Potentials - Challenges - Directions: Heavy-Flavor Theory

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What can we learn from heavy-flavor observables?
What are the challenges to reach a quantitative level?
Which directions to go to make progress?
Heavy quarks as probes of the QGP

- Probes should not thermalize with the medium, e.g. dileptons, high-pT jets,...
- The mass of heavy quarks (HQ) sets another scale: $m_c, m_b$
- HQ vacuum shower terminates much earlier: $E/Q_H^2$ with $Q_H = \sqrt{Q_0^2 + m_Q^2}$.
- Number of thermally excited HQ is negligibly small.
- HQ as leading parton is always tagged.

probe the entire momentum range from

low $p_T \sim m_Q$ to high $p_T \gg m_Q$ dynamics
Diffusion coefficient from lattice QCD

- Lattice QCD at finite $T$ is performed in Euclidean space ⇒ notoriously difficult to calculate dynamical quantities.
- Relate the current-current correlations (calculated on the lattice) to spectral functions by the inversion of the spectral representation by an initial assumption for the spectral function, Maximal Entropy Method, etc.
- Obtain transport coefficients from the slope of spectral function $\rho_E$ at $\omega = 0$ (Kubo formula).

momentum diffusion:

$$\frac{\kappa}{T^3} = \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega}$$

spatial diffusion: $D_s = \frac{2T^2}{\kappa}$

Approximations/limitations:
quenched QCD, heavy quark vs. charm quark, continuum extrapolation, ...

Current lattice QCD estimates are between $D_s \sim 2 - 7...$
The observables $R_{AA}$ and $v_2$ reflect the dependence of the in-medium energy loss and "partial" thermalization on the heavy-quark diffusion coefficient!
Collisional (elastic) energy loss - pQCD inspired

LO Feynmann diagrams for perturbative heavy quark scattering off a light parton

- $t$-channel IR singularity, regulated by the Debye screening mass $m_D$
- HTL energy loss: resummed propagator for $|t| \ll t^*$, bare propagator $|t| \gg t^*$
- Relevant separation of scales $g^2T^2 \ll T^2$ probably not fulfilled at RHIC/LHC.

- One-gluon exchange model: reduced IR regulator $\lambda m_D^2$ in the hard propagator
- Running coupling $\alpha_{\text{eff}}(t)$ and self-consistent
  $m_D^2 = (1 + 6n_f)4\pi\alpha_s(m_D^2)T^2$

Radiative energy loss - pQCD inspired

- Extention of Gunion-Bertsch approximation beyond mid-rapidity and to finite mass $m_Q$ ⇒ distribution of induced gluon radiation ($E_{rad}^{\text{loss}} \propto E L$):

$$P_g(x, k_\perp, q_\perp, m_Q) = \frac{3\alpha_s}{\pi^2} \frac{1 - x}{x} \left( \frac{k_\perp}{k_\perp^2 + x^2 m_Q^2} - \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2 + x^2 m_Q^2} \right)^2$$

J. Gunion, PRD25 (1982); B. Zakharov, JETPL 63/65 (1996/7); O. Fochler et al. PRD88 (2013); J. Aichelin et al. PRD89 (2014)

- Coherent (LPM) emission if $\tau_{\text{form}} = \sqrt{\frac{\omega}{q}} > l_{\text{mfp}}$

- $E_{rad}^{\text{loss}} \propto \sqrt{E} L$, if $\tau_{\text{form}} > L$ then $E_{rad}^{\text{loss}} \propto L^2$

- Dynamical realization challenging

K. Zapp et al. PRL103 (2009), JHEP 1107 (2011)

- heavy vs light probes different regions of coherence
... and nonperturbative approaches!

