Including resummation in the NLO BK equation

T. Lappi

University of Jyväskylä, Finland

Hard Probes 2016, Wuhan

Outline

Outline of the talk:

- NLO BK equation
- Numerical result: $\ln r$ divergence
- Double and single log resummation
- Numerical results: resummation only and resummation + finite terms

Literature

- "Next-to-leading order evolution of color dipoles," I. Balitsky and G. A. Chirilli, Phys. Rev. D **77** (2008) 014019, [arXiv:0710.4330 [hep-ph]] .
- "Direct numerical solution of the coordinate space Balitsky-Kovchegov equation at next to leading order," T. L., **H. Mäntysaari**,
  Phys. Rev. D **91** (2015) 074016, [arXiv:1502.02400 [hep-ph]]
- "Resumming double logarithms in the QCD evolution of color dipoles,"
  E. Iancu et al Phys. Lett. B **744** (2015) 293 , [arXiv:1502.05642 [hep-ph]] .
- "Collinearly-improved BK evolution meets the HERA data," E. Iancu et al Phys. Lett. B **750** (2015) 643 , [arXiv:1507.03651 [hep-ph]] .
- "Next-to-leading order Balitsky-Kovchegov equation with resummation,"
  T. L., **H. Mäntysaari**,
  Phys. Rev. D **93** (2016) 094004, [arXiv:1601.06598 [hep-ph]] .

Literature

- "Next-to-leading order evolution of color dipoles," I. Balitsky and G. A. Chirilli, Phys. Rev. D **77** (2008) 014019, [arXiv:0710.4330 [hep-ph]] .
- "Direct numerical solution of the coordinate space Balitsky-Kovchegov equation at next to leading order," T. L., **H. Mäntysaari**, Phys. Rev. D **91** (2015) 074016, [arXiv:1502.02400 [hep-ph]]
- "Resumming double logarithms in the QCD evolution of color dipoles," E. Iancu et al Phys. Lett. B **744** (2015) 293 , [arXiv:1502.05642 [hep-ph]] .
- "Collinearly-improved BK evolution meets the HERA data," E. Iancu et al Phys. Lett. B **750** (2015) 643 , [arXiv:1507.03651 [hep-ph]] .
- "Next-to-leading order Balitsky-Kovchegov equation with resummation," T. L., **H. Mäntysaari**, Phys. Rev. D **93** (2016) 094004, [arXiv:1601.06598 [hep-ph]] .

Motivation

Many ingredients available for NLO small-x calculations:

- NLO BK equation
- NLO JIMWLK equation
- NLO γ^* impact factor for DIS
- NLO single inclusive cross section for forward pA
- ...

Armed with these, want phanomenology @ NLO!
But first need to solve the evolution equation(s)!

The NLO BK equation

as derived by Balitsky and Chirilli, 2007

Equation

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} \boldsymbol{K_1} \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} \boldsymbol{K_f} \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} \boldsymbol{K_2} \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$

Notations & approximations

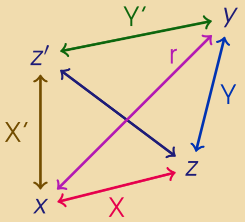
$$S(x-y) \equiv \langle \mathrm{Tr}\, U^\dagger(x) U(y) \rangle$$

$$\otimes = \int d^2 z \quad \Big/ \quad \int d^2 z\, d^2 z'$$

- Here large N_c & mean field:

$$\langle \mathrm{Tr}\, U^\dagger U\, \mathrm{Tr}\, U^\dagger U \rangle \to \langle \mathrm{Tr}\, U^\dagger U \rangle \langle \mathrm{Tr}\, U^\dagger U \rangle$$

Coordinates



The NLO BK equation

as derived by Balitsky and Chirilli, 2007

Equation

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2} K_1 \otimes [S(X)S(Y) - S(r)] + \frac{\alpha_s^2 N_F N_c}{8\pi^4} K_f \otimes S(Y)[S(X') - S(X)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)]$$

