Universität Bielefeld

Hard probes on the Lattice

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A.Francis, OK, M.Laine, T.Neuhaus, H.Ohno, PRD92 (2015) 116003,
Nonperturbative estimate of the heavy quark momentum diffusion coefficient

H-T.Ding, F.Meyer, OK, PRD94 (2016) 034504,
Thermal dilepton rates and electrical conductivity of the QGP from the lattice

J.Ghiglieri, OK, M.Laine, F.Meyer, PRD94 (2016) 016005,
Lattice constraints on the thermal photon rate

Hard Probes 2016, Wuhan
23.09.2016

Motivation – electromagnetic probes

Dilepton rates

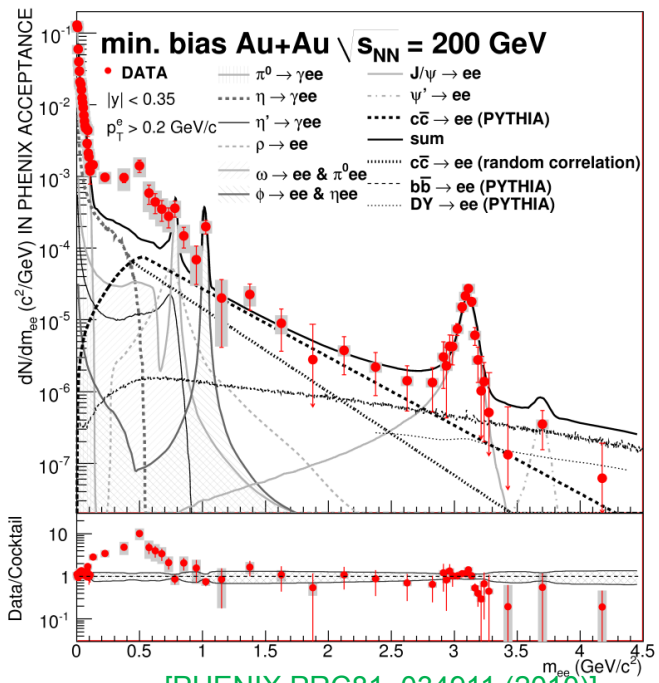
large enhancement between 150-750 MeV

Photon rates

possible window for photons from QGP

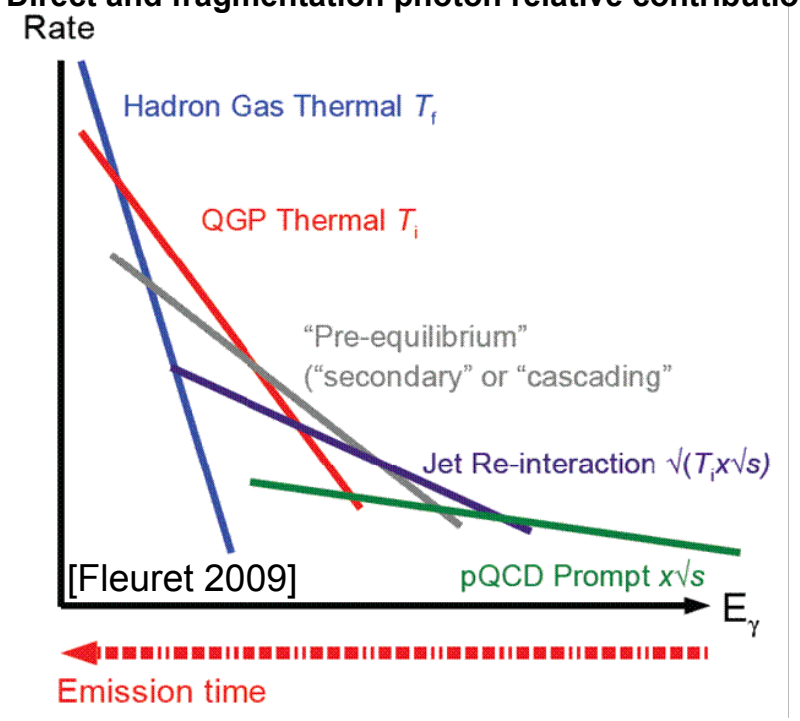
indications for thermal effects!?

Need to understand the contribution from QGP



[PHENIX PRC81, 034911 (2010)]

Direct and fragmentation photon relative contribution



[Fleuret 2009]

both directly related to vector-meson spectral function:

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{(\omega^2-\vec{p}^2)(e^{\omega/T}-1)}\rho_V(\omega,\vec{p},T)$$

$$\omega\frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\sigma_V(\omega=|\vec{p}|,T)$$

Motivation - Transport Coefficients

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

Need to be determined from QCD using first principle lattice calculations!

for heavy flavour:

Heavy Quark Diffusion Constant D

[H.T.Ding, OK et al., PRD86(2012)014509]

Heavy Quark Momentum Diffusion κ

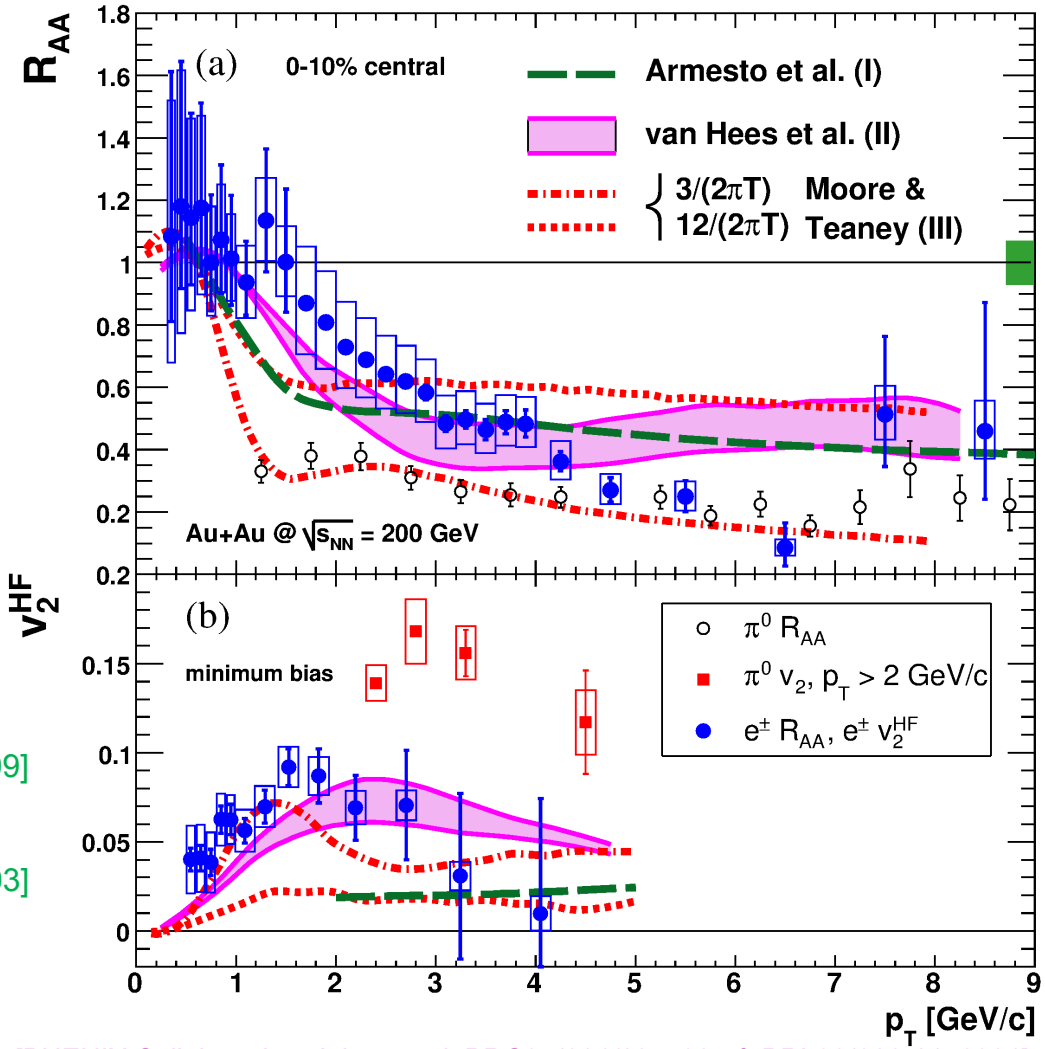
[A.Francis, OK, et al., PRD92(2015)116003]

for light quarks:

Light quark flavour diffusion /

Electrical conductivity

[A.Francis, OK et al., PRD83(2011)034504
H-T.Ding, F.Meyer, OK, PRD94 (2016) 034504]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

Vector meson spectral function – hard to separate different scales

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

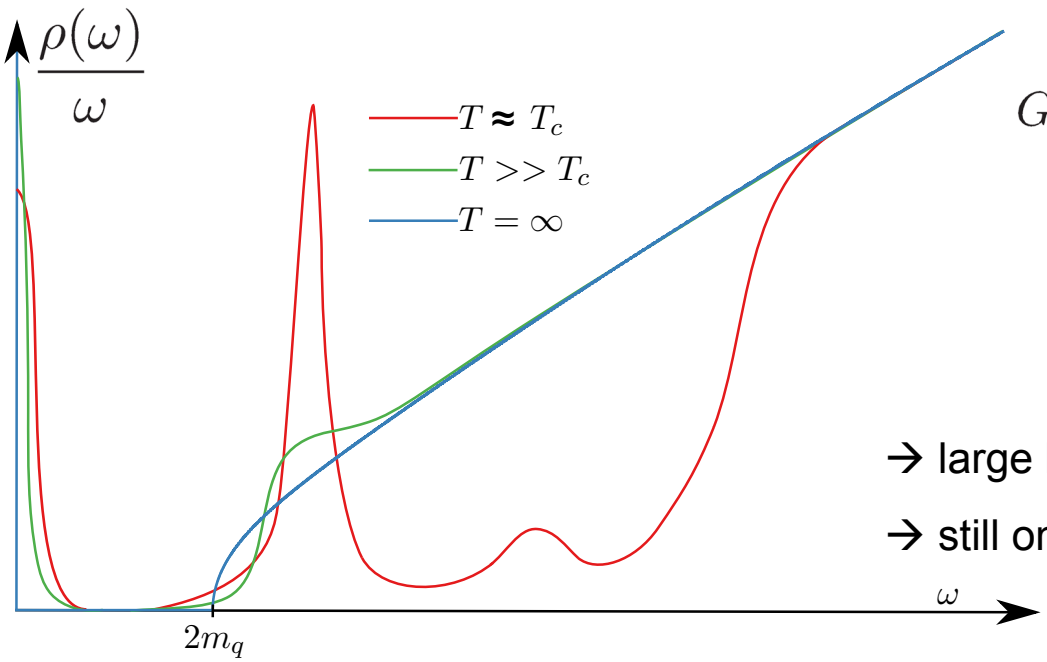
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Different contributions and scales enter in the spectral function

- **continuum at large frequencies**
- **possible bound states at intermediate frequencies**
- **transport contributions at small frequencies**
- **in addition cut-off effects on the lattice**

notoriously difficult to extract from correlation functions

Spectral functions in the QGP



$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$
$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

→ large lattices and continuum extrapolation needed

→ still only possible in the quenched approximation

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}, \qquad \eta = \frac{T}{MD}$

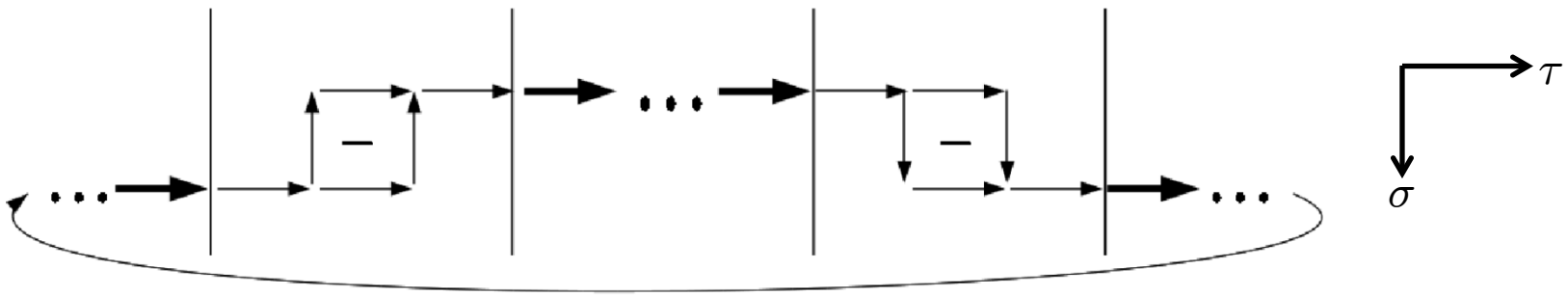
Heavy Quark Momentum Diffusion Constant – Single Quark in the Medium

Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012,
S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\frac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\frac{1}{T};0)]\right\rangle}$$

