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A Novel Approach For Event-By-Event Early Gluon Fields

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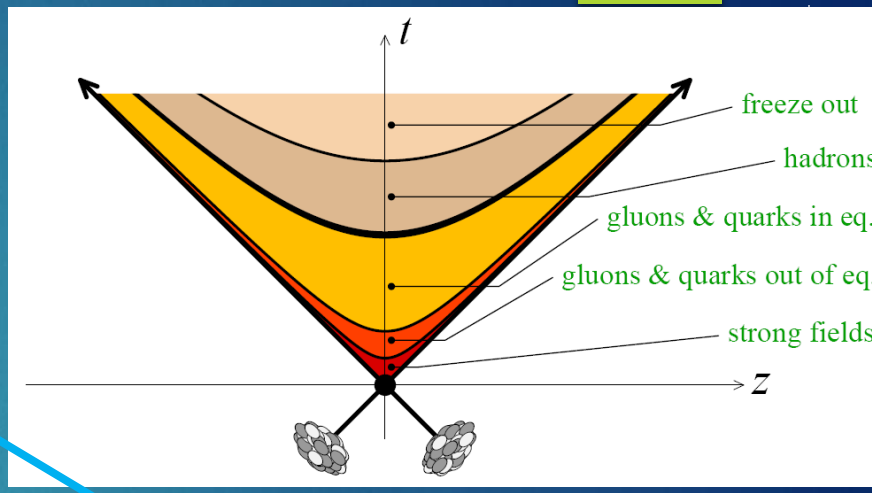
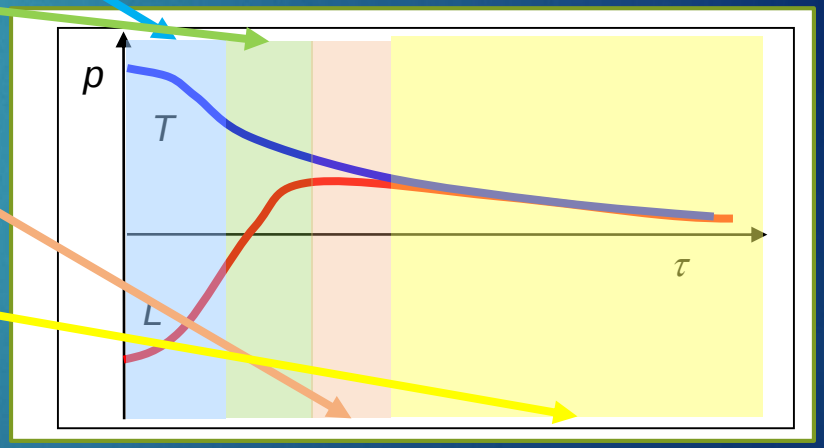
Brief Outline

- Introduction and Motivation
- Classical Gluon Fields: Formalism
- Review of Analytic Results for Event Averages
- Semi-Analytic Approach to Event-By-Event Simulations

Little Bang: Macroscopic Evolution

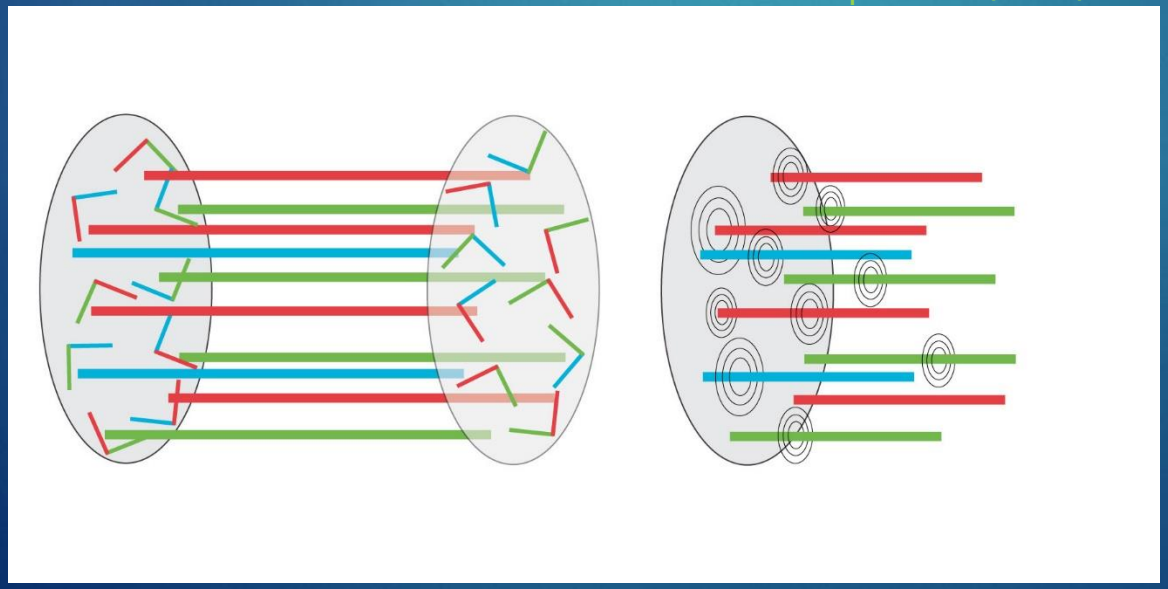
- Initial nuclear wave functions
- Strong classical gluon fields
- Decoherence
- Local equilibration/isotropization
- QGP/HG fluid close to local equilibrium
- Hadron gas in the kinetic regime and freeze-out





Early Time Evolution

- Initial wave function can be calculated approximately using Color Glass picture.
- Overlap mechanism of nuclear fields known (in the light cone limit) = boundary conditions for further time evolution. Kovner, McLerran, Weigert (1995)
- Dominance of longitudinal (chromo) electric and magnetic fields ("flux tubes") McLerran, Lappi (2006) RJF, Kapusta, Li (2006)



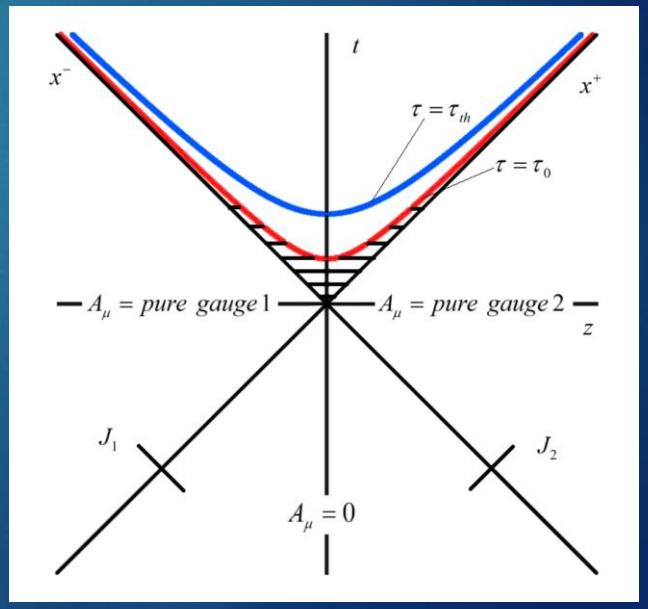
CGC In The Classical Approximation

- Field sources = light cone current *J* (created by transverse color charge distributions ρ).
- ρ from *Gaussian* color fluctuations of a color-neutral nucleus.

$$\left\langle \rho_i^a(x) \right\rangle = 0$$

$$\left\langle \rho_i^a(x_1)\rho_j^b(x_2) \right\rangle = \frac{g^2}{N_c^2-1}\delta_{ij}\delta^{ab}\lambda_i\left(x_1^{\mp}\right)\delta\left(x_1^{\mp}-x_2^{\mp}\right)\delta^2\left(\mathbf{x}_{1T}-\mathbf{x}_{2T}\right) \qquad \mu_i = \int dx^{\mp}\lambda_i\left(x^{\mp}\right)$$

- Solve Yang-Mills equations $[D^\mu, F^{\mu\nu}] = J^\nu$ for gluon field $A^\mu(\rho)$.
- From $A^\mu(\rho)$ calculate $F^{\mu\nu}(\rho)$ and $T^{\mu\nu}(\rho)$.



Solving Yang-Mills for $\tau \geq 0$

- Numerical Solutions → Successful Phenomenology, available EBE.

  Krasnitz, Nara, Venugopalan (2003); Lappi (2003); ...
  Fukushima, Gelis (2012); Schenke, Tribedy, Venugopalan (2012)

- Here: Analytic solution, expansion of gauge field in time τ

  RJF, Kapusta, Li (2006)
  Chen, Fries, Kapusta, Li (2015)

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

$$A^i_\perp(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A^i_{\perp(n)}(x_\perp)$$

- Boundary conditions

$$A^i_{\perp(0)}(x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

- Recursive solution:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D^i_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]$$

$$A^i_{\perp(n)} = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D^j_{(k)}, F^{ji}_{(l)}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]\right)$$

Solving Yang-Mills for $\tau \geq 0$

- Gauge field in terms of colliding fields

$$A(\tau, x_\perp) = A_{(0)} + \frac{\tau^2}{8}[D^j, [D^j, A_{(0)}]] + \frac{\tau^4}{192}[D^k, [D^k, [D^j, [D^j, A_{(0)}]]]] + \frac{ig\tau^4}{48}\epsilon^{ij}[D^i A_{(0)}, D^j B_0] + \mathcal{O}(\tau^6)\,,$$

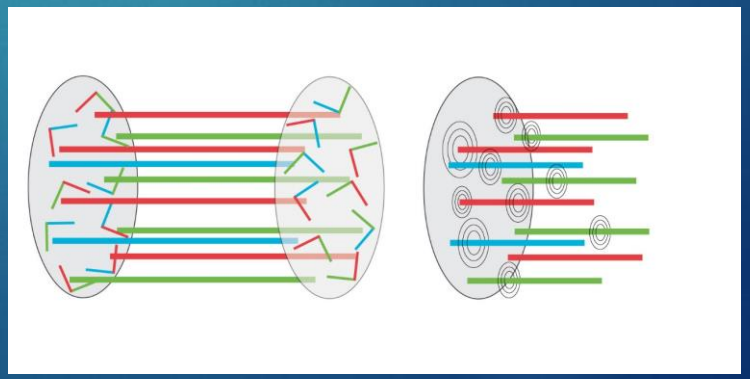
$$A^i_\perp(\tau, x_\perp) = A^i_{\perp(0)} + \frac{\tau^2}{4}\epsilon^{ij}[D^j, B_0] + \frac{\tau^4}{64}\epsilon^{ij} D^j D^k D^k B_0 - \frac{ig\tau^4}{64}[B_0, D^i B_0] + \frac{ig\tau^4}{16}[A_{(0)}, [D^i, A_{(0)}]] + \mathcal{O}(\tau^6)$$

- 0th order (boundary conditions at $\tau = 0$): Immediately leads to the known initial longitudinal field

McLerran, Lappi (2006)
RJF, Kapusta, Li (2006)

$$F^{+-}_{(0)} = ig\left[A^i_1, A^i_2\right]$$
$$F^{21}_{(0)} = ig\varepsilon^{ij}\left[A^i_1, A^j_2\right]$$

- Transverse fields build up through the QCD versions of Gauss' Law, Ampere's Law, Faraday's Law.



