Transport coefficients from energy loss studies in an expanding QGP

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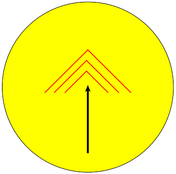
- Characterize events in terms of an observable: Energy loss that can be quantified in terms of missing p_t
- **Message is that medium cannot be described with a single value of η/s nor of $\hat{q}$ but values for these coefficients can be obtained for events classified in terms of a given ΔE**

Fast parton moving in medium



$$J^\nu(x) = \langle \frac{dE}{dx} \rangle \delta(\mathbf{x} - \mathbf{v}t) v^\nu$$

- $v^\nu \equiv (1, \mathbf{v})$, $\mathbf{v}$ is the corresponding parton velocity
- $\langle \frac{dE}{dx} \rangle$ is the average energy-loss per unit length

R. B. Neufeld, T. Renk. Phys. Rev. C. **82***. 044903 (2010).*

Energy-momentum deposited described by linear, small viscosity hydro

Medium's total energy-momentum $T^{\mu\nu}$ form initial $T_0^{\mu\nu}$ perturbed by fast parton

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$$

Small deviations from equilibrium hydro equations

$$\begin{aligned} \partial_\mu T_0^{\mu\nu} &= 0 \\ \partial_\mu \delta T^{\mu\nu} &= J^\nu \end{aligned}$$

Express tensor components in terms of energy density $\delta\epsilon$ and momentum density $\mathbf{g}$ transferred by fast parton to the medium

$$\begin{aligned} \delta T_0^{00} &\equiv \delta\epsilon \\ \delta T^{0i} &\equiv g^i \\ \delta T^{ij} &\equiv c_s^2 \delta\epsilon \delta^{ij} - \frac{3}{4}\Gamma_s (\partial^i g^j + \partial^j g^i - \frac{2}{3} \nabla \cdot \mathbf{g} \delta^{ij}) \\ \Gamma_s &\equiv \frac{4\eta}{3\epsilon_0 (1 + c_s^2)} = 4\eta/3sT \quad \text{sound attenuation length} \end{aligned}$$

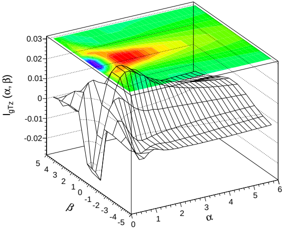
Initial medium's energy density and temperature: $\epsilon_0, \; T_0$

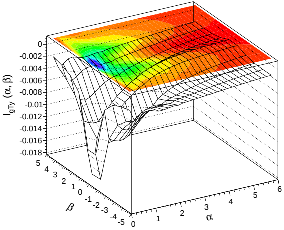
Equations more easily solved in Fourier space

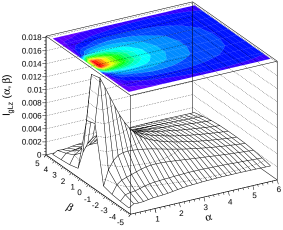
$$\begin{aligned}
\delta\epsilon(\omega,k) &= \frac{ik\mathbf{J}_L + \mathbf{J}^0(i\omega - \Gamma_s k^2)}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2} \\
\mathbf{g}_L(\omega,k) &= \frac{i\omega\mathbf{J}_L + ic_s^2\hat{\mathbf{k}}J^0}{\omega^2 - c_s^2 k^2 + i\Gamma_s \omega k^2} \\
\mathbf{g}_T(\omega,k) &= \frac{i\mathbf{J}_T}{\omega + \frac{3}{4}i\Gamma_s k^2} \\
\mathbf{J}_L &\equiv (\mathbf{J}\cdot\hat{\mathbf{k}})\,\hat{\mathbf{k}} \\
\mathbf{J}_T &\equiv \mathbf{J} - \mathbf{J}_L
\end{aligned}$$

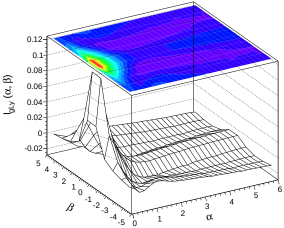
Solutions in coordinate space $[\alpha = |z - vt| / \left(\frac{3\Gamma_s}{2v}\right), \; \beta = x_T / \left(\frac{3\Gamma_s}{2v}\right)]$

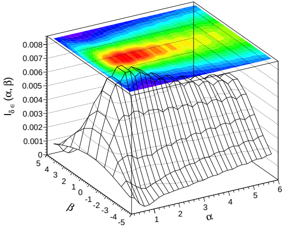
A. A., I. Dominguez, M. E. Tejeda-Yeomans, Phys. Rev. C **88***, 025203 (2013).*











Particle multiplicity

Particle momentum distribution (Cooper-Frye)

$$E\frac{dN}{d^3p} = \frac{1}{(2\pi)^3}\int d\Sigma_\mu p^\mu [f(p\cdot u) - f(p_0)]$$

Constant time freeze-out hyper surface: $d\Sigma_\mu p^\mu = E\ d^3r$.