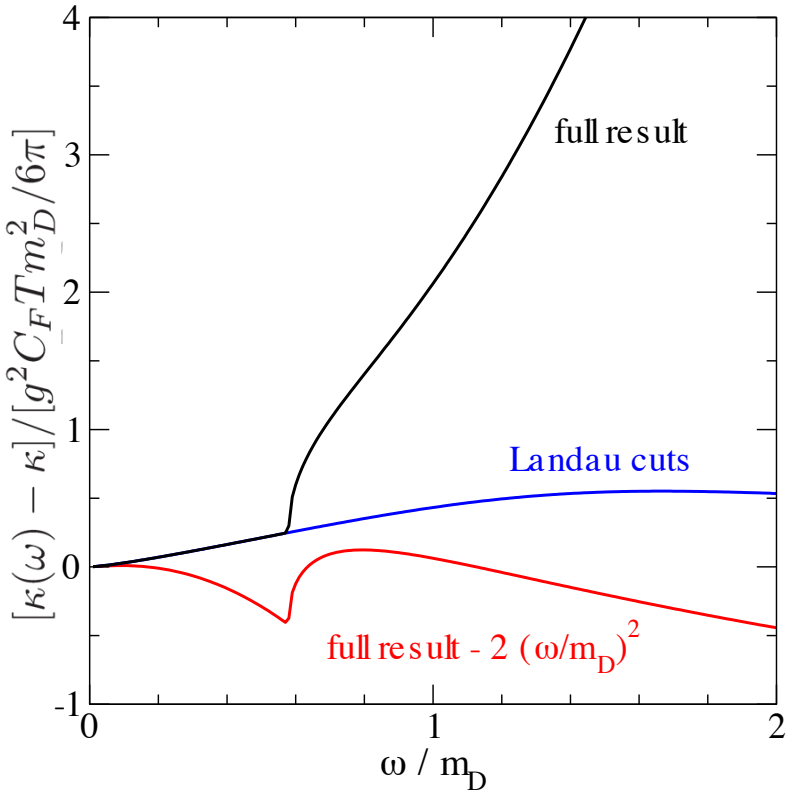
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant

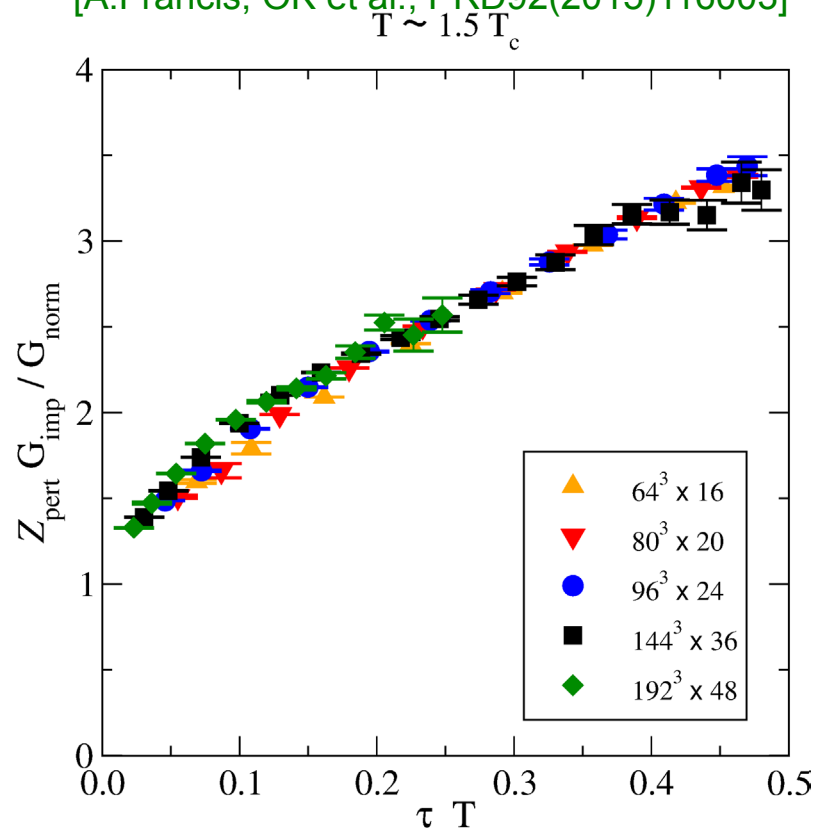
NLO spectral function in perturbation theory:

[Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



Lattice QCD correlation functions:

[A.Francis, OK et al., PRD92(2015)116003]



in contrast to a narrow transport peak, from this a smooth limit is expected

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega}$$

qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

no bound state contributions in this operator

Heavy Quark Momentum Diffusion Constant – IR and UV asymptotics

$\omega \ll T$: linear behavior motivated at small frequencies

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

$\omega \gg T$: vacuum perturbative results and leading order thermal correction:

$$\rho_{\mathrm{UV}}(\omega) = \left[\rho_{\mathrm{UV}}(\omega)\right]_{T=0} + \mathcal{O}\left(\frac{g^4T^4}{\omega}\right)$$

using a renormalization scale $\bar{\mu}_\omega = \omega$ for $\omega \gg \Lambda_{\overline{MS}}$ leading order becomes

$$\rho_{\mathrm{UV}}(\omega) = \Phi_{UV}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{MS})}\right)\right]$$

$$\Phi_{\mathrm{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega)C_F\omega^3}{6\pi} \quad , \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

We use Ansätze that are consistent with these asymptotic behaviors and model corrections to ρ_{IR} by a power series in ω

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

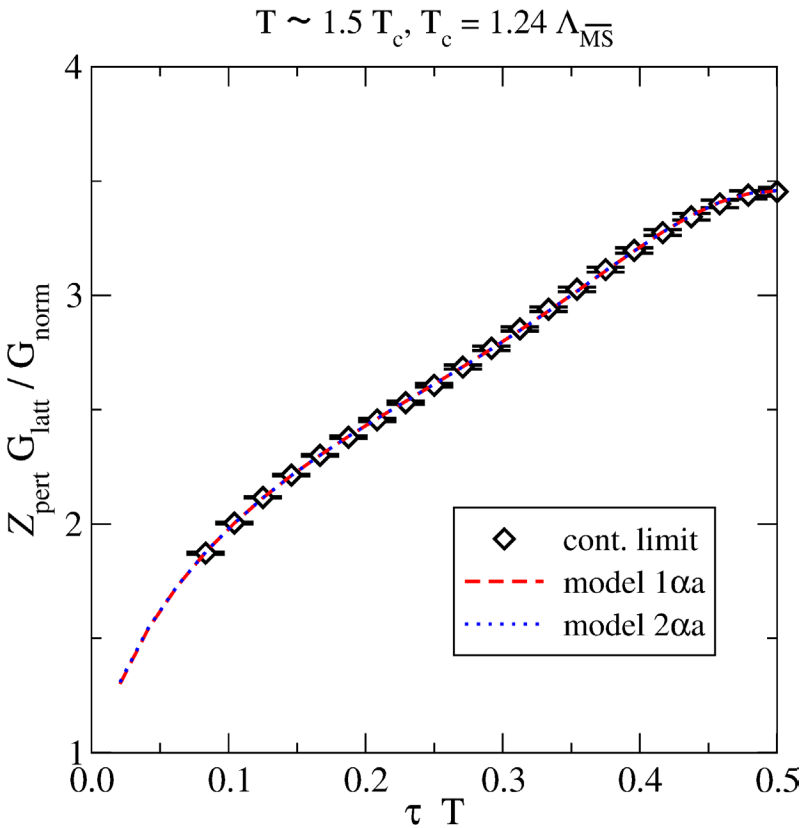
[A.Francis, OK et al., PRD92(2015)116003]

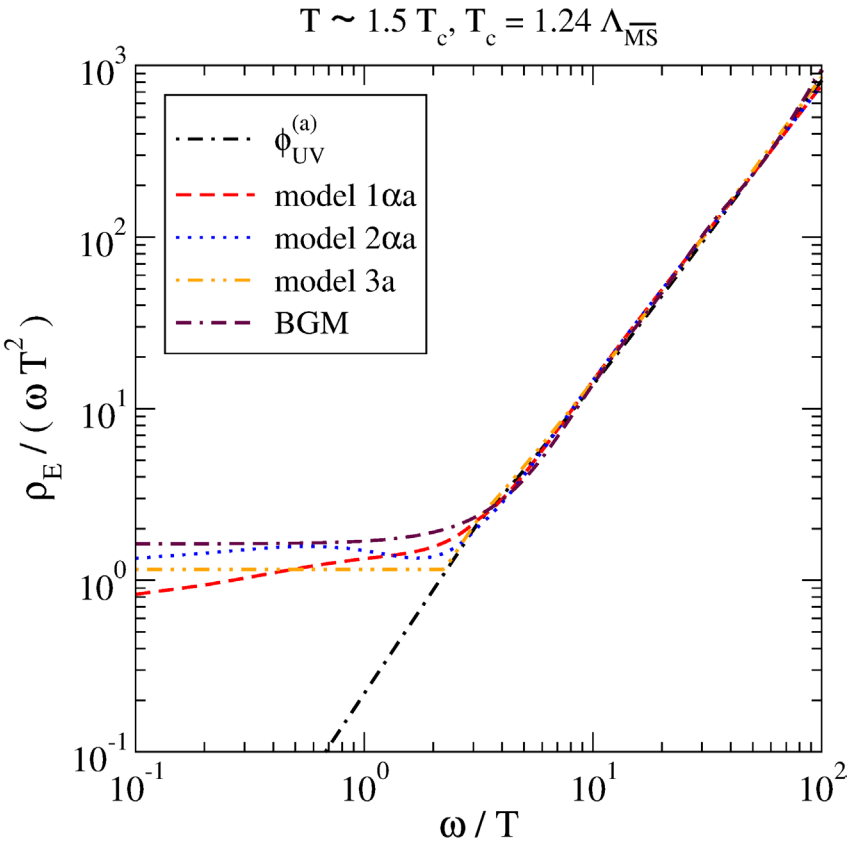
analysis of the systematic uncertainties by

using the best perturbative knowledge in the UV part of the spectral function

modeling corrections to ρ_{IR} by a power series in ω

and fitting to continuum extrapolated correlation function





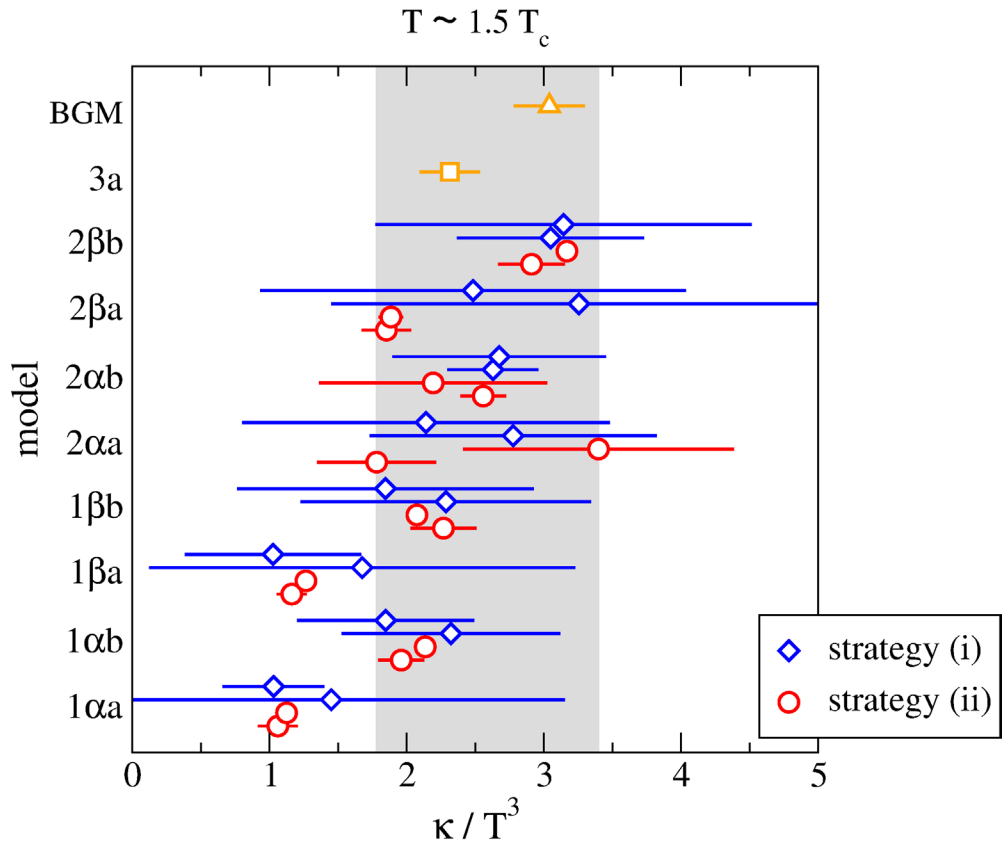
$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi} \rho_{\rm model}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

[A.Francis, OK et al., PRD92(2015)116003]

using various model corrections to ρ_{IR}



we analysed the systematic uncertainties

→ **continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} = 1.8...3.4$$

Equilibration time scale for heavy quarks

in the non-relativistic limit, κ is related to the diffusion coefficient D:

$$2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7...7.0$$

and to the drag coefficient η_D:

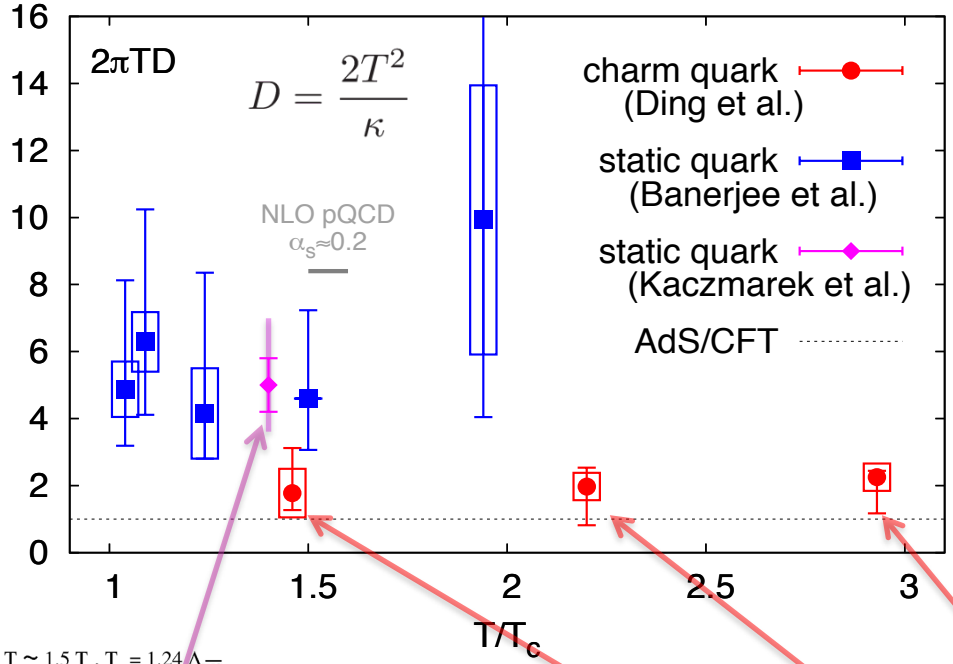
$$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

used to estimate the time scale associated with kinetic equilibration of heavy quarks:

$$\tau_{\rm kin} = \frac{1}{\eta_D} = (1.8\ldots3.4)\left(\frac{T_{\rm c}}{T}\right)^2\left(\frac{M}{1.5\ {\rm GeV}}\right){\rm fm/c}$$

close to T_c, $\tau_{kin} \simeq$ 1fm/c and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.

Lattice QCD results on heavy quark diffusion coefficients

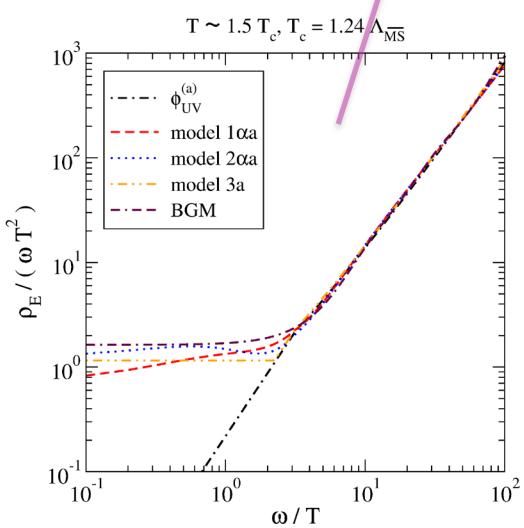


[H.T.Ding, OK, H.Satz et al., PRD86(2012)014509]

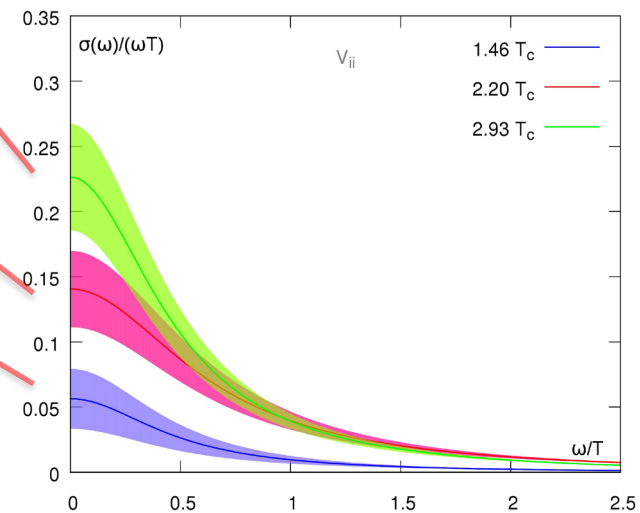
[Banerjee et al., PRD85(2012) 014510]

[A.Francis, OK et al., PRD92(2015)116003]

charm quark mass
quenched approximation
no continuum extrapolation yet



heavy quark mass limit
quenched approximation
continuum extrapolated



next goals: continuum extrapolation for charm and bottom vector-meson correlator

→ quark mass dependence of diffusion coefficients + sequential melting of quarkonia

Vector-meson correlation function for light quarks on large & fine lattices

[H-T.Ding, F.Meyer, OK, PRD94(2016)034504, H.T.Ding, A.Francis, OK et al., PRD83(2011)034504]

quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

non-perturbative renormalization constants and quark masses close to the chiral limit

	N_τ	N_σ	β	κ	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$	confs
1.1 T_c	32	96	7.192	0.13440	0.2796	1.12	0.8164	1.09	314
	48	144	7.544	0.13383	0.2843	1.14	0.8169	1.10	358
	64	192	7.793	0.13345	0.2862	1.15	0.8127	1.09	242
1.3 T_c	28	96	7.192	0.13440	0.3195	1.28	0.9330	1.25	232
	42	144	7.544	0.13383	0.3249	1.31	0.9336	1.25	417
	56	192	7.793	0.13345	0.3271	1.31	0.9288	1.25	273
1.5 T_c	24	128	7.192	0.13440	0.3728	1.50	1.0886	1.46	340
	32	128	7.457	0.13390	0.3846	1.55	1.1093	1.49	255
	48	128	7.793	0.13340	0.3817	1.53	1.0836	1.45	456

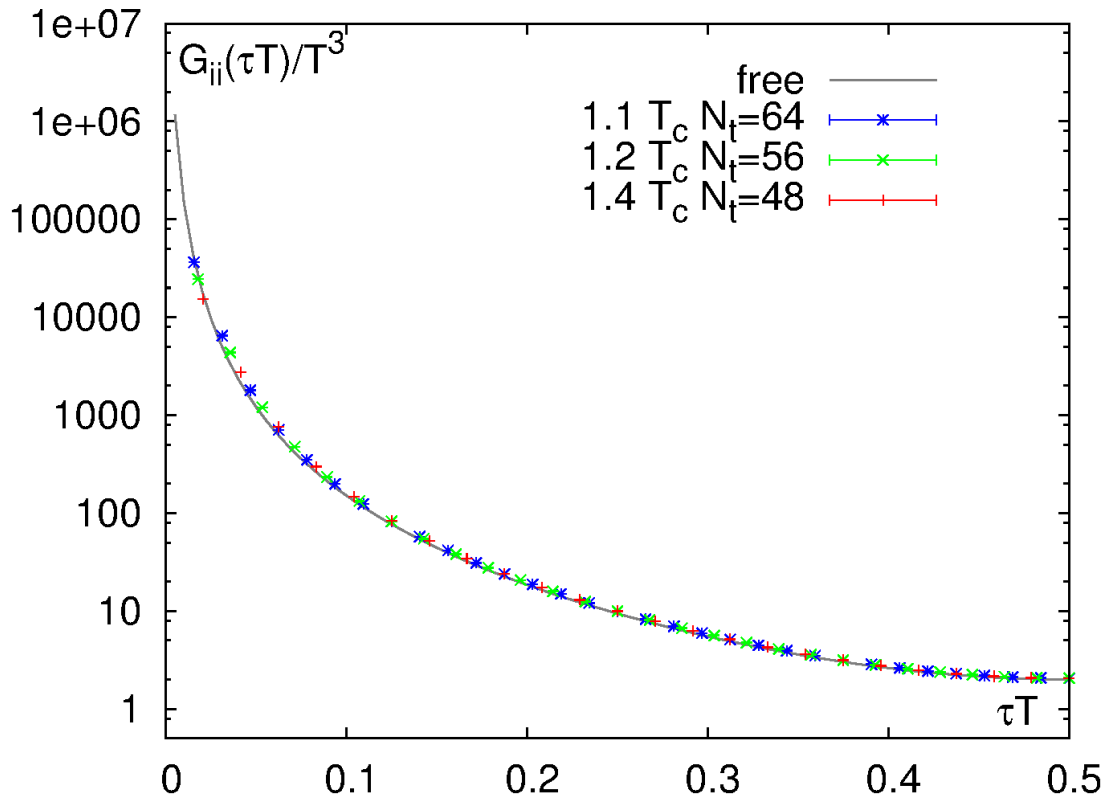
Scale setting using r_0 and t_0 [A.Francis, M.Laine, T.Neuhaus, H.Ohno PRD92(2015)116003]

fixed aspect ratio N_σ/N_τ = 3 and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi\vec{k}\frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

Vector-meson correlation function



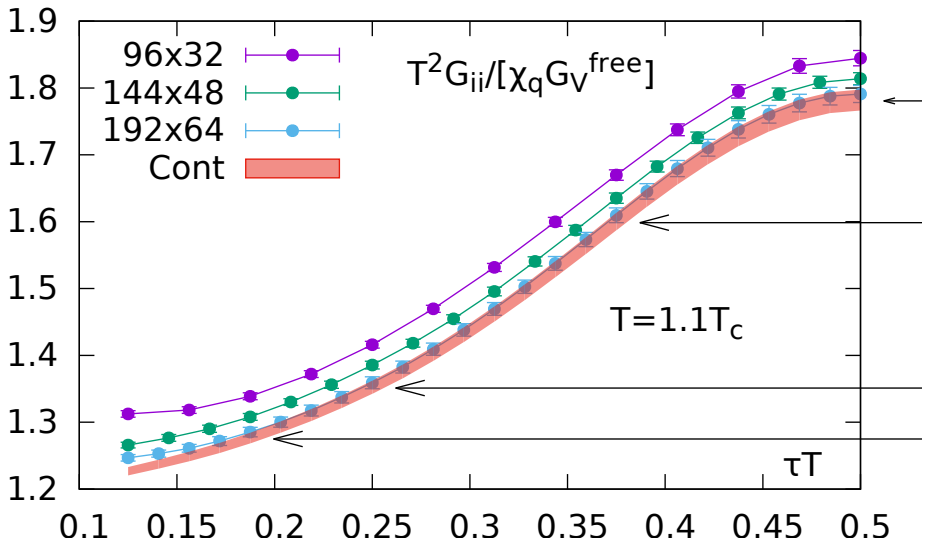
compared to free (non-interacting) correlator:

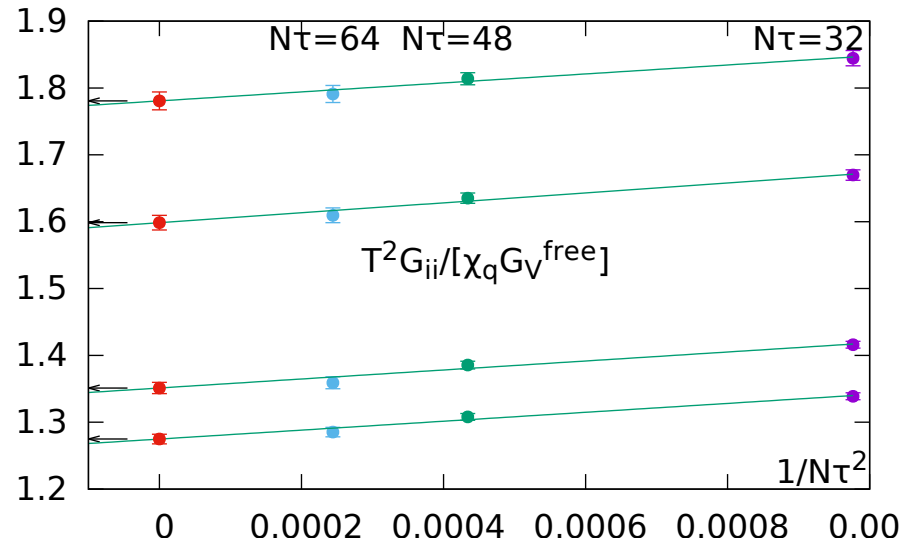
$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T) \frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

hard to distinguish differences due to different orders of magnitude in the correlator

→ in the following we will use $G_V^{free}(\tau)$ as a normalization

Continuum extrapolation





correlators normalized by quark number susceptibility χ_q independent of renormalization

