LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

Hard Probes
Wuhan, September 2016

Jet Substructure

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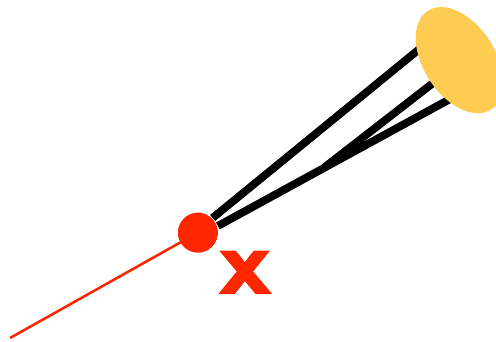
Includes material from
Gavin Salam and Grégory Soyez

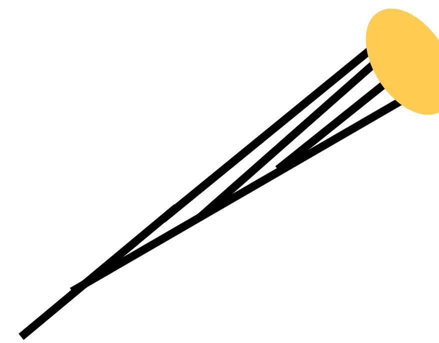
Not all jets are created equal

For instance, you may want to be able to tell



Decay of a heavy (boosted) object

from



Light parton fragmentation

Or, more generally, you may want to be able to tell something about how the jet originated (e.g. quark/gluon discrimination, quenching, ...)

How to ‘look’ inside a jet?

- Use the clustering history of a ‘physical’ sequential recombination clustering algorithm
- Study jet shape-variables sensitive to specific distributions of radiation inside the jet
- Literally ‘look’ at the distribution of radiation inside the jet (machine-learning techniques)
- .....

The challenge

The structure of a jet is usually obscured by soft, large-angle noise (underlying event, pileup,...)

Grooming and **background subtraction** go hand in hand in 'cleaning it up' and facilitating the **tagging** of the relevant features

(aim: limit contamination from background while retaining bulk of perturbative radiation)

(Boosted) jet studies at the LHC

Lily Asquith, summary talk at BOOST 2015

Boost is about:

1. Tagging high pT objects (SM and BSM)
2. Improving measurements (pileup, mass resolution etc)

ATLAS and CMS have taken different approaches to these things from day one.

ATLAS:
AKT4 CA12 split-fitered (BDRS)
AKT10 trimmed (R3/R2)
N-subjettiness WTA
JVT/ ρ
D2

CMS:
AKT5 CA8 pruned (p510)
CA15 HTT
N-subjettiness one-pass
Puppi
Soft drop

Essentially none of these tools existed as late as seven years ago

jet algorithms
groomers/taggers/jet shapes
background subtraction tools

Glossary

What	i.e.	When	Ref.
AKT	Anti-kt algorithm	2008	0802.1189
CA	Cambridge/Aachen algorithm	1999	9907280
BDRS	mass-drop tagger, includes filtering	2008	0802.2470
trimmed	Trimming, tagger/groomer	2009	0912.1342
pruned	Pruning, tagger/groomer	2009	0903.5081
HTT	HepTopTagger	2009	0910.5472
N-subjettiness	jet shape function, used in tagging	2010	1011.2268
WTA	Winner-Take-All (recombination scheme)	2013	1310.7584
one-pass	choice of axis for N-subjettiness	2010	
JVT	Jet Vertex Tagger (used in pileup subtr.)	2014	
ρ	background density (used in pileup subtr.)	2007	0707.1378
D2	jet shape function, used in tagging	2014	1409.6298
PUPPI	particle-by-particle pileup subtr.	2014	1407.6013
Soft Drop	tagger/groomer	2014	1402.2657

Jet algorithms

Sequential recombination algorithms

1. Define a **distance d_{ij}** between two particles and a **beam distance d_{iB}**. Calculate them for all particles in event

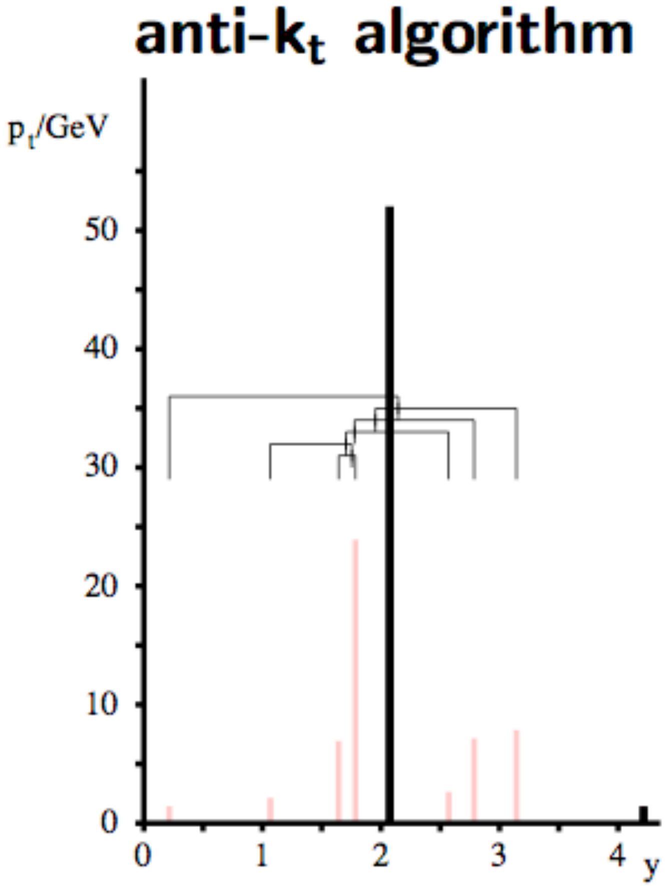
2. Find **smallest** of all d_{ij}, d_{iB}

3. If it's a d_{ij}, **recombine** particles *i* and *j*.
   If it's a d_{iB}, call particle *i* a jet

4. **Repeat** from step 2 until no particles are left

5. Only use jets with $p_t > p_{t,min}$

Hierarchical substructure



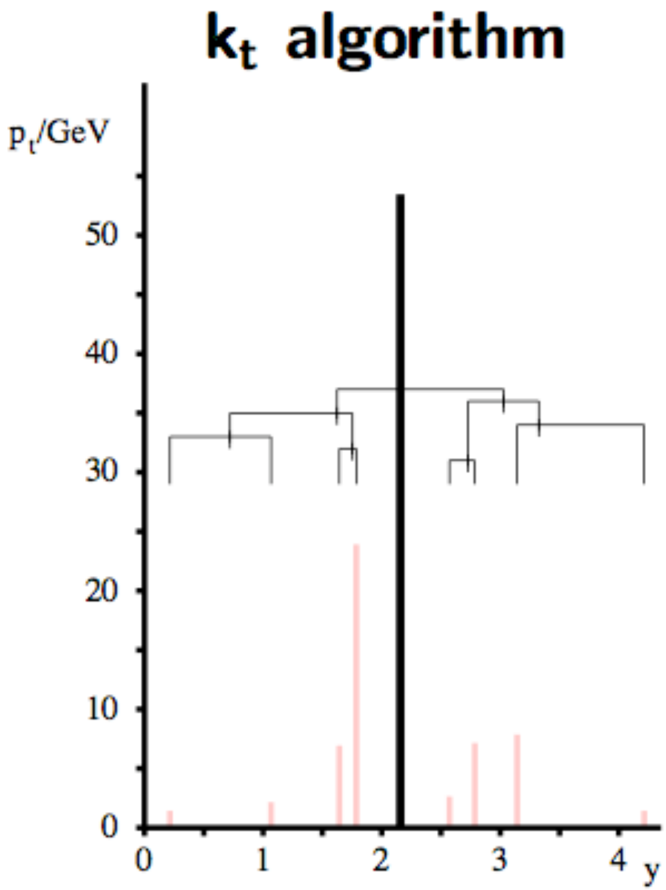
Anti-k_t distance measure

$$d_{ij} = \min\left(\frac{1}{p_{ti}^2}, \frac{1}{p_{tj}^2}\right)\frac{\Delta y^2 + \Delta\phi^2}{R^2}$$

Cluster by merging
to the **hardest/closest** particle

MC, G. Salam and G. Soyez, arXiv:0802.1189

Hierarchical substructure



k_t distance measure

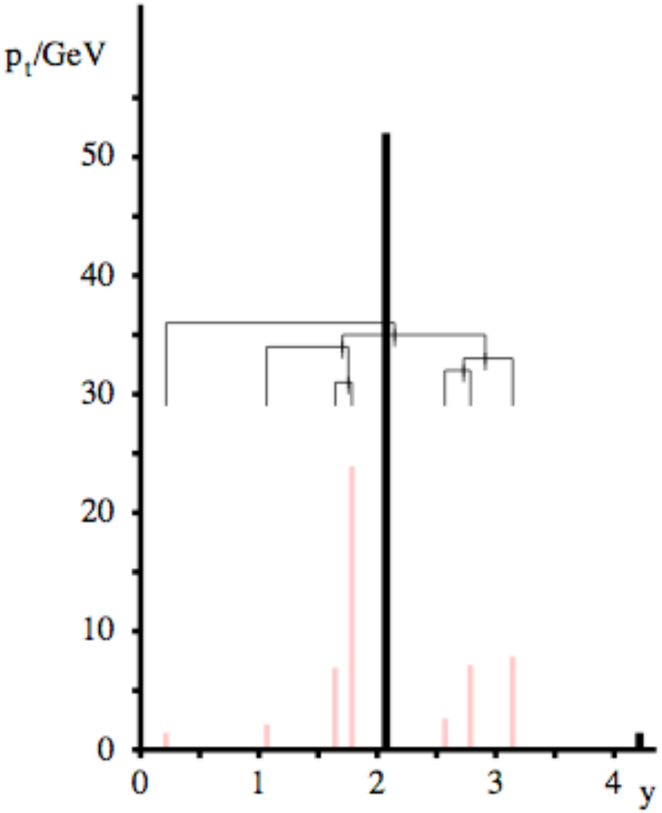
$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \frac{\Delta y^2 + \Delta \phi^2}{R^2}$$

