

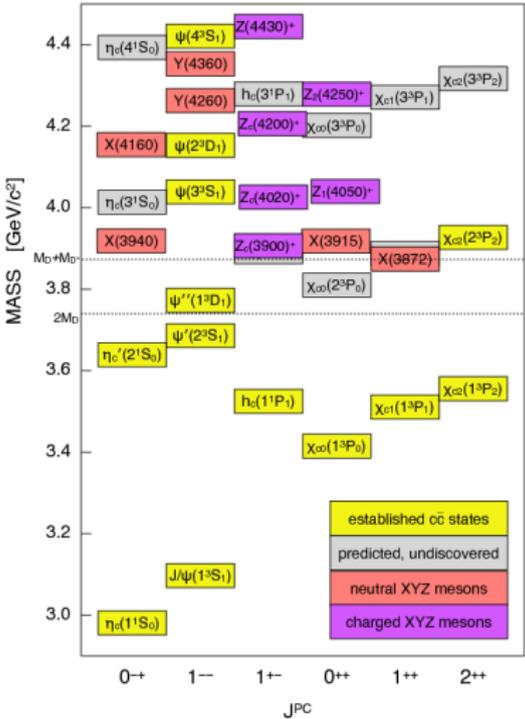
Hybrid and Exotic Spectroscopy with Charm Quarks in Lattice QCD

Gavin Cheung
for the Hadron Spectrum Collaboration

DAMTP, University of Cambridge

7 September 2016

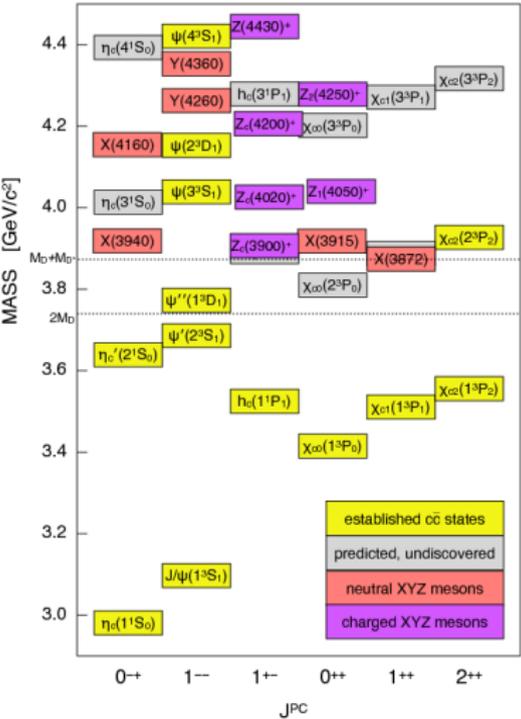
Introduction



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S. Olsen, arxiv:1511.01589

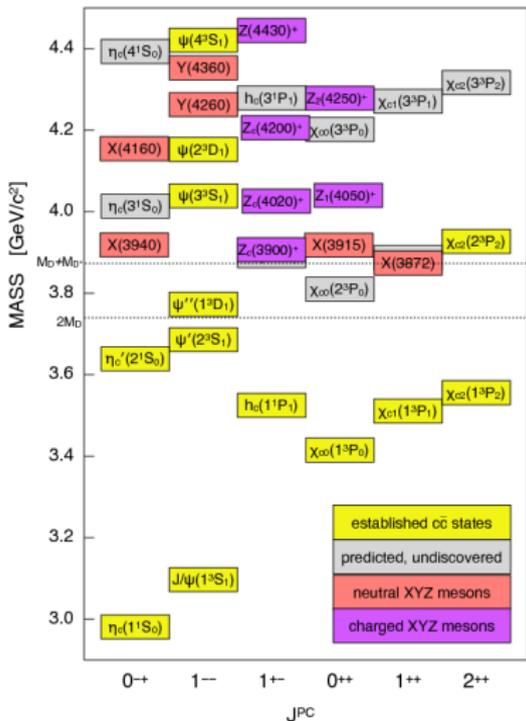
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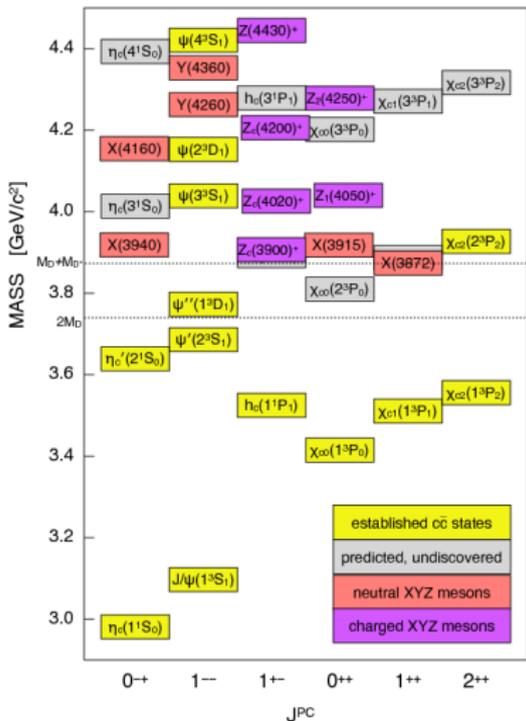
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- ▶ Tetraquarks? Molecules? Cusps? Hybrids?
- ▶ First principles calculations using lattice QCD to understand these states.

Calculation Details

- ▶ Determine spectrum of hidden and open charmed states including excitations and states with intrinsic gluonic component on the lattice at pion mass $M_\pi \sim 240$ MeV.

[G.C., C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, D. Tims, to appear]

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Lattice Volume	M_π (MeV)	N_{cfgs}
$24^3 \times 128$	391	553
$32^3 \times 256$	236	484

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- ▶ Finite volume errors $\sim e^{-M_\pi L}$.

