Hard probes on the Lattice

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A. Francis, OK, M. Laine, T. Neuhaus, H. Ohno, PRD92 (2015) 116003,
Nonperturbative estimate of the heavy quark momentum diffusion coefficient

H-T. Ding, F. Meyer, OK, PRD94 (2016) 034504,
Thermal dilepton rates and electrical conductivity of the QGP from the lattice

J. Ghiglieri, OK, M. Laine, F. Meyer, PRD94 (2016) 016005,
Lattice constraints on the thermal photon rate

Hard Probes 2016, Wuhan
23.09.2016
Motivation – electromagnetic probes

Dilepton rates

large enhancement between 150-750 MeV

possible window for photons from QGP

indications for thermal effects!?

Need to understand the contribution from QGP

both directly related to vector-meson spectral function:

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \rho_v(\omega, \vec{p}, T)$$

$$\omega \frac{dN_\gamma}{d^4xd^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_v(\omega = |\vec{p}|, T)$$

Direct and fragmentation photon relative contribution

Hadron Gas Thermal $T_i$

QGP Thermal $T_i$

“Pre-equilibrium” ("secondary" or "cascading"

Jet Re-interaction $\sqrt{T_x/\sqrt{s}}$

pQCD Prompt $x/\sqrt{s}$

Emission time

[PHENIX PRC81, 034911 (2010)]

[PHENIX PRC81, 034911 (2010)]
**Transport Coefficients** are important ingredients into **hydro/transport models** for the evolution of the system.

Usually determined by matching to experiment (see right plot)

Need to be determined from QCD using first principle lattice calculations!

for heavy flavour:

**Heavy Quark Diffusion Constant D**

[H.T.Ding, OK et al., PRD86(2012)014509]

**Heavy Quark Momentum Diffusion \( \kappa \)**


for light quarks:

Light quark flavour diffusion /

Electrical conductivity

[A.Francis, OK et al., PRD83(2011)034504
Vector meson spectral function – hard to separate different scales

\[ G(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T) \]

\[ K(\tau, \omega, T) = \frac{\cosh \left( \frac{\omega(\tau - \frac{1}{2T})}{2T} \right)}{\sinh \left( \frac{\omega}{2T} \right)} \]

Different contributions and scales enter in the spectral function
- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

Spectral functions in the QGP

\[ \rho(\omega) \]

- \( T \approx T_c \)
- \( T >> T_c \)
- \( T = \infty \)

\( G_{\mu\nu}(\tau, \vec{x}) = \langle J_\mu(\tau, \vec{x}) J^\dagger_\nu(0, \vec{0}) \rangle \]
\[ J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \Gamma_\mu \psi(\tau, \vec{x}) \]

\[ \rho(\omega \ll T) \simeq 2\chi_0 \frac{T}{M} \frac{\omega \eta}{\omega^2 + \eta^2}, \quad \eta = \frac{T}{M \Delta} \]

→ large lattices and continuum extrapolation needed
→ still only possible in the quenched approximation
Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) “color-electric correlator”

\[ G_E(\tau) \equiv -\frac{1}{3} \sum_{i=1}^{3} \frac{\left\langle \text{Re} \text{Tr} \left[ U \left( \frac{1}{T}; \tau \right) gE_i(\tau, 0) U(\tau; 0) gE_i(0, 0) \right] \right\rangle}{\left\langle \text{Re} \text{Tr}[U(\frac{1}{T}; 0)] \right\rangle} \]

Heavy quark (momentum) diffusion:

\[ \kappa = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} \quad D = \frac{2T^2}{\kappa} \]
NLO spectral function in perturbation theory:

in contrast to a narrow transport peak, from this a smooth limit is expected

qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

no bound state contributions in this operator

Lattice QCD correlation functions:
[A.Francis, OK et al., PRD92(2015)116003]
$\omega \ll T$: linear behavior motivated at small frequencies

$$\rho_{\text{IR}}(\omega) = \frac{\kappa \omega}{2T}$$

$\omega \gg T$: vacuum perturbative results and leading order thermal correction:

$$\rho_{\text{UV}}(\omega) = \left[ \rho_{\text{UV}}(\omega) \right]_{T=0} + \mathcal{O} \left( \frac{g^4 T^4}{\omega} \right)$$

using a renormalization scale $\mu_\omega = \omega$ for $\omega \gg \Lambda_{\overline{\text{MS}}}$ leading order becomes

$$\rho_{\text{UV}}(\omega) = \Phi_{\text{UV}}(\omega) \left[ 1 + \mathcal{O} \left( \frac{1}{\ln(\omega/\Lambda_{\overline{\text{MS}}})} \right) \right]$$

$$\Phi_{\text{UV}}(\omega) = \frac{g^2 (\mu_\omega) C_F \omega^3}{6\pi} , \quad \mu_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

