Evolution equations and factorization in pA collisions

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Outline

- Introduction
- High-energy evolution equations at NLL and collinear resummations

  $\longrightarrow$ talk by Lappi
- DIS at NLO in the dipole factorization
- Forward single inclusive hadron production in pA at NLO in the hybrid factorization

  $\longrightarrow$ talk by Zhu

Collinear factorization in pA

In the Bjorken limit ($p_\perp^2 \sim s \to +\infty$):

$$\frac{d^2\sigma_{pA\to h+X}}{d^2p_\perp} = \sum_{i,j,l=q_f,\bar{q}_f,g} \int_0^1 dx_1 \int_0^1 dx_2\ f_{i/p}(x_1;\mu_F^2)\ f_{j/A}(x_2;\mu_F^2)$$
$$\times \int_0^1 \frac{dz}{z^2}\ D_{h/l}(z;\mu_F^2)\ \frac{d\sigma_{ij\to l+X'}}{d^2k_\perp}(x_1x_2s, k_\perp = p_\perp/z;\mu_F^2)\ \left(1+O\left(\frac{1}{p_\perp^2}\right)\right)$$

Partonic cross-section calculable in pQCD: short range QCD interaction

PDFs $f_{i/p,A}(z,\mu_F^2)$ and FFs $D_{h/i}(z;\mu_F^2)$ non-perturbative but universal: process independent

Independence of observables on $\mu_F^2 \Rightarrow$ DGLAP equations for $f_i(z,\mu_F^2)$

In the end: natural choice: $\mu_F^2 \sim p_\perp^2$, in order to resum large $\log(p_\perp^2/\mu_F^2)$.

Initial condition for $f_{i/p,A}(z,\mu_F^2)$ obtained from fit on DIS, DY, ...

Initial condition for $D_{h/i}(z;\mu_F^2)$ obtained from fit on $e^+e^- \to h$, SIDIS, ...

Other types of factorization

Collinear factorization not enough for processes with several well-separated large scales, for example:

- Quasi back-to-back dijet production: require TMD factorization, with CSS resummation (which includes Sudakov double log resummation)

- Regge-Gribov limit: for $s \gg p_\perp, \ldots$, high-energy $\log(s/p_\perp^2)$ more important than DGLAP $\log(p_\perp^2/\mu_F^2)$

  High-energy resummation performed with
  - the BFKL evolution in the dilute hadron case
  - the B-JIMWLK or BK evolution in the dense hadron or nucleus case (gluon saturation/CGC). $\rightarrow$ Main focus of this talk.

A few recent works

Relevant recent works that I will not discuss further in this talk:

- Sudakov resummation for multi-scale high-energy processes with gluon saturation
  Mueller, Xiao, Yuan (2012-2013)

- Compatibility of TMD, BFKL and CGC factorization formalisms for forward dijets in pA collisions
  Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren (2015-2016)

- Unification of low-x and CSS/Sudakov evolutions for TMD PDFs
  Balitsky, Tarasov (2015-2016); Kovchegov, Sievert (2015); Zhou (2016)

- $+$ Many articles about spin physics at low-x

Universality of high-energy/CGC factorization

Many processes can be written in terms of the same non-perturbative objects like the dipole-target amplitude.

$\Rightarrow$ 3 steps program:

ep, eA : Fits of the dipole-target amplitude, using high-energy evolution equations

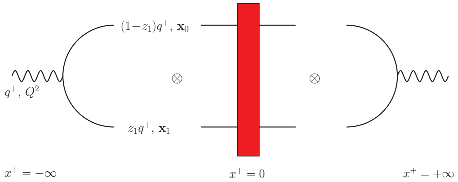
pp, pA : Check of the universality of the high-energy factorization, and further constraints

AA : Calculate *Glasma* initial conditions from first principles and from previous experimental constraints
$\rightarrow$ Use JIMWLK factorization formulae for AA from Gelis, Lappi, Venugopalan (2008-2009)

Example for the first two steps: talk by Mäntysaari

Preliminary realization of the complete programm: IP-Glasma model Schenke, Tribedy, Venugopalan (2012)

Dipole factorization for DIS at LO



$$\sigma_{T,L}^{\gamma p \to X}(x_{Bj}, Q^2) \;=\; \frac{4N_c\, \alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\, \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1$$

$$\times\, \mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01}, z_1, Q^2)\, \Big[1 - \langle \mathcal{S}_{01} \rangle_Y \Big]$$

Bjorken, Kogut, Soper (1971); Nikolaev, Zakharov (1990)

Dipole operator: $\quad \mathcal{S}_{01} = \dfrac{1}{N_c} \mathrm{Tr}\Big(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1) \Big)$

with "rapidity" $Y \sim \log(1/x_{Bj})$ for $x_{Bj} \to 0$.
$\to$ Dependence of $\langle \mathcal{S}_{01} \rangle_Y$ on Y comes from high-energy (low-x_{Bj}) LL resummation.

