

Amplitude analysis of $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays using CLEO-c data

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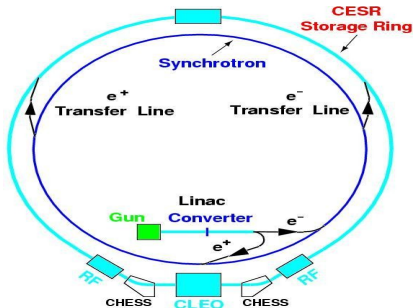
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Amplitude analysis of $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

- Provides insights into strong phase difference between D and \bar{D} decays
→ Input for measurement of CKM angle γ in $B^- \rightarrow D K^-$
- Understanding of hadron dynamics
- Search for direct CP -violation

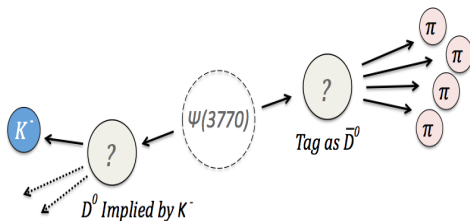
Introduction to CLEO-c



CLEO-c

- 4π detector at the Cornell Electron Storage Ring (CESR)
- Symmetric $e^+ e^-$ collider
- Operated at $\psi(3770)$ resonance
- $L_{int} = 818 \text{ 1/pb}$ collected

Why CLEO-c ?



$$e^+ e^- \rightarrow \psi(3770) \rightarrow D_a D_b$$

- **Flavour tag:**

$$D_b \rightarrow K^- \pi^+ \Rightarrow D_a = \overline{D^0}$$

- **CP tag:**

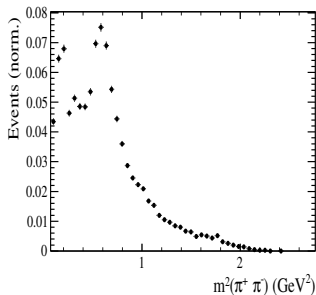
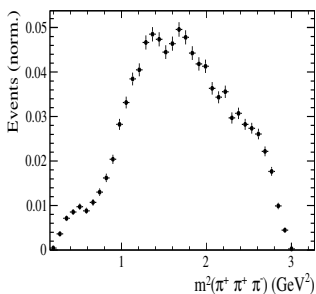
$$D_b \rightarrow K^- K^+ (CP+) \Rightarrow D_a = CP-$$

$$\mathcal{A}_{CP\pm} \approx (\mathcal{A}_{D^0} \pm \mathcal{A}_{\overline{D^0}})$$

$$\Rightarrow |\mathcal{A}_{CP\pm}|^2 \approx |\mathcal{A}_{D^0}|^2 + |\mathcal{A}_{\overline{D^0}}|^2 \pm 2|\mathcal{A}_{D^0}||\mathcal{A}_{\overline{D^0}}| \cos(\delta_D)$$

Look at CLEO-c data

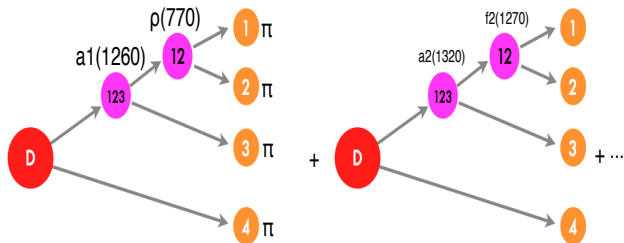
$\approx 8k$ $D \rightarrow 4\pi$ flavour tagged candidates



Resonances ?

- $a_1(1260) \rightarrow \pi \pi \pi$
- $\rho(770) \rightarrow \pi \pi$

Amplitude analysis



Isobar formalism

- Amplitude $\mathcal{A}_i \approx BW_{a_1}(m_{123}^2) \cdot BW_{\rho}(m_{12}^2) \cdot S_f$
- $PDF \approx |\sum_i a_i \mathcal{A}_i|^2$
- Complex coefficients a_i

Decay channels

Plenty of possible decay channels ! How to select them ?

