Parton propagation in a dense medium

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Outline

- Introduction
- High energy eikonal trajectories
- Beyond eikonal propagation
- In-medium gluon cascade
- Q & A

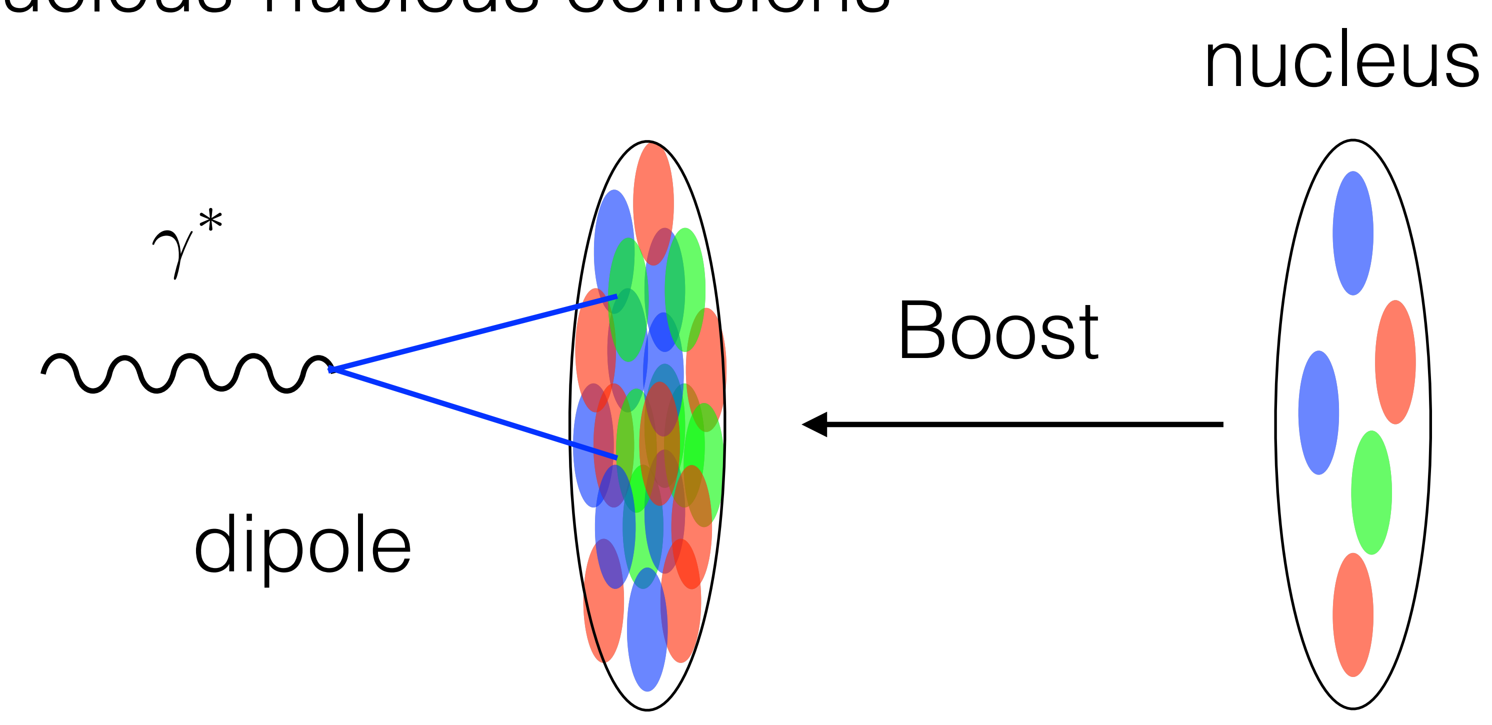
Introduction

Heavy Ion Collisions

- Study QCD matter under **extreme conditions**; high energy and particle densities, strong gauge fields, …

- **In cold nuclear matter:** at high energy the nuclear wave function is characterized by a large number of small x (soft) gluons that might reach the point of saturation $\longrightarrow$ DIS, proton-Nucleus, initial state of nucleus-nucleus collisions

The dipole **rescatters** several times when propagating through a saturated nucleus

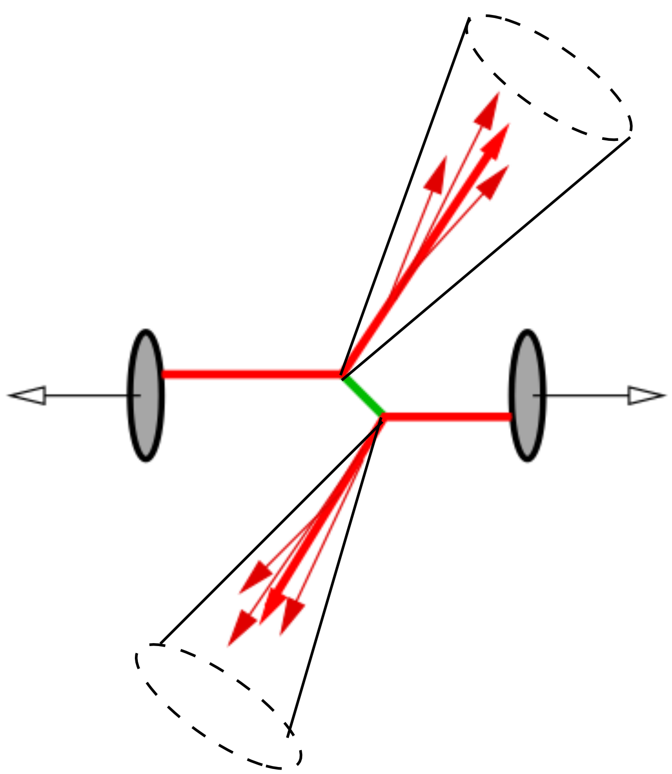


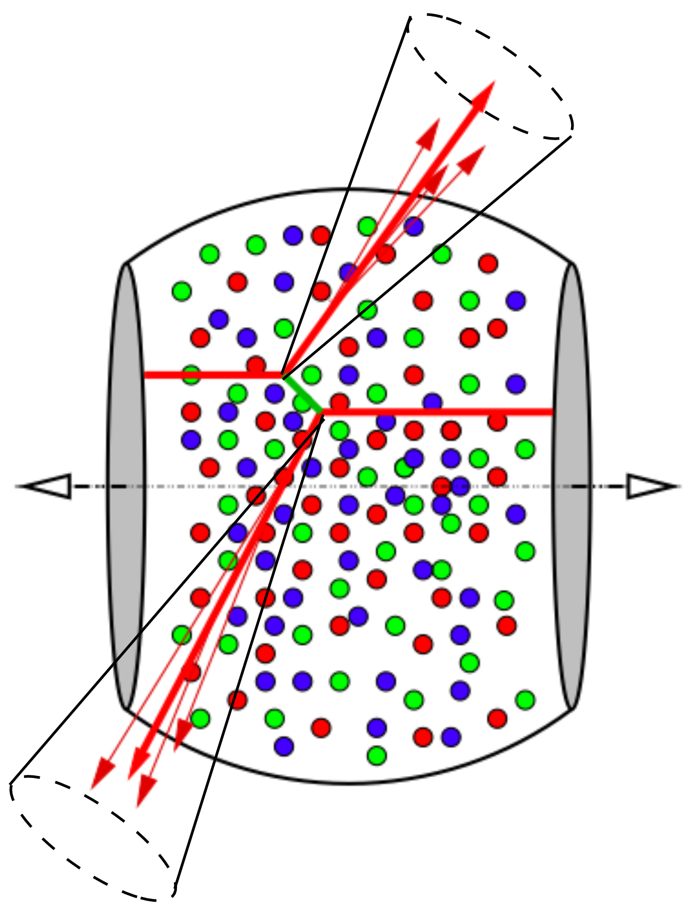
[Mueller, McLerran, Venugopalan, Balitsky, Kovchegov, Kovner, Leonidov, Weigert, Jalilian-Marian, Iancu (1992-2001)]

Heavy Ion Collisions

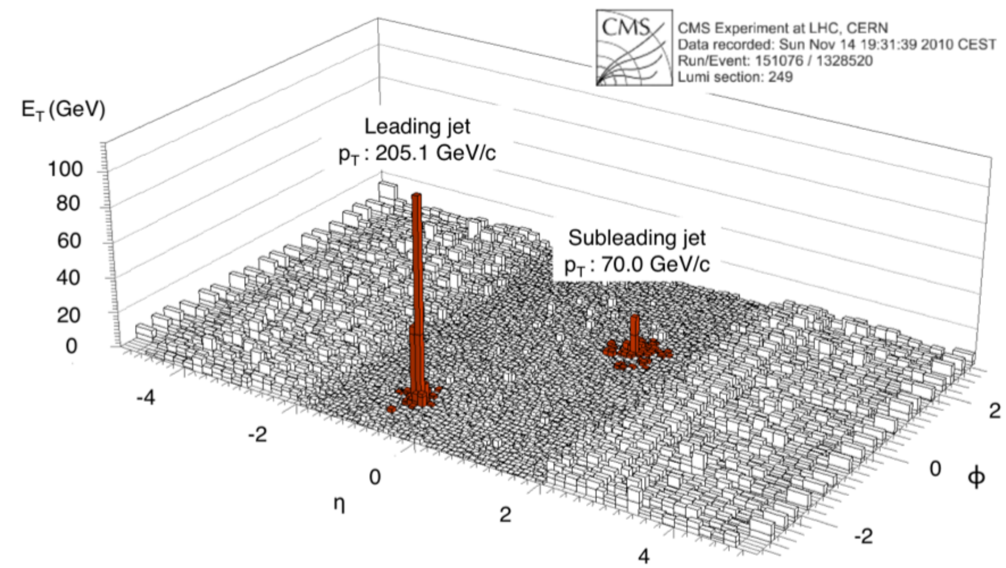
- **Hot nuclear matter:** probe the properties of the QGP produced in HIC, using bulk observables or hard probes such as: quarkonia, high pt hadrons, and **jets**

Dijet in: p-p…





and A-A…



The high energy jets propagate in the QGP and lose energy to the medium

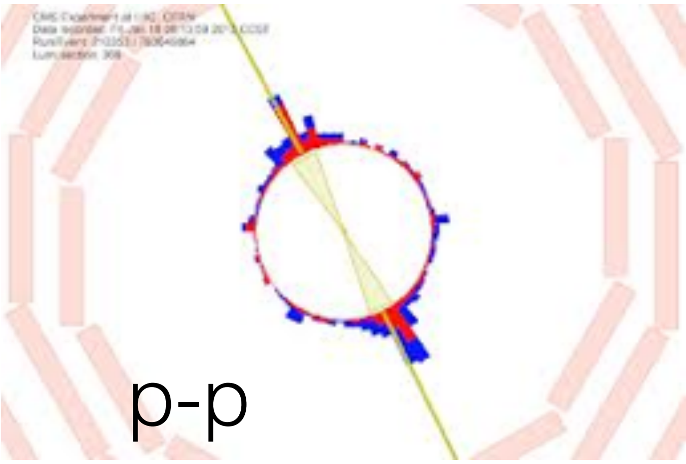
Extending pQCD Toolbox

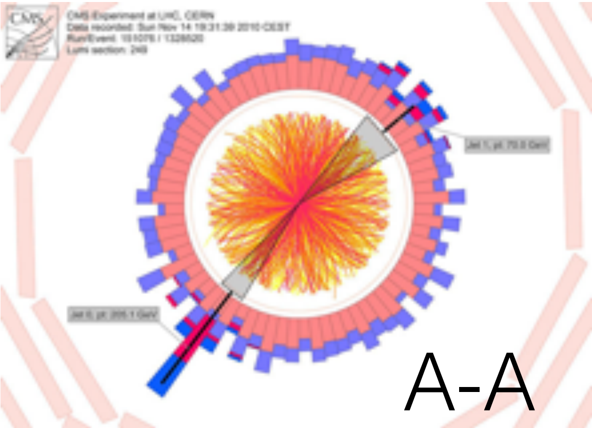
- **Asymptotic Freedom:** at short distances QCD is a weekly coupled theory. Therefore, provided we have a large scale, $Q \gg \Lambda_{\mathrm{QCD}}$, in the problem, one can use perturbation theory to compute inclusive observables. exp: $e^+ e^-$ total cross section, jet event shapes , etc,

- **Factorization theorems:** if hadrons are involved in the initial state or measured in the final state one can factorize short distance physics (calculable) from large distance non-perturbative physics encoded in universal (process independent) parton distributions (PDF's, Fragmentation functions). Exp: DIS, Drell-Yan, hadron production, etc.