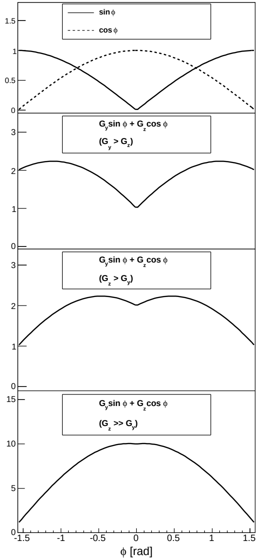
Equilibrium distribution (Boltzmann): $f_0 = e^{-p_T/T_0}$.

Distribution generated by deposited energy-momentum:

$$f(x_\perp, p_\perp) - f_0 \simeq \left(\frac{p_T}{T_0\epsilon_0}\right)\left(\frac{\delta\epsilon}{\epsilon_0} + \frac{g_y(x_\perp)\sin\phi + g_z(x_\perp)\cos\phi}{\epsilon_0(1+c_s^2)}\right) e^{-p_T/T_0}$$

Angle between $\mathbf{g}$ and $\hat{\mathbf{z}}$: ϕ

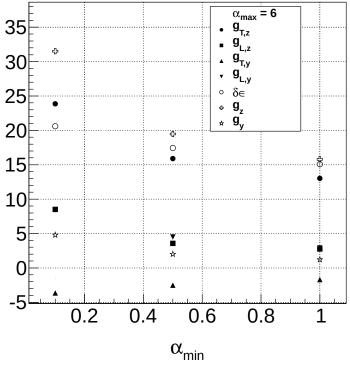
Particle distribution around parton direction of motion depends on the relative strength of g_y (the coefficient of $\sin\phi$) and g_z (the coefficient of $\cos\phi$)



$$\delta\epsilon = \left(\frac{1}{4\pi}\right)\left(\frac{dE}{dx}\right)\left(\frac{2v}{3\Gamma_s}\right)^2\left(\frac{9}{8v}\right) I_{\delta\epsilon}(\alpha,\beta)$$

$$\mathbf{g}_i = \left(\frac{1}{4\pi}\right)\left(\frac{dE}{dx}\right)\left(\frac{2v}{3\Gamma_s}\right)^2 I_{\mathbf{g}_i}(\alpha,\beta)$$

$-$



Probability density to produce particles around fast parton's direction of motion

Use Cooper-Frye distribution and divide by total number of particles N

$$\mathcal{P}(p_T, r, \phi) = \frac{1}{N}\frac{dN}{p_T dp_T d^2 r}$$

The average momentum squared carried by the disturbance, transverse to the direction of the fast parton is

$$\langle q^2 \rangle \equiv 2 \int d^2 r \int dp_T p_T \int_0^{\pi/2} d\phi \; \mathcal{P}(p_T, r, \phi) p_T^2 \sin^2 \phi$$

Explicitly

$$\langle q^2 \rangle = 20 \;\; T_0^2 \;\; \frac{\int d^2 r \left[\frac{\pi}{8}\delta\epsilon + \frac{(4/3)\mathbf{g}_y + (2/3)\mathbf{g}_z}{(1+c_s^2)}\right]}{\int d^2 r \left[\frac{\pi}{4}\delta\epsilon + \frac{2\mathbf{g}_y + 2\mathbf{g}_z}{(1+c_s^2)}\right]}$$

$\hat{q}$ from $\langle q^2 \rangle$ and medium's length L

- Can $\langle q^2 \rangle$ be identified with the average momentum squared *given to the fast parton* by the medium and therefore with $\hat{q}$ upon dividing by the medium's length L?
- The calculation refers to the average momentum squared given to the medium by the fast parton.
- If the parton's change in energy is small the main effect on the fast parton is a deflection of its original trajectory.
- As a result of energy and momentum conservation during this deflection, the momentum **put into the medium** should compensate the momentum **given to the fast parton**.
- Since in a hydrodynamical picture, the energy-momentum is described in terms of $\delta\epsilon$ and $\mathbf{g}$, we can write for the parameter $\hat{q}$