- cascade decays

$$D \rightarrow \pi^- [a_1(1270)^+ [S, D] \rightarrow \rho(770) \pi^+]$$

$$D \rightarrow \pi^- [a_1(1270)^+ \rightarrow \sigma \pi^+]$$

- quasi-two-body

$$D \rightarrow \sigma \sigma$$

$$D[S, P, D] \rightarrow \rho(770) \rho(770)$$

- single resonance

$$D \rightarrow \sigma (\pi\pi)_S$$

$$D \rightarrow \rho(770) (\pi\pi)_S$$

- non-resonant

$$D \rightarrow (\pi\pi)_S (\pi\pi)_S$$

$$D[P] \rightarrow (\pi\pi)_V (\pi\pi)_S$$

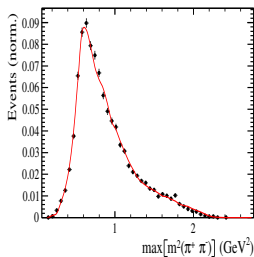
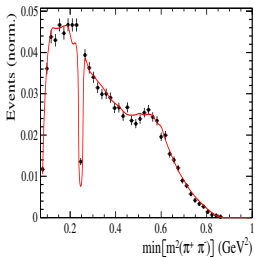
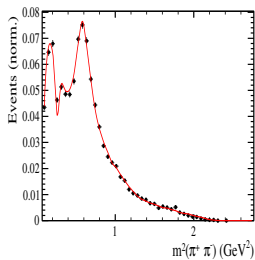
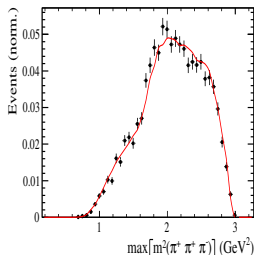
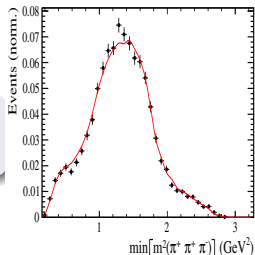
LASSO

- **LASSO:** Data-driven method for model selection (M. Williams, JINST 10 (2015) no.09, P09002)
- Include “all” amplitudes, but penalize complexity in the likelihood:

$$-2 \cdot \log(L) \rightarrow -2 \cdot \log(L) + \lambda \cdot \sum_i |a_i|$$

Amplitude fit

18 amplitudes selected
 $\chi^2/\nu = 1.33$



Decay mode	Fit fraction F_i [%]
$D^0 \rightarrow \pi^- [a_1(1260)^+ \rightarrow \pi^+ \rho(770)]$	36.7 ± 2.4
$D^0 \rightarrow \pi^- [a_1(1260)^+ \rightarrow \pi^+ \sigma]$	10.9 ± 1.5
$D^0 \rightarrow \pi^+ a_1(1260)^-$	5.3 ± 0.5
$D^0 \rightarrow \pi^- [\pi(1300)^+ \rightarrow \pi^+ \sigma]$	4.2 ± 1.0
$D^0 \rightarrow \pi^- [\pi(1300)^+ \rightarrow \pi^+ (\pi\pi)_P]$	6.1 ± 0.7
$D^0 \rightarrow \pi^- \pi(1300)^-$	3.3 ± 0.6
$D^0 \rightarrow \pi^- [a_1(1640)^+[D] \rightarrow \pi^+ \rho(770)]$	3.6 ± 0.6
$D^0 \rightarrow \pi^- [a_1(1640)^+ \rightarrow \pi^+ \sigma]$	1.2 ± 0.5
$D^0 \rightarrow \pi^- [\pi_2(1670)^+ \rightarrow \pi^+ f_2(1270)]$	1.5 ± 0.3
$D^0 \rightarrow \pi^- [\pi_2(1670)^+ \rightarrow \pi^+ \sigma]$	3.3 ± 0.6
$D^0 \rightarrow \sigma f_0(1370)$	18.4 ± 1.4
$D^0 \rightarrow \sigma \rho(770)$	4.4 ± 1.0
$D^0 \rightarrow \rho(770) \rho(770)$	0.9 ± 0.3
$D^0[P] \rightarrow \rho(770) \rho(770)$	7.1 ± 0.5
$D^0[D] \rightarrow \rho(770) \rho(770)$	15.5 ± 1.2
$D^0 \rightarrow f_2(1270) f_2(1270)$	1.1 ± 0.3
Sum	123.7 ± 6.8