Cluster by merging
the **softest/closest** particles

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

Hierarchical substructure

Cambridge/Aachen



C/A distance measure

$$d_{ij} = \frac{\Delta y^2 + \Delta\phi^2}{R^2}$$

Cluster by merging the **closest** particles

Y. Dokshitzer, G. Leder, S.Moretti and B. Webber, JHEP 08 (1997) 001
M.Wobisch and T.Wengler, hep-ph/9907280

Tagging and Grooming

Tagging and Grooming

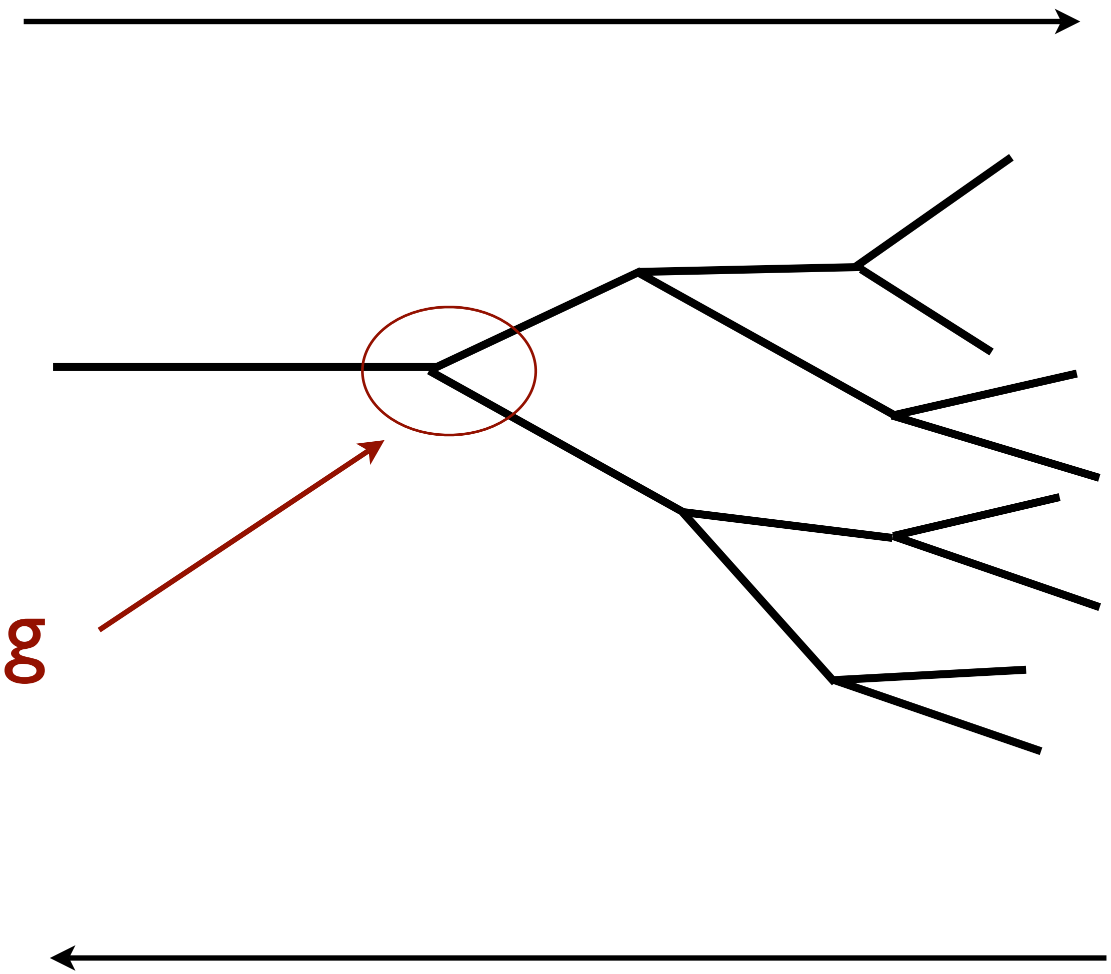
The substructure of a jet can be exploited to

- *remove background contamination from the jet or its components, while keeping the bulk of the perturbative radiation, and without affecting overall jet production rates (often generically denoted as* ***grooming****)*
  - *First examples: filtering, trimming, pruning*
- ***tag*** *a particular structure inside the jet, e.g. a massive particle decaying or a specific parton splitting*
  - *First examples: Higgs (2-prong decay), top (3-prong decay)*

This can lead to the ability to reconstruct a 'relevant splitting'

Parton shower: in theory....

direction of shower



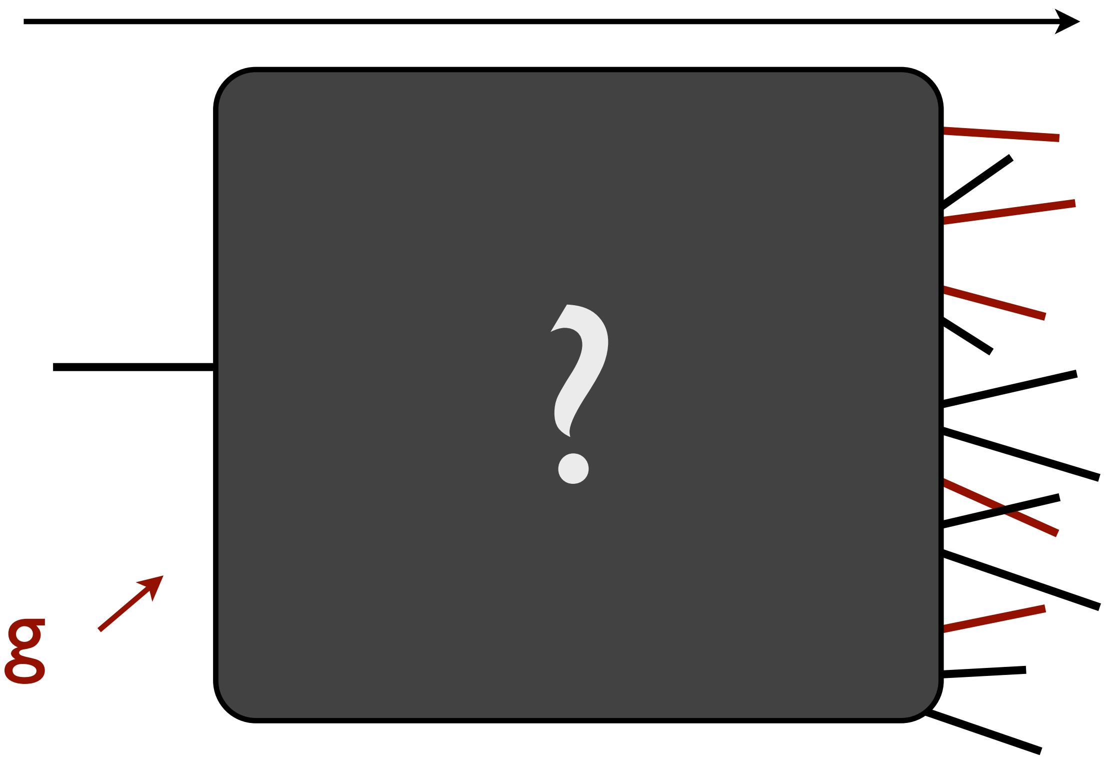
First splitting

direction of clustering

Parton shower: in practice

direction of shower

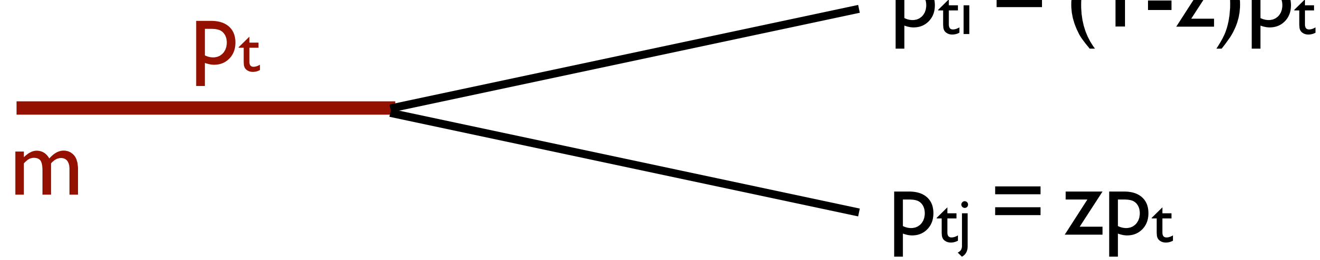
First splitting



direction of clustering

Splittings and distances

Quasi-collinear splitting ($p_{tj} < p_{ti}$)



Invariant mass: $$m^2 \simeq p_{ti} p_{tj} \Delta R_{ij}^2 = (1-z) z p_t^2 \Delta R_{ij}^2$$

k_t distance: $$d_{ij} \overset{(p_{tj} < p_{ti})}{=} z^2 p_t^2 \Delta R_{ij}^2 \simeq \frac{z}{1-z} m^2$$

For a given mass, the **background** (parton shower) will have smaller distance d_{ij} than the **signal** (massive decay, symmetric 1→2 decay), i.e. it will tend to **cluster earlier** in the k_t algorithm

Potential tagger: last clustering in k_t algorithm

This is where the hierarchy of the k_t algorithm becomes relevant. QCD radiation is mainly clustered first, and only at the end the more symmetric due to decays are reclustered