Resonance scattering (TAMU):

- Basic assumption: for $T \lesssim 3T_c$ two-body interactions $\rightarrow$ potential $V(t)$
- Spatial diffusion coefficient comparable to quenched IQCD.
- Smooth transition to hadronic medium with minimum close to $T_c$

Talk by M. He


Strong coupling:

- In AdS/CFT a heavy quark is represented by a string connected to a D7 brane.
  C. Herzog et al. JHEP2006; S. Gubser PRD74 (2006)
- Leading-order drag coefficients were excluded by comparison to data.
- Momentum-kicks are multiplicative and grow with the HQ velocity $\rightarrow$ important toward higher $p_T$!
- At larger momenta HQ in strong-coupling reach a speed limit $\rightarrow$ expected to work in an intermediate $p_T$ regime! W. Horowitz, PRD (2015)
Mass dependence: light vs heavy flavor

\[ R_{AA}(g) < R_{AA}(u, d, s) < R_{AA}(c) < R_{AA}(b) < R_{AA}(t?) \]

Dead cone effect: Dokshitzer et al., PLB 519 (2001)

\[ \frac{d\sigma_{\text{rad}}}{d\theta} \propto \frac{\theta^2}{(\theta^2 + M_Q^2/E_Q^2)} \]


When the hard scattering assumption is relaxed, emission at low \( k_\perp \) is significantly less suppressed. J. Aichelin et al. PRD89 (2014)

M. Djordjevic et al. PLB737 (2014)
Temperature dependence: from $\sqrt{s} = 0.062$ to 5 TeV

- QGP becomes hotter from $\sqrt{s} = 62$ GeV to $\sqrt{s} = 5$ TeV.

- Temperatures in the space-time evolution have more weight on the probed transport coefficient.

- Better handle on the temperature dependence of the diffusion coefficient!
• strong initial magnetic field in heavy-ion collisions \( \approx \mathcal{O}(10^{19}) \text{ Gauss} \approx 10m^2_T \)
• fast decay of the magnetic field within the first 0.1 fm

**Talk by V. Greco:** sizable effect on heavy quark \( v_1 \) due to the Lorentz force

S. Das et al. 1608.02231

• short formation time of heavy quarks!
• long equilibration times!
(for light quarks effect is smaller U. Gursoy, K. Rajagopal, D. Kharzeev, PRC89 (2014))

**Talk by K. Hattori:** effects on heavy quark \( v_2 \)
challenges
Challenge to describe $R_{AA}$ and $v_2$ simultaneously "puzzle"
(Too) many models describe $R_{AA}$ and $v_2$
Heavy-quark dynamics in HIC

- LO pQCD, e.g. FONLL → inclusive spectra, no azimuthal $Q\bar{Q}$ correlations
  
  M. Cacciari et al. PRL 95 (2005), JHEP 1210 (2012)

- NLO pQCD matrix elements plus parton shower, e.g. POWHEG or MC@NLO → exclusive spectra, like $Q\bar{Q}$ correlations
  

- Cold nuclear matter effects, i.e. shadowing, $p_T$ broadening, Cronin effect, etc.
  

- Consistent initialization of HF and LF sectors!
Scattering off light QGP constituents, sampled from fluid dynamics or given within microscopic transport [BAMPS J. Uphoff et al. PRL114 (2015) or PHSD T. Song et al. PRC92 (2015)]

Any model with $P(\Delta E)$ produces the generic $p_T$ shape of $R_{AA}$, magnitude depends strongly on the bulk evolution model! [T. Renk, PRC85 (2012)]

Proper modeling of the QGP evolution is important! Should be well tested in the light hadron sector!
Heavy-quark dynamics in HIC

- Coalescence/Recombination – predominantly at small $p_T$. Parameter-dependent!
  
  e.g. C. B. Dover et al., PRC 44 (1991)

- Fragmentation – predominantly at large $p_T$. Medium-modification?
  
  e.g. M. Cacciari et al., PRL 95 (2005)

- After hadronization: final hadronic interactions of $D$ mesons.
  
  L. Tolos et al., PRD88 (2013); J. Torres-Rincon et al., PRD89 (2014)
Consistent coupling of the HQ to medium

Does the EoS match the representation of the medium (quasiparticles)?

- HQ scatter off (thermal) QP in the medium
- **Inconsistent:** Massless partons or $m(T)$ from pQCD DO NOT reproduce the lattice EoS!