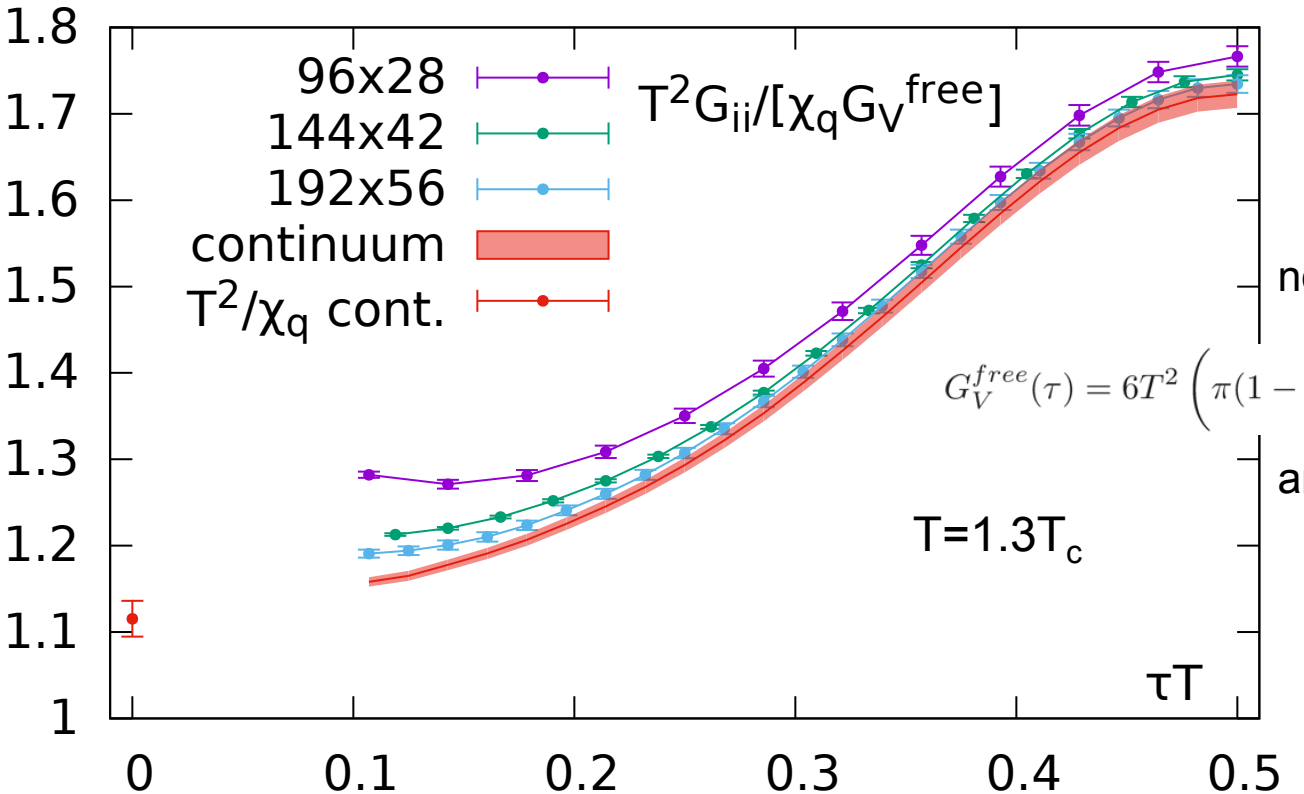
and by the free non-interacting correlator $G_V^{free}(\tau)$

we interpolate the correlator for each lattice spacing

and perform the continuum limit a→0 at each distance τT

cut-off effects are visible at all distances on finite lattices

Continuum extrapolation



normalized by free correlator

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T) \frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

and quark number susceptibility

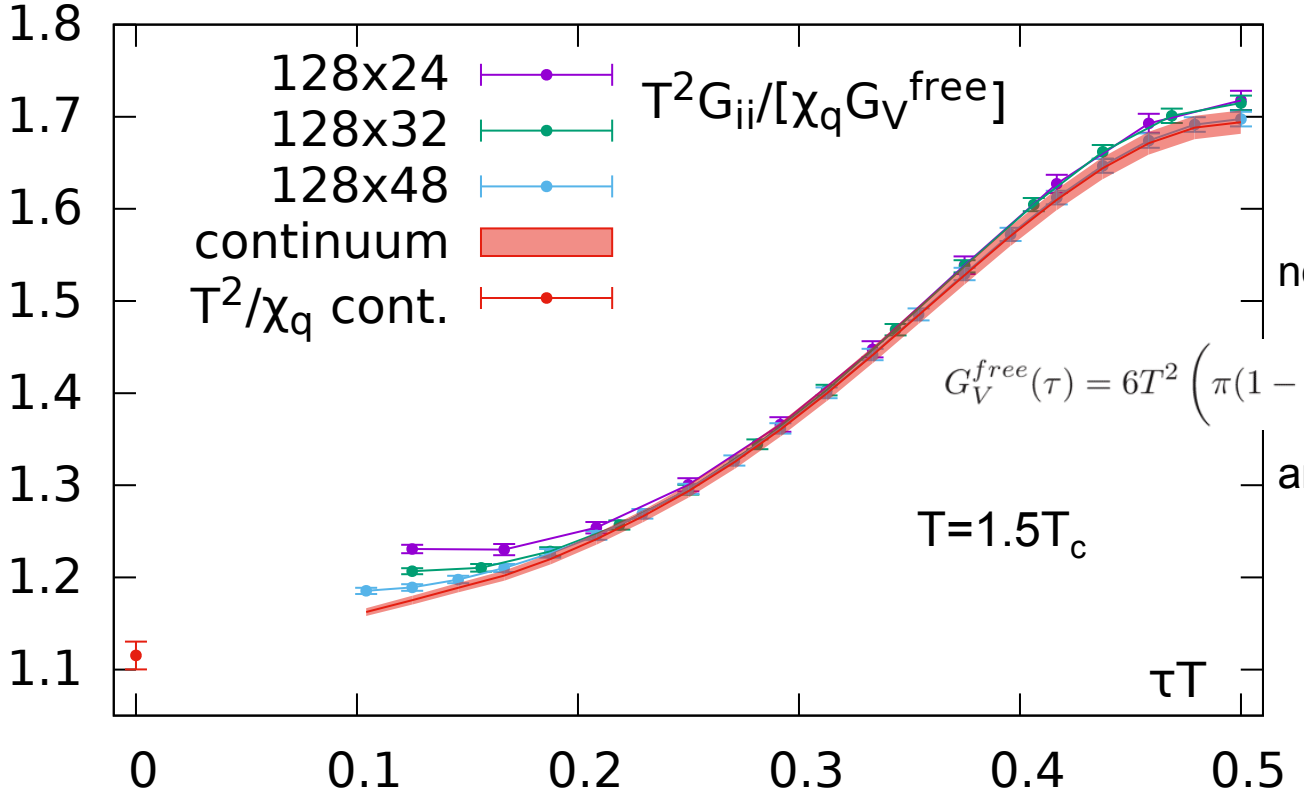
cut-off effects are visible at all distances on finite lattices but

well defined continuum limit on the correlator level

well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

Continuum extrapolation



normalized by free correlator

$$G_V^{free}(\tau) = 6T^2 \left(\pi(1-2\tau T) \frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2\frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

and quark number susceptibility

cut-off effects are visible at all distances on finite lattices but

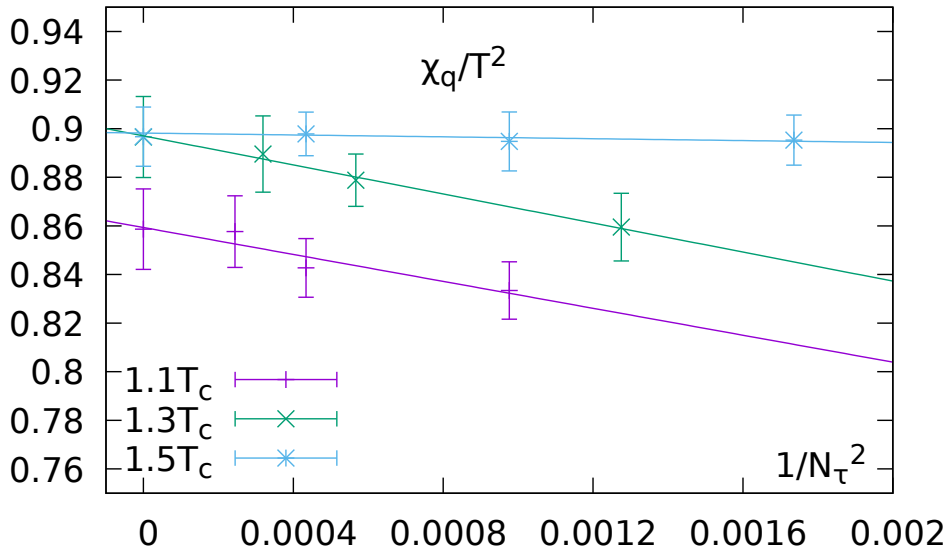
well defined continuum limit on the correlator level

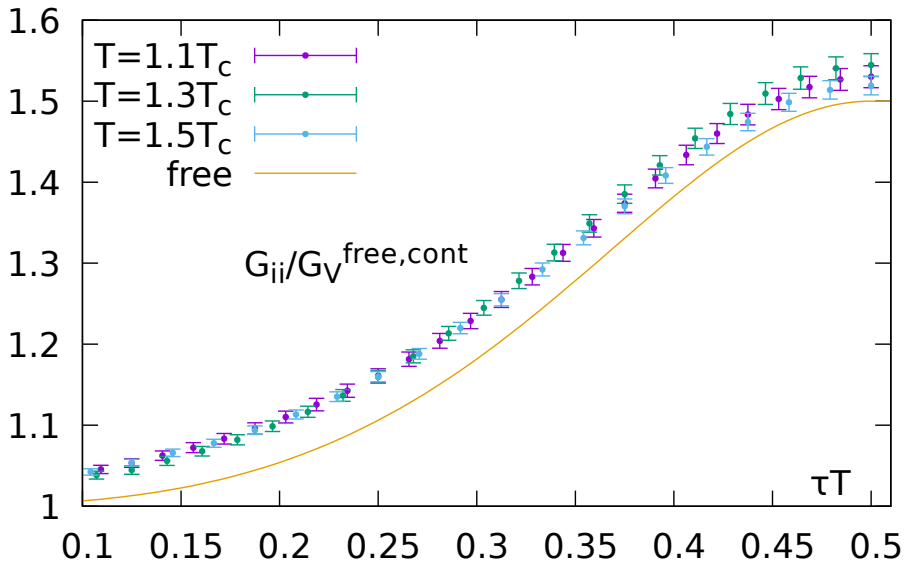
well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

Continuum extrapolated vector-meson correlation function

continuum extrapolated results available for three temperatures in the QGP





similar behavior in this temperature region

main difference due to different quark number susceptibility χ_q/T^2

→ indications for a weak T-dependence of the temperature scaled

electrical conductivity and thermal dilepton rates

Spectral functions at high temperature

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\rho_{00}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega)$$
$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)$$

δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{oo}(\omega)$+$\rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{oo} is protected, the δ-funtion in ρ_{ii} gets smeared → transport peak:

Ansatz:

$$\kappa = \frac{\alpha_s}{\pi}$$

at leading order

$$\rho_{00}(\omega) = 2\pi\chi_q \omega \delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

→ electrical conductivity: $$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T} \frac{\sigma}{T} = \frac{2}{3}\frac{\chi_q}{T^2}\frac{T}{\Gamma}\, c_{BW}\, C_{em}$$

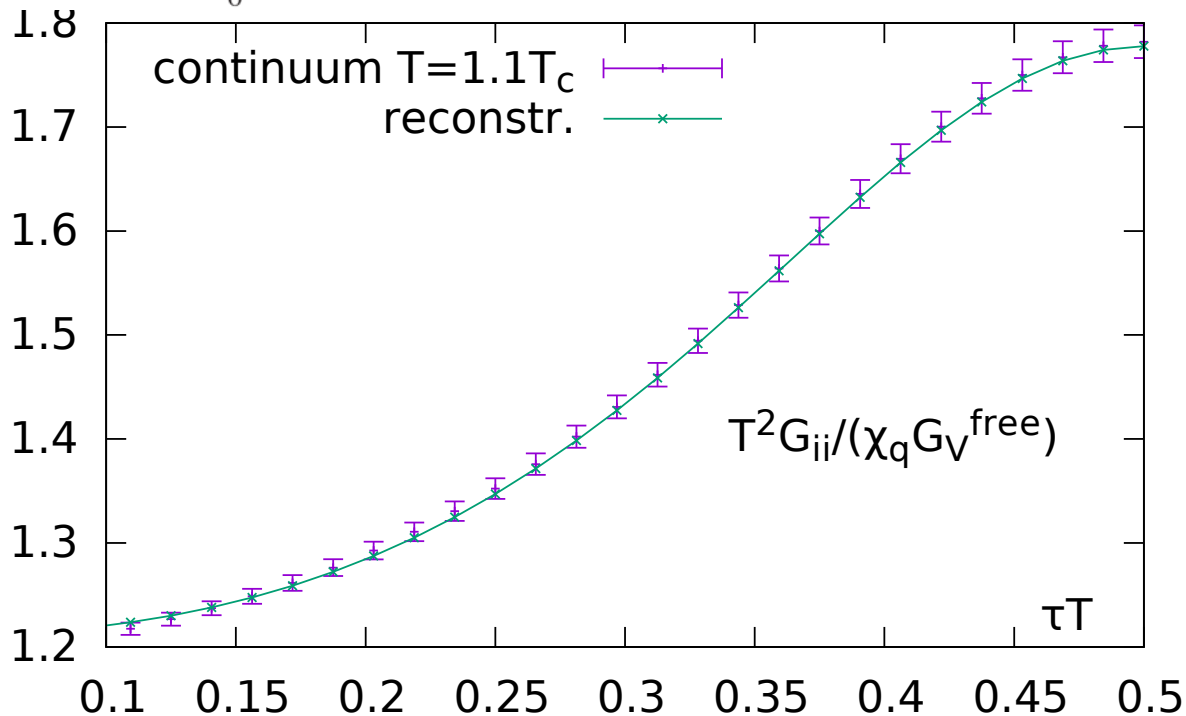
Spectral function and electrical conductivity

use our Ansatz for the spectral function and fit to the continuum extrapolated correlators

