Studies of $D^+ \to K_S/K_L K^+ (\pi^0)$ and $D^0 \to K_S/K_L \pi^0 (\pi^0)$

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BESIII

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Overview

- BEPCII and BESIII
- Data Sample
- Analysis Technique
- Study of $D^+ \rightarrow K_S/K_L K^+ (\pi^0)$ decays
- Study of $D^0 \rightarrow K_S/K_L \pi^0 (\pi^0)$ decays
- Summary
First physics run starts from 2009!

Beam energy: 1.0 -- 2.3 GeV

Luminosity reached the design value (04/05/2016)
\[1.0 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}\]
BESIII detector

Drift Chamber (MDC)
\[ \sigma_{p/p}^{} (^{0}/0) = 0.5\% (1\text{GeV}) \]
\[ \sigma_{dE/dx}^{} (^{0}/0) = 6\% \]

Time Of Flight (TOF)
\[ \sigma_{T}^{} : 90 \text{ ps Barrel} \]
\[ 110 \text{ ps endcap} \]

Super-conducting magnet (1.0 Tesla)

\[ \mu \text{ Counter} \]
8- 9 layers RPC
\[ \delta R\Phi = 1.4 \text{ cm} \sim 1.7 \text{ cm} \]

EMC:
\[ \sigma_{E/VE}^{} (^{0}/0) = 2.5 \% (1 \text{GeV}) \]
\[ (\text{CsI}) \quad \sigma_{z,\phi}^{} (\text{cm}) = 0.5 - 0.7 \text{ cm}/\text{VE} \]
• We have the largest $\psi(3770)$ dataset

• Near the peak of $\psi(3770)$ resonance, only $D\bar{D}$ pairs are produced

• Analysis technique
  • Single tag (ST)
  • Double tag (DT)
  • Flavor tag (CF)
  • CP tag (CP)

• Useful variables
  • Beam constraint mass ($M_{BC}$)
    \[ M_{BC} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_D|^2/c^2} \]
  • Energy difference ($\Delta E$)
    \[ \Delta E = E_D - E_{\text{beam}} \]
Reconstruction Method of $K_L$

- $K_L$ is hard to be well reconstructed by BESIII as result of its long flight distance and rare decay rate in the MDC.

- The idea is to use the information in EMC as the $K_L$s leave the hadronic showers in the CsI

- The position information is used to perform the kinematic fit, while constraining the $K_L$ mass. The $K_L$ momentum can be obtained from the kinematic fit.

- $K_L$ reconstruction efficiencies are corrected to data. Verified by the $D \rightarrow K_L e \nu$ measurement

- GEANT4 does not involve different nuclear cross section for $K^0$ and $K^0$, and the effects due to $K^0 \rightarrow K^0$ oscillations.
In the Standard Model (SM), the singly Cabibbo suppressed (SCS) D meson hadronic decays are predicted to exhibit CP asymmetries of the order of $10^{-3}$.

Direct CP violation in SCS decays could arise from the interference between tree-level and penguin decay processes.

DCS and CF decays are expected to be CP invariant in the SM because they are dominated by a single weak amplitude.

Measurements of CP asymmetries in SCS processes greater than $O(10^{-3})$ would be evidence of physics beyond the SM.

We can test CP asymmetry in SCS decays $D^+ \rightarrow K_S/K_L K^+(\pi^0)$ by doing a charge-dependent measurement of branching fraction

$$A_{CP} = \frac{B(D^+ \rightarrow K_{S,L}^0 K^+(\pi^0)) - B(D^- \rightarrow K_{S,L}^0 K^-(\pi^0))}{B(D^+ \rightarrow K_{S,L}^0 K^+(\pi^0)) + B(D^- \rightarrow K_{S,L}^0 K^-(\pi^0))}$$
Study of $D^+ \to K_S/K_L K^+(\pi^0)$ decays

- **Absolute Branching Fraction (BF)**

$$B_{\text{sig}} = \frac{N_{DT}/\epsilon_{DT}}{N_{ST}/\epsilon_{ST}} = \frac{N_{DT}/\epsilon}{N_{ST}}$$

where $\epsilon = \epsilon_{DT}/\epsilon_{ST}$ is the efficiency of reconstructing the signal decay.