Extracting Observables

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$$\begin{aligned}C_{ij}(t) &= \langle 0 | O_i(t) O_j^\dagger(0) | 0 \rangle \\ &= \sum_n \langle 0 | e^{Ht} O_i(0) e^{-Ht} \frac{|n\rangle\langle n|}{2E_n} O_j^\dagger(0) | 0 \rangle \\ &= \sum_n \frac{1}{2E_n} e^{-E_n t} \langle 0 | O_i | n \rangle \langle n | O_j^\dagger | 0 \rangle.\end{aligned}$$

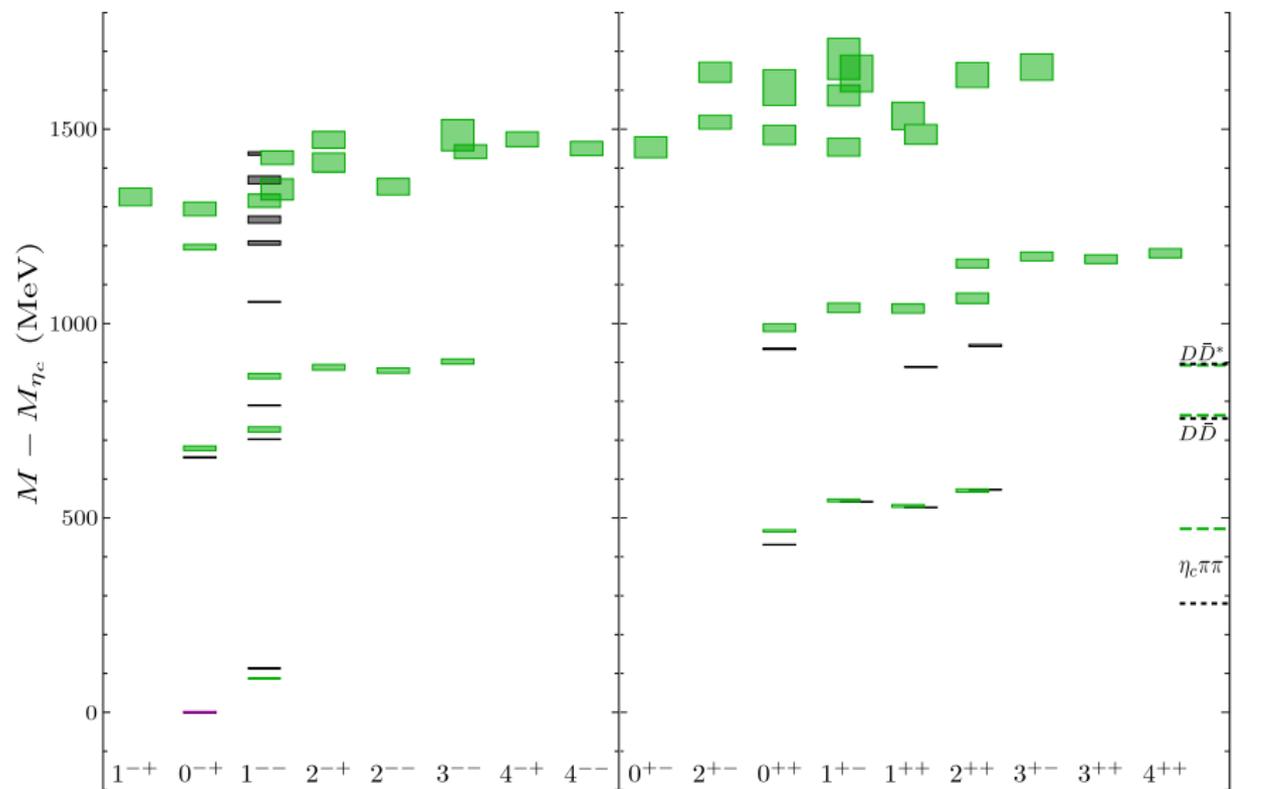
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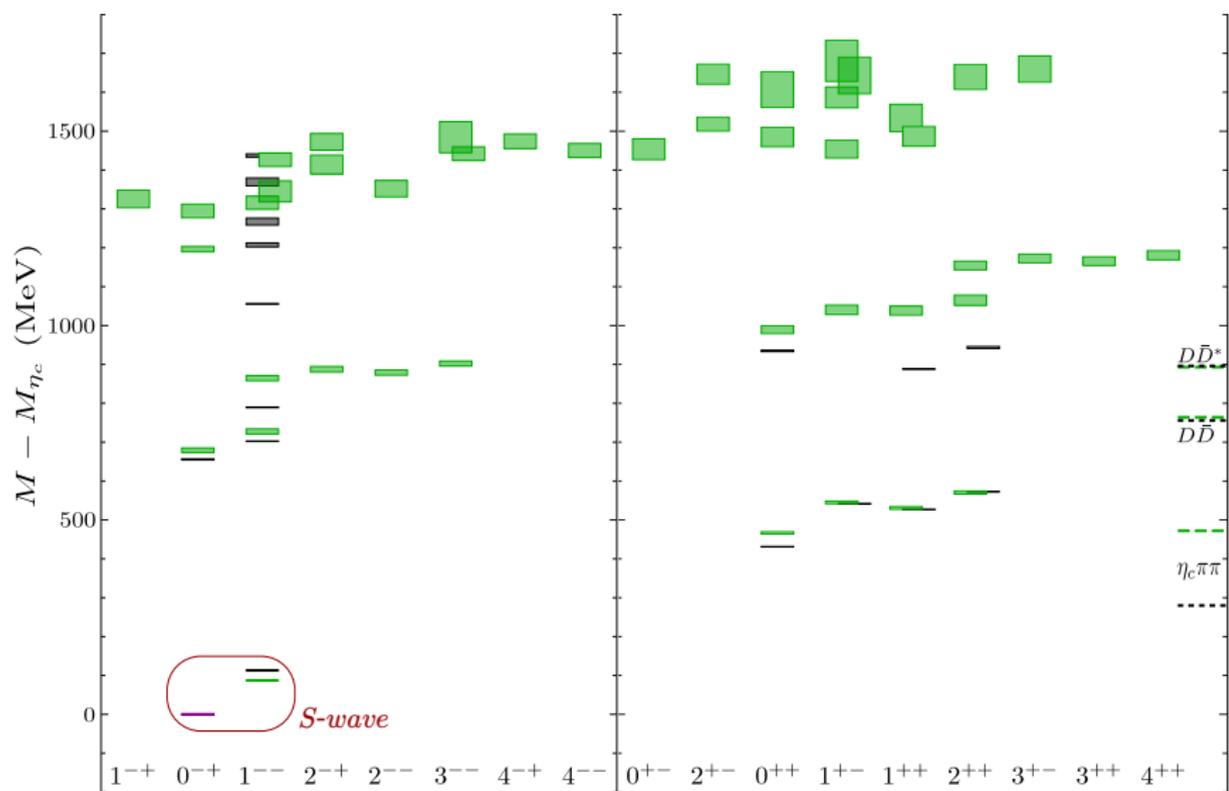
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- ▶ The spectrum is contained in $C_{ij}(t)$. In practice, we use many $O_i^\dagger \sim \bar{c} \Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} c$.