Energy Momentum Tensor at $O(\tau^2)$

- Minkowski Components

$\vec{S} = \vec{E} \times \vec{B}$

$$T_{\mathrm{f}}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O(\tau^2) & \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O(\tau^2) & O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) & \varepsilon_0 + O(\tau^2) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O(\tau^2) \end{pmatrix}$$

- 0th order: Color Capacitor $\varepsilon_0 = \frac{1}{2}(E_0^2 + B_0^2)$
- 1st order: Poynting vector → Transverse Flow

RJF, Kapusta, Li (2006)
Chen, RJF (2013)
Chen, RJF, Kapusta, Li (2015)

$$\alpha^i = -\frac{\tau}{2}\nabla^i \varepsilon_0$$

Like hydrodynamic flow, gradient of transverse pressure $p_T = \varepsilon_0$; even in rapidity.

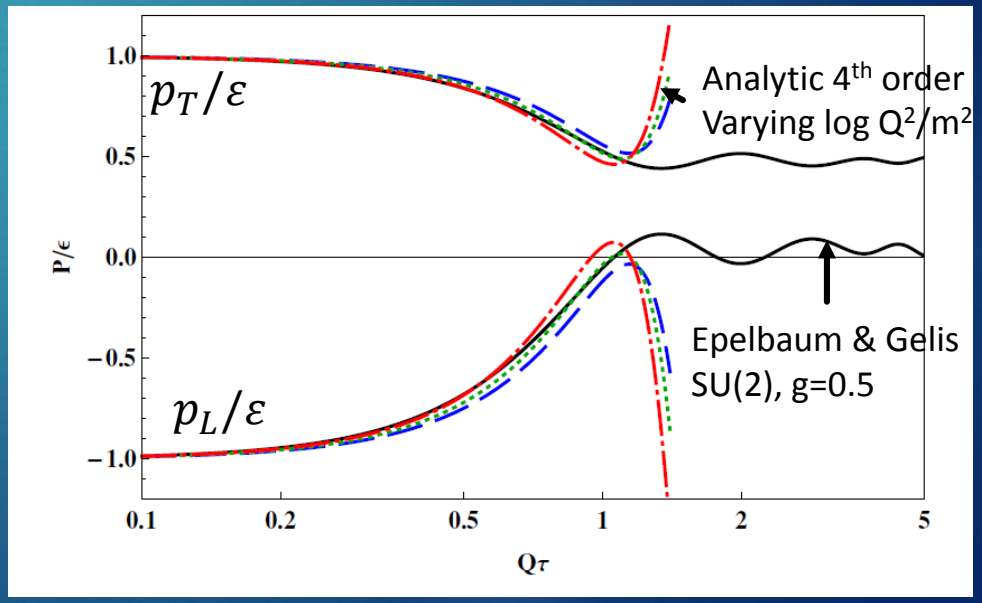
$$\beta^i = \frac{\tau}{2}\varepsilon^{ij}\left([D^j, B_0]E_0 - [D^j, E_0]B_0\right)$$

Beyond hydro; odd in rapidity

Early Energy Density and Pressure

- Event averages can be calculated analytically, $\langle \rho_1^2 \rangle \sim \mu_1, \langle \rho_2^2 \rangle \sim \mu_2$
  - Initial energy density $\varepsilon_0 = \frac{g^6 N_c \left(N_c^2 - 1\right)}{8\pi} \mu_1 \mu_2 \ln^2 \frac{Q^2}{m^2}$ Lappi (2006)
  - Initial flow $\alpha^i = -\varepsilon_0 \frac{\nabla^i (\mu_1 \mu_2)}{\mu_1 \mu_2}$ $\beta^i = -\varepsilon_0 \frac{\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2}{\mu_1 \mu_2}$ Chen, RJF (2013) Chen, RJF, Kapusta, Li (2015)
- Early time evolution of pressure and energy density, consistent with numerical evaluations.
- Simplified pocket formulas
  - $\varepsilon = \varepsilon_0 \left(1 - \frac{1}{2}(Q\tau)^2 + O(\tau^4)\right)$
  - $p_T = \varepsilon_0 \left(1 - (Q\tau)^2 + O(\tau^4)\right)$
  - $p_L = \varepsilon_0 \left(1 - \frac{3}{2}(Q\tau)^2 + O(\tau^4)\right)$
- Resumming $(Q\tau)^k$ terms

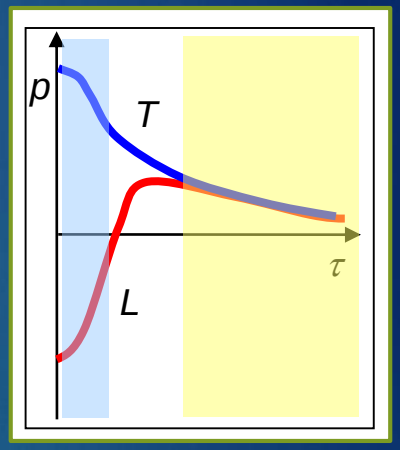
Li, Kapusta, 1602:0906



Switching to Hydro

- No equilibration in classical YM.
- Instantanous matching between clYM and viscous fluid dynamics, enforce conservation laws.

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$$



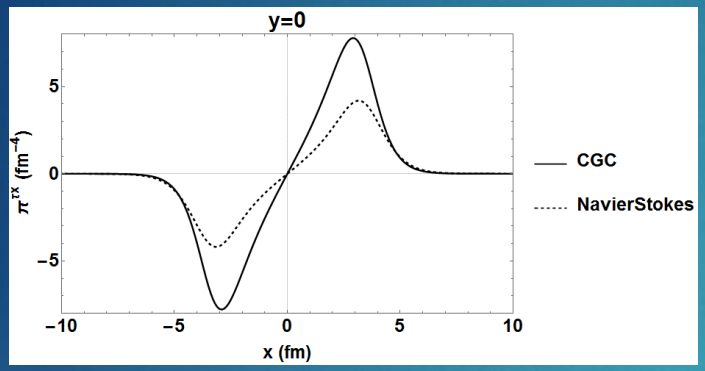
- Equivalent to direct decomposition of YM energy momentum tensor into hydro fields at some matching time τ_0:

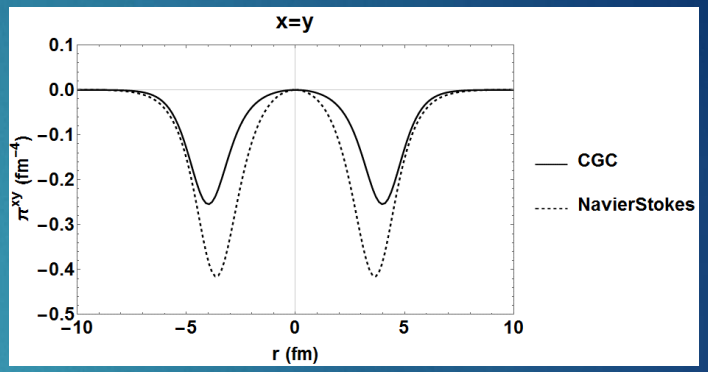
$$T_f^{\mu\nu} = (\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

- Keep viscous stress even if it is large (otherwise violate energy+momentum conservation at $\tau = \tau_0$).
- Viscous stress extracted from YM generally follows Navier-Stokes behavior.

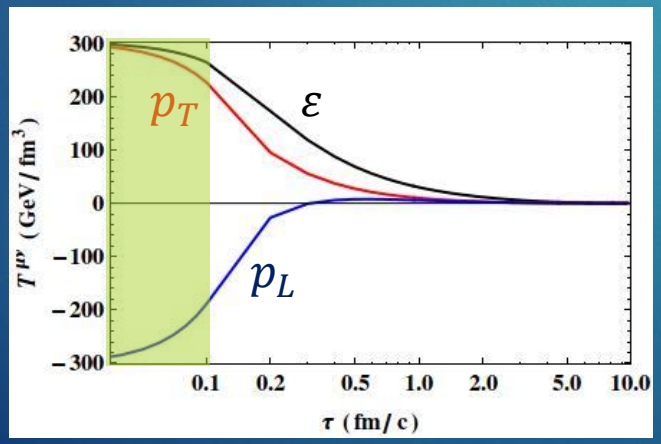
Yang Mills in Terms of Hydro Fields

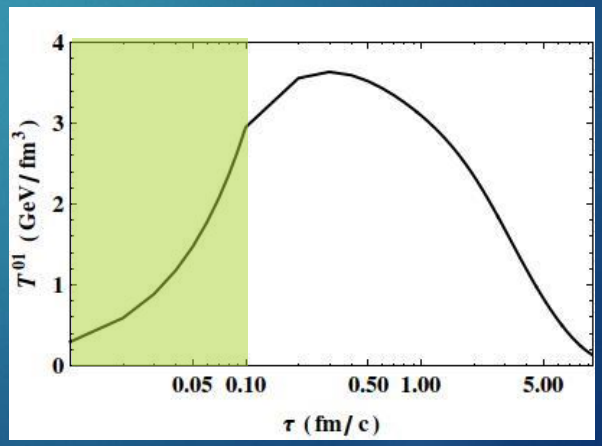
- Viscous stress extracted from YM generally follows Navier-Stokes behavior.





- Pressure and flow fields evolve smoothly from YM to vHydro as expected.
  - Here Pb+Pb, b = 6 fm, $\eta/s = 1/4\pi$, matched at $\tau = 0.1$ fm.