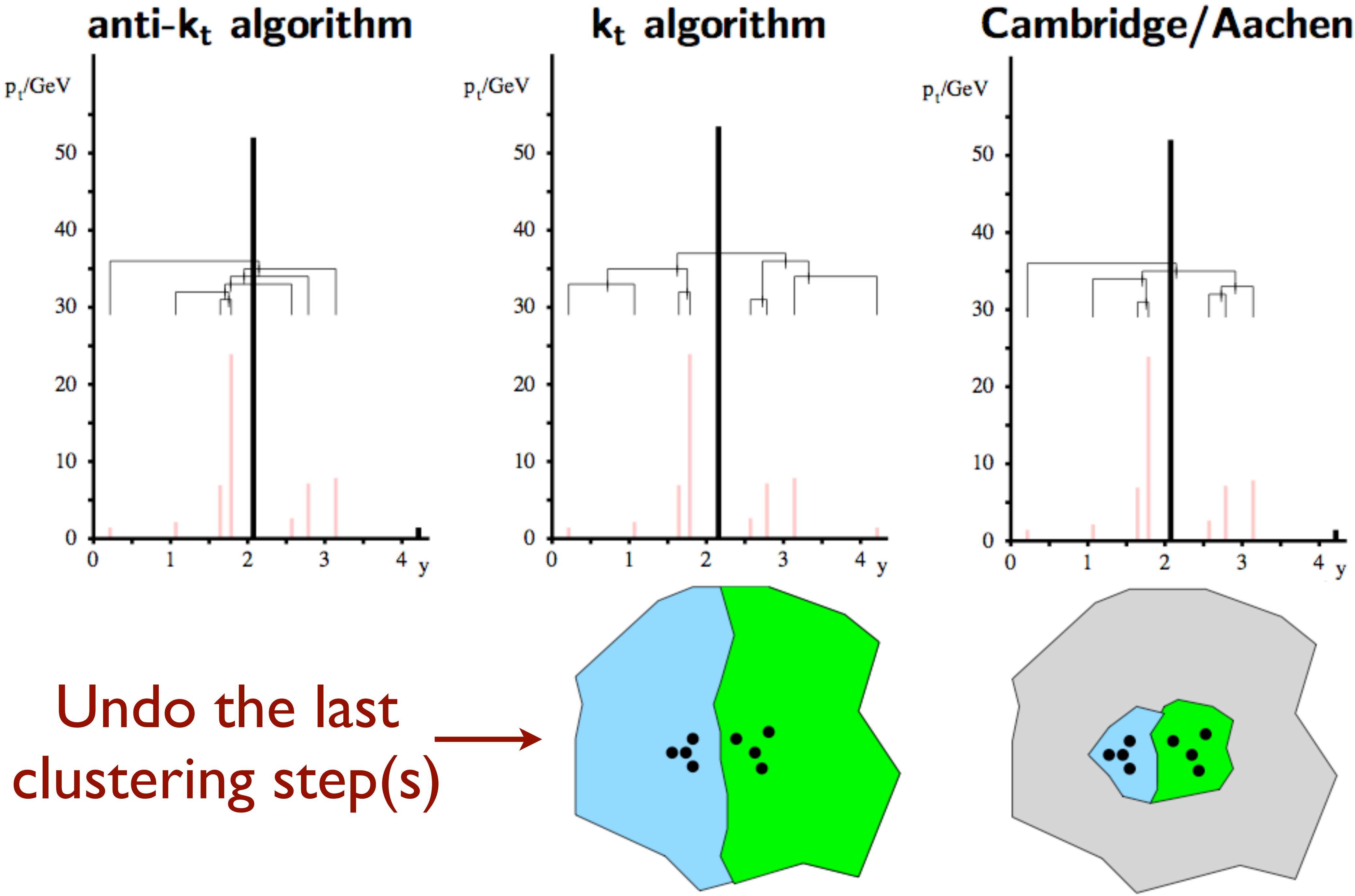
What jet algorithm to use

k_t can seem simple (just decluster last step), but the presence of large-angle soft noise in subjets can **degrade signal efficiency**

Cambridge/Aachen behaves better since it adapts to the angular distance of the relevant subjets. However, one needs to **iteratively decluster** in order to find the right splitting

Slide by
Gavin Salam

Hierarchical substructure



Undo the last clustering step(s) →

Tagging and Grooming

Declustering until the relevant splitting is found is a form of grooming/tagging (e.g. **MassDrop**, **SoftDrop**)

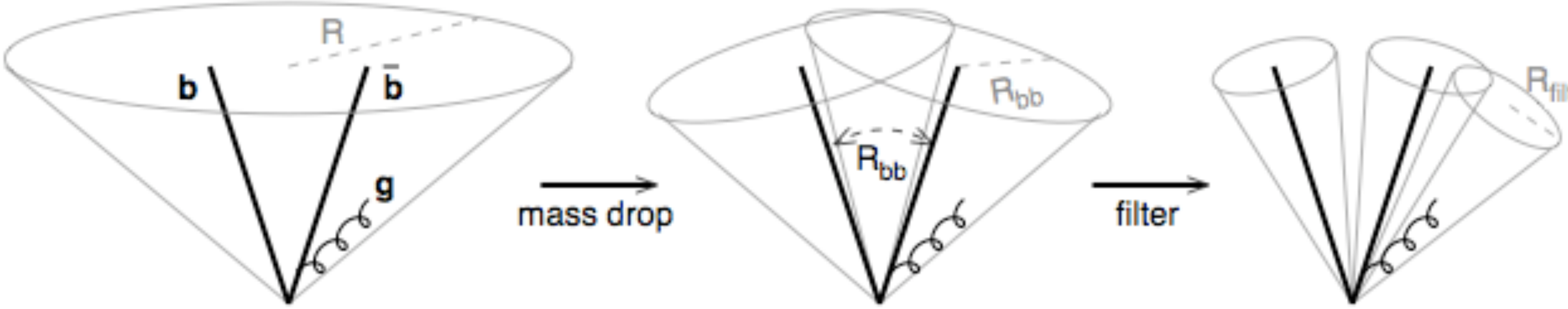
Other approaches include reclustering jet constituents with a smaller radius and keeping only some hardest subjets (e.g. **Filtering**, **Trimming**), or veto some soft, large-angle recombinations while clustering (e.g. **Pruning**)

Butterworth, Davison, Rubin, Salam, 2008
Larkoski, Marzani, Soyez, Thaler, 2014
Krohn, Thaler, Wang, 2009
S. Ellis, Vermilion, Walsh, 2009

$pp \to ZH \to \nu\bar{\nu}b\bar{b}$

The BDRS tagger/groomer

Butterworth, Davison, Rubin, Salam, 2008



- A two-prong tagger/groomer for boosted Higgs, which
  - Uses the **Cambridge/Aachen** algorithm (because it's 'physical')
  - Employs a **Mass-Drop** condition, as well as an **asymmetry cut** to find the **relevant splitting** (i.e. '**tag**' the heavy particle)
  - Includes a post-processing step, using 'filtering' (introduced in the same paper) to clean as much as possible the resulting jets of UE contamination ('**grooming**')

Soft Drop declustering

Larkoski, Marzani, Soyez, Thaler, 2014

Soft Drop Condition: $$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left(\frac{\Delta R_{12}}{R_0}\right)^{\beta}$$

i.e. (for $\beta>0$) remove large-angle soft radiation from a jet of radius R_0

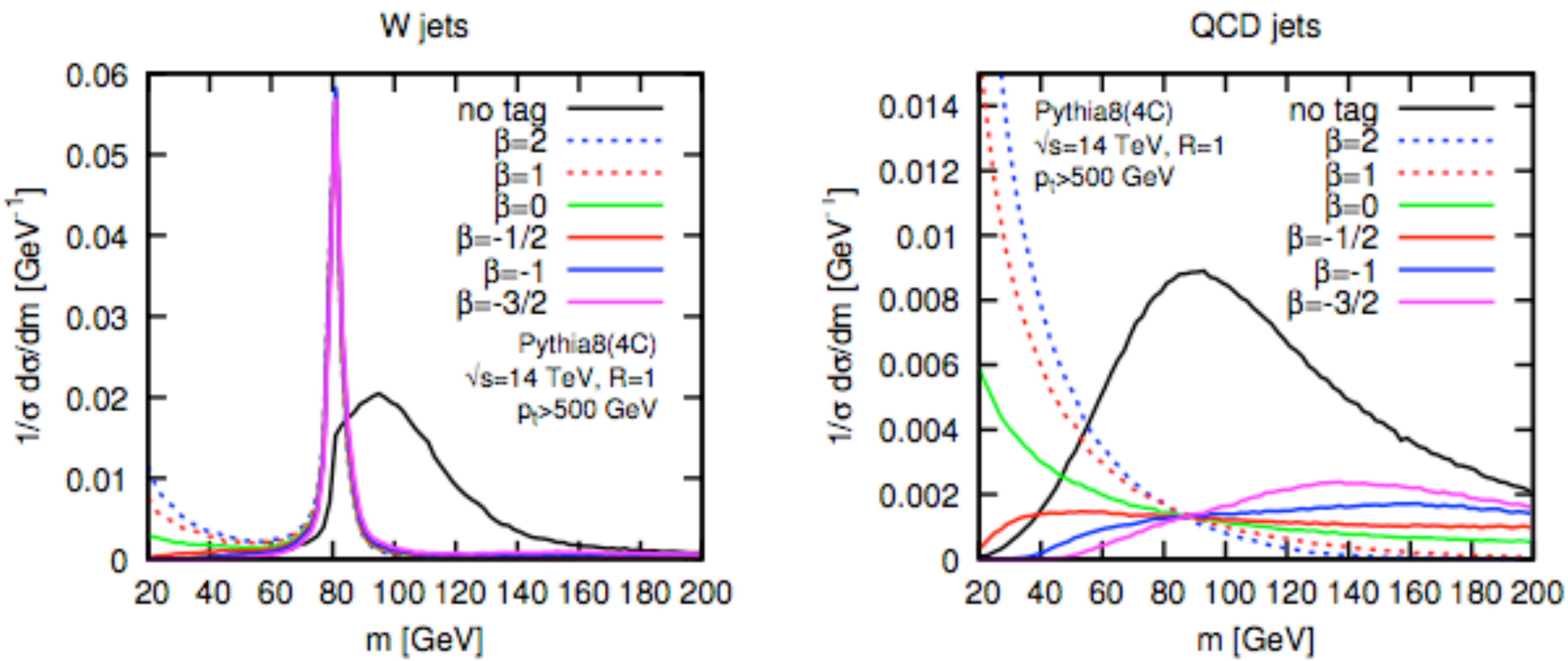
1. Break the jet *j* into two subjets by undoing the last stage of C/A clustering. Label the resulting two subjets as j_1 and j_2
2. If the subjets pass the soft drop condition (i.e. they are both sufficiently hard) then deem j to be the final soft-drop jet
3. Otherwise, redefine *j* to be equal to the subjet with larger p_T and iterate the procedure from point 1
4. If *j* is a singleton and can no longer be declustered, then one can either remove *j* from consideration ("tagging mode") or leave *j* as the final soft-drop jet ("grooming mode")

