

# Single inclusive forward hadron production at next-to-leading order

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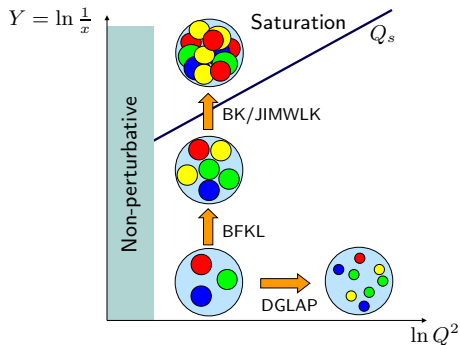


B. Ducloué, T. Lappi, YZ, Phys.Rev. D93 (2016) 114016 [arXiv:1604.00225 [hep-ph]]

24 Sept 2016, HP16, Wuhan

## Motivations

Our goal is to study QCD in the saturation regime



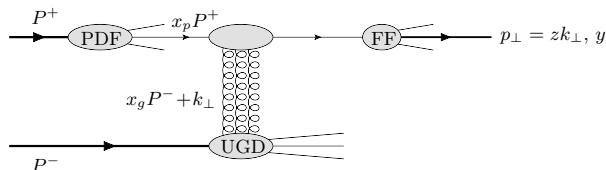
The production of **forward** particles is a crucial tool to probe small  $x$  values

Saturation effects should be enhanced by the higher densities in **pA** collisions

Here we study the inclusive production of a forward hadron in proton-nucleus collisions:  $pA \rightarrow hX$

## Motivations

Single inclusive forward hadron production at LO in the  $q \rightarrow q$  channel:



The values of  $x_p$  and  $x_g$  probed in the projectile and the target are given by  
 $x_p = \frac{p_\perp}{\sqrt{s}} e^y$ ,  $x_g = \frac{p_\perp}{\sqrt{s}} e^{-y}$

The **dilute projectile** is described in terms of well known collinear PDFs

The **dense target** is described by an unintegrated gluon distribution  $\mathcal{F}$ , which is the Fourier-transform of the fundamental representation dipole correlator:

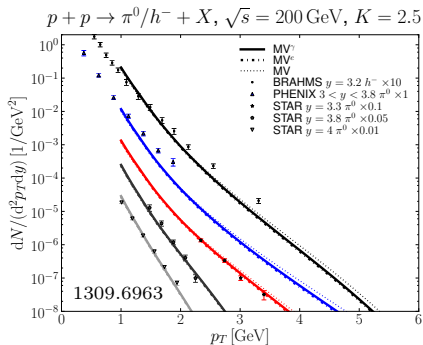
$$\mathcal{F}(k_\perp) = \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^2} e^{-ik_\perp \cdot (\mathbf{x}-\mathbf{y})} S(\mathbf{x}, \mathbf{y}), \quad S(\mathbf{x}, \mathbf{y}) = \left\langle \frac{1}{N_c} \text{Tr} U(\mathbf{x}) U^\dagger(\mathbf{y}) \right\rangle$$

The LO cross section reads  $\frac{d\sigma}{d^2\mathbf{p}dy} = \sum_q \int_\tau^1 \frac{dz}{z^2} x_p q(x_p) \mathcal{F}(k_\perp) D_{h/q}(z)$ ,

where  $D_{h/q}(z)$  are the fragmentation functions

## Motivations

Several LO calculations achieved a quite good description of experimental data, but often with rather large  $K$  factors to get the correct normalization



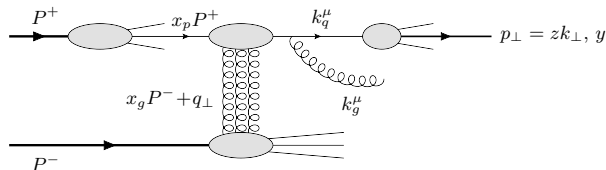
Lappi, Mäntysaari

It is important to extend these calculations to higher orders to check the stability of the perturbative expansion and to have more accurate predictions

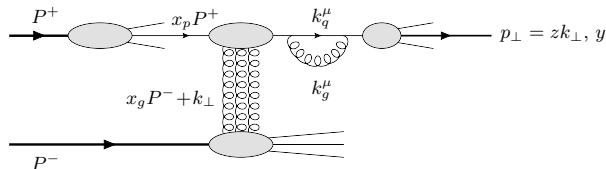
## Motivations

The expression for the NLO cross section has been computed by [Chirilli, Xiao, Yuan](#)