The BF satisfies $BF(D^+ \to K_S K^+(\pi^0)) = BF(D^+ \to K_L K^+(\pi^0))$ because $K^0$ is 50% $K_S$ and 50% $K_L$.

- **CP asymmetry**

$$A_{CP} = \frac{B(D^+ \to K^0_{S,L} K^+(\pi^0)) - B(D^- \to K^0_{S,L} K^-(\pi^0))}{B(D^+ \to K^0_{S,L} K^+(\pi^0)) + B(D^- \to K^0_{S,L} K^-(\pi^0))}$$
Fits to the ST $M_{BC}$ distribution in data

\[ N_{ST} = 2N_{D^+D^-}B_{\text{tag}}\epsilon_{ST} \]

| ST mode | \( D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm \) | \( D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm \pi^0 \) | \( D^\pm \rightarrow K^0_S\pi^\pm \) | \( D^\pm \rightarrow K^0_S\pi^\pm \pi^0 \) | \( D^\pm \rightarrow K^0_S\pi^\pm \pi^\pm \pi^\mp \) | \( D^\pm \rightarrow K^\mp K^\pm \pi^\pm \) |

- Binned maximum likelihood fit
- MC determined signal shape + ARGUS background
Fits to the DT $M_{BC}$ distribution in data

- DT yields of data are from binned maximum likelihood fits to the two-dimensional $M_{BC}(\text{signal})$ versus $M_{BC}(\text{tag})$ distribution.

- **Signal region:**
  $M_{BC}(\text{tag}) = M_{BC}(\text{signal}) = M_D$
  beam energy spread,
  ISR,
  the $\psi(3770)$ resonance line shape momentum resolution.

- **The horizontal and vertical bands:**
  one of the D mesons is correctly reconstructed, and the second is incorrectly reconstructed.

- **The diagonal band:**
  mis-partitioning $D\bar{D}$ candidates continuum events.
Fits to the DT $M_{BC}$ distribution in data

- Projections of double tag candidate masses on the $M_{BC}$ (signal), with the six tag modes.

- MC-based signal shape and background shape
Sources of systematic uncertainties in the measured BFs and CP asymmetries.

<table>
<thead>
<tr>
<th>Source</th>
<th>$K_S^0 K^+$</th>
<th>$K_S^0 K^-$</th>
<th>$K_S^0 K^+ \pi^0$</th>
<th>$K_S^0 K^- \pi^0$</th>
<th>$K_L^0 K^+$</th>
<th>$K_L^0 K^-$</th>
<th>$K_L^0 K^+ \pi^0$</th>
<th>$K_L^0 K^- \pi^0$</th>
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</thead>
<tbody>
<tr>
<td>$K^\pm$ tracking</td>
<td>0.7</td>
<td>0.9</td>
<td>1.8</td>
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<td>$K^\pm$ PID</td>
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<tr>
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<td>1.2</td>
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<tr>
<td>Extra $\chi^2$ cut</td>
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<td>1.8</td>
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<td>Peaking backgrounds in DT</td>
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<td>-</td>
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<td>Sub-resonances</td>
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<td>1.1</td>
<td>-</td>
<td>-</td>
<td>1.5</td>
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<td>$M_{BC}$ fit in ST, Negligible</td>
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<td>Negligible</td>
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<tr>
<td>$M_{BC}$ fit in DT</td>
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<td>$\Delta E$ requirement</td>
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<td>-</td>
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<tr>
<td>Total (for $B$)</td>
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<tr>
<td>Total (for $A_{CP}$)</td>
<td>2.1</td>
<td>2.2</td>
<td>3.5</td>
<td>3.2</td>
<td>1.5</td>
<td>1.6</td>
<td>2.3</td>
<td>2.1</td>
</tr>
</tbody>
</table>

The main sources are $K_{S,L}$ correction and $\chi^2$ cut in $K_L$ selection, which studied in \textit{PRD 92, 112008}. Systematic uncertainties are determined by weighting residual uncertainties of the data-MC differences using $K_{S,L}$ momentum of signal events.
Results of BFs and CP asymmetries

The first and second uncertainties are statistical and systematic errors.  