$$\hat{q} = \langle q^2 \rangle / L$$

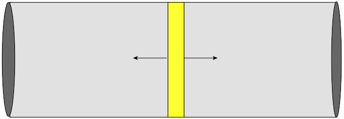
$\hat{q}$: Static vs expanding medium

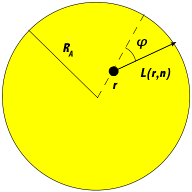
$$\hat{q} = 20\ \frac{T_0^2}{L}\ \frac{\int d^2r \left[\frac{\pi}{8}\delta\epsilon + \frac{(4/3)\mathbf{g}_y+(2/3)\mathbf{g}_z}{(1+c_s^2)}\right]}{\int d^2r \left[\frac{\pi}{4}\delta\epsilon + \frac{2\mathbf{g}_y+2\mathbf{g}_z}{(1+c_s^2)}\right]}$$

- For the static case, the expression for $\hat{q}$ is essentially independent of η/s.
- What happens if the medium is expanding?
- Classify events according to energy loss ΔE and in-medium travelled length $L(\mathbf{r},\hat{\mathbf{n}})$, where $\mathbf{r}$ and $\hat{\mathbf{n}}$ are the location of hard scattering and direction of propagation of fast parton.

$$\hat{q}_{\Delta E} \;=\; \frac{20\ T_0^2 \int d^2r \left(\frac{\pi}{8}\delta\epsilon + \frac{(4/3)\mathbf{g}_y+(2/3)\mathbf{g}_z}{(1+c_s^2)}\right)_{\Delta E}}{\sum_{\Delta E} L(\mathbf{r},\hat{\mathbf{n}}) \int d^2r \left(\frac{\pi}{4}\delta\epsilon + \frac{2\mathbf{g}_y+2\mathbf{g}_z}{(1+c_s^2)}\right)_{\Delta E}}.$$

Longitudinally expanding medium





$$L(\mathbf{r}, \hat{\mathbf{n}}) \;=\; \frac{1}{2}\left(\sqrt{R_A^2 - r^2 \sin\varphi} - r\cos\varphi\right)$$

Take as the energy loss per unit length

$$\left(\frac{dE}{dx}\right) \;=\; \frac{\Delta E}{L(\mathbf{r}, \hat{\mathbf{n}})}$$

Model for longitudinally expanding medium

H. Zhang, J. F. Owens, E. Wang, and X.-N Wang, Phys. Rev. Lett. **98**, *212301 (2007)*

- ρ_g: gluon density;
  ρ_0: central gluon density;
  τ_0: formation time; λ_0: mean free path

medium's influence time

$$\Delta E = \langle L/\lambda_0 \rangle = \left\langle \frac{dE}{dx} \right\rangle_{1d} \int_{\tau_0}^{\infty} d\tau \frac{\tau - \tau_0}{\tau_0\, \rho_0}\, \rho_g(\tau, \mathbf{b}, \mathbf{r} + \hat{\mathbf{n}}\tau)$$

$$\langle n \rangle \equiv \left\langle \frac{dE}{dx} \right\rangle_{1d} \int_{\tau_0}^{\infty} d\tau \frac{1}{\lambda_0\, \rho_0}\, \rho_g(\tau, \mathbf{b}, \mathbf{r} + \hat{\mathbf{n}}\tau)$$

average number of scatterings

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- ρ_g: gluon density; ρ_0: central gluon density;
  T_A: Thickness function; $\mathbf{b}$: impact parameter;
  R_A: nuclear radius; λ_0: mean free path.

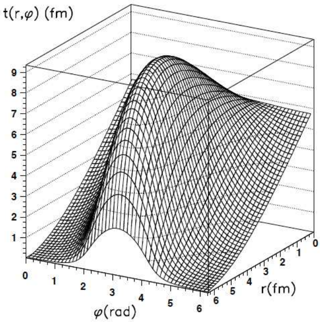
$$\rho_g(\tau, \mathbf{b}, \mathbf{r}, \hat{\mathbf{n}}) = \frac{\tau_0\, \rho_0}{\tau}\, \frac{\pi R_A^2}{2A} \left[T_A(|\mathbf{r} + \hat{\mathbf{n}}\tau|) + T_A(|\mathbf{b} - \mathbf{r} - \hat{\mathbf{n}}\tau|) \right]$$

$$\left\langle \frac{dE}{dx} \right\rangle_{1d} = \epsilon_0 \left[\frac{E}{\mu_0} - 1.6 \right]^{1.2} \left[7.5 + \frac{E}{\mu_0} \right]^{-1}$$