B-JIMWLK and BK evolutions

RG evolution for the dipole amplitude at LL accuracy:

$$\begin{aligned}\partial_Y \langle \mathbf{S}_{01}\rangle_Y &= \frac{2\alpha_s C_F}{\pi}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\langle \mathbf{S}_{012}-\mathbf{S}_{01}\rangle_Y\\ &= \bar{\alpha}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\,\langle \mathbf{S}_{02}\mathbf{S}_{21}-\mathbf{S}_{01}\rangle_Y\end{aligned}$$

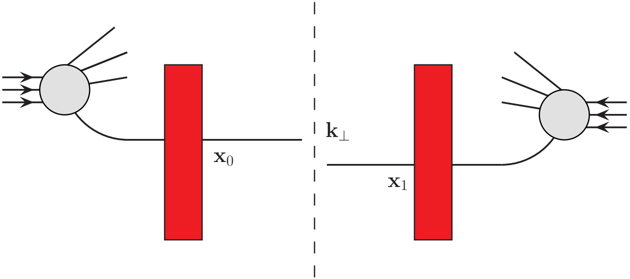
with $\bar{\alpha} = N_c\alpha_s/\pi$, and the $q\bar{q}g$ tripole operator

$$\mathbf{S}_{012} \equiv \frac{1}{N_c C_F}\mathrm{Tr}\Big(U_F(\mathbf{x}_0)t^a U_F^\dagger(\mathbf{x}_1)t^b\Big)\,U_A^{ba}(\mathbf{x}_2) = \frac{N_c}{2C_F}\left[\mathbf{S}_{02}\,\mathbf{S}_{21} - \frac{1}{N_c^2}\mathbf{S}_{01}\right]$$

New operator $\langle \mathbf{S}_{012}\rangle_Y$ or $\langle \mathbf{S}_{02}\mathbf{S}_{21}\rangle_Y$ appears $\Rightarrow$ only the first equation in B-JIMWLK infinite hierarchy.

In practice: truncate the hierarchy with the approx $\langle \mathbf{S}_{02}\mathbf{S}_{21}\rangle_Y \simeq \langle \mathbf{S}_{02}\rangle_Y\,\langle \mathbf{S}_{21}\rangle_Y$ to get the BK equation.
Balitsky (1996); Kovchegov (1999)

Forward single-inclusive particle production in pA at LO

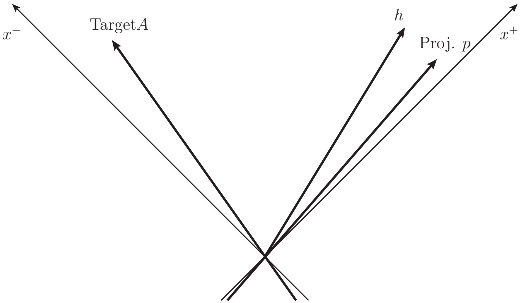


$$\frac{\mathrm{d}\sigma^{pA\to q+X}}{\mathrm{d}y\,\mathrm{d}^2\mathbf{k}_\perp} = \frac{1}{(2\pi^2)}\sum_f x\,q_f(x,\mu_F^2)\int \mathrm{d}^2\mathbf{x}_0\int \mathrm{d}^2\mathbf{x}_1\; e^{-i\mathbf{k}_\perp\cdot(\mathbf{x}_0-\mathbf{x}_1)}\;\langle\mathcal{S}_{01}\rangle_Y$$

with $x = e^y\,|\mathbf{k}_\perp|/\sqrt{s}$ and $Y = y + \log(|\mathbf{k}_\perp|/\sqrt{s})$

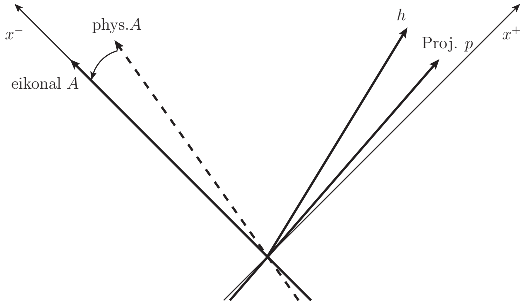
Fragmentation functions and gluon channel can be included easily.
$\rightarrow$ *Hybrid factorization*
Dumitru, Hayashigaki, Jalilian-Marian (2002-2006)

Eikonal approximation and factorization schemes



Light-cone kinematics for forward hadron production in pA

Eikonal approximation and factorization schemes



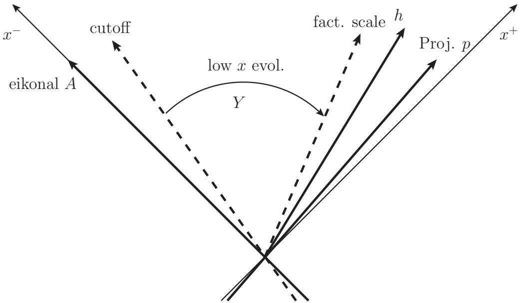
High-energy/eikonal approximation:

⋆ Power corrections in s dropped

⋆ Target put on an light-like trajectory to simplify calculations

⇒ Unphysical *rapidity divergences* are induced from high-energy LLs.

