

Jet and Heavy Flavor Production from Soft Collinear Effective Theory

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Hard Probes '16, Wuhan



Outline

- Open heavy flavor production
- Inclusive jet observables
- Conclusions

Kang, FR, Vitev '16

arXiv:1609.04908

Kang, FR, Vitev '16

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Kang, FR, Vitev '16

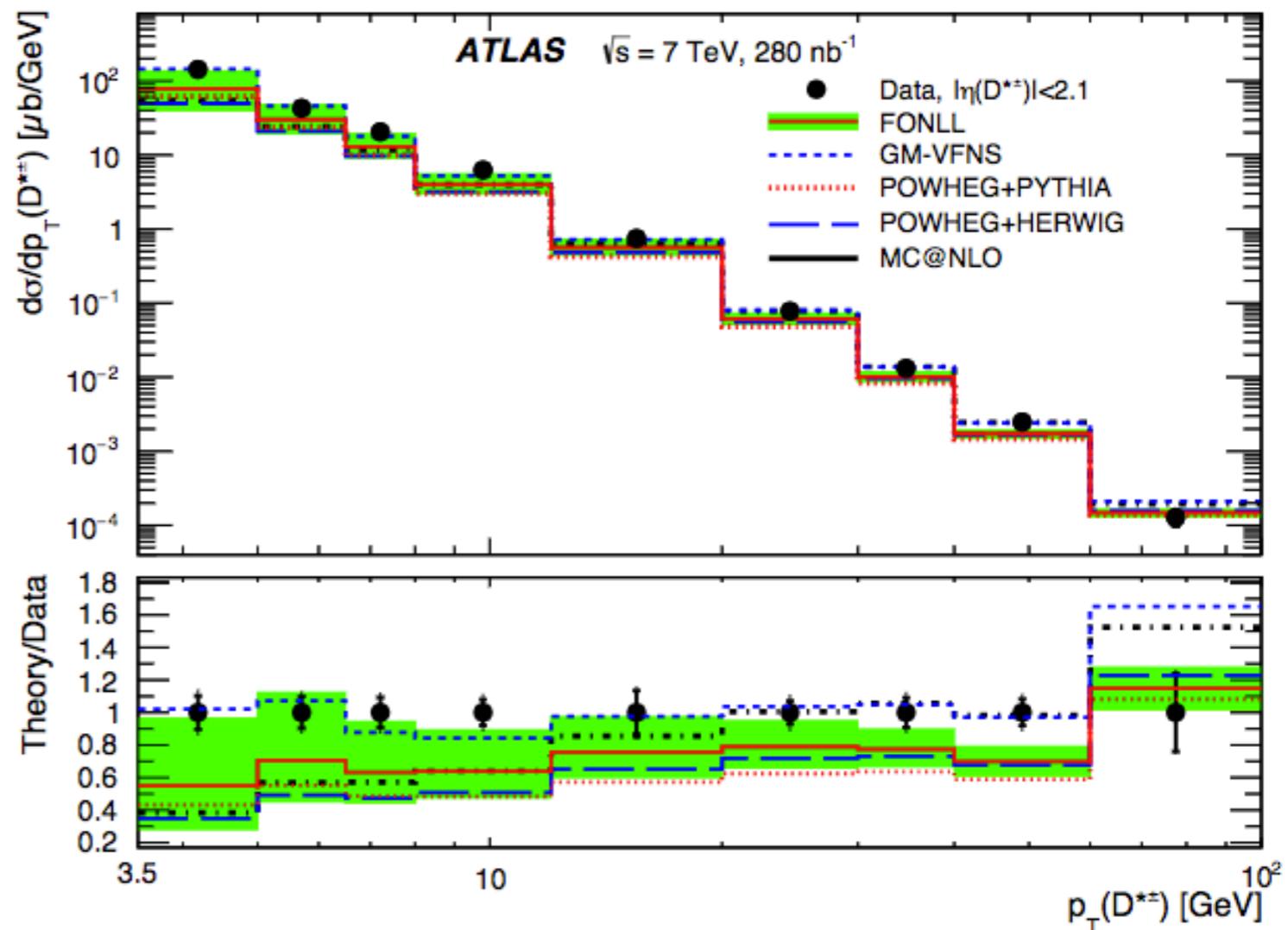
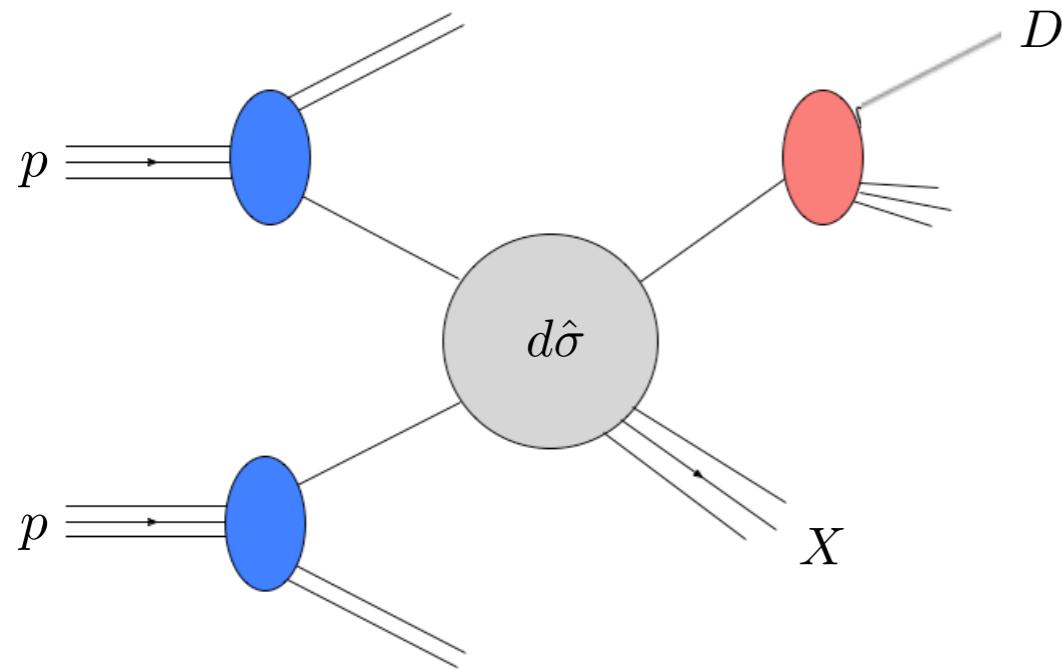
arXiv:1609.04908

Kang, FR, Vitev '16

arXiv:1606.06732

D and B-meson production $pp \rightarrow HX$

Inclusive D-meson
data taken at the LHC



Nucl. Phys. B 907 (2016) 717
Similarly CMS, ALICE

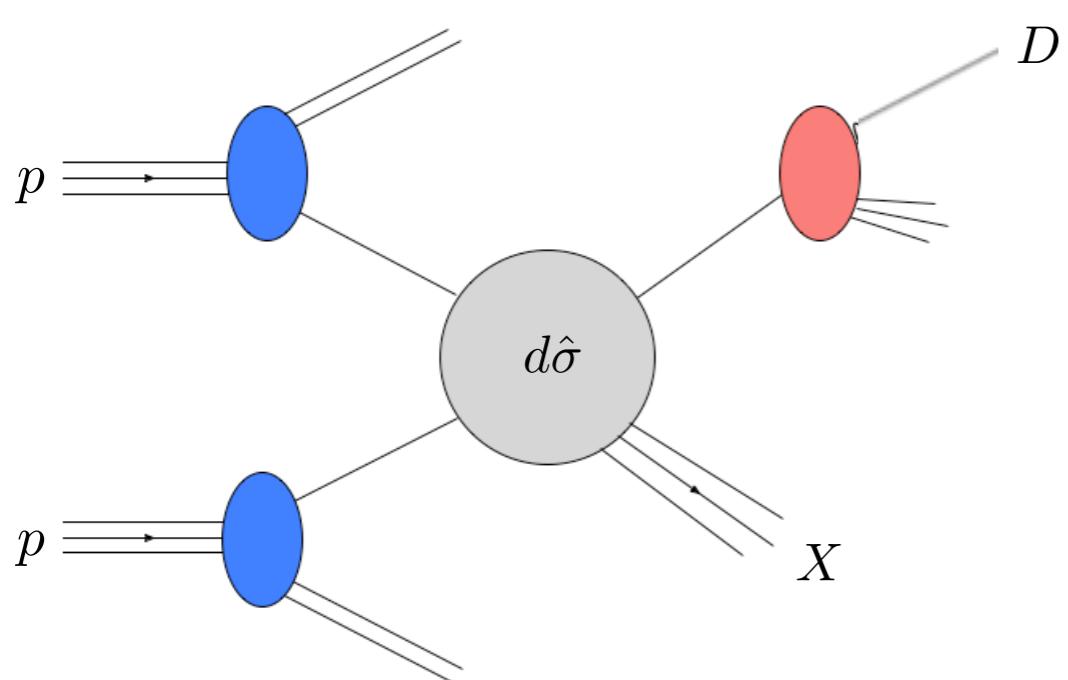
D and B-meson production $pp \rightarrow HX$

Next-to-leading order in QCD

Jäger, Stratmann, Vogelsang '02

$$\frac{d\sigma^{pp \rightarrow DX}}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dv dz} D_c^D(z_c, \mu)$$

where $v = 1 - \frac{2\hat{p}_T}{\sqrt{\hat{s}}} e^{-\hat{\eta}}, \quad z = \frac{2\hat{p}_T}{\sqrt{s}} \cosh \hat{\eta}$



$$\hat{\eta} = \eta - \ln(x_a/x_b)/2$$

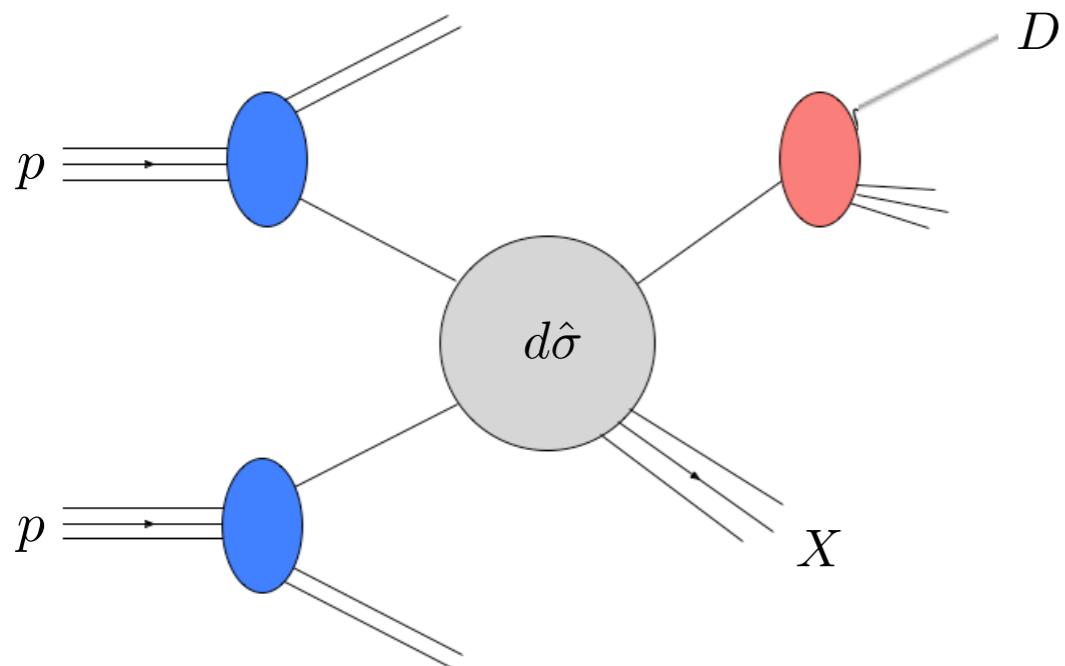
NLO: $\frac{d\hat{\sigma}_{ab}^c}{dv dz} = \frac{d\hat{\sigma}_{ab}^{c,(0)}}{dv} \delta(1-z) + \frac{\alpha_s(\mu)}{2\pi} \frac{d\hat{\sigma}_{ab}^{c,(1)}}{dv dz}$

D and B-meson production $pp \rightarrow HX$

Next-to-leading order in QCD

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$$\frac{d\sigma^{pp \rightarrow DX}}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dv dz} D_c^D(z_c, \mu)$$