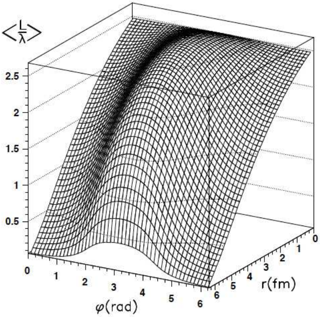
- $\mu_0 = 1.5$ GeV; $\epsilon_0 = 2$ GeV/fm, tuned to describe R_{AA} at LHC

Medium's influence time and average number of scatterings, central collisions

A. A., J. Jalilian-Marian, A. Ortiz, G. Paic, J. Magnin, M. E. Tejeda-Yeomans, Phys. Rev. C **84**, *024915 (2011)*



medium's influence time



average number of scatterings

Relating η/s and $\hat{q}$

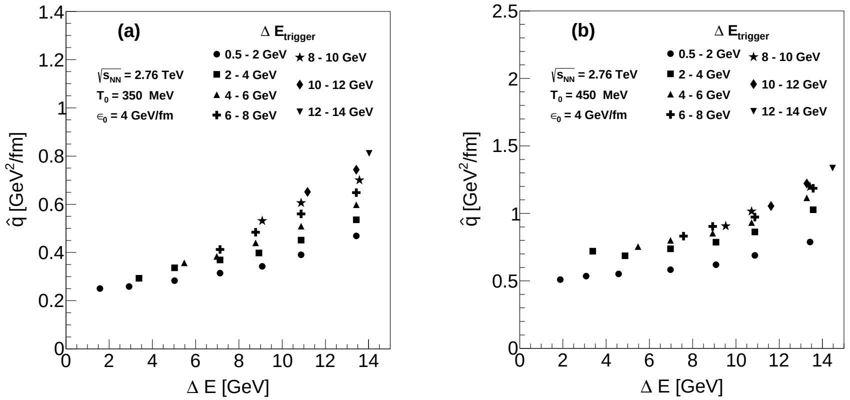
$$\hat{q}_{\Delta E} = \frac{20\ T_0^2 \int d^2r \left(\frac{\pi}{8}\delta\epsilon + \frac{(4/3)\mathbf{g}_y + (2/3)\mathbf{g}_z}{(1+c_s^2)}\right)_{\Delta E}}{\sum_{\Delta E} L(\mathbf{r},\hat{\mathbf{n}}) \int d^2r \left(\frac{\pi}{4}\delta\epsilon + \frac{2\mathbf{g}_y + 2\mathbf{g}_z}{(1+c_s^2)}\right)_{\Delta E}}$$

$$\frac{\eta}{s} \sim \frac{\text{mean free path}}{\text{thermal wavelength}} = T\ \frac{L(\mathbf{r},\hat{\mathbf{n}})}{\langle n \rangle}$$

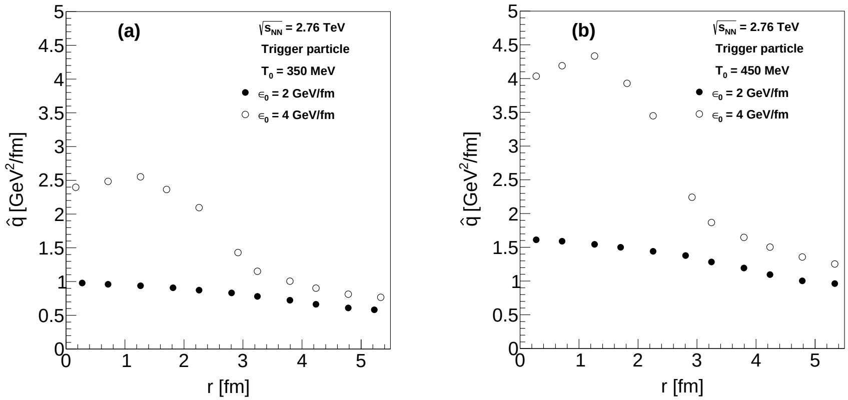
$\hat{q}$ vs ΔE (trigger particle)