Eikonal approximation and factorization schemes

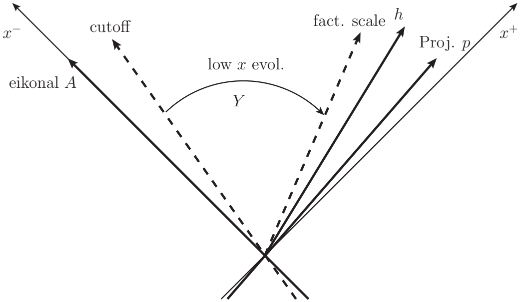


High-energy LL $\rightarrow$ large log range Y between:

$\star$ Cutoff for the rapidity div., set by a physical scale of the target

$\star$ Factorization scale, close to the produced hadron

Eikonal approximation and factorization schemes



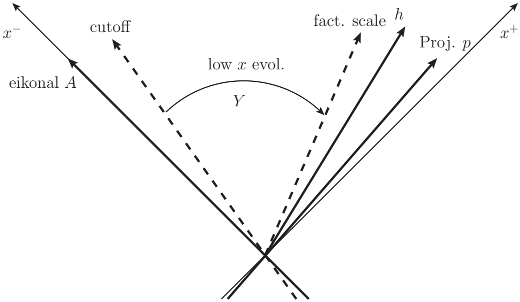
Ex. 1: k^- factorization scheme

$\star$ Cutoff $k^-_{\mathrm{max}} = x_0\, P^-_T$

$\star$ Factorization scale k^-_f

$\Rightarrow$ Range for the evolution: $Y^-_f = \log(k^-_{\mathrm{max}}/k^-_f)$.

Eikonal approximation and factorization schemes



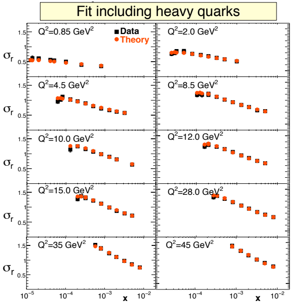
Ex. 2: k^+ factorization scheme

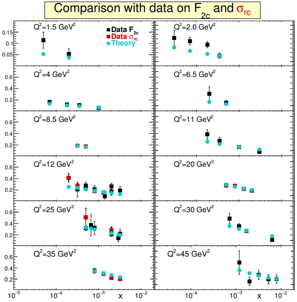
⋆ Cutoff $k^+_{\rm min} = Q_0^2/2x_0\, P_T^- = Q_0^2\, P_P^+/x_0\, s$

⋆ Factorization scale k_f^+

$\Rightarrow$ Range for the evolution: $Y_f^+ = \log(k_f^+/k^+_{\rm min}) = \log(x_0\, s\, k_f^+/Q_0^2\, P_P^+)$.

DIS phenomenology





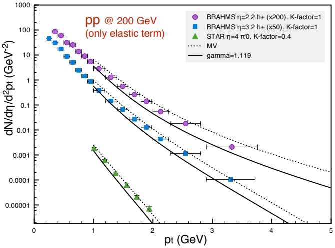
Fits of the reduced DIS cross-section σ_r and its charm contribution σ_{rc} at HERA data with numerical solutions of the running coupling BK equation.

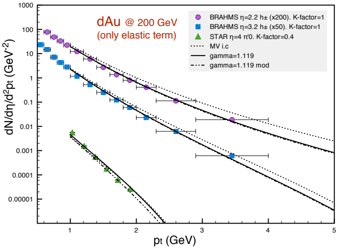
Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
see also: Kuokkanen, Rummukainen, Weigert (2012);
Lappi, Mäntysaari (2013); . . .

Good fit, but require a big rescaling of Λ_{QCD} as extra parameter, to slow down the BK evolution.

$\rightarrow$ Mimics missing higher order contributions, like a K-factor.

Phenomenology for single-inclusive particle production





Fits of the single-inclusive hadron or pion production cross-section at forward rapidity in p-p and d-Au collisions at RHIC, using the hybrid factorization at LO, and running coupling BK evolution.

Similar results at LHC (p-p and p-Pb) and Tevatron (p-p) at central rapidity, using $k_\perp$-factorization.

Albacete, Dumitru, Fujii, Nara (2013)

see also: Albacete, Marquet (2010); Lappi, Mäntysaari (2013); . . .

Status of the calculations for the NLL evolution

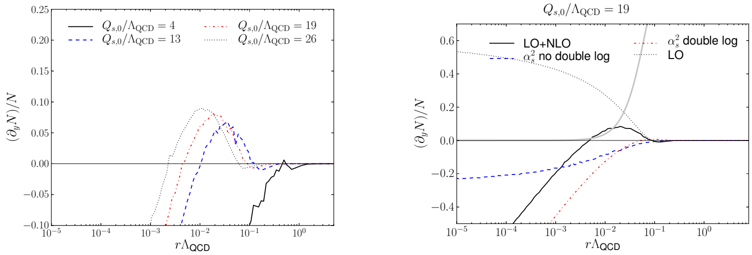
The NLO corrections to the evolution equations are known, allowing in principle NLL resummations: $\bar{\alpha}(\bar{\alpha}\log(s))^n$