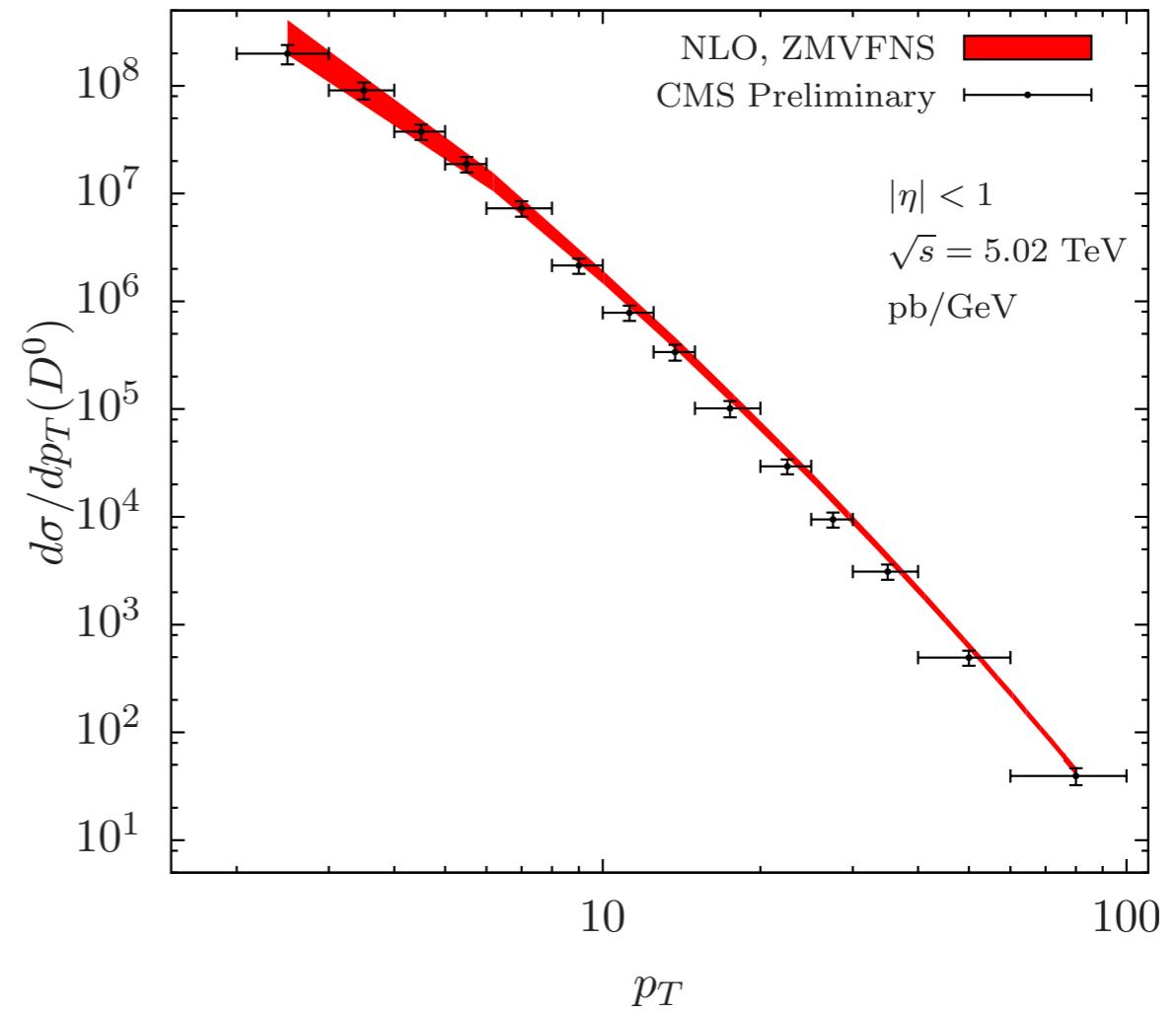
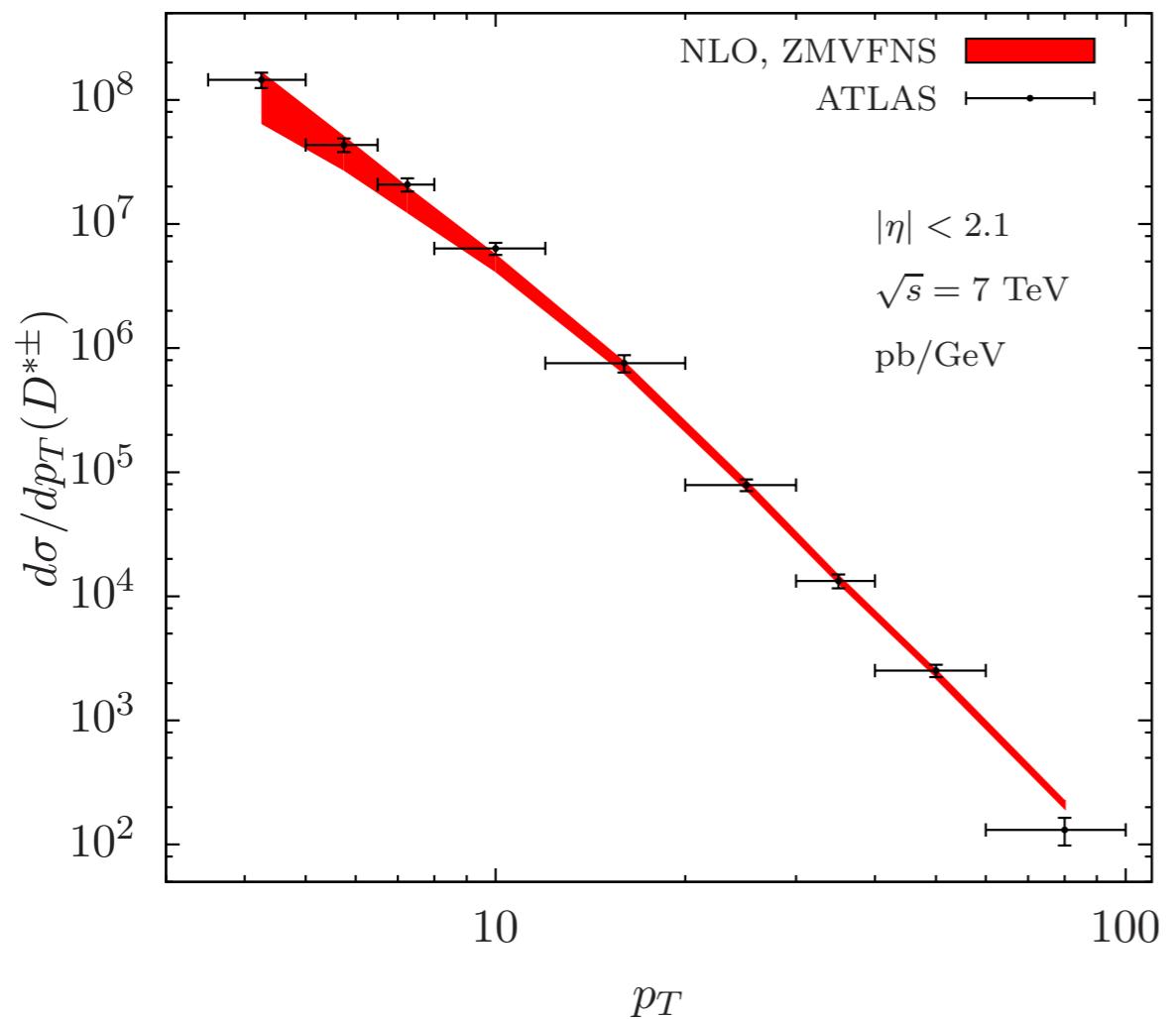


Using FFs from
Kneesch, Kniehl, Kramer, Schienbein '08

- Zero mass variable flavor scheme
- General mass scheme
- fit from $e^+e^- \rightarrow DX$ data

D-meson production $pp \rightarrow HX$

Data taken at the LHC

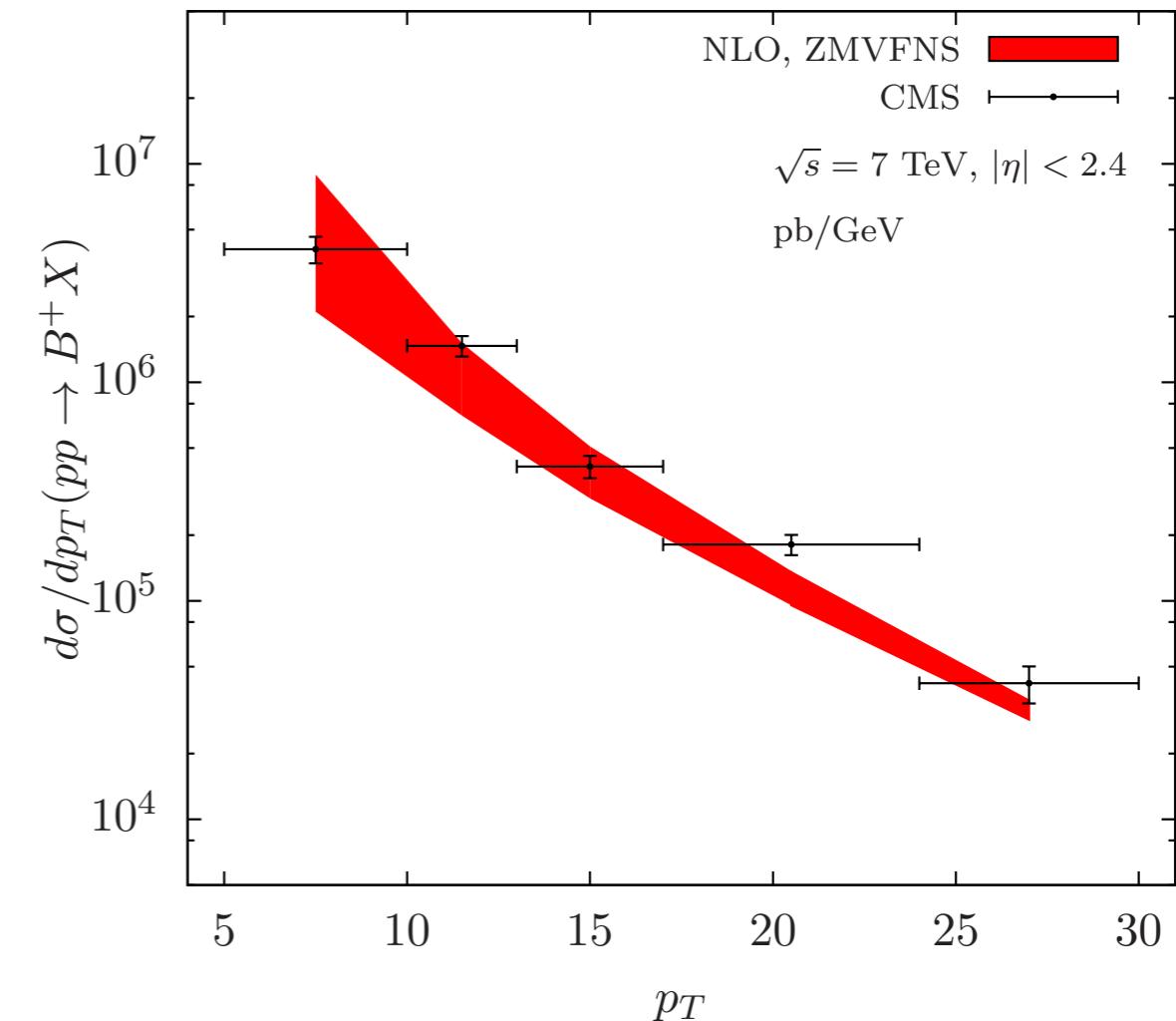
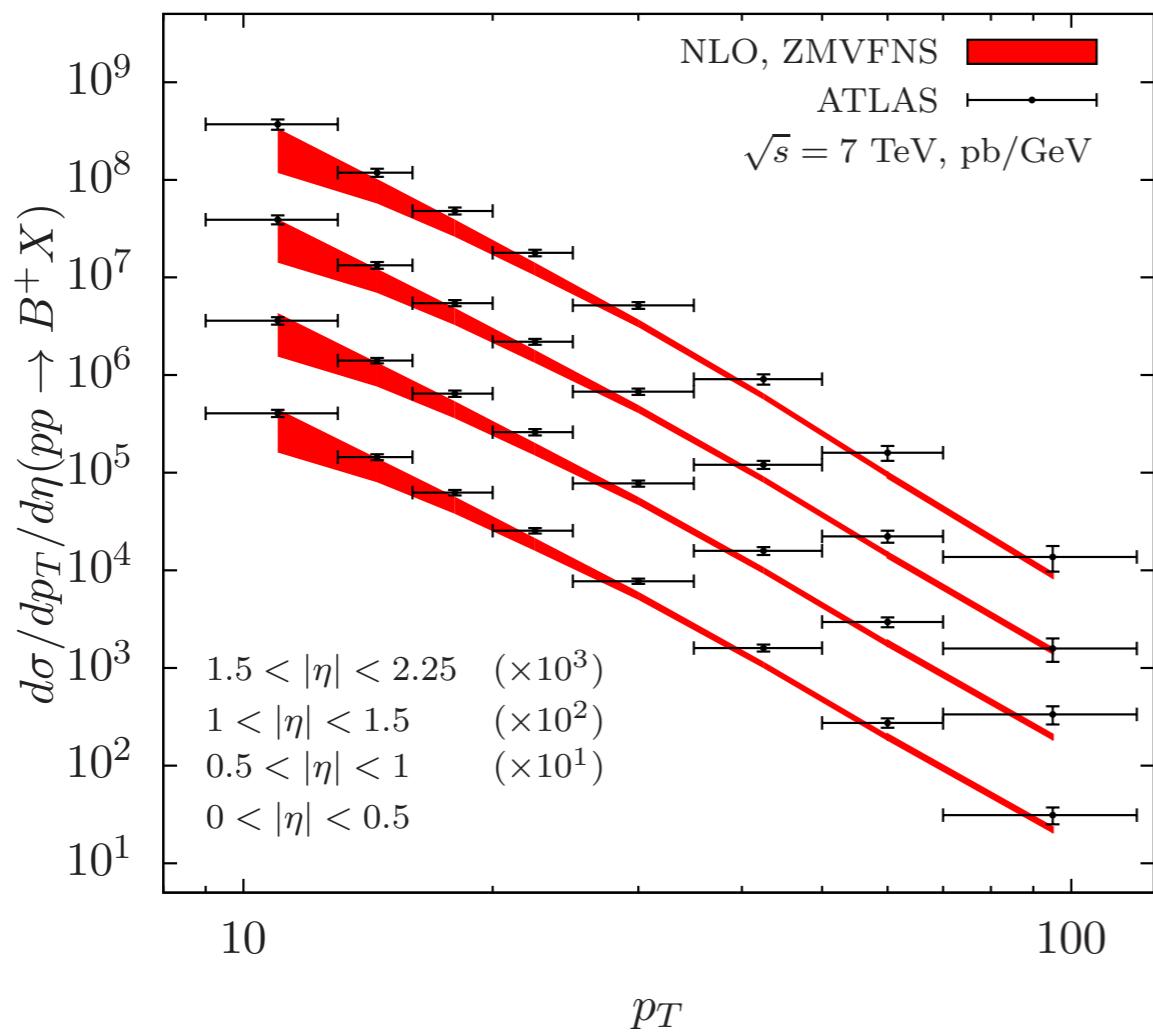