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\mathcal{B}(D^+) \times 10^{-3}$</th>
<th>$\mathcal{B}(D^-) \times 10^{-3}$</th>
<th>$\overline{\mathcal{B}} \times 10^{-3}$</th>
<th>$A_{CP} %$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0_S K^\pm$</td>
<td>$3.01 \pm 0.12 \pm 0.10$</td>
<td>$3.10 \pm 0.12 \pm 0.10$</td>
<td>$3.06 \pm 0.09 \pm 0.10$</td>
<td>$-1.5 \pm 2.8 \pm 1.6$</td>
</tr>
<tr>
<td>$K^0_S K^\pm \pi^0$</td>
<td>$5.23 \pm 0.28 \pm 0.24$</td>
<td>$5.09 \pm 0.29 \pm 0.22$</td>
<td>$5.16 \pm 0.21 \pm 0.23$</td>
<td>$1.4 \pm 4.0 \pm 2.4$</td>
</tr>
<tr>
<td>$K^0_L K^\pm$</td>
<td>$3.13 \pm 0.14 \pm 0.13$</td>
<td>$3.32 \pm 0.15 \pm 0.13$</td>
<td>$3.23 \pm 0.11 \pm 0.13$</td>
<td>$-3.0 \pm 3.2 \pm 1.2$</td>
</tr>
<tr>
<td>$K^0_L K^\pm \pi^0$</td>
<td>$5.17 \pm 0.30 \pm 0.21$</td>
<td>$5.26 \pm 0.30 \pm 0.20$</td>
<td>$5.22 \pm 0.22 \pm 0.21$</td>
<td>$-0.9 \pm 4.1 \pm 1.6$</td>
</tr>
</tbody>
</table>

- The BF of $D^+ \to K_S K^+$ agrees with the CLEO result.
  
  $\mathcal{B}(D^+ \to K_S^0 K^+)$

  CLEO $(3.14 \pm 0.09 \pm 0.08) \times 10^{-3}$

  
  Single tag method

- The BFs of $D^+ \to K_S K^+ \pi^0$, $D^+ \to K_L K^+$ and $D^+ \to K_L K^+ \pi^0$ are measured for the first time.

- No evidence for CP asymmetry in the four SCS decays.
Study of $D^0 \rightarrow K_S/K_L \pi^0 (\pi^0)$ decays

- As first pointed out by I. I. Bigi, the BFs of $D \rightarrow K_S\pi$’s and $D \rightarrow K_L\pi$’s are not the same because of the interference of the Cabibbo favored (CF) component $D \rightarrow K^0\pi$’s with the doubly Cabibbo suppressed (DCS) $D \rightarrow \bar{K}^0\pi$’s component.

- Denote a parameter by
  \[ R(D \rightarrow K_{S,L} + \pi') = \frac{B(D \rightarrow K_S^0 + \pi') - B(D \rightarrow K_L^0 + \pi')}{B(D \rightarrow K_S^0 + \pi') + B(D \rightarrow K_L^0 + \pi')} \]
  to describe this BF asymmetry.

- Previous CLEO-c’s results by using 281 pb$^{-1}$ of $\psi(3770)$ data
  
  \[ R(D^0 \rightarrow K_{S,L}\pi^0) = 0.108 \pm 0.025 \pm 0.024 \]
  
  \[ R(D^+ \rightarrow K_{S,L}\pi^+) = 0.022 \pm 0.016 \pm 0.018 \]

- We can update the results with a much larger data sample.
Study of $D^0 \rightarrow K_S/K_L \pi^0 (\pi^0)$ decays

- BF $(\text{CP} \pm)$ can be extracted by ratios of yields of $(\text{CF}, \text{CP} \pm)$ and (CF)

$$
N_{ST}(\text{CF}) = (1 + r_i^2) \cdot 2N_{D^0 \bar{D}^0} B_{\text{tag}} \epsilon_{ST}
$$

$$
N_{DT}(\text{CF}, \text{CP} \pm) = (1 + r_i^2 \pm 2r_i \cos \delta_i) \cdot 2N_{D^0 \bar{D}^0} B_{\text{tag}} B_{\text{sig}} \epsilon_{DT}
$$

$$
B_{\text{sig}}(\text{CP} \pm) = \frac{1}{1 + C_f} \frac{N_{DT}/\epsilon}{N_{ST}}, \quad (C_f = \frac{2r_i \cos \delta_i}{1 + r_i^2})
$$