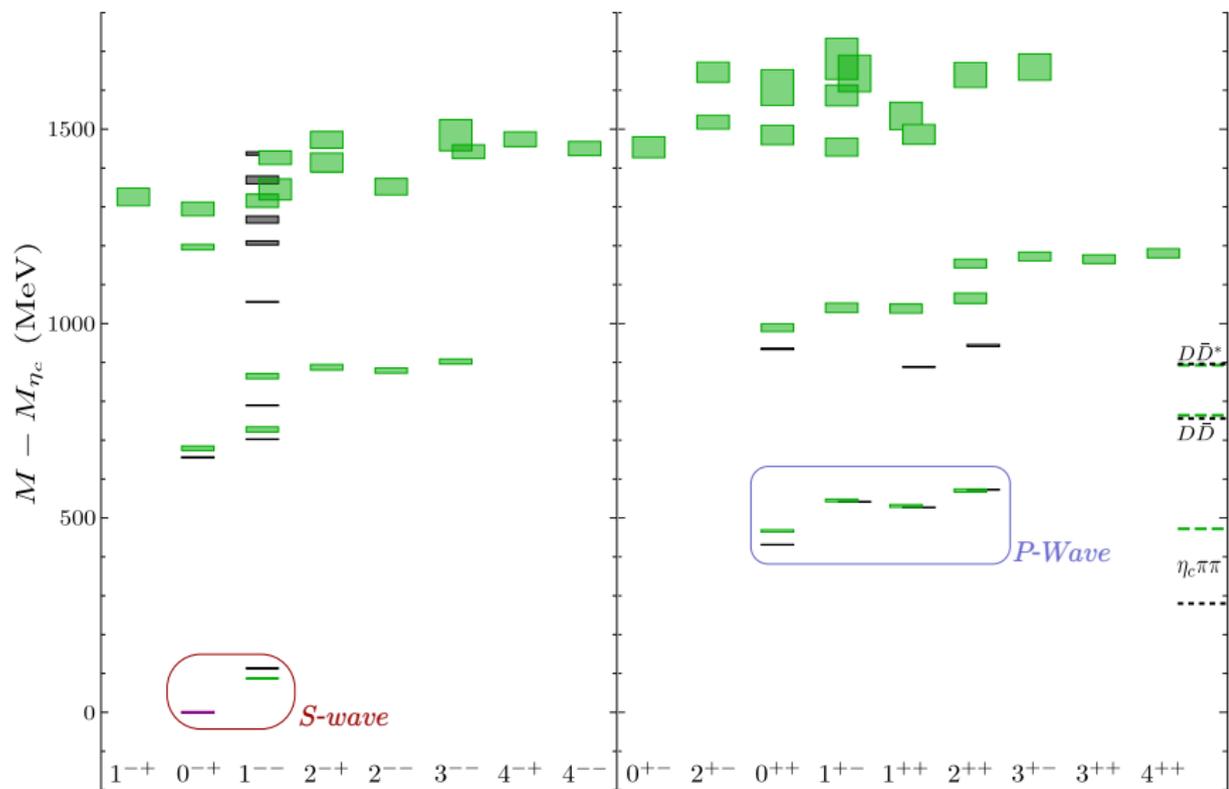
Charmonium Spectrum at $M_\pi \sim 240$ MeV



