Factorization of in-medium parton branching beyond the eikonal approximation

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 manuscipt under preparation…

Hard Probes 2016

Wuhan, China, 2016
Introduction

- Heavy-Ion Collisions:
  - Perfect laboratory to evaluate QCD behaviour at high temperatures and densities
Introduction

✦ Heavy-Ion Collisions:

✦ Perfect laboratory to evaluate QCD behaviour at high temperatures and densities

Modified probes (energy loss, broadening, …)

Macroscopic properties of the created medium

QCD dynamics in dense regime

High momentum particles, jets, …

Quark-Gluon Plasma

Confinement/Deconfinement

Asymptotic freedom

QCD Phase transitions

Bulk of the collision

Stages of a nucleus-nucleus collision

$t$

$z$

Strong Fields

Freeze out

Hadrons in eq.

quarks and gluon in eq.

quarks & gluons out of eq.

L. Apolinário et al, Hard Probes 2016
Introduction

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  - Perfect laboratory to evaluate QCD behaviour at high temperatures and densities

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Stages of a nucleus-nucleus collision

- Strong Fields
  - quarks & gluons out of eq.

Freeze out

Hadrons in eq.

Strong Fields
Vacuum Parton Shower

- Starting by what we know:

- pQCD description of a high virtuality parton

- Phase space limited to: $\sqrt{Q^2} \geq \omega \geq k_\perp >> \Lambda_{\text{QCD}}$

$$dP^{q\rightarrow qg}_{\omega, k_\perp} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{dk_\perp^2}{k_\perp^2}$$

Double Log enhancement
Vacuum Parton Shower

✦ Starting by what we know:

✦ pQCD description of a high virtuality parton

✦ Phase space limited to: $\sqrt{Q^2} \geq \omega \geq k_\perp >> \Lambda_{\text{QCD}}$

$\omega, k_\perp$

$\theta$

$\frac{dP}{d\omega dk_\perp^2}$

Double Log enhancement

✦ Ordering variable $t \sim \ln Q$

✦ DGLAP evolution equation

$$f(z,Q^2) = \text{p.d.f}$$

$$P(x) = \text{splitting function}$$

$$\frac{\partial f_i(z, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_1^1 dx \sum_j P_{i\rightarrow j} \left( \frac{z}{x} \right) f_j(x, Q^2)$$

$L. \text{Apolinário et al, Hard Probes 2016}$
Vacuum Parton Shower

✧ Generalising for multiple gluon emissions:

✧ Need to check interferences between subsequent emissions

\[ dN_{q}^{\omega \rightarrow 0} \sim \alpha_{s} C_{R} \frac{d\omega}{\omega} \frac{\sin\theta d\theta}{1 - \cos \theta} \Theta(\cos \theta_{1} - \cos \theta) \]

\[ \Rightarrow \theta > \theta_{1} > \theta_{2} > \cdots \]
Vacuum Parton Shower

- Generalising for multiple gluon emissions:
- Need to check interferences between subsequent emissions

\[ \sqrt{Q^2} \]

\[ \Lambda_{\text{QCD}} \]

Hadronization
(non pQCD)

d\text{N}_{\omega \rightarrow 0} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{\sin \theta \, d\theta}{1 - \cos \theta} \Theta(\cos \theta_1 - \cos \theta)

\Rightarrow \theta > \theta_1 > \theta_2 > \cdots

Ordering variable: \( t \sim \ln \theta^2 \)
(MLLA Approximation)
Includes Single Logs
In-Medium Interactions

- Medium = Quark-Gluon Plasma: strongly coupled fluid (non pQCD)

- Hard probe: $\sqrt{Q^2} \gg \Lambda_{\text{QCD}}$, $T$ (pQCD evolution)
In-Medium Interactions

✦ Medium = Quark-Gluon Plasma: strongly coupled fluid (non pQCD)

✦ Hard probe: $\sqrt{Q^2} \gg \Lambda_{\text{QCD}}$, $T$ (pQCD evolution)

✦ Medium - hard probe interaction: ???

✦ Approximation: medium as a collection of independent and static scattering centres:

Medium Length: $L$
Mean free path: $\lambda_{\text{mfp}}$
Debye Mass: $m_D^{-1}$

$L \gg \lambda_{\text{mfp}} \gg m_D^{-1}$

Transport coefficient:
$$\hat{q} = \frac{<k_{\perp}^2>}{\lambda_{\text{mfp}}}$$
In-Medium Propagation

- High-energy approximation: \( p_+ \gg p_\perp \)
- No energy loss in the longitudinal direction;
- Transverse momentum “kick”: Brownian motion in the transverse direction
- Color phase rotation:

\[
\int A_a(x_n) = \frac{1}{2\pi^2} \int dq \sqrt{2} e^{i\cdot q_n} \left( A_a(q) \right)
\]
In-Medium Propagation

- High-energy approximation: $p_+ >> p_\perp$
- No energy loss in the longitudinal direction;
- Transverse momentum “kick”: Brownian motion in the transverse direction
- Color phase rotation:

\[ G(x_{0+}, x_{0\perp}; L_+, x_\perp | p_+) = \int_{r_\perp(x_{0+})=x_{0\perp}}^{r_\perp(L_+)=x_\perp} DR_\perp(\xi) \exp \left\{ \frac{i p_+}{2} \int_{x_{0+}}^{L_+} d\xi \left( \frac{dr_\perp}{d\xi} \right)^2 \right\} \times W(x_{0+}, L_+; r_\perp(\xi)), \]
In-Medium Propagation

- High-energy approximation: $p_+ \gg p_\perp$
- No energy loss in the longitudinal direction;
- Transverse momentum “kick”: Brownian motion in the transverse direction
- Color phase rotation:

$$W(x_0^+, L^+; x_\perp) = \mathcal{P}\exp\left\{ig \int_{x_0^+}^{L^+} dx_+ A_-(x_+, x_\perp)\right\}$$