Notations & approximations

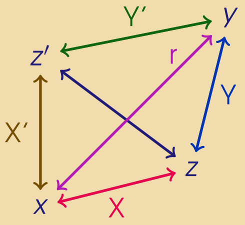
$$S(x-y) \equiv \langle \mathrm{Tr}\, U^\dagger(x) U(y) \rangle$$

$$\otimes = \int d^2 z \quad \Big/ \quad \int d^2 z\, d^2 z'$$

- Here large N_c & mean field:

$$\langle \mathrm{Tr}\, U^\dagger U \,\mathrm{Tr}\, U^\dagger U \rangle \to \langle \mathrm{Tr}\, U^\dagger U \rangle \langle \mathrm{Tr}\, U^\dagger U \rangle$$

Coordinates



Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

$$K_f = \frac{2}{(z-z')^4} - \frac{X'^2 Y^2 + Y'^2 X^2 - r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)}\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2 (X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

$$K_f = \frac{2}{(z-z')^4} - \frac{X'^2 Y^2 + Y'^2 X^2 - r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)}\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

- Leading order

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)} + \frac{r^4}{X^2Y'^2(X^2Y'^2 - X'^2Y^2)} + \frac{r^2}{X^2Y'^2(z-z')^2}\right]\ln\frac{X^2Y'^2}{X'^2Y^2}$$

$$K_f = \frac{2}{(z-z')^4} - \frac{X'^2Y^2 + Y'^2X^2 - r^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2}$$

- Leading order
- Running coupling (Terms with β function coefficient)

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

$$K_f = \frac{2}{(z-z')^4} - \frac{X'^2 Y^2 + Y'^2 X^2 - r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)}\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)

Kernels

$$K_1 = \frac{r^2}{X^2 Y^2}\left[1 + \frac{\alpha_s N_c}{4\pi}\left(\frac{\beta}{N_c}\ln r^2\mu^2 - \frac{\beta}{N_c}\frac{X^2 - Y^2}{r^2}\ln\frac{X^2}{Y^2} + \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right)\right]$$

$$K_2 = -\frac{2}{(z-z')^4} + \left[\frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^4}{X^2 Y'^2(X^2 Y'^2 - X'^2 Y^2)} + \frac{r^2}{X^2 Y'^2 (z-z')^2}\right]\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

$$K_f = \frac{2}{(z-z')^4} - \frac{X'^2 Y^2 + Y'^2 X^2 - r^2(z-z')^2}{(z-z')^4(X^2 Y'^2 - X'^2 Y^2)}\ln\frac{X^2 Y'^2}{X'^2 Y^2}$$

- Leading order
- Running coupling (Terms with β function coefficient)
- Conformal logs $\Longrightarrow$ vanish for $r = 0$ ($X = Y$ & $X' = Y'$)
- Nonconformal double log $\Longrightarrow$ blows up for $r = 0$

Running coupling

Absorb the β-terms into

- "Balitsky" running for LO term
- Parent dipole running for NLO terms

Now:

$$\frac{\alpha_s N_c}{2\pi^2} K_1 = \frac{\alpha_s(r) N_c}{2\pi^2}\left[\frac{r^2}{X^2 Y^2} + \frac{1}{X^2}\left(\frac{\alpha_s(X)}{\alpha_s(Y)} - 1\right) + \frac{1}{Y^2}\left(\frac{\alpha_s(Y)}{\alpha_s(X)} - 1\right)\right]$$
$$+ \frac{\alpha_s(r)^2 N_c{}^2}{8\pi^3}\frac{r^2}{X^2 Y^2}\left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{N_F}{N_c} - 2\ln\frac{X^2}{r^2}\ln\frac{Y^2}{r^2}\right]$$