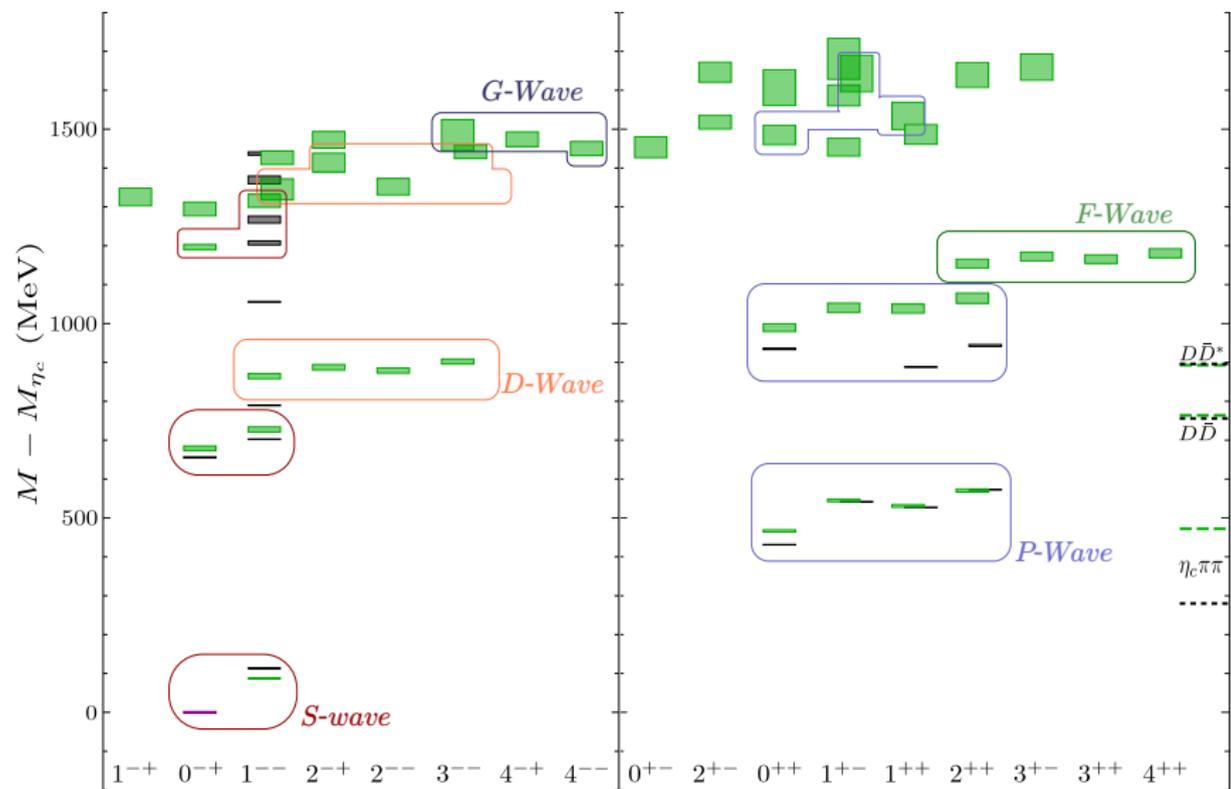
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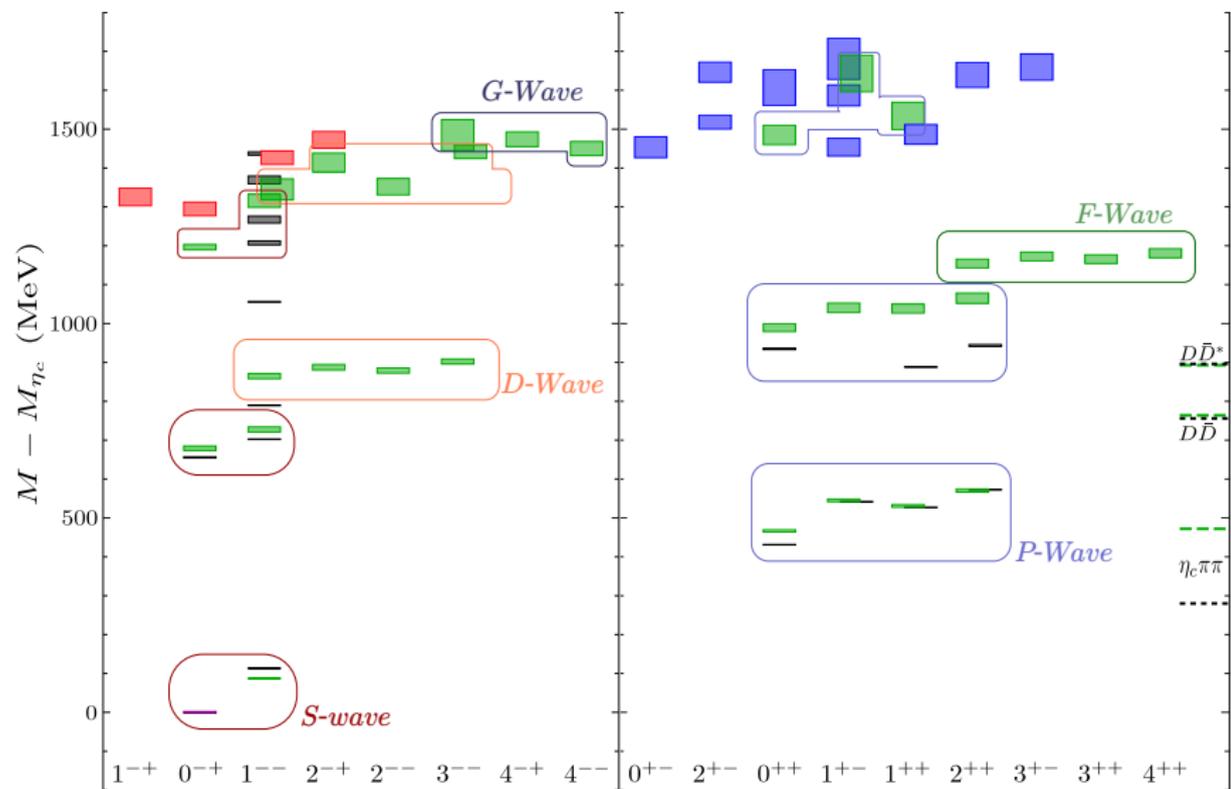
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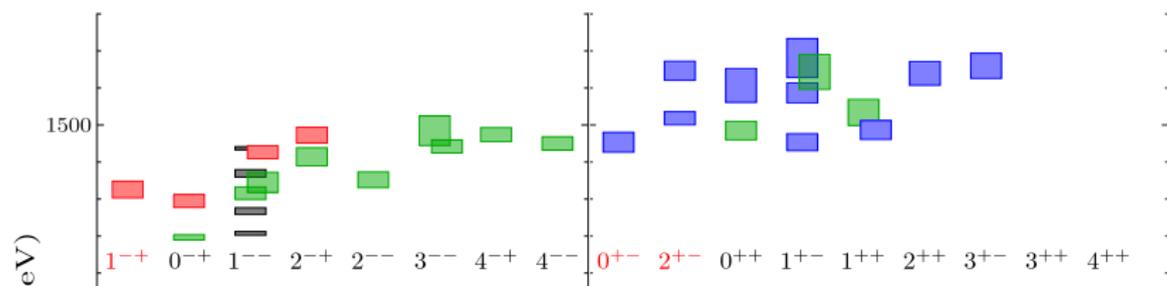
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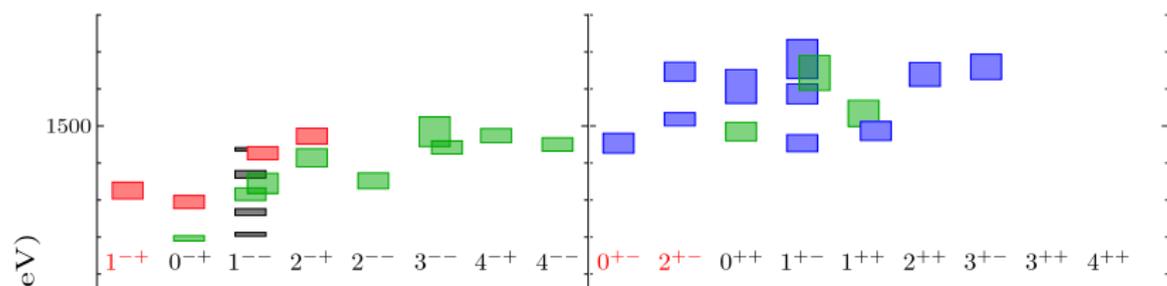