We use Ansätze that are consistent with these asymptotic behaviors and model corrections to $\rho_{\text{IR}}$ by a power series in $\omega$
analysis of the systematic uncertainties by using the best perturbative knowledge in the UV part of the spectral function modeling corrections to $\rho_{IR}$ by a power series in $\omega$ and fitting to continuum extrapolated correlation function

\[ G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi} \rho_{\text{model}}(\omega) \cosh\left(\frac{\tau}{2} - \tau T\right) \frac{\omega}{\sinh\frac{\omega}{2T}} \]

\[ \kappa/T^3 = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} \]
using various model corrections to $\rho_{IR}$

we analysed the systematic uncertainties

$\rightarrow$ continuum estimate of $\kappa$:

$$\kappa/T^3 = \lim_{\omega \to 0} \frac{2T \rho_E(\omega)}{\omega} = 1.8...3.4$$
in the non-relativistic limit, \( \kappa \) is related to the diffusion coefficient \( D \):

\[
2\pi T D = 4\pi \frac{T^3}{\kappa} = 3.7...7.0
\]

and to the drag coefficient \( \eta_D \):

\[
\eta_D = \frac{\kappa}{2M_{\text{kin}}T} \left( 1 + O \left( \frac{\alpha_s^{3/2}T}{M_{\text{kin}}} \right) \right)
\]

used to estimate the time scale associated with kinetic equilibration of heavy quarks:

\[
\tau_{\text{kin}} = \frac{1}{\eta_D} = (1.8...3.4) \left( \frac{T_c}{T} \right)^2 \left( \frac{M}{1.5 \text{ GeV}} \right) \text{fm/c}
\]

close to \( T_c \), \( \tau_{\text{kin}} \approx 1 \text{fm/c} \) and therefore charm quark kinetic equilibration appears to be almost as fast as that of light partons.
Lattice QCD results on heavy quark diffusion coefficients

\[ D = \frac{2T^2}{\kappa} \]

\[ \alpha_s \approx 0.2 \]

NLO pQCD

\[ \frac{T}{T_c} \]

charm quark mass

quenched approximation

no continuum extrapolation yet

next goals: continuum extrapolation for charm and bottom vector-meson correlator

\[ \Rightarrow \text{quark mass dependence of diffusion coefficients} + \text{sequential melting of quarkonia} \]
Vector-meson correlation function for light quarks on large & fine lattices


quenched SU(3) gauge configurations (separated by 500 updates)

non-perturbatively O(a) clover improved Wilson fermion valence quarks

non-perturbative renormalization constants and quark masses close to the chiral limit

| $N_{\tau}$ | $N_{\sigma}$ | $\beta$ | $\kappa$ | $T\sqrt{t_0}$ | $T/T_c |_{t_0}$ | $T r_0$ | $T/T_c |_{r_0}$ | confs |
|---|---|---|---|---|---|---|---|---|
| 32 | 96 | 7.192 | 0.13440 | 0.2796 | 1.12 | 0.8164 | 1.09 | 314 |
| 48 | 144 | 7.544 | 0.13383 | 0.2843 | 1.14 | 0.8169 | 1.10 | 358 |
| 64 | 192 | 7.793 | 0.13345 | 0.2862 | 1.15 | 0.8127 | 1.09 | 242 |
| 28 | 96 | 7.192 | 0.13440 | 0.3195 | 1.28 | 0.9330 | 1.25 | 232 |
| 42 | 144 | 7.544 | 0.13383 | 0.3249 | 1.31 | 0.9336 | 1.25 | 417 |
| 56 | 192 | 7.793 | 0.13345 | 0.3271 | 1.31 | 0.9288 | 1.25 | 273 |
| 24 | 128 | 7.192 | 0.13440 | 0.3728 | 1.50 | 1.0886 | 1.46 | 340 |
| 32 | 128 | 7.457 | 0.13390 | 0.3846 | 1.55 | 1.1093 | 1.49 | 255 |
| 48 | 128 | 7.793 | 0.13340 | 0.3817 | 1.53 | 1.0836 | 1.45 | 456 |

