Data-driven analysis of the temperature and momentum dependence of the heavy-quark transport coefficient

Yingru Xu

Duke University

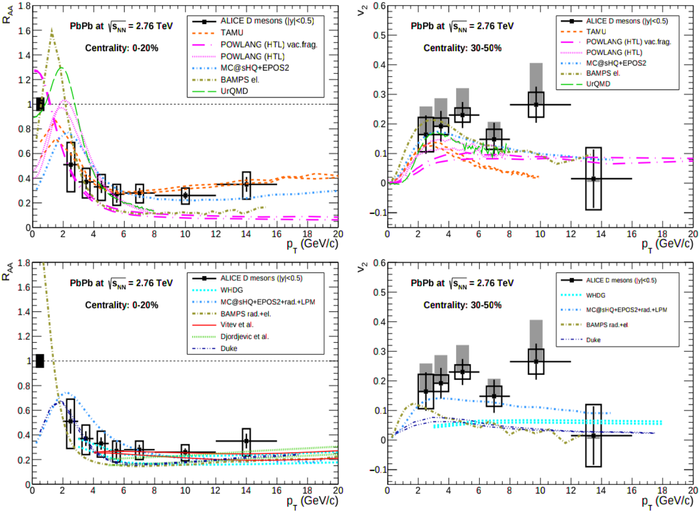
yx59@phy.duke.edu

Sep, 2016

In collaboration with :
Shanshan Cao
Marlene Nahrgang
Steffen A. Bass

HQ in-medium evolution

a challenge for simultaneously describing R_{AA} and v_2 results!

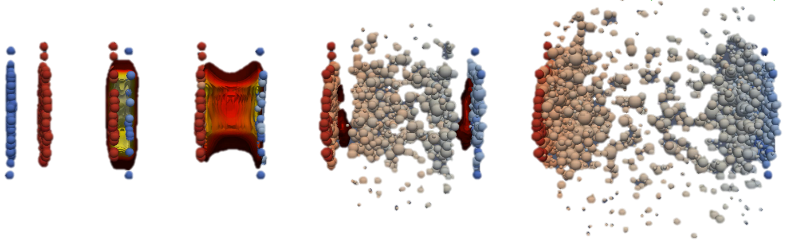


arxiv: 1506.03981

Figure : PbPb collision at the LHC: comparison between models and experimental observables

HQ transport model

figure credit: Hannah Petersen(Au-Au collisions)



initial condition

spatial IC: T_RENTo
momentum IC: pQCD

HQ in-medium

HQ transport:
Langevin (col + rad)
medium:
hydrodynamic

hadronization

hybrid model:
fragmentation +
recombination

initial condition

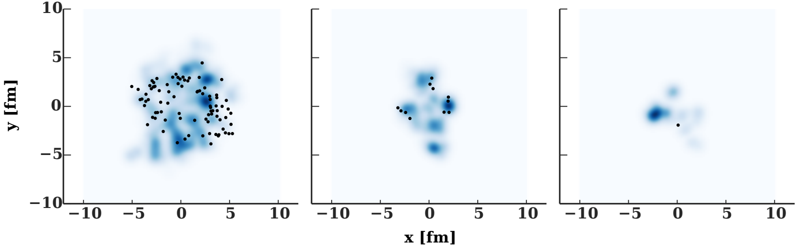
position space: T$_R$ENTo (A parametric IC model)

- entropy deposition proportional to reduced thickness function
  $\frac{ds}{dy}\Big|_{\tau=\tau_0} \propto \left(\frac{T_A+T_B}{2}\right)^{1/p}$ J.S.Moreland,J.Bernhard, and S.A.Bass, Phys.Rev.C 92, 011901(2015)
- $p=0$ (geometric mean), $\frac{ds}{dy} \propto \sqrt{T_A T_B}$ (mimic the behavior of IP-glasma model)
- heavy quark initial production probability: $\frac{dN}{dy}\Big|_{\tau=\tau_0} \propto T_A T_B$

momentum space: Leading order pQCD

S.Cao, G.Qin, and S.A.Bass, Phys.Rev.C 92, 024907(2015)

- parton distribution funciton: CTEQ5
- nuclear shadowing effect: EPS09



calibration of the medium

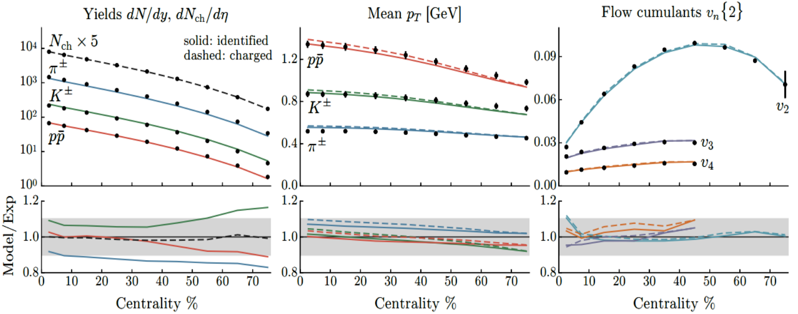
medium evolution

- (2+1)D viscous hydro: iEbE-VishNU

H.Song and U.W.Heinz, Phys.Rev.C 77, 064901(2008)

- temperature-dependent shear vis + bulk vis correction
- $(\eta/s)(T) = (\eta/s)_{min} + (\eta/s)_{slope}(T - T_c)$
- all the initial/medium related parameters are calibrated by Bayesian model-to-data comparison with experimental observables (yields, mean p_T, flow cumulants $v_n\{2\}$)

J.Bernhard,J.S.Moreland,S.A.Bass, J.Liu, and U.Heinz Phys.Rev.C 94, 024907(2015)



HQ in-medium evolution

HQ propagation

S.Cao, G.Qin, and S.A.Bass, Phys.Rev.C 92, 024907(2015)