Example of real  $q \rightarrow q$  contribution:



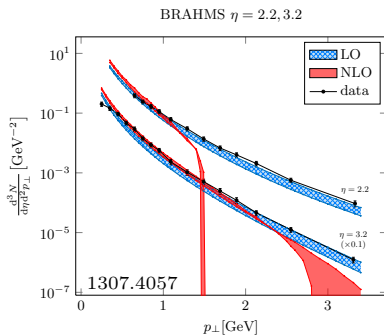
Example of virtual  $q \rightarrow q$  contribution:



$1 - \xi = \frac{k_g^+}{x_p P^+}$  is the momentum fraction of the incoming quark carried by the gluon

# Motivations

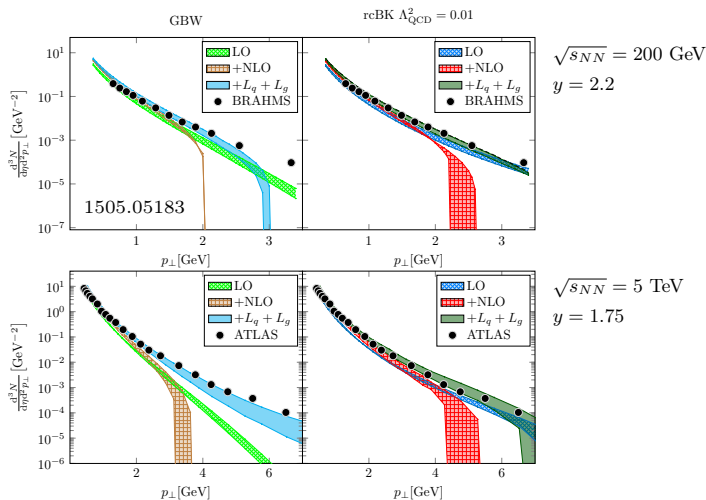
First numerical implementation of the NLO cross section: [Stařto, Xiao, Zaslavsky](#)



The cross section becomes negative above some transverse momentum

## Motivations

Several proposals to solve this issue, for example the kinematical constraint/Ioffe time cutoff (Altioluk, Armesto, Beuf, Kovner, Lublinsky). Numerical implementation: Watanabe, Xiao, Yuan, Zaslavsky:



The negativity problem is less severe but still present in some cases

# Motivations

The purpose of this work:

- ▶ Identify the origin of the negativity at large transverse momentum
- ▶ See if we can find a way to cure it

For this we make some simplifications

- ▶ We consider only the  $q \rightarrow q$  channel
- ▶ We use a simple gaussian form for the dipole cross section

Golec-Biernat and Wüsthoff (GBW) model:  $S(\mathbf{r}) = e^{-\frac{\mathbf{r}^2 Q_s^2}{4}}$

Our goal is **not** (yet) to make predictions to compare to experimental data



## The NLO cross section

The expression for the multiplicity at NLO reads

$$\begin{aligned}
 \frac{dN^{pA \rightarrow hX}}{d^2\mathbf{p} dy_h} &= \int_{\tau}^1 \frac{dz}{z^2} D_{h/q}(z) x_p q(x_p) \frac{S^{(0)}(k_{\perp})}{(2\pi)^2} && \leftarrow \text{LO term} \\
 &+ \frac{\alpha_s}{2\pi^2} \int \frac{dz}{z^2} D_{h/q}(z) \int_{\tau/z}^1 d\xi \frac{1+\xi^2}{1-\xi} \frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right) \left\{ C_F \mathcal{I}(k_{\perp}, \xi) + \frac{N_c}{2} \mathcal{J}(k_{\perp}, \xi) \right\} && \leftarrow \text{real NLO term} \\
 &- \frac{\alpha_s}{2\pi^2} \int \frac{dz}{z^2} D_{h/q}(z) \int_0^1 d\xi \frac{1+\xi^2}{1-\xi} x_p q(x_p) \left\{ C_F \mathcal{I}_v(k_{\perp}, \xi) + \frac{N_c}{2} \mathcal{J}_v(k_{\perp}, \xi) \right\} && \leftarrow \text{virtual NLO term}
 \end{aligned}$$