Nucl. Phys. B 907 (2016) 717
CMS-PAS-HIN-16-010

Jäger, Stratmann, Vogelsang '02
ZMVFS Kneesch, Kniehl, Kramer, Schienbein - '08

B-meson production $pp \rightarrow HX$

Data taken at the LHC



JHEP 10 (2013) 042

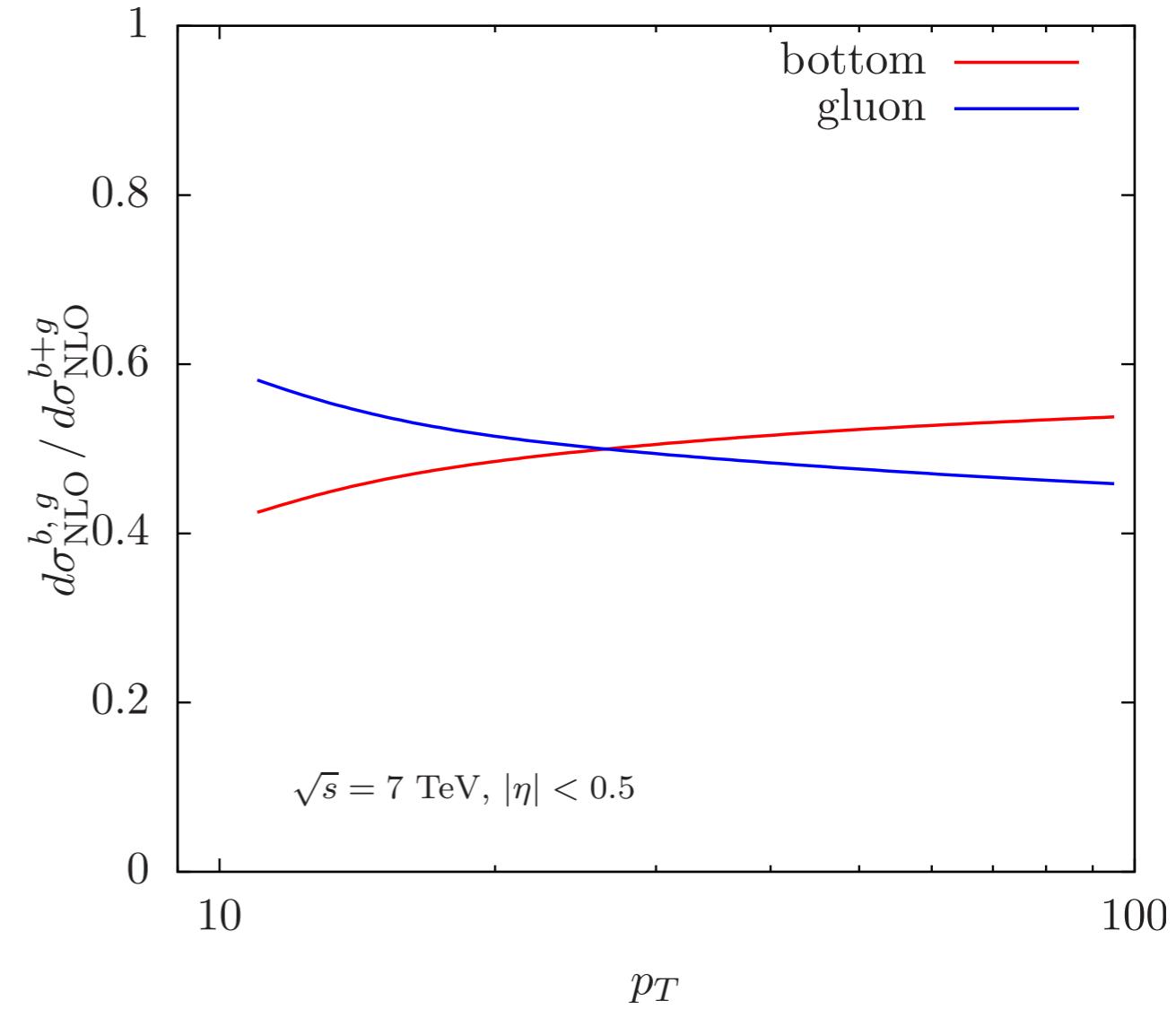
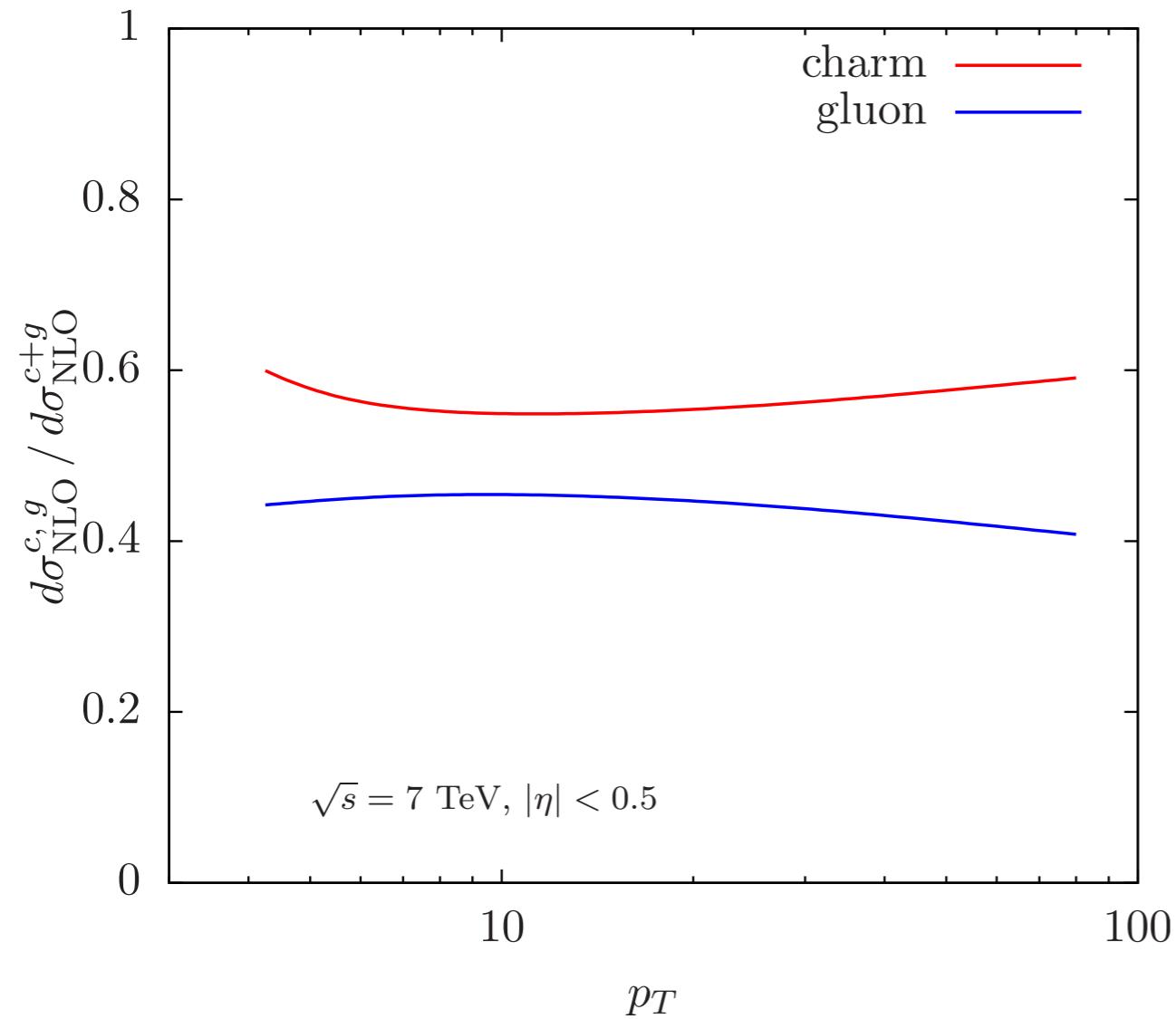
Phys. Rev. Lett 106 (2011) 112001

Jäger, Stratmann, Vogelsang '02

ZMVFS Kniehl, Kramer, Schienbein, Spiesberger - '08

D and B-meson production $pp \rightarrow HX$

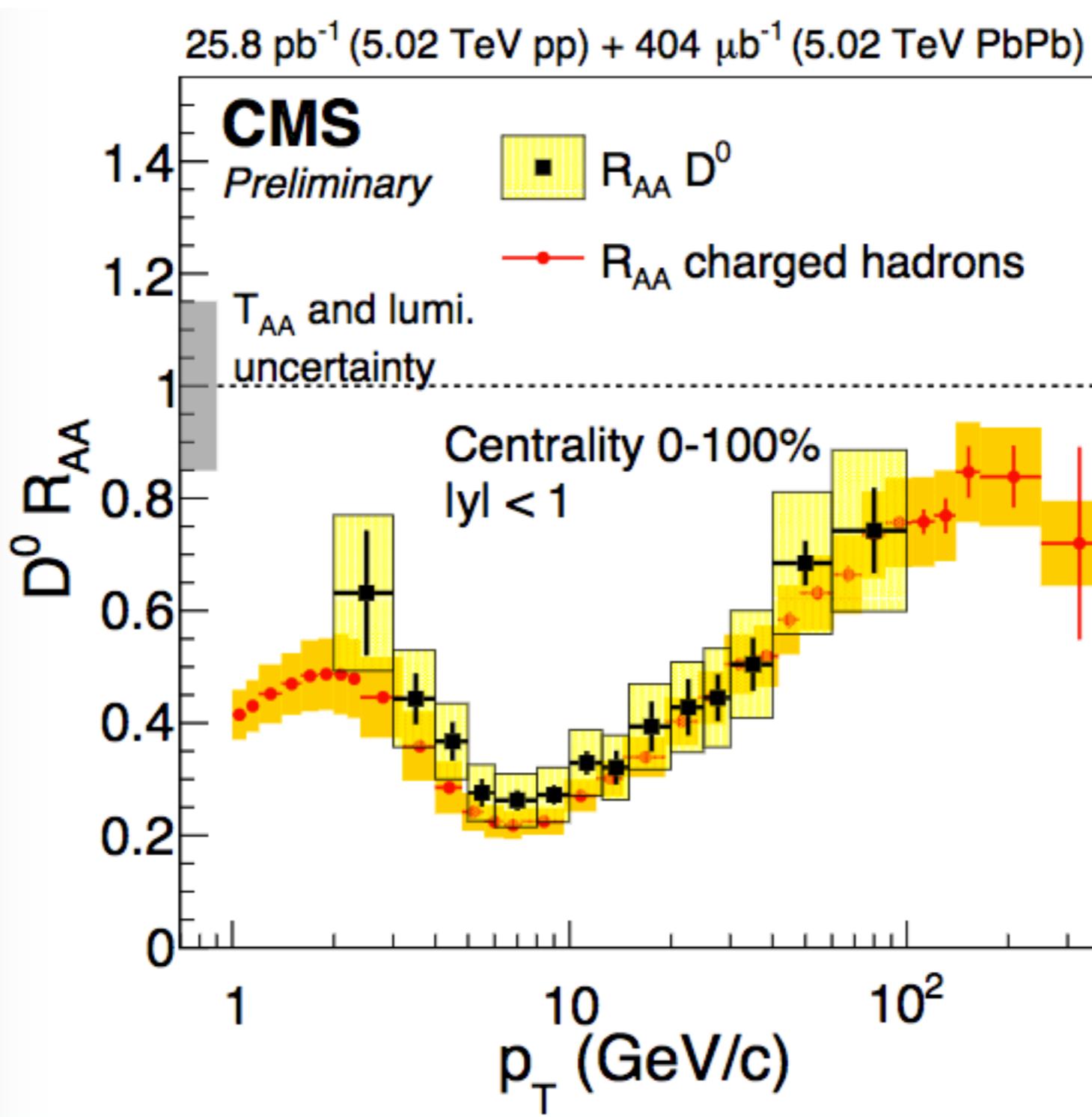
Heavy quark-gluon contribution



Jäger, Stratmann, Vogelsang '02

ZMVFS Kneesch, Kniehl, Kramer, Schienbein - '08
Kniehl, Kramer, Schienbein, Spiesberger - '08

D-meson suppression in PbPb



CMS-PAS-HIN-16-001

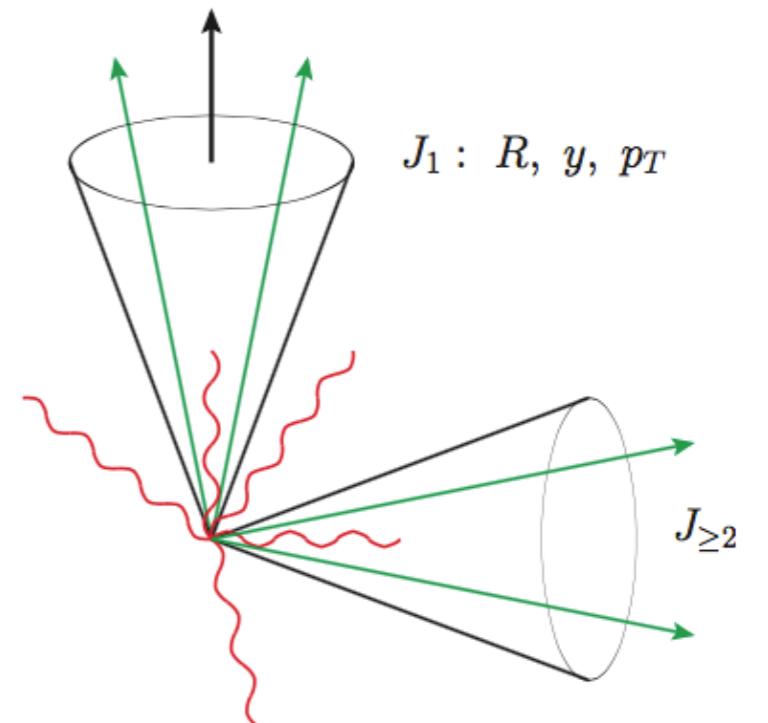
Soft Collinear Effective Theory

- Invaluable tool for high precision LHC phenomenology
- Identify collinear, soft and hard modes at the level of the Lagrangian
- Factorization
- Resummation for multi-scale problems

Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart '00-'02

- Include medium interaction via Glauber gluon exchange
- Full in-medium splitting functions

*Idilbi, Majumder '08, D'Eramo, Liu, Rajagopal '10
Ovanesyan, Vitev '12*



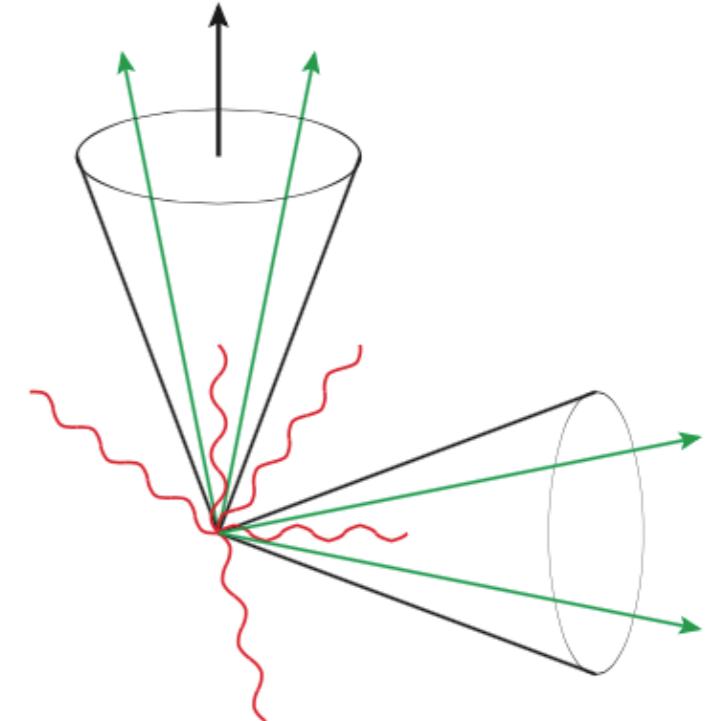
$$d\sigma \sim H(p_T, \mu) J_1(z, \mu) \dots J_N(\mu) S_{n_1 \dots n_N}(\mu)$$

SCET_{M, G}

$$\mathcal{L}_{\text{SCET}_{M,G}} = \mathcal{L}_{\text{SCET}_M} + \mathcal{L}_G(\xi_n, A_n, A_G)$$

Kang, FRVitev '16

- $\mathcal{L}_{\text{SCET}_M} = \bar{\xi}_{n,p'} \left\{ i n \cdot \partial + (\not{P}_\perp + g \not{A}_{n,q}^\perp) W_n \frac{1}{\not{P}} W_n^\dagger (\not{P}_\perp + g \not{A}_{n,q'}^\perp) \right\} \frac{\not{k}}{2} \xi_{n,p} + m \bar{\xi}_{n,p'} \left[(\not{P}_\perp + g \not{A}_{n,q}^\perp), W_n \frac{1}{\not{P}} W_n^\dagger \right] \frac{\not{k}}{2} \xi_{n,p} - m^2 \bar{\xi}_{n,p'} W_n \frac{1}{\not{P}} W_n^\dagger \frac{\not{k}}{2} \xi_{n,p}$



Leibovich, Ligeti, Wise '03

- $\mathcal{L}_G(\xi_n, A_n, A_G) = \sum_{p,p'} e^{-i(p-p')x} \left(\bar{\xi}_{n,p'} \Gamma_{q\bar{q}A_G}^{\mu,a} \frac{\not{k}}{2} \xi_{n,p} - i \Gamma_{ggA_G}^{\mu\nu\lambda,abc} (A_{n,p'}^c)_\lambda (A_{n,p}^b)_\nu \right) A_{G\mu,a}(x)$

Ovanesyan, Vitev '12

Feynman rules for interaction with the medium
do not depend on the mass to leading-power!