- improved Langevin transport model

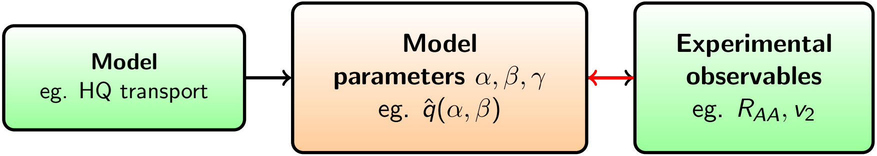
$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f_g} \qquad (1)$$

- drag force: $\eta_D(p) = \kappa/(2TE)$
- thermal random force: $\langle \xi^i(t)\xi^j(t') = \kappa\delta^{ij}\delta(t-t')\rangle$
- recoil force from gluon radiation: $\vec{f_g} = -d\vec{p_g}/dt$
- gluon emission probability:

$$\frac{dN_g}{dxdk_\perp^2 dt} = \frac{2\alpha_s P(x)\hat{q}_g}{\pi k_\perp^4} \sin^2(\frac{t-t_i}{2\tau_f})(\frac{k_\perp^2}{k_\perp^2 + x^2M^2})^4 \qquad (2)$$

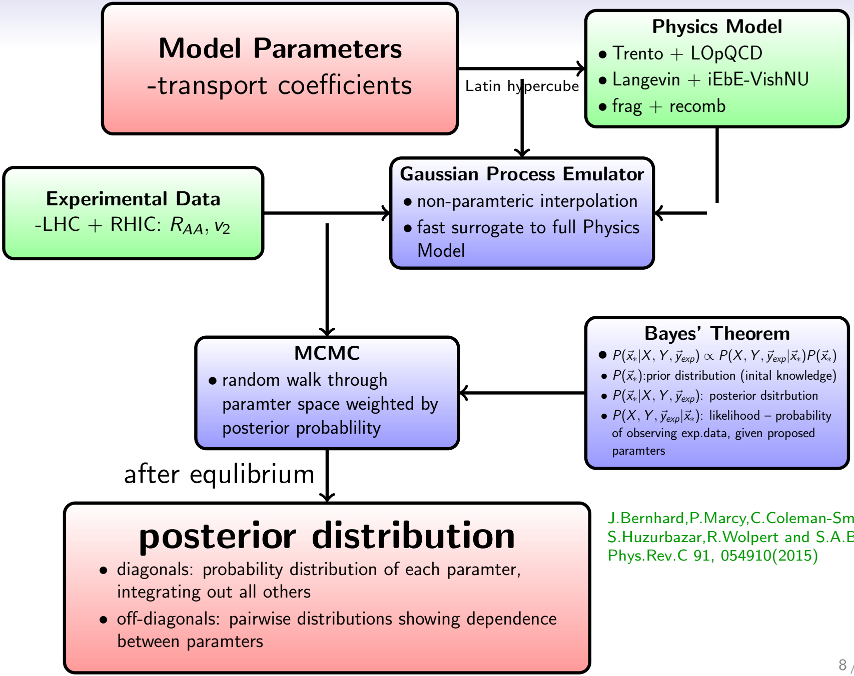
- $\hat{q}_g = \hat{q} C_A/C_F = 2\kappa C_A/C_F$, $D_s = 2T^2/\kappa$

Bayesian model-to-data analysis



Bayesian analysis:

- physical properties of the system encapsulated in parameters of the model
- Bayesian analysis allows us to simultaneously calibrate all model parameters through model-to-data comparison
- find the optimal parameters such that the model best describes the experimental observables
- extract the probability distribution of all parameters



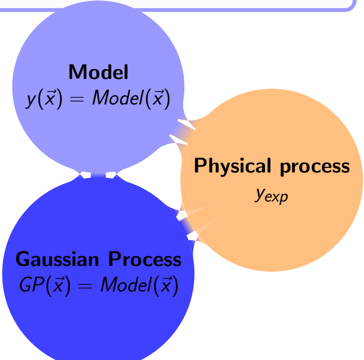
J.Bernhard,P.Marcy,C.Coleman-Smith, S.Huzurbazar,R.Wolpert and S.A.Bass, Phys.Rev.C 91, 054910(2015)

temp-dependent parameterization of diffusion coefficient

$$D_s = T^2/\hat{q},\ \hat{q} = \hat{q}_{pQCD} * preK * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$$

difficulties

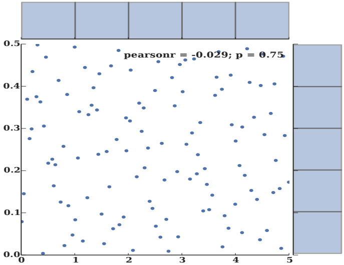
HQ transport model run $\propto$ 2hrs for 10 events produced; 10000 events needed for event-by-event study $\Rightarrow O(10^4)$ CPU hours to evaluate one input $\vec{x}_*$



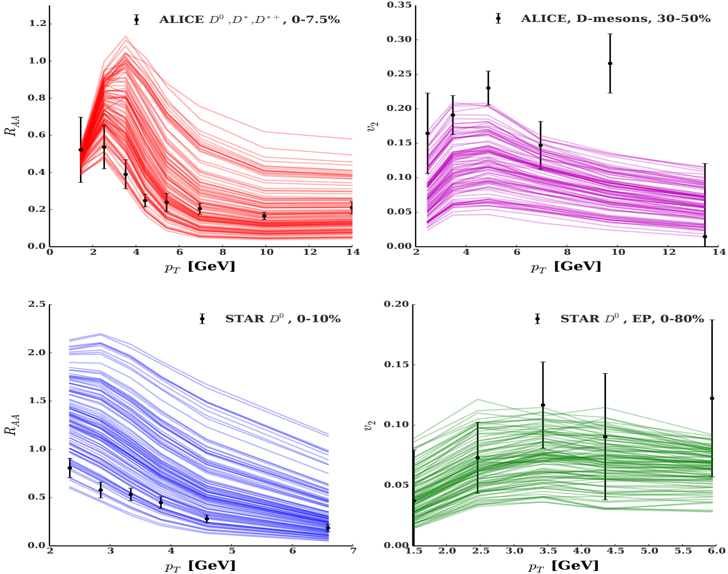
param	k_T	σ_T	preK
range	0-5	0.001-0.5	0.1-1.4