Preliminary results

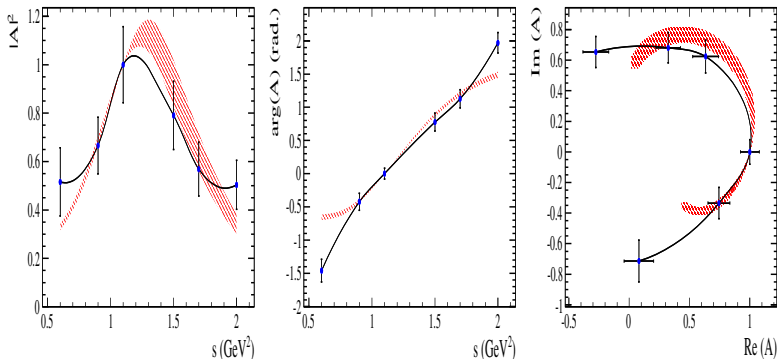
Results

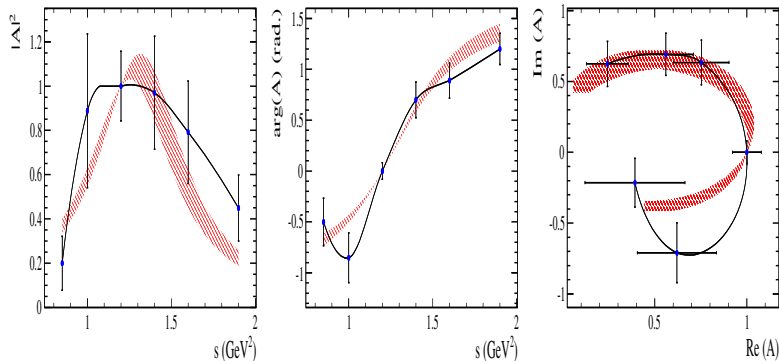
Resonance		Our result	PDG
$a_1(1260)$	m [MeV]	1231 ± 8	1230 ± 40
	Γ [MeV]	459 ± 18	$250 - 600$
$\pi(1300)$	m [MeV]	1180 ± 12	1185 ± 18
	Γ [MeV]	297 ± 20	$200 - 600$
$a_1(1640)$	m [MeV]	1644 ± 16	1647 ± 22
	Γ [MeV]	222 ± 56	254 ± 27

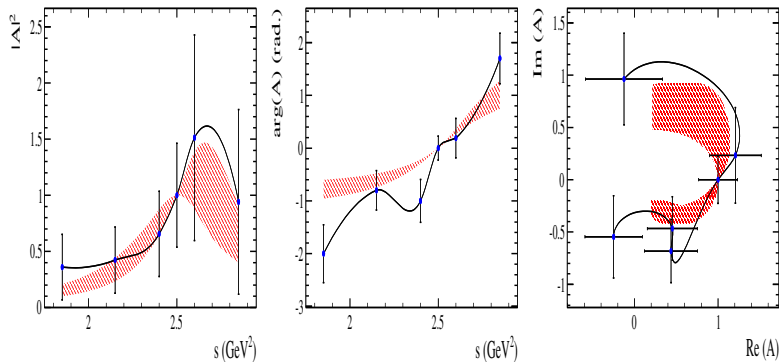
Preliminary results

Model-independent cross-check: $a_1(1260)$

- Replace BW by cubic spline
- 6 independent complex knots placed in $m^2(\pi\pi\pi)$
- Compare with BW expectation



Model-independent cross-check: $\pi(1300)$ 

Model-independent cross-check: $a_1(1640)$ 

CP even fraction

$$F_{+}^{4\pi} = \frac{N_{CP+}}{N_{CP+} + N_{CP-}} = \frac{1}{2} \left(1 + \frac{1}{N} \int |A_D| |A_{\bar{D}}| \cos(\delta_D) d\phi_4 \right)$$