Bauer, Fleming, Luke, Pirjol, Rothstein, Stewart '00-'02
Idilbi, Majumder '08, D'Eramo, Liu, Rajagopal '10

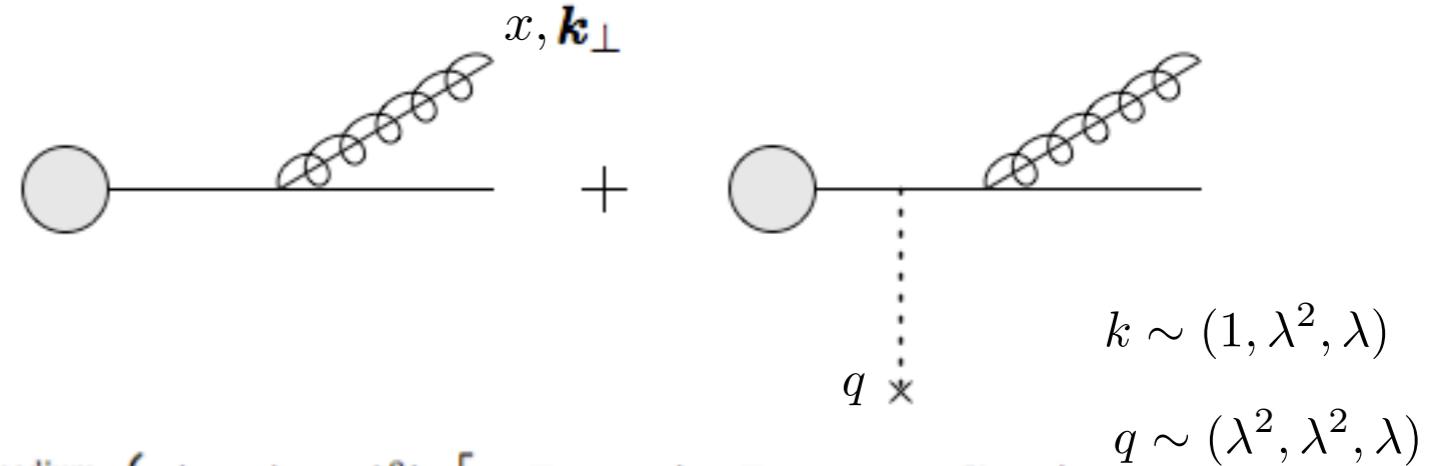
$$\frac{p}{\frac{q_1}{\vdots}} = i v(q_{1\perp}) (b_1)_R (b_1)_{T_i} \frac{\not{k}}{2}$$

SCET_{M, G} splitting kernels

Basic ingredients for the calculation
of the modification in AA collisions:

Final state - massive

- medium: e.g.



$$\begin{aligned} \left(\frac{dN}{dxd^2\mathbf{k}_\perp} \right)_{Q \rightarrow Qg} &= \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp} \left\{ \left(\frac{1 + (1-x)^2}{x} \right) \left[\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \cdot \left(\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} - \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} \right) \right. \right. \\ &\times (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} \cdot \left(2 \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} - \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} - \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) \\ &+ \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \cdot \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} (1 - \cos[(\Omega_2 - \Omega_3)\Delta z]) + \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} \cdot \left(\frac{\mathbf{D}_\perp}{\mathbf{D}_\perp^2 + \nu^2} - \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} \right) (1 - \cos[\Omega_4 \Delta z]) \\ &- \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} \cdot \frac{\mathbf{D}_\perp}{\mathbf{D}_\perp^2 + \nu^2} (1 - \cos[\Omega_5 \Delta z]) + \frac{1}{N_c^2} \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \cdot \left(\frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} - \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) \Big], \\ &+ x^3 m^2 \left[\frac{1}{\mathbf{B}_\perp^2 + \nu^2} \cdot \left(\frac{1}{\mathbf{B}_\perp^2 + \nu^2} - \frac{1}{\mathbf{C}_\perp^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \dots \right] \Big\} \end{aligned}$$

where: $\nu = xm$, $\mathbf{A}_\perp = \mathbf{k}_\perp$, $\mathbf{B}_\perp = \mathbf{k}_\perp + x\mathbf{q}_\perp$, $\mathbf{C}_\perp = \mathbf{k}_\perp - (1-x)\mathbf{q}_\perp$, $\mathbf{D}_\perp = \mathbf{k}_\perp - \mathbf{q}_\perp$

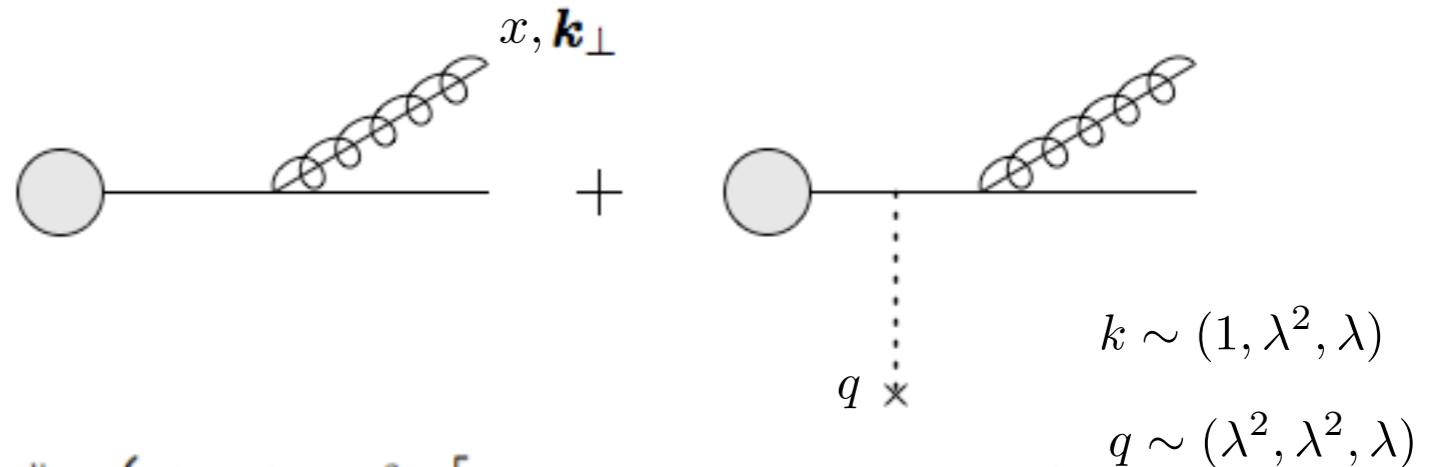
$$\Omega_1 - \Omega_2 = \frac{\mathbf{B}_\perp^2 + \nu^2}{p_0^+ x(1-x)}, \Omega_1 - \Omega_3 = \frac{\mathbf{C}_\perp^2 + \nu^2}{p_0^+ x(1-x)}, \dots$$

SCET_{M, G} splitting kernels

Basic ingredients for the calculation
of the modification in AA collisions:

Final state - massive

- medium: e.g.

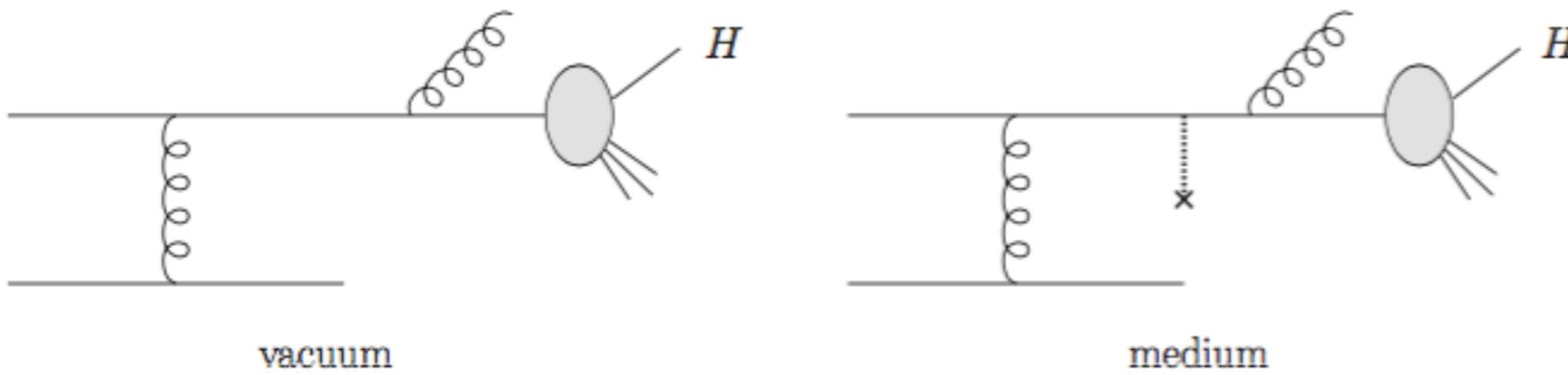


$$\left(\frac{dN}{dxd^2\mathbf{k}_\perp} \right)_{Q \rightarrow Qg} = \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp} \left\{ \left(\frac{1 + (1-x)^2}{x} \right) \left[\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \cdot \left(\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} - \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} \right) \right. \right. \\ \times (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} \cdot \left(2 \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} - \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} - \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) \\ + \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \cdot \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2 + \nu^2} (1 - \cos[(\Omega_2 - \Omega_3)\Delta z]) + \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} \cdot \left(\frac{\mathbf{D}_\perp}{\mathbf{D}_\perp^2 + \nu^2} - \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} \right) (1 - \cos[\Omega_4 \Delta z]) \\ - \frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} \cdot \frac{\mathbf{D}_\perp}{\mathbf{D}_\perp^2 + \nu^2} (1 - \cos[\Omega_5 \Delta z]) + \frac{1}{N_c^2} \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \cdot \left(\frac{\mathbf{A}_\perp}{\mathbf{A}_\perp^2 + \nu^2} - \frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) \Big], \\ \left. + x^3 m^2 \left[\frac{1}{\mathbf{B}_\perp^2 + \nu^2} \cdot \left(\frac{1}{\mathbf{B}_\perp^2 + \nu^2} - \frac{1}{\mathbf{C}_\perp^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \dots \right] \right\}$$

Soft emission limit is consistent with Gyulassy, Levai, Vitev '00
Djordjevic, Gyulassy '03

Application of in-medium splitting functions

- SCET is an important tool to understand the structure of cross sections, e.g. jets Kang, FR, Vitev '16, '16
- Hadron cross sections



$$\sum_j \hat{\sigma}_i^{(0)} \otimes \mathcal{P}_{i \rightarrow jk} \otimes D_j^H$$



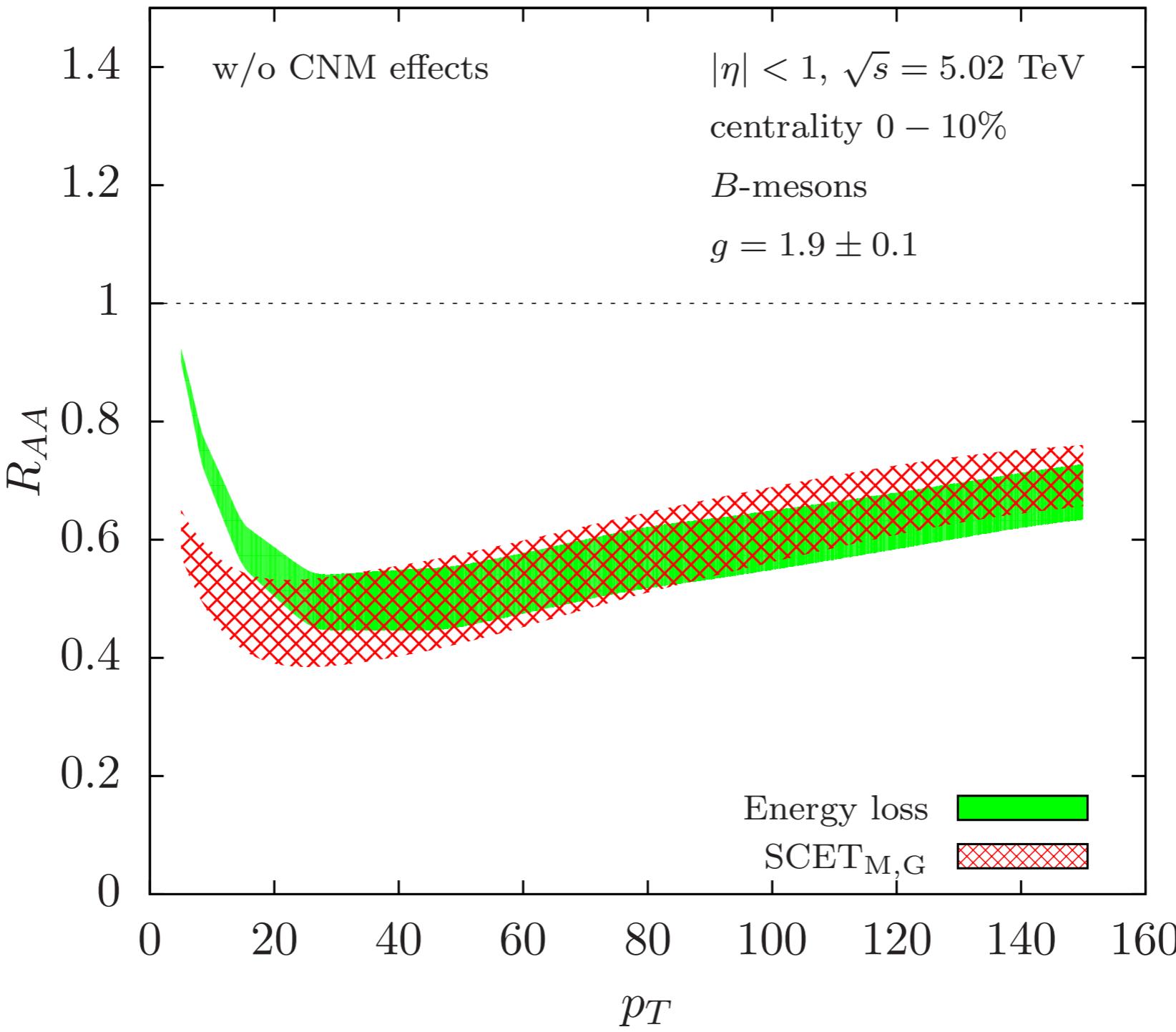
vacuum + medium splitting function

$$\mathcal{P}_{i \rightarrow jk}(z, \mu) = \mathcal{P}_{i \rightarrow jk}^{\text{vac}}(z, \mu) + \mathcal{P}_{i \rightarrow jk}^{\text{med}}(z, \mu)$$

$$d\sigma_{\text{PbPb}}^H = d\sigma_{pp}^{H,\text{NLO}} + d\sigma_{\text{PbPb}}^{H,\text{med}}$$