Scale setting using $r_0$ and $t_0$ [A.Francis, M.Laine, T.Neuhaus, H.Ohno PRD92(2015)116003]

fixed aspect ratio $N_{\sigma}/N_{\tau} = 3$ and 3.43 to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi k \frac{N_{\tau}}{N_{\sigma}}$$

constant physical volume $(1.9\text{fm})^3$
Vector-meson correlation function

compared to free (non-interacting) correlator:

\[ G_V^{\text{free}}(\tau) = 6T^2 \left( \pi(1-2\tau T) \frac{1 + \cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2 \frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right) \]

hard to distinguish differences due to different orders of magnitude in the correlator

→ in the following we will use \( G_V^{\text{free}}(\tau) \) as a normalization
Continuum extrapolation

correlators normalized by quark number susceptibility $\chi_q$ independent of renormalization

and by the free non-interacting correlator $G_{VV}^{\text{free}}(\tau)$

we interpolate the correlator for each lattice spacing

and perform the continuum limit $a \to 0$ at each distance $\tau T$

cut-off effects are visible at all distances on finite lattices
Continuum extrapolation

Continuum extrapolation cut-off effects are visible at all distances on finite lattices but well defined continuum limit on the correlator level well behaved continuum correlator down to small distances approaching the correct asymptotic limit for $\tau \to 0$
Continuum extrapolation

Cut-off effects are visible at all distances on finite lattices but well defined continuum limit on the correlator level. Well behaved continuum correlator down to small distances approaching the correct asymptotic limit for $\tau \to 0$.
continuum extrapolated results available for three temperatures in the QGP

similar behavior in this temperature region

main difference due to different quark number susceptibility $\chi_q/T^2$

→ indications for a weak T-dependence of the temperature scaled electrical conductivity and thermal dilepton rates
Spectral functions at high temperature

**Free theory (massless case):**

free non-interacting vector spectral function (infinite temperature):

\[
\rho_{00}^{\text{free}}(\omega) = 2\pi T^2 \omega \delta(\omega)
\]

\[
\rho_{ii}^{\text{free}}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi} \omega^2 \tanh(\omega/4T)
\]

\(\delta\)-functions exactly cancel in \(\rho_V(\omega) = -\rho_{00}(\omega) + \rho_{ii}(\omega)\)

**With interactions (but without bound states):**

while \(\rho_{00}\) is protected, the \(\delta\)-funtion in \(\rho_{ii}\) gets smeared \(\rightarrow\) transport peak:

**Ansatz:**

\[
\rho_{00}(\omega) = 2\pi \chi_q \omega \delta(\omega)
\]

\[
\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega \Gamma / 2}{\omega^2 + (\Gamma / 2)^2} + \frac{3}{2\pi} (1 + \kappa) \omega^2 \tanh(\omega/4T)
\]

Ansatz with 3-4 parameters: \((\chi_q), c_{BW}, \Gamma, \kappa\)

\(\rightarrow\) electrical conductivity:

\[
\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T} \frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW} C_{em}
\]
use our Ansatz for the spectral function and fit to the continuum extrapolated correlators

\[
\begin{align*}
\rho_{00}(\omega) &= 2\pi \chi_q \omega \delta(\omega) \\
\rho_{ii}(\omega) &= 2\chi_q c_{BW} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1 + \kappa) \omega^2 \tanh(\omega/4T) \\
G(\tau, \vec{p}, T) &= \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega, \vec{p}, T) K(\tau, \omega, T)
\end{align*}
\]

all three temperatures well described by this rather simple Ansatz
Use our Ansatz for the spectral function

\[ \rho_{00}(\omega) = 2\pi \chi_q \omega \delta(\omega) \]
\[ \rho_{ii}(\omega) = 2\chi_q e_B w \frac{\omega \Gamma / 2}{\omega^2 + (\Gamma / 2)^2} + \frac{3}{2\pi} (1 + \kappa) \omega^2 \tanh(\omega / 4T) \times \Theta(\omega_0, \Delta_\omega) \]