Hybrid Mesons



Consistent with adding an effective gluonic degree of freedom
 $J^{PC} = 1^{+-}$ to the quark model.

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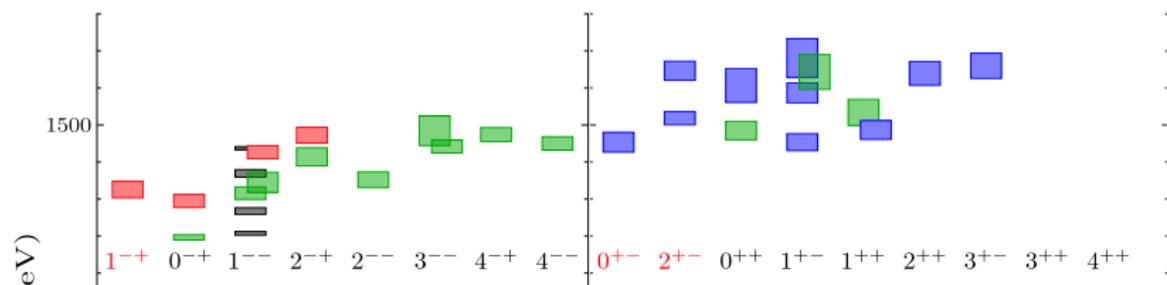


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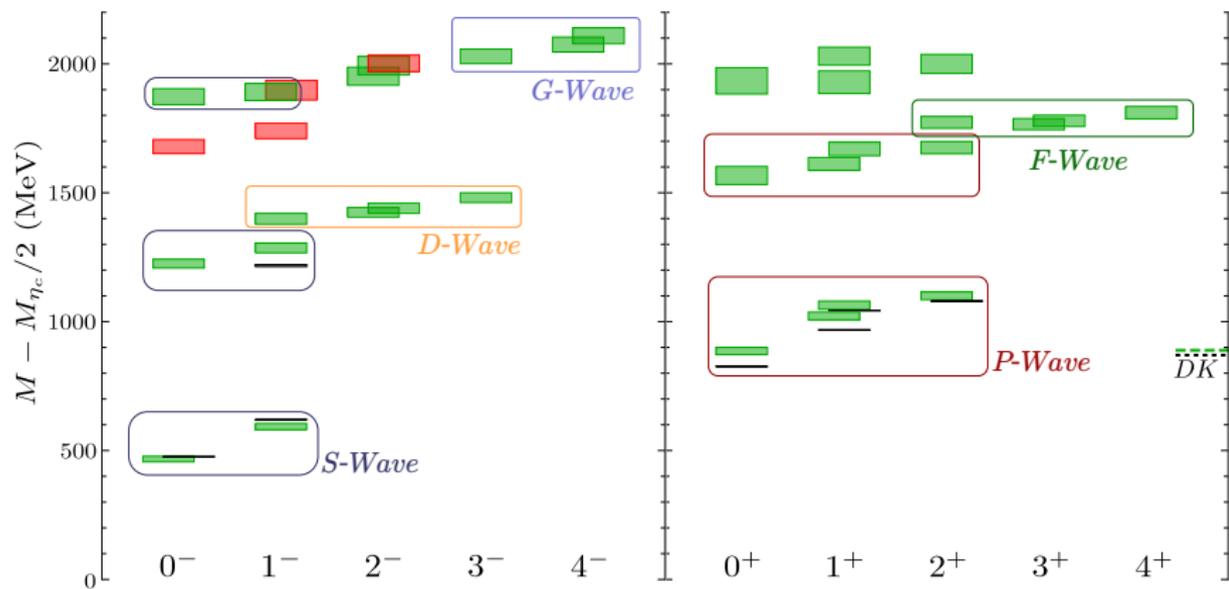
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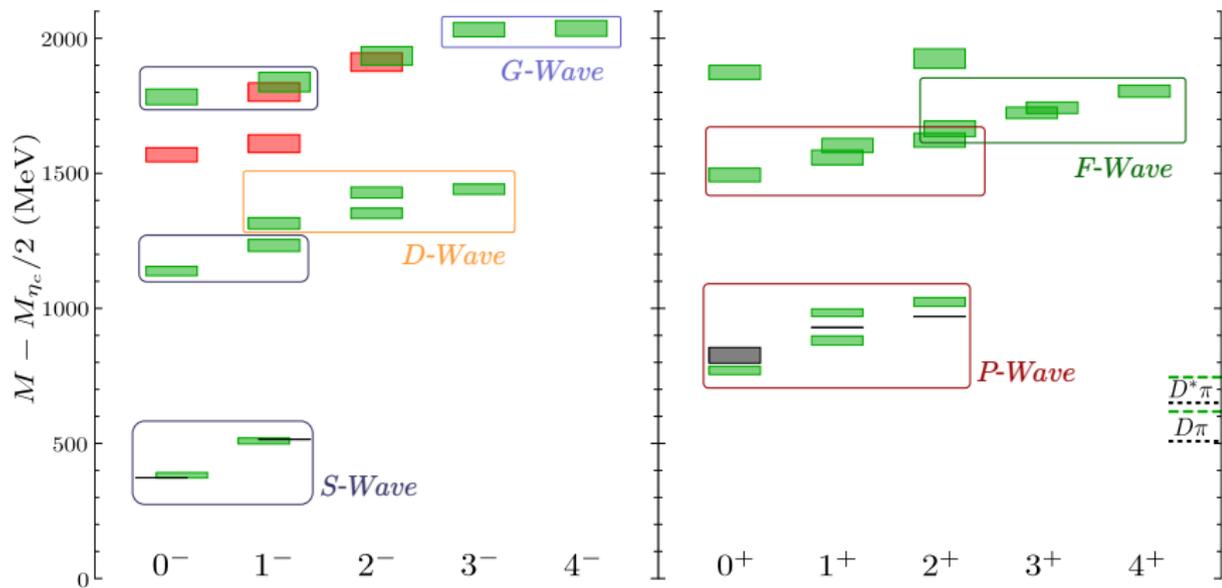
$q\bar{q}$ $L = 1$