with

$$\begin{aligned}
 \mathcal{I}(k_{\perp}, \xi) &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} \mathcal{S}(q_{\perp}) \left[ \frac{\mathbf{k} - \mathbf{q}}{(\mathbf{k} - \mathbf{q})^2} - \frac{\mathbf{k} - \xi\mathbf{q}}{(\mathbf{k} - \xi\mathbf{q})^2} \right]^2 \\
 \mathcal{J}(k_{\perp}, \xi) &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{2(\mathbf{k} - \xi\mathbf{q}) \cdot (\mathbf{k} - \mathbf{q})}{(\mathbf{k} - \xi\mathbf{q})^2 (\mathbf{k} - \mathbf{q})^2} \mathcal{S}(q_{\perp}) - \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} \frac{2(\mathbf{k} - \xi\mathbf{q}) \cdot (\mathbf{k} - \mathbf{l})}{(\mathbf{k} - \xi\mathbf{q})^2 (\mathbf{k} - \mathbf{l})^2} \mathcal{S}(q_{\perp}) \mathcal{S}(l_{\perp}) \\
 \mathcal{I}_v(k_{\perp}, \xi) &= \mathcal{S}(k_{\perp}) \int \frac{d^2\mathbf{q}}{(2\pi)^2} \left[ \frac{\mathbf{k} - \mathbf{q}}{(\mathbf{k} - \mathbf{q})^2} - \frac{\xi\mathbf{k} - \mathbf{q}}{(\xi\mathbf{k} - \mathbf{q})^2} \right]^2 \\
 \mathcal{J}_v(k_{\perp}, \xi) &= \mathcal{S}(k_{\perp}) \left[ \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{2(\xi\mathbf{k} - \mathbf{q}) \cdot (\mathbf{k} - \mathbf{q})}{(\xi\mathbf{k} - \mathbf{q})^2 (\mathbf{k} - \mathbf{q})^2} - \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} \frac{2(\xi\mathbf{k} - \mathbf{q}) \cdot (\mathbf{l} - \mathbf{q})}{(\xi\mathbf{k} - \mathbf{q})^2 (\mathbf{l} - \mathbf{q})^2} \mathcal{S}(l_{\perp}) \right]
 \end{aligned}$$

Here and in the following we study the multiplicity which is related to the cross section by an integral over the impact parameter:

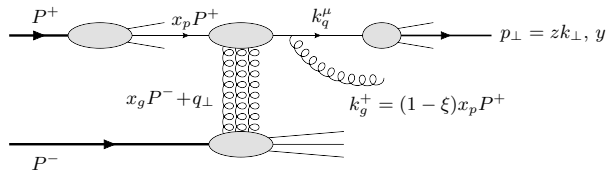
$$\frac{d\sigma^{pA \rightarrow hX}}{d^2\mathbf{p} dy_h} = \int d^2\mathbf{b} \frac{dN^{pA \rightarrow hX}}{d^2\mathbf{p} dy_h}$$

and we have defined  $\mathcal{S}(k_{\perp})$  such that  $\mathcal{F}(k_{\perp}) = \int \frac{d^2\mathbf{b}}{(2\pi)^2} \mathcal{S}(k_{\perp})$

## Divergences

After summing the real and virtual contributions, two types of divergences remain in the NLO cross section:

- ▶ The **collinear** divergence
  - Occurs when the additional gluon is collinear to either the incoming or outgoing quark
  - Affects only the NLO corrections proportional to  $C_F$
- ▶ The **rapidity** divergence
  - Occurs when  $\xi \rightarrow 1 \Leftrightarrow$  the rapidity of the unobserved gluon  $\rightarrow -\infty$   
 $\Leftrightarrow$  this gluon is collinear to the target
  - Affects only the NLO corrections proportional to  $N_c$



## The collinear divergence

For the collinear divergence we follow the same treatment as [Chirilli, Xiao, Yuan](#):

Using dimensional regularization in  $4 - 2\epsilon$  dimensions:  $\int \frac{d^2\mathbf{q}}{(2\pi)^2} \rightarrow \mu^{2\epsilon} \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}}$ , the divergent part of the **real**  $C_F$  term reads

$$-\frac{1}{\hat{\epsilon}} \frac{\alpha_s}{2\pi} C_F \int \frac{dz}{z^2} D_{h/q}(z) \int_{\tau/z}^1 d\xi \frac{1+\xi^2}{1-\xi} \frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right) \left[ \mathcal{F}(k_\perp) + \frac{1}{\xi^2} \mathcal{F}\left(\frac{k_\perp}{\xi}\right) \right]$$

And the divergent part of the **virtual**  $C_F$  term is

$$\frac{1}{\hat{\epsilon}} \frac{\alpha_s}{\pi} C_F \int \frac{dz}{z^2} D_{h/q}(z) x_p q(x_p) \int_0^1 d\xi \frac{1+\xi^2}{1-\xi} \mathcal{F}(k_\perp)$$

where  $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$ .