Analysis of the systematic errors using truncation of the large \( \omega \) contribution

\[ \Theta(\omega_0, \Delta_\omega) = \left(1 + e^{(\omega_0^2 - \omega^2) / \omega \Delta_\omega}\right)^{-1} \]

\[ \frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T} \]

electrical conductivity

systematic uncertainties (within this Ansatz) estimated by varying the truncation with similar \( \chi^2 / \text{dof} \sim 0.5-1.1 \)

Use a flat transport Ansatz for the spectral function

\[ \rho_{\text{flat}}(\omega) = a \chi q \omega \left( 1 - \tilde{\Theta}(\omega_0, \Delta_0) \right) + (1 + k) \rho_{\text{free}}(\omega) \tilde{\Theta}(\omega_1, \Delta_1) \]

\[ \tilde{\Theta}(\omega, \omega_i, \Delta_i) = \left( 1 + \exp \left( \frac{\omega_i^2 - \omega^2}{\omega \Delta_i} \right) \right)^{-1} \]

**Analysis of the systematic errors**

using a flat behavior at small \( \omega \)

systematic uncertainties (within this Ansatz) estimated by varying the truncation with similar \( \chi^2/\text{dof} \approx 0.5-1.1 \) still consistent with our data \( \rightarrow \) lower limit for \( \sigma/T \)
Improve the UV behavior of the spectral function using perturbation theory:

At very high energies, due to asymptotic freedom:

→ perturbation should be working
→ thermal effects should be suppressed
→ “vacuum physics”

5-loop vacuum spectral function:

\[
\rho_V(\omega) = \frac{3\omega^2}{4\pi} R(\omega^2)
\]

\[
R(\omega^2) = r_{0,0} + r_{1,0} a_s + (r_{2,0} + r_{2,1} \ell) a_s^2 \\
+ (r_{3,0} + r_{3,1} \ell + r_{3,2} \ell^2) a_s^3 \\
+ (r_{4,0} + r_{4,1} \ell + r_{4,2} \ell^2 + r_{4,3} \ell^3) a_s^4 + \mathcal{O}(a_s^5)
\]

using 3-loop \(\alpha_s\) and \(l = \log(\mu^2/\omega^2)\)

using a renormalization scale \(\mu = (1..5)\max(\pi T, \omega)\)

taking leading order thermal effect into account

\[
\rho^{(T)}_{ii}(\omega) \equiv \frac{3\omega^2}{4\pi} \left[1 - 2n_F(\omega/2)\right] R(\omega^2) + \pi \chi^\text{free}_q \omega \delta(\omega)
\]

Spectral function and electrical conductivity

include improved large $\omega$ behavior from vacuum perturbation theory

\[
\rho_{\text{impr}}(\omega, T) = \rho_{\text{free}}^V(\omega, T) R(\omega^2)
\]

\[
\rho_R(\omega, T) = \rho_{\text{BW}}(\omega, T) + C \rho_{\text{impr}}(\omega, T).
\]

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\sigma/(C_{\text{em}} T)$</th>
<th>$\Gamma/T$</th>
<th>$c_{\text{BW}} T/\Gamma$</th>
<th>$C$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.1T_c$</td>
<td>0.452(251)</td>
<td>1.62(1.09)</td>
<td>0.790(438)</td>
<td>0.993(7)</td>
<td>1.11</td>
</tr>
<tr>
<td>$1.3T_c$</td>
<td>0.301(87)</td>
<td>2.89(1.18)</td>
<td>0.504(145)</td>
<td>0.984(8)</td>
<td>0.53</td>
</tr>
<tr>
<td>$1.5T_c$</td>
<td>0.326(87)</td>
<td>2.38(85)</td>
<td>0.548(146)</td>
<td>0.996(7)</td>
<td>1.12</td>
</tr>
</tbody>
</table>
Electrical conductivity

continuum estimate for the

T-dependence of the electrical conductivity:

\[ \frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p} = 0, T)}{\omega T} \]

lower and upper limits from the systematic analysis of our classes of spectral functions:

other studies using dynamical clover Wilson or staggered fermions (all w/o continuum limit):
Dilepton rate directly related to vector spectral function:

\[
\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T} - 1)} \rho_V(\omega, T)
\]
Photon rate directly related to vector spectral function (at finite momentum):

\[
\omega \frac{dN_\gamma}{d^4x d^3q} = \frac{5\alpha}{6\pi^2} \frac{1}{e^{\omega/T} - 1} \sigma_V (\omega = |\vec{k}|, T)
\]
Non-interacting limit, “Born rate” for large invariant mass $M \gg \pi T$, with $M^2 = \omega^2 + k^2$

$$\rho_\nu(\omega, k) = \frac{N_c T M^2}{2\pi k} \left\{ \ln \left[ \frac{\cosh(\frac{\omega+k}{4T})}{\cosh(\frac{\omega-k}{4T})} \right] - \frac{\omega \theta(k - \omega)}{2T} \right\},$$


Leading-log order for invariant mass $M=0$:

$$\rho_\nu(k, k) = \frac{\alpha_s N_c C_F T^2}{4} \ln \left( \frac{1}{\alpha_s} \right) \left[ 1 - 2n_F(k) \right] + O(\alpha_s T^2),$$


Complete leading order for invariant mass $M=0$:


NLO at $M = 0$:

[J.Ghiglieri et al., JHEP 1305 (2013) 010]

NLO at $M \sim gT$:

[J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029]

NLO at $M \sim \pi T$:

[M.Laine, JHEP 1311 (2013) 120]

$N^4$LO at $M \gg \pi T$:

Vector spectral function in the hydrodynamic regime for $\omega, k \lesssim \alpha_s^2 T$:

$$\frac{\rho_V(\omega, k)}{\omega} = \left( \frac{\omega^2 - k^2}{\omega^2 + D^2 k^4} + 2 \right) \chi_q D$$

with the quark number susceptibility:

$$\chi_q \equiv \int_0^\beta d\tau \int_x \langle V^0(\tau, x) V^0(0) \rangle$$

and the diffusion coefficient:

$$D \equiv \frac{1}{3 \chi_q} \lim_{\omega \to 0^+} \sum_{i=1}^3 \rho_{ii}(\omega, 0)$$

which relate to the electric conductivity:

$$\sigma = e^2 \sum_{f=1}^{N_f} Q_f^2 \chi_q D$$

In this limit the (soft) photon rate becomes:

$$\frac{d\Gamma_\gamma(k)}{d^3k} \lesssim \frac{\alpha_s^2 T}{(2\pi)^3 k}$$

In the AdS/CFT framework the vector spectral function has the same infrared structure and here numerical result can make predictions beyond the hydro regime [S. Caron-Huot, JHEP12 (2006) 015]

Although not QCD, it may give qualitative insight into the structures at small $\omega$ and $k$
Thermal corrections in the intermediate frequency regime required and proper treatment of the small frequency regime

interpolation between different regimes

progress in perturbation theory in the past years → compare to lattice QCD results
pQCD spectral function used in our analysis

to allow for non-perturbative effects

and to analyze how far pQCD can be trusted

we model the infrared behavior

assuming smoothness at the light cone

and fit to continuum extrapolated lattice correlators

3T < \omega < 10T: \quad [\text{J.Ghiglieri and G.D.Moore, JHEP 1412 (2014) 029}]


\omega >> 10T: \quad [\text{M.Laine, JHEP 1311 (2013) 120}]

interpolation between the different regimes: \quad \text{www.laine.itp.unibe.ch/dilepton-lattice}
Modeling the spectral function

\[(5+2\ n_{\text{max}})^{\text{th}}\ \text{order polynomial Ansatz at small } \omega:\]

\[
\rho_{\text{fit}} \equiv \frac{\beta}{2\omega_0^3} \left( 5 - \frac{3\omega^2}{\omega_0^2} \right) - \frac{\gamma}{2\omega_0^3} \left( 1 - \frac{\omega^2}{\omega_0^2} \right) + \sum_{n=0}^{n_{\text{max}}} \frac{\delta_n}{\omega_0^{1+2n}} \left( 1 - \frac{\omega^2}{\omega_0^2} \right)^2
\]

with the constraints to match smoothly with pQCD at \( \omega_0 \)

\[
\rho_\nu(\omega_0, k) \equiv \beta, \quad \rho'_\nu(\omega_0, k) \equiv \gamma,
\]

and \( n_{\text{max}} + 1 \) free parameters

starting with a linear behavior at \( \omega \ll T \)

smoothly matched to the perturbative spectral function at \( \omega_0 \approx \sqrt{k^2 + (\pi T)^2} \)