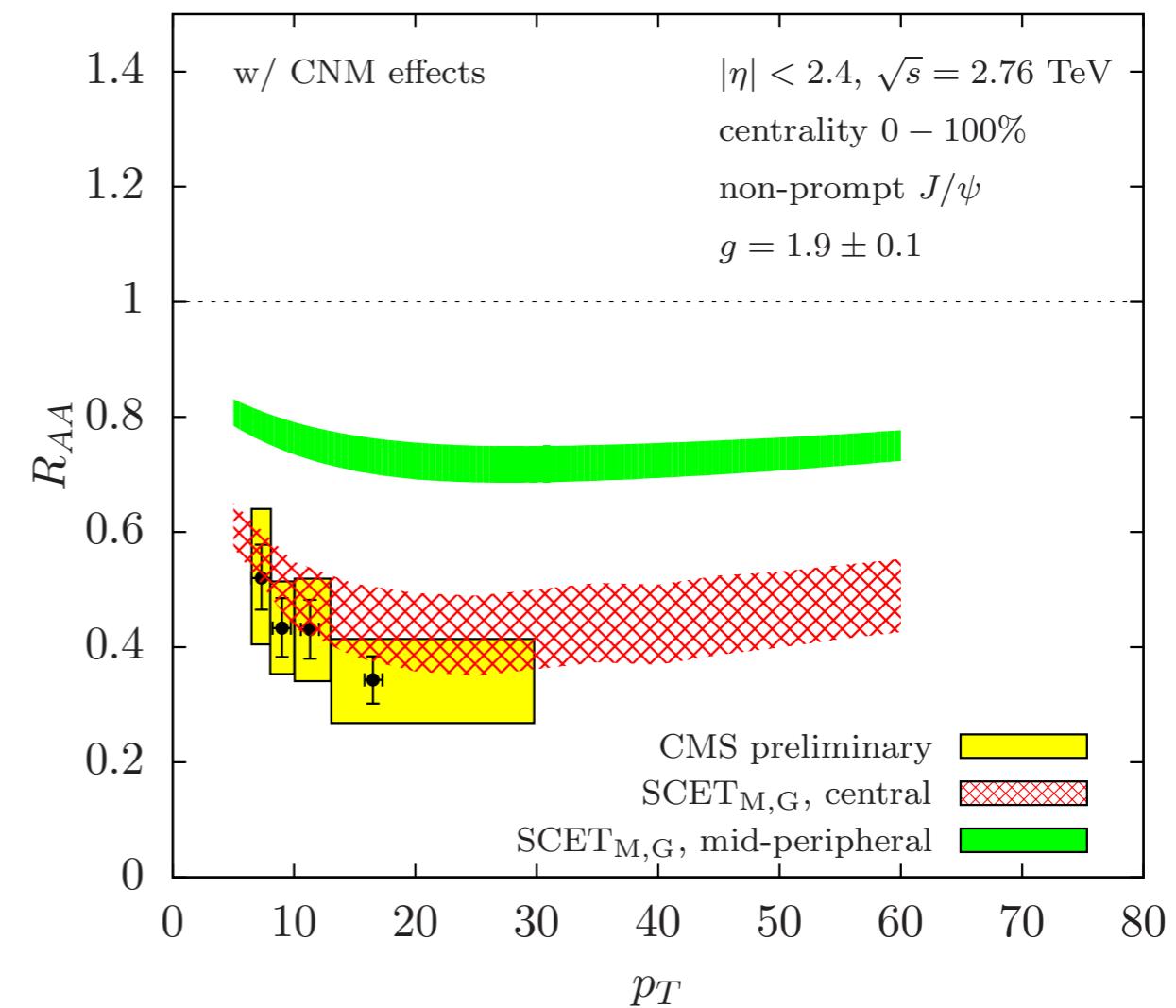
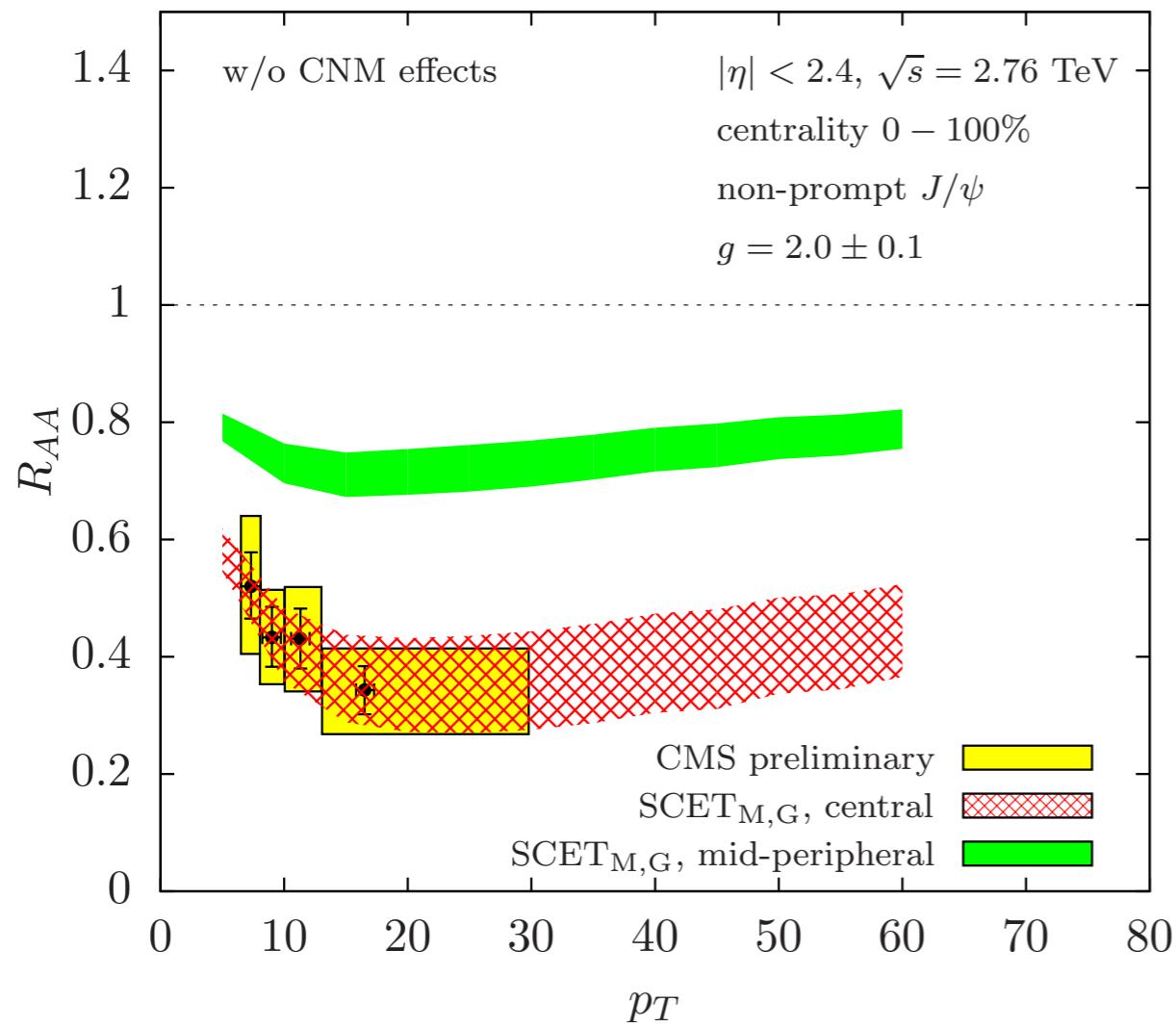
Traditional energy loss and SCET_{M,G}

B-mesons



Comparison to LHC data

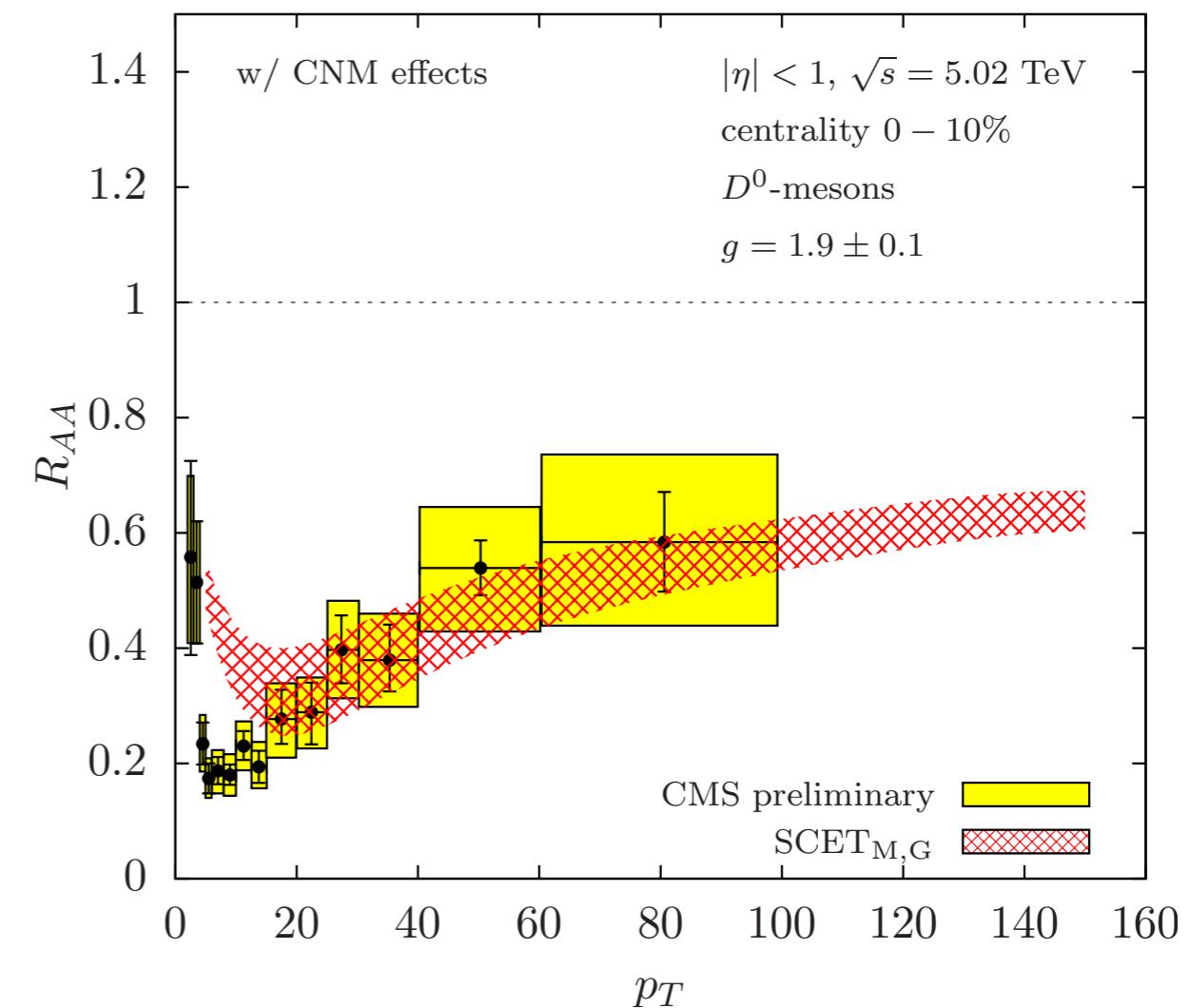
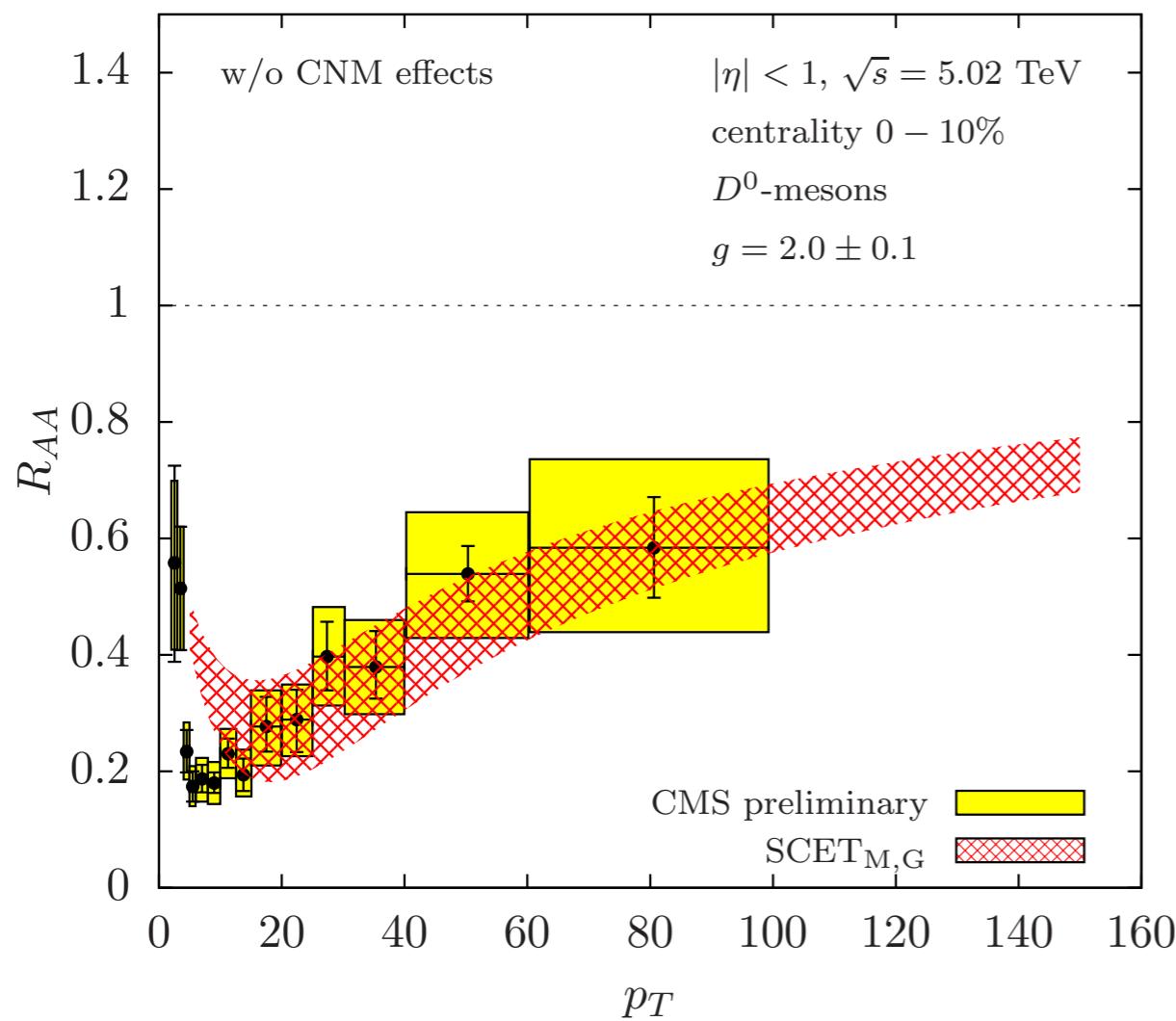
non-prompt J/ψ , from B-meson decays
minimum bias data