MN et al. PRC93 (2016) 1602.03544;
H. Berrehrah et al. 1604.02343

$m(T)$ and $\alpha_s(q)$

$m(T)$ and $\alpha_s(q, T)$
Boltzmann equation for HQ phase-space distribution

\[
\frac{d}{dt} f_Q(t, \vec{x}, \vec{p}) = C[f_Q] \quad \text{with} \quad C[f_Q] = \int d\vec{k} \left[ w(\vec{p} + \vec{k}, \vec{k}) f_Q(\vec{p} + \vec{k}) - w(\vec{p}, \vec{k}) f_Q(\vec{p}) \right]
\]

expanding \( C \) for small momentum transfer \( k \ll p \) (in the medium \( k \sim \mathcal{O}(gT) \)) and keeping lowest 2 terms \( \Rightarrow \) Fokker-Planck equation

\[
\frac{\partial}{\partial t} f_Q(t, \vec{p}) = \frac{\partial}{\partial p^i} \left( A^i(\vec{p}) f_Q(t, \vec{p}) + \frac{\partial}{\partial p^j} \left[ B^{ij}(\vec{p}) f_Q(t, \vec{p}) \right] \right)
\]

friction (drag) \quad momentum diffusion

Recast to Langevin equation (probably good for bottom, but for charm?)

\[
\frac{d}{dt} \vec{p} = -\eta_D(p) \vec{p} + \vec{\xi} \quad \text{with} \quad \langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')
\]

Transport coefficients connected by fluctuation-dissipation theorem (Einstein relation):

\[
\eta_D = \frac{\kappa}{2m_Q T}, \quad D_s = \frac{T}{m_Q \eta_D} \quad \text{spatial diffusion}
\]

D. Walton et al., PRL84 (2000); G. Moore et al., PRC71 (2005)
• Under which conditions should Brownian motion be a valid approximation for relativistic particles?
• Calculations of transport coefficients from the underlying theory do not necessarily fulfil FDT.
• Langevin leads to Gaussian momentum distribution, Boltzmann very different.

S. Das et al, PRC90 (2014)
Mass dependence: light vs heavy flavor

Delicate interplay between energy loss and fragmentation can lead to similar $R_{AA}$ of light hadrons and $D$ mesons: M. Djordjevic et al. PRL112 (2014)

- Similar effects seen in LBL-Boltzmann transport with (scaled) LO pQCD cross sections and radiative energy loss according to higher-twist formulation.
  

- Currently no well accepted theoretical description gives $R_{AA}^D \sim R_{AA}^B$...
Complete models and theoretical improvements

Continuous improvement on the theory side is needed, many ingredients contribute, eg. for the pQCD-based description by M. Djordjevic:

- dynamical scattering centers,
- finite size QCD medium,
- radiative and collisional energy loss,
- finite magnetic mass,
- running coupling,
- ...


How to connect

- high-\(p_T\) jet shower evolution to leading-parton energy loss to low-\(p_T\) diffusion?
- perturbative and nonperturbative regimes?
- weak- and strong-coupling scenarios?
- coherent and incoherent radiation pattern?
Systematic comparison of model ingredients

Participation of many different models brought together by JET-HQ collaboration/EMMI Task Force - more are welcome to join!

Different models in infinite static matter (aka “brick” problem)

drag coefficient

 charms \( R_{AA} \)

To come next: evolution through the same background QGP evolution...
Bayesian model-to-data statistical analysis

- HQ Langevin dynamics + 2+1d fluid dynamics
  \((\eta/s(T), \zeta/s(T))\) constraint by bulk observables via Bayesian analysis
  

\[ D_s(T) = \frac{T^2 K^{-1}}{\hat{q}_{\text{pQCD}}} \left( 1 + K_T \exp\left( \frac{- (T - T_c)^2}{2\sigma_T^2} \right) \right)^{-1} \]

- Assume a parametrization:

Talk by Yingru Xu

Know the probability distributions of all parameters and correlations

⇒ temperature dependence of charm quark diffusion coefficient!