Event-By-Event Calculations

- Event-by-event calculations (semi-analytic) $\rho_1, \rho_2 \rightarrow A_1^i, A_2^i$
- No need to solve 3+1D differential equations numerically
  - Use recursion relation to solve the time evolution $\longrightarrow$ 2-D problem
  - Solve 2-D problem through tabulated Greens functions
- Work Flow

MC Sampling of charge distributions ρ_1, ρ_2
Set UV scale Q at this step through coarse graining

Solve for gauge fields α_1, α_2 in covariant gauge

$$\Delta\alpha_i = -\rho_i$$

Set IR scale m In the Greens function

Gauge transformation to physical gauge A_1^i, A_2^i

$A^k = \frac{i}{g} U \partial^k U^\dagger$ where $U = \mathbb{P}\exp\left[-ig\int \alpha dz^{\mp}\right]$

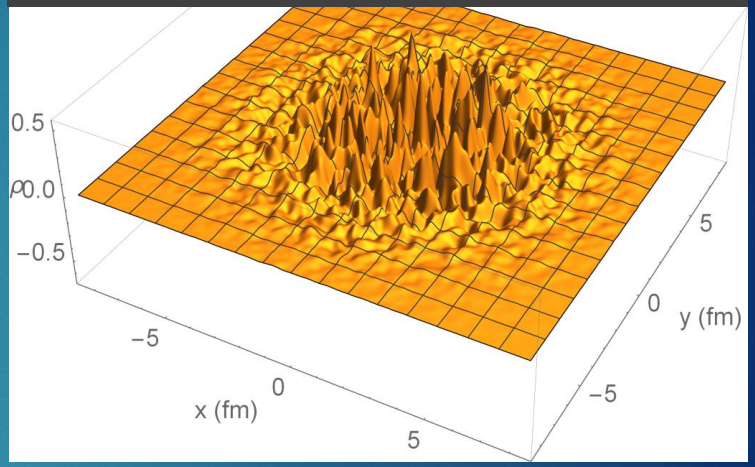
Calculate coefficients in our power series.

- First 3 steps similar to earlier work.

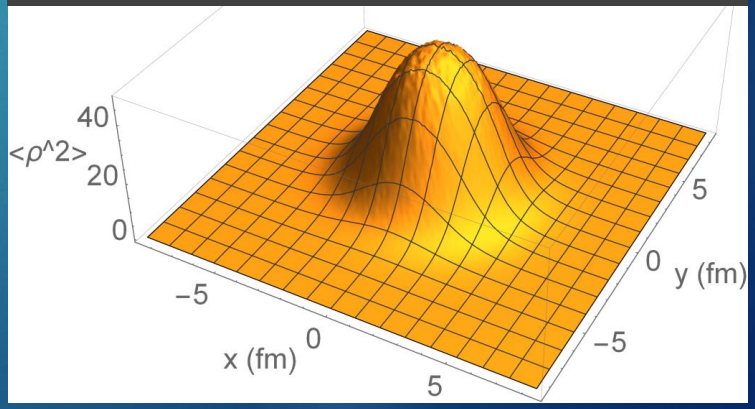
Charge Distributions

- Sample nuclei $\langle \rho_1^2 \rangle \sim \mu_1, \langle \rho_2^2 \rangle \sim \mu_2$
- Here: $\mu_i \sim$ Woods-Saxon nuclear profile, integrated in longitudinal direction.
  - No correlations from nucleons, fixed positions, fixed sizes
- Coarse graining of charges on a scale $1/Q$. UV/coarse graining scale Q fixed explicitly.
- Simplest possible modeling of charges, will be improved in the near future.

Color charge density $\rho^{\mathbf{1}}$; R = 3 fm, Q=1 GeV; typical event



Average charge squared $\langle \rho^{\mathbf{c}} \rho^{\mathbf{c}} \rangle$; R = 3 fm, Q=1 GeV ; 7800 events



Single Nucleus Potentials

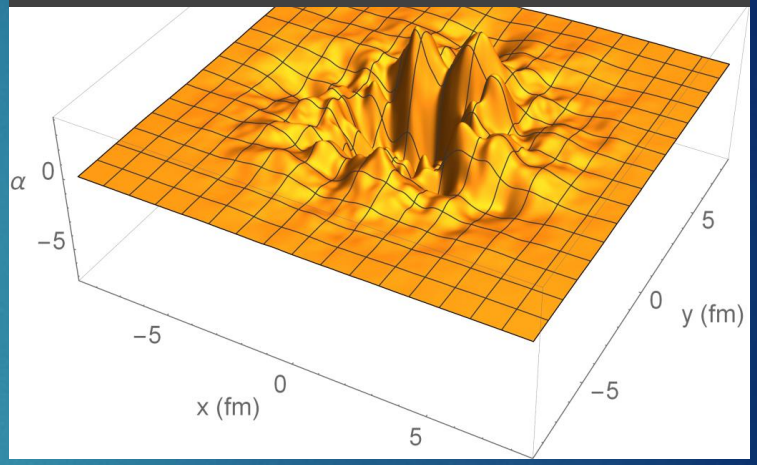
- Solve using pre-tabulated Green's function.

- For speed-up the coarse graining of the charges is actually carried out through a coarse grained Green's function

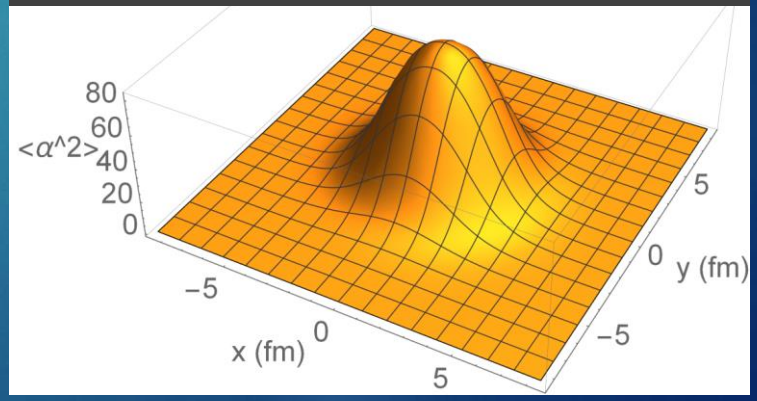
$$G_C(\vec{x}_\perp) = \frac{1}{2\pi}\int K_0(m|\vec{x}_\perp - \vec{z}|)e^{-Q^2(\vec{x}_\perp-\vec{z})^2}d^2z$$

- All scale dependences in one formula
- No dependence on size or grid constant of the transverse grid.

Covariant potential α^1 ; Q=1 GeV, m=0.2 GeV; typical event

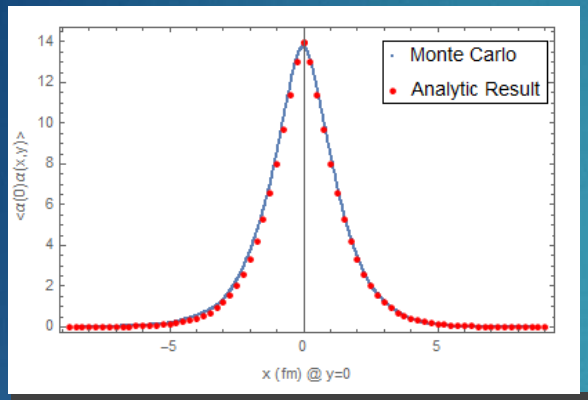


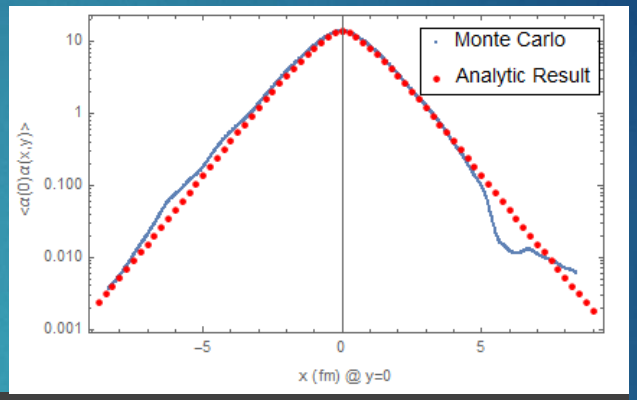
Average potential squared $\langle \alpha^c \alpha^c \rangle$; Q=1 GeV, m=0.2 GeV ; 7800 events



Scales and Correlation Functions

- One easily reproduces analytic correlation functions with given m.





Correlation function $\langle \alpha^{\mathbf{c}}(0,0)\alpha^{\mathbf{c}}(x,0)\rangle$; Q=1 GeV, m=0.2 GeV ; 7800 events

- Infrared scale well controlled.

- We still need to systematically study the dependence on the UV scale.

Single Nucleus LC Fields to

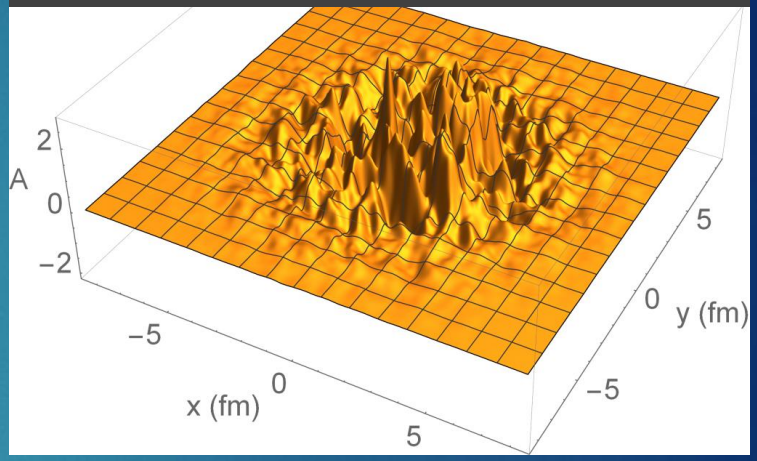
- Gauge transformation $\longrightarrow$ obtain fields A_1^i, A_2^i in each nucleus.