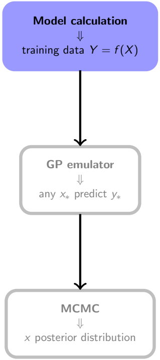
Latin hypercube design

120 input parameters $X = (\vec{x}_1, \vec{x}_2, ..., \vec{x}_{120})$



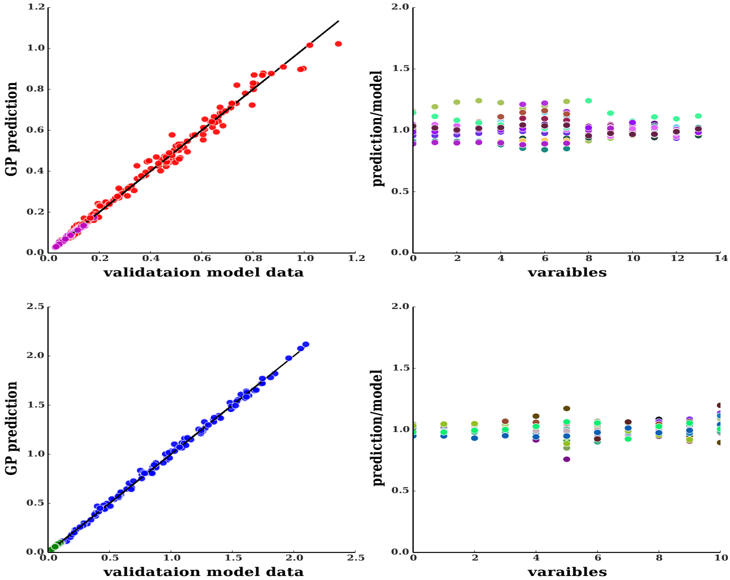
T-dependence results: Model outputs

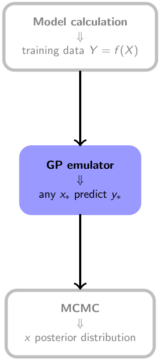




T-dependence results: GP training

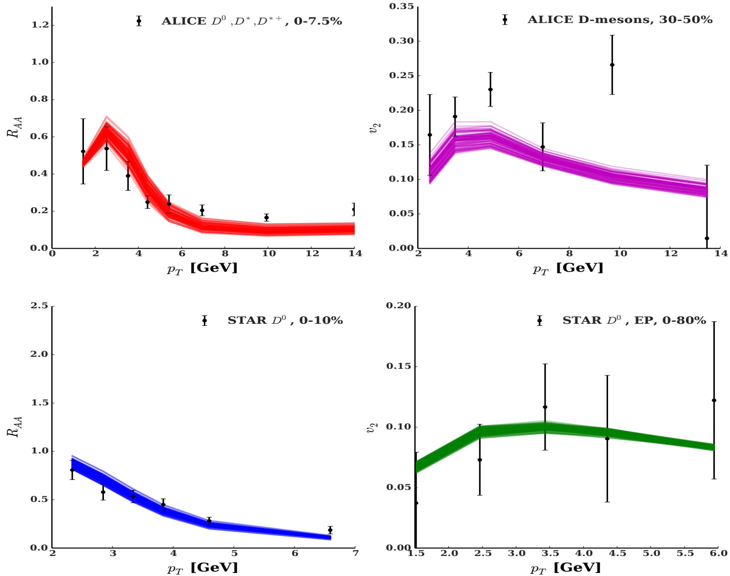
GP emulator prediction, validated by model outputs

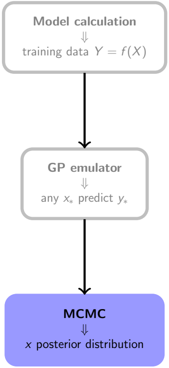




T-dependence results: calibation

outputs after calibration

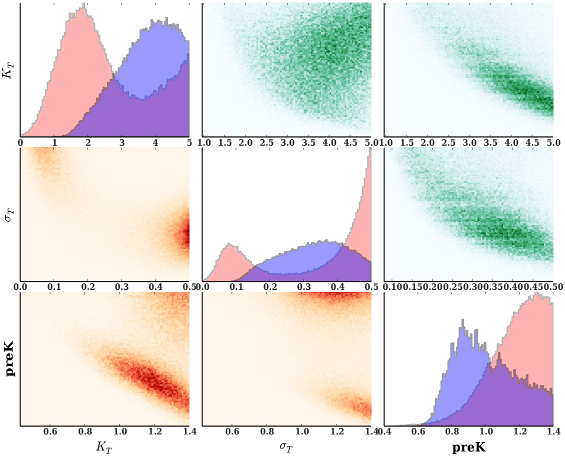


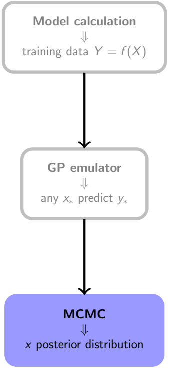


T-dependence results: MCMC (calibration)