- where $C_f$ is extracted by ratios of yields of $(\text{CF}, \text{CP} \pm)$ and (CP±)

$$
\text{CP}^+, \text{CF} : \quad A = (1 + r^2 - 2r \cos \delta) B_{\text{CF}} = \frac{N_{\text{CP}^+, \text{CF}}/\epsilon}{N_{\text{CP}^+}}
$$

$$
\text{CP}^-, \text{CF} : \quad B = (1 + r^2 + 2r \cos \delta) B_{\text{CF}} = \frac{N_{\text{CP}^-, \text{CF}}/\epsilon}{N_{\text{CP}^-}}
$$

$$
C_f = \frac{2r \cos \delta}{1 + r^2} = \frac{B - A}{B + A}
$$

<table>
<thead>
<tr>
<th>Type</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CP}^+$</td>
<td>$K_L \pi^0, K_S \pi^0 \pi^0$</td>
</tr>
<tr>
<td>$\text{CP}^-$</td>
<td>$K_S \pi^0, K_L \pi^0 \pi^0$</td>
</tr>
<tr>
<td>$\text{CF}$</td>
<td>$K \pi, K \pi \pi^0, K \pi \pi \pi$</td>
</tr>
</tbody>
</table>
BF: ST Yields

- ST yields are from $M_{BC}$ fit.
- Binned maximum likelihood fit
- MC determined signal shape + ARGUS background

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**Diagrams:**

- $D^0 \rightarrow K\pi$
- $D^0 \rightarrow K\pi\pi\pi$
- $D^0 \rightarrow K\pi^0$
- $D^0 \rightarrow KK$
- $D^0 \rightarrow \pi\pi$
- $D^0 \rightarrow K_S^0\pi^0$
BF: DT Yields

- Two dimensional fits to $M_{BC}(\text{tag})$ versus $M_{BC}(\text{signal})$
- Projections of double tag candidate masses on the $M_{BC}(\text{signal})$, with the tag mode $K\pi\pi^0$. 

*BEH preliminary*
Results of BFs and the asymmetries (R)

- BFs of \((K_S\pi^0, K_L\pi^0, K_S\pi^0\pi^0)\) are consistent with values in PDG.

- BF of \(K_L\pi^0\pi^0\) is the first accurate measurement.
  - \(\text{Br}(K_S\pi^0) = (1.230 \pm 0.020)\%
  - \(\text{Br}(K_L\pi^0) = (0.991 \pm 0.019)\%
  - \(\text{Br}(K_S\pi^0\pi^0) = (0.975 \pm 0.024)\%
  - \(\text{Br}(K_L\pi^0\pi^0) = (1.175 \pm 0.040)\%

- The asymmetries are consistent with CLEO-c’s measurement but with better statistical uncertainties
Oscillations between meson and anti-meson, also called mixing, can occur when the flavor eigenstates differ from the physical mass eigenstates. The oscillations are conventionally characterized by two dimensionless parameters $x = \Delta m / \Gamma$ and $y = \Delta \Gamma / \Gamma$. In the absence of CP violation, one has $y_{CP} = y$.

We measure $y_{CP}$ using $D^0 \to K_S \pi^0$ and $D^0 \to K_L \pi^0$ versus $D^0 \to K e^\nu_e$ (CP± + SL).

\[
N_{ST}(CP^\pm) = (1 \mp y_{CP}) \cdot 2N_{D^0 \bar{D}^0} B_{tag} \epsilon_{ST}
\]
\[
N_{DT}(CP^\pm,Ke\nu_e) = (1 + y_{CP}^2) \cdot 2N_{D^0 \bar{D}^0} B_{tag} B_{sig} \epsilon_{DT}
\]

\[
\begin{align*}
\alpha &= \frac{N_{DT}(CP^+,Ke\nu_e) / \epsilon}{N_{ST}(CP^+)} \\
\beta &= \frac{N_{DT}(CP^-,Ke\nu_e) / \epsilon}{N_{ST}(CP^-)}
\end{align*}
\]
\[
y_{CP} = \frac{\alpha - \beta}{\alpha + \beta}
\]