- Using discretized Wilson lines

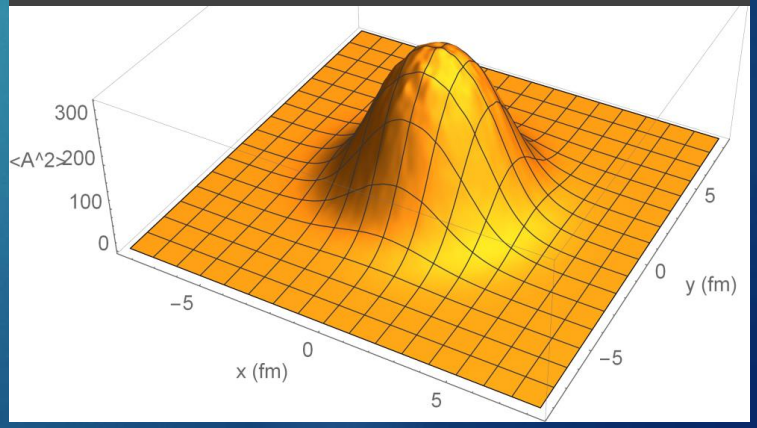
$$U\left(x_{(i,j)}\right) = \prod_{k=1}^{N_\pm} e^{-ig\alpha_a\left(z^\pm{}_k, x_{(i,j)}\right)\Delta z^\pm}$$

- Numerical derivatives (here: 4th order)

Covariant field $A^{1,x}$; Q=1 GeV, m=0.2 GeV; typical event



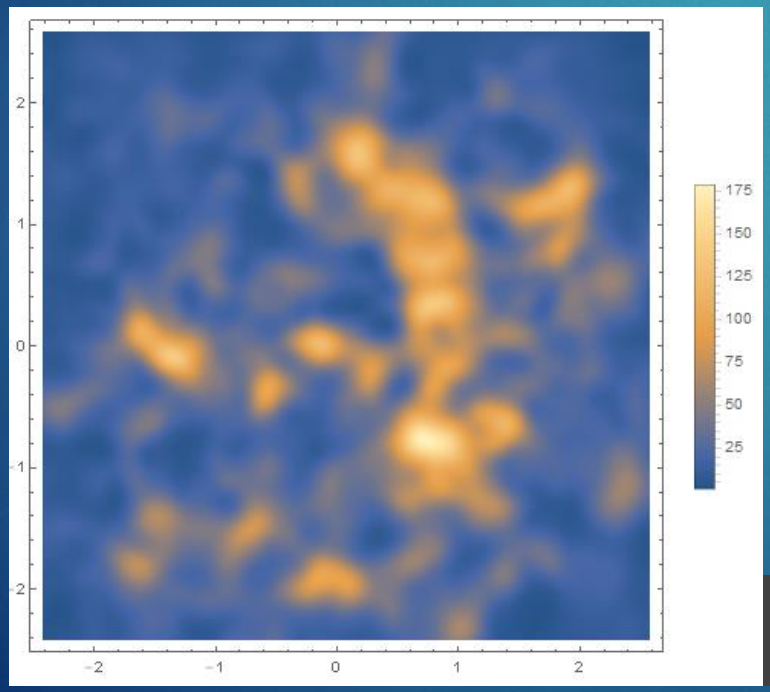
Average potential squared $\langle A^{c,i}A^{c,i}\rangle$; Q=1 GeV, m=0.2 GeV ; 7800 events



After The Collision

- Initial longitudinal fields $E_0 = ig\delta^{ij}[A^i, A^j]$, $B_0 = ig\varepsilon^{ij}[A^i, A^j]$

- Initial energy density $\varepsilon_0 = \frac{1}{2}(E_0^2 + B_0^2)$



Initial energy density ε_0; b=0; Q=1 GeV, m=0.2 GeV; typical event

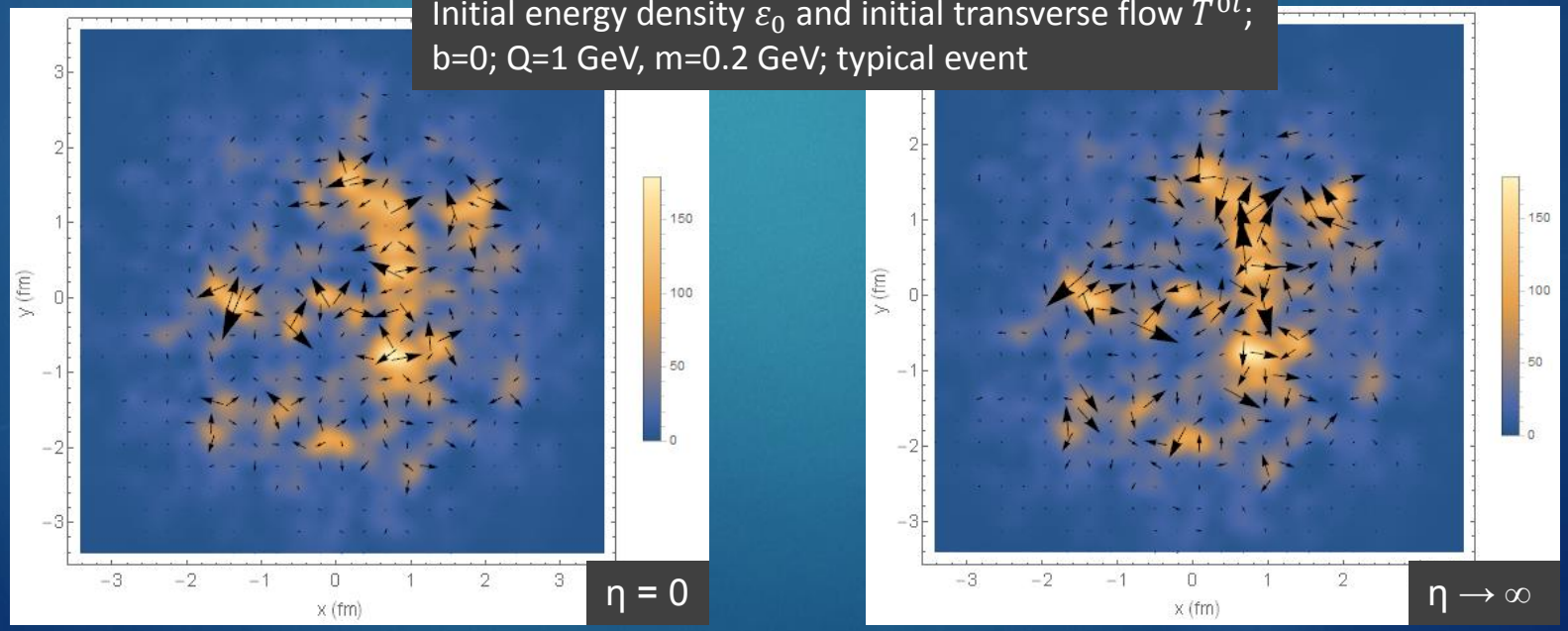
Initial Flow

- Initial transverse flow

$$T^{0i} = -\frac{\tau}{2}\nabla^i \varepsilon_0 \cosh\eta + \frac{\tau}{2}\varepsilon^{ij}\left([D^j, B_0]E_0 - [D^j, E_0]B_0\right)\sinh\eta + O(\tau^3),$$

- $\langle \beta^i \rangle$ vanishes for central collisions but not event-by-event.
- Non-abelian effects present in individual events.
- Effects to be explored in the future.

Initial energy density ε_0 and initial transverse flow T^{0i}; b=0; Q=1 GeV, m=0.2 GeV; typical event



Higher Order Coefficients

- Second order: response to the flow field. For example energy density

$$T^{00} = \varepsilon_0 - \frac{\tau^2}{4}\left(\nabla^i \alpha^i + \delta\right) - \frac{\tau^2}{8}\nabla^i \beta^i \sinh(2\eta) + \frac{\tau^2}{8}\delta \cosh(2\eta)$$

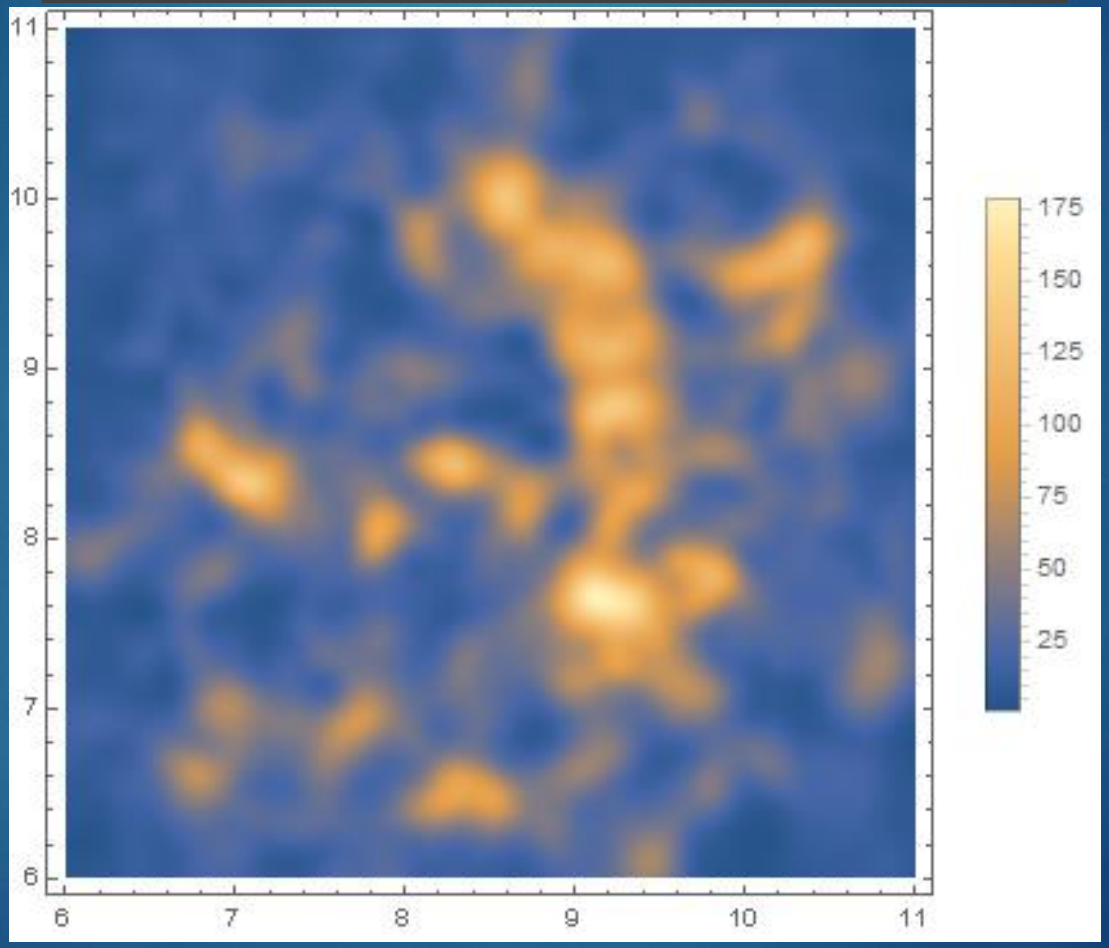
$$\delta = \left[D^i, E_0\right]\left[D^i, E_0\right] + \left[D^i, B_0\right]\left[D^i, B_0\right]$$

- Generally: explicit expressions available, need to compute commutators and covariant derivatives.