- Calculation of the NLL BK equation:
  Balitsky, Chirilli (2008)
- Construction of the NLL Balitsky's hierarchy and the NLL JIMWLK equation
  Balitsky, Chirilli (2013); Kovner, Lublinsky, Mulian (2013)

  (Use NLL BK and previous calculations of corrections to 3 quarks scattering on a target: Grabovsky (2013))
- Direct calculation of the NLL JIMWLK equation:
  Mulian, Lublinsky : *to appear*

Moreover: Proof that observables like DIS or like particle production obey the same NLL equation (despite crossing of Wilson lines from the complex conjugate amplitude to the amplitude)
Mueller, Munier (2012)

Problems with the NLL evolution



First numerical simulations of the NLL BK equation show pathologies:

- The probability of interaction of a very small dipole with the target decreases with energy, and becomes negative!

  $\Rightarrow$ Unphysical behavior, and makes the numerics completely unstable...

  $\Rightarrow$ Issues come from double (and single ?) collinear logs appearing in the NLL BK kernel

Lappi, Mäntysaari (2015)

Problems with the NLL evolution: same as BFKL

Similar large unphysical corrections were found in the NLL BFKL equation, induced by:

- Kinematical inconsistencies in the LL evolution (double logs).
- Non-optimal running coupling prescription in the LL evolution (single logs).
- Dynamical corrections induced from DGLAP evolutions of the colliding particles, due to the duality between low x_{Bj} and high Q^2 evolutions (single logs).

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$\rightarrow$ All these corrections have been resummed (*collinear resummations*) in order to get sensible results with BFKL at NLL accuracy.
Ciafaloni, Colferai, Salam, Staśto (1998-2007)
Altarelli, Ball, Forte (1999-2008)

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And gluon saturation cannot help to avoid theses problems, as shown numerically in a simplified setup.
Avsar, Staśto, Triantafyllopoulos, Zaslavsky (2011)

Kinematical improvement for BFKL

Usual kinematical regime considered to derive the (LL) BFKL equation:
for example

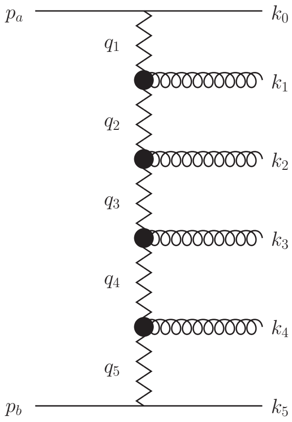
$$k_0^+ \gg k_1^+ \gg \cdots \gg k_n^+ \gg \ldots$$

and

$$\mathbf{k}_0{}^2 \simeq \mathbf{k}_1{}^2 \simeq \cdots \simeq \mathbf{k}_n{}^2 \simeq \ldots$$

But the $\mathbf{k}_n$ are then integrated over without restriction.

$\Rightarrow$ Second condition not consistent nor meaningful.



Kinematical improvement for BFKL

Approximations required in the derivation of the BFKL equation are valid only if successive gluon are strongly ordered in k^+ and in k^- simultaneously:

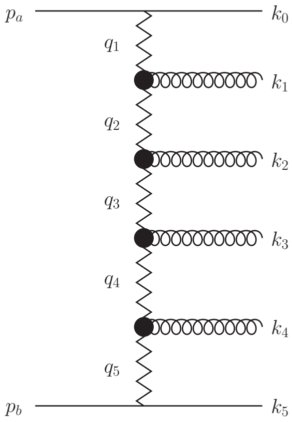
$$k_0^+ \gg k_1^+ \gg \cdots \gg k_n^+ \gg \ldots$$

and

$$k_0^- \ll k_1^- \ll \cdots \ll k_n^- \ll \ldots$$

$\Rightarrow$ Successive gluons are ordered in lifetime both from the projectile (k^-) and from the target (k^+) point of view.

$\Rightarrow$ Defines the correct kinematical phase space for high-energy LL.



Kinematical improvement for BFKL

In each factorization scheme, only the ordering along the chosen evolution variable is guarantied.

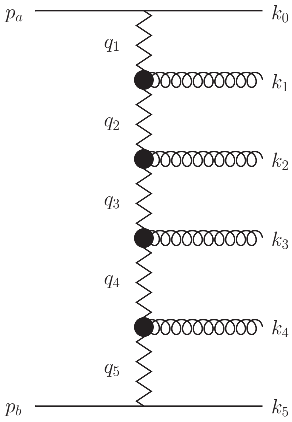
Example: factorization scheme with regulator $k^+_{\min}$ and evolution over $Y_f^+ = \log(k_f^+/k^+_{\min})$
$\Rightarrow$ strong ordering in k^+.

Then, ordering in k^- has to be imposed in the BFKL equation, by a restriction on the $\mathbf{k}_\perp$ integration, since $k_n^- = \mathbf{k}^2_{n\perp}/2k_n^+$.