Freeze coupling in the IR to $\alpha_s(r \to \infty) = 0.76$:

$$\alpha_s(r) = \frac{4\pi}{\beta \ln\left\{\left[(2.5)^{10} + \left(\left(4e^{-2\gamma_E}\right) / \left(r^2 \Lambda_{QCD}^2\right)\right)^5\right]^{0.2}\right\}} \sim \frac{4\pi}{\beta \ln\left\{\left(\frac{4e^{-2\gamma_E}}{r^2\Lambda_{QCD}^2}\right)\right\}}$$

Initial condition

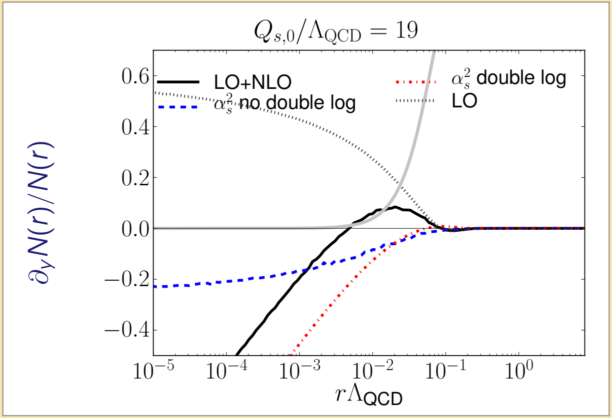
$$N(r) \equiv 1 - S(r) = 1 - \exp\left[-\frac{(r^2 Q_{s0}^2)^{\gamma}}{4} \ln\left(\frac{1}{r\Lambda_{\rm QCD}} + e\right)\right],$$

2 tunable parameters

- $\frac{Q_{s0}}{\Lambda_{\rm QCD}}$ $\Longrightarrow$ basically determines value of α_s at $y = 0$
- γ: anomalous dimension: shape $N(r) \sim r^{2\gamma}$
  - LO phenomenology prefers $\gamma \gtrsim 1$ at $y = 0$
  - At LO this eventually evolves into $\gamma \sim 0.8$ (running α_s)

Large double log

Relative change of $N(r)$ in one step dy diverges for small r



Behavior caused by the nonconformal double log term

T.L., H. Mäntysaari 2015

Double log resummation

Integral form of BK: double log $\sim$ shift of rapidity variable

$$\partial_y S_y(r) = \alpha_s \int_z \frac{r^2}{X^2 Y^2} \left[1 + \frac{\alpha_s N_c}{2\pi} \ln\frac{X^2}{r^2} \ln\frac{Y^2}{r^2}\right] [S_y(X) S_y(Y) - S_y(r)]$$

$$\Longleftrightarrow$$

$$S_y(r) = \alpha_s \int^y \mathrm{d}y' \int_z \frac{r^2}{X^2 Y^2} \left[1 + \frac{\alpha_s N_c}{2\pi} \ln\frac{X^2}{r^2} \ln\frac{Y^2}{r^2}\right] [S_{y'}(X) S_{y'}(Y) - S_{y'}(r)]$$

$$\Longleftrightarrow$$

$$S_y(r) = \alpha_s \int \mathrm{d}^2 z \int^{\mathbf{y - \ln z^2/r^2}} \mathrm{d}y' \frac{r^2}{X^2 Y^2} [S_{y'}(X) S_{y'}(Y) - S_{y'}(r)]$$

For $r \ll z \sim 1/Q_s$. $\Longrightarrow$ "Kinematical constraint" Beuf

Iancu et al 2015 rewrite as practical rapidity-local form

$$\left[1 + \frac{\alpha_s N_c}{2\pi} \ln\frac{X^2}{r^2} \ln\frac{Y^2}{r^2}\right] \Longrightarrow \frac{J_1\left(2\sqrt{\frac{\alpha_s N_c}{\pi} \ln\frac{X^2}{r^2} \ln\frac{Y^2}{r^2}}\right)}{\sqrt{\frac{\alpha_s N_c}{\pi} \ln\frac{X^2}{r^2} \ln\frac{Y^2}{r^2}}} \quad \text{with} \quad \int^y \mathrm{d}y'$$