- Initial goal: 6th or 8th order with good derivative schemes.

Time Evolution (Preliminary)

Energy density T^{00}; b=0; $\tau = 0$ to $\tau = 0.10$ fm @ $\eta = 0$
Q=1 GeV, m=0.2 GeV; typical event



Summary

- Analytic results available for classical gluon fields in nuclear collisions at very early time evolution.
  - Power series in time, convergence up to ~ $1/Q_s$.
- Flow driven by QCD versions of Gauss/Ampere/Faraday Laws.
  - Rapidity odd (but boost invariant) contributions
  - Matching to fluid dynamics: smooth evolution of energy density and pressure, discontinuities for microscopic mechanisms.
- New: semi-analytic event-by-event calculations
  - Good control of scales; analytic insights into mechanisms possible (e.g. β^i)
  - Numerically inexpensive

Outlook

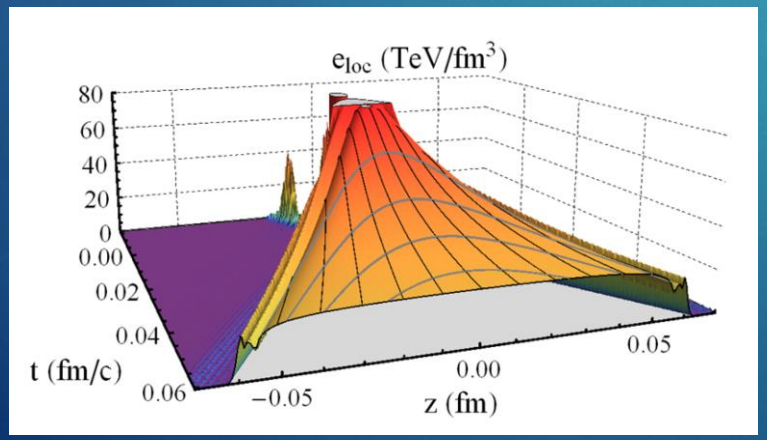
- More realistic modeling of charge distributions
- More quantitative checks (comparisons with analytic results for event averages)
- Other checks?
- Match to hydro $\longrightarrow$ phenomenology

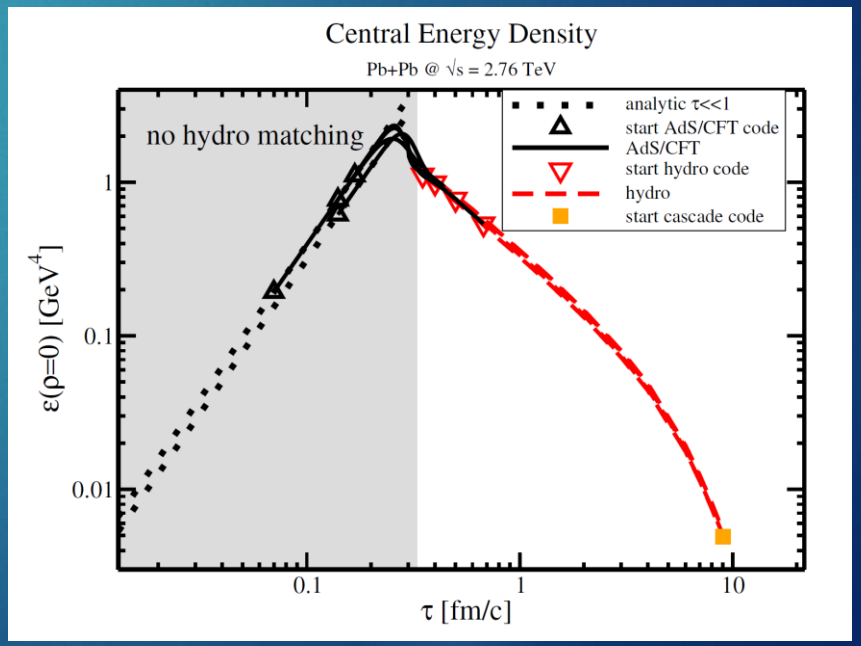
Backup

Early Time Evolution

- Large body of work on early time dynamics
  - Phenomenological models
  - Weak coupling limit
  - Color glass condensate
- Recently much progress on strong coupling scenarios

Chesler, Kilbertus, van der Schee (2015)
van der Schee, Schenke (2015)
van der Schee, Romatschke, Pratt (2013)
…





Origin of Flow in YM

- Initial longitudinal fields E_0, B_0 → transverse fields through QCD versions of Ampere's, Faraday's and Gauss' Law.
  - Here *abelian* version for simplicity.
- Gauss' Law at fixed time *t*
  - Difference in long. flux ⟶ transverse flux
  - *rapidity-odd* and *radial*
- Ampere/Faraday as function of *t*:
  - Decreasing long. flux ⟶ transverse field
  - *rapidity-even* and *curling* field
- Full classical QCD at $O(\tau^1)$:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\, \varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\, \varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

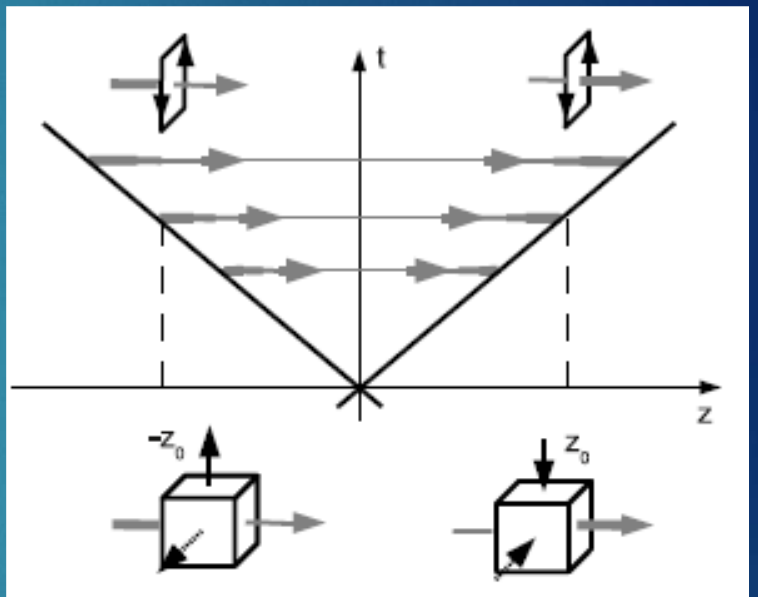
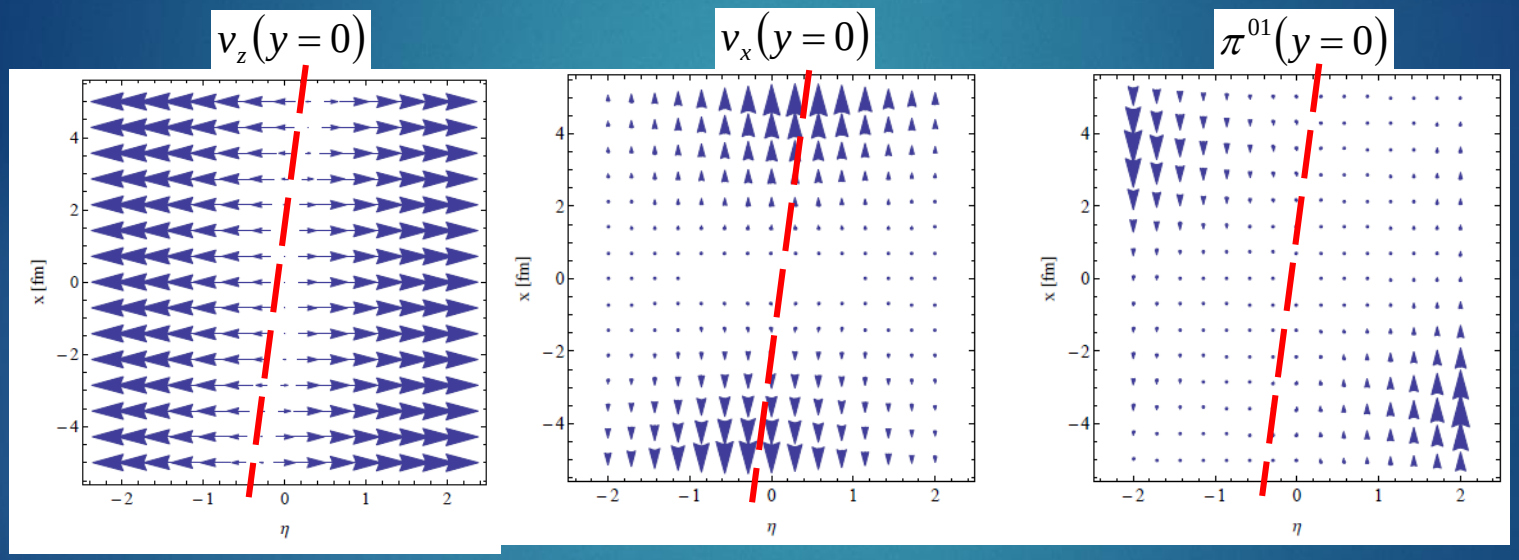


Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

Chen, RJF (2013)