CNM here: Cold nuclear matter energy loss, Cronin and isospin effects

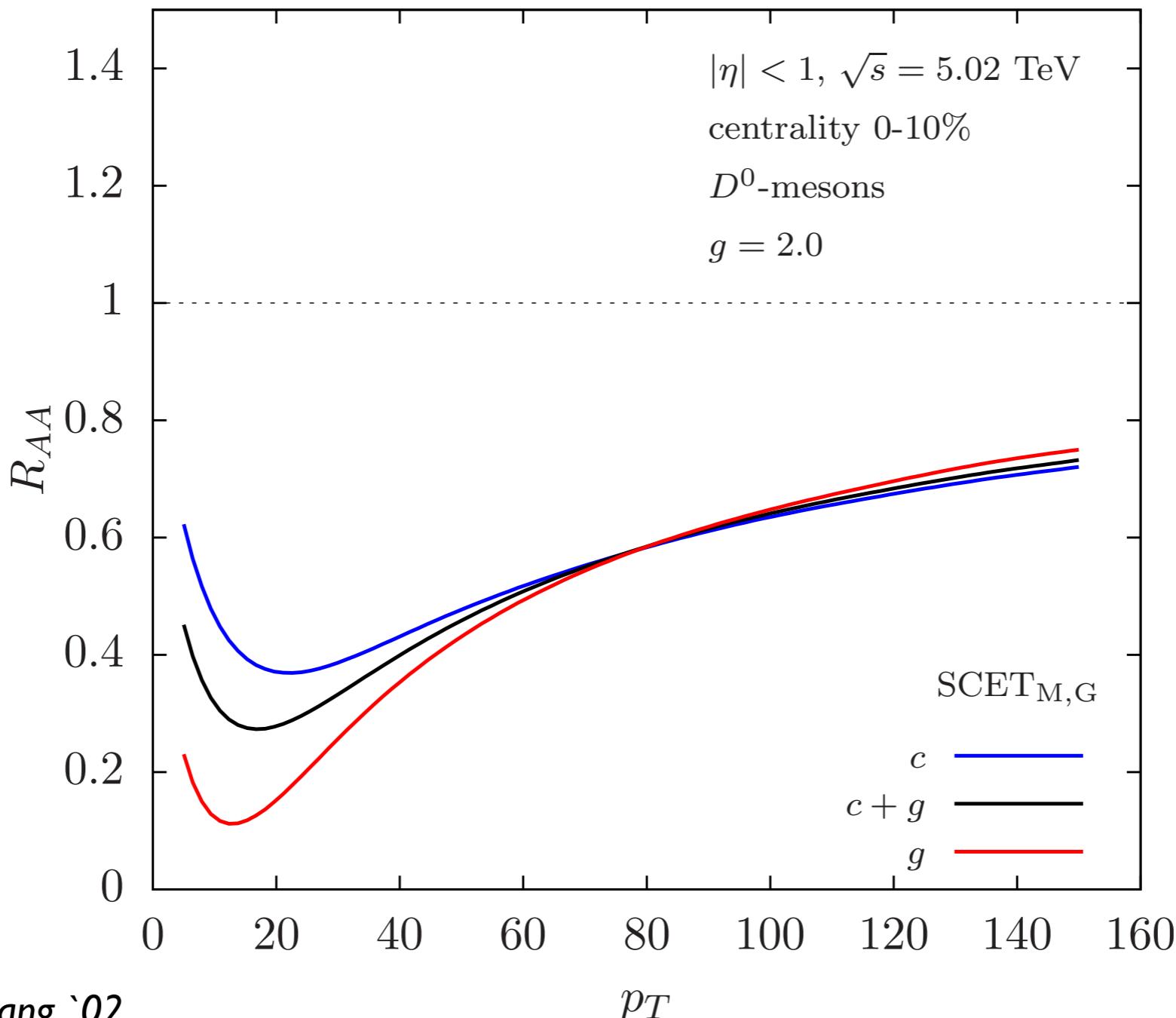
Comparison to LHC data

D-mesons



D-meson R_{AA}

Heavy quark-gluon suppression



Jäger, Stratmann, Vogelsang '02

ZMVFS Kneesch, Kniehl, Kramer, Schienbein - '08
 Kniehl, Kramer, Schienbein, Spiesberger - '08

The Jet Fragmentation Function $pp \rightarrow (\text{jeth})X$

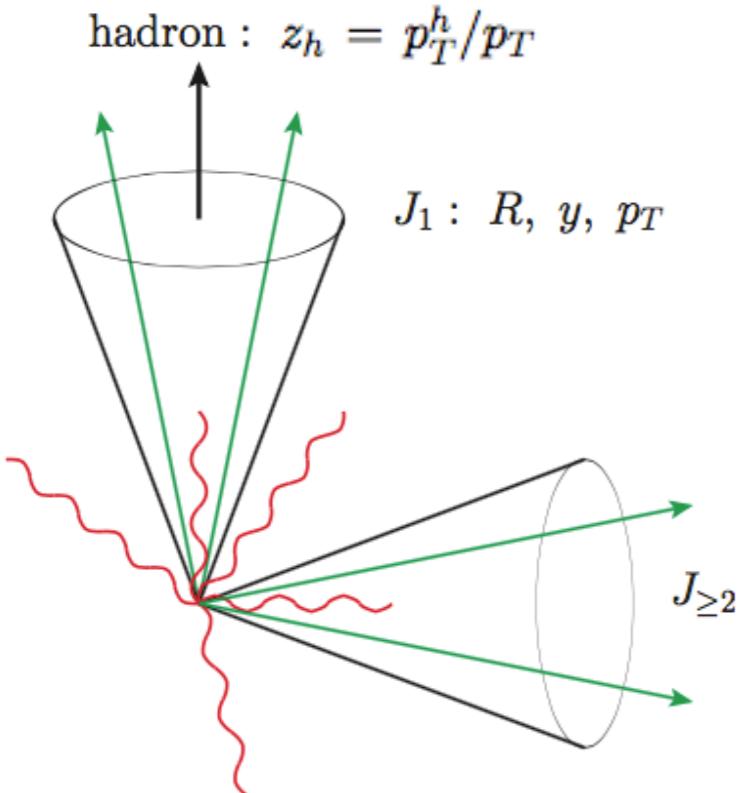
Chien, Kang, FR, Vitev, Xing '15

Kang, FR, Vitev '16, '16

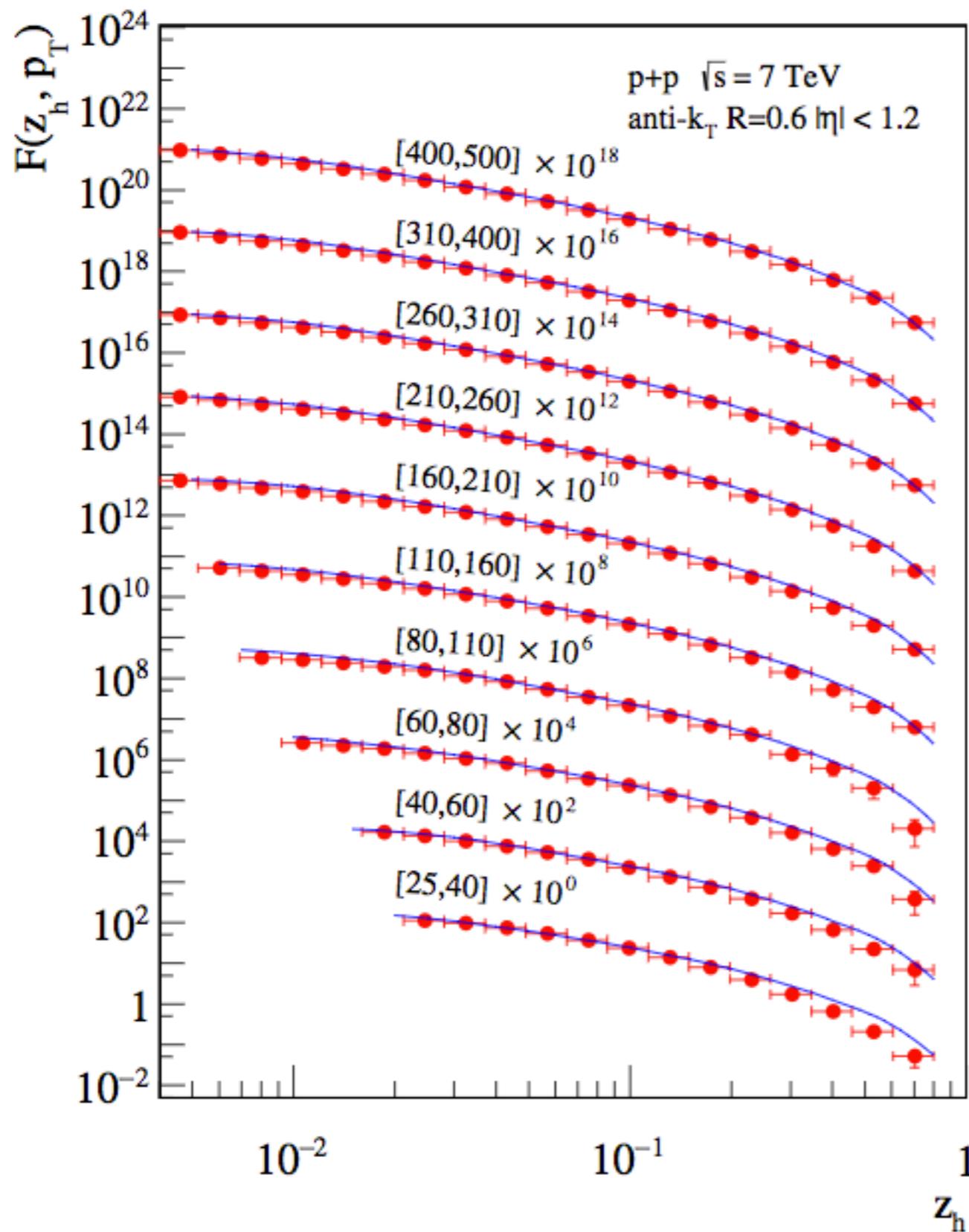
$$F(z_h, p_T) = \frac{d\sigma^{pp \rightarrow (\text{jeth})X}}{dp_T d\eta dz_h} / \frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta}$$

where

$$\frac{d\sigma^{pp \rightarrow (\text{jeth})X}}{dp_T d\eta dz_h} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dv dz} G_c^h(z_c, z_h, \omega_J, \mu)$$



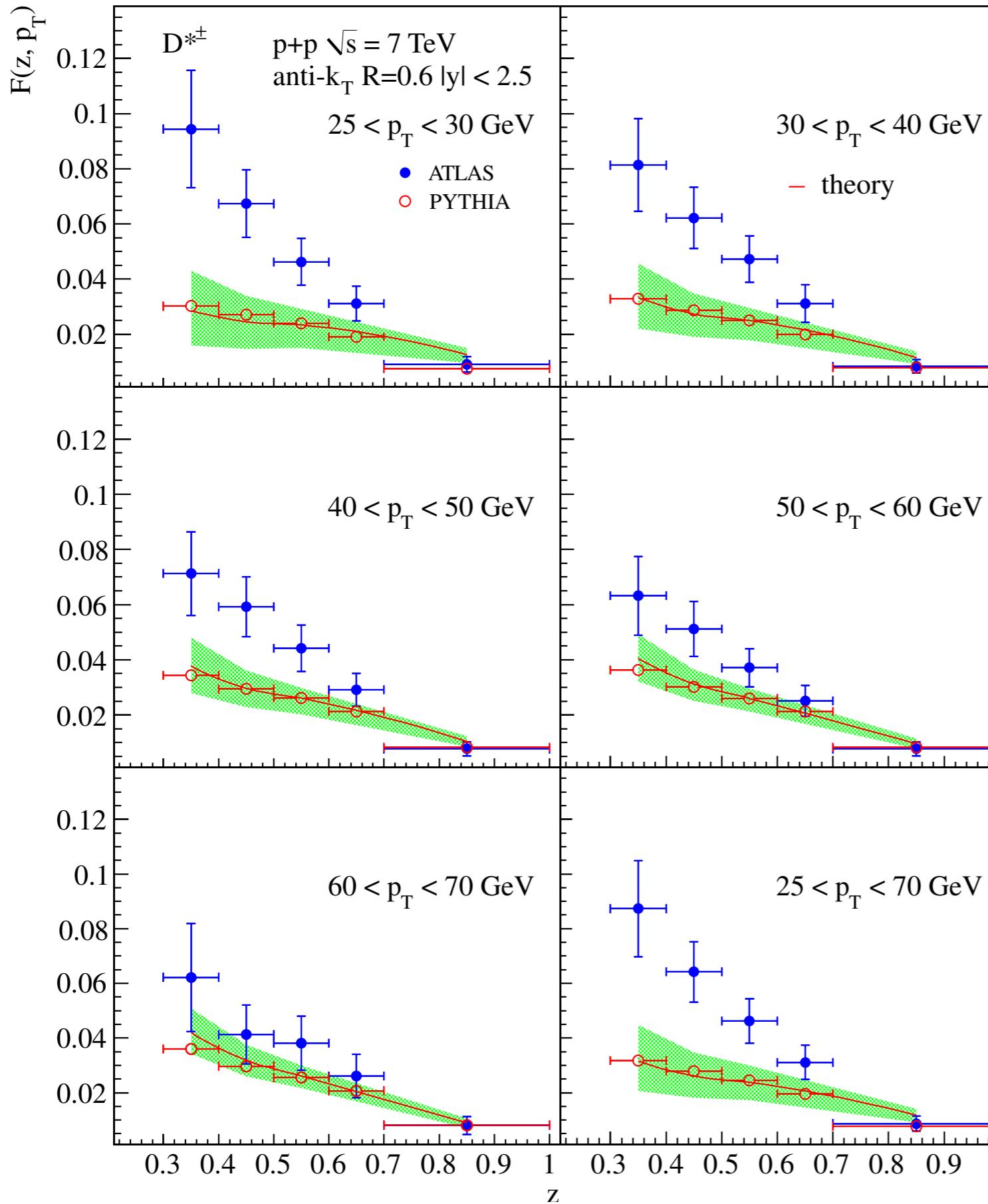
“semi-inclusive fragmenting jet function” in SCET
resummation of $\ln R$, i.e. NLO + NLL_R