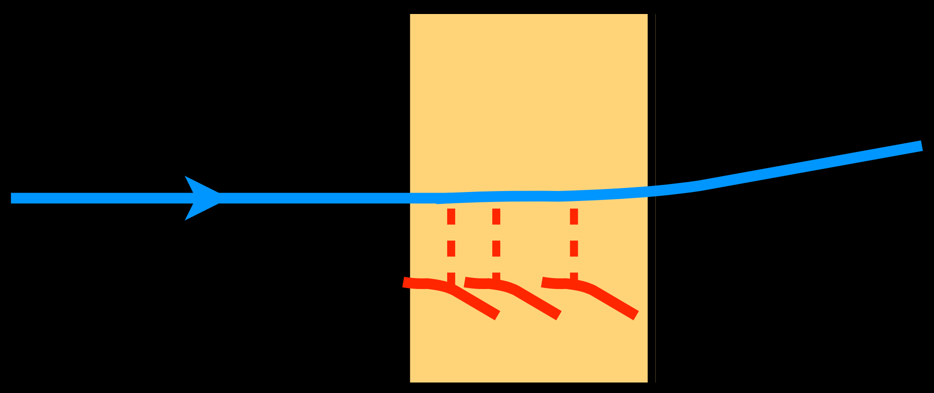
Extending pQCD Toolbox

- In Heavy Ion Collisions, standard perturbation techniques break down because of **final state interactions** **(large particle density)**
- **Thermalization, hydrodynamization, jet-quenching, …**
- Many scales in the problem: from the hard partons to the soft constituents of the thermal bath
- Standard approach in jet-quenching: treat jets as perturbations on top of a **classical background field** (QGP)





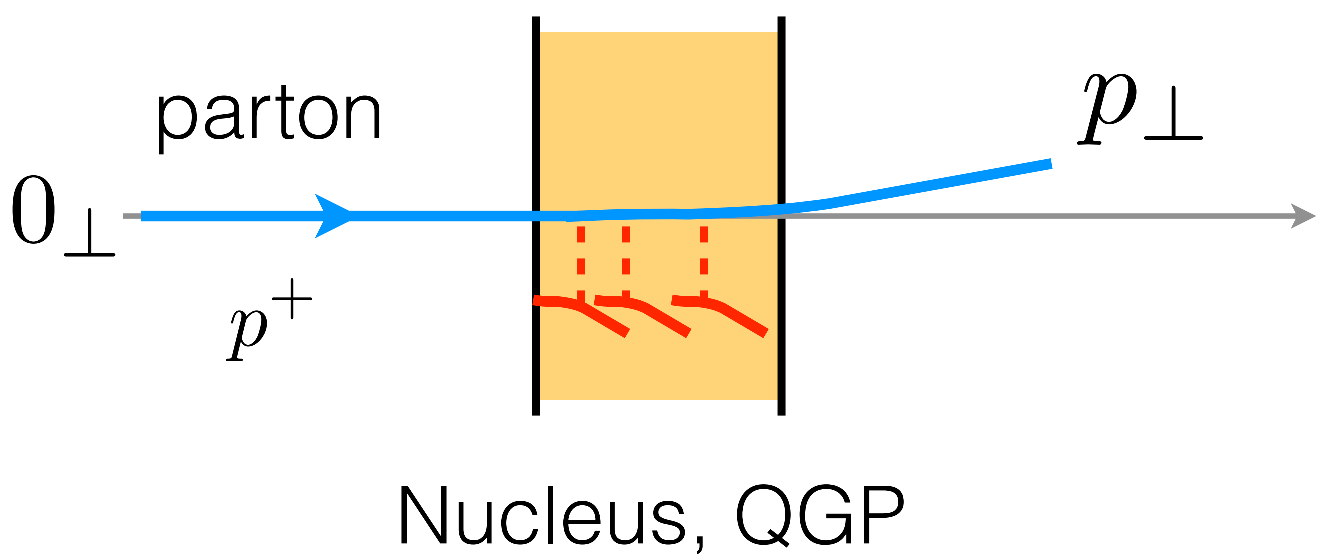
Eikonal trajectories

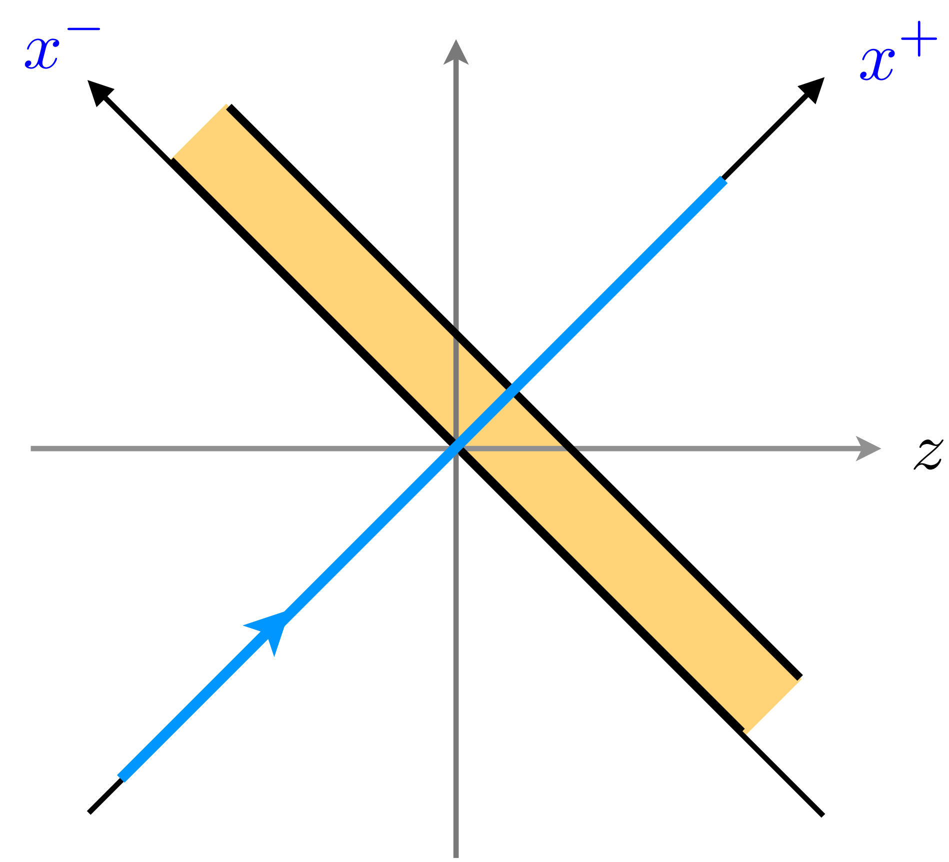


kinematics of high energy scatterings

- We want to compute the amplitude for a high energy parton passing through cold or hot nuclear matter
- In the high energy limit (Regge limit), the projectile is barely deflected from its trajectory

$$\sqrt{s} \sim p_z \gg p_\perp$$

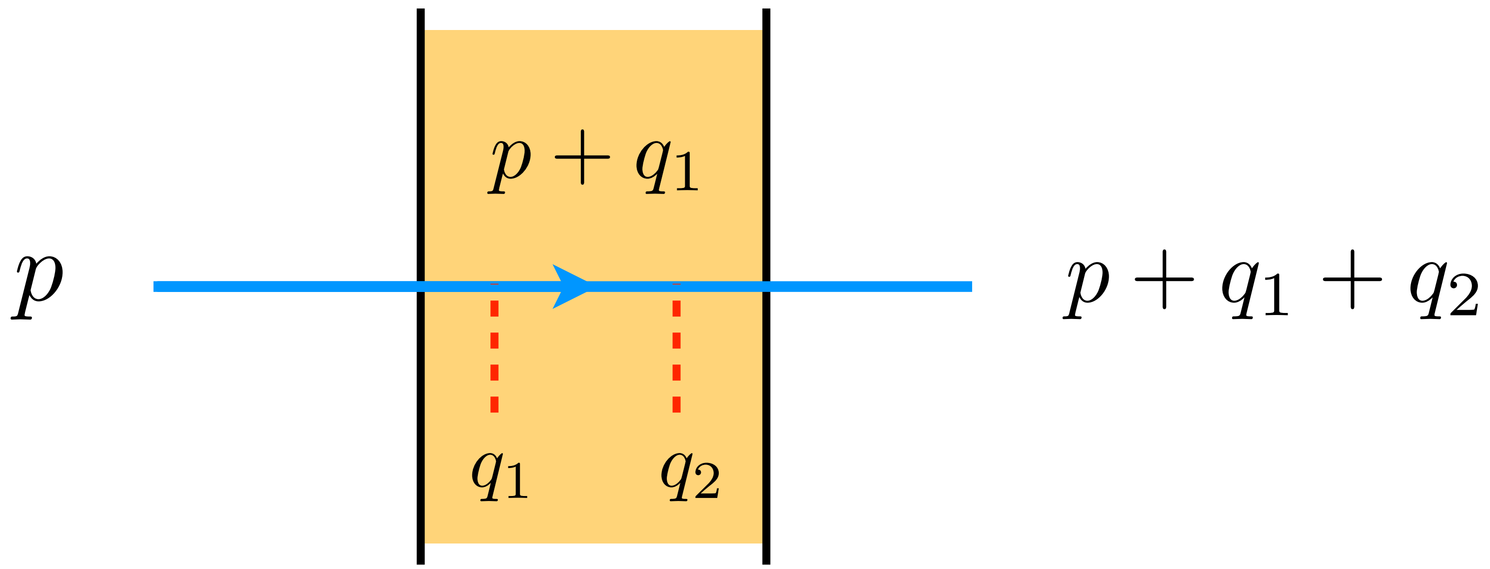




light-cone variables: $p^\pm \equiv (E \pm p_z)/\sqrt{2}$ $\quad x^\pm \equiv (t \pm z)/\sqrt{2}$

Wilson line trajectories

Let us compute the amplitude for high energy double scattering to illustrate the simplifications that follow from the high energy limit



$$(ig)^2 \bar{u}(p) \not{A}(q_1) \frac{(\not{p} + \not{q}_1)}{(p+q_1)^2 + i\epsilon} \not{A}(q_2) u(p + q_1 + q_2)$$

Wilson line trajectories

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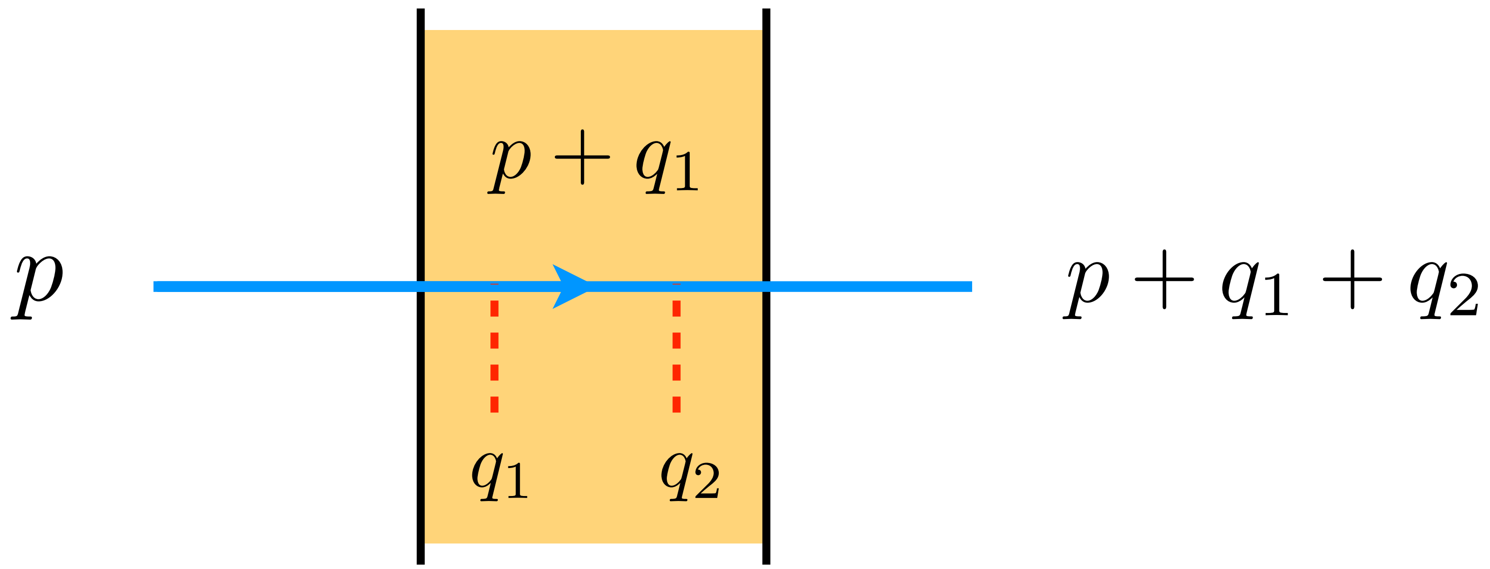

$$(ig)^2\bar{u}(p)\not{A}(q_1)\frac{\sum_s u(p+q_1)\bar{u}(p+q_1)}{(p+q_1)^2+i\epsilon}\not{A}(q_2)u(p+q_1+q_2)$$