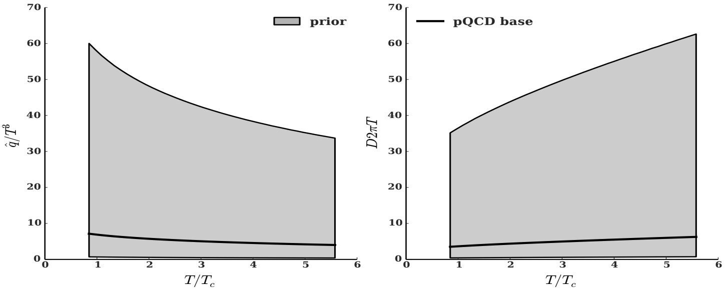
posterior probability distribution of parameters

param	range	80% CR (PbPb)	80%CR (AuAu)	mean (PbPb)	mean(AuAu)
k_T	0-5	1.08 - 2.51	2.51-4.73	2.52	3.71
σ_T	0.001-0.5	0.07-0.48	0.19-0.45	.033	0.33
prek	0.1-1.4	0.96-1.36	0.78-1.28	1.18	1.00





(momentum) diffusion coefficient – prior range

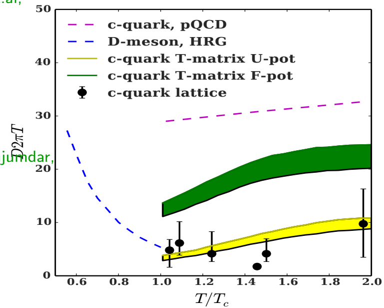


H.Ding,A.Francis,O.Kaczmarek,et.al, Phys.Rev.D 86,014509(2012)

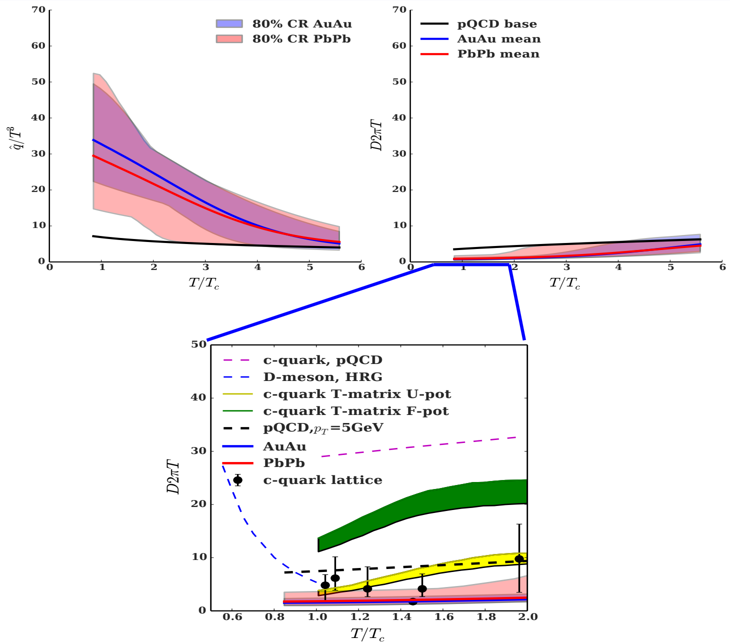
M.He,R.J.Fries,and R.Rapp, Phys.Rev.Lett 11,112301(2013)

F.Riek,and R.Rapp, Phys.Rev.C 82,035201(2010)

D.Banerjee,S.Datta,R.Gavai,P.Majumdar, Phys.Rev.D 85,014510(2012)

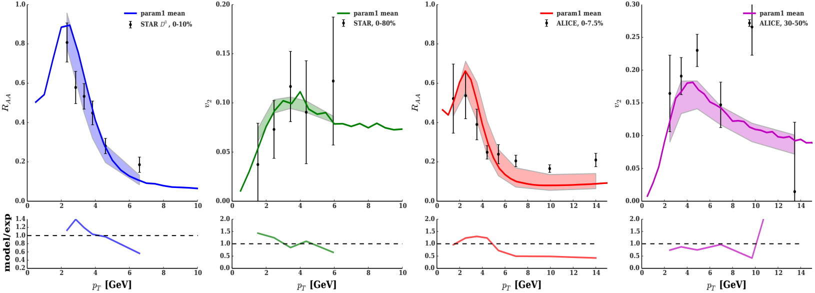


(momentum) diffusion coefficient– posterior range



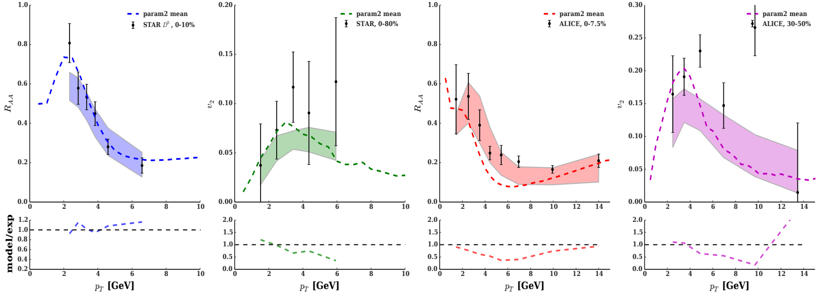
other parameterization

- T-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$
- p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}})$
- T,p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}}) * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$



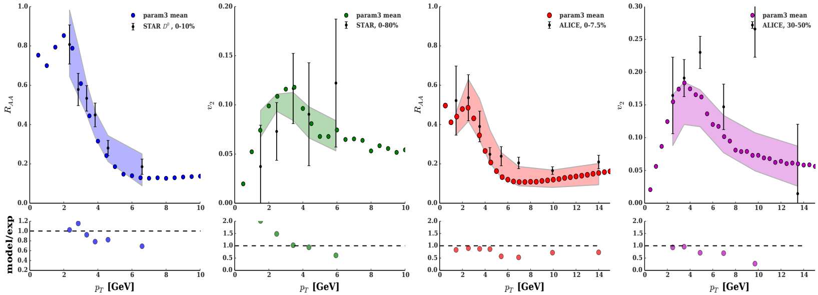
other paramterization

- T-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$
- p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}})$
- T,p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}}) * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$