Matching to Hydrodynamics

- Hydro fields in Minkowski components

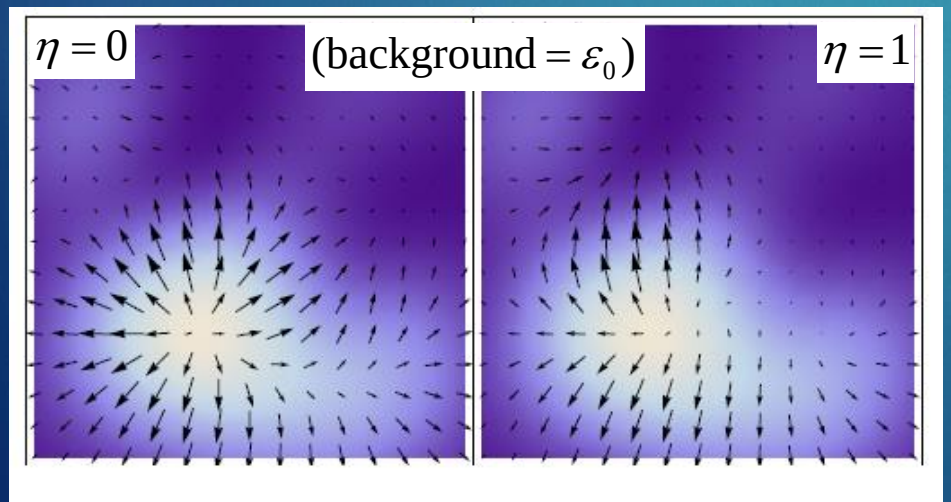


Field And Flow Patterns

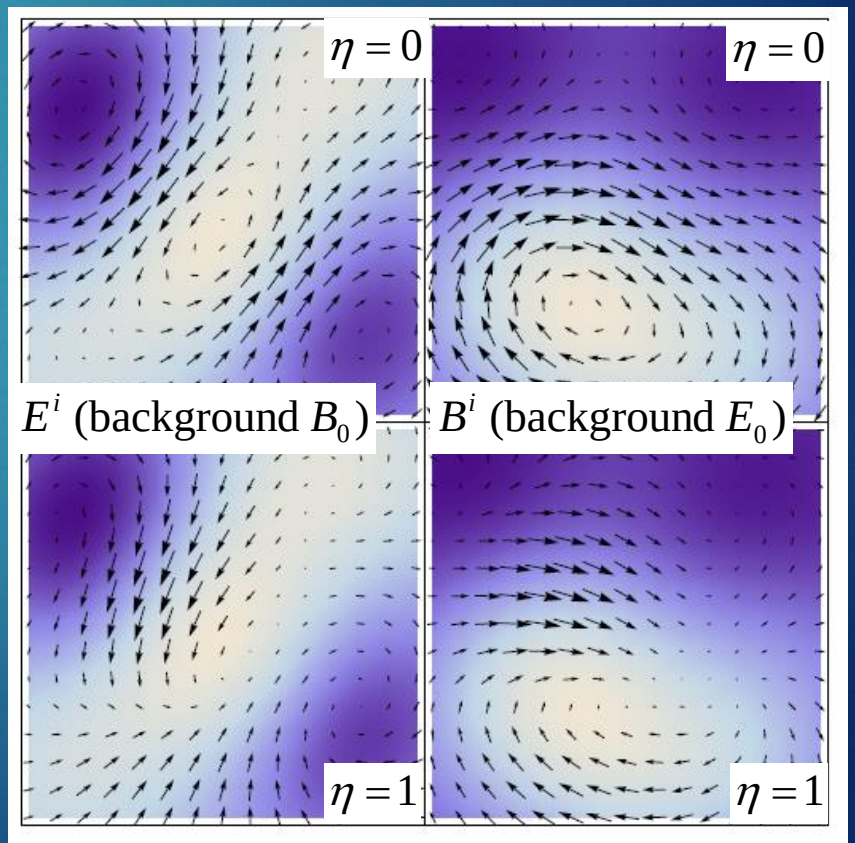
- Transverse Poynting vector for random initial fields (abelian example).
- $\eta = 0$: "Hydro-like" flow from large to small energy density
- $\eta \neq 0$: Quenching/amplification of flow due to the underlying field structure (from Gauss Law),

Energy Flow

Chen, RJF (2013)



Transverse Fields

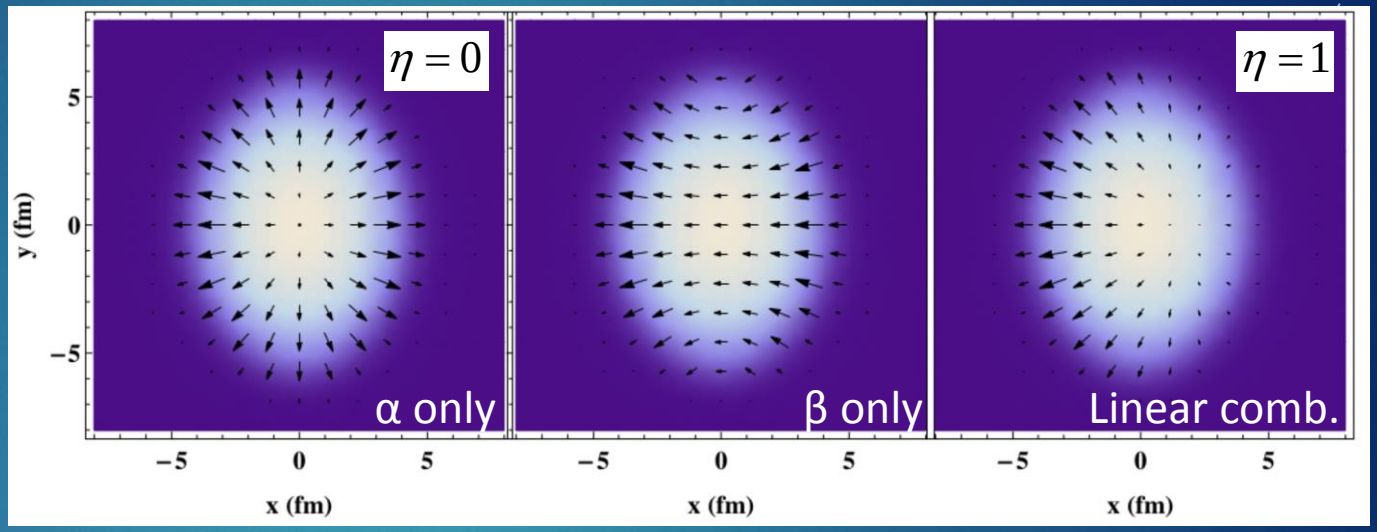


Transverse Flow

- Transverse Poynting vector: Transverse plane

Chen, RJF (2013)

Pb+Pb, b=6 fm

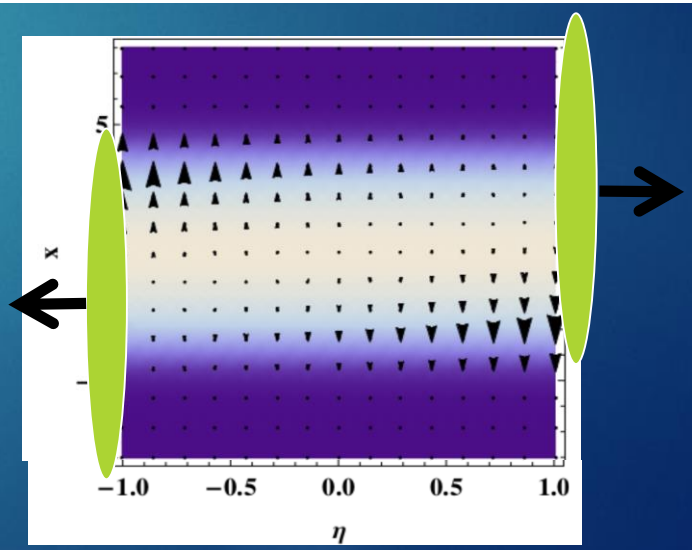


$$\alpha^i = -\varepsilon_0 \frac{\nabla^i (\mu_1 \mu_2)}{\mu_1 \mu_2}$$

$$\beta^i = -\varepsilon_0 \frac{\mu_2 \nabla^i \mu_1 - \mu_1 \nabla^i \mu_2}{\mu_1 \mu_2}$$

- Transverse Poynting vector: Event Plane

  (long. component suppressed)



- Radial and elliptic flow
- Rapidity-odd directed flow (from Gauss Law)
- Angular momentum

Resummation of the Time Evolution

- Generic arguments: convergence radius of the recursive solution ~$1/Q_s$.
- "Weak field" approximation to Yang Mills

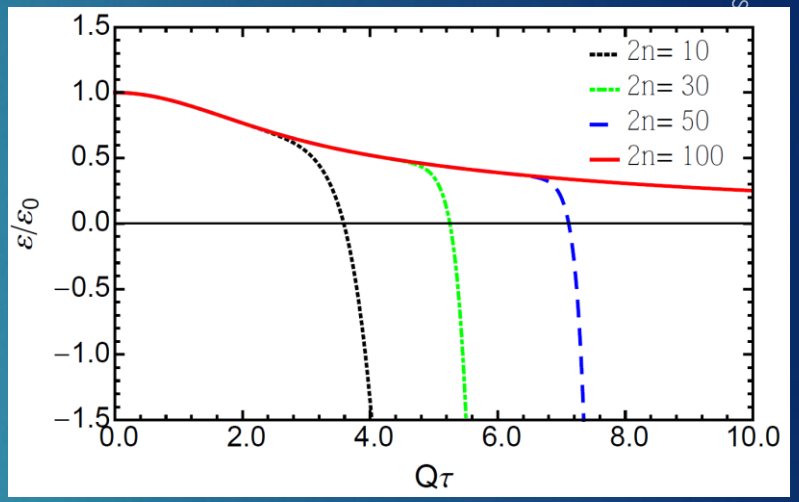
$$A^{\mathrm{LO}}(\tau,\mathbf{k}_\perp)=\frac{2A_{(0)}(\mathbf{k}_\perp)}{k_\perp\tau}J_1(k_\perp\tau)$$

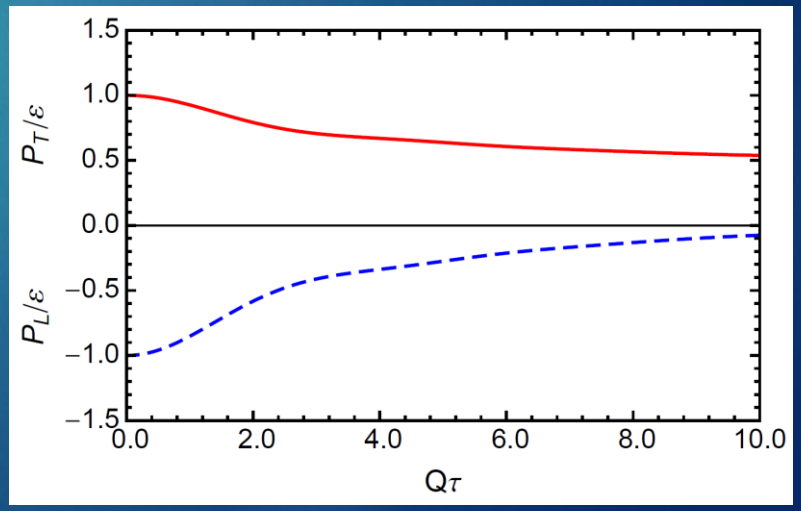
$$A_\perp^{i\,\mathrm{LO}}(\tau,\mathbf{k}_\perp)=A^i_{\perp(0)}(\mathbf{k}_\perp)J_0(k_\perp\tau)$$