A. A., I. Dominguez, J. Jalilian-Marian, M. E. Tejeda-Yeomans, Phys. Rev. C **94**, 024913 (2016)

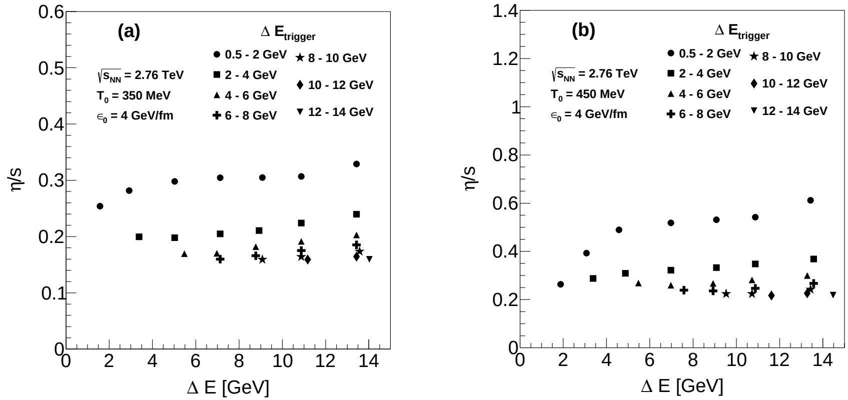
Use **MadGraph 5** to generate parton events at random positions **r**, moving in random directions $\hat{\mathbf{n}}$



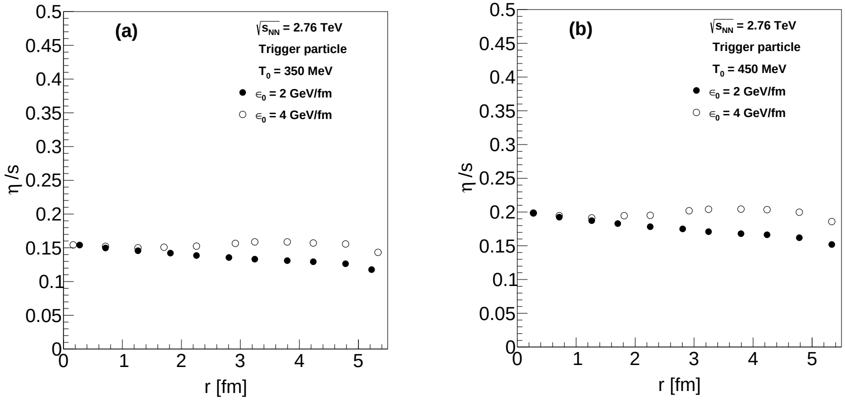
$\hat{q}$ vs r (trigger particle)



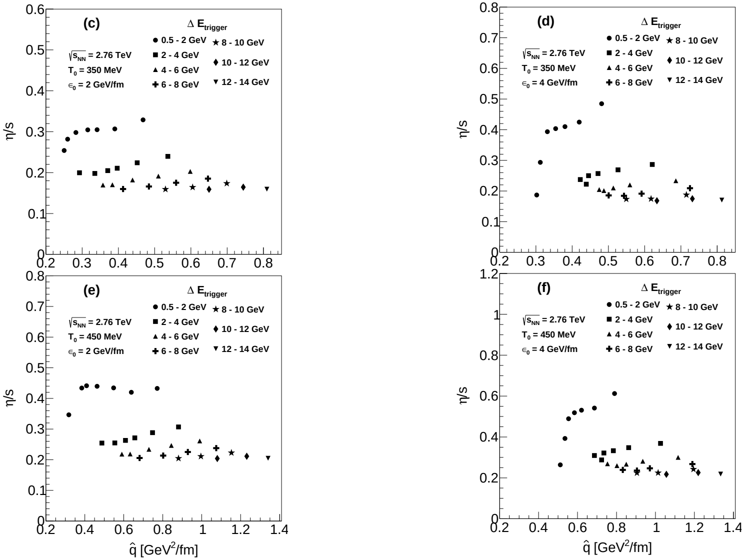
η/s vs ΔE (trigger particle)



η/s vs r (trigger particle)



$\hat{q}$ vs η/s (trigger particle)



SUMMARY AND CONCLUSIONS

- A non-trivial behavior of the transport coefficients $\hat{q}$ and η/s with the the location of the hard scattering and with the energy loss characterizing the events is obtained for an expanding medium.
- Model the amount of energy and momentum given to the medium by a fast moving parton in terms of **linear viscous hydrodynamics** and the amount of particles produced by this energy-momentum in terms of the **Cooper-Frye formula.**
- Events characterized by energy loss or by location of the hard scattering within the medium.
- The expanding medium **cannot** be characterized by single values of $\hat{q}$ or η/s, though the second one of these coefficients shows a milder dependence on r or ΔE.
- For conditions present in nuclear collisions at high energies, it is important to **characterize the events in terms of a given observable**, such as the amount of energy loss (missing p_t), **before extracting a particular value for the transport coefficients.**