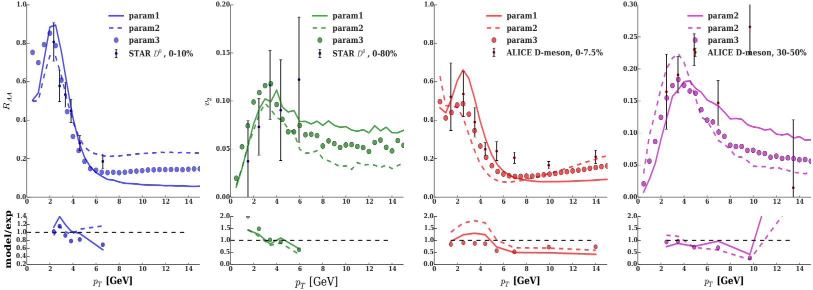
other paramterization

- T-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$
- p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}})$
- T,p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}}) * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$



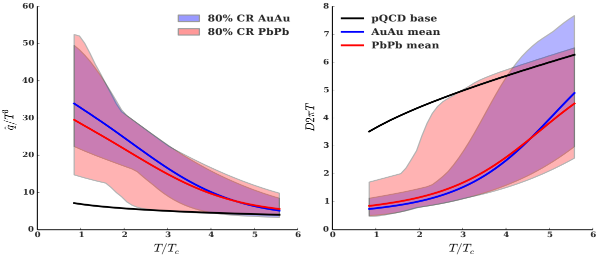
other paramterization

- T-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$
- p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}})$
- T,p-dependence: $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}}) * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$



summary

- By applying Bayesian model-to-data analysis, we are able extract the temperature and momentum dependence of $\hat{q}$, D_s from data



- Simultaneous agreement of R_{AA} and v_2 compared to data
- Discrepancies of the parameter posterior distribution between RHIC and the LHC energies:

  possible hint of temperature and momentum dependence that is not fully captured in our parameterization
- Improve uncertainty analysis (systematic/statistic error); more extension on other experimental observables, etc..

backups

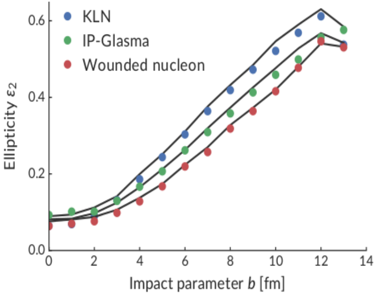
$\mathbf{T_R}$**ENTo** (A parametric IC model)

Ansatz:

entropy density proportional to generalized mean of local nuclear density:

$$s \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

- p= -1, $\frac{2T_A T_B}{T_A + T_N B}$
- p = 0, $\sqrt{T_A T_B}$
- p = 1, $\frac{T_A + T_B}{2}$



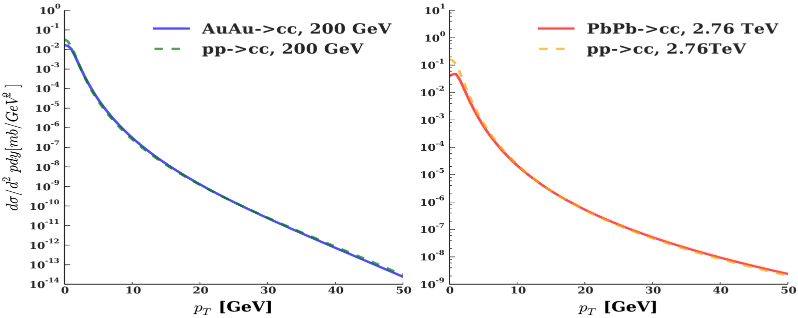
initial condition

position space: $\mathbf{T_R ENTo}$ (A parametric IC model)

- entropy deposition: $\frac{ds}{dy} \propto \sqrt{T_A T_B}$

momentum space: Leading order pQCD

- parton distribution funciton: CTEQ5
- nuclear shadowing effect: EPS09



Bayes' theorem

$$P(\vec{x}_*|X, Y, \vec{y}_{exp}) \propto P(X, Y, \vec{y}_{exp}|\vec{x}_*)P(\vec{x}_*) \quad (3)$$

- $X = (\vec{x}_1, \vec{x}_2, ..., \vec{x}_m)$: input parameters
  $Y = (\vec{y}_1, \vec{y}_2, ..., \vec{y}_m)$: output of the model
- $P(\vec{x}_*|X, Y, \vec{y}_{exp})$: posterior possibility distribution for $\vec{x}_*$ for given $X, Y, \vec{y}_{exp}$
- $P(X, Y, \vec{y}_{exp}|\vec{x}_*)$: likelihood

$$P(X, Y, \vec{y}_{exp}|\vec{x}_*) \propto \exp\left(-\frac{1}{2}(\vec{y}_* - \vec{y}_{exp})^T \Sigma^{-1} (\vec{y}_* - \vec{y}_{exp})\right) \quad (4)$$

- $P(\vec{x}_*)$: prior possibility distribution of $\vec{x}_*$ (Our initial knowledge of the input parameters)
- with experimental statistical error as uncertainty:$\Sigma = diag(\sigma^2_{exp} \vec{y_{exp}})$

Bayesian model-to-data comparison

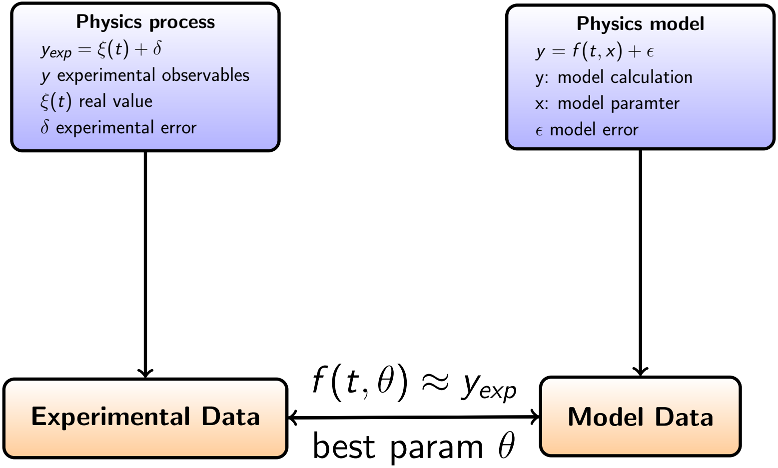
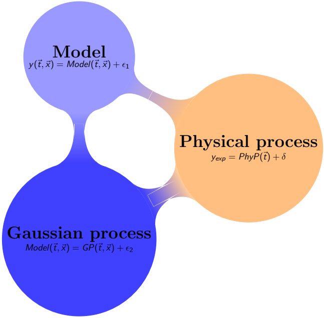