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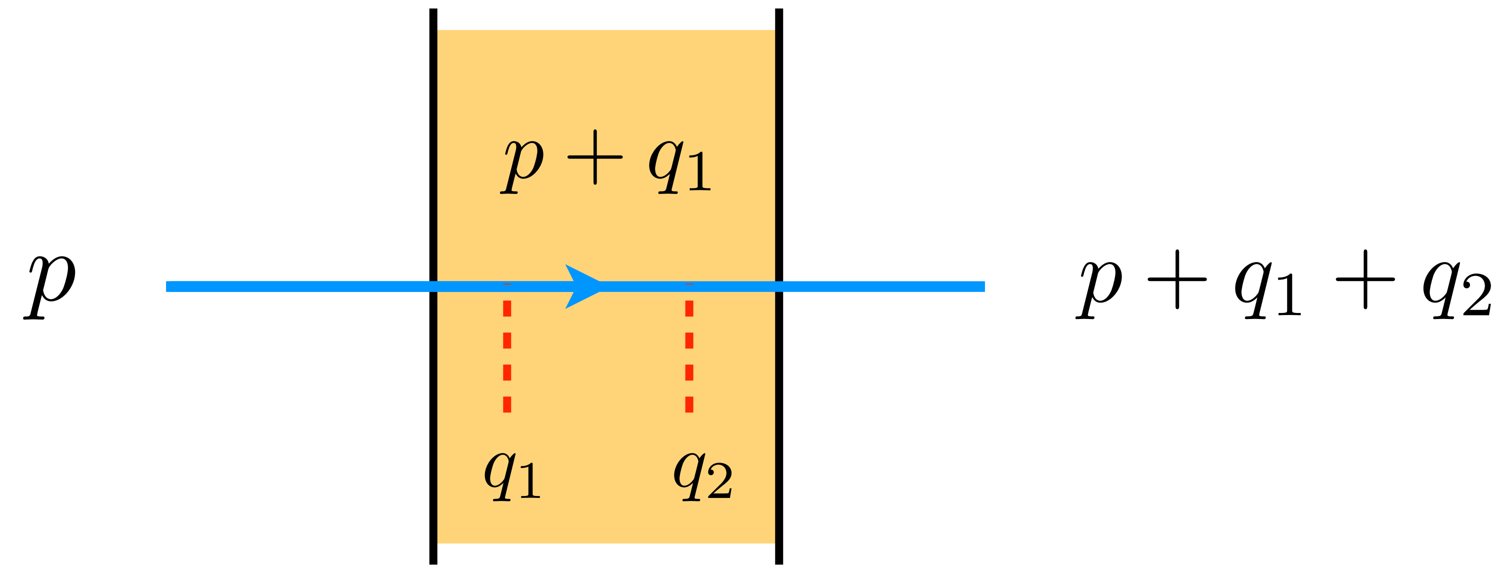

$$(ig)^2 \bar{u}(p) \not{A}(q_1) \frac{\sum_s u(p)\bar{u}(p)}{(p+q_1)^2 + i\epsilon} \not{A}(q_2) u(p+q_1+q_2)$$

Wilson line trajectories

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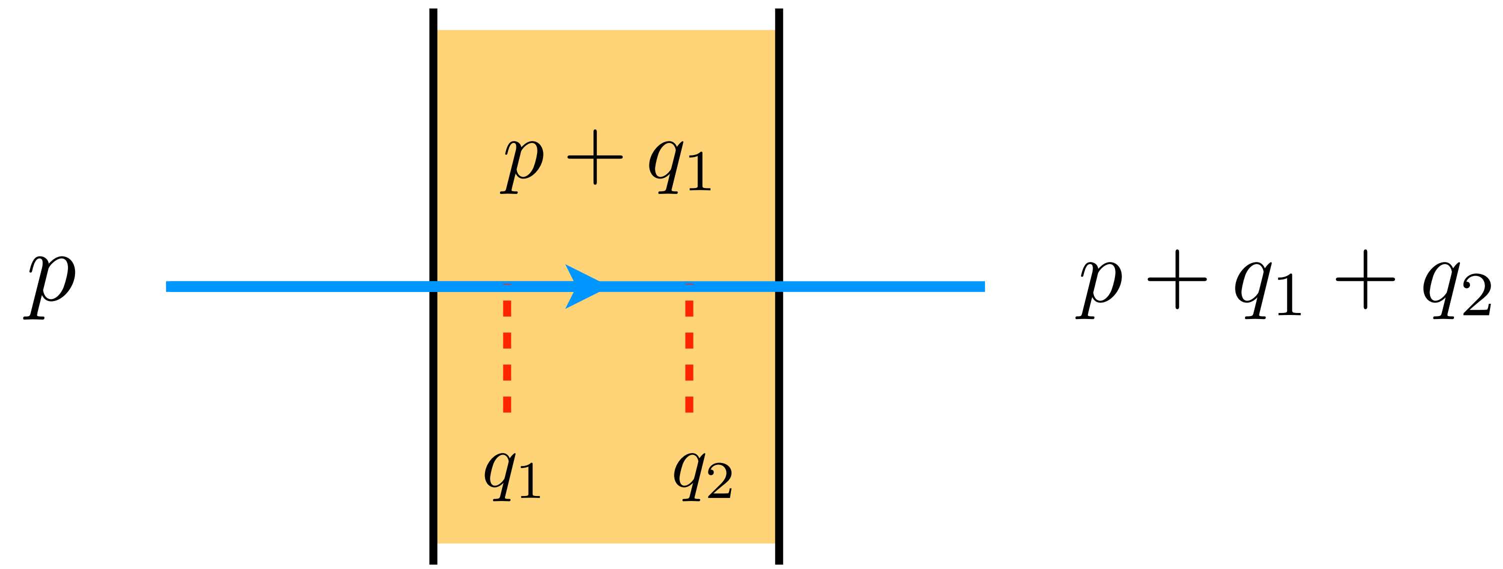


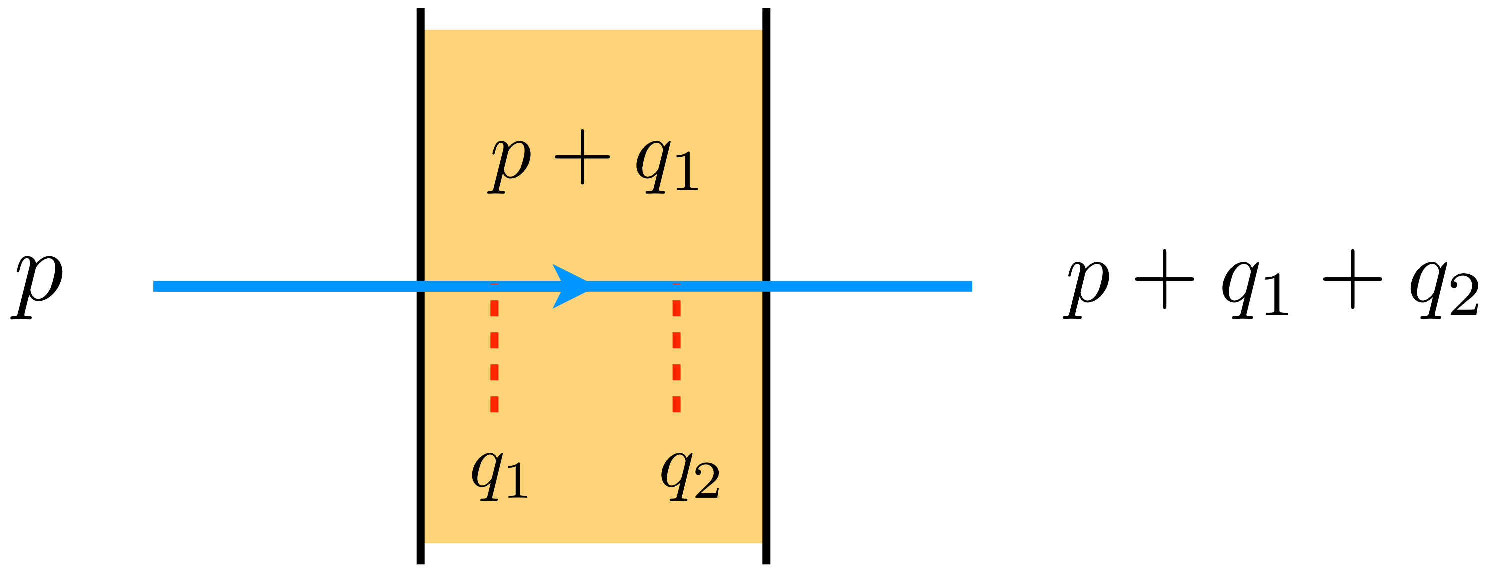
$$(ig)^2 \bar{u}(p) \not{A}(q_1) \frac{\sum_s u(p)\bar{u}(p)}{(p+q_1)^2 + i\epsilon} \not{A}(q_2) u(p+q_1+q_2)$$

using the Gordon's identity and neglecting spin flip:

$$\bar{u}(p) \not{A}(q_1) u(p) \approx 2p^+ A^-(q_1)$$

Wilson line trajectories

Let us compute the amplitude for high energy double scattering to illustrate the simplifications that follow from the high energy limit



$$(ig)^2 \frac{2p^+}{(p+q_1)^- + i\epsilon} A^-(q_1) A^-(q_2)$$

we recognize the Fourier transform of the Heaviside distribution

$$\frac{1}{(p+q_1)^- + i\epsilon} = \int_{-\infty}^{+\infty} \mathrm{d}x^+ \, \theta(x^+) \, \mathrm{e}^{ix^+(p+q_1)^-}$$

Wilson line trajectories

…and after some more algebra we obtain

$$2p^+ \int \mathrm{d}^2\boldsymbol{x} \left[(ig)^2 \int_{-\infty}^{+\infty} \mathrm{d}x_2^+ \int_{-\infty}^{x_2^+} \mathrm{d}x_1^+ A^-(x_1^+, \boldsymbol{x}) A^-(x_2^+, \boldsymbol{x}) \right] \mathrm{e}^{-i(\boldsymbol{q}_1+\boldsymbol{q}_2)\cdot\boldsymbol{x}}$$

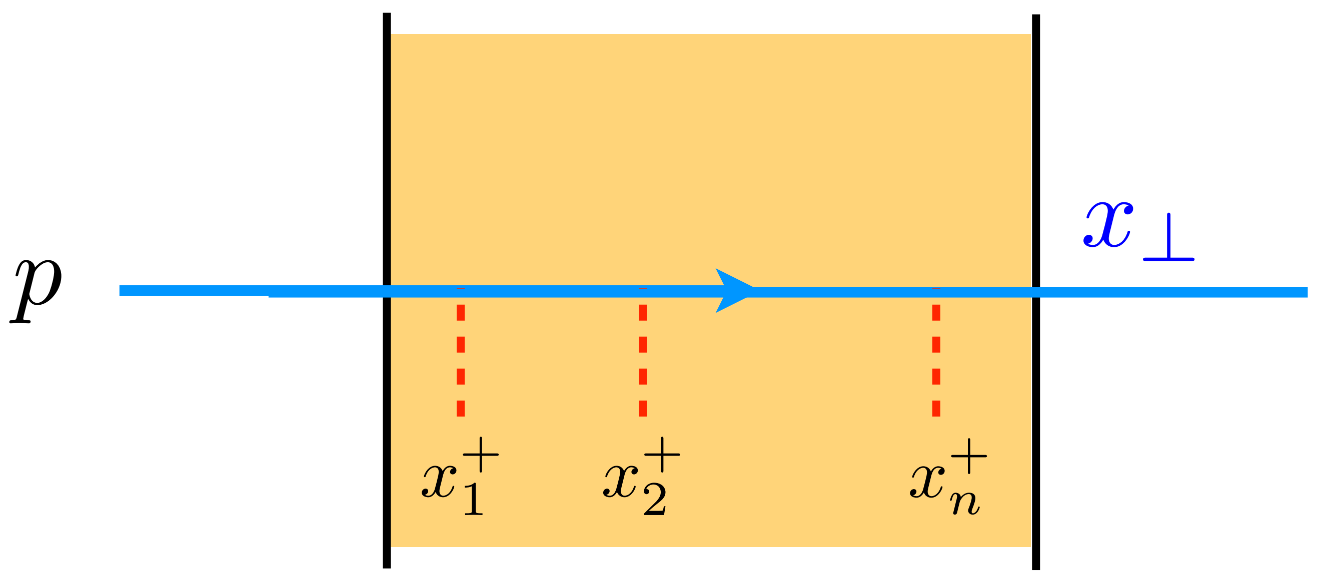
summing over arbitrary numbers of path ordered scatterings the quantity between brackets can be expressed as

$$\boxed{U(\boldsymbol{x}) \equiv \mathcal{P}_+ \exp\left[ig \int_{-\infty}^{+\infty} \mathrm{d}x^+ A_a^-(x^+, \boldsymbol{x}) T^a \right]}$$

Wilson line trajectories

$$U(\boldsymbol{x}) \equiv \mathcal{P}_+ \exp\left[ig\int_{-\infty}^{+\infty} \mathrm{d}x^+ A_a^-(x^+,\boldsymbol{x})T^a\right]$$

This Wilson line encodes the **color precession** of a high energy parton along its **straight line trajectory**



it is the building block of high energy scattering processes:
DIS, hadron production in proton-nucleus, Jet-quenching…