Y. Xu, S. Cao, MN, S. Bass, in preparation
Bayesian model-to-data statistical analysis

- HQ Langevin dynamics + 2+1d fluid dynamics
  
  \((\eta/s(T), \zeta/s(T)\) constraint by bulk observables via Bayesian analysis\)


**BEFORE**

**AFTER**

![Graphs showing data before and after analysis](image)

assume a parametrization:

\[
D_s(T) = \frac{T^2 K^{-1}}{\hat{q}_{pQCD}} \left( 1 + K_T \exp\left( \frac{-(T-T_c)^2}{2\sigma_T^2} \right) \right)^{-1}
\]

Talk by Yingru Xu

know the probability distributions of all parameters and correlations

⇒ **temperature dependence of charm quark diffusion coefficient!**

Y. Xu, S. Cao, MN, S. Bass, in preparation
Beyond traditional observables: $Q\bar{Q}$ azimuthal correlations

- High discriminating power between different interaction mechanisms: collisional vs. radiative energy loss.

$\langle p_{\perp} \rangle$ from MC@sHQ+EPOS2:

- Low $p_T$ pairs more likely to remain correlated for strong than for weak coupling.
- Already the $c\bar{c}$ proton-proton baseline is not well understood theoretically ...

see also: S. Cao, G.Y. Qin, S. Bass, PRC92 (2015); A. Beraudo EPJC75 (2015)
Beyond traditional observables: from correlations to flow

- Fourier transform of $DD$ azimuthal correlations $\Rightarrow V_n$ of $D$ mesons:

  $\text{coll.}, K = 1.5$

First calculation of higher-order flow harmonics for heavy flavor!

MN et al., SQM2013
Beyond traditional observables: higher-order flow harmonics

- Most models give a $\tau_{\text{relax}}$ for charm quarks much longer than the evolution of the QGP, but $v_2(\text{HF}) \lesssim v_2(\text{LF}) \rightarrow$ indication for “partial” thermalization?
- Higher-order Fourier coefficients were important for understanding charged hadron flow $\Rightarrow$ What about heavy-flavor $v_3$, $v_4$, ...?

- Expectation: $v_3$ and higher-order coefficients (and centrality dependence) show the incomplete coupling of HQ to the medium!

From my last talk: “Looking forward to $v_3$ data from LHC and RHIC!”
Beyond traditional observables: higher-order flow harmonics

(dashed lines) predictions for $v_3$!

MN et al. PRC91 (2015) 1410.5396

Talks by D. Gülhan, Jian Sun
Beyond traditional observables: higher-order flow harmonics

- $v_3$ of charm is due to the LF flow, small effect from $L$ differences!

- EP method ($\approx$ SP method CMS)

- Sophisticated energy loss model, HF dynamics and coupling to the soft sector (medium flow)!

Study HF - LF correlations by consistently coupling heavy quark and medium dynamics!

(like upcoming EPOS3+HQ - successor of MC@shQ+EPOS2 by the Subatech group Nantes → to be ready for QM17!)

- Flow due to $L$ difference (no deflection due to medium flow)

- SP method (CMS)

- Simple energy loss models, no HQ dynamics...
potentials

probe medium properties
and the HQ-medium interaction: $T, m, L, E$ dependence

challenges

hadronization

weak vs. strong

hadronization

consistent HQ-medium coupling

(in)coherent

non-perturbative regime

model improvements

model-to-data analysis

new observables

Directions

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extra
Radiative energy loss

- LO pQCD matrix element for $2 \to 3$ process Kunszt et al. PRD21 (1980)
- Gunion-Bertsch approximation derived in the high-energy limit, where the radiated gluon $k_\perp$ and the momentum transfer $q_\perp$ are soft $\ll \sqrt{s}$.
- Incoherent radiation off a massless parton, mid-rapidity
- Extention beyond mid-rapidity and to finite mass $m_Q$ (heavy quarks!)
  $\Rightarrow$ distribution of induced gluon radiation:

$$P_g(x, k_\perp, q_\perp, m_Q) = \frac{3\alpha_s}{\pi^2} \frac{1-x}{x} \left(\frac{k_\perp}{k_\perp^2 + x^2 m_Q^2} - \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2 + x^2 m_Q^2}\right)^2$$