<table>
<thead>
<tr>
<th>Type</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CP^+$</td>
<td>$K_L^0 \pi^0$</td>
</tr>
<tr>
<td>$CP^-$</td>
<td>$K_S^0 \pi^0$</td>
</tr>
<tr>
<td>semi-leptonic</td>
<td>$Ke\nu_e$</td>
</tr>
</tbody>
</table>
ST yields can be got from $K_S\pi^0$, $K_L\pi^0$ branching fraction measurement results before. DT yields are from $U_{\text{miss}}$ fit.

\[
\begin{align*}
D^0 \to K^0_S\pi^0 & \quad D^0 \to K^{0}\nu_e \\
D^0 \to K^0_L\pi^0 & \quad D^0 \to K^{0}\nu_e \\
\end{align*}
\]

\[
U_{\text{miss}}(\text{tag}) = E_{\text{miss}}(\text{tag}) - p_{\text{miss}}(\text{tag}) \\
P_{\text{miss}}(\text{tag}) = P_{e^+e^-} - P_{\text{signal}} - P_{\text{track}}(\text{tag})
\]

statistical error only

\[
\begin{array}{ll}
\alpha \ (K^0_L\pi^0, K^{0}\nu_e) & 3.603 \pm 0.142 \\
\beta \ (K^0_S\pi^0, K^{0}\nu_e) & 3.533 \pm 0.100 \ \\
y_{\text{CP}} & \frac{\alpha - \beta}{\alpha + \beta} \ (0.98 \pm 2.43)\%
\end{array}
\]
Ø

The BF of \( D^+ \rightarrow K_S K^+ \) agrees with the CLEO result. The BFs of \( D^+ \rightarrow K_S K^+ \pi^0 \), \( D^+ \rightarrow K_L K^+ \) and \( D^+ \rightarrow K_L K^+ \pi^0 \) are measured for the first time.

\[
\begin{align*}
\text{BF}(D^+ \rightarrow K_S K^+) & = (3.06 \pm 0.09 \pm 0.10) \times 10^{-3} \\
\text{BF}(D^+ \rightarrow K_S K^+ \pi^0) & = (5.16 \pm 0.21 \pm 0.23) \times 10^{-3} \\
\text{BF}(D^+ \rightarrow K_L K^+) & = (3.23 \pm 0.11 \pm 0.13) \times 10^{-3} \\
\text{BF}(D^+ \rightarrow K_L K^+ \pi^0) & = (5.22 \pm 0.22 \pm 0.21) \times 10^{-3}
\end{align*}
\]

The results of BFs satisfy \( \text{BF}(D^+ \rightarrow K_S K^+(\pi^0)) = \text{BF}(D^+ \rightarrow K_L K^+(\pi^0)) \) in 1σ. No evidence for CP asymmetry is found in the four SCS decays.

Ø

BFs of \( (K_S \pi^0, K_L \pi^0, K_S \pi^0 \pi^0) \) are consistent with values in PDG. BF of \( K_L \pi^0 \pi^0 \) is the first accurate measurement. The asymmetries are consistent with CLEO-c’s measurement but with better statistical uncertainties.

\[
\begin{align*}
\text{BF}(D^0 \rightarrow K_S \pi^0) & = (1.230 \pm 0.020)\% \\
\text{BF}(D^0 \rightarrow K_S \pi^0 \pi^0) & = (0.975 \pm 0.024)\% \\
\text{BF}(D^0 \rightarrow K_L \pi^0) & = (0.991 \pm 0.019)\% \\
\text{BF}(D^0 \rightarrow K_L \pi^0 \pi^0) & = (1.175 \pm 0.040)\% \\
R(D^0 \rightarrow K_{S,L} \pi^0) & = (10.77 \pm 1.25)\% \\
R(D^0 \rightarrow K_{S,L} \pi^0 \pi^0) & = (-9.29 \pm 2.09)\%
\end{align*}
\]

\( \gamma_{CP} = (0.98 \pm 2.43)\% \), using \( K_S \pi^0, K_L \pi^0 \) versus \( K e\nu_e \).

Thank you!
K_L selection

We take D^- → K^+π^-π^-, D^+ → K_LK^+ as an example channel:

- By using four-momentum of the tag D^-, four-momentum of K^+, and the position information of extra neutral shower deposit in EMC, we perform kinematic fit for the whole event to select candidate of K_L shower with minimal χ^2 value of kinematic fit.