Beyond eikonal in-medium propagation

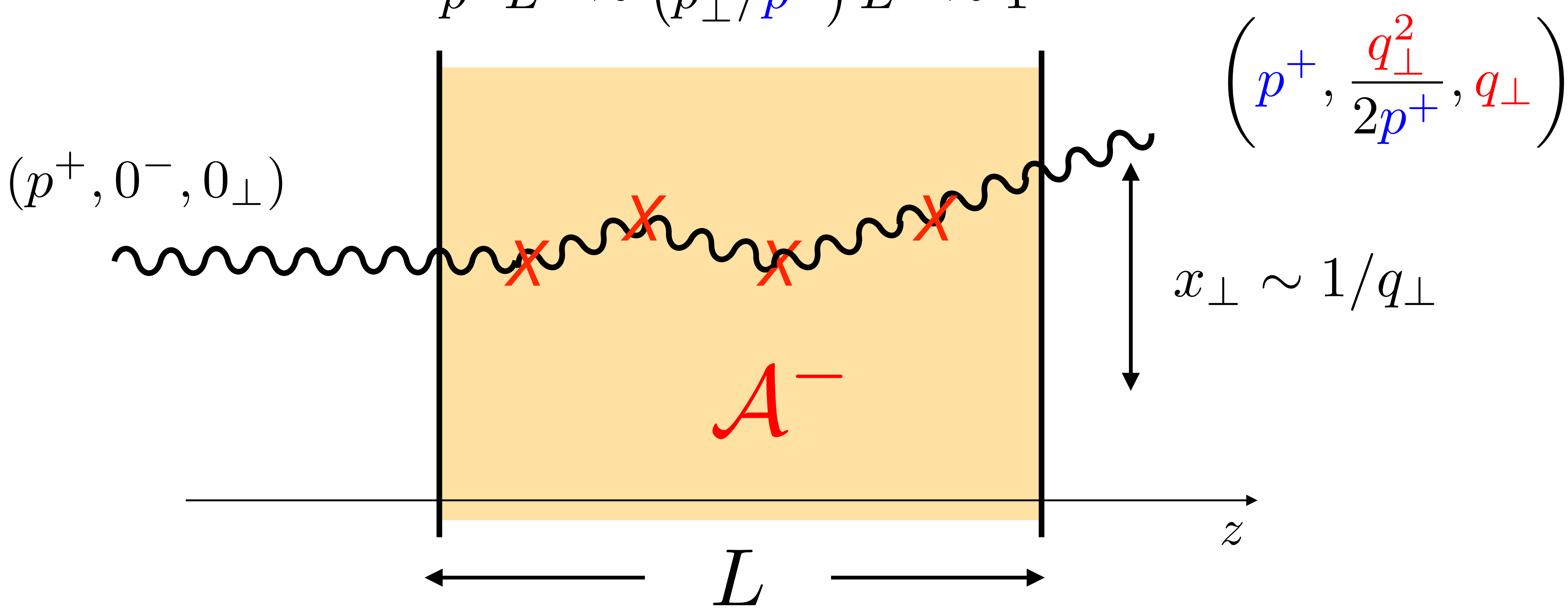
Non-eikonal propagation

- **The scattering remains eikonal:** neglect power corrections of the **small momentum transfer** $q^+ \ll p^+$:

$$\text{eikonal vertex} \sim \delta(q^+)\, p^\mu \;\Leftrightarrow\; \mathcal{A}^-(x^+, x_\perp)$$

- **Large medium:** but allow the gluon to **explore the transverse plan** between two scatterings

$$p^- L^+ \sim \left(p_\perp^2/p^+\right) L^+ \sim 1$$



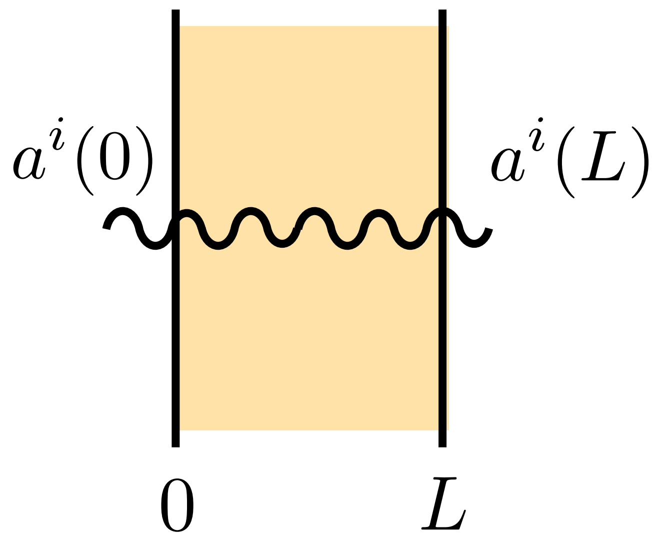
Gluon propagator in light-cone gauge

We are interested in the propagation of **small fluctuations** on top of the **medium background field**:

$$A^\mu \equiv \mathcal{A}^\mu + a^\mu$$

In the light-cone gauge, $A^+ = 0$, the **linearized** Yang-Mills equation $[D_\mu, F^{\mu\nu}] = 0$ yields

$$a^i(L) = \int_{y_\perp, y^-} G_R(x,y)\, \partial^+ a^i(0)$$



where the retarded propagator (diagonal in helicity)

$$\left[\Box_x + 2ig\left(\mathcal{A}^- \cdot T\right)\partial_x^+\right] G_R(x,y) = 0$$

Gluon propagator in light-cone gauge

The background field explicitly breaks boost invariance. Therefore, it is more convenient to work in a mixed representation

$$(p^+, p^-, p_\perp) \to (p^+, x^+, p_\perp)$$

As a result the dynamics is equivalent to a **2-D non-relativistic quantum mechanics.** The relevant Green's function reads

$$\mathcal{G}_{p^+}(p_\perp, t\,|\,p'_\perp, t') \equiv \int_{\boldsymbol{x},x^-} G_R(x,y)\ \mathrm{e}^{i(x-y)^- p^+ - i\boldsymbol{x}\cdot\boldsymbol{p}}$$

which obeys a Schrödinger-like equation, with $t \equiv x^+$

$$\left[i\partial_t + \frac{\boldsymbol{p}_\perp^2}{2p^+} + g\,\mathcal{A}\cdot T\otimes\right]\mathcal{G}_{p^+}(\boldsymbol{p}_\perp, t\,|\,\boldsymbol{p}'_\perp, t') = 1$$

Gluon propagator in light-cone gauge

The solution can be written formally as a path integral

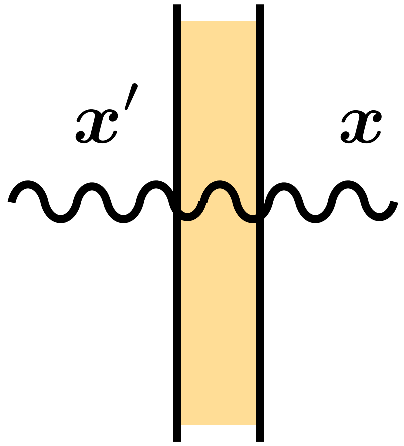
$$\boxed{\mathcal{G}_{p^+}(\boldsymbol{x}_\perp, t \,|\, \boldsymbol{x}'_\perp, t') = \int_{\boldsymbol{x}}^{\boldsymbol{x}'} \mathcal{D}\boldsymbol{r}\, \mathrm{e}^{i\int_t^{t'} \mathrm{d}\xi (\dot{\boldsymbol{r}}(\xi))^2}\; U[\mathcal{A}(\xi, \boldsymbol{r}(\xi))]}$$

$\Rightarrow$ the free propagator in Fourier space is simply

$$\mathcal{G}^0_{p^+}(\boldsymbol{p}_\perp, t - t') = \mathrm{e}^{i\frac{\boldsymbol{p}_\perp^2}{2p^+}(t-t')}$$

$\Rightarrow$ in the limit $L \to 0$ or $p^+ \to \infty$ one recovers the straight-line trajectory

$$\mathcal{G}_\infty(\boldsymbol{x}_\perp, t \,|\, \boldsymbol{x}'_\perp, t') = \frac{1}{2}\delta(\boldsymbol{x} - \boldsymbol{x}')\, U(\boldsymbol{x})$$



Medium average

- Owing to large number of medium DOF, a great simplification comes from a **statistical treatment**
- Moreover, in weakly coupled plasmas for which the **correlation length** is much smaller than the **mean free path**:

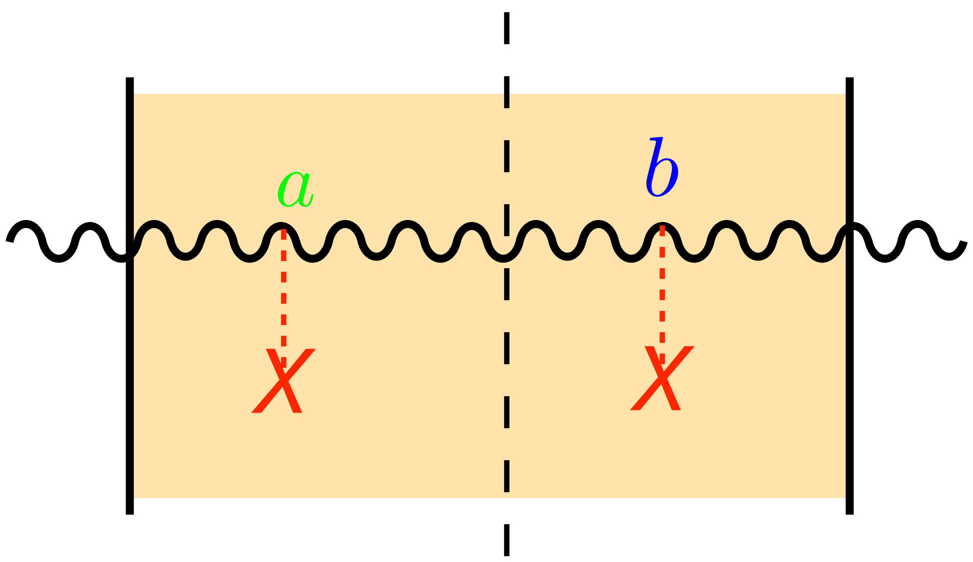
$$m_D^{-1} \ll \ell_{\mathrm{mfp}} \ll L$$

$$m_D^{-1} \sim (gT)^{-1}$$

$$\ell_{\mathrm{mfp}} \sim (g^2T)^{-1}$$

- Hence, the medium field can be treated as a **Gaussian random variable**:

$$\langle\, \mathcal{A}^a(\boldsymbol{p},t)\mathcal{A}^b(\boldsymbol{p}',t')\,\rangle \equiv \delta^{ab}\delta(t-t')\delta(\boldsymbol{p}-\boldsymbol{p}')\gamma(\boldsymbol{p})$$

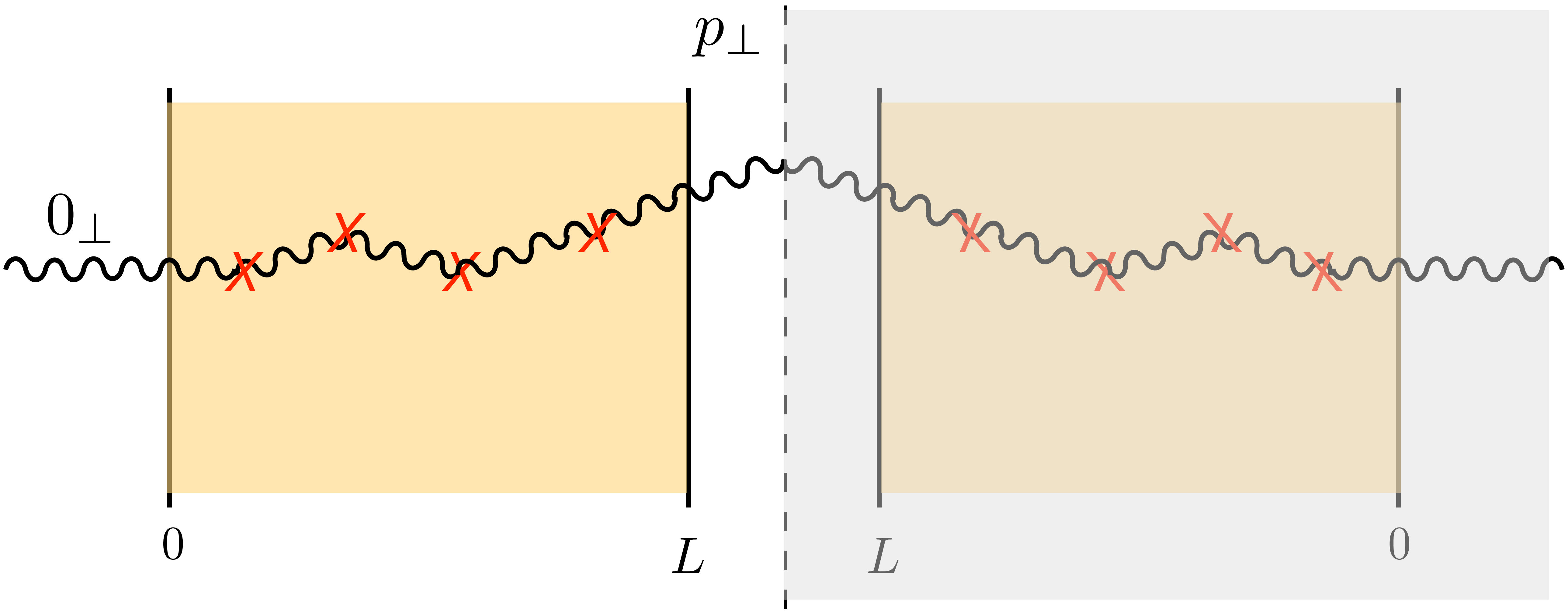


related to the (2 to 2) elastic cross-section

$$\gamma(\boldsymbol{q}) \sim \frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}} = \frac{\alpha_s^2}{\boldsymbol{q}^4}$$

pt-broadening

- As an application one could compute the pt broadening probability for a high energy quark (gluon) passing through a medium



amplitude

c.c. amplitude

pt-broadening

- As an application one could compute the pt broadening probability for a high energy quark (gluon) passing through a medium