Comparison to ATLAS data
at $\sqrt{s} = 7 \text{ TeV}$

Light charged hadrons $h = \pi + K + p$

Using DSS FFs
de Florian, Sassot, Stratmann - '07

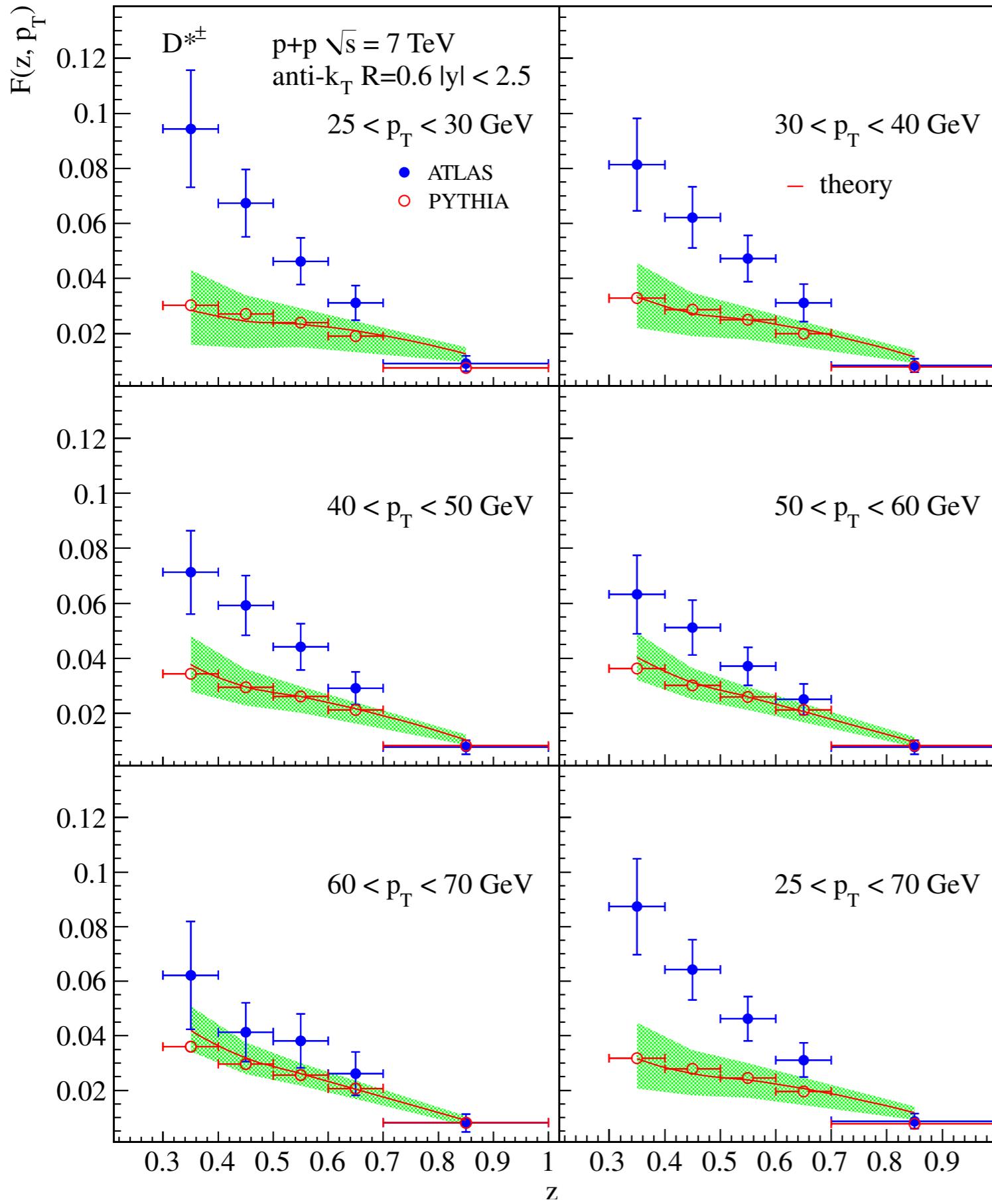


D-meson
jet fragmentation function

Comparison to ATLAS data
and PYTHIA simulations
at $\sqrt{s} = 7$ TeV

Using FFs from
Kneesch, Kniehl, Kramer, Schienbein - '08

ZMVNFS, $e^+e^- \rightarrow DX$
 $\mu, \mu_J, \mu_g \gg m_Q$



D-meson
jet fragmentation function

Motivates the need for
global fits of heavy meson
fragmentation functions!

Outline

- Open heavy flavor production
- Inclusive jet observables
- Conclusions

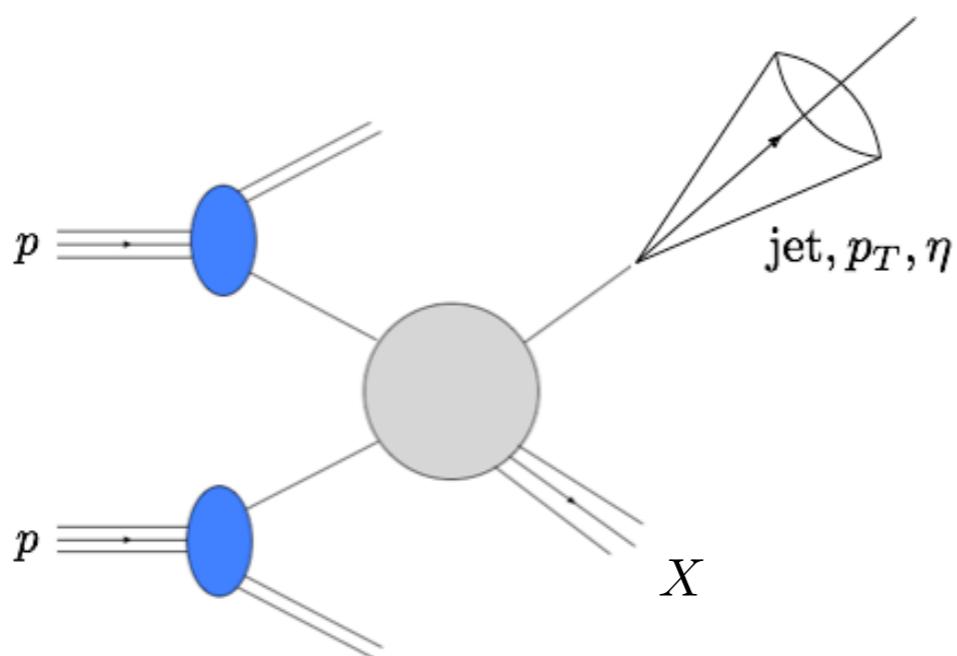
Kang, FR, Vitev '16 arXiv:1609.04908

Kang, FR, Vitev '16, '16 arXiv:1606.06732

Inclusive Jet Production in SCET $pp \rightarrow \text{jet}X$

Kang, FR, Vitev '16, '16

$$\frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dv dz} J_c(z_c, \omega_J, \mu)$$



“semi-inclusive jet function” in SCET

see also:

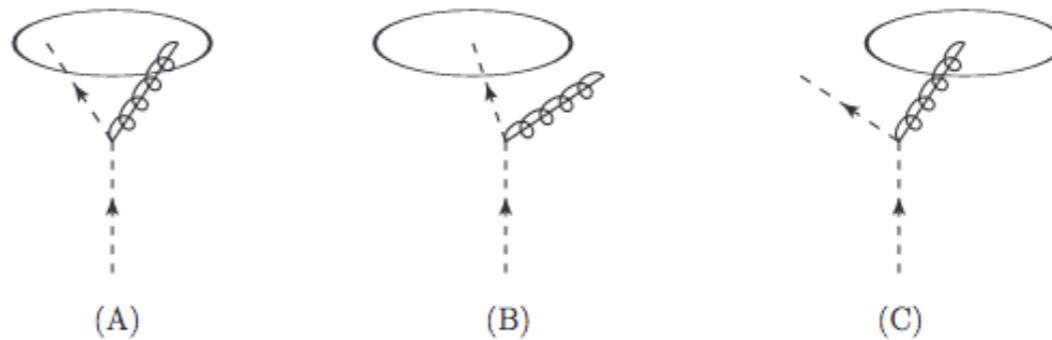
Jäger, Stratmann, Vogelsang '04, Mukherjee, Vogelsang '12, Kaufmann, Mukherjee, Vogelsang '15, Dasgupta, Dreyer, Salam, Soyez '14, '16

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Definition similar to FFs
but perturbatively calculable:



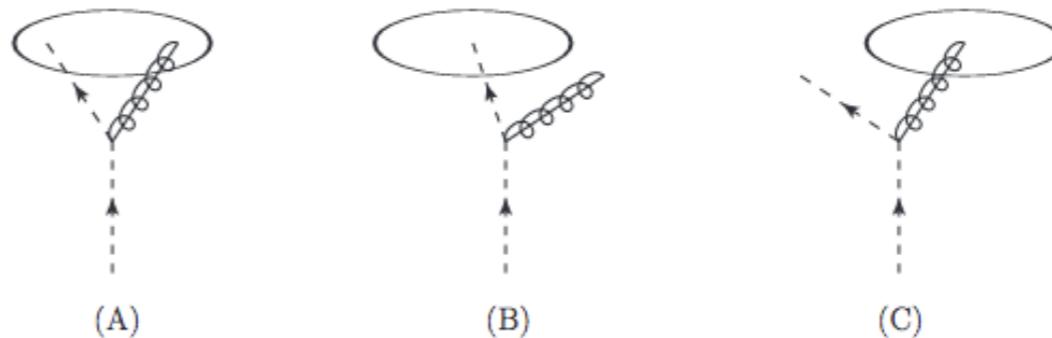
Jet cross section at NLO: $d\sigma \sim \mathcal{A} \ln R + \mathcal{B} + \mathcal{O}(R^2)$

Inclusive Jet Production in SCET $pp \rightarrow \text{jet}X$

Kang, FRVitev '16, '16

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Definition similar to FFs
but perturbatively calculable:



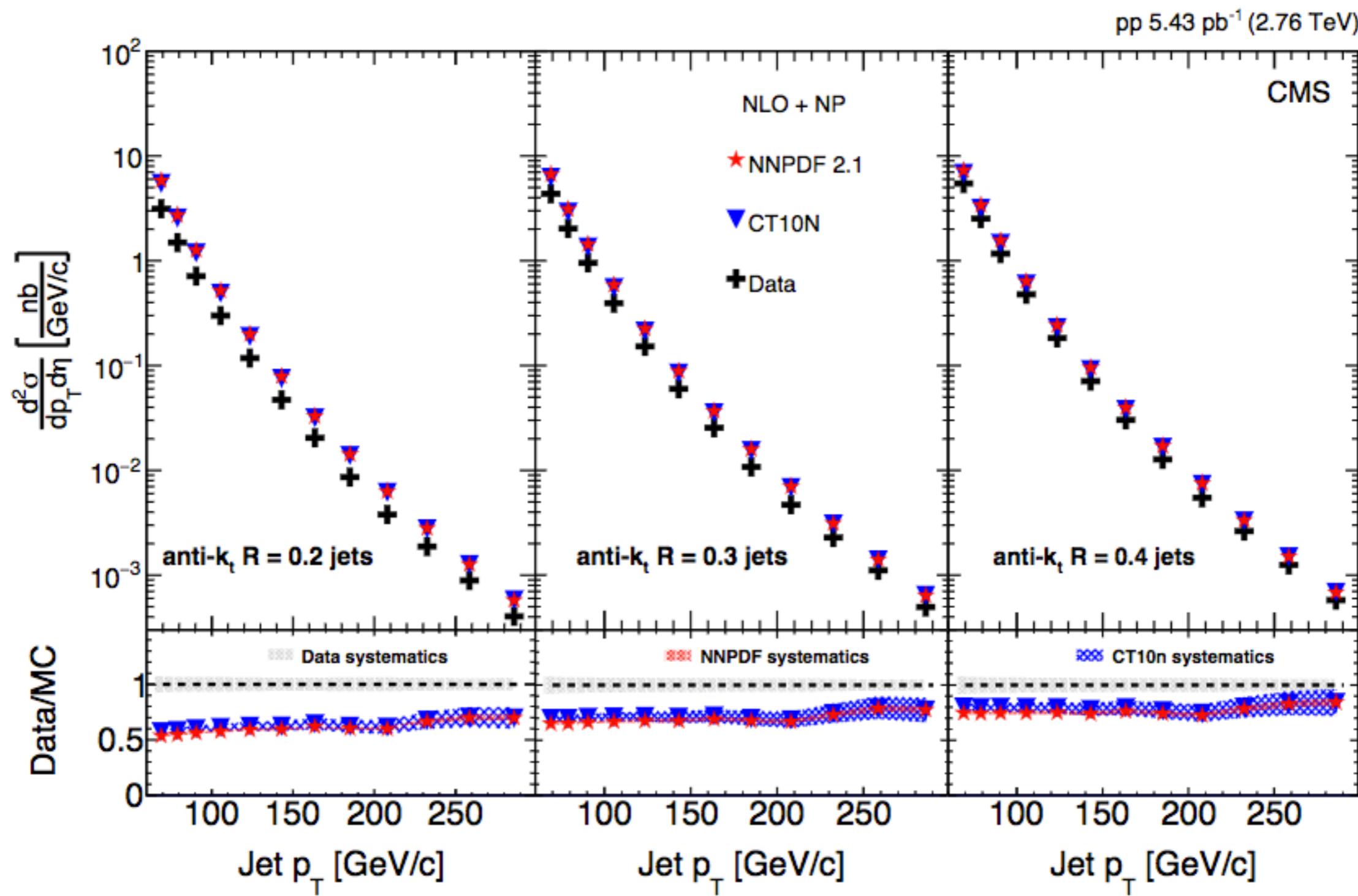
Follows standard timelike DGLAP

$$\mu \frac{d}{d\mu} J_i(z, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}, \mu\right) J_j(z', \omega_J, \mu)$$

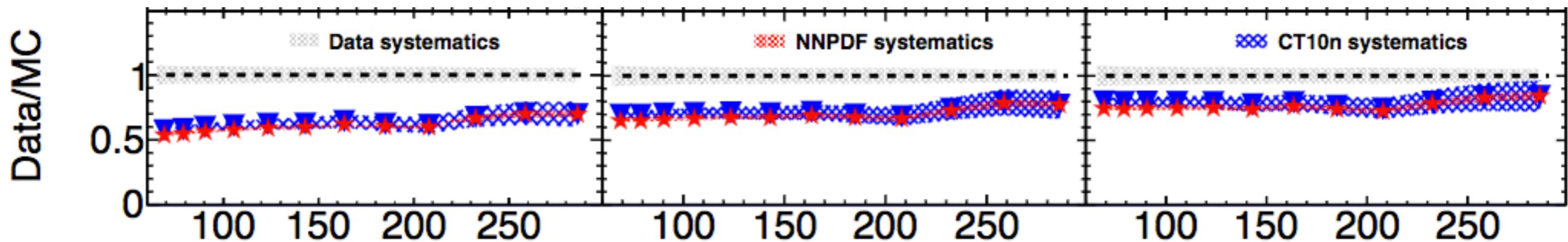
→ resummation of single logarithms $\ln R$, i.e. NLO + NLL_R

Especially relevant for heavy-ion phenomenology!