Figure : find the 'true' paramter θ, extract the shape of paramters by doing model-to-data analysis

Guassian Process emulator



A substitution of the model to rapidly calculate the output

- physics process: $y_{exp} = PhyP(\vec{t}) + \delta$, $\vec{t}$ is known variables , eg. $\sqrt{s_{NN}}$
- model simulation: $y = Model(\vec{t},\vec{x}) + \epsilon_1 \Rightarrow y_{exp} \sim Model(\vec{t},\vec{\theta})$
- Gaussian process emulator: $Model(\vec{t},\vec{x}) = GP(\vec{t},\vec{x}) + \epsilon_2$ $\Rightarrow y \sim GP(\mu(\vec{x}), \sigma(\vec{x},\vec{x'}))$
- $\delta, \epsilon_{1,2}$ are the errors(sys, stats)
- $\mu(\vec{x})$ mean vector, $\sigma(\vec{x},\vec{x'})$ the covariance function of each pair $(\vec{x},\vec{x'})$

Gaussian process

Definition:
A Gaussian process is a collection of random variables, any finite number of which have a joint Gaussian distribution.

- Stochastic function: $\vec{x} \to y$
- $\vec{x}$: n-dimensional input vector; y: normally distributed output
- specified by:
  - mean function $\mu(\vec{x})$
  - covariance function $\sigma(\vec{x}, \vec{x}')$
  - this study: $\sigma(\vec{x}, \vec{x}') = \sigma_{GP}^2 exp\left[-\frac{\vec{x}-\vec{x}'}{2l^2}\right] + \sigma_n^2\delta_{xx'}$

conditioning a Gaussian process

Given: training inputs points X and training outputs Y at X
predict: $\vec{x}_* \Rightarrow y_*$
The predictive distribution at arbitrary test points $\vec{x}_*$ is the multivariate-normal distribution

- $y_* = N(\mu, \Sigma)$
- $\mu = \sigma(X, X_*)\sigma(X, X)^{-1}y$
- $\Sigma = \sigma(X, X_*) - \sigma(X_*, X)\sigma(X, X)^{-1}\sigma(X, X_*)$

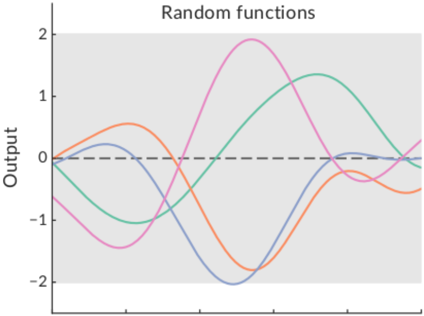
Likelihood function:

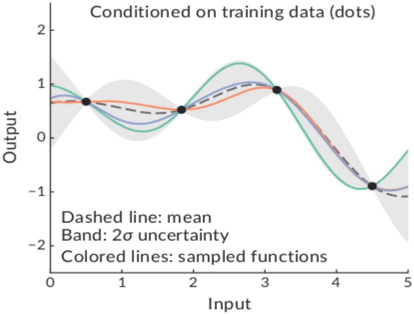
$$logP(Y|X, \vec{\theta}) = -\frac{1}{2}Y^T\Sigma^(-1)(x, \vec{\theta})Y - \frac{1}{2}|\Sigma(X, \vec{\theta})| - \frac{N}{2}log(2\pi)$$
(5)

Gaussian Process emulator

Gaussian Process:

- stochastic function: maps inputs to normally-distributed outputs
- specified by mean and covariance functions
- non-parametric interpolation
- predicts probabilities distributions: narrow near training points, wide in gaps
- fast surrogate to real physical model





principle component analysis

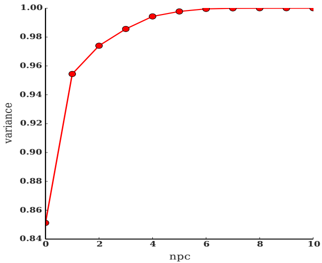
Many highly correlated outputs ⇒ principle component analysis
PCs = eigenvectors of outputs covariance matrix

$$Y = USV^T \tag{6}$$

$$Y^T Y = V \Lambda V^T \tag{7}$$

transform data into orthogonal,uncorrelated linear combinations:

$$Z = \sqrt{m} YV, Y = \frac{1}{\sqrt{m}} ZV^T \tag{8}$$



Markov Chain Monte Carlo

posterior distribution is sampled with MCMC method

- Random walk in parameter space, where each step is accepted or rejected based on a relative likelihood
- Converges to posterior distribution as number of steps $N \to \infty$
- accptance fraction α_f of steps measures the quality of random walk
  - $\alpha_f \simeq 0 \Rightarrow$walker "stuck"
  - $\alpha_f \simeq 1 \Rightarrow$pure random walk
  - aim for 0.2-0.5
- autocorrelation time = Number of steps between indepedence samples "Burn-in" talks a few correlations, gathering enough samples $\simeq 0(10)$autocorrelations

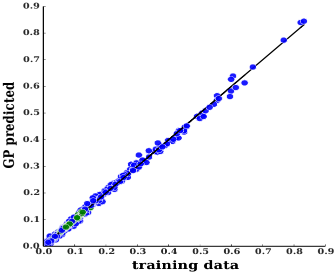
input parameters $\vec{x}$:

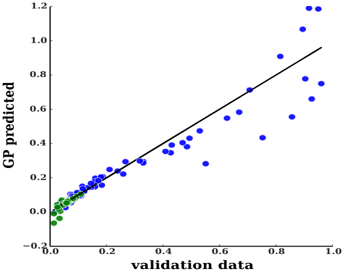
- $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}})$
- $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$
- $\hat{q} = \hat{q}_{pQCD} * preK * (1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}}) * (1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}})$

param	range1	range2	range3
k_p	N/A	0-15	0-12
σ_p	N/A	0.1-10.5	0.1-10.5
k_T	0-5	N/A	1-5
σ_T	0.001-0.5	N/A	0.001-0.5
$preK$	0.1-1.4	0.1-2.0	0.3-1.4

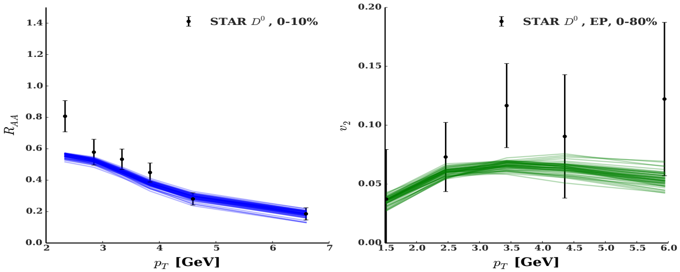
param 2 results: GP training

20 validation $\vec{x} \Rightarrow \vec{y} = Model(\vec{x})$ compare with $\vec{y} = GP(\vec{x})$



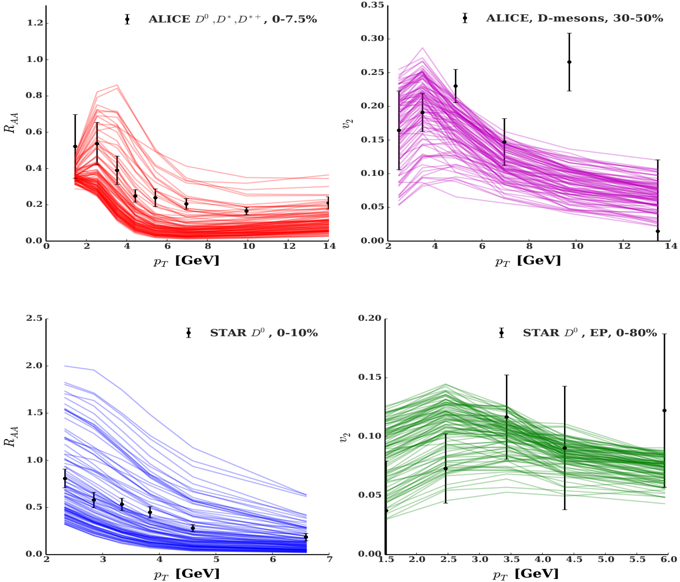


output from calibration $\vec{y^*}$:



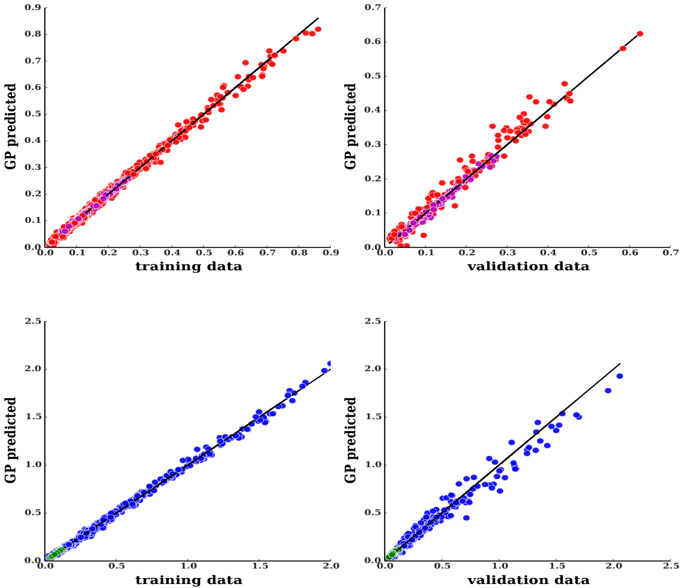
param 3 results: model output

output from the model $\vec{y}$: training data

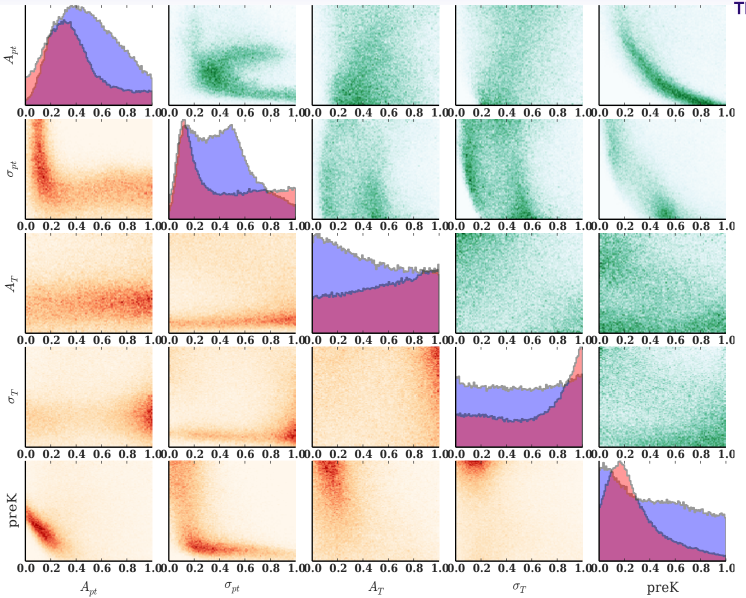


param 3 results: GP training

20 validation $\vec{x} \Rightarrow \vec{y} = Model(\vec{x})$ compare with $\vec{y} = GP(\vec{x})$