  - Can be rederived from the recursive solution.
- Resumming $(Q\tau)^k$ terms: semi-closed form

$$\mathcal{A}=\varepsilon_0+\frac{2\varepsilon_0}{\ln^2(Q^2/m^2)}\mathrm{G}_A(Q\tau)$$
$$-\frac{\varepsilon_0}{\ln(Q^2/m^2)}(Q\tau)^2\left[{}_3F_4(1,1,\frac{3}{2};2,2,2,2;-(Q\tau)^2)\right]$$

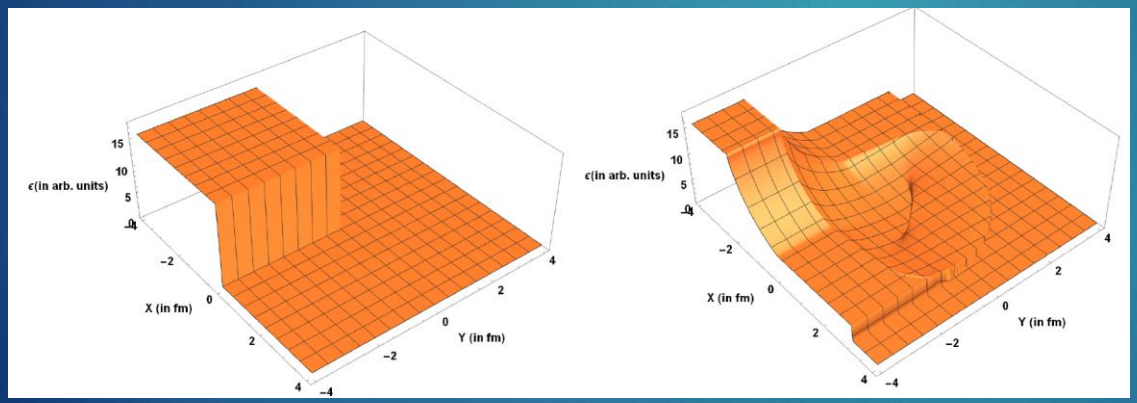
Li, Kapusta, 1602:09060

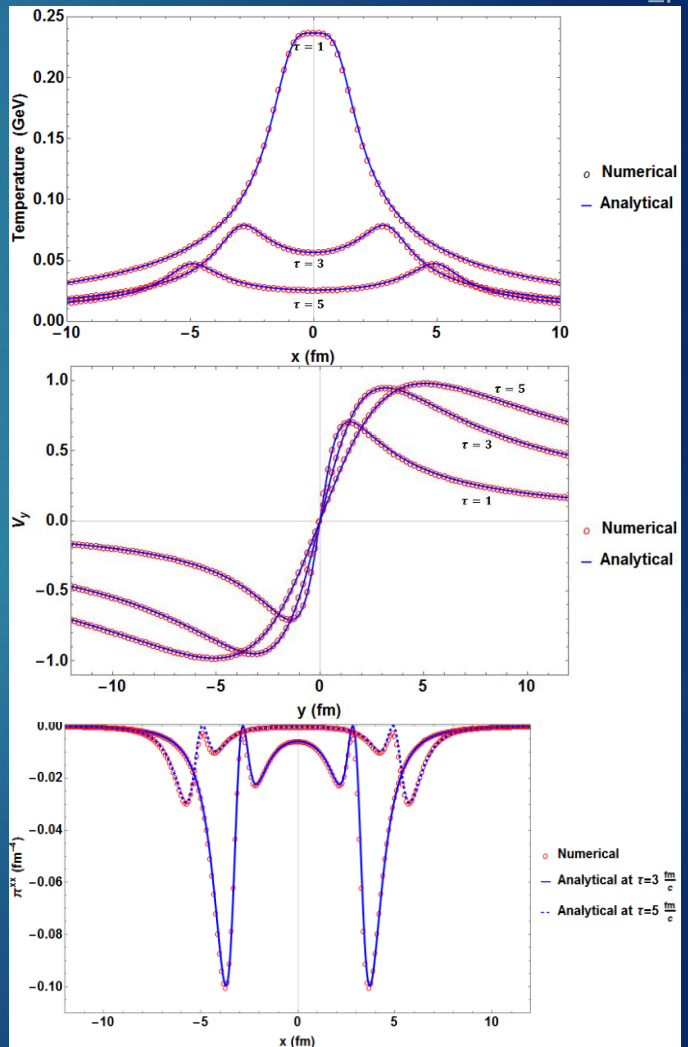




Texas 3+1 D Fluid Code

- KT for fluxes, 5th order WENO for spatial derivatives, 3rd order TVD Runge Kutta for time integration.
- Bulk and shear stress, vorticity
- Gubser and Sod-type tests:

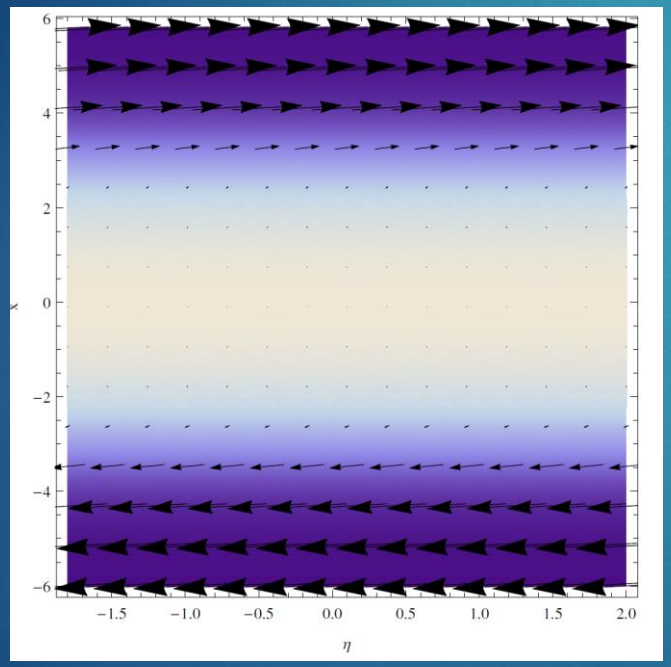




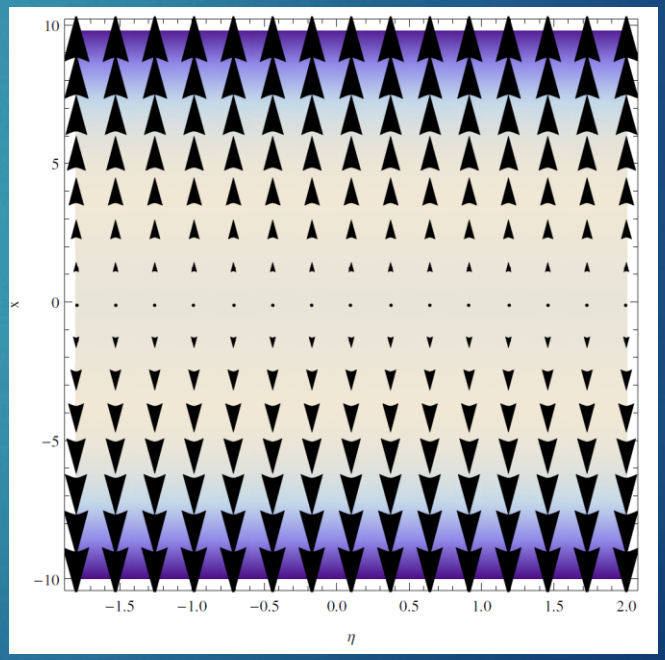
Gauss Law vs Shear Viscosity

- Angular momentum realized as shear flow, not rotation (due to boost invariance)
- $\eta - x$ (event plane) velocity vector in Milne coordinates:

$\tau = 0.1$ fm



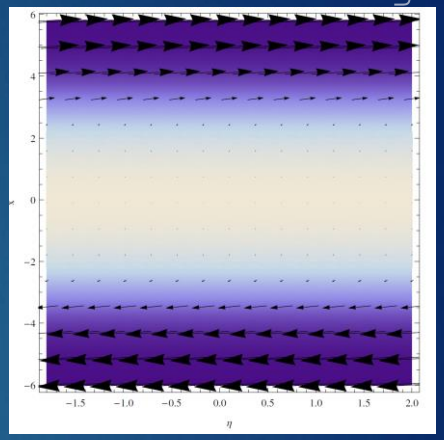
$\tau = 9.6$ fm



- Shear flow decreases rapidly in the hydro evolution.

Gauss Law vs Shear Viscosity

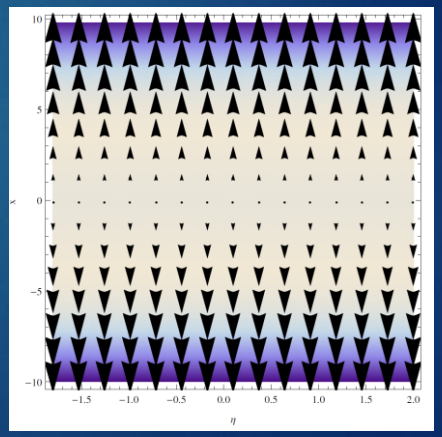
- Yang-Mills phase: Gauss Law builds up shear flow.



- Viscous fluid dynamics works against the gradient.

$$\frac{\partial v_z}{\partial t} = \frac{\eta}{\varepsilon + p}\frac{\partial^2 v_z}{\partial x^2}$$

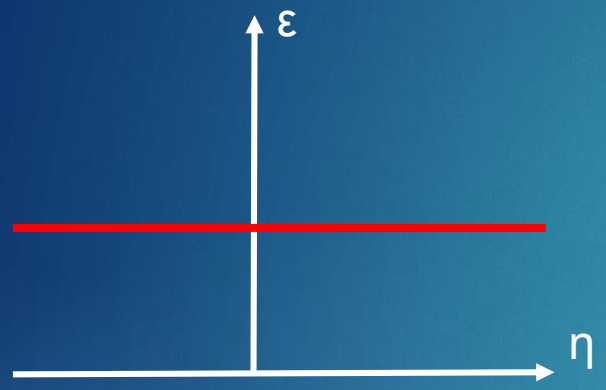
- Discontinuity in the time evolution.
- Is this something that has observable consequences?