$\rightarrow$ Kinematical consistency constraint

Ciafaloni (1988); Andersson, Gustafson, Kharraziha, Samuelsson (1996); Kwieciński, Martin, Sutton (1996)

Analog in Mellin space: Salam (1998)



Kinematical consistency constraint in the dipole picture

Real contribution to BK (in k^+ factorization scheme):

Dipole splitting $\mathbf{x}_{01} \mapsto \mathbf{x}_{02} + \mathbf{x}_{21}$ by emission of a soft gluon $(k_2^+, \mathbf{x}_2)$

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With consistent treatment of kinematics:

No contribution to LLs from gluons with small but finite k_2^+ emitted at parametrically large distances, as $k_2^+ x_{02}^2 \simeq k_2^+ x_{21}^2 \gtrsim \sqrt{k_0^+ k_1^+ x_{01}^2}$.

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Physical interpretation: splitting of the parent dipole into too large daugther dipoles violate lifetime ordering of the fluctuations in the projectile.

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Physical interpretation: splitting of the parent dipole into too large daugther dipoles violate lifetime ordering of the fluctuations in the projectile.

Need to include a restriction $\theta\left(k_f^+ \, x_{01}^2 - k_2^+ \, |\mathbf{x}_{02} \cdot \mathbf{x}_{21}|\right)$ for the real term in the integral version of BK.

Modification of the virtual term then obtained by unitarity.

G.B. (2014) (see also Motyka, Staśto (2009))

Kinematically consistent BK equation

Rewriting this improved BK equation as an integro-differential equation:

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y^+} \right\}$$

G.B. (2014)

$$\Delta_{012} = \max\left\{0,\ \log\left(\frac{|\mathbf{x}_{02}\cdot\mathbf{x}_{21}|}{x_{01}^2}\right)\right\}$$

so that

$$\begin{aligned}
\Delta_{012} &= 0 \quad \text{for} \quad x_{02}^2 \lesssim x_{01}^2 \quad \text{and} \quad x_{21}^2 \lesssim x_{01}^2 \\
\Delta_{012} &\sim \log\left(\frac{x_{02}^2}{x_{01}^2}\right) \sim \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \quad \text{for} \quad x_{01}^2 \ll x_{02}^2 \sim x_{21}^2
\end{aligned}$$

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$$\times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y^+} \right\}$$

G.B. (2014)

⋆ Reduction of the phase space by the theta function

⋆ Non-locality of the real emission term

$\Rightarrow$ Both modifications slow down the BK evolution, especially at smaller Y^+.

Moreoever: Taylor-re-expanding around $\Delta_{012} = 0$:
one reproduces the problematic terms $\sim \Delta_{012}^2$ present in the BK equation at NLL, plus a tower of higher order terms of that type.

$\Rightarrow$ kcBK provides a more accurate LL resummation than standard BK.

Other prescription for kinematical improvement of BK

The resummation of the same kinematical double logs can also be done for BK (still in k^+ scheme) keeping transverse and LC variables separated:

$$\partial_{Y^+}\widetilde{\langle \mathcal{S}_{01}\rangle}_{Y^+} = \bar{\alpha}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\left[\frac{\mathrm{J}_1\big(2\sqrt{\bar{\alpha}\rho_{012}^2}\big)}{\sqrt{\bar{\alpha}\rho_{012}^2}}\right]\left[\widetilde{\langle \mathcal{S}_{02}\rangle}_{Y^+}\widetilde{\langle \mathcal{S}_{21}\rangle}_{Y^+}-\widetilde{\langle \mathcal{S}_{01}\rangle}_{Y^+}\right]$$

where $\rho_{012}^2 \equiv \log\left(\frac{x_{02}^2}{x_{01}^2}\right)\,\log\left(\frac{x_{21}^2}{x_{01}^2}\right)$

Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015)

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where $\rho_{012}^2 \equiv \log\left(\frac{x_{02}^2}{x_{01}^2}\right)\,\log\left(\frac{x_{21}^2}{x_{01}^2}\right)$

Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015)

Caveat: The solution $\widetilde{\langle \mathcal{S}_{01}\rangle}_{Y^+}$ obeys a modified initial condition, and coincide with the physical $\langle \mathcal{S}_{01}\rangle_{Y^+}$ only in the most interesting range $Y^+ > \rho_{012}$.

Running coupling: Balitsky's prescription

Running coupling log terms in NLL BK kernel:

$$\bar{\alpha}_\mu \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \left\{1 + \bar{\alpha}_\mu \left[b \log\left(\frac{x_{01}^2\, \mu^2}{4}\right) - 2b\, \Psi(1) - b\frac{(x_{02}^2 - x_{21}^2)}{x_{01}^2} \log\left(\frac{x_{02}^2}{x_{21}^2}\right) + \cdots\right]\right\}$$

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Balitsky's running coupling prescription:

$$\bar{\alpha}_\mu \, \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \quad \mapsto \quad \bar{\alpha}(x_{01}) \left\{ \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} + \frac{1}{x_{02}^2}\left(\frac{\bar{\alpha}(x_{02})}{\bar{\alpha}(x_{21})} - 1\right) + \frac{1}{x_{21}^2}\left(\frac{\bar{\alpha}(x_{21})}{\bar{\alpha}(x_{02})} - 1\right) \right\}$$