Single log resummation

Iancu et al 2015 : there is also a single log,
with DGLAP anomalous dimension $A_1 = 11/12$:

$$\partial_Y S(r) = \alpha_s^2 \int_{zz'} \left[-\frac{2}{(z-z')^4} + \frac{X^2 Y'^2 + X'^2 Y^2 - 4r^2(z-z')^2}{(z-z')^4 (X^2 Y'^2 - X'^2 Y^2)} \ln \frac{X^2 Y'^2}{X'^2 Y^2} \right]$$
$$\times [S(X)S(z-z')S(Y') - S(X)S(Y)] + \ldots$$
$$\sim \alpha_s^2 A_1 \int_z \frac{r^2}{X^2 Y^2} \ln \frac{\min\{X^2, Y^2\}}{r^2} + \ldots$$

Should be resummed into

$$K_{STL} = \exp\left\{ -\frac{\alpha_s N_c A_1}{\pi} \left| \ln \frac{C_{sub} r^2}{\min\{X^2, Y^2\}} r^2 \right| \right\}$$

- Iancu et al take $C_{sub} = 1$ and throw other NLO terms away
- This captures leading logs, but leaves other α_s^2-terms unknown, and result dependent on arbitrary C_{sub}

Full equation

$$\partial_Y S(r) = \frac{\alpha_s N_c}{2\pi^2} K_1 \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c{}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)] + N_F\text{-part}$$

Now solve resummed equation

$$\partial_Y S(r) = \frac{\alpha_s N_c}{2\pi^2} \left[K_{DLA} K_{STL} K_{Bal} - K_{\text{sub}} + K_1^{\text{fin}}\right] \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 N_c{}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)] + N_F\text{-part}$$

- $K_{DLA} \sim J_1(x)/x$ would give double log in K_1 if expanded in α_s
- K_{Bal} resums β-function terms in K_1
- $K_{STL} = \exp\left\{-\alpha_s N_c A_1 \# \ln r^2\right\}$ resums single log
- K_{sub} subtracts α_s-part of K_{STL}, which is already in K_2
- K_1^{fin} is rest (nonlog) part in K_1

Resummation and rest

$$\partial_y S(r) = \frac{\alpha_s N_c}{2\pi^2}\left[K_{DLA}K_{STL}K_{Bal} - K_{sub} + K_1^{fin}\right] \otimes [S(X)S(Y) - S(r)]$$
$$+ \frac{\alpha_s^2 {N_c}^2}{8\pi^4} K_2 \otimes [S(X)S(z-z')S(Y') - S(X)S(Y)] + N_F\text{-part}$$

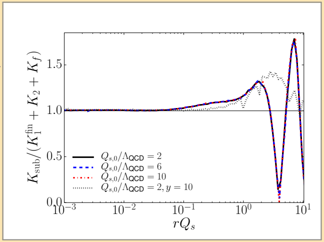
Split this into three parts

LO (running coupling) $\frac{\alpha_s N_c}{2\pi^2} K_{Bal}$: used so far

Resummation $\frac{\alpha_s N_c}{2\pi^2} K_{Bal} [K_{DLA}K_{STL} - 1]$: ($\sim$ used by Iancu et al)

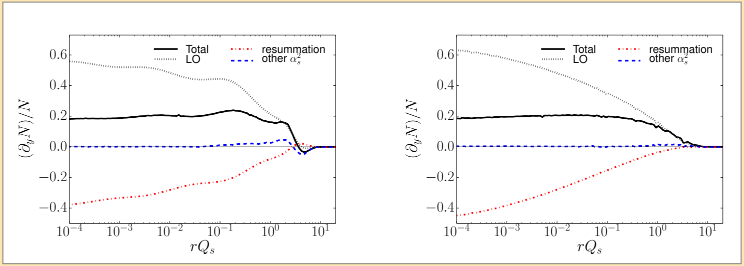
Finite NLO rest: $-K_{sub} + K_1^{fin}$ and K_2, K_f