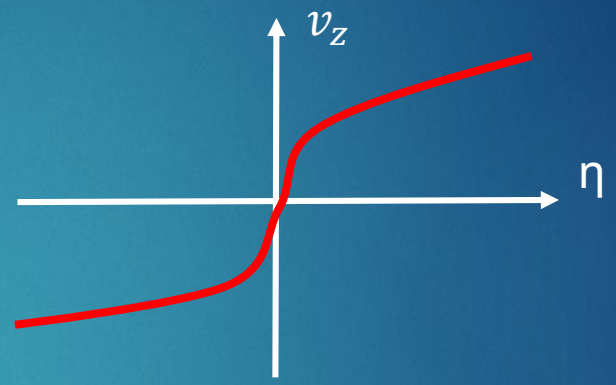


- Angular momentum conserved locally but not globally (sources on the light cone!)

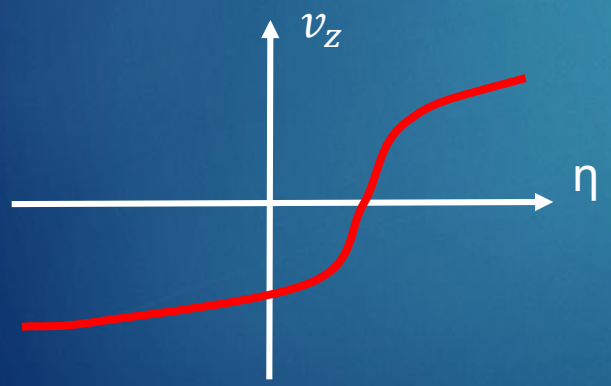
On Boost Invariance

- Naïve boost invariance:





- This is also allowed:



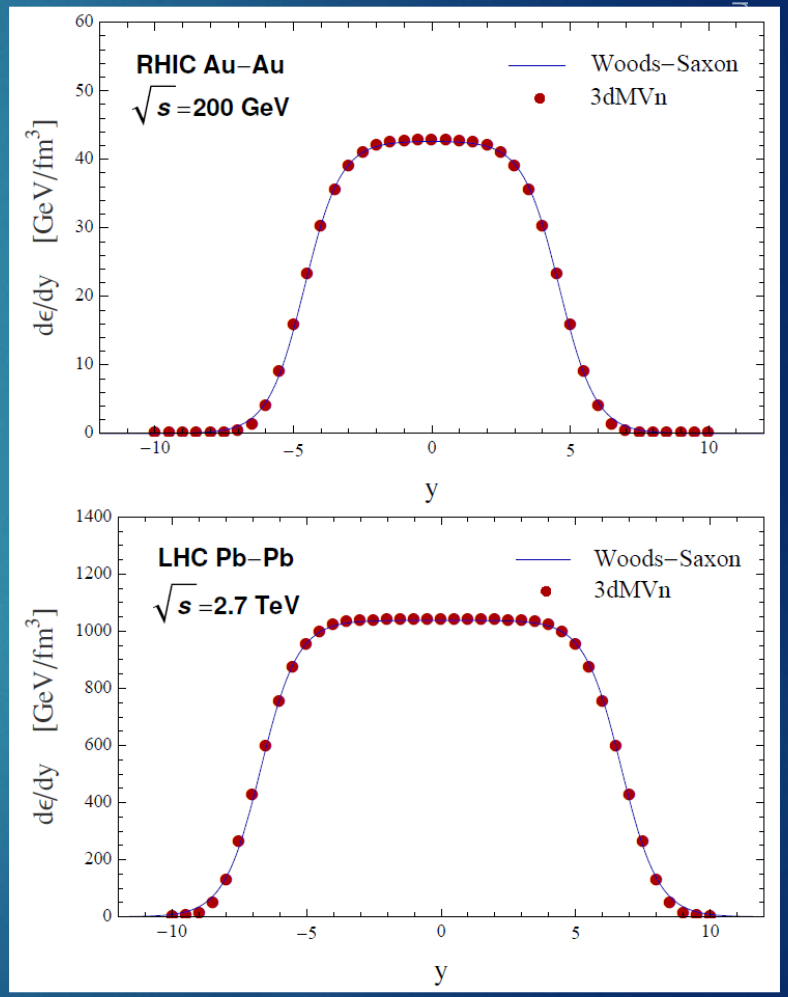
- Boost invariance does not have to pick the lab frame for the node $v_z = 0$.
- An asymmetry between ρ_1 and ρ_2 can lead to a shift of the node away from the lab frame.
- Locally determined for each point in the transverse plane.

Beyond Boost Invariance

- Real nuclei are slightly off the light cone.
- Need to break boost-invariance in a controlled way.
- Using approximations valid for $R_A/\gamma << 1/Q_s$ we estimated the rapidity dependence of the initial energy density ε_0.

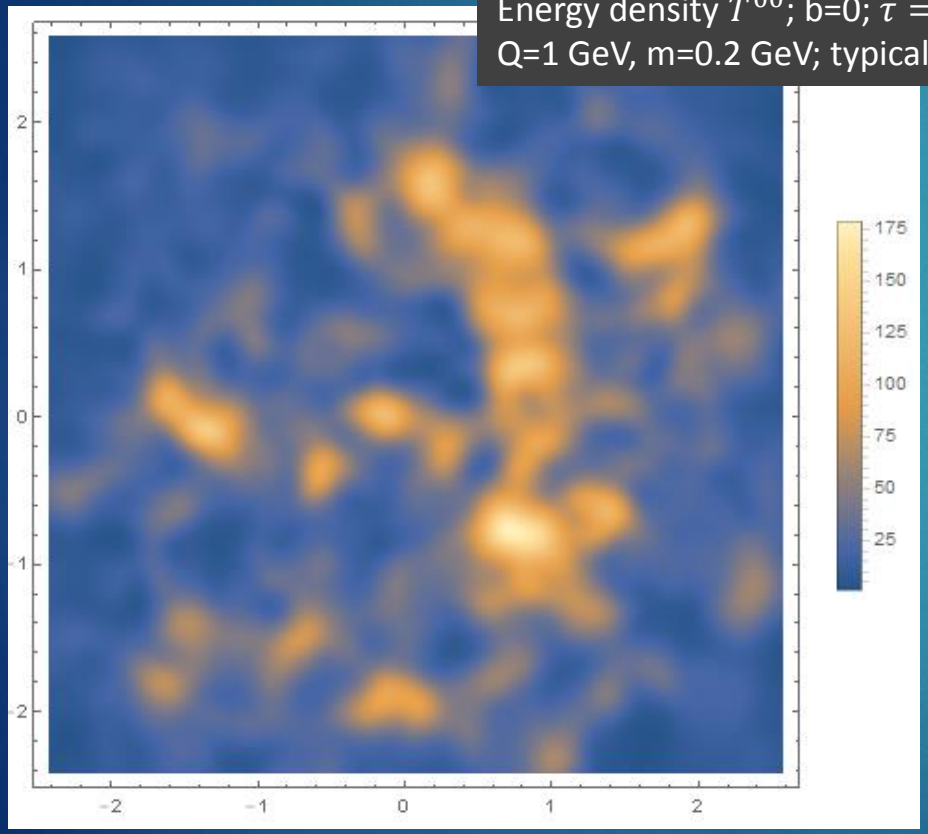
  Ozonder, RJF (2014)

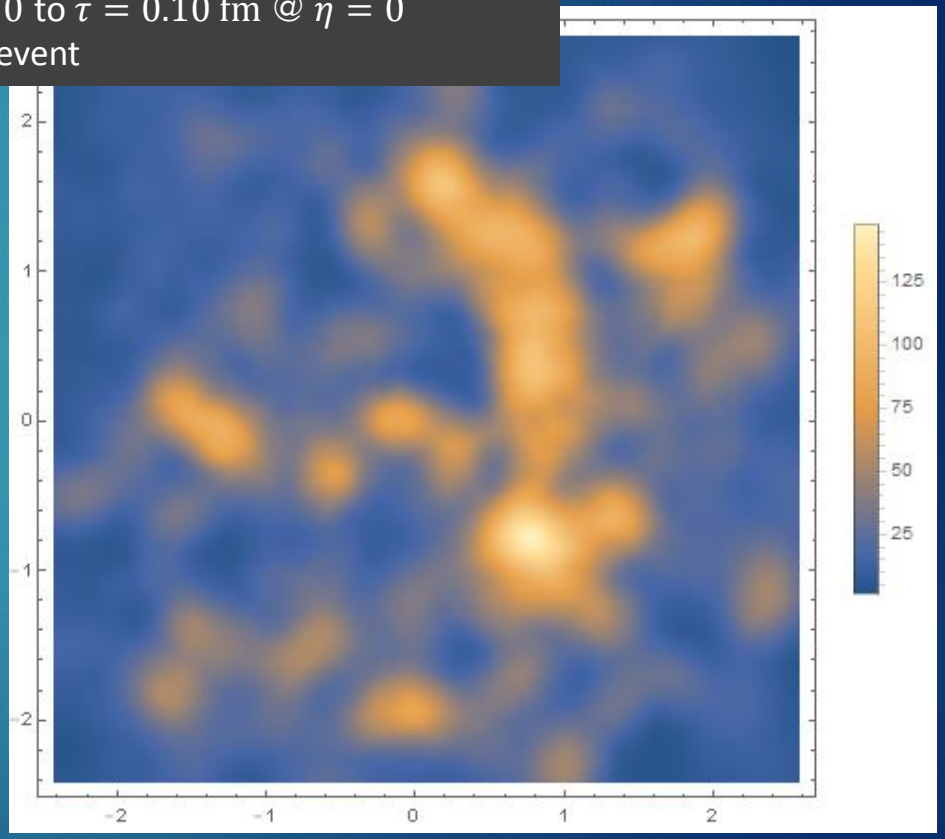
- It would enable us to study angular momentum in the initial color glass. Stay tuned.



Time Evolution (Preliminary)

Energy density T^{00}; b=0; $\tau = 0$ to $\tau = 0.10$ fm @ $\eta = 0$
Q=1 GeV, m=0.2 GeV; typical event





$\tau = 0$

$\tau = 0.10$ fm