- Given that we only used kinematic fit to select the candidate of K_L shower, the momentum value of K_L is determined by requiring ΔE of signal D to equal to zero. In this way, the quantity of M_BC(tag) and M_BC(signal) will be not highly correlated.

- The further selection requirement is given here
  - At least one(three) good neutral shower for K_LK^+(π^0)
  - At least three(five) good neutral shower for K_Lπ^0(π^0)
  - No extra γγ combinations satisfied 0.110 < M_{γγ} < 0.155 GeV/c^2
  - E_{dep} > 0.1 GeV
Measurement of $C_f$

\[
\mathcal{B}_{\text{sig}(\text{CP}^\pm)} = \frac{1}{1 \mp C_f} \frac{N_{DT}/\epsilon}{N_{ST}}, \quad (C_f = \frac{2r_i \cos \delta_i}{1 + r_i^2})
\]

\[
\begin{align*}
N_{CP-} &= 2 \cdot N_{D^0 \bar{D}^0} \cdot \mathcal{B}_{CP-} \cdot \epsilon_{CP-} \\
N_{CP+} &= 2 \cdot N_{D^0 \bar{D}^0} \cdot \mathcal{B}_{CP+} \cdot \epsilon_{CP+} \\
N_{CP-,CF} &= (1 + r_i^2 + 2r_i \cos \delta_i) \cdot 2 \cdot N_{D^0 \bar{D}^0} \cdot \mathcal{B}_{CF} \cdot \mathcal{B}_{CP-} \cdot \epsilon_{CP-,CF} \\
N_{CP+,CF} &= (1 + r_i^2 - 2r_i \cos \delta_i) \cdot 2 \cdot N_{D^0 \bar{D}^0} \cdot \mathcal{B}_{CF} \cdot \mathcal{B}_{CP+} \cdot \epsilon_{CP+,CF}
\end{align*}
\]

\[
\begin{align*}
CP+,CF: \quad A &= (1 + r^2 - 2r \cos \delta) \quad \mathcal{B}_{CF} = \frac{N_{CP+,CF}/\epsilon}{N_{CP+}} \\
CP-,CF: \quad B &= (1 + r^2 + 2r \cos \delta) \quad \mathcal{B}_{CF} = \frac{N_{CP-,CF}/\epsilon}{N_{CP-}}
\end{align*}
\]

\[
C_f = \frac{2r \cos \delta}{1 + r^2} = \frac{B - A}{B + A}
\]

<table>
<thead>
<tr>
<th>Type</th>
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</tr>
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<tbody>
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<td>$CP^+$</td>
<td>$KK$, $\pi\pi$</td>
</tr>
<tr>
<td>$CP^-$</td>
<td>$K_S^0\pi^0$</td>
</tr>
<tr>
<td>$CF$</td>
<td>$K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$</td>
</tr>
</tbody>
</table>
Measurement of $\frac{N_{ST}(K_L^0\pi^0)}{\epsilon_{ST}(K_L^0\pi^0)}$

Because the $K_L$ reconstructed method needed the whole event information, which can provide DT yield and DT efficiency only. We need to obtain the ratio of ST yield and ST efficiency indirectly.

\[
N_{ST}(CP^+) = (1 - y_{CP}) \cdot 2N_{D^0\bar{D}^0}B_{tag}\epsilon_{ST}
\]

\[
N_{DT}(CP^+, CF) = (1 + y_{CP})(1 + r^2 \pm 2r \cos \delta) \cdot 2N_{D^0\bar{D}^0}B_{tag}B_{sig}\epsilon_{DT}
\]

\[
f = \frac{(1 + y_{CP})(1 + r^2 \pm 2r \cos \delta)}{1 - y_{CP}}
\]

\[
f \cdot B_{sig} = \frac{N_{DT}(KK, CF)/\epsilon_{DT}(KK, CF)}{N_{ST}(KK)/\epsilon_{ST}(KK)}
\]

\[
= \frac{N_{DT}(K_L^0\pi^0, CF)/\epsilon_{DT}(K_L^0\pi^0, CF)}{N_{ST}(K_L^0\pi^0)/\epsilon_{ST}(K_L^0\pi^0)}
\]

CP+: KK, ππ, K_Lπ^0

\[
\frac{N_{ST}(K_L^0\pi^0)}{\epsilon_{ST}(K_L^0\pi^0)}
\]