- Separation depends on C_{sub} in $K_{STL} = \exp\left\{-\frac{\alpha_s N_c A_1}{\pi}\left|\ln \frac{C_{sub} r^2}{\min\{X^2, Y^2\}} r^2\right|\right\}$
- Numerically **choose** C_{sub} to **minimize** the finite NLO part: result $C_{sub} = 0.65$



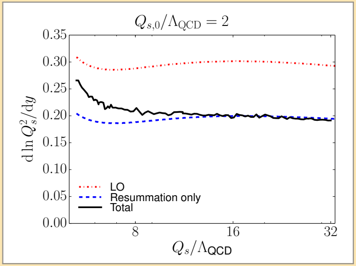
Contribution to evolution speed

y=0 : finite NLO small　　　y=10 : finite NLO negligible



Finite NLO terms

- Significant for small y
  $\Longrightarrow$ phenomenology
- Negligible at $y \to \infty$



Anomalous dimension

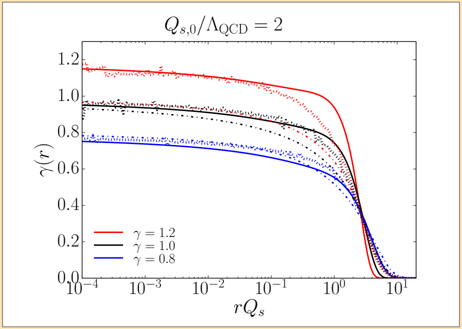
Recall initial condition: $N(r) = 1 - e^{-\frac{(r^2 Q_{s0}^2)^\gamma}{4} \ln\left(\frac{1}{r\Lambda_{QCD}} + e\right)}$,

Define

$$\gamma(r) \equiv -\frac{d \ln N(r)}{d \ln r^2}$$

Geometric scaling?

- LO: fast to $\gamma \sim 0.8$
- NLO: stay at initial γ



- Solid: initial condition
- Dotted: $y = 5$ NLO
- Dot-dashed: $y = 5$ LO (rc)

Anomalous dimension

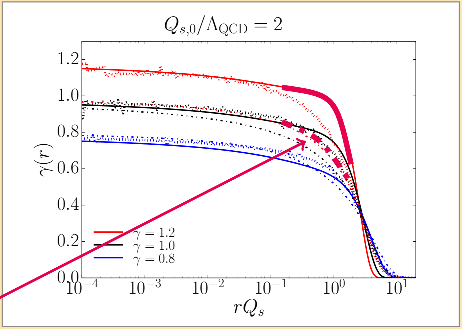
Recall initial condition: $N(r) = 1 - e^{-\frac{(r^2 Q_{s0}^2)^\gamma}{4} \ln\left(\frac{1}{r\Lambda_{QCD}} + e\right)}$,

Define

$$\gamma(r) \equiv -\frac{d \ln N(r)}{d \ln r^2}$$

Geometric scaling?

- LO: fast to $\gamma \sim 0.8$
- NLO: stay at initial γ



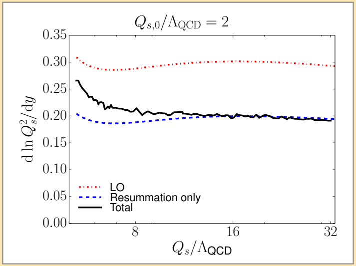
LO $y = 0$ to $y = 5$

- Solid: initial condition
- Dotted: $y = 5$ NLO
- Dot-dashed: $y = 5$ LO (rc)

Conclusions

Numerical solution of NLO BK equation

- Resummation
  - Resummation of double ⊥ logs stabilizes equation
  - Can also resum single ⊥ logs, ambiguity in constant C_{sub}
  - Numerically: $C_{\mathrm{sub}} = 0.65$ minimizes finite NLO
- Properties of solution
  - Finite NLO terms important for realistic $Q_s/\Lambda_{\mathrm{QCD}}$
  - Geometric scaling: sets in slowly if at all



Outlook:

- Fit DIS
- Apply to pA