- NLL terms contained in Balitsky's prescription are exactly the b terms in NLL BK
  $\Rightarrow$ Balitsky's prescription enough to resum RC single logs
- Higher order terms in the prescription guessed from renormalon arguments.
- However: non positive-definite kernel
- Strong sensitivity to freezing prescription in the IR

Running coupling: BLM prescription

Other possible running coupling prescription: BLM

$\Rightarrow$ Choose $\mu = Q_{BLM}$ to cancel the RC terms in NLL BK:

$$\bar{\alpha}_{\mu} \quad \mapsto \quad \bar{\alpha}(Q_{BLM}) \quad \text{where} \quad Q^2_{BLM} \equiv \frac{4e^{2\Psi(1)}}{x^2_{01}} \left(\frac{x^2_{02}}{x^2_{21}}\right)^{(x^2_{02}-x^2_{21})/x^2_{01}}$$

- Includes the same NLL terms as Balitsky's prescription $\Rightarrow$ ok for RC single log resummation
- Differ only by terms of order NNLL and higher
- Leads to a positive-definite rcBK kernel
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- Very weak sensitivity on the details of the IR freezing of $\bar{\alpha}$

$\rightarrow$ Perturbatively equivalent to the RC prescription mistakenly called FAC given in
Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015)
written there in a more IR sensitive way.

Dealing with the last single logs in NLL BK

Last step missing for the collinear resummation of NLL BK:

Single logs induced by the non-sigular part of DGLAP splitting functions

By very far the most difficult part of the problem! No genuine all order resummation known yet.

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However, simple exponentiation prescription enough to deal with the single logs explicitly appearing in NLL BK at large N_c:

$$\frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \quad \mapsto \quad \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\ \exp\left\{-\frac{11}{12}\,\bar{\alpha}\,\left|\log\left(\frac{x_{01}^2}{\min\{x_{02}^2, x_{21}^2\}}\right)\right|\right\}$$

Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015)

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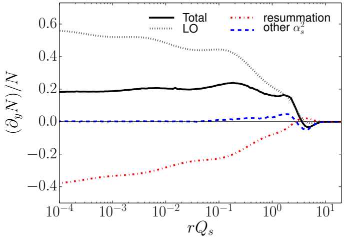
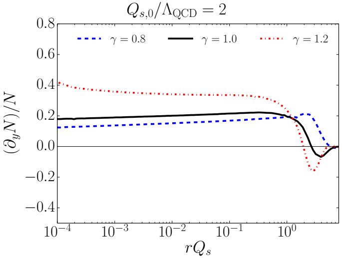
Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015)

$\rightarrow$ Analog to the prescription given in
Gotsman, Levin, Maor, Naftali (2004)
but should be much more stable numerically.

Numerics for NLL BK with collinear resummations



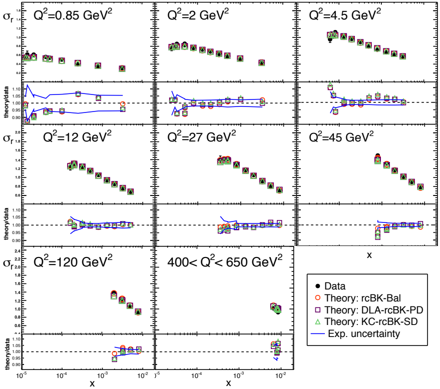


Including these collinear resummations, numerical simulations of the NLL BK equation now stable.

Lappi, Mäntysaari (2016)

$\Rightarrow$ NLL BK can in principle be used in future phenomenological studies and global fits at NLO+NLL accuracy. Important milestone!

DIS phenomenology at LO + kcLL accuracy



Albacete (2015)

Good fits to HERA data can be obtained with both the non-local and local implementations of the kinematical improvement of BK.

$\Rightarrow$ Good starting point for further studies at NLO and/or NLL.

See also Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos (2015)

DIS at NLO in the dipole factorization

Earlier calculations of NLO corrections to DIS cross-section:
Balitsky, Chirilli (2011); G.B. (2012)
However, in both papers only $q\bar{q}g$ NLO contributions to DIS were calculated explicitly, whereas $q\bar{q}$ NLO corrections were guessed.

- Results from Balitsky, Chirilli (2011) more general but not available in a form convenient for numerical studies
- Guess for the $q\bar{q}$ contribution at one loop in G.B. (2012) not correct: $\Rightarrow$ explicit one-loop calculations required

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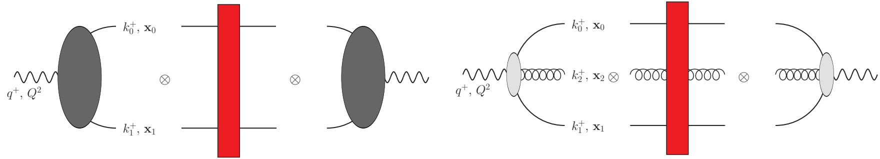
- Guess for the $q\bar{q}$ contribution at one loop in G.B. (2012) not correct: $\Rightarrow$ explicit one-loop calculations required
- Calculation of $\gamma^*_{T,L} \to q\bar{q}$ LF wavefunctions at one loop
  G.B. (2016)
- Combination of $q\bar{q}$ and $q\bar{q}g$ parts of the NLO corrections to DIS structure functions
  G.B., *in preparation*

$\Rightarrow$ Final results for DIS at NLO in the dipole factorization soon available in a convenient form!