Soft Drop declustering

Larkoski, Marzani, Soyez, Thaler, 2014

The paper contains

- ✓ analytical calculations and comparisons to Monte Carlos
- ✓ study of effect of non-perturbative corrections
- ✓ performance studies



Example of SoftDrop performance when used as a boosted W tagger

Background subtraction

Background subtraction

Observable level

- Determination of *susceptibility to contamination* of each specific observable needed
- Possibility to get unbiased subtraction by construction
- Basic example: transverse momentum
  $\mathbf{p_t^{sub} = p_t^{raw} - \rho A}$ (MC, Salam 0707.1378)

- Other examples:
  - Analytical calculations of susceptibility for selected jet shapes (Sapeta et al. 1009.1143, Alon et al. 1101.3002)
  - Moments of jet fragmentation functions (MC, Quiroga, Salam, Soyez, 1209.6086)
  - Generic (numerical) approach to susceptibility determination for any shape (Soyet et al, 1211.2811)
  - Cleansing (Krohn, Schwartz, Low, Wang, 1309.4777)
  - Neutral-proportional-to-Charged (MC, Salam, Soyez 1404.7353)

Event (= particle) level

Background subtraction

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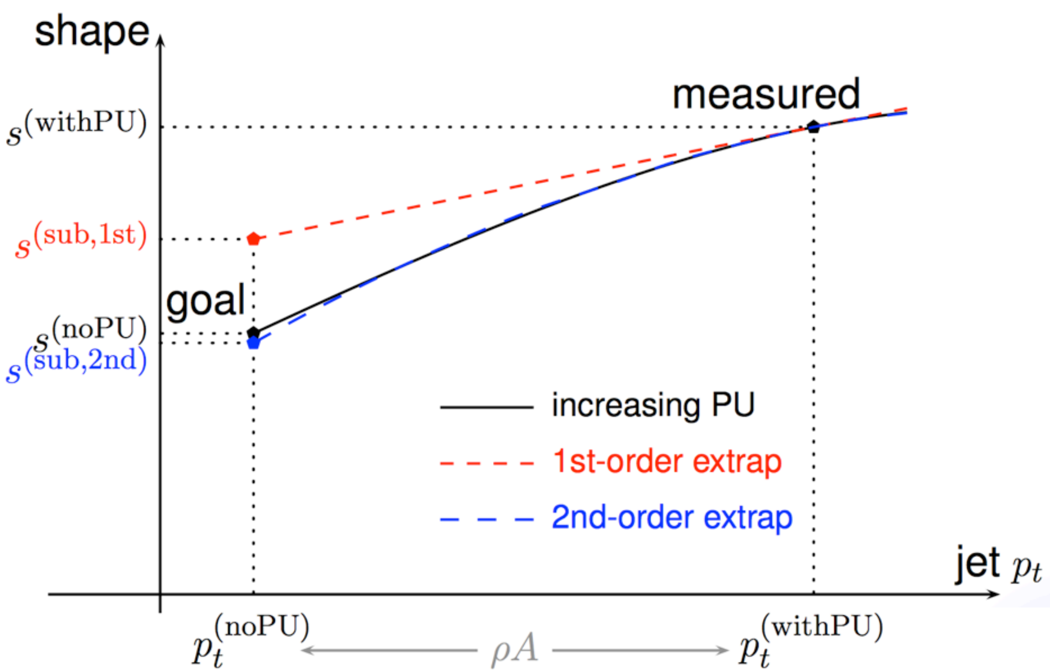
Event (= particle) level

- The event is modified before calculating observables (jets, shapes, etc)
- Method not naturally unbiased, but can often be tuned
- Final dispersion potentially lower, as effective number of particles usually reduced

- Examples:
    - CMS Voronoi method (Lai, unpubl.)
    - Constituent Subtraction (Berta, Spousta, Miller, Leitner, 1403.3108)
    - PUPPI (Bertolini, Harris, Low, Tran, 1407.6013)
    - SoftKiller (MC, Salam, Soyez, 1407.0408)
    - ....

Numerical jet shape correction



Soyez et al. 1211.2811

A generic **jet shape** (a function of the momenta of all constituents of a jet) is modified by the addition of pileup

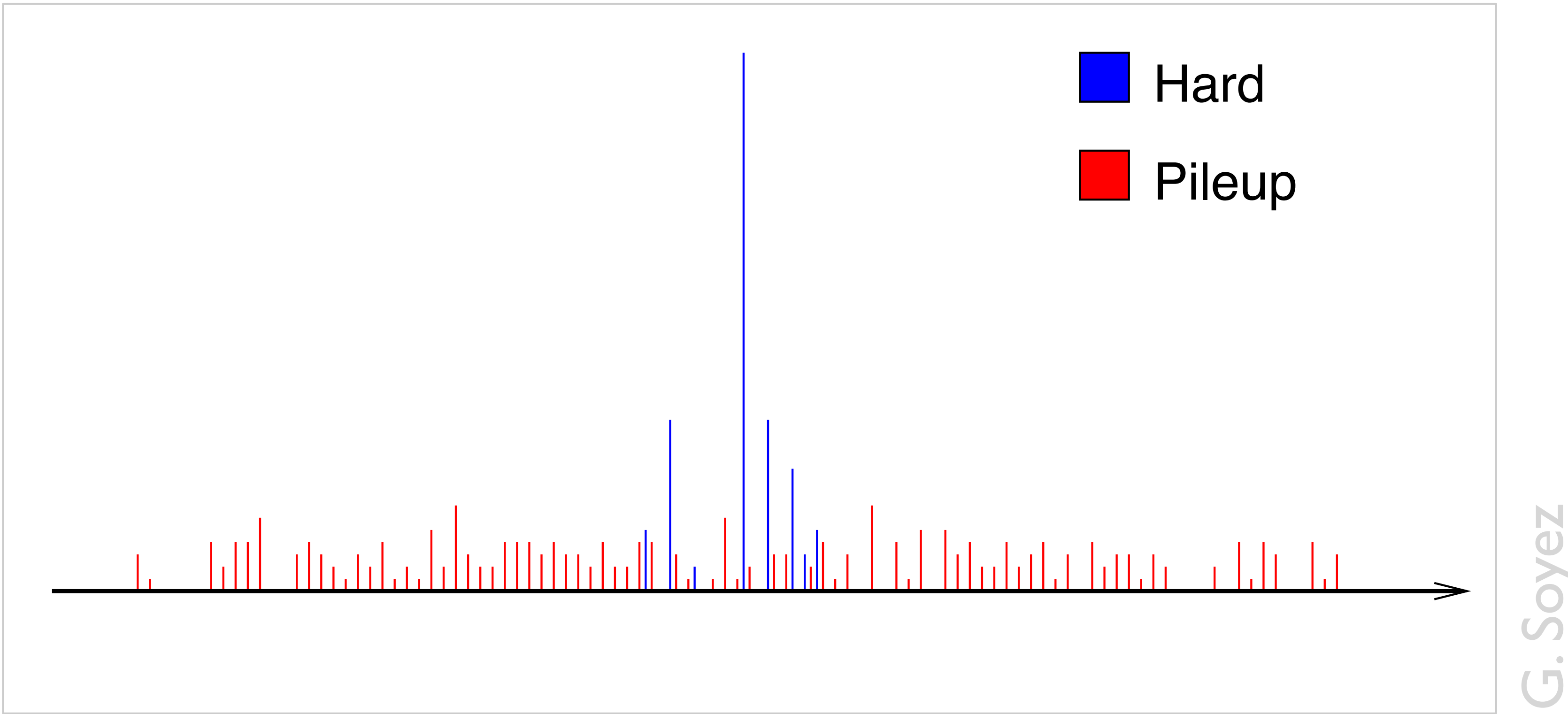
Correct it by calculating numerically the derivatives that enter its Taylor expansion and subtracting (this generalises the jet area/median subtraction for transverse mom.)

Pileup momentum density

Numerical derivative w.r.t. ghosts momenta

$$V_{\mathrm{jet,sub}} = V_{\mathrm{jet}} - \rho V_{\mathrm{jet}}^{[1]} + \frac{1}{2}\rho^2 V_{\mathrm{jet}}^{[2]} + \cdots$$