In the following we will use \( n_{\text{max}} = 0 \) and \( n_{\text{max}} = 1 \) for the fits to the lattice data

and to estimate the systematic uncertainties
Lattice constraints on photon rates

Fixed aspect ratio used to perform continuum extrapolation at finite $p$

$$\frac{\vec{p}}{T} = 2\pi k \frac{N_\tau}{N_\sigma}$$

use perturbation theory at large $\omega$

and fit a polynomial at small $\omega$ to extract the spectral function

| $\beta_0$ | $N_s^3 \times N_\tau$ | confs | $T\sqrt{t_0}$ | $T/T_c|_{t_0}$ | $Tr_0$ | $T/T_c|_{r_0}$ |
|-----------|-------------------------|-------|---------------|-----------------|--------|-----------------|
| 7.192     | $96^3 \times 32$        | 314   | 0.2796        | 1.12            | 0.816  | 1.09            |
| 7.544     | $144^3 \times 48$       | 358   | 0.2843        | 1.14            | 0.817  | 1.10            |
| 7.793     | $192^3 \times 64$       | 242   | 0.2862        | 1.15            | 0.813  | 1.09            |
| 7.192     | $96^3 \times 28$        | 232   | 0.3195        | 1.28            | 0.933  | 1.25            |
| 7.544     | $144^3 \times 42$       | 417   | 0.3249        | 1.31            | 0.934  | 1.25            |
| 7.793     | $192^3 \times 56$       | 273   | 0.3271        | 1.31            | 0.929  | 1.25            |

T = 1.1T_c

$G_\nu / G_{norm, \nu}$

$k = 2.094T$

$k = 4.189T$

$k = 6.283T$

$\rho_\nu / \omega T$

$k = 2.094T$

$k = 4.189T$

$k = 6.283T$
Fixed aspect ratio used to perform continuum extrapolation at finite $p$

\[ \frac{\vec{p}}{T} = 2\pi k \frac{N_\tau}{N_\sigma} \]

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The spectral function at the photon point $\omega = k$

$$D_{\text{eff}}(k) \equiv \begin{cases} 
\frac{\rho_\nu(k, k)}{2\chi^2_q k} & , \ k > 0 \\
\lim_{\omega \to 0^+} \frac{\rho^{ii}(\omega, 0)}{3\chi_q \omega} & , \ k = 0 
\end{cases}$$

can be used to calculate the photon rate

$$\frac{d\Gamma_\gamma(k)}{d^3k} = \frac{2\alpha_{\text{em}}\chi_q}{3\pi^2} n_B(k) D_{\text{eff}}(k) + \mathcal{O}(\alpha_{\text{em}}^2)$$

becomes more perturbative at larger $k$, approaching the NLO prediction (valid for $k \gg gT$)

but non-perturbative for $k/T < 3$

Electrical conductivity obtained in the limit $k \to 0$ between the results from

AdS/CFT: $DT = \frac{1}{2\pi}$ [Arnold, Moore Yaffe, JHEP 05 (2003)]

Conclusions and Outlook

using continuum extrapolated correlation functions from Lattice QCD and
using phenomenologically inspired and perturbatively improved Ansätze
allows to extracted transport properties and spectral properties

These results for the

→ Electrical conductivity / Flavor diffusion coefficients
→ Thermal dilepton rates
→ Thermal photon rates

should be included in hydro models for the evolution of the medium

all parameters and pQCD spectral functions are available from

The methodology developed in this studies within the quenched approximation

shall be extended to full QCD calculations for a realistic QGP medium

as close to $T_c$ dynamical fermion degrees of freedom will become important