- One finds that the broadening probability is related to the 2-point function correlator (forward dipole scattering amplitude)

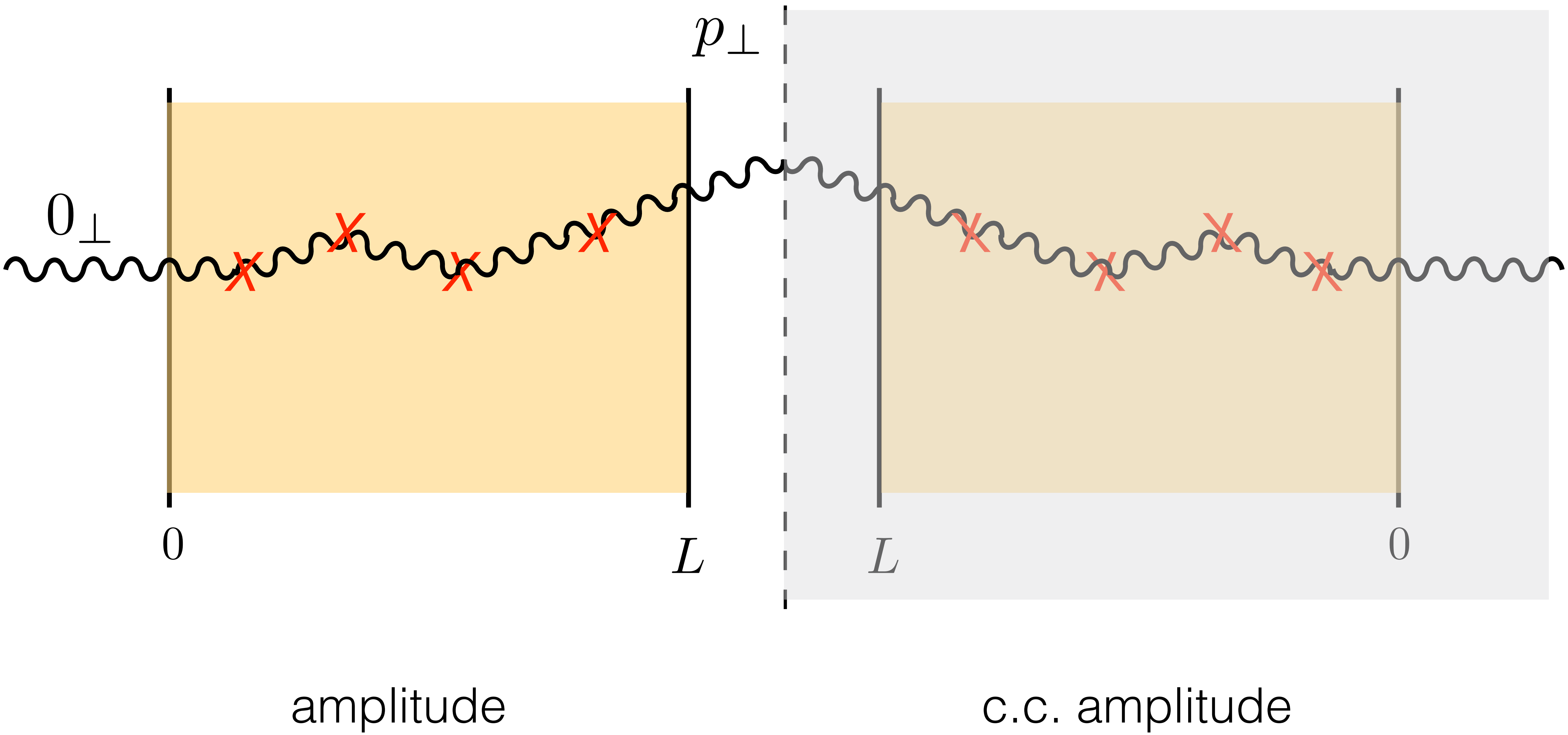
$$\mathcal{P}(\boldsymbol{p}) \equiv \frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \int_{\boldsymbol{x}} \langle \mathrm{Tr}\, U(0) U^\dagger(\boldsymbol{x}) \rangle \, \mathrm{e}^{i\boldsymbol{x}\cdot\boldsymbol{p}}$$

- Recall the path ordered Wilson-line reads

$$U(\boldsymbol{x}) \equiv \mathcal{T}_+ \exp\left[ig \int_0^L \mathrm{d}t \, \mathcal{A}(t, \boldsymbol{x}) \cdot T \right]$$

pt-broadening (medium average)

- Because of the equal time correlation function it is convenient to fold the amplitude and the complex conjugate amplitude



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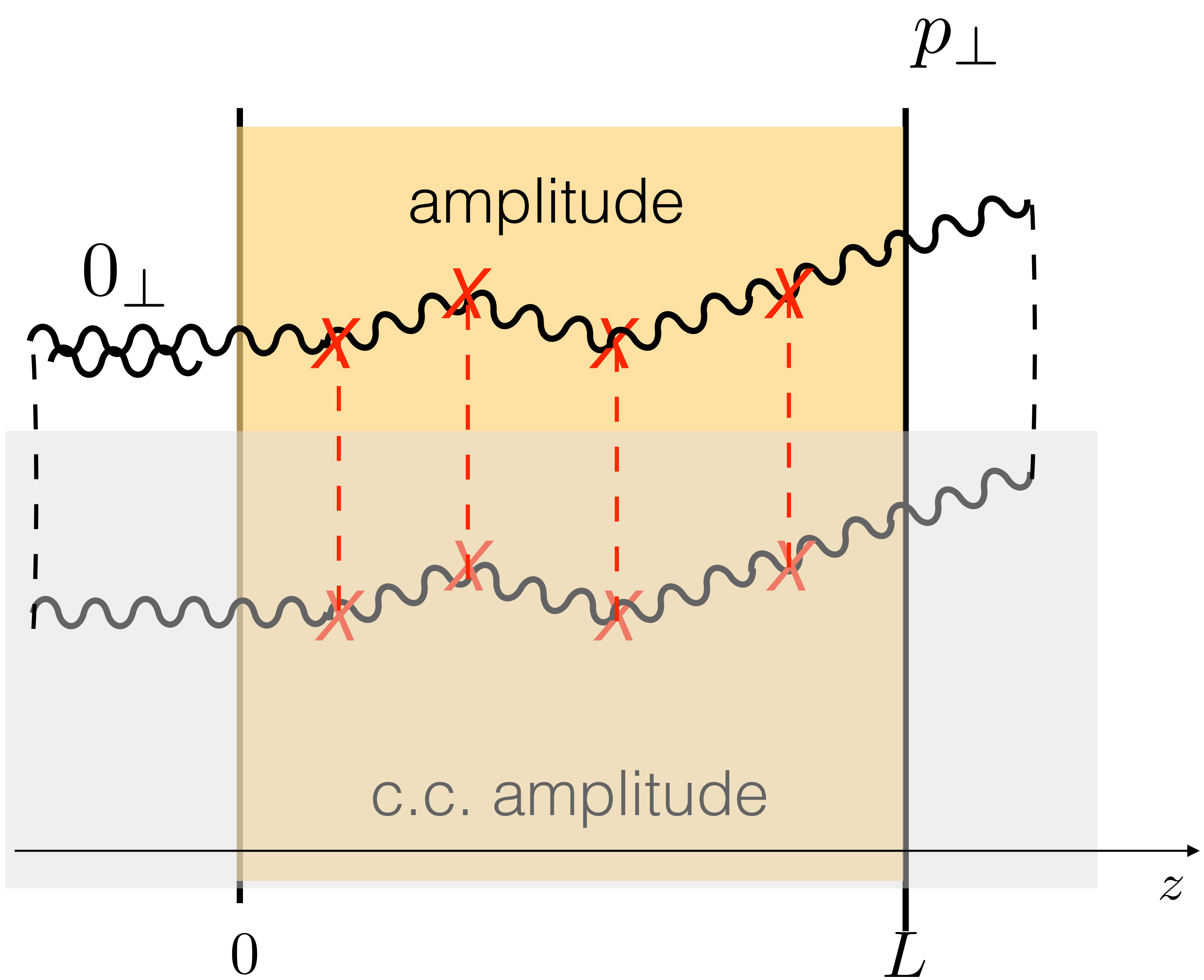
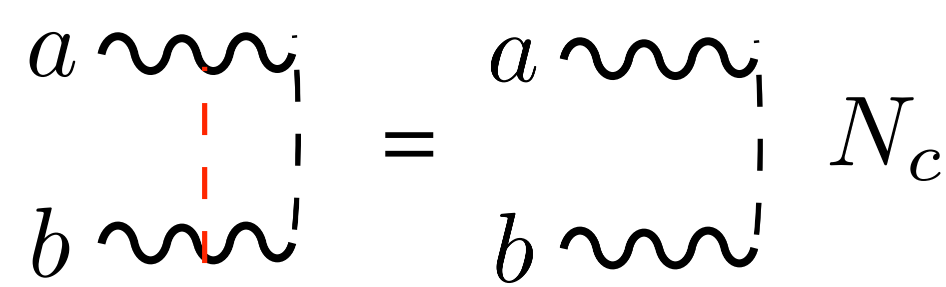


- Color algebra is trivial: each vertical line yields an N_c color factor



$$(T^c T^c)^{ab} = \delta^{ab}\, N_c$$

pt-broadening

- The average over the medium configurations yields the **Master Equation:**

$$\partial_t \mathcal{P}(\boldsymbol{p},t) = -\int \mathrm{d}^2\boldsymbol{q}\,\gamma(\boldsymbol{q})\,\mathcal{P}(\boldsymbol{p}-\boldsymbol{q},t) + \int \mathrm{d}^2\boldsymbol{q}\,\gamma(\boldsymbol{q})\,\mathcal{P}(\boldsymbol{p},t)$$

- which, to gain more insight, can be reduced to a Fokker-Planck equation **(diffusion in momentum space)** in the regime of multiple soft scattering: $q_\perp \ll p_\perp$

$$\boxed{\partial_t \mathcal{P}(\boldsymbol{p},t) = \frac{1}{4}\hat{q}\,\nabla^2_{\boldsymbol{q}}\mathcal{P}(\boldsymbol{p},t)}$$

- where the diffusion coefficient, $\hat{q}$, is the so-called **jet-quenching parameter,** which encodes medium properties

$$\boxed{\hat{q} \equiv \frac{\langle p_\perp^2\rangle}{L} = \int_{\boldsymbol{q}} \boldsymbol{q}^2\gamma(\boldsymbol{q}) = \frac{g^4 N_c n}{4\pi}\ln\frac{Q^2}{m_D^2}}$$