An event: particle level

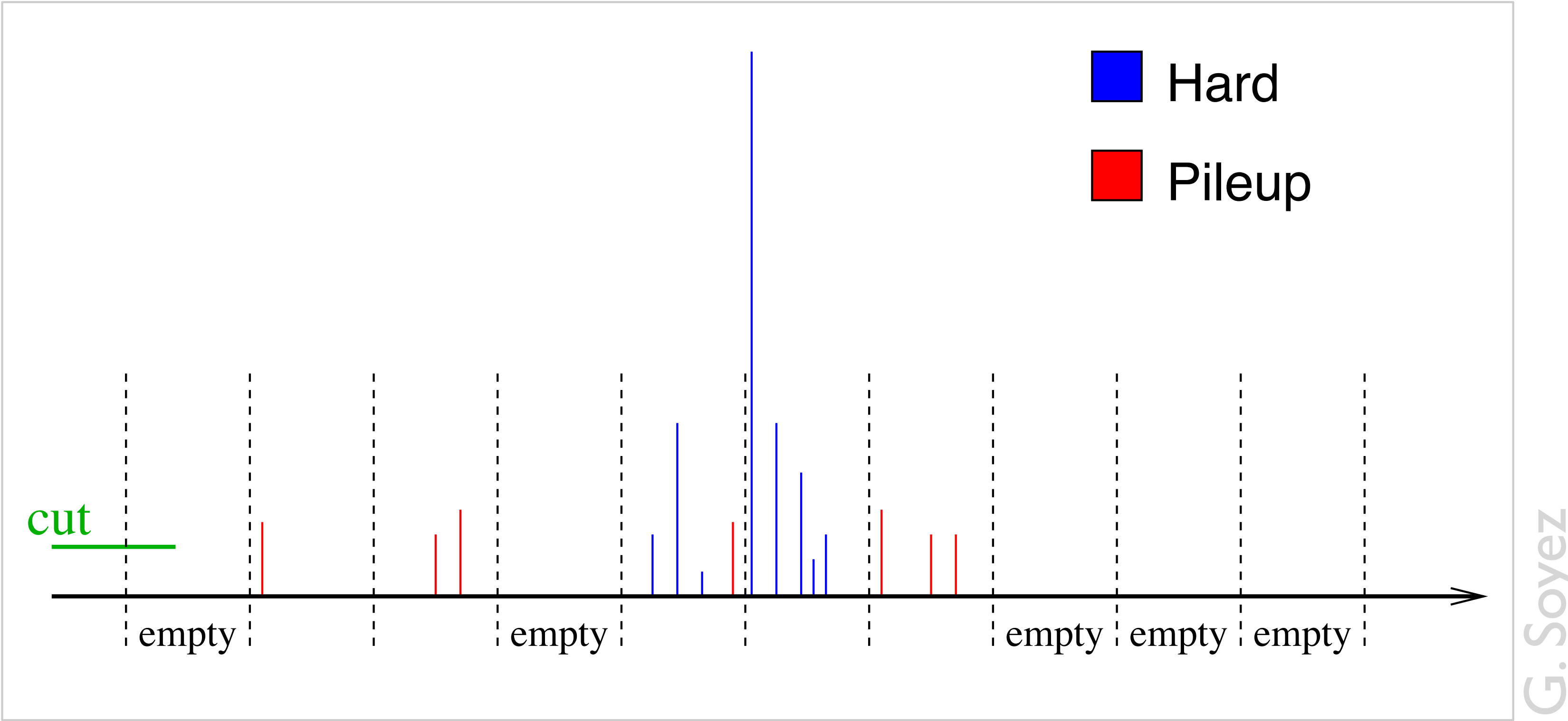


Soft Killer introduces a **particle momentum cut** such that the median momentum density (ρ) of the event is zero

Constituent Subtractor subtracts each constituent using iterative **local pairings to ghosts** whose momentum is set by ρ

Soft Killer

MC, Salam, Soyez, 1407.0408

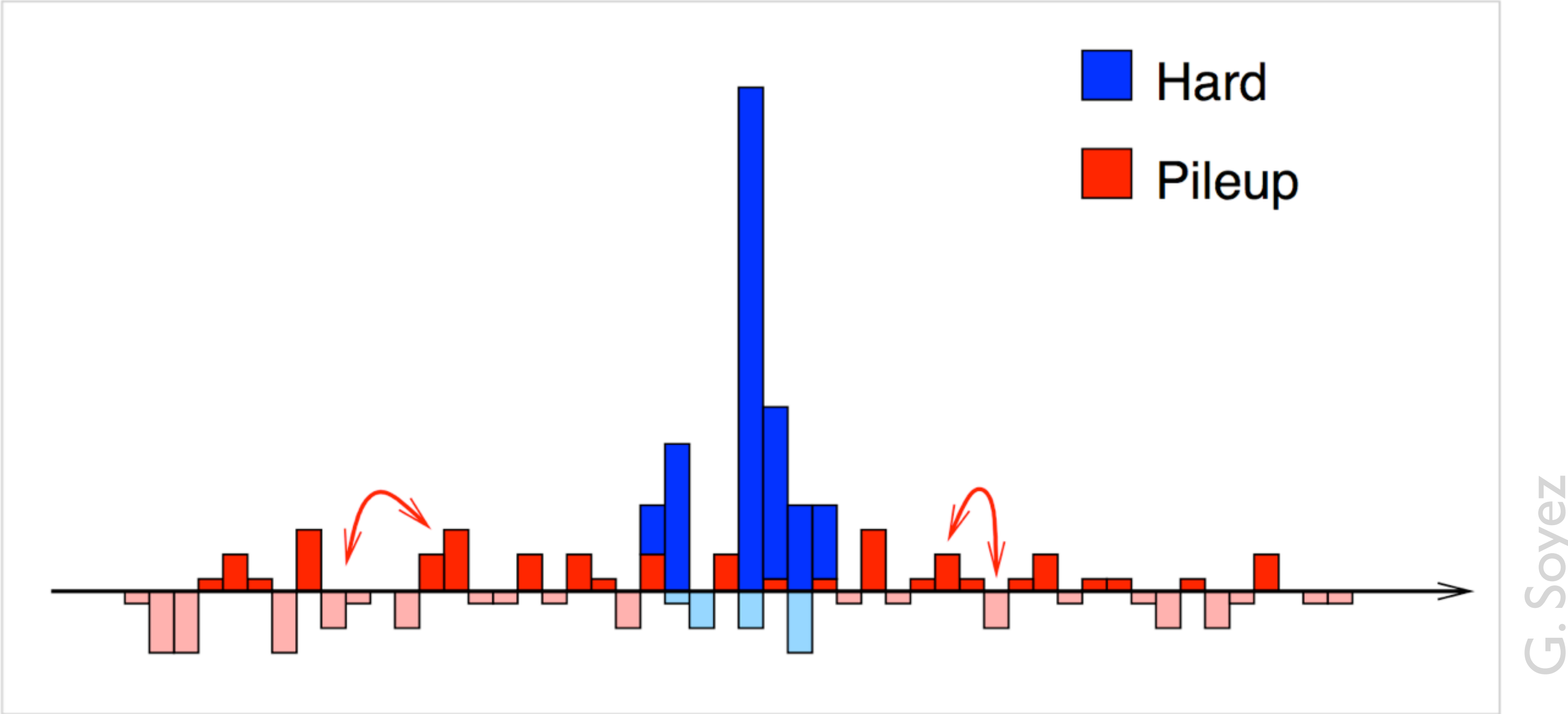


Half of the event is empty ⇒ **ρ = 0** (because it's the median)

NB. SK needs tuning of the size of the patches used to calculate ρ.
0.4 was found to be a good choice for R=0.4 jets

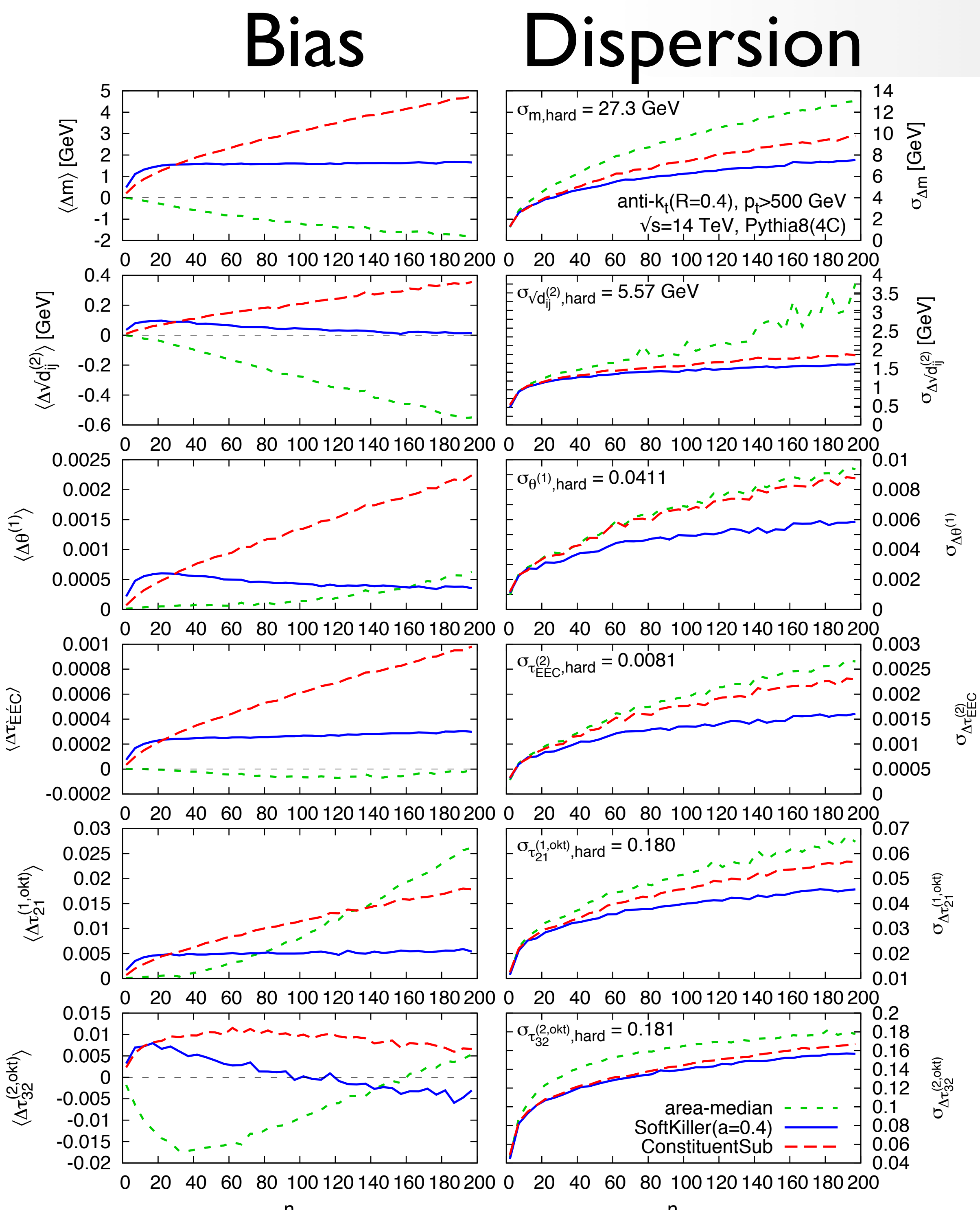
Constituent Subtractor

Berta, Spousta, Miller, Leitner, 1403.3108



Constituent Subtractor uses local pairings to ghosts to subtract iteratively momentum from constituents, reshuffling it to ghosts when oversubtracting, so as to maintain overall balance