$$\{1^{+-}; 0^{++}, 1^{++}, 2^{++}\} \rightarrow \{0^{++}, 1^{++}, 2^{++}; 0^{+-}, 1^{+-}(3), 2^{+-}(2), 3^{+-}\}$$

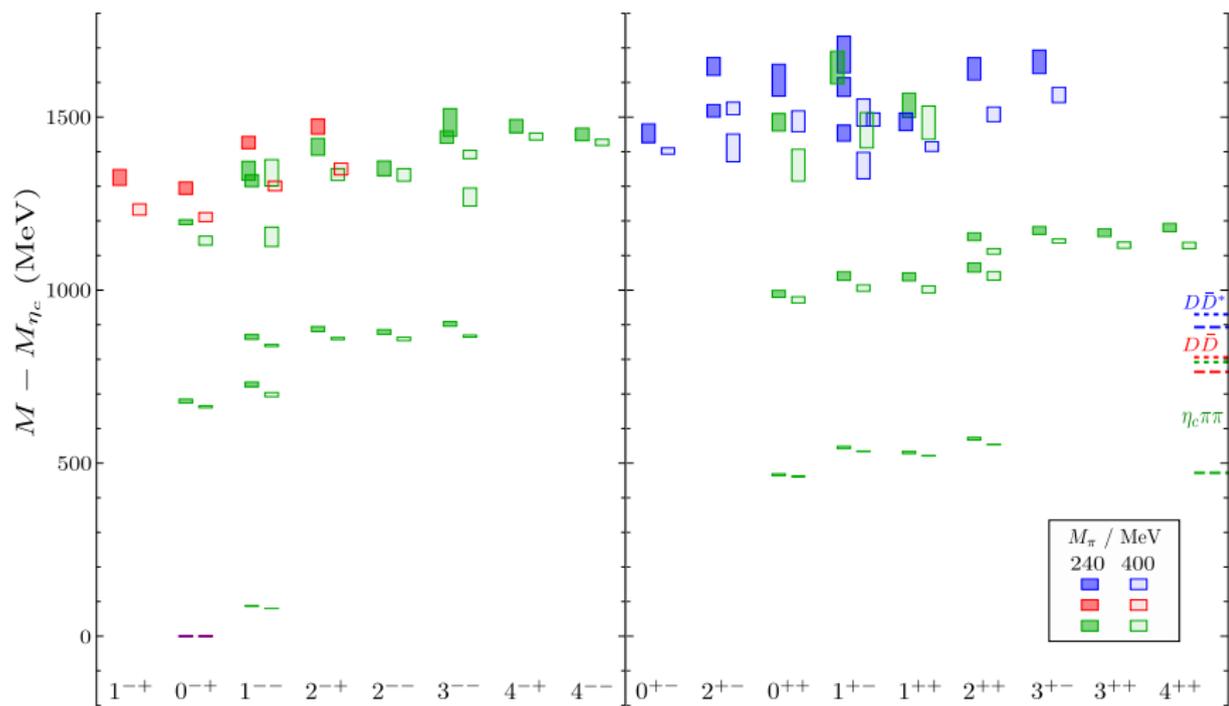
D_s Spectrum at $M_\pi \sim 240$ MeV



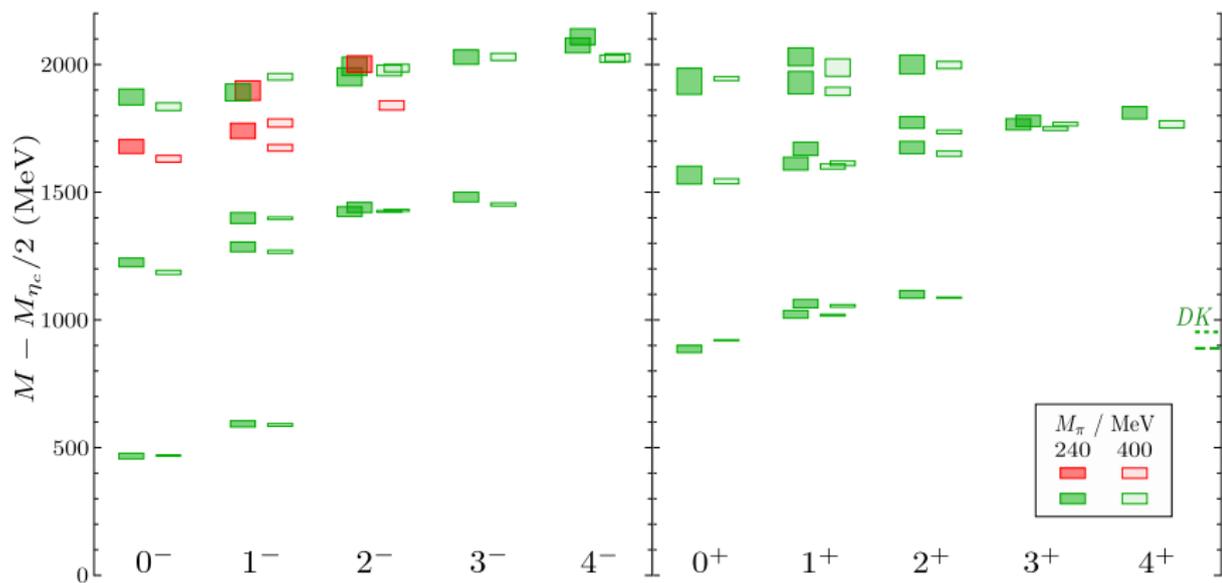
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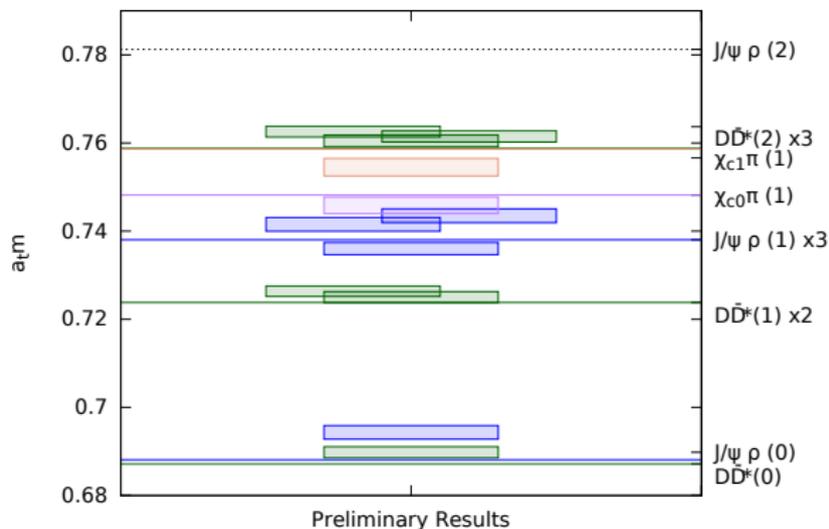
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- ▶ Comparing with previous results at $M_\pi \sim 400$ MeV, we find minor differences and conclude that the overall qualitative structure is the same.
- ▶ This work has set the foundation for future scattering calculations.

Next Steps?

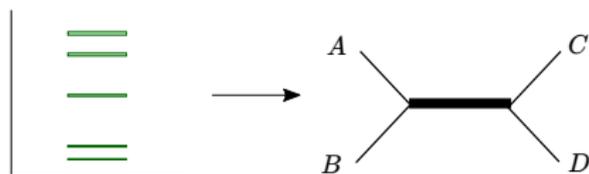
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- ▶ Need to include multibody operators. Example of spectrum using meson-meson and tetraquark operators in $c\bar{c} 1^{++}$ isospin-1 channel relevant for charged Z states.