These divergences can be factorized into the **DGLAP** evolution of the quark PDF  $q(x)$  and the fragmentation function  $D_{h/q}(z)$  in the  $\overline{\text{MS}}$  scheme:

$$q(x, \mu) = q^{(0)}(x) - \frac{1}{\hat{\epsilon}} \frac{\alpha_s(\mu)}{2\pi} \int_x^1 \frac{d\xi}{\xi} \mathcal{P}_{qq}(\xi) q\left(\frac{x}{\xi}\right)$$
$$D_{h/q}(z, \mu) = D_{h/q}^{(0)}(z) - \frac{1}{\hat{\epsilon}} \frac{\alpha_s(\mu)}{2\pi} \int_z^1 \frac{d\xi}{\xi} \mathcal{P}_{qq}(\xi) D_{h/q}\left(\frac{z}{\xi}\right)$$

## The rapidity divergence

The  $N_c$  part of the NLO corrections is divergent when  $\xi \rightarrow 1$

This corresponds to a gluon which is almost collinear to the target

Therefore it is natural to absorb this contribution in the gluon field of the target

Chirilli, Xiao, Yuan: define the renormalized gluon distribution of the target as

$$\mathcal{S}(k_\perp) = \mathcal{S}^{(0)}(k_\perp) + 2\alpha_s N_c \int_0^1 \frac{d\xi}{1-\xi} [\mathcal{J}(k_\perp, 1) - \mathcal{J}_v(k_\perp, 1)]$$

In position space this can be written as

$$S(\mathbf{x}-\mathbf{y}) = S^{(0)}(\mathbf{x}-\mathbf{y}) - \frac{\alpha_s N_c}{2\pi^2} \int_0^1 \frac{d\xi}{1-\xi} \int d^2\mathbf{z} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2} [S(\mathbf{x}-\mathbf{y}) - S(\mathbf{x}-\mathbf{z})S(\mathbf{z}-\mathbf{y})]$$

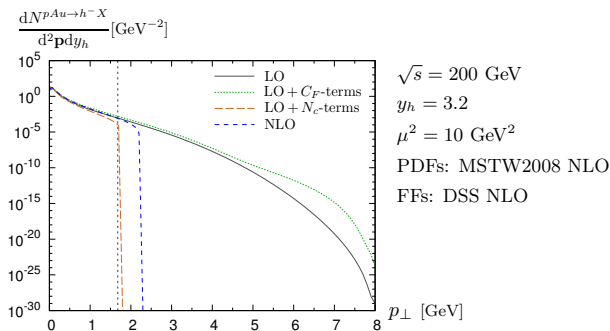
or, if we differentiate with respect to  $Y$ ,

$$\frac{\partial}{\partial Y} S(\mathbf{x}-\mathbf{y}) = -\frac{\alpha_s N_c}{2\pi^2} \int d^2\mathbf{z} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2} [S(\mathbf{x}-\mathbf{y}) - S(\mathbf{x}-\mathbf{z})S(\mathbf{z}-\mathbf{y})]$$

Which is the well-known Balitsky-Kovchegov evolution equation for  $S$

## The subtracted cross section

After the divergences have been subtracted, the multiplicity is finite...



...but negative above some  $p_\perp$ . This is similar to the results obtained when including all the channels (Stasto, Xiao, Zaslavsky)

At large  $p_\perp$  the  $C_F$  term is positive  $\rightarrow$  the negativity comes from the  $N_c$  term

## The origin of the negativity

The fact that the  $N_c$  term is negative at large  $p_\perp$  can be understood by looking at its large- $k_\perp$  limit:

$$\frac{N_c}{2} \frac{\alpha_s}{2\pi^2} \int \frac{dz}{z^2} D_{h/q}(z) \int_{\tau/z}^1 \frac{d\xi}{(1-\xi)_+} \mathcal{K}(\xi), \quad \mathcal{K}(\xi) = (1+\xi^2) \frac{x_p}{\xi} q \left( \frac{x_p}{\xi} \right) \mathcal{J}(k_\perp, \xi)$$

At large  $k_\perp$ ,  $\mathcal{K}(\xi)$  behaves like  $\mathcal{K}(\xi) \approx (1+\xi^2) \frac{x_p}{\xi} q \left( \frac{x_p}{\xi} \right) \frac{2\xi}{k_\perp^4} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \mathbf{q}^2 \mathcal{S}(q_\perp)$ , which is positive and generally increasing with  $\xi$ .