Comparisons



Area-median
Soft Killer
Constituent Subtractor

Various jet shapes:

- jet mass
- kt clustering scale
- jet width (= broadening, = girth)
- energy-energy correlation moment
- τ_{21} and τ_{32} N-subjettiness ratios

Substructure studies in HI

Substructure in HI

Generic **experimental characterisation of jets in HI** collisions, even in the absence of universally valid theoretical descriptions, can and should be a **priority**

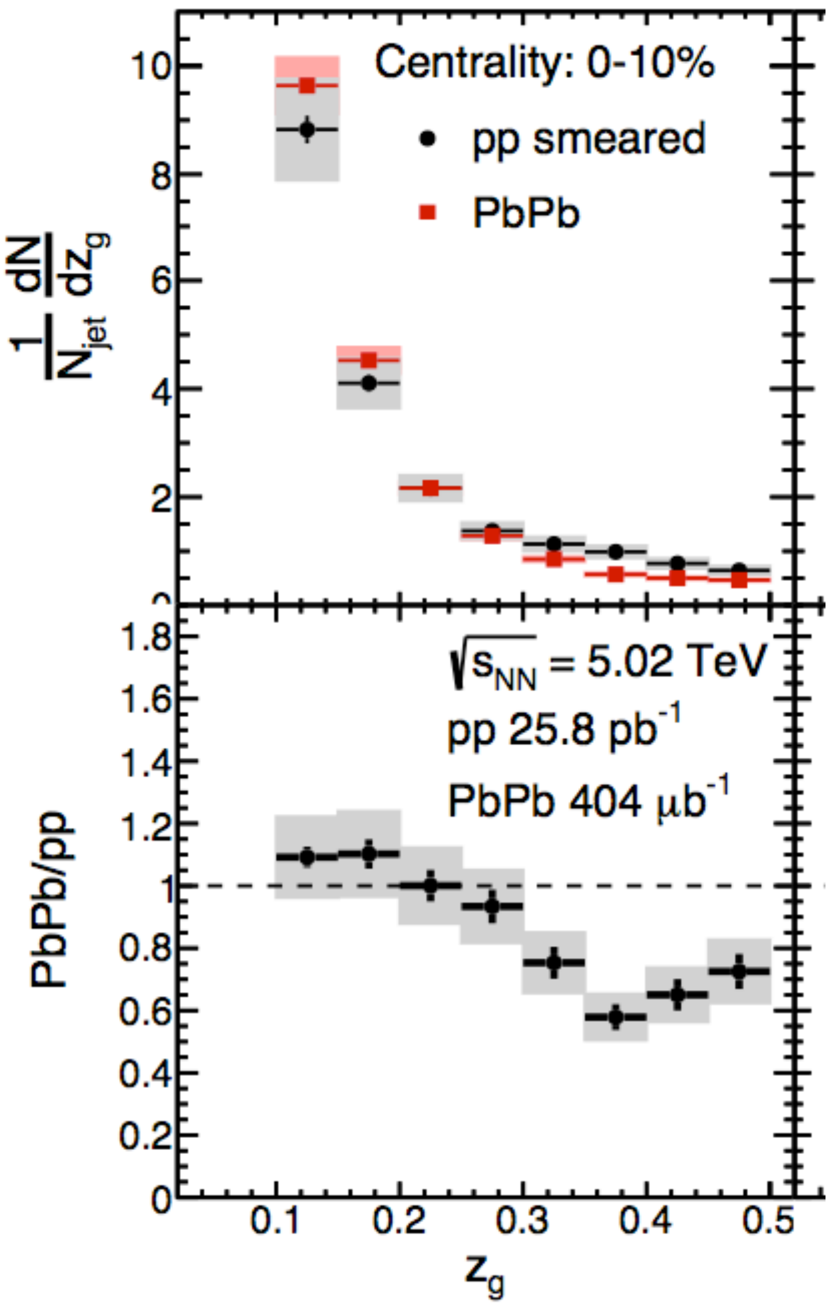
Measurements exist for

- Longitudinal fragmentation functions
- Radial distributions
- Splitting functions
- Other jet shapes
- ....

Ideally, a coherent, motivated and well defined (small) set of distributions and shapes is agreed upon, and measurements and predictions are systematically improved and refined

CMS splitting function

CMS PAS HIN-16-006



CMS has measured the momentum fraction of the ‘first splitting’,

$$z_g = \frac{p_{T2}}{p_{T1} + p_{T2}}$$

Definition:
reduction of event using **Constituent Subtractor**, then grooming using **Soft Drop (β=0, z_{cut}=0.1)**

Robustness? Calculability?

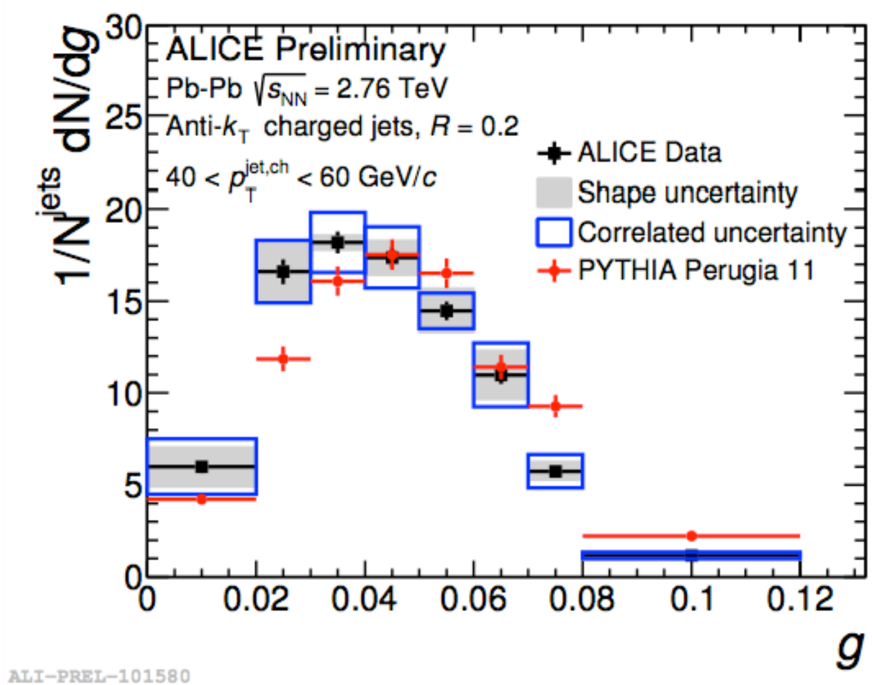
ALICE jet shapes

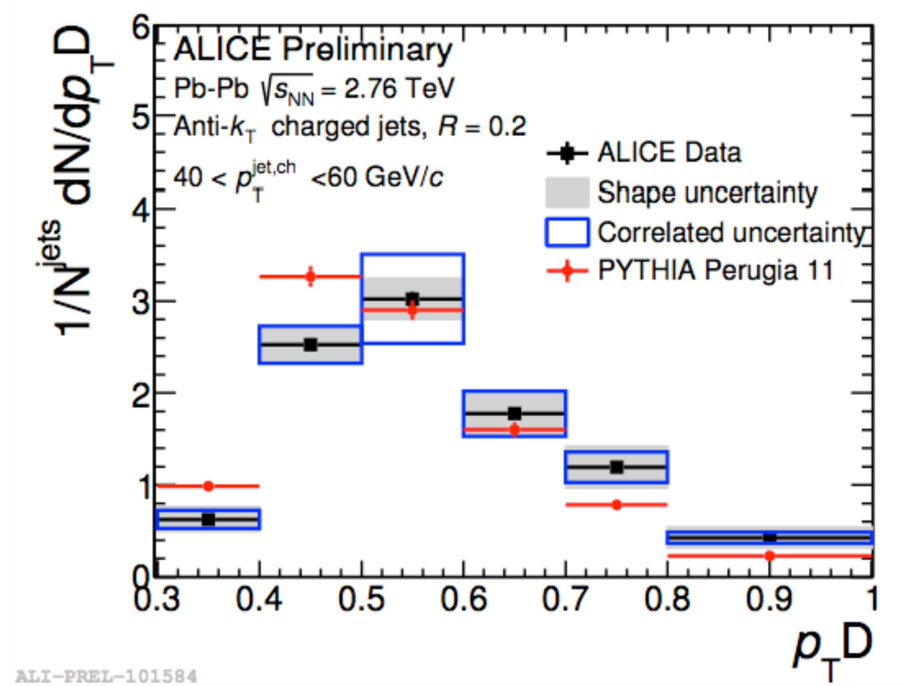
1512.0788

ALICE has measured the first radial moment and the second moment of the constituent momentum distribution in jets

$$g = \sum_{i \in jet} \frac{p_T^i}{p_{T,jet}} |\Delta R_{i,jet}|$$

$$p_T D = \frac{\sqrt{\sum_{i \in jet} p_{T,i}^2}}{\sum_{i \in jet} p_{T,i}}$$





Approach: **numerical area-median** correction for shapes, cross-checked with **Constituent Subtraction**, plus unfolding

Conclusions

- Jet substructure techniques are quite young in general. They are probably even younger HI. There is likely room for improvement

- To avoid fragmenting the field, and make progress efficient, we should
  - Introduce techniques motivated by analytical arguments, not simply MC testing
  - Ensure that they enjoy a **good analytical calculability**
    - very little reason to introduce today a novel substructure technique that does not enjoy a decent calculability, unless HUGE improvement can be shown (and still, it should be justifiable and robust)
  - Provide a **public implementation** (e.g. in the FastJet contrib project, **http://fastjet.hepforge.org/contrib**, public repository for third-party contributions)
  - Choose for measurement and calculation robust and meaningful observables