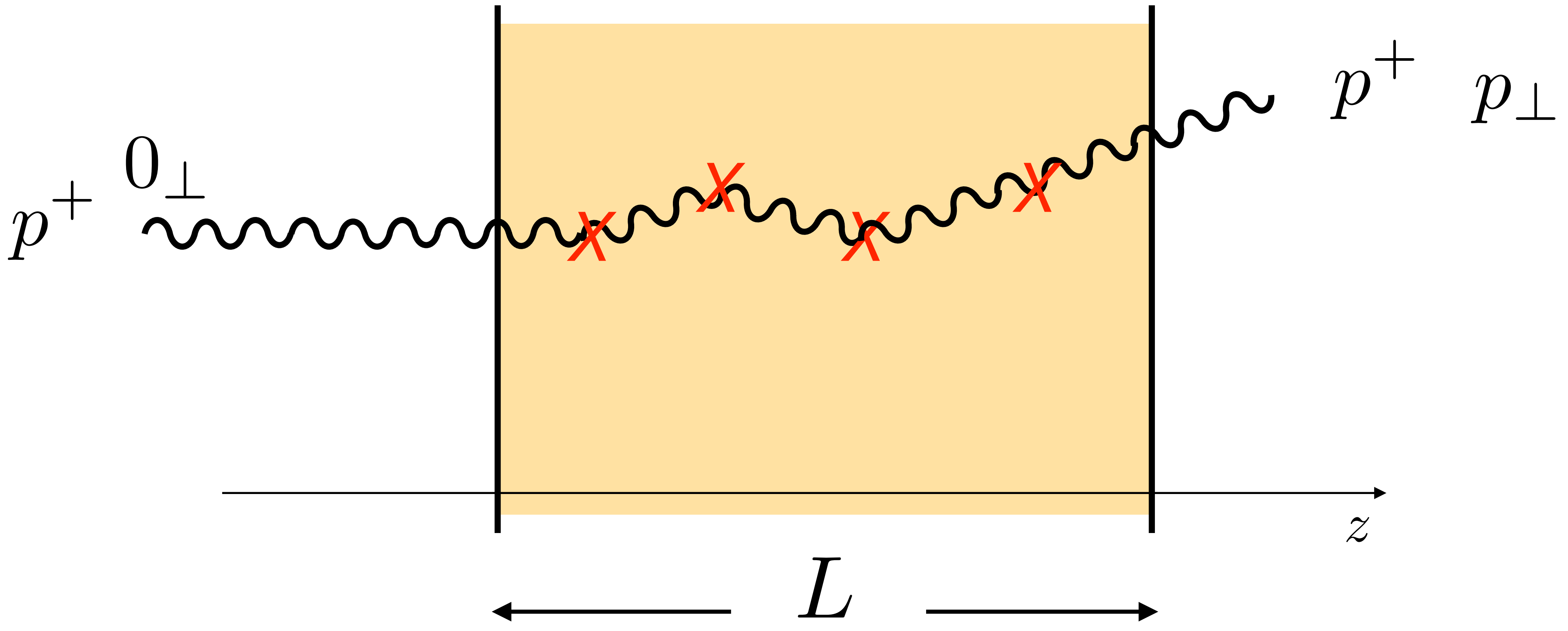
pt-broadening

- The broadening probability in the regime $p_\perp \lesssim \hat{q}L$

$$\boxed{\mathcal{P}(p_\perp, L) \equiv \frac{4\pi}{\hat{q}\,L}\mathrm{e}^{-\frac{p_\perp^2}{\hat{q}\,L}}}$$

typical transverse momentum squared

$$\langle p_\perp^2 \rangle_{\rm typ} \equiv \hat{q}L$$

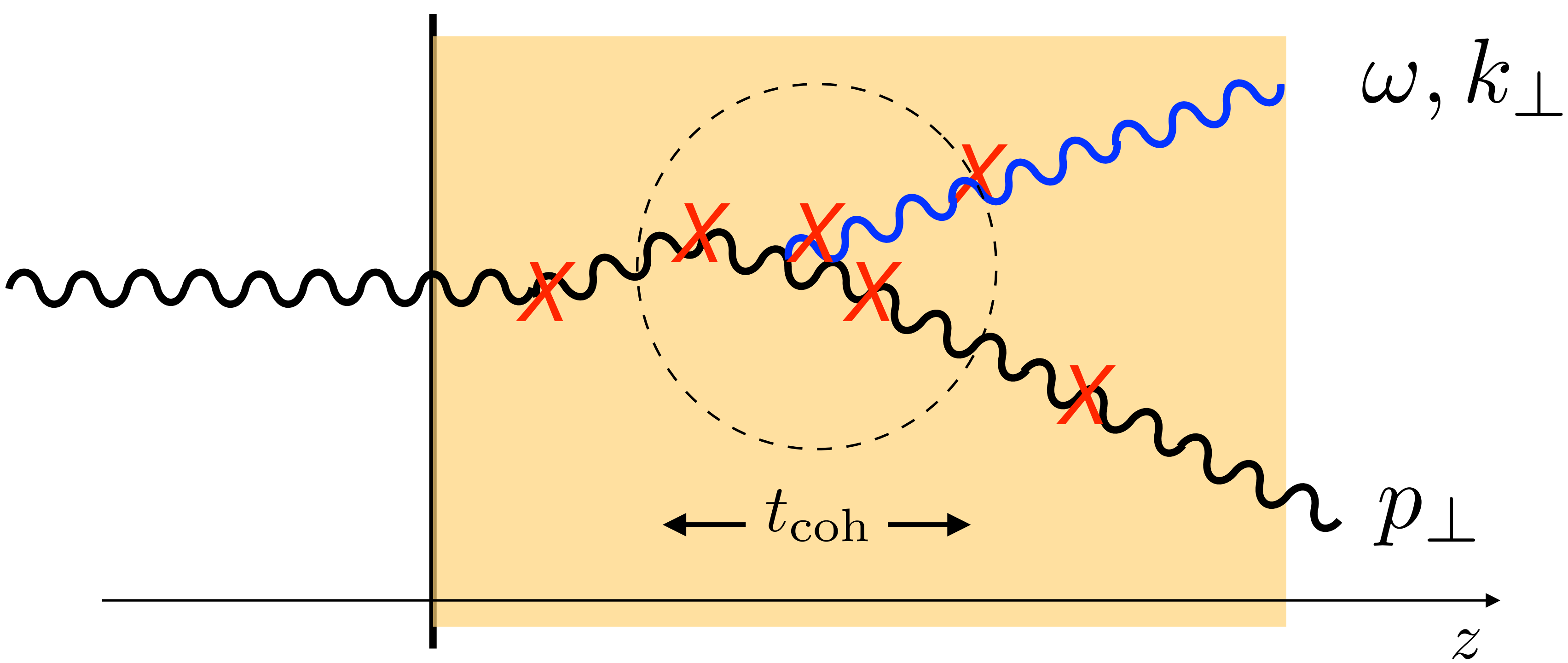


In-medium gluon cascade and jet-quenching

Medium-induced splittings

- Multiple scattering can trigger gluon radiation
- **Laudau-Pomeranchuk-Migdal effect:** during the splitting time many scattering centers act coherently as a single one and thus, suppressing the radiation rate

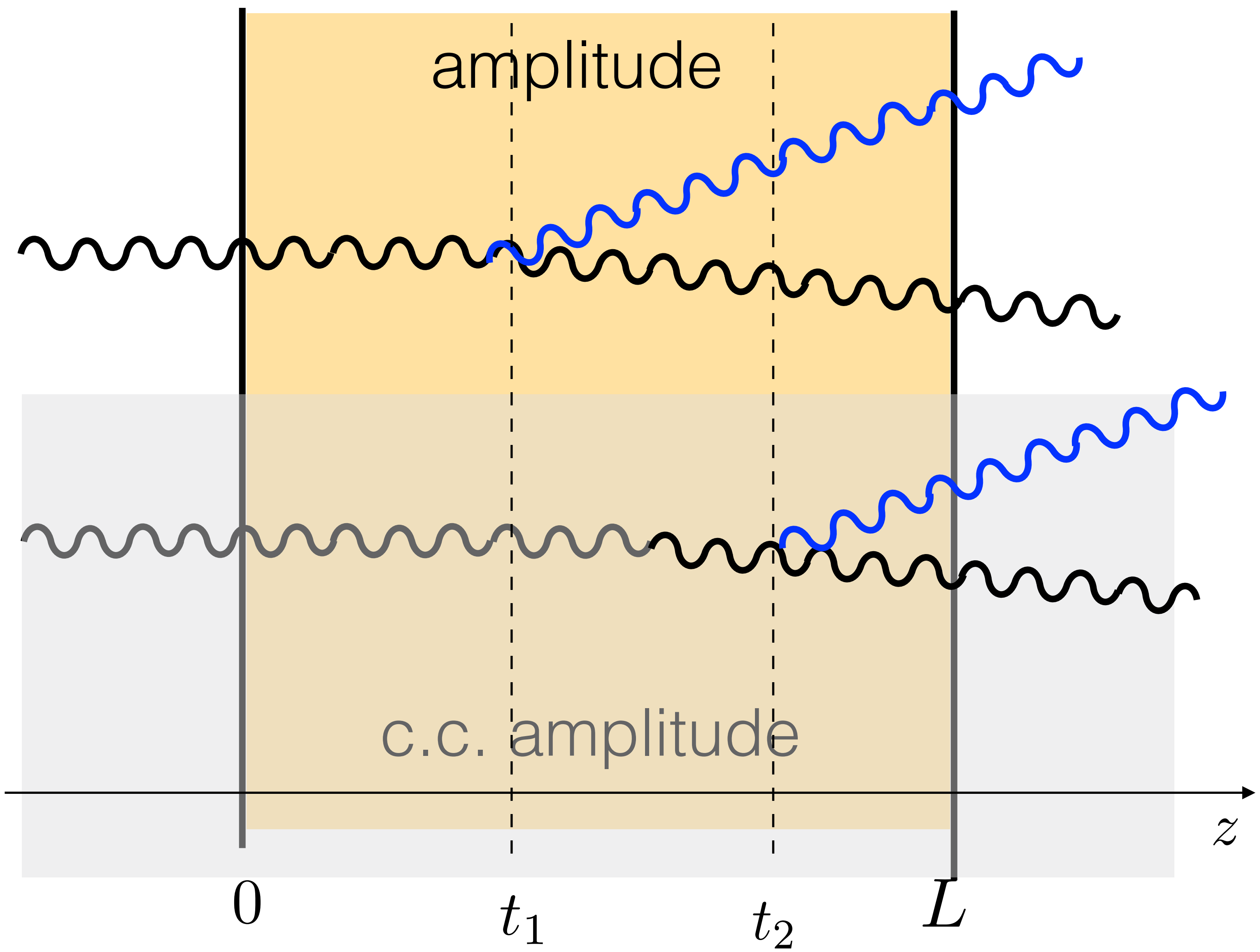
$$t_{\rm coh} = \frac{\omega}{k_\perp^2} \sim \frac{\omega}{\hat{q}\, t_{\rm coh}} \qquad \Rightarrow \qquad t_{\rm coh} \sim \sqrt{\frac{\omega}{\hat{q}}}$$



[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

Medium-induced splittings

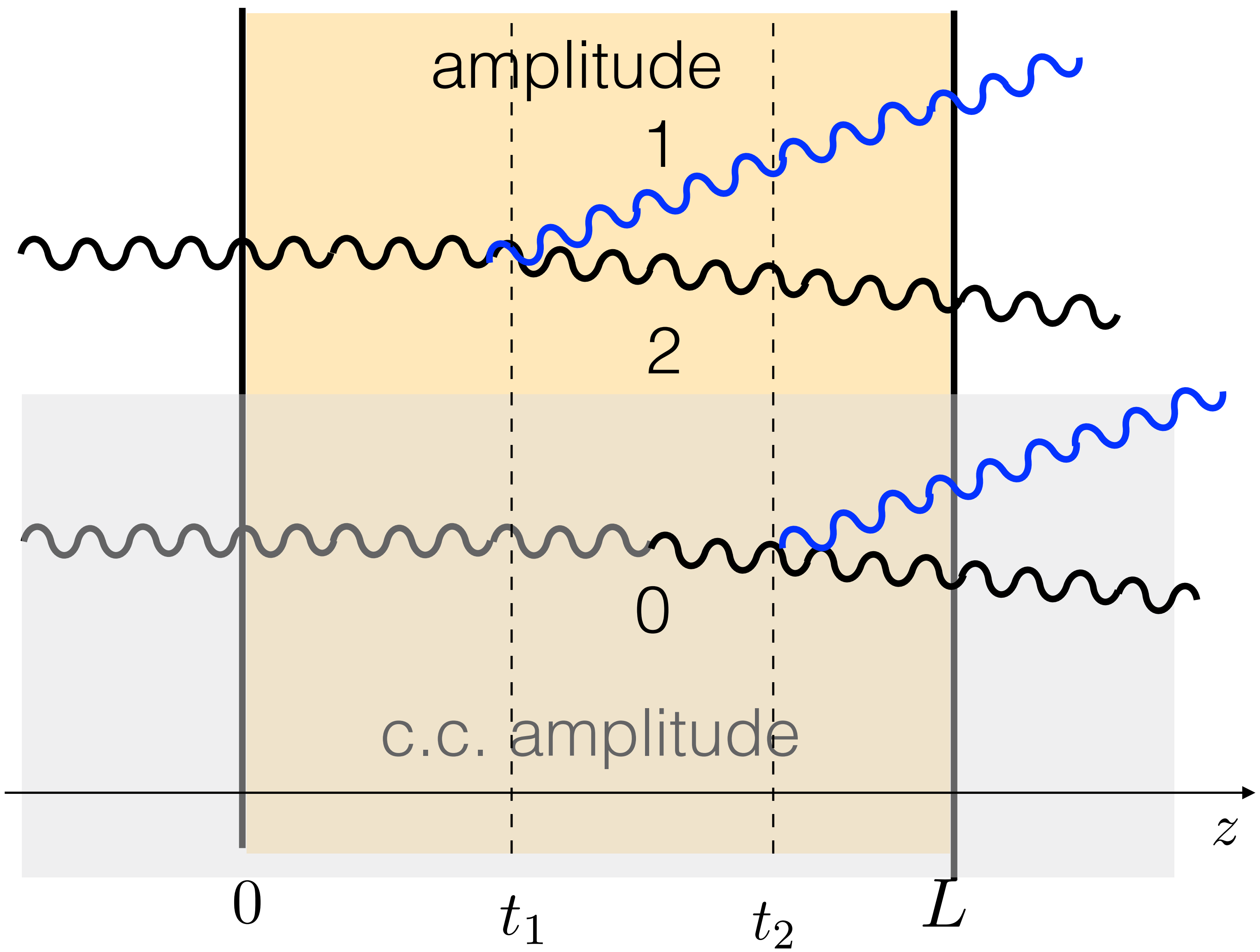
- Going back to folding the complex conjugate amplitude on the amplitude we see that one has to integrate over the difference between the radiation time in the amplitude and complex conjugate