Inclusive Jet Production in SCET $pp \rightarrow \text{jet}X$

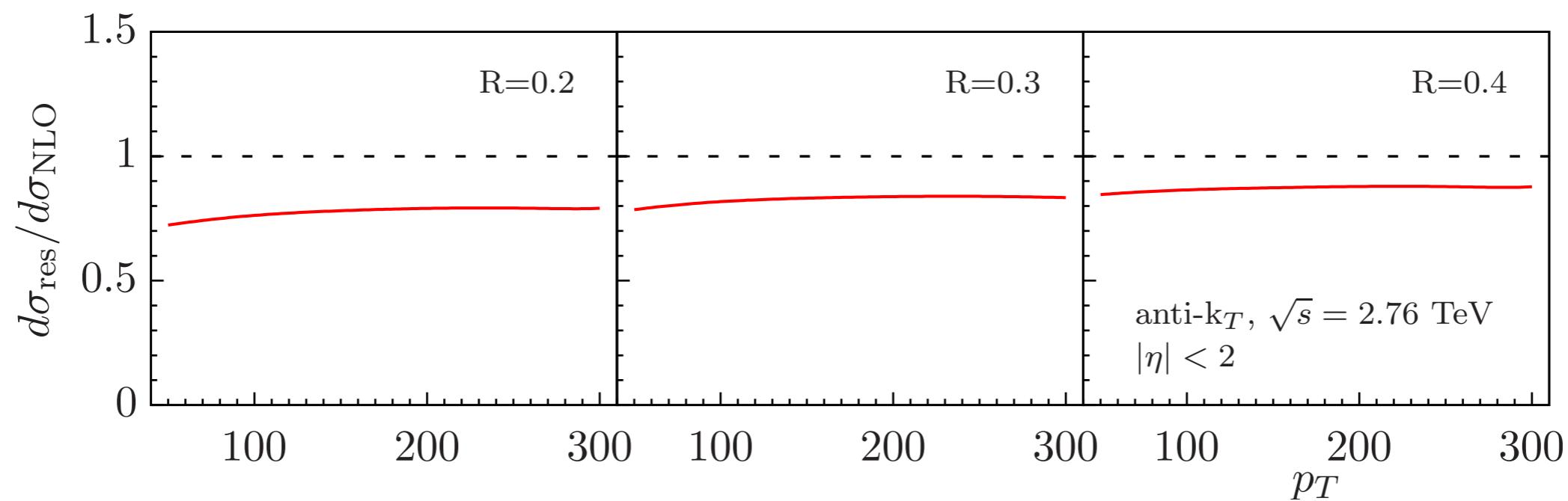
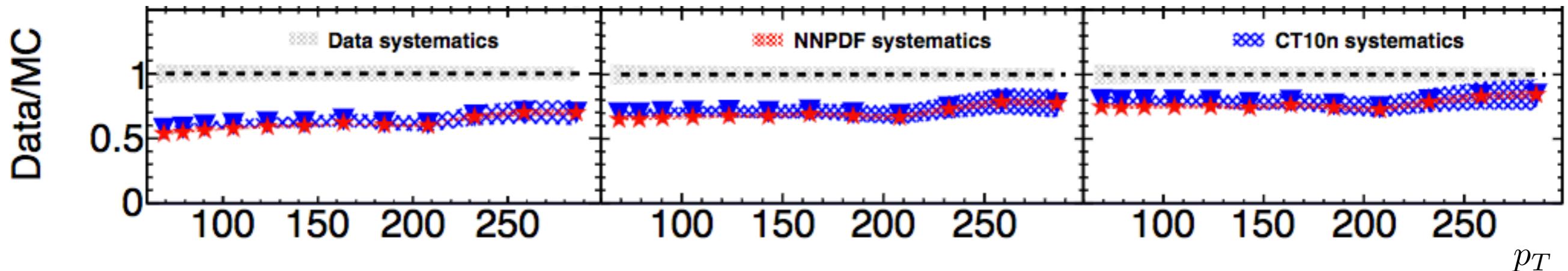


Inclusive Jet Production in SCET $pp \rightarrow \text{jet}X$



Inclusive Jet Production in SCET $pp \rightarrow \text{jet}X$

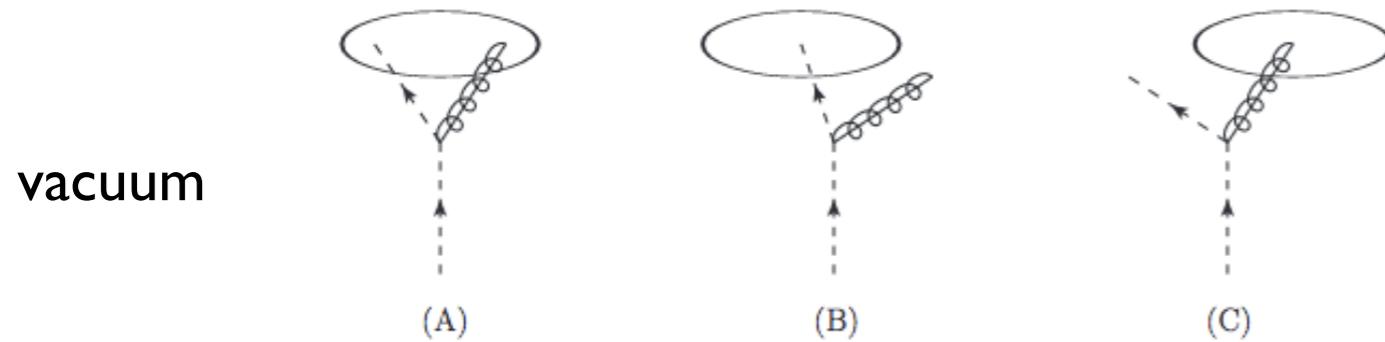
Kang, FR, Vitev '16, '16



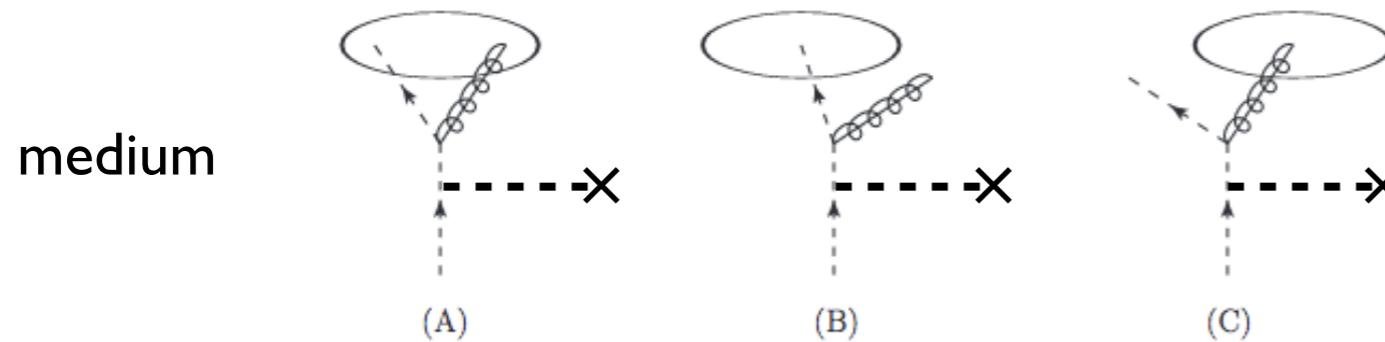
Inclusive Jet Production in SCET PbPb \rightarrow jetX

Kang, FRVitev '16, '16

$$\frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dv dz} J_c(z_c, \omega_J, \mu)$$



$$J_c^{\text{vac}}(z, \omega_J) + J_c^{\text{med}}(z, \omega_J)$$

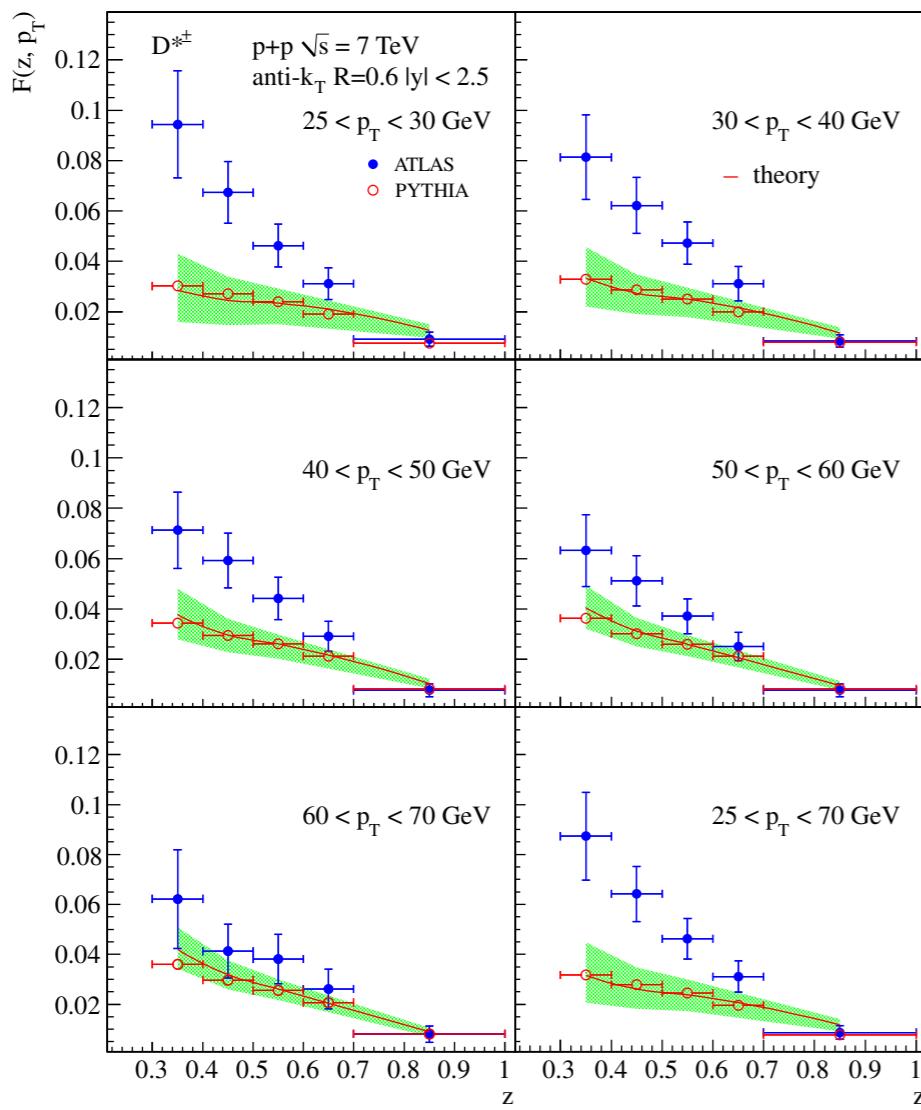


Jet substructure within SCET

e.g. the jet fragmentation function

Kang, FR, Vitev '16, '16
Chien, Kang, FR, Vitev, Xing '15

$$\frac{d\sigma^{pp \rightarrow (\text{jeth})X}}{dp_T d\eta dz_h} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dv dz} \mathcal{G}_c^h(z_c, z_h, \omega_J, \mu)$$



semi-inclusive fragmenting jet function

- More differential probe of the fragmentation process
- study modification in the medium

Outline

- Open heavy flavor production
- Inclusive jet observables
- Conclusions

Kang, FR, Vitev '16 arXiv:1609.04908

Kang, FR, Vitev '16, '16 arXiv:1606.06732

Conclusions

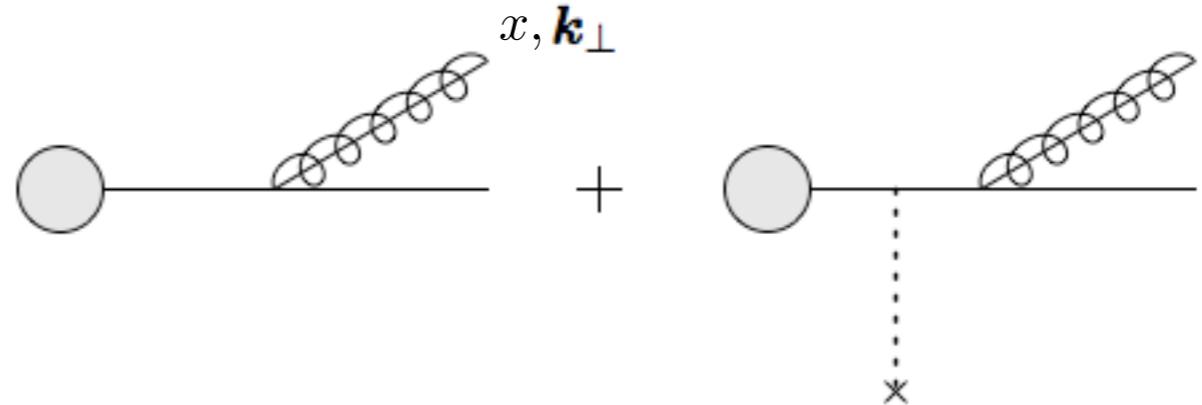
- Soft Collinear Effective Theory is an invaluable tool for heavy-ion physics
- New massive in-medium splitting functions
- Good description of D- and B-meson R_{AA}
- Results motivate global fits of heavy meson FFs
- $\ln R$ resummation for inclusive jet spectra
- Consistent treatment at NLO in QCD for inclusive hadron and jet observables

... please stay tuned!

SCET_{M, G} splitting kernels

Basic ingredients for the calculation
of the modification in AA collisions:

Final state - massive



- medium: Soft gluon approximation

$$x \left(\frac{dN}{dx} \right)_{Q \rightarrow Qg} = \frac{\alpha_s}{\pi^2} C_F \int d\Delta z \frac{1}{\lambda_g(z)} \int d^2 \mathbf{k}_\perp d^2 \mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2 \mathbf{q}_\perp} \frac{2 \mathbf{k}_\perp \cdot \mathbf{q}_\perp}{[\mathbf{k}_\perp^2 + x^2 m^2][(k_\perp - q_\perp)^2 + x^2 m^2]} \left[1 - \cos \frac{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2}{xp_0^+} \Delta z \right]$$

Soft gluon limit is consistent with
Gyulassy, Levai, Vitev '00
Djordjevic, Gyulassy '03

Numerical results