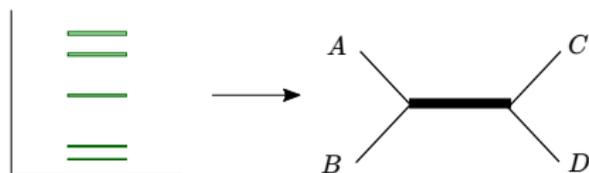


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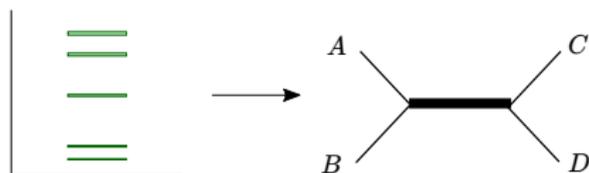


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- ▶ Recently performed coupled channel $D\pi, D\eta, D_s\bar{K}$ scattering analysis on $M_\pi = 391$ MeV lattices.

[Graham Moir 11:40am Weds. [arXiv:1607.07093]]

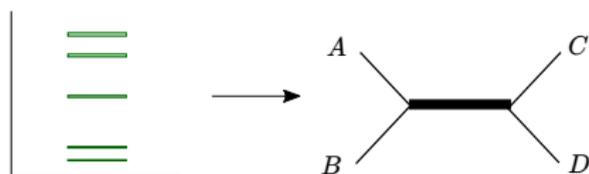
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Many interesting channels, $DK, D\bar{D}, D\bar{D}^*, \dots$ to understand $D_{s0}^*(2317), X(3872)$, etc.
- ▶ Radiative transitions between charmonium states can also be calculated on the lattice and are underway.
- ▶ Lattice QCD will play a crucial role in understanding current and future exotic states.