- $F_{+}^{4\pi}$ (flavour-tagged, model-dependent) = $(73.5 \pm 0.9)\%$
(**Preliminary result**)
- $F_{+}^{4\pi}$ (CP-tagged, model-independent) = $(73.7 \pm 2.8)\%$
(Malde et al., PLB 747 (2015) 9)

Search for direct CP-violation

$$A_i^{CP} = \frac{F_i - \bar{F}_i}{F_i + \bar{F}_i}$$

Decay mode	A_i^{CP} [%]
$D^0 \rightarrow \pi^- a_1(1260)^+$	5.0 ± 3.2
$D^0 \rightarrow \pi^+ a_1(1260)^-$	6.8 ± 13.2
$D^0 \rightarrow \pi^- \pi(1300)^+$	-7.4 ± 8.0
$D^0 \rightarrow \pi^+ \pi(1300)^-$	-9.6 ± 16.5
$D^0 \rightarrow \pi^- a_1(1640)^+$	7.8 ± 12.5
$D^0 \rightarrow \pi^- \pi_2(1670)^+$	6.7 ± 14.0
$D^0 \rightarrow \sigma f_0(1370)$	-8.7 ± 4.5
$D^0 \rightarrow \sigma \rho(770)$	26.3 ± 15.2
$D^0 \rightarrow \rho(770) \rho(770)$	-46.7 ± 34.0
$D^0[P] \rightarrow \rho(770) \rho(770)$	-9.1 ± 7.9
$D^0[D] \rightarrow \rho(770) \rho(770)$	-7.9 ± 8.3
$D^0 \rightarrow f_2(1270) f_2(1270)$	-28.7 ± 20.7
$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$	-5.5 ± 3.3

No evidence for CP-violation (**Preliminary results**)

Conclusion

Summary

- First flavour tagged amplitude analysis of $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ [preliminary results]
- Verified resonant phase motion of observed $a_1(1260)$, $\pi(1300)$ and $a_1(1640)$ states
- CP -even fraction consistent with model-independent measurement
- No evidence for CP -violation

Follow-up analyses

- Model-independent measurement of δ_D in multiple phase-space bins for $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ [in preparation]
- Will allow measurement of γ in $B^- \rightarrow DK^-$, $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ (like for $K_S \pi \pi$)

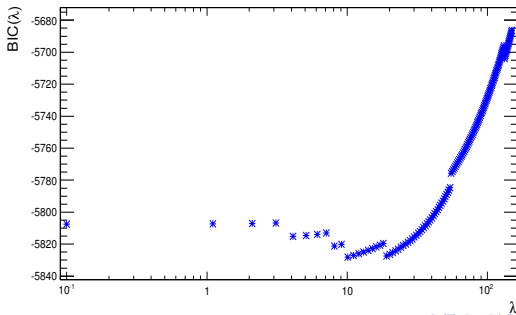
Acknowledgement

We thank the former CLEO collaboration for the privilege to use their precious data !

Backup: LASSO

How to choose λ ?

- $\text{BIC}(\lambda) = -2 \cdot \log(L) + r \cdot \log(N_{\text{events}})$
 r = amplitudes with fit fraction $>$ threshold (0.5 %)
- Balances **gain in fit quality vs. complexity**
- Optimal value $\lambda \approx 20$