- $\Rightarrow E_{\text{rad}}^{\text{loss}} \propto E \Lambda$

J. Gunion, PRD25 (1982); O. Fochler et al. PRD88 (2013); J. Aichelin et al. PRD89 (2014)
Coherent emission - LPM

- Coherent emission if $\tau_{\text{form}} = \sqrt{\frac{3}{q}} > l_{\text{mfp}}$
- QCD analogon to the Landau-Pomeranchuk-Migdal (LPM) effect
- Important in QCD: rescattering of the forming gluon with medium partons $\Rightarrow$ less suppression than in QED
- At large energies in BDMPS-Z: $\Rightarrow E_{\text{loss}}^{\text{rad}} \propto \sqrt{E L}$
- For very energetic partons $\tau_{\text{form}} > L$, then $E_{\text{rad}}^{\text{loss}} \propto L^2$, estimate for the LHC ($L \sim 2\text{ fm}, \hat{q} \sim 2 \text{ GeV/fm} \Rightarrow \omega_c \sim 20 \text{ GeV}$)


Baier et al. PLB 345 (1995); NPB 483 (1997); ibid. 484 (1997); B. G. Zakharov, JETP Lett. 63 (1996) 952
Dead cone effect

suppression of high-energetic (small angle) gluon emission by the heavy quark mass:

\[
\frac{d\sigma_{\text{rad}}}{\theta d\theta} \propto \frac{\theta^2}{(\theta^2 + M_Q^2/E_Q^2)}
\]

Dokshitzer et al., PLB 519 (2001)

- Suppresses gluon emission in the dead cone \( \theta_D = M_Q/E_Q \)
- Introduces a mass hierarchy in the radiative energy loss.
- But: assumes hard scatterings!

- When the hard scattering assumption is relaxed, emission at low \( k_{\perp} \) is significantly less suppressed:

\[
\frac{P_g(x,k_{\perp};M)}{P_g(x,k_{\perp};0)}
\]

hard-scattering approximation

all scatterings
Charm production (and diffusion?) in pPb collisions

- $3 + 1d$ fluid dynamical evolution + Langevin dynamics, initial shadowing.

- Centrality dependence of $R_{pPb}$ expected due to energy loss.
  (Note, that experimentally $Q_{pPb}$!)

- Indications that $v_2$ of $D$ mesons decouples from medium flow - unlike in AA collisions - and decreases with centrality.

- Can HF measurements in pPb help answering the question of initial vs final state effects?

Y. Xu et al, Duke University, in preparation

see also: A. Beraudo, JHEP1603 (2016)
Modeling of heavy-quark dynamics in the QGP

- Model the QGP: a locally thermalized medium provides the scattering partners.
- Input from a fluid dynamical description of the bulk QGP medium: temperatures and fluid velocities.
- Use a fluid dynamical description which describes well the bulk observables!

smooth initial conditions

fluctuating initial conditions
• Due to the radial flow of the matter low-\(p_T\) \(c\bar{c}\)-pairs are pushed into the same direction.

• Initial correlations at \(\Delta \phi \sim \pi\) are washed out but additional correlations at small opening angles appear.

• This happens only in the purely collisional interaction mechanism!

• No “partonic wind” effect observed in collisional+radiative(+LPM) interaction mechanism!

\[ \vec{P}_{T,Q}^{\text{ini}}, \vec{P}_{T,Q}^{\text{fin}}, \vec{P}_{T,\text{cm}}^{\text{fin}} \]

\[ dN_{c\bar{c}}/d\Delta \phi \]
QGP: initial state and bulk flow (2)

average temperature and overlap area

centrality dependence:

+ increase of initial eccentricities
+ decrease of interaction rate and medium size

⇒ expectation: heavy-flavor flow shows a weaker dependence on centrality, especially for $v_3$

MN et al. PRC91 (2015)
At small $p_T$: relative enhancement of flow in purely collisional scenario over collisional+radiative($+$LPM) larger for $v_3$ than for $v_2$
Contribution to the flow from hadronization.

For low $p_T$ the charm flow is predominantly due to the flow of the bulk.