DIS at NLO: full fixed-order results



$$\sigma_{T,L}^{\gamma p}(Q^2, x_{Bj}) = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1$$

$$\times\Bigg\{\left[1+\frac{\alpha_s C_F}{\pi}\,\widetilde{\mathcal{V}}(z_{\min})\right]\mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01}, z_1, Q^2)\,\Big[1-\langle\mathcal{S}_{01}\rangle_0\Big]$$

$$+\frac{2\alpha_s C_F}{\pi}\int_{z_{\min}}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\mathcal{I}_{T,L}^{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2)\,\Big[1-\langle\mathcal{S}_{012}\rangle_0\Big]\Bigg\}$$

with $z_n = k_n^+/q^+$ and $z_{\min} = k_{\min}^+/q^+ = \frac{x_{Bj}}{Q^2}\,\frac{Q_0^2}{x_0}$.

G.B. (2012-2016)

DIS at NLO: LL resummation

Method for the LL resummation in the k^+ scheme for NLO DIS:

1. Assign $k^+_{\min}$ to the scale set by the target: $k^+_{\min} = \frac{Q_0^2}{2x_0\, P^-} = \frac{x_{Bj}\, Q_0^2}{x_0\, Q^2}\, q^+$
2. Choose a factorization scale $k_f^+ \lesssim k_0^+, k_1^+$, corresponding to a range for the high-energy evolution $Y_f^+ \equiv \log\left(\frac{k_f^+}{k^+_{\min}}\right) = \log\left(\frac{x_0\, Q^2\, k_f^+}{x_{Bj}\, Q_0^2\, q^+}\right)$
3. In the LO term, make the replacement

$$\langle \mathcal{S}_{012}\rangle_0 \quad = \quad \langle \mathcal{S}_{012}\rangle_{Y_f^+} - \int_0^{Y_f^+} dY^+ \left(\partial_{Y^+}\langle \mathcal{S}_{012}\rangle_{Y^+}\right)$$

   with both terms calculated with the same evolution equation

4. Combine the second term with the NLO correction to cancel its $k^+_{\min}$ dependence and the associated large logs.

$\Rightarrow$ More accurate subtraction of LL from NLO with the kinematically consistent LL BK equation than with the naive LL BK equation

Calculations for single inclusive production at NLO

NLO corrections for the hybrid factorization of forward single inclusive hadron production in pA

- Massless partons contributions:
  Chirilli, Xiao, Yuan (2012) (see also Altinoluk, Kovner (2011)))
- Heavy quark contributions (in FFNS):
  Altinoluk, Armesto, G.B., Lublinsky, Kovner (2016)

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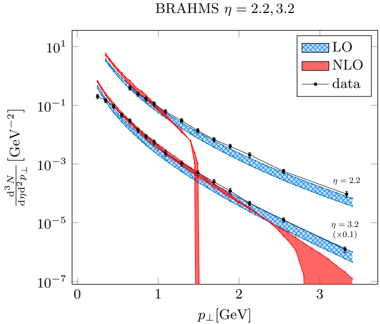
In the massless case, the NLO corrections have various log contributions (and divergences) which have to be disentangled and resummed:

- Initial state collinear radiation, associated with the DGLAP evolution for the projectile PDF.
- Final state collinear radiation, associated with the DGLAP evolution for the FF into the produced hadron.
- Low-x radiation, associated with the high-energy JIMWLK/BK evolution of the target

After subtracting and resumming these, the leftover NLO correction should be well-behaved, but...

Problem with numerical results

First numerical implemention of NLO corrections (with LL resummation):



Forward single inclusive hadron production in d-Au at RHIC
Staśto, Xiao, Zaslavsky (2013)

Good at small $p_\perp$, but large negative NLO corrections at large $p_\perp$! The cross-section even becomes negative!

$\Rightarrow$ Calculation of NLO corrections needs to be revisited...

Isolating the problem

- Perturbative NLO calculations have been redone independently: ok!
  Altinoluk, Armesto, G.B., Lublinsky, Kovner (2015)
- IS (projectile) and FS DGLAP resummation: ok!
- High-energy LL resummation not done in a consistent factorization scheme in the initial calculation:
  Unrelated to the regularization of the rapidity divergence!

Trying to solve the problem

Many proposals to improve the situation:

Staśto, Yuan, Xiao, Zaslavsky (2014)
Altinoluk, Armesto, G.B., Lublinsky, Kovner (2015)
Watanabe, Yuan, Xiao, Zaslavsky (2015)
Ducloué, Lappi, Zhu (2016)
Iancu, Mueller, Triantafyllopoulos (2016)

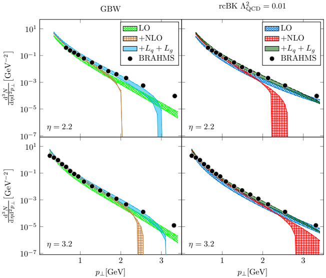
Main ingredients in most of these:

- Try improve consistency between :
  1. Regularization of *rapidity divergence*
  2. Subtraction of high-energy LL from NLO results
  3. Resummation of high-energy LL into LO term
- Use factorization scheme along k^- (a.k.a. *Ioffe time*) in order to optimize matching between BK and DGLAP for the target

However, many details differ between these prescriptions.