Thank you for listening!

The Variational Method

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n t}$$

Solve the generalised eigenvalue problem

$$C_{ij}(t) v_j^n = \lambda^n(t) C_{ij}(t_0) v_j^n.$$

Then the eigenvalue is related to the mass by

$$\lambda^n(t) = e^{-(t-t_0)E_n} + \mathcal{O}(e^{-(t-t_0)E_m})$$

and the overlap is related to the eigenvector by

$$Z_i^n = \sqrt{2E_n} e^{E_n t_0/2} v_j^{n*} C_{ji}(t_0).$$

Constructing the Charmonium Operator

1. Construct a fermion bilinear with a gamma matrix Γ and a number of derivatives D all with the same quantum numbers.

$$O(t) = \bar{c}\Gamma D_1 D_2 \dots c.$$

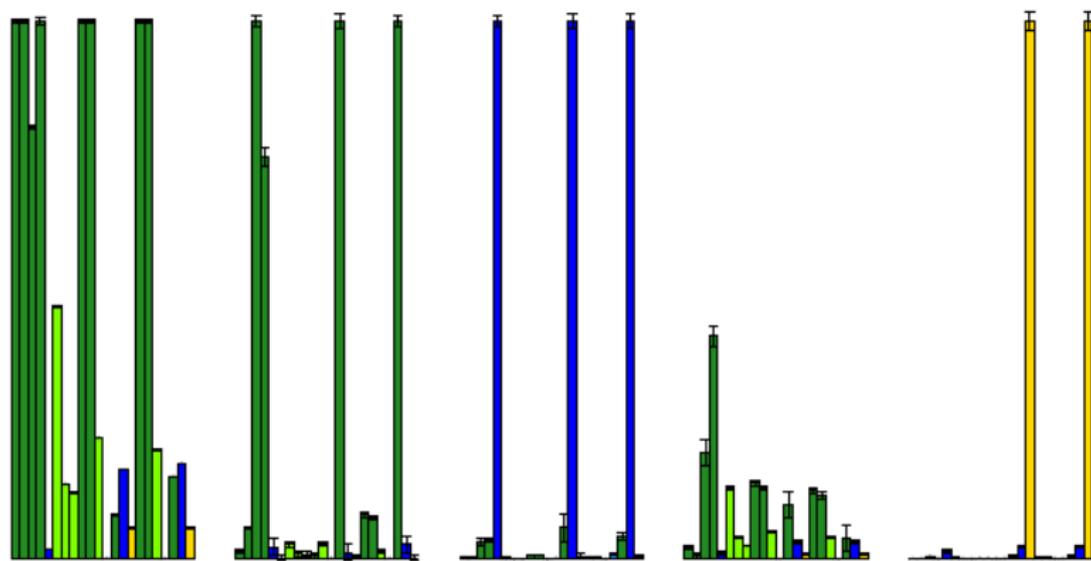
2. Couple to a continuum angular momentum irrep using Clebsch-Gordan coefficients. For example, for one gamma matrix and one derivative,

$$O^{J,M}(t) = \sum_{m_1, m_2} \langle J_1, m_1; J_2, m_2 | J, M \rangle \bar{c}\Gamma_{m_1} D_{m_2} c.$$

3. Project onto a lattice irrep using 'subduction' coefficients.

$$O_{\Lambda, \lambda}^{[J]}(t) = \sum_M S_{\Lambda, \lambda}^{J, M} O^{J, M}(t).$$

Spin Identification



[0]0.5858(4)

[1]0.6828(18)

[2]0.6856(16)

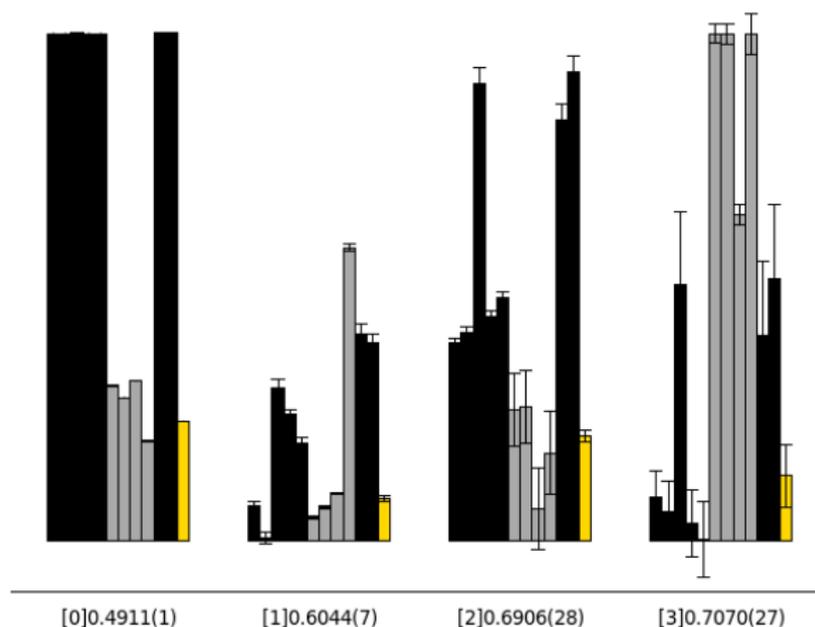
[3]0.6670(25)

[4]0.6829(34)

Overlaps for first 5 states in T_2^{++} .
J=2 (green), J=3 (blue), J=4 (yellow)

Hybrid Identification

Look for high overlap onto operators proportional to $[D_1, D_2]$.



Overlaps for first 4 states in 0^{-+} .

J=0 (black), Hybrid J=0 (grey), J=4 (yellow)