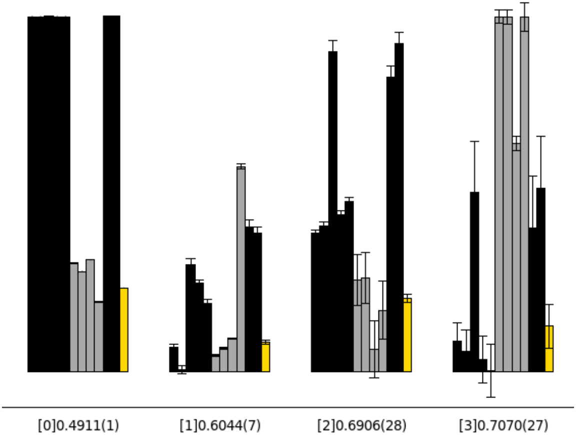
Spin Identification



Overlaps for first 5 states in T_2^{++}.
J=2 (green), J=3 (blue), J=4 (yellow)

Hybrid Identification

Look for high overlap onto operators proportional to $[D_1, D_2]$.



Overlaps for first 4 states in 0^{-+}.
J=0 (black), Hybrid J=0 (grey), J=4 (yellow)