Medium-induced splittings

- It follows that the splitting kernel depends on the correlator of 3 Wilson lines

$$\mathcal{K}(t_1 - t_2) \sim \langle \, \mathrm{Tr} \, \mathcal{G}_1^{ab} \mathcal{G}_2 \, T^b \, \mathcal{G}_0^\dagger \, T^a \, \rangle$$



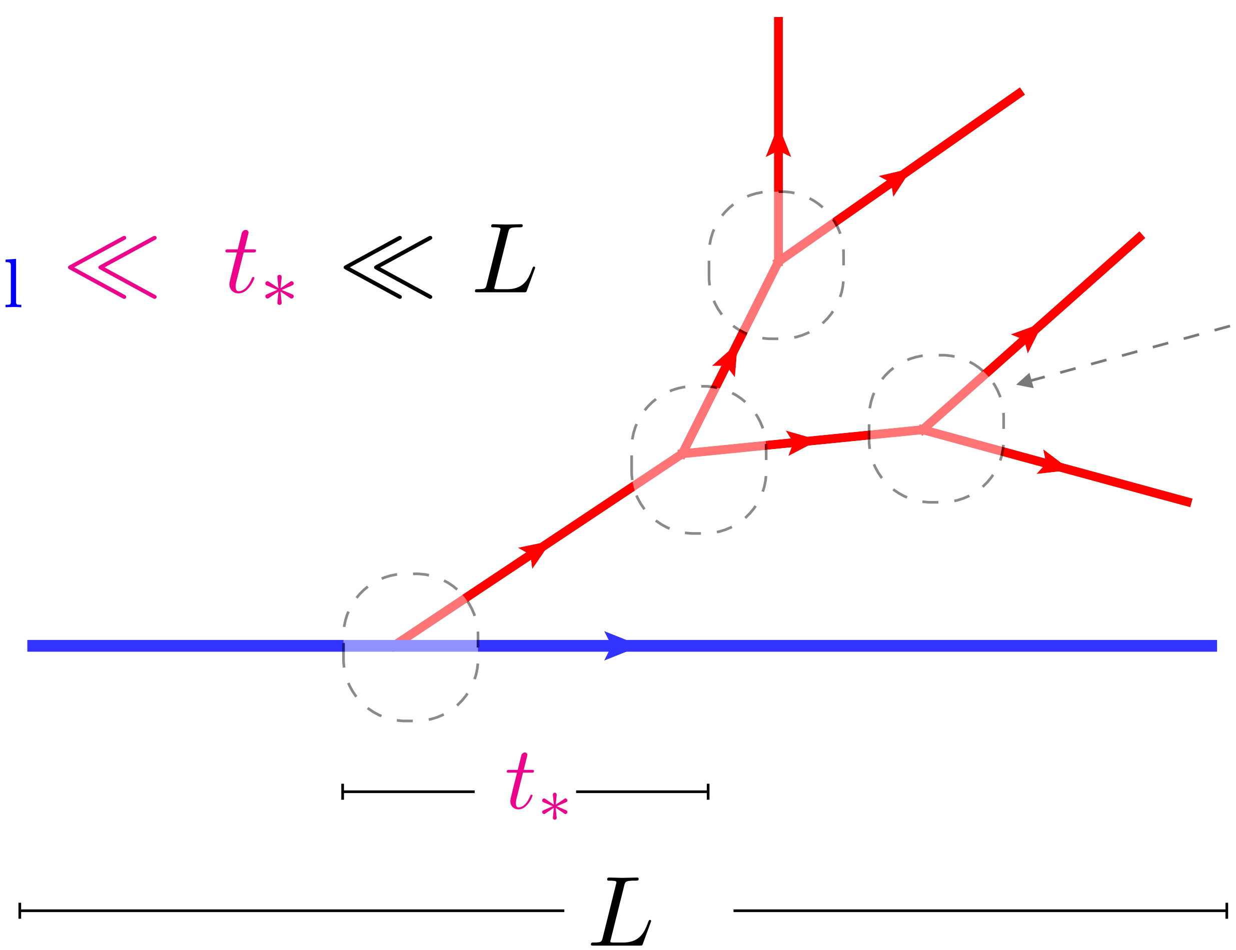
In-medium gluon cascade

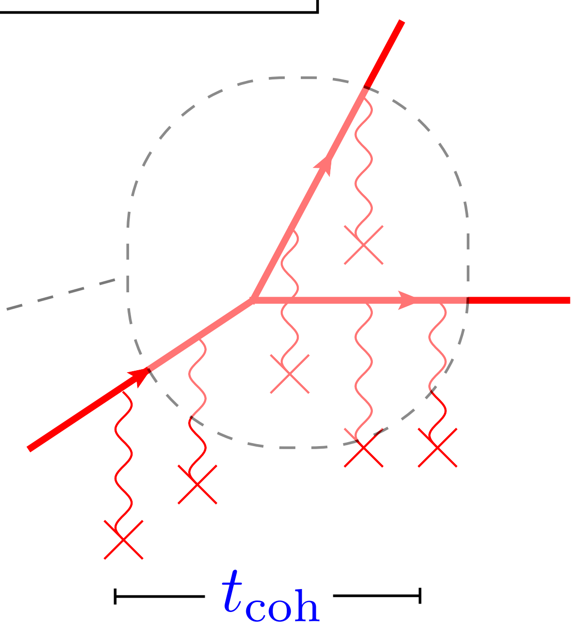
- **Probabilistic picture:** large probability for **soft, rapid and independent multiple gluon branching**

$$\boxed{\omega \frac{\mathrm{d}P}{\mathrm{d}\omega \mathrm{d}t} \equiv \frac{\alpha_s}{t_{\mathrm{coh}}} \equiv \frac{1}{t_*}}$$

$$\boxed{t_*(\omega) = \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}}}$$

$$t_{\mathrm{col}} \ll t_* \ll L$$

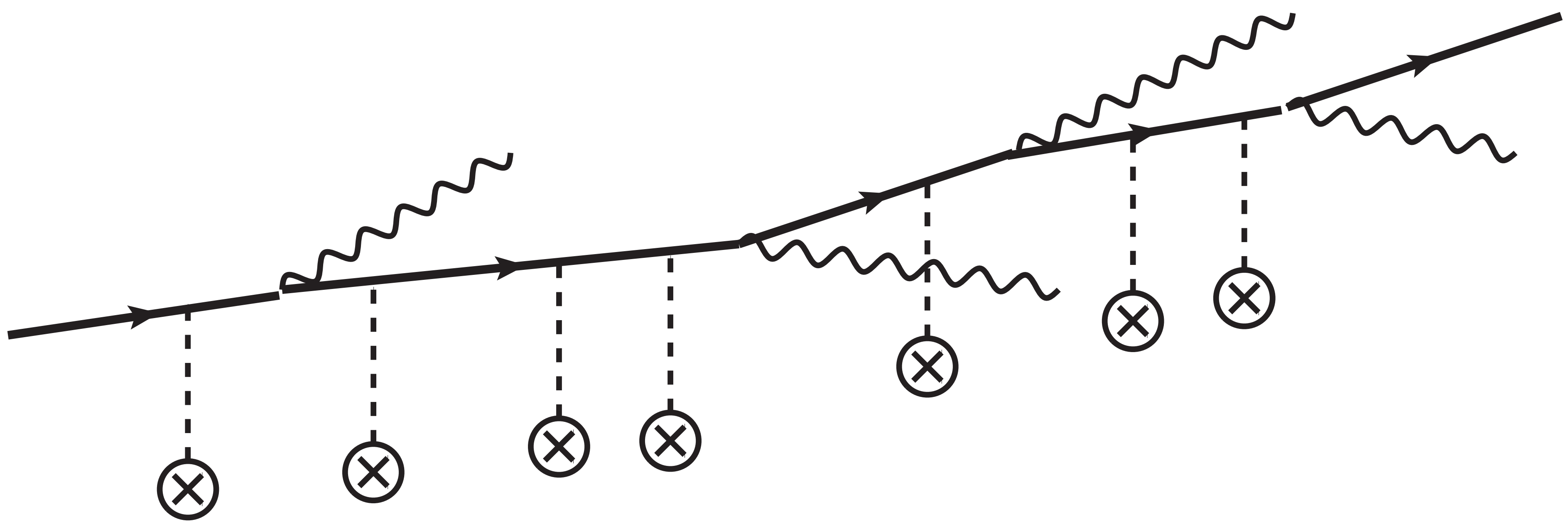




[Baier, Dokshitzer, Schiff, Mueller (2001)
Jeon, Moore (2003)
Blaizot, Dominguez, Iancu, MT (2013-2014)]

In-medium gluon cascade

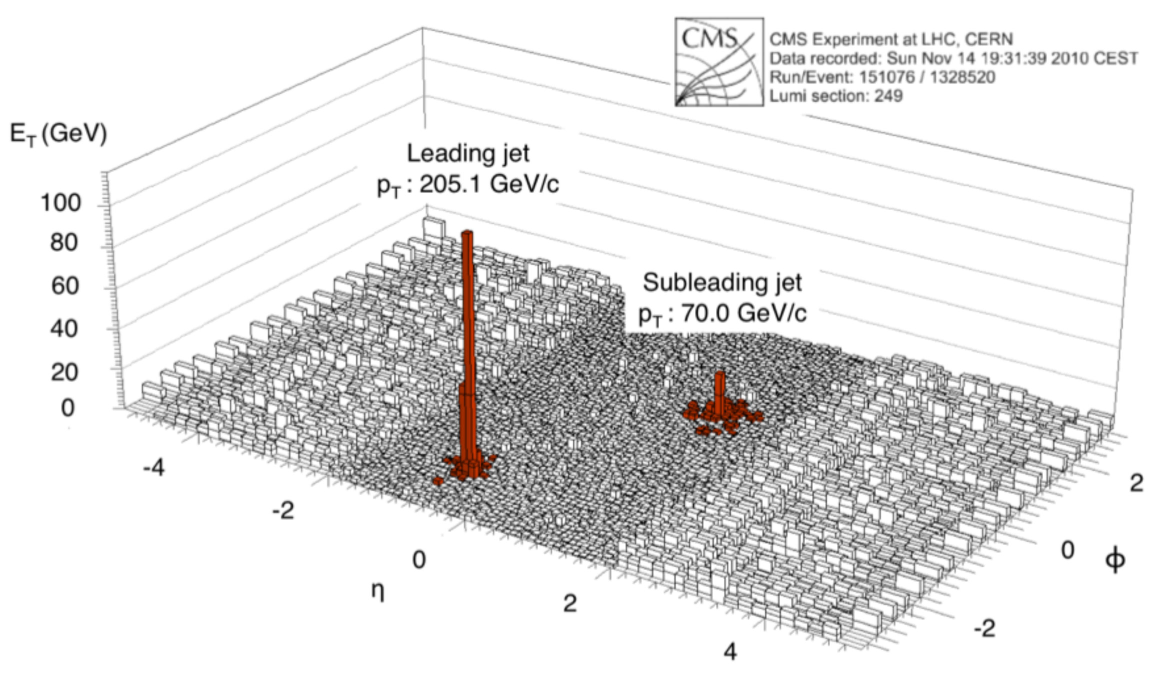
- Hence, in addition to **momentum broadening**, a high energy parton undergoes **inelastic scattering**



- For large media, multiple soft gluon radiation is the dominant mechanism for energy loss and causes jet quenching