Therefore the **plus-distribution** will lead to a negative contribution.

This plus-distribution comes from the subtraction of the **rapidity divergence**

Let us come back to the renormalized UGD as defined by **Chirilli, Xiao, Yuan**:

$$\mathcal{S}(k_\perp) = \mathcal{S}^{(0)}(k_\perp) + 2\alpha_s N_c \int_0^1 \frac{d\xi}{1-\xi} [\mathcal{J}(k_\perp, 1) - \mathcal{J}_v(k_\perp, 1)]$$

The rapidity divergence occurs at  $\xi = 1$  so this point should be included in the subtraction term. But the choice of the **lower limit** is rather arbitrary

## The rapidity divergence subtraction

More generally one could use

$$\mathcal{S}(k_{\perp}) = \mathcal{S}^{(0)}(k_{\perp}) + 2\alpha_s N_c \int_{\xi_f}^1 \frac{d\xi}{1-\xi} [\mathcal{J}(k_{\perp}, 1) - \mathcal{J}_v(k_{\perp}, 1)]$$

where we have introduced  $\xi_f \in [0 : 1]$  which plays the role of a (rapidity) **factorization** scale, arbitrary at this stage. It determines how much of the **finite** contribution is considered to be part of the evolution of the target

At large  $k_{\perp}$  the  $N_c$  term now reads

$$\frac{N_c}{2} \frac{\alpha_s}{2\pi^2} \int \frac{dz}{z^2} D_{h/q}(z) \left( \int_{\tau/z}^{\xi_f} \frac{d\xi}{1-\xi} \mathcal{K}(\xi) + \int_{\xi_f}^1 \frac{d\xi}{1-\xi} [\mathcal{K}(\xi) - \mathcal{K}(1)] \right).$$

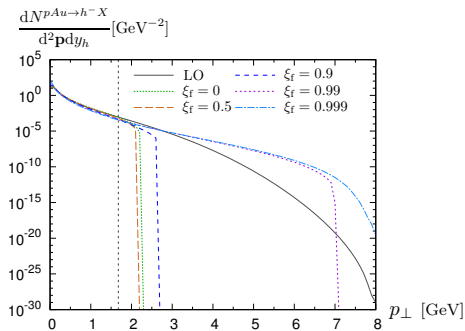
Since  $\mathcal{K}(\xi)$  is positive and increases with  $\xi$ , the first term yields a positive contribution while the second one yields a negative contribution

If we increase  $\xi_f$ , we make the **positive** contribution larger and the **negative** contribution smaller  $\rightarrow$  increase of the cross section

Like for other arbitrary scales, physical quantities should not depend on  $\xi_f$

## Dependence of the cross section on $\xi_f$

Multiplicity for several values of  $\xi_f$  between 0 and 1:



As expected, larger values of  $\xi_f$  lead to positive cross sections up to larger  $p_{\perp}$

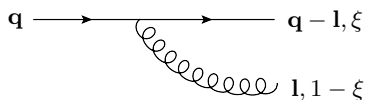
The results depend strongly on the choice of  $\xi_f$

Here we have varied  $\xi_f$  in a very wide range. We need to fix it to a “physical” value and then vary it in a reasonable range to estimate the remaining uncertainty



## How to choose the value of $\xi_f$

We need a condition to specify which contributions will be part of the evolution of the target. Let us consider a typical NLO diagram:



The light cone energy introduced from the gluon emission is

$$\Delta k^- = \frac{1}{2x_p P^+} \left[ \frac{\mathbf{l}^2}{1 - \xi} + \frac{(\mathbf{q} - \mathbf{l})^2}{\xi} - \mathbf{q}^2 \right] = \frac{x_g^{\text{LO}} P^-}{\mathbf{k}^2} \frac{(\mathbf{1} - (\mathbf{1} - \xi)\mathbf{q})^2}{\xi(1 - \xi)}$$