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)$$

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$



all three temperatures well described by this rather simple Ansatz

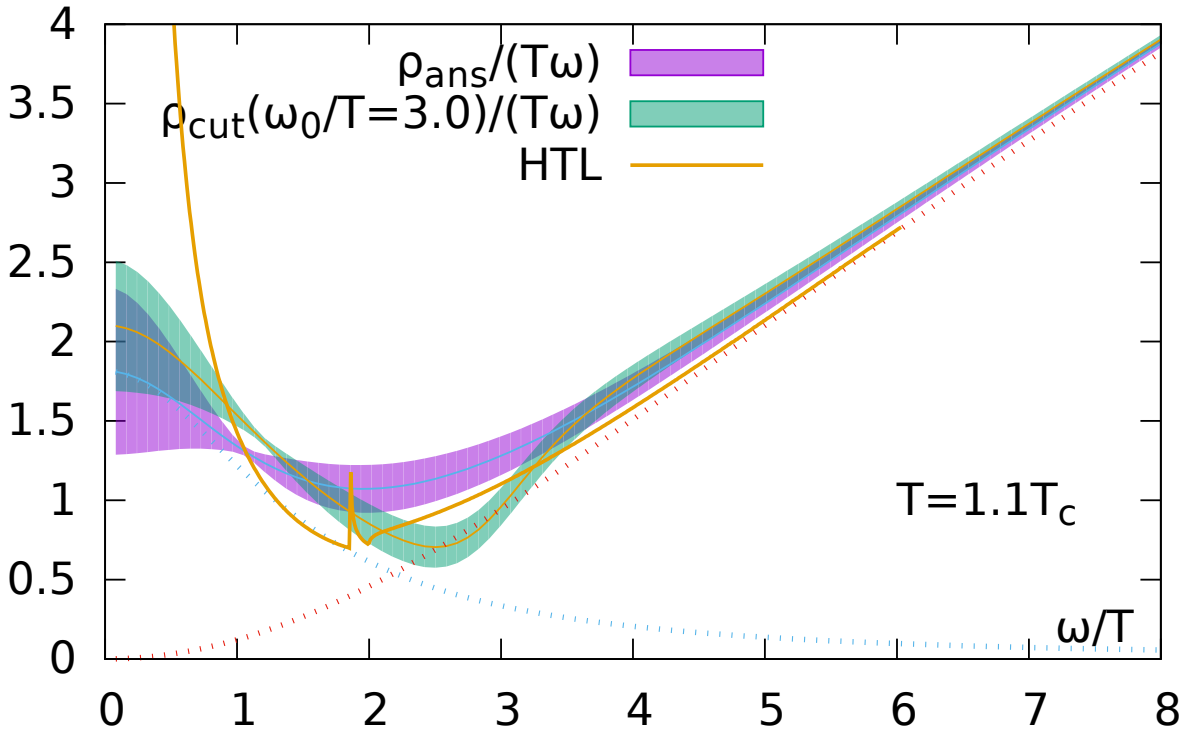
T	$\sigma/(C_{\rm em}T)$	Γ/T	$c_{BW}T/\Gamma$	k	χ^2/dof
$1.1T_c$	0.302(88)	2.86(1.16)	0.528(154)	0.038(8)	1.15
$1.3T_c$	0.254(51)	3.91(1.25)	0.425(85)	0.029(9)	0.52
$1.5T_c$	0.266(48)	3.33(89)	0.445(80)	0.040(7)	1.13

Spectral function and electrical conductivity

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

systematic uncertainties (within this Ansatz) estimated by varying the truncation

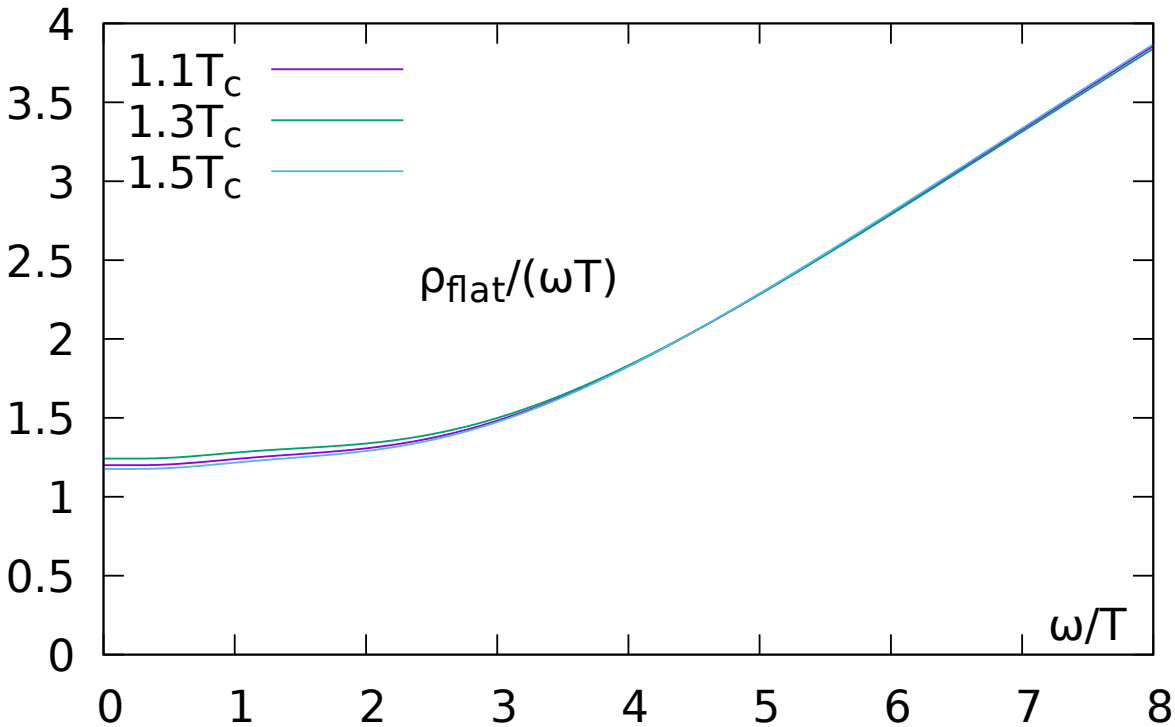
with similar χ^2/dof ~ 0.5-1.1

Spectral function and electrical conductivity

Use a flat transport Ansatz for the spectral function

$$\rho_{\rm flat}(\omega) = a\chi_q\omega\left(1-\widetilde{\Theta}(\omega_0,\Delta_0)\right)+(1+k)\rho_{\rm free}(\omega)\widetilde{\Theta}(\omega_1,\Delta_1)$$

$$\widetilde{\Theta}(\omega,\omega_i,\Delta_i)=\left(1+\exp\left(\frac{\omega_i^2-\omega^2}{\omega\Delta_i}\right)\right)^{-1}$$



Analysis of the systematic errors

using a flat behavior at small ω

$$\frac{\sigma}{T}=\frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

systematic uncertainties (within this Ansatz) estimated by varying the truncation

with similar χ^2/dof ~ 0.5-1.1 still consistent with our data → lower limit for σ/T

perturbation theory – vacuum spectral function

Improve the UV behavior of the spectral function using perturbation theory:

At very high energies, due to asymptotic freedom

→ perturbation should be working

→ thermal effects should be suppressed

→ "vacuum physics"

5-loop vacuum spectral function:

$$\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)$$

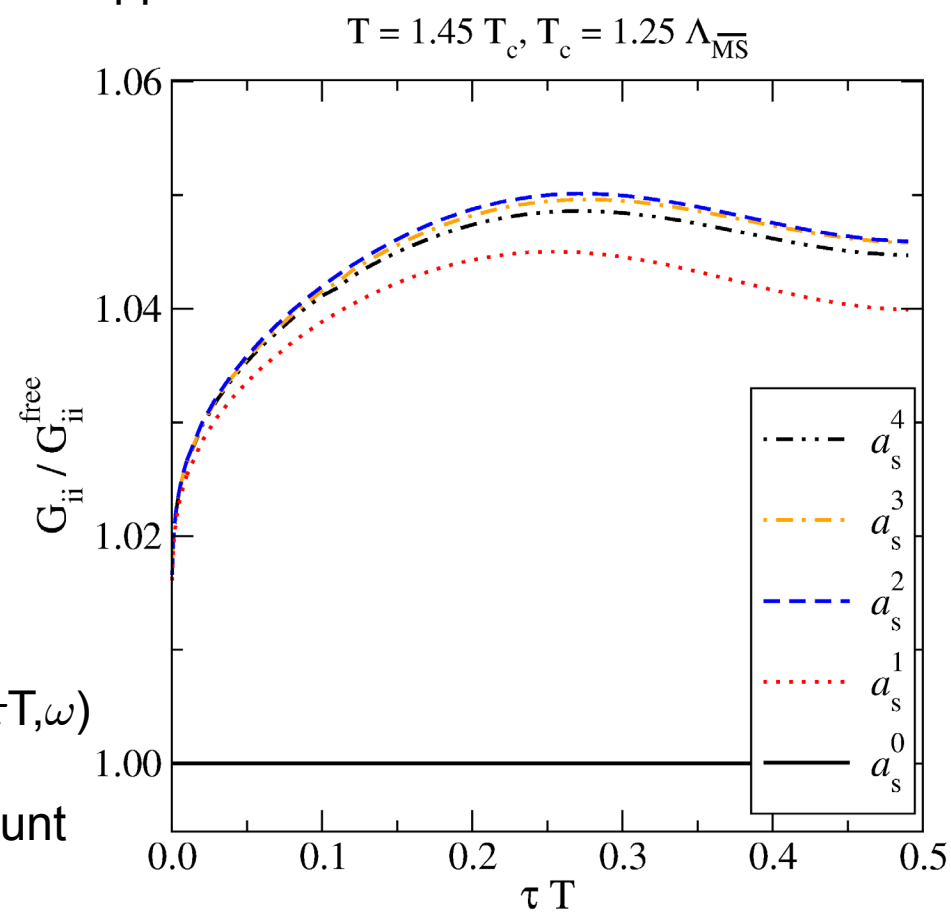
$$\begin{aligned} R(\omega^2) &= r_{0,0} + r_{1,0}\, a_s + \left(r_{2,0} + r_{2,1}\,\ell\right) a_s^2 \\ &+ \left(r_{3,0} + r_{3,1}\,\ell + r_{3,2}\,\ell^2\right) a_s^3 \\ &+ \left(r_{4,0} + r_{4,1}\,\ell + r_{4,2}\,\ell^2 + r_{4,3}\,\ell^3\right) a_s^4 + \mathcal{O}(a_s^5) \end{aligned}$$

using 3-loop α_s and l=log(μ^2/ω^2)

using a renormalization scale μ=(1..5)max(πT,ω)

taking leading order thermal effect into account

$$\rho_{ii}^{(T)}(\omega) \equiv \frac{3\omega^2}{4\pi}[1 - 2n_{\rm F}(\frac{\omega}{2})]R(\omega^2) + \pi\chi_{\rm q}^{\rm free}\omega\delta(\omega)$$

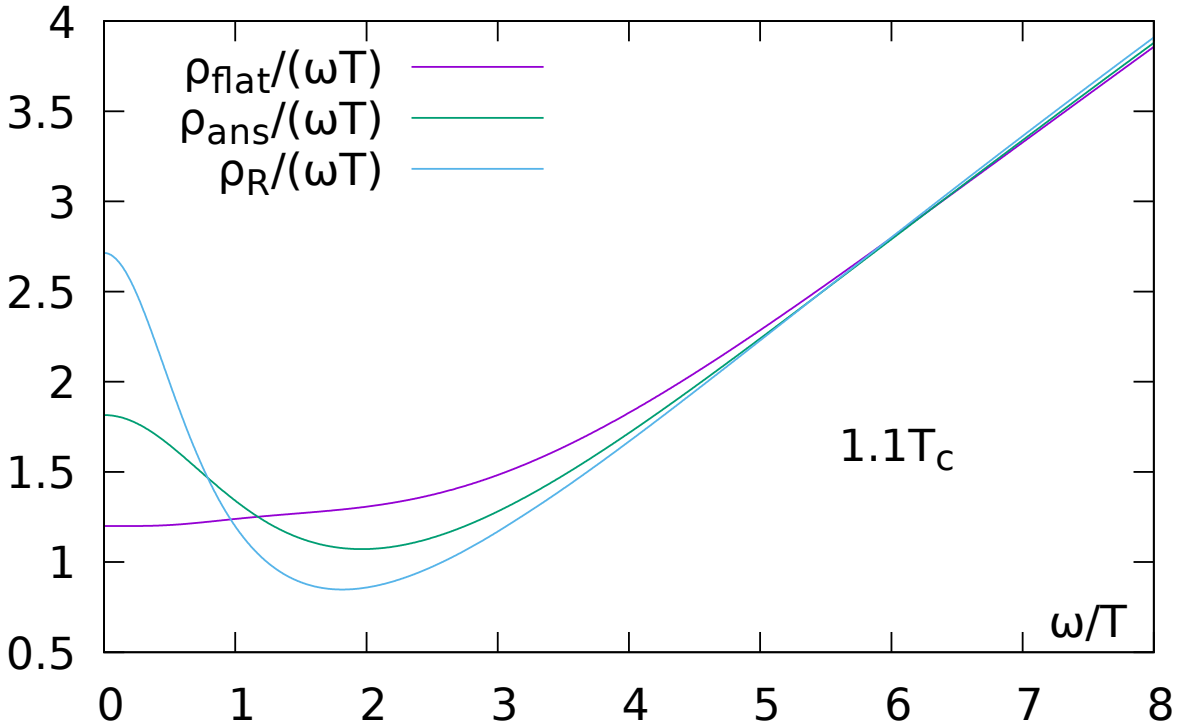


[Y.Burnier and M.Laine, arXiv 1201.1994]