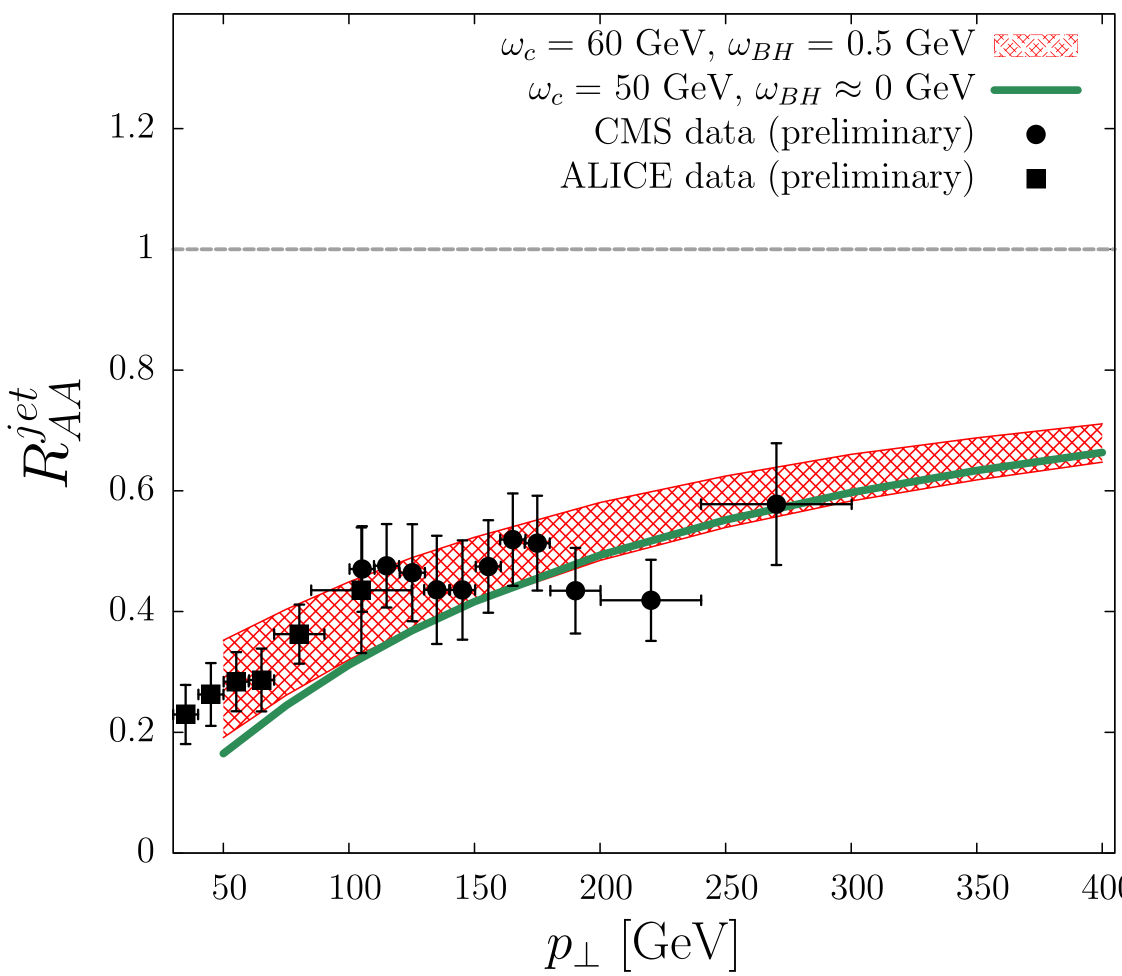
Jet-quenching: where does energy go?

- Strong suppression of jets have been observe at the LHC (and previously at RIHC)
- CMS collaboration recently recovered the missing energy at very large angles



- Nuclear modification factor about 0.5 for 300 GeV jets

$$R_{AA} \equiv \frac{1}{N_{\rm coll}} \frac{{\rm d}N_{AA}}{{\rm d}N_{pp}}$$

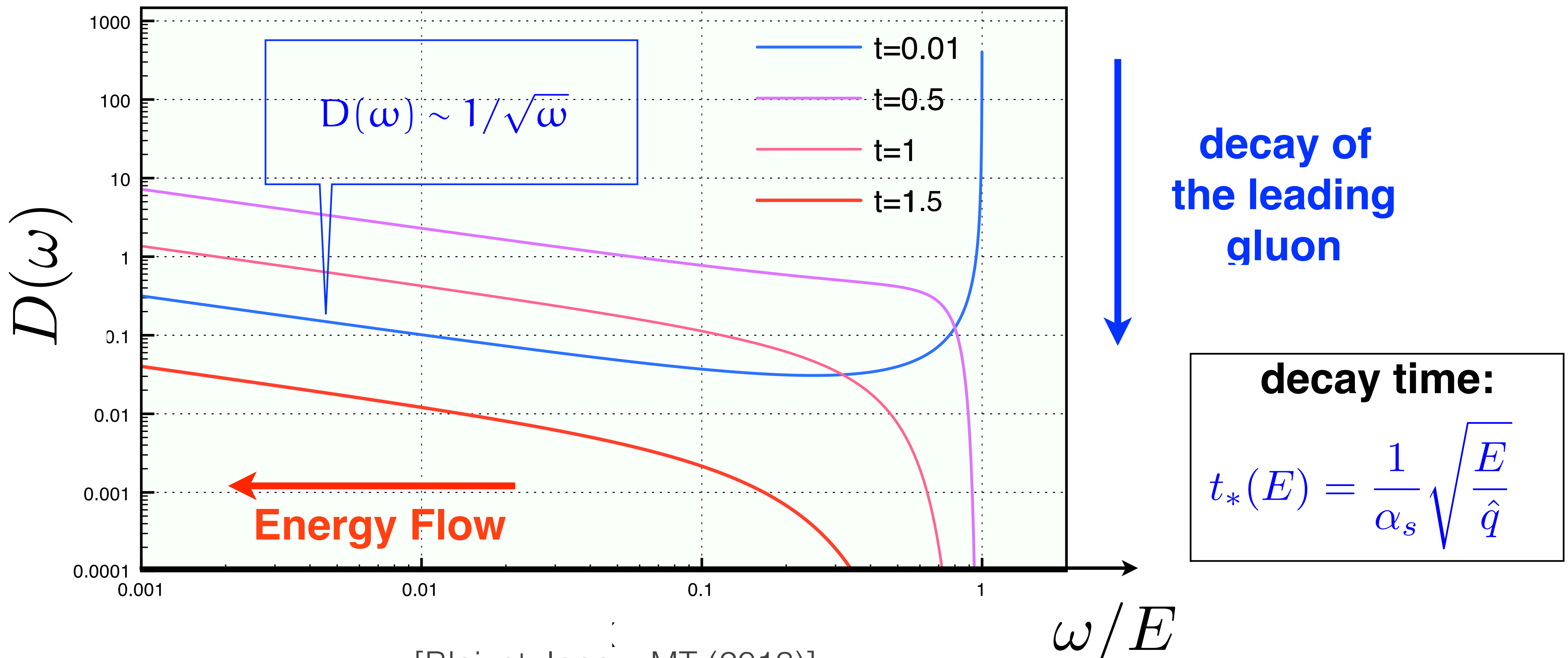


Jet-quenching: where does energy go?

- Initially we have a single gluon with energy E

$$D(\omega) \equiv \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} = E\,\delta(\omega - E)$$

- **In-medium cascade** $\longrightarrow$ time evolution of the energy distribution



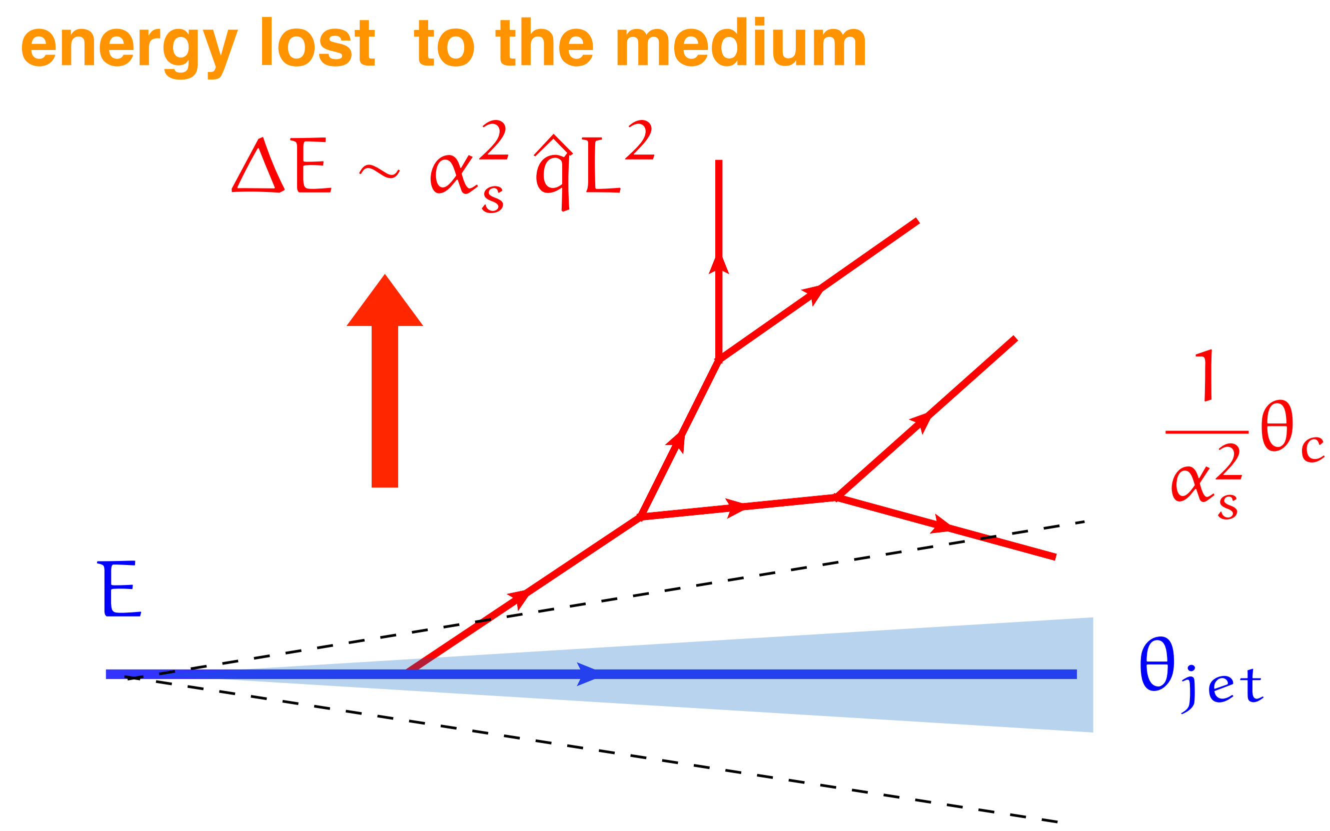
[Blaizot, Iancu, MT (2013)]

Jet-quenching: where does energy go?

- Multiple branchings occur at parametrically **large angle**

$$\theta_s \equiv \frac{1}{\alpha_s^2 \sqrt{\hat{q} L^3}} \gg \theta_{\rm jet}$$

- **Constant energy flow** from jet energy scale E down to the medium temperature scale $T \sim \mathcal{O}(1 GeV)$



Q & A

Q&A

Q1: In most high energy calculation the light-cone gauge is used. Why is it a better choice than covariant gauge?

In the light cone gauge certain Feynman diagrams vanish identically. In the gauge $A^+ = 0$, for instance, the coupling of a high energy parton moving in the $-z$ direction is strongly suppressed:

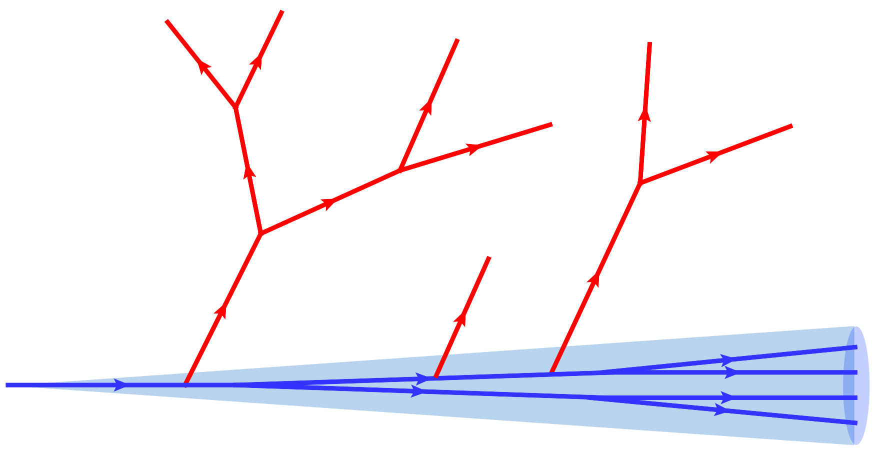
$$J^- A^+ = 0$$

Q&A

Q2: How does the in-medium cascade compare to the vacuum cascade?

The vacuum cascade is characterized by collinear splittings which are triggered by a single hard event. Moreover, because of color coherence of the jet large angle soft gluon radiation are strongly suppressed, hence, the collimation of QCD jets.

In-medium cascade exhibits an opposite behavior since small angle radiation is suppressed due to the LPM effect and coherence is destroyed along the cascade die to rapid in-medium color randomization



Q&A

Q3: What is a Monte Carlo Event Eenerator? and why is it useful for jet observables?

An event generator (such as PYTHIA, HERWIG, SHERPA, JEWEL, MARTINI, etc), is a numerical implementation, using Monte Carlo techniques, of a probabilistic picture, in which a parton cascade is described by independent elementary branching processes

It is a flexible tool that allows to compute any type of jet observables by accounting for experimental cuts for instance, and therefore, it is complementary to analytical techniques

$\longrightarrow$ Of course, event generators have the limitations of their underlying theoretical assumptions

Q&A

Q4: The medium modifies the jet structure but what about medium response?

Back reaction such as the recoil of medium partons might play a role in jet observables that are sensitive to soft particles, such as fragmentation functions

Q&A

Q5: Can one tell a medium parton from a jet parton?

The short answer to this question is no, owing to the fact that there is a continuum of scales between the hard jet scale and the characteristic medium scale and the importance of fluctuations. Nevertheless, if one is interested in hard enough partons produced in a jet event they are most likely not the result of background fluctuation