Newer numerical tests

Some of these new prescriptions have been tested numerically, for example:



Forward single inclusive hadron production in d-Au at RHIC
Watanabe, Yuan, Xiao, Zaslavsky (2015)

$\Rightarrow$ Quantitative improvement, but seem to rather delay than fully solve the negativity problem...

Similar numerical results have been obtained in:
Ducloué, Lappi, Zhu (2016)

2 Additional issues for pA at NLO

- Small problem: NLL BK and collinear resummations available in the k^+ factorization scheme but not in the k^- scheme.
  $\Rightarrow$ We should perform LL subtractions/resummations for NLO corrections in pA in the k^+ scheme, not the k^- one, when attempting to reach NLL precision.

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- Big problem: incoming and outgoing partons on-shell in the hybrid factorization
  $\Rightarrow$ No k^- restriction on the phase-space from the projectile side
  $\Rightarrow$ High-energy LL term ecountered in the NLO corrections in pA does not obeys kinematical improvement, by contrast to the LL contribution in DIS at NLO
  $\Rightarrow$ High-energy factorization on the target side broken by the collinear factorization on the projectile side ????
  $\Rightarrow$ Need to accommodate parton virtualities or to switch from collinear factorization to TMD factorization in the hybrid formalism for pA ?

Conclusions

High-energy factorization/CGC well on the way towards NLO+NLL accuracy:

- Partial collinear resummations have been performed for the NLL BK equation, making its numerical solution possible and now available.
- Final results for the DIS structure functions at NLO in the dipole factorization will appear soon
- Despite many efforts, the situation is still unclear concerning high-energy resummations for the forward single-inclusive hadron production in pA at NLO in the hybrid factorization

  $\Rightarrow$ More analytical and numerical work and discussions needed to reach a consensus on the correct implementation of the hybrid factorization beyond LO+LL.

γ_L total cross section at NLO

$$\sigma_L = 4N_c\,\alpha_{em}\,\mathrm{Re}\sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+ \, \frac{4Q^2}{q^+} \left(\frac{k_0^+}{q^+}\right)^2 \left(\frac{k_1^+}{q^+}\right)^2$$

$$\times\Bigg\{ \delta(k_0^+ + k_1^+ - q^+) \left[\mathrm{K}_0\left(Q x_{01} \frac{\sqrt{k_0^+ k_1^+}}{q^+}\right)\right]^2 \left[1 + \frac{\alpha_s C_F}{\pi}\, \widetilde{\mathcal{V}}^L_{\mathrm{reg.}}\right] \left[1 - \langle \mathcal{S}_{01}\rangle_0\right]$$

$$+ \frac{\alpha_s C_F}{\pi} \int_{k_{\min}^+}^{+\infty} \frac{dk_2^+}{k_2^+}\, \delta(k_0^+ + k_1^+ + k_2^+ - q^+) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \left[q \text{ term} + \bar{q} \text{ term} + \text{leftover}\right] \Bigg\}$$

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With:

$$q\ \mathrm{term} = \left[2+\left(\frac{2k_2^+}{k_0^+}\right)+\left(\frac{k_2^+}{k_0^+}\right)^2\right]\left[\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2}-\frac{\mathbf{x}_{21}}{x_{21}^2}\right)\right]$$

$$\times\left\{\Big[\mathrm{K}_0(QX_{012})\Big]^2\Big[1-\langle\mathcal{S}_{012}\rangle_0\Big]-\left[\mathrm{K}_0\left(Qx_{01}\frac{\sqrt{(k_0^++k_2^+)k_1^+}}{q^+}\right)\right]^2\Big[1-\langle\mathcal{S}_{01}\rangle_0\Big]\right\}$$

$$X_{012}^2 \equiv \frac{1}{(q^+)^2}\Big[k_0^+k_1^+x_{01}^2 + k_0^+k_2^+x_{02}^2 + k_1^+k_2^+x_{12}^2\Big] = \frac{q\bar{q}g\ \mathrm{form.\ time}}{2q^+}$$

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With:

$$\mathrm{leftover} \;=\; \left[\left(\frac{k_2^+}{k_0^+}\right)^2 + \left(\frac{k_2^+}{k_1^+}\right)^2\right]\Big[\mathrm{K}_0(QX_{012})\Big]^2\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\,x_{21}^2}\right)\Big[1-\langle\mathcal{S}_{012}\rangle_0\Big]$$

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With:

$$\widetilde{\mathcal{V}}^L_{\text{reg.}} = \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2}$$

UV and soft divergent terms have been moved from $\widetilde{\mathcal{V}}^L$ to the q and $\bar{q}$ terms, as well as a constant $1/2$ (rational term $(D-4)/(D-4)$)