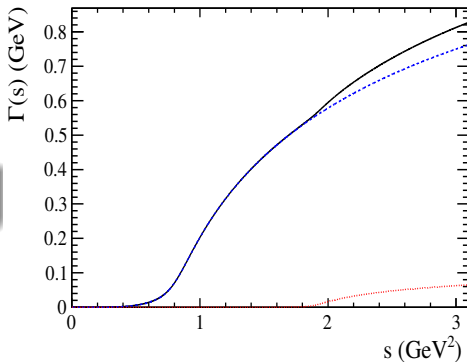


Decay mode	LASSO	No $\pi(1300)$	No $a_1(1640)$
$D^0 \rightarrow \pi^- [a_1(1260)^+ \rightarrow \pi^+ \rho(770)]$	36.7 ± 2.4	44.4 ± 3.3	46.2 ± 2.7
$D^0 \rightarrow \pi^- [a_1(1260)^+[D] \rightarrow \pi^+ \rho(770)]$	-	-	4.4 ± 0.5
$D^0 \rightarrow \pi^- [a_1(1260)^+ \rightarrow \pi^+ \sigma]$	10.9 ± 1.5	5.1 ± 0.7	8.3 ± 1.0
$D^0 \rightarrow \pi^+ a_1(1260)^-$	5.3 ± 0.5	3.0 ± 0.5	2.3 ± 0.4
$D^0 \rightarrow \pi^- [\pi(1300)^+ \rightarrow \pi^+ (\pi^+ \pi^-)_P]$	6.1 ± 0.7	-	7.0 ± 1.2
$D^0 \rightarrow \pi^- [\pi(1300)^+ \rightarrow \pi^+ \sigma]$	4.2 ± 1.0	-	10.0 ± 2.0
$D^0 \rightarrow \pi^+ \pi(1300)^-$	3.9 ± 0.6	-	9.2 ± 1.3
$D^0 \rightarrow \pi^- [a_1(1640)^+[D] \rightarrow \pi^+ \rho(770)]$	3.6 ± 0.6	4.3 ± 1.2	-
$D^0 \rightarrow \pi^- [a_1(1640)^+ \rightarrow \pi^+ \sigma]$	1.2 ± 0.5	-	-
$D^0 \rightarrow \pi^- [a_1(1640)^+ \rightarrow \pi^+ f_2(1270)]$	-	2.4 ± 0.6	-
$D^0 \rightarrow \pi^- [\pi_2(1670)^+ \rightarrow \pi^+ f_2(1270)]$	1.5 ± 0.3	8.1 ± 1.9	0.4 ± 0.1
$D^0 \rightarrow \pi^- [\pi_2(1670)^+ \rightarrow \pi^+ \sigma]$	3.3 ± 0.6	3.9 ± 0.6	0.5 ± 0.2
$D^0 \rightarrow \pi^+ \pi_2(1670)^-$	-	-	1.6 ± 0.6
$D^0 \rightarrow \sigma f_0(1370)$	18.4 ± 1.4	4.4 ± 0.5	-
$D^0 \rightarrow f_0(1370) f_0(1370)$	-	-	12.7 ± 1.4
$D^0 \rightarrow \sigma \rho(770)$	4.4 ± 1.0	7.5 ± 0.9	-
$D^0 \rightarrow f_0(1370) \rho(770)$	-	-	4.7 ± 0.9
$D^0 \rightarrow \rho(770) \rho(770)$	0.9 ± 0.3	3.4 ± 0.8	-
$D^0[P] \rightarrow \rho(770) \rho(770)$	7.1 ± 0.5	8.1 ± 0.6	7.1 ± 0.7
$D^0[D] \rightarrow \rho(770) \rho(770)$	15.5 ± 1.2	7.0 ± 1.0	17.5 ± 1.6
$D^0 \rightarrow f_2(1270) \sigma$	-	4.2 ± 1.2	2.07 ± 0.6
$D^0 \rightarrow f_2(1270) f_0(980)$	-	3.0 ± 0.7	0.6 ± 0.3
$D^0 \rightarrow f_2(1270) (\pi \pi)_P$	-	4.3 ± 0.9	-
$D^0 \rightarrow f_2(1270) f_2(1270)$	1.1 ± 0.3	9.6 ± 1.9	-
$D^0[D] \rightarrow (\pi^+ \pi^-)_P (\pi^+ \pi^-)_P$	-	7.8 ± 1.0	-

Backup: Running width

- $\Gamma_{a_1}(s) = \Gamma_{\pi\pi\pi}(s) + \Gamma_{KK^*}(s)$
- $\Gamma_{\pi\pi\pi}(s) = \frac{1}{s} \int |A(s_{12}, s_{23})|^2 ds_{12} ds_{23}$

97% : $a_1(1260) \rightarrow \pi\pi\pi$
 3% : $a_1(1260) \rightarrow K^*(892)K$



Backup: Systematics

Error sources

- $f_{bkg} = 18.5 \pm 0.8\%$
- background shape
- mistag prob. $\omega = 4.5 \pm 0.5\%$
- efficiency
- fitter bias
- resonance parameters
- form factors

$$\sigma_{sys} \approx 1 \text{ to } 3 \sigma_{stat}$$