Spectral function and electrical conductivity

include improved large ω behavior from vacuum perturbation theory

$$\begin{aligned}\rho_{\rm impr}(\omega,T) &= \rho_V^{\rm free}(\omega,T)R(\omega^2)\\ \rho_{\rm R}(\omega,T) &= \rho_{\rm BW}(\omega,T)+C\rho_{\rm impr}(\omega,T).\end{aligned}$$



T	$\sigma/(C_{\rm em}T)$	Γ/T	$c_{BW}T/\Gamma$	C	$\chi^2/{\rm dof}$
$1.1T_c$	0.452(251)	1.62(1.09)	0.790(438)	0.993(7)	1.11
$1.3T_c$	0.301(87)	2.89(1.18)	0.504(145)	0.984(8)	0.53
$1.5T_c$	0.326(87)	2.38(85)	0.548(146)	0.996(7)	1.12

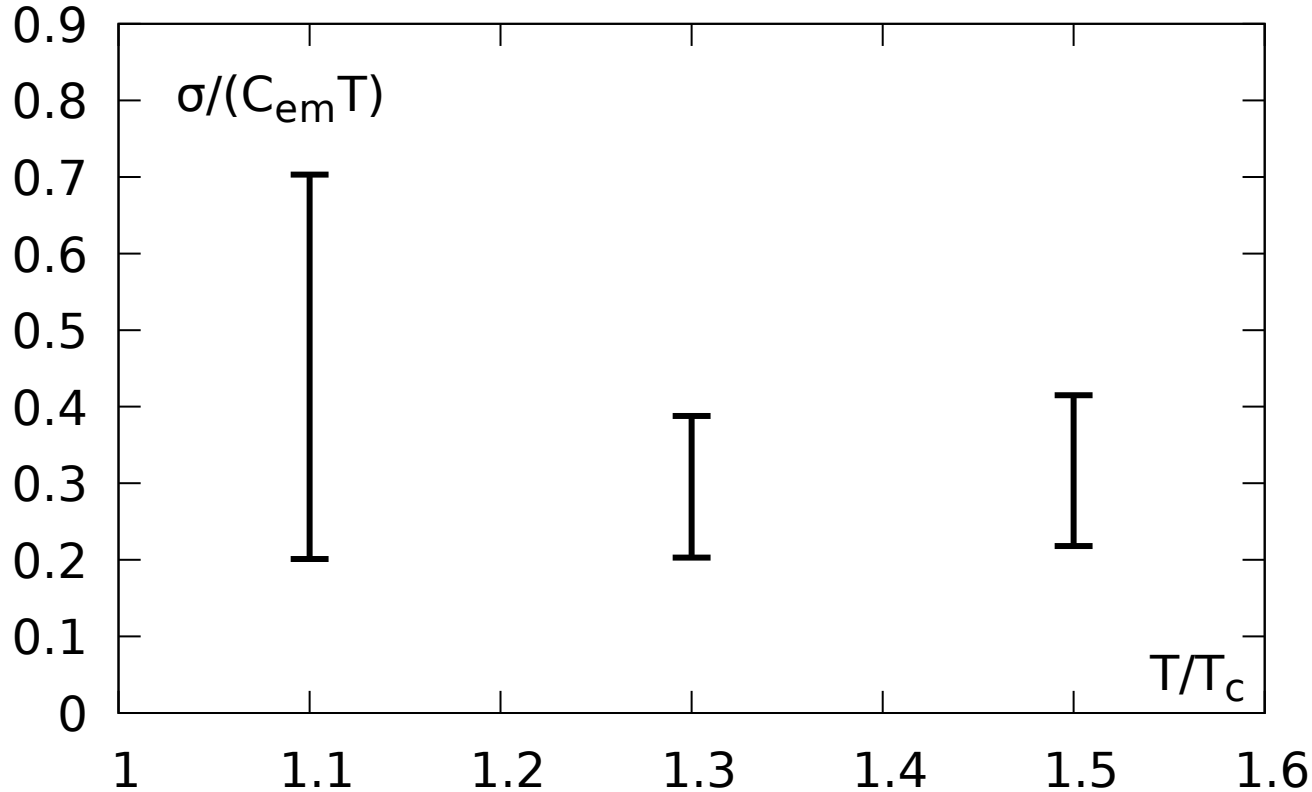
Electrical conductivity

continuum estimate for the

T-dependence of the electrical conductivity:

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T}$$

lower and upper limits from the systematic analysis of our classes of spectral functions:

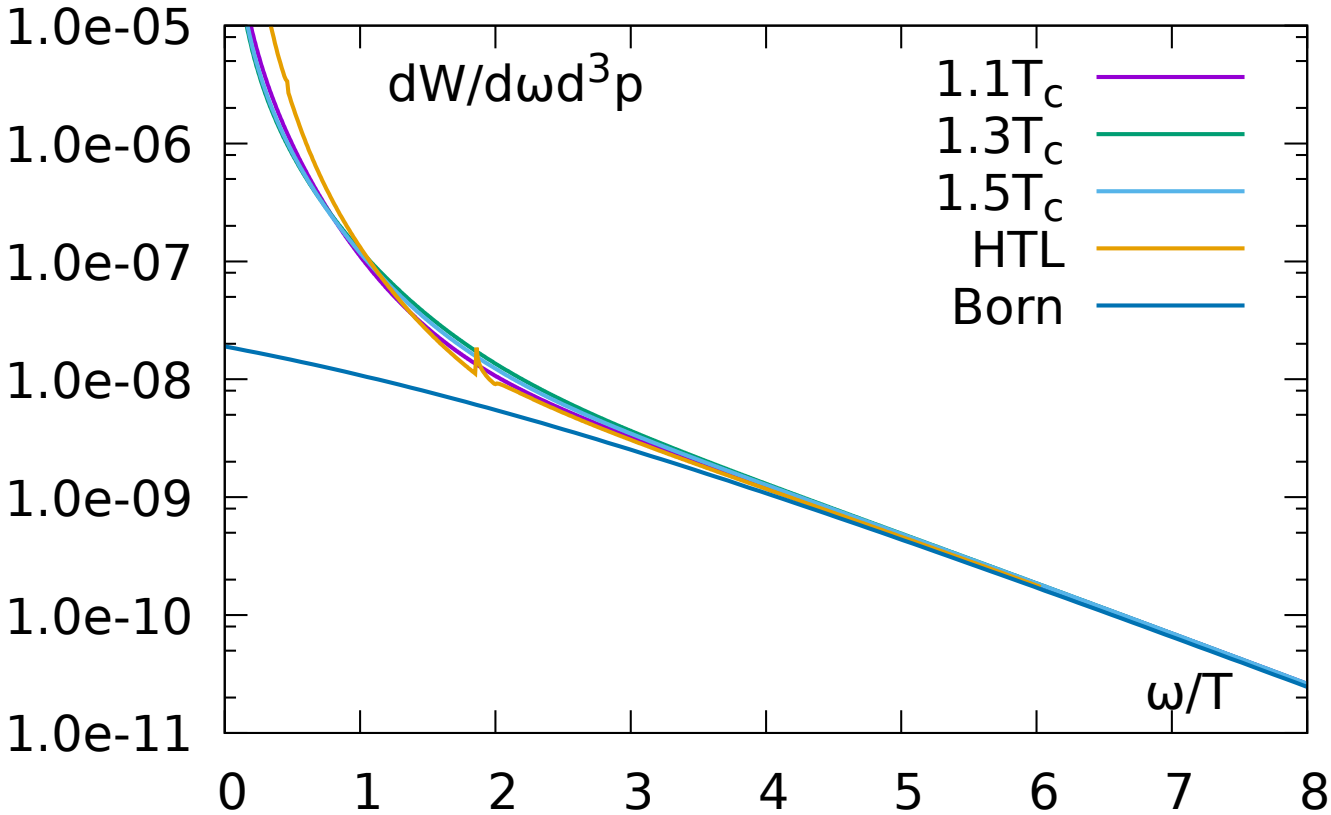


other studies using dynamical clover Wilson or staggered fermions (all w/o continuum limit):
A.Amato et al., arXiv:1307.6763, B.B.Brandt et al., JHEP 1303 (2013) 100, S.Gupta, PLB 597 (2004) 57
G.Aarts et al., PRL 99 (2007) 022002, Brandt et al., PRD93 (2016) 054510, G. Aarts et al., JHEP02 (2015)186

Dilepton rates

Dileptonrate directly related to vector spectral function:

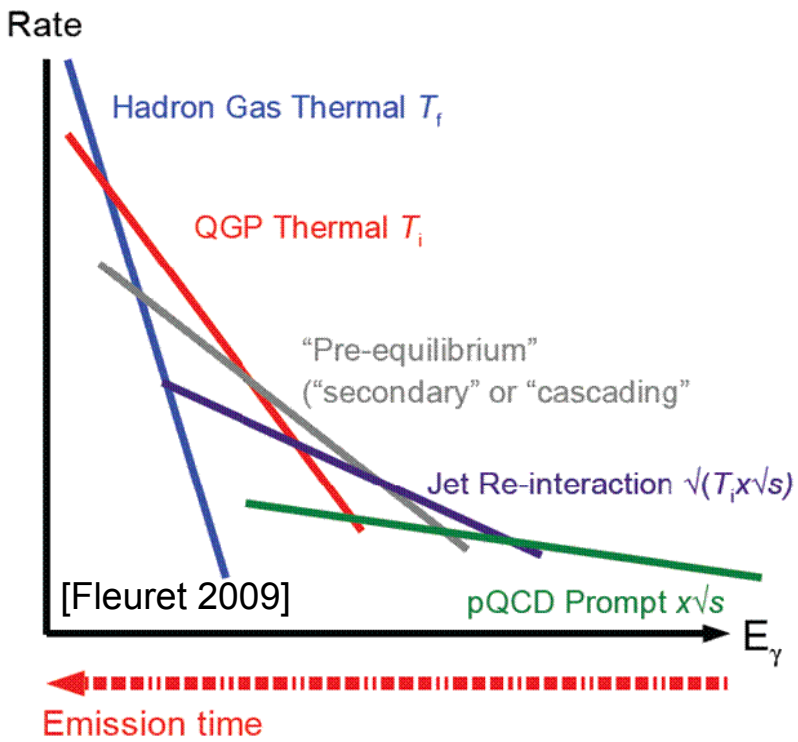
$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\mathbf{T})$$

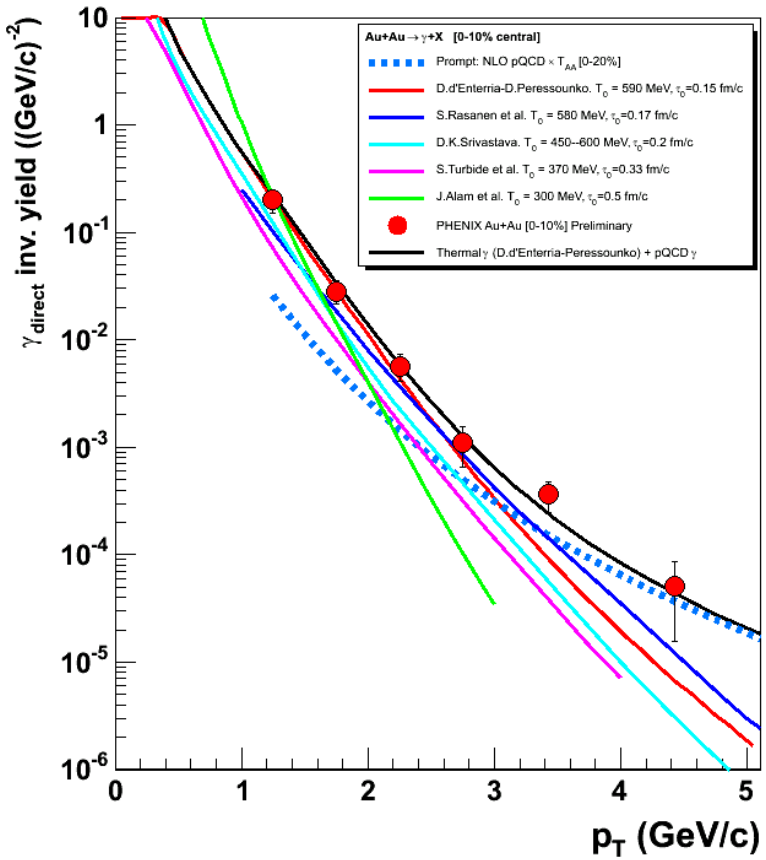


Hard thermal loop (HTL)
[E.Braaten, R.D.Pisarski, NP B337 (1990) 569]