Here we decide to absorb fluctuations with  $\Delta k^-$  larger than a certain factorization scale  $x_f$  in the evolution of the target. At large  $k_\perp$  this leads to

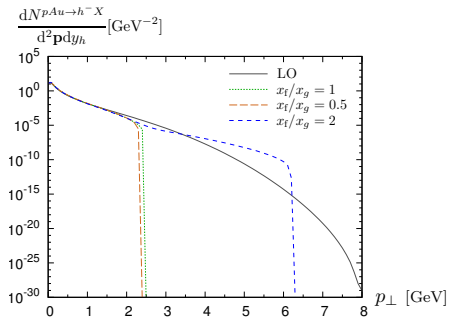
$$\Delta k^- \approx \frac{x_g^{\text{LO}} P^-}{\mathbf{k}^2} \frac{Q_s^2}{1 - \xi} \geq x_f P^- \Leftrightarrow 1 - \xi \leq \frac{Q_s^2 x_g^{\text{LO}}}{\mathbf{k}^2 x_f} \Rightarrow \xi_f = 1 - \frac{Q_s^2 x_g^{\text{LO}}}{\mathbf{k}^2 x_f},$$

with a “natural” value  $x_f \sim x_g^{\text{LO}}$ . In practice we use  $\xi_f = \frac{k_\perp^2}{k_\perp^2 + \frac{x_g^{\text{LO}}}{x_f} Q_s^2}$ ,

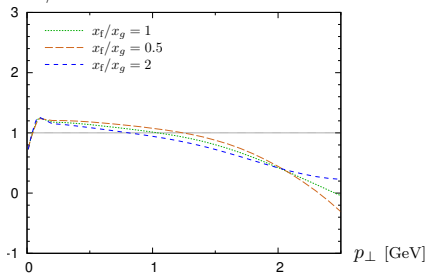
which has the same large  $k_\perp$  behaviour and goes smoothly to  $\xi_f = 0$  at  $k_\perp = 0$

## Results with a $k_{\perp}$ -dependent $\xi_f$

Multiplicity for  $\frac{x_f}{x_g} \in \{1, \frac{1}{2}, 2\}$ :



NLO/LO



At small  $p_{\perp}$  the dependence of the cross section on  $\frac{x_f}{x_g}$  is rather small

Values of  $\frac{x_f}{x_g}$  in  $[\frac{1}{2} : 2]$  still lead to negative cross sections at large  $p_{\perp}$

However the  $p_{\perp}$  value where this occurs depends strongly on this ratio

In particular a value of  $\frac{x_f}{x_g} = 2$  extends significantly the range of positivity

## Future improvements

These results may not seem very promising but they were obtained in a very simplistic approach.

Future directions that may lead to improvements:

- ▶ **Implement the light cone ordering condition in an exact way** in the transverse momentum integrals. For now we have used the external transverse scales  $k_{\perp}$  and  $Q_s$ , which allows us to reuse many results of [Chirilli, Xiao, Yuan](#)
- ▶ **Use a more physical dipole cross section**  
The [GBW](#) model leads to simple analytical expressions. However in this model the NLO cross section is completely governed by the NLO corrections ( $\sim k_{\perp}^{-4}$ ) at large  $p_{\perp}$ . A dipole cross section obtained by solving the [Balitsky-Kovchegov](#) equation should lead to a power-law behaviour of the LO contribution at large  $p_{\perp}$  and so less sensitivity to the NLO corrections

## Conclusions

We proposed to modify the subtraction procedure of the rapidity divergence to solve the issue of large negative NLO corrections at large  $p_{\perp}$  in this process

- ▶ We introduced a **rapidity factorization scale**  $\xi_f$ 
  - The NLO cross section at large  $p_{\perp}$  is very sensitive to the choice of  $\xi_f$
  - By increasing  $\xi_f$  it is possible to make the cross section positive up to arbitrarily large values of  $p_{\perp}$
- ▶ We proposed to fix  $\xi_f$  by imposing **light cone ordering**
  - The cross section still becomes negative at some  $p_{\perp}$  when  $\xi_f$  is varied in its “natural” range
  - The  $p_{\perp}$  value at which this occurs changes a lot in this “natural” range

Directions for future work:

- ▶ Implement the light cone ordering condition in an exact way
- ▶ Use more physical dipole cross sections

These steps are necessary before drawing definitive conclusions on this approach