Backup

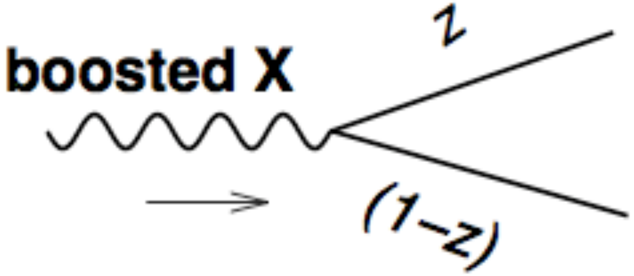
The IRC safe algorithms

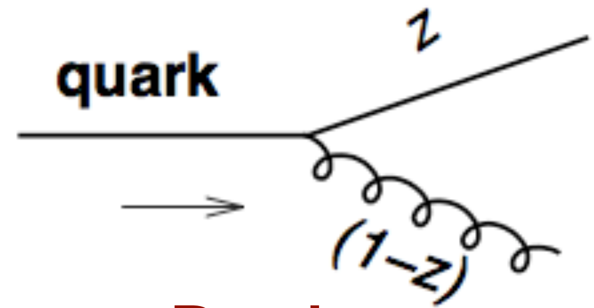
	Speed	Regularity	UE contamination	Backreaction	Hierarchical substructure
k_t	☺ ☺ ☺	☂	☂☂	☁☁	☺ ☺
Cambridge /Aachen	☺ ☺ ☺	☂	☂	☁☁	☺ ☺ ☺
anti-k_t	☺ ☺ ☺	☺ ☺	☁/ ☺	☺ ☺	✘
SISCone	☺	☁	☺ ☺	☁	✘

Array of tools with different characteristics.
Pick the right one for the job

QCD v. heavy decay

A possible approach for reducing the QCD background is to identify the two prongs of the heavy particle decay, and put a cut on their momentum fraction





Signal:

$$P(z) \sim 1$$

Will split mainly **symmetrically**

Background:

$$P(z) \sim \frac{1+z^2}{1-z} \qquad P(z) \sim \frac{1+(1-z)^2}{z}$$

Will split mainly **asymmetrically**

Potential tagger: asymmetric splitting

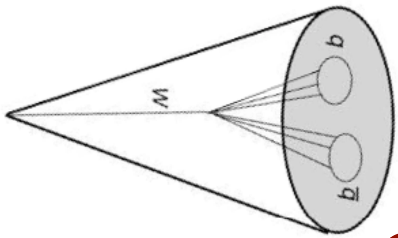
Possibly implemented via a cut on

$$y = min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{m^2} \simeq \frac{min(p_{ti}, p_{tj})}{max(p_{ti}, p_{tj})}$$

Jet substructure

Goal: tag a boosted massive particle whose decay products end up in a single 'fattish' jet

W



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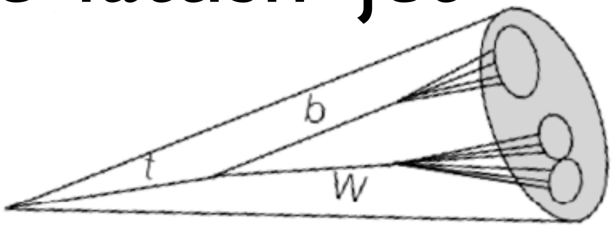

Cone aperture: $R \sim 2m/p_T$

- Electroweak-scale particles (m ~ 100 GeV) boosted to a few hundreds GeV (e.g. coming from the decay of a TeV-scale BSM particle) mean **R ~ 1**
  - Too large for a single 'standard' jet with R=0.4-0.7 to catch all decay products
  - Too small for 'standard jets' to give separate jets for the decay products
  - Using smaller jets (R=0.1-0.2) over all event gives huge combinatorial issues

Need a completely new strategy

BDRS in FastJet

In FastJet

```
#include “fastjet/tools/MassDropTagger.hh”
#include “fastjet/tools/Filter.hh”

JetDefinition jet_def(cambridge_algorithm, 1.2);
ClusterSequence cs(input_particles, jet_def);

// define the tagger and use it
MassDropTagger md_tagger(0.667, 0.09);
PseudoJet tagged = md_tagger(jets[0]);

// define the filter and use it
Filter filter(0.3,SelectorNHardest(3));
Pseudojet higgs = filter(tagged);     // this is the Higgs!!
```

The real analysis is slightly more refined (b-tagging, dynamical filter radius, etc) but the main features are already present here

First taggers/groomers

- **Mass Drop + Filtering** — Butterworth, Davison, Rubin, Salam, 2008

  Decluster with mass drop and asymmetry conditions
  Recluster constituents into subjets at distance scale R_{filt}, retain n_{filt} hardest subjets

- **Jet 'trimming'** — Krohn, Thaler, Wang, 2009

  Recluster constituents into subjets at distance scale R_{trim},
  retain subjets with $p_{t,subjet} > \varepsilon_{trim}\, p_{t,jet}$

- **Jet 'pruning'** — S. Ellis, Vermilion, Walsh, 2009

  While building up the jet, discard softer subjets when $\Delta R > R_{prune}$
  and $\min(p_{t1},p_{t2}) < \varepsilon_{prune}\, (p_{t1}+p_{t2})$

Aim: limit contamination from QCD background while retaining bulk of perturbative radiation

Trimming and pruner are a priori groomers, but can become taggers when combined with an invariant mass window test
(if you can groom away everything then there's no heavy particle in the jet)

Jet trimming

Krohn, Thaler, Wang, 2009

1. Cluster all cells/tracks into jets using any clustering algorithm. The resulting jets are called the seed jets.

2. Within each seed jet, recluster the constituents using a (possibly different) jet algorithm into subjets with a characteristic radius R_{sub} smaller than that of the seed jet.

3. Consider each subjet, and discard the contributions of subjet i to the associated seed jet if $p_{Ti} < f_{cut} \cdot \Lambda_{hard}$, where f_{cut} is a fixed dimensionless parameter, and Λ_{hard} is some hard scale chosen depending upon the kinematics of the event.

4. Assemble the remaining subjets into the trimmed jet.

Different condition for retaining jets (p_T-cut rather than n_{filt} hardest) with respect to filtering, but otherwise identical

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Different condition for retaining jets (p_T-cut rather than n_{filt} hardest) with respect to filtering, but otherwise identical

```
#include "fastjet/tools/Filter.hh"

// define trimmer
Filter trimmer(0.3,SelectorPtFractionMin(0.03));
```

Jet pruning

S. Ellis, Vermilion, Walsh, 2009

0. Start with a jet found by any jet algorithm, and collect the objects (such as calorimeter towers) in the jet into a list L. Define parameters $D_{\rm cut}$ and $z_{\rm cut}$ for the pruning procedure.

1. Rerun a jet algorithm on the list L, checking for the following condition in each recombination $i, j \to p$:

$$z = \frac{\min(p_{Ti}, p_{Tj})}{p_{Tp}} < z_{\rm cut} \quad \text{and} \quad \Delta R_{ij} > D_{\rm cut}.$$

This algorithm must be a recombination algorithm such as the CA or k_T algorithms, and should give a "useful" jet substructure (one where we can meaningfully interpret recombinations in terms of the physics of the jet).

2. If the conditions in 1. are met, do not merge the two branches 1 and 2 into p. Instead, discard the softer branch, i.e., veto on the merging. Proceed with the algorithm.

3. The resulting jet is the *pruned jet*, and can be compared with the jet found in Step 0.

True in general for substructure studies

Exclude soft stuff and large angle recombinations from clustering

Pruning in FastJet

In FastJet

```
#include "fastjet/tools/Pruner.hh"

JetDefinition jet_def(cambridge_algorithm, 1.2);
ClusterSequence cs(input_particles, jet_def);

// define the pruner and use it
double zcut = 0.1;
double rcut_factor = 0.5;

Pruner pruner(cambridge_algorithm, zcut, rcut_factor);

PseudoJet tagged = pruner(jets[0]);
```

Alternatives to hierarchical substruct.

- If what we are interested in is the structure of the constituents of a jet, the "jet" itself is not the most important feature.
- A different algorithm, or simply the study of the constituents in a certain patch will also do. Selected alternatives are:
  - Use of jet-shapes to characterise certain features
    - e.g. *N-subjettiness*: how many subjets a jets appears to have

      Thaler, van Tilburg, 2011
  - Alternative ways of clustering
    - e.g. *Qjets*: the clustering history not deterministic, but controlled by random probabilities of merging. Can be combined with, e.g. pruning

      Ellis, Hornig, Roy, Krohn, Schwartz, 2012
  - Use information from matrix element
    - e.g. *shower deconstruction*: use analytic shower calculations to estimate probability that a certain configuration comes from signal or from background

      Soper, Spannowsky, 2011
  - Use event shapes mimicking jet properties
    - e.g. *JetsWithoutJets*, mimicking trimming

      Bertolini, Chen, Thaler, 2013

N-subjettiness

Thaler, van Tilburg, 2010

$$\tau_N^{(\beta)} = \sum_i p_{Ti} \min \left\{ R_{1,i}^{\beta}, R_{2,i}^{\beta}, \ldots, R_{N,i}^{\beta} \right\}$$

Sum over constituents of a jet

Distances to axes of N subjets

τ_N measures departure from N-parton energy flow:

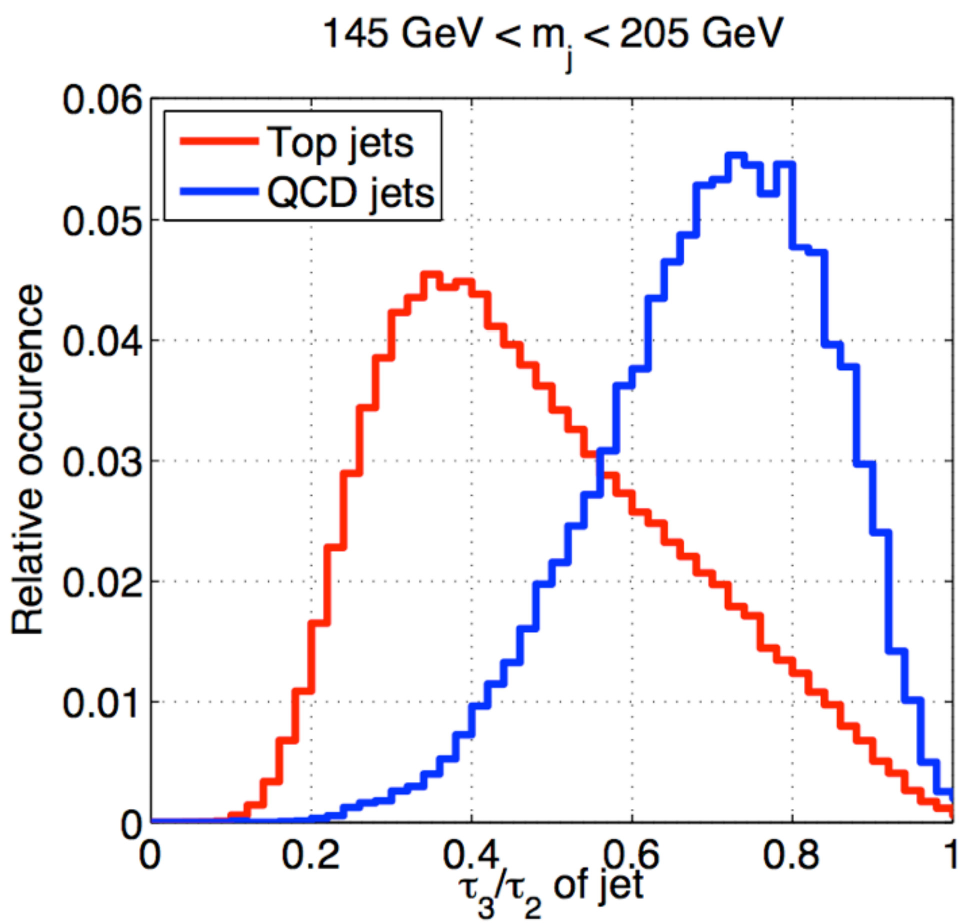
if a jet has N subjets, τ_{N-1} should be much larger than τ_N

N-subjettiness

Thaler, van Tilburg, 2010

$$\tau_{N,N-1}^{(\beta)} \equiv \frac{\tau_N^{(\beta)}}{\tau_{N-1}^{(\beta)}}$$

A jet with a **small** $\tau_{N,N-1}$ is more likely to have N than N-1 subjets



(from 1011.2268, with β=1)

Larkoski, Salam, Thaler 2013

Energy correlation functions

Probes of N-prong structures without requiring identification of subjets

$$\mathrm{ECF}(N,\beta) = \sum_{i_1<i_2<\ldots<i_N\in J} \left(\prod_{a=1}^{N} p_{T i_a}\right) \left(\prod_{b=1}^{N-1} \prod_{c=b+1}^{N} R_{i_b i_c}\right)^{\beta}$$

Angular (y-φ) distances between constituents

ECF(N+1) is zero if there are only N particles

More generally, *if there are N subjets one expects ECF(N+1) to be much smaller than ECF(N)*

[because radiation will be mainly soft/collinear to subjets]

Larkoski, Salam, Thaler 2013

Discriminators

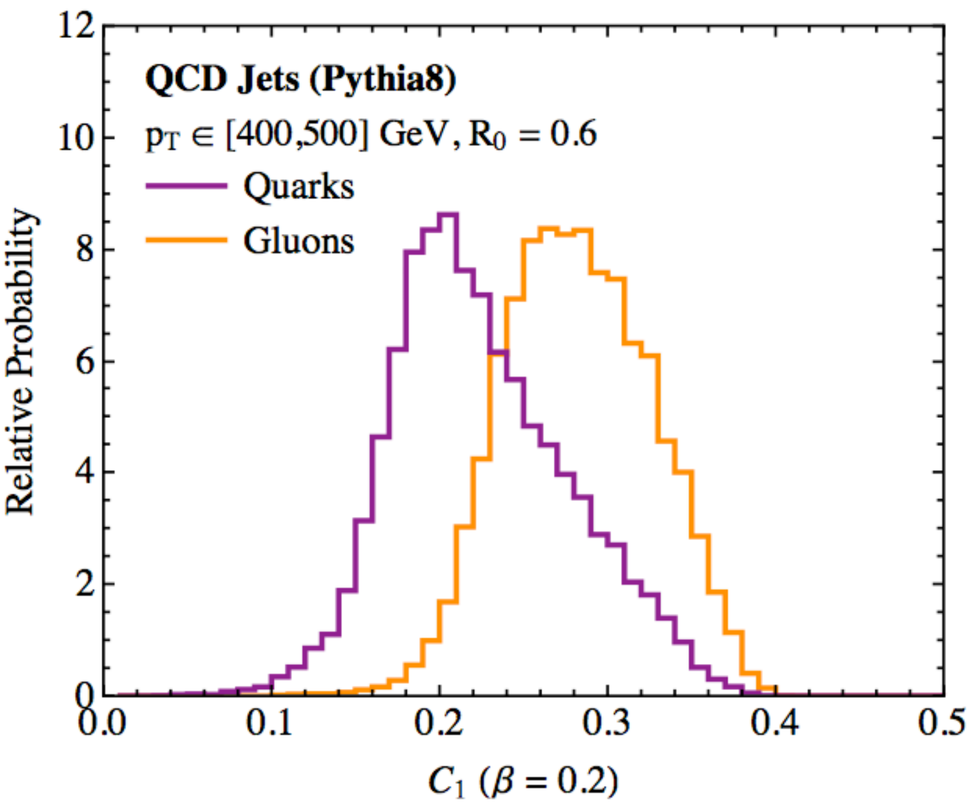
$$r_N^{(\beta)} \equiv \frac{\mathrm{ECF}(N+1,\beta)}{\mathrm{ECF}(N,\beta)}$$

small for N prongs:
if N hard partons, small if radiation
only soft-collinear

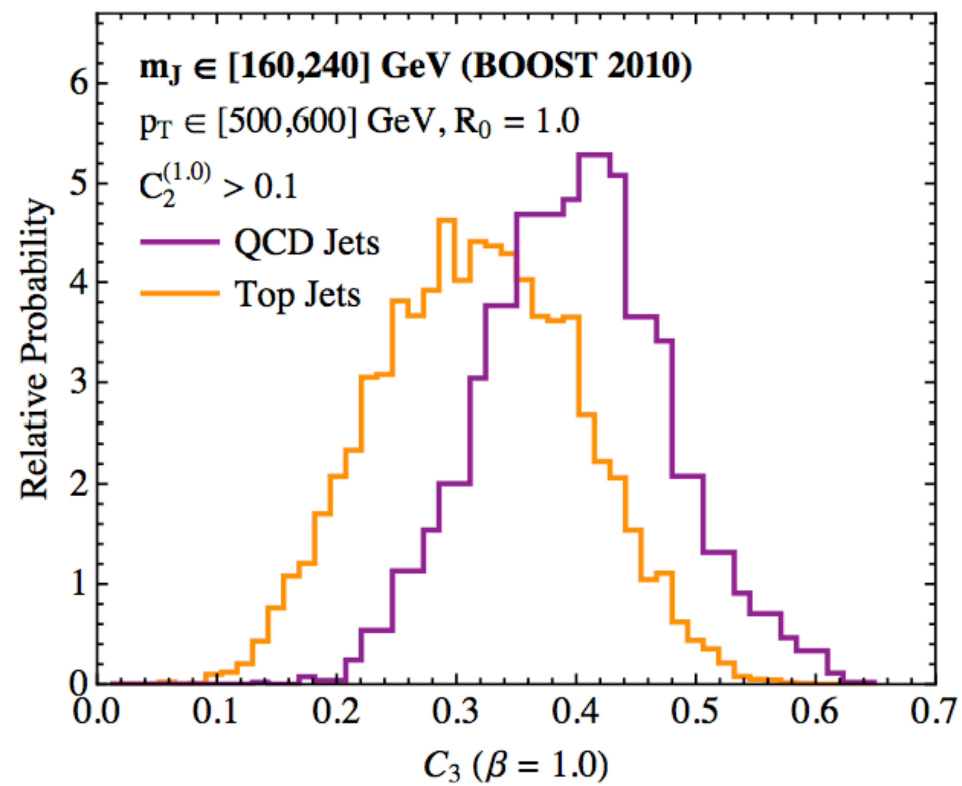
$$C_N^{(\beta)} \equiv \frac{r_N^{(\beta)}}{r_{N-1}^{(\beta)}} = \frac{\mathrm{ECF}(N+1,\beta)\,\mathrm{ECF}(N-1,\beta)}{\mathrm{ECF}(N,\beta)^2}$$

A jet with a **small** C_N is more likely
to have N prongs and at most soft/coll radiation

C_1
quark-gluon discriminator



C_3
top tagging



Note different values of β
(chosen to maximise discriminating power)

Background estimation and subtraction

```
// constructor for a background estimator
JetMedianBackgroundEstimator bge(Selector sel,
                                 JetDefinition jet_def,
                                 AreaDefinition area_def);

// an alternative (faster) background estimator
//GridMedianBackgroundEstimator bge(Selector sel, grid_step);

bge.set_particles(input_particles);
....
double rho = bge.rho(jet);   // extract rho estimation
```

```
// define a subtractor
Subtractor sub(&bge);

// apply it to a jet (or a vector of jets)
PseudoJet subtracted_jet = sub(jet);
```

Pilup subtraction from jet shapes using GenericSubtractor from fjcontrib

```
#include "ExampleShapes.hh"
#include "GenericSubtractor.hh"

// define a specific jet shape
contrib::Angularity ang(1.0); // angularity with alpha=1.0

// define a generic subtractor
... construct a background estimator bge_rho....
contrib::GenericSubtractor gen_sub(&bge_rho);

// compute the subtracted shape
double subtracted_ang = gen_sub(ang, jet);
```

Particle-level pilup removal

Pilup removal using SoftKiller from fjcontrib

(SoftKiller progressively removes soft particles until ρ of event is zero)

```
#include "SoftKiller.hh"

// define SoftKiller
double grid_size = 0.4;
contrib::SoftKiller soft_killer(rapmax, grid_size);

// apply it to the full event
double pt_thresh //returns threshold for killed particles
vector<PseudoJet> soft_killed_event;
soft_killer.apply(full_event, soft_killed_event, pt_thresh);

// proceed with clustering and calculating shapes with
// the reduced soft_killed_event
ClusterSequence(soft_killed_event,.....)
```