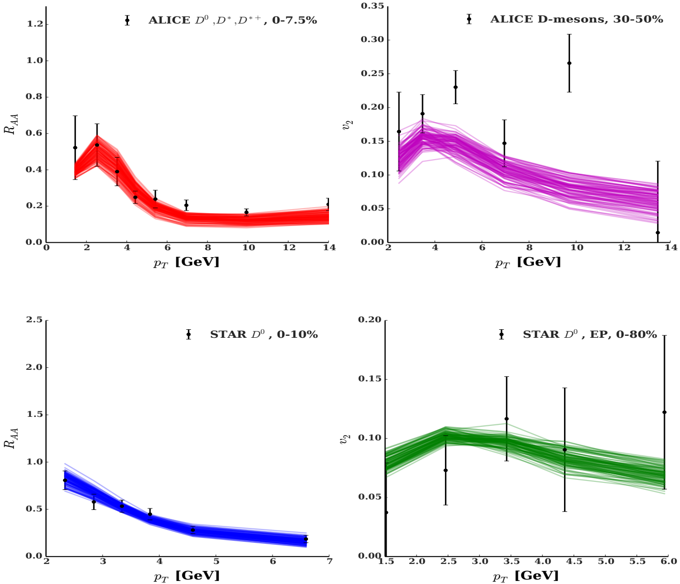


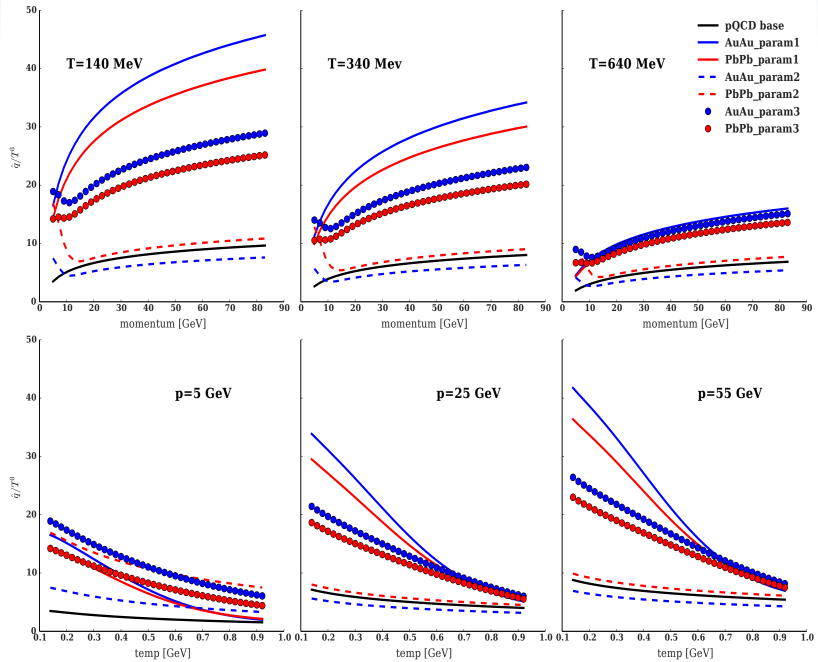
param 3 results: MCMC (calibration)



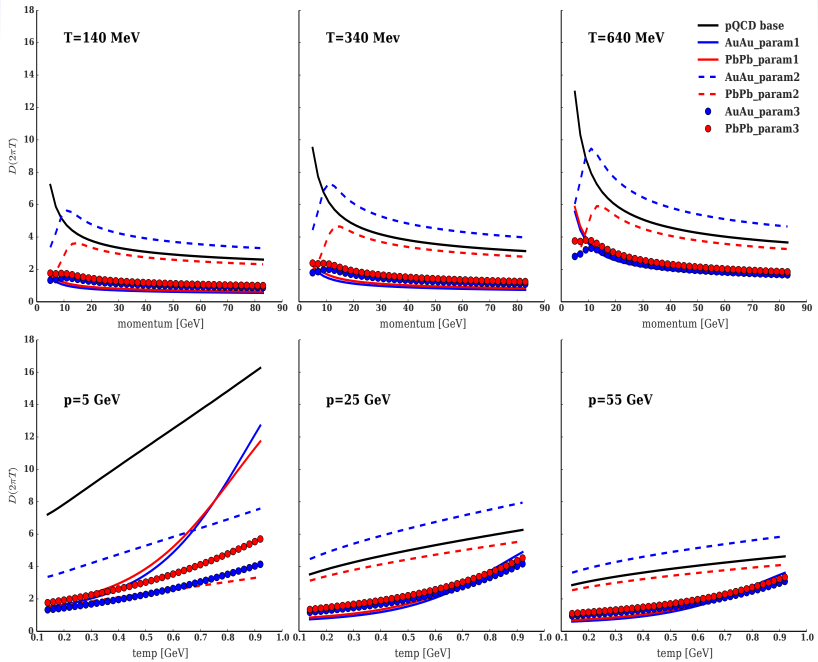
param 3 results: calibration output

output from calibration $\vec{y}^*$:





T=140 MeV
T=340 Mev
T=640 MeV
p=5 GeV
p=25 GeV
p=55 GeV
pQCD base
AuAu_param1
PbPb_param1
AuAu_param2
PbPb_param2
AuAu_param3
PbPb_param3
$\hat{q}/T^3$
momentum [GeV]
temp [GeV]


T=140 MeV
T=340 Mev
T=640 MeV
pQCD base
AuAu_param1
PbPb_param1
AuAu_param2
PbPb_param2
AuAu_param3
PbPb_param3
$D(2\pi T)$
momentum [GeV]
p=5 GeV
p=25 GeV
p=55 GeV
temp [GeV]