Hard Probes in Heavy Ion Collisions - Photons

Direct and fragmentation photon relative contribution





Photonrate directly related to vector spectral function (at finite momentum):

$$\omega \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4x\mathrm{d}^3q} = \frac{5\alpha}{6\pi^2}\frac{1}{e^{\omega/T}-1}\sigma_V(\omega = |\vec{k}|, T)$$

Perturbative knowledge on the vector spectral function

Non-interacting limit, "Born rate" for large invariant mass M>>πT, with $M^2=\omega^2+k^2$

$$\rho_{\rm V}(\omega,\mathbf{k}) \;=\; \frac{N_{\rm c}TM^2}{2\pi k}\left\{\ln\left[\frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})}\right]-\frac{\omega\,\theta(k-\omega)}{2T}\right\},$$

[G. Aarts and J.M. Martinez Resco, NPB 726 (2005) 93]

Leading-log order for invariant mass M=0: [J.I. Kapusta et al.,PRD44 (1991) 2774, R. Baier et al. Z.Phys.C53 (1992) 433]

$$\rho_{\rm V}(k,\mathbf{k}) \;=\; \frac{\alpha_{\rm s}N_{\rm c}C_{\rm F}T^2}{4}\ln\left(\frac{1}{\alpha_{\rm s}}\right)\Big[1-2n_{\rm F}(k)\Big]+\mathcal{O}(\alpha_{\rm s}T^2)\;,$$

Complete leading order for invariant mass M=0:

[Arnold, Moore, Yaffe, JHEP11(2001)57 and JHEP12(2001)9]

NLO at M = 0: [J.Ghiglieri et al., JHEP 1305 (2013) 010]

NLO at M $\sim$ gT: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

NLO at M $\sim \pi$T: [M.Laine, JHEP 1311 (2013) 120]

N^4LO at M >> πT: [S. Caron-Huot, PRD79 (2009) 125009, P.A.Baikov et al. PRL101 (2008) 012002]

Hydrodynamic regime

Vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$:

$$\frac{\rho_{\rm V}(\omega, \mathbf{k})}{\omega} = \left(\frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2 \right) \chi_{\rm q} D$$

with the quark number susceptibility: $\chi_{\rm q} \equiv \int_0^\beta {\rm d}\tau \int_{\mathbf{x}} \langle V^0(\tau, \mathbf{x}) V^0(0) \rangle$

and the diffusion coefficient: $D \equiv \frac{1}{3\chi_{\rm q}} \lim_{\omega \to 0^+} \sum_{i=1}^{3} \frac{\rho^{ii}(\omega, \mathbf{0})}{\omega}$

which relate to the electric conductivity: $\sigma = e^2 \sum_{f=1}^{N_{\rm f}} Q_f^2 \chi_{\rm q} D$

In this limit the (soft) photon rate becomes: $\frac{{\rm d}\Gamma_\gamma(\mathbf{k})}{{\rm d}^3\mathbf{k}} \overset{k \lesssim \alpha_{\rm s}^2 T}{\approx} \frac{2T\sigma}{(2\pi)^3 k}$

In the AdS/CFT framework the vector spectral function has the same infrared structure

and here numerical result can make predictions beyond the hydro regime

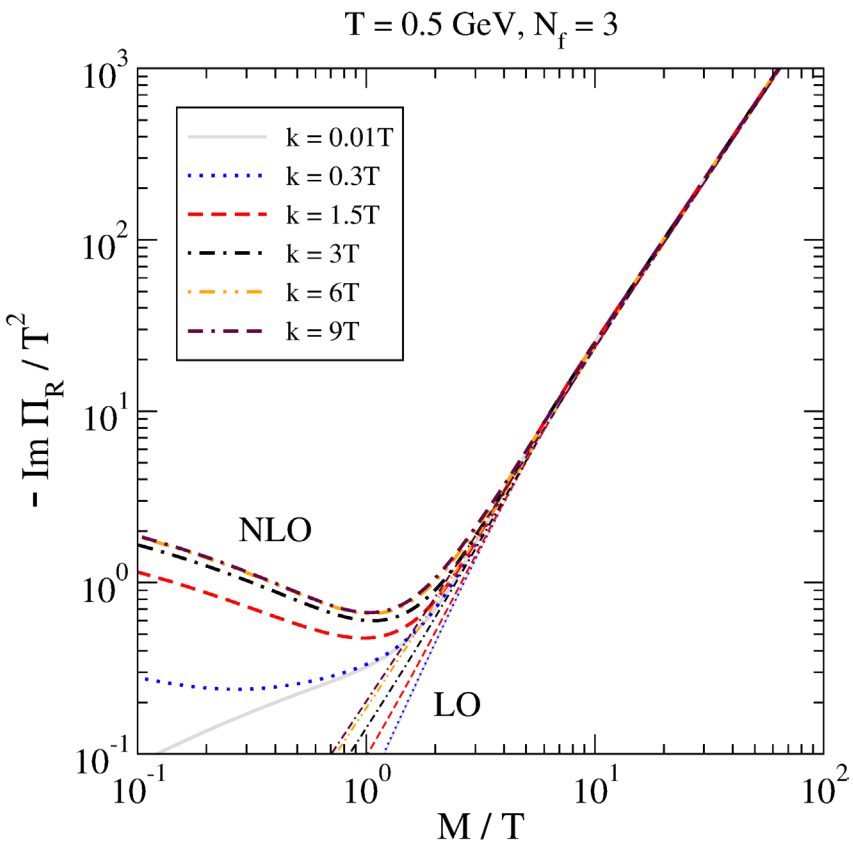
[S.Caron-Huot, JHEP12 (2006) 015]

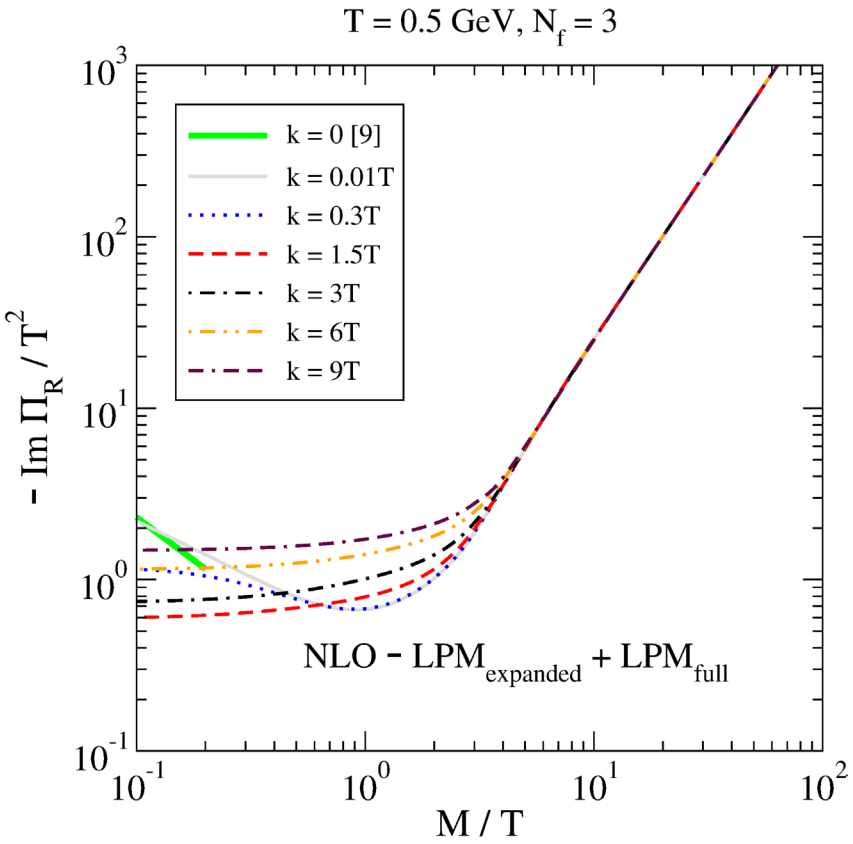
Although not QCD, it may give qualitative insight into the structures at small ω and k

perturbation theory – thermal corrections

Thermal corrections in the intermediate frequency regime required [Altherr+Aurenche 1989]

and proper treatment of the small frequency regime [Ghiglieri+Moore 2014] [Moore+Robert 2006]





interpolation between different regimes [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

progress in perturbation theory in the past years → compare to lattice QCD results

pQCD spectral functions

pQCD spectral function used in our analysis

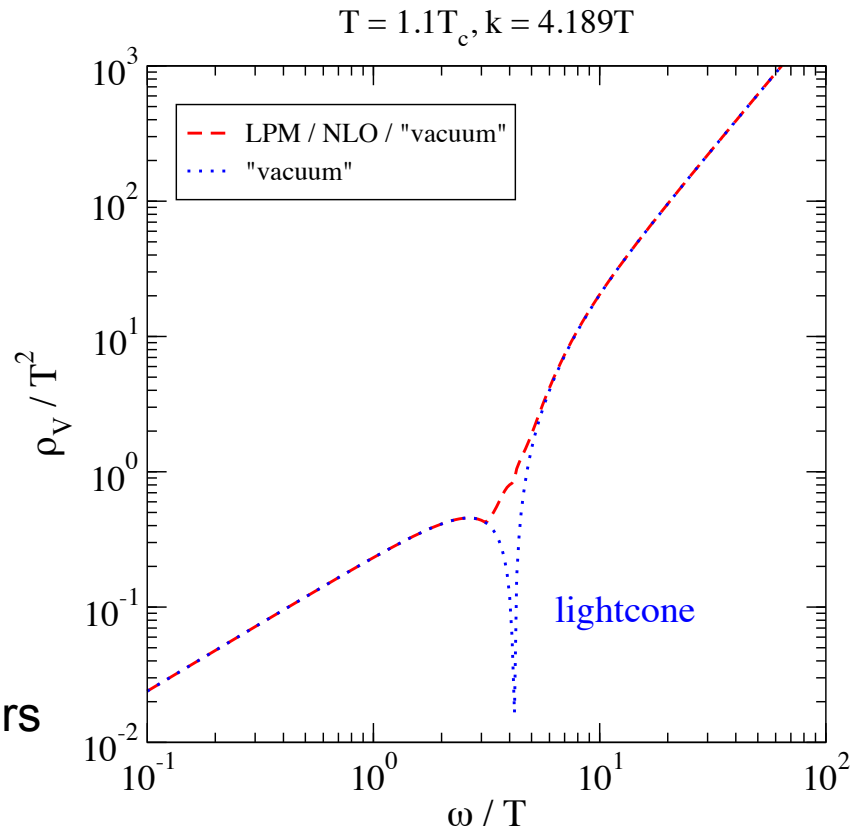
to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior

assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators



$3T < \omega < 10T$: [J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

$\omega > 10T$: [I. Ghisoiu and M.Laine, JHEP 10 (2014) 84, arXiv:1407.7955]

$\omega >> 10T$: [M.Laine, JHEP 1311 (2013) 120]

interpolation between the different regimes: www.laine.itp.unibe.ch/dilepton-lattice

Modeling the spectral function

(5+2 n_{max})th order polynomial Ansatz at small ω:

$$\rho_{\rm fit} \equiv \frac{\beta\,\omega^3}{2\omega_0^3}\left(5-\frac{3\omega^2}{\omega_0^2}\right)-\frac{\gamma\,\omega^3}{2\omega_0^2}\left(1-\frac{\omega^2}{\omega_0^2}\right)+\sum_{n=0}^{n_{\rm max}}\frac{\delta_n\omega^{1+2n}}{\omega_0^{1+2n}}\left(1-\frac{\omega^2}{\omega_0^2}\right)^2$$

with the constraints to match smoothly with pQCD at $\omega_{\rm o}$

$$\rho_{\rm V}(\omega_0,\mathbf{k}) \equiv \beta\,, \quad \rho'_{\rm V}(\omega_0,\mathbf{k}) \equiv \gamma\,,$$

and n_{max}+1 free parameters

starting with a linear behavior at $\omega \ll$ T

smoothly matched to the perturbative spectral function at $\omega_0 \simeq \sqrt{k^2+(\pi T)^2}$

In the following we will use n_{max} = 0 and n_{max} = 1 for the fits to the lattice data

and to estimate the systematic uncertainties

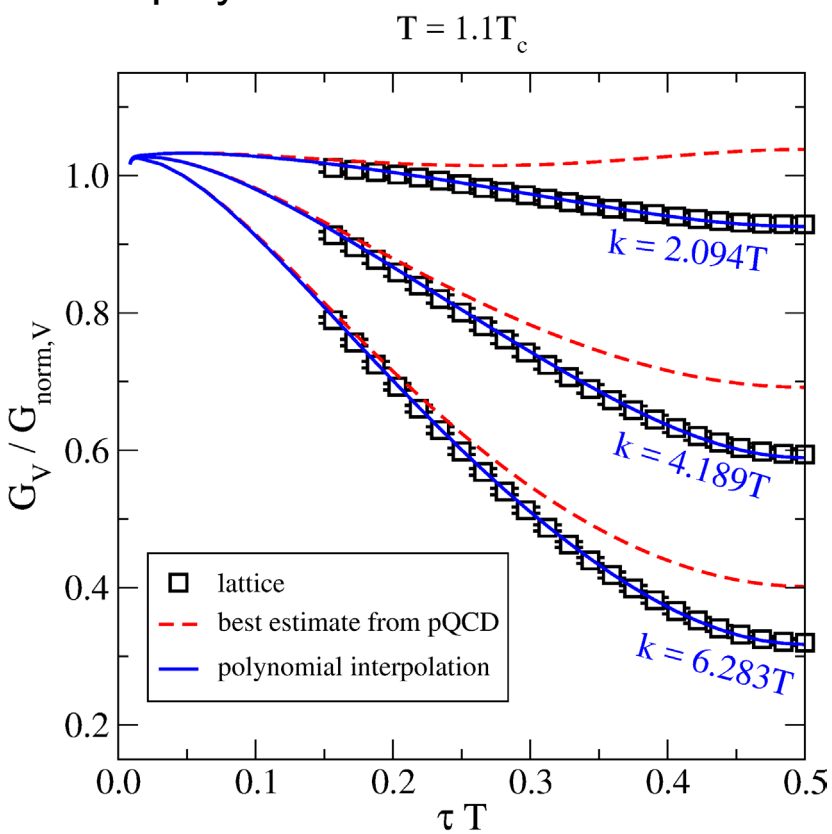
Lattice constraints on photon rates

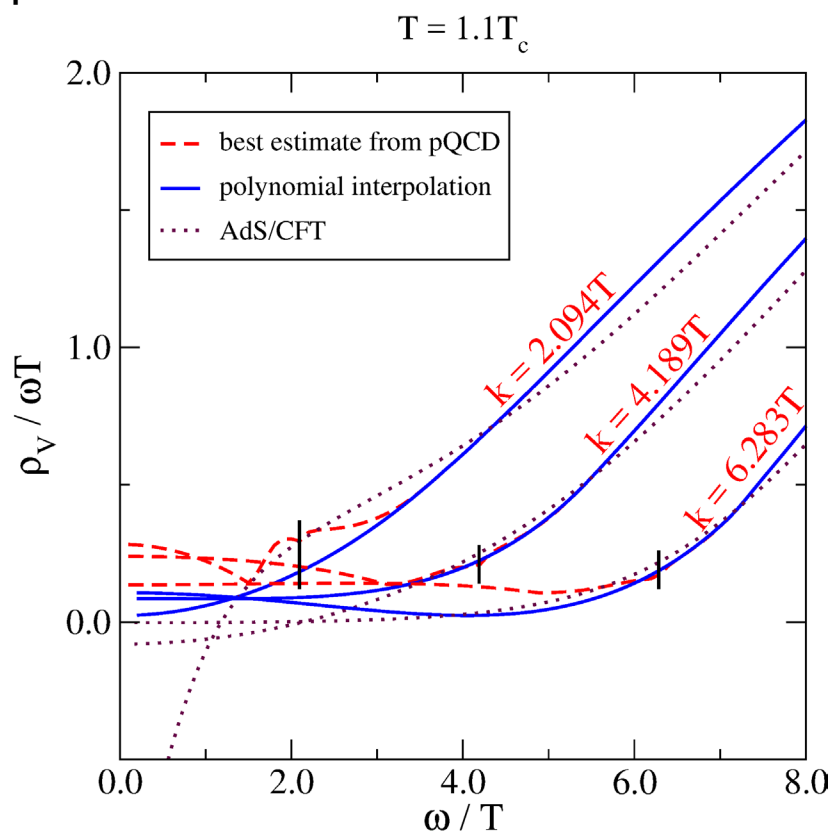
Fixed aspect ratio used to perform continuum extrapolation at finite p

$$\frac{\vec{p}}{T} = 2\pi \vec{k} \frac{N_\tau}{N_\sigma}$$

β_0	$N_s^3 \times N_\tau$	confs	$T\sqrt{t_0}$	$T/T_c\|_{t_0}$	Tr_0	$T/T_c\|_{r_0}$
7.192	$96^3 \times 32$	314	0.2796	1.12	0.816	1.09
7.544	$144^3 \times 48$	358	0.2843	1.14	0.817	1.10
7.793	$192^3 \times 64$	242	0.2862	1.15	0.813	1.09
7.192	$96^3 \times 28$	232	0.3195	1.28	0.933	1.25
7.544	$144^3 \times 42$	417	0.3249	1.31	0.934	1.25
7.793	$192^3 \times 56$	273	0.3271	1.31	0.929	1.25

use perturbation theory at large ω and fit a polynomial at small ω to extract the spectral function





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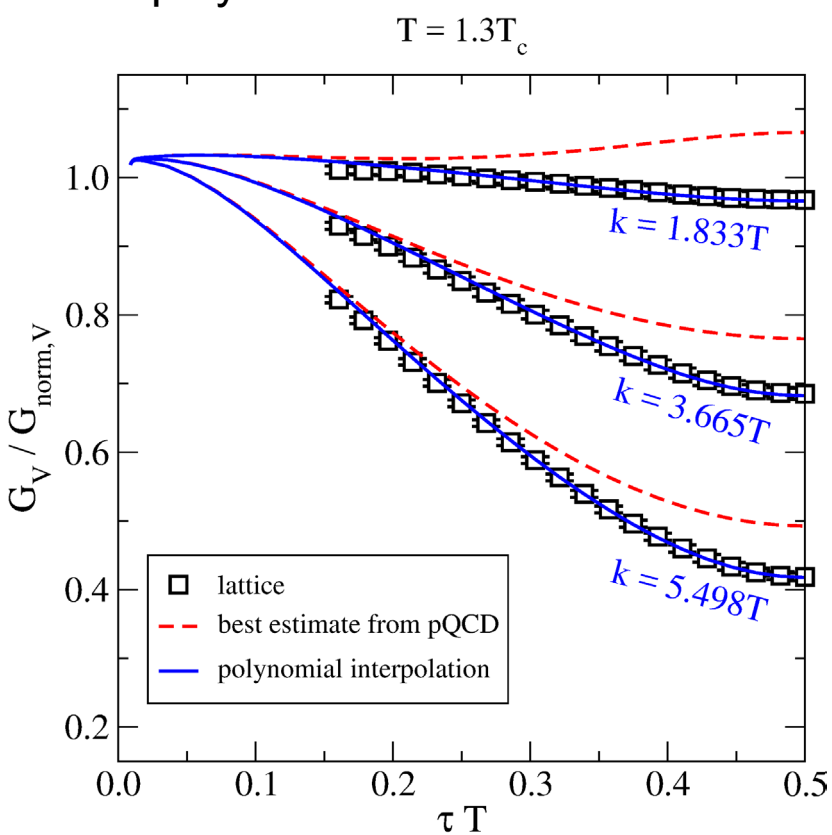
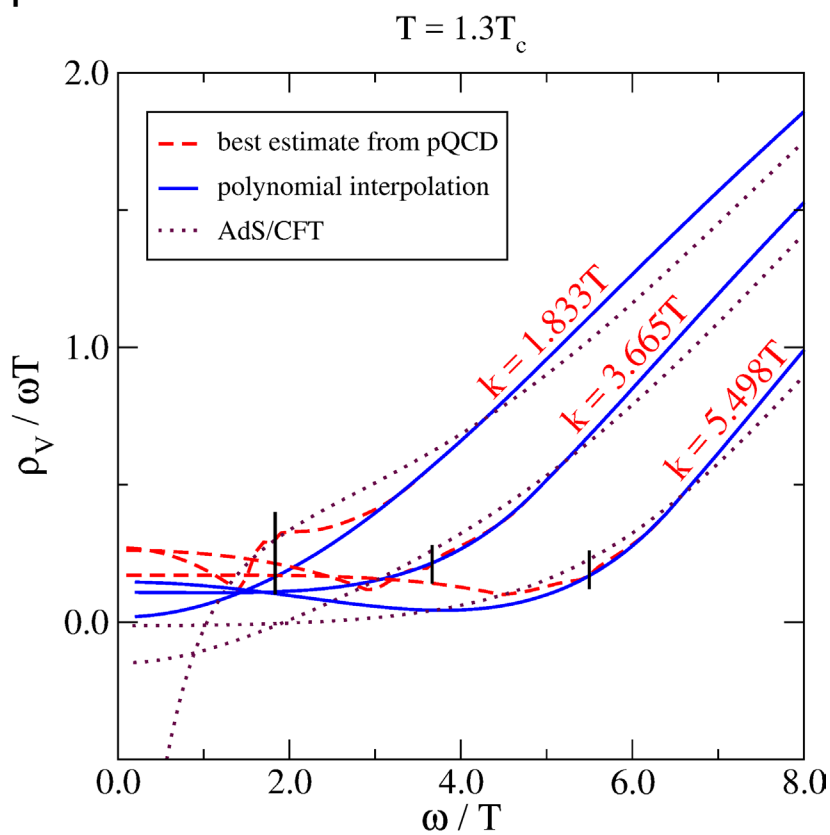
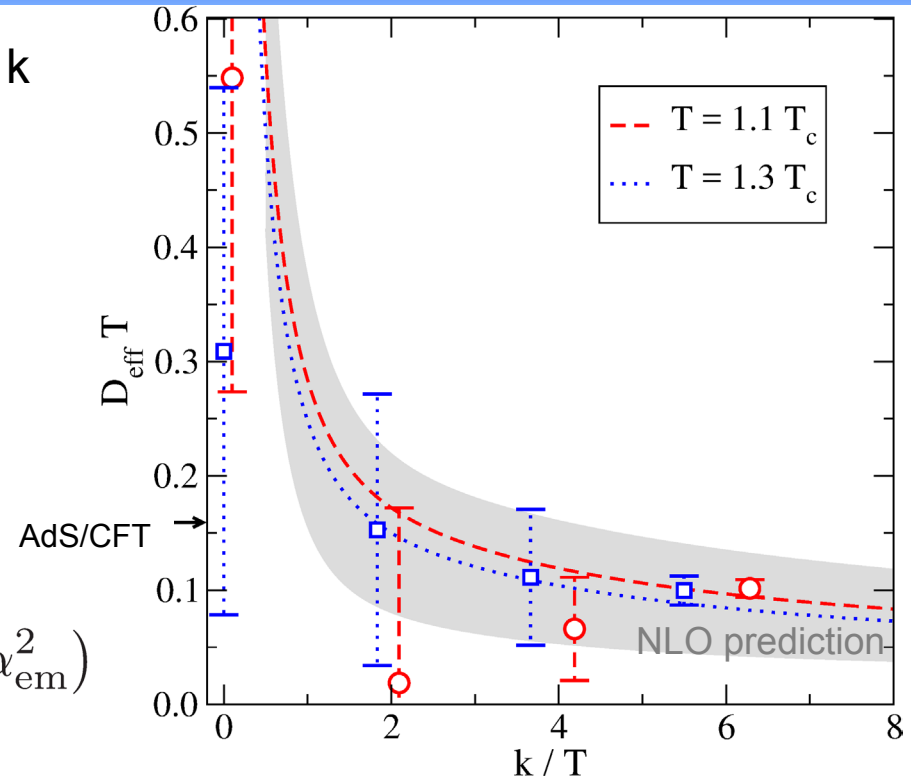




Lattice constraints on photon rates

The spectral function at the photon point ω = k

$$D_{\rm eff}(k) \equiv \begin{cases} \dfrac{\rho_{\rm V}(k,\mathbf{k})}{2\chi_{\rm q} k} & , \quad k>0 \\ \lim\limits_{\omega\to 0^+} \dfrac{\rho^{ii}(\omega,\mathbf{0})}{3\chi_{\rm q}\omega} & , \quad k=0 \end{cases} .$$



can be used to calculate the <u>photon rate</u>

$$\frac{{\rm d}\Gamma_\gamma(\mathbf{k})}{{\rm d}^3\mathbf{k}} = \frac{2\alpha_{\rm em}\chi_{\rm q}}{3\pi^2} n_{\rm B}(k) D_{\rm eff}(k) + \mathcal{O}\left(\alpha_{\rm em}^2\right)$$

becomes more perturbative at larger k, approaching the NLO prediction (valid for k>>gT)

[J. Ghiglieri, G.D. Moore, JHEP12 (2014) 029]

but non-perturbative for k/T < 3

<u>Electrical conductivity</u> obtained in the limit k→0 between the results from

AdS/CFT: $DT = \dfrac{1}{2\pi}$

[Arnold, Moore Yaffe, JHEP 05 (2003)]

LO perturbation theory [G.Policastro, D.T.Son,A.O.Starinets, JHEP09(2002)043]

using lattice value for χ_q/T^2: $DT = 2.9 - 3.1$

Conclusions and Outlook

using continuum extrapolated correlation functions from Lattice QCD and

using phenomenologically inspired and perturbatively improved Ansätze

allows to extracted transport properties and spectral properties

These results for the

→ Electrical conductivity / Flavor diffusion coefficients

→ Thermal dilepton rates

→ Thermal photon rates

should be included in hydro models for the evolution of the medium

all parameters and pQCD spectral functions are available from
[PRD94(2016)016005, PRD94(2016)034504, www.laine.itp.unibe.ch/dilepton-lattice]

The methodology developed in this studies within the quenched approximation

shall be extended to full QCD calculations for a realistic QGP medium

as close to T_c dynamical fermion degrees of freedom will become important