Path-ordering

Medium colour field

Transverse coordinate

Initial/Final coordinates

$$G(x_0^+, x_0^\perp; L^+, x_\perp|p_+) = \int_{r_\perp(x_0^+)=x_0^\perp}^{r_\perp(L^+)=x_\perp} \mathcal{D}r_\perp(\xi) \exp\left\{\frac{ip_+}{2} \int_{x_0^+}^{L^+} d\xi \left(\frac{dr_\perp}{d\xi}\right)^2\right\}$$

$$\times W(x_0^+, L^+; r_\perp(\xi)),$$
In-Medium Energy Loss

- Medium interactions induces accumulation of momenta
- Enhancement of gluon radiation

\[ p \quad \rightarrow \quad k = z \, p \quad \rightarrow \quad q = (1-z) \, p \]

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000)]
[Zakharov (1996)]
[Wiedemann (2001)]
[Arnold, Moore, Yaffe (2002)]
[LA, Armesto, Salgado, (2012)]
In-Medium Energy Loss

- Medium interactions induces accumulation of momenta
- Enhancement of gluon radiation
- Timescale ~ $Q^{-1}$:
  - Frozen medium color configuration during propagation
  - Average over the configurations ensemble

\[
p = k = z \, p \\
q = (1-z) \, p
\]

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000)]
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Medium Averages

- Color information in the $W$:

- N-field correlators up to second order in fields: $< A(x_\perp)A(y_\perp) >$

- Dipole approximation

$$C_F n(\xi) \sigma(r) \simeq \frac{1}{2} q_F r^2 + O(r^2 \ln r^2)$$
Medium Averages

✦ Color information in the $W$:

✦ N-field correlators up to second order in fields: $< A(x_\perp)A(y_\perp) >$

✦ Dipole approximation $C_F n(\xi) \sigma(r) \simeq \frac{1}{2} \hat{q}_F r^2 + O(r^2 \ln r^2)$

✦ Kinematic information on the $G_0$:

✦ Path-integral evaluation using a semi-classical approximation:

$$G_0(x_+, x; y_+, y) = \int_{r(x_+)=x}^{r(y_+)=y} \mathcal{D}r(\xi) \exp \left\{ \frac{ip_+}{2} \int_{x_+}^{y_+} d\xi \left( \frac{dr}{d\xi} \right)^2 \right\}$$

$$= \frac{1}{(2\pi i)^{D/2}} \left| \det \left( -\frac{\partial^2 R_{cl}}{\partial y_i \partial x_i} \right) \right|^{1/2} e^{iR_{cl}(x_+,x;y_+,y)}$$
Medium Averages

- Color information in the $W$

- N-field correlators up to second order in fields: $\langle A(x_\perp)A(y_\perp) \rangle$

- Dipole approximation

$$C_F n(\xi) \sigma(r) \simeq \frac{1}{2} \hat{q}_F r^2 + \mathcal{O}(r^2 \ln r^2)$$

- Kinematic information on the $G_0$:

Path-integral evaluation using a semi-classical approximation:

$$G_0(x_+, x; y_+, y) = \int_{r(x_+)=x}^{r(y_+)=y} D\mathbf{r}(\xi) \exp \left\{ \frac{ip_+}{2} \int_{x_+}^{y_+} d\xi \left( \frac{dr}{d\xi} \right)^2 \right\}$$

$$= \frac{1}{(2\pi i)^{D/2}} \det \left( - \frac{\partial^2 R_{cl}}{\partial y_i \partial x_i} \right)^{1/2} e^{iR_{cl}(x_+, x; y_+, y)}$$

Dominant contribution for the average trajectory given by the classical path
Medium Averages

- Color information in the W:
- N-field correlators up to second order in fields: \( <A(x_\perp)A(y_\perp)> \)
- Dipole approximation: \( C_F n(\xi) \sigma(r) \approx \frac{1}{2} \hat{q}_F r^2 + O(r^2 \ln r^2) \)
- Kinematic information on the \( G_0 \):
  - Path-integral evaluation using a semi-classical approximation:
    \[
    G_0(x_+, x; y_+, y) = \int_{r(x_+) = x}^{r(y_+) = y} \mathcal{D}r(\xi) \exp \left\{ \frac{i p_+}{2} \int_{x_+}^{y_+} d\xi \left( \frac{dr}{d\xi} \right)^2 \right\}
    \]
  - Fluctuations of the classical action
    \[
    = \frac{1}{(2\pi i)^{D/2}} \left| \det \left( -\frac{\partial^2 R_{cl}}{\partial y_i \partial x_i} \right) \right|^{1/2} e^{iR_{cl}(x_+, x; y_+, y)}
    \]
In-Medium Radiation

- Single medium-induced gluon radiation:

[Blaizot, Dominguez, Iancu, Mehtar-Tani (2013-2014)]
[LA, Armesto, Milhano, Salgado, (2015)]
In-Medium Radiation

- Single medium-induced gluon radiation:

\[ \mathcal{P}(p_{\perp}) \]

Momentum broadening:

\[
\mathcal{P}(k_{\perp}) = \frac{4\pi}{\hat{q}\Delta L_+} \exp \left\{ -\frac{\Delta k_{\perp}^2}{\hat{q}\Delta L_+} \right\}
\]

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

\[
x_q(\bar{q}) / x_{\bar{g}}(g)
\]
In-Medium Radiation

- Single medium-induced gluon radiation:

\[ P(p_{\perp}) = \frac{4\pi}{\hat{q}\Delta L_{+}} \exp\left\{ -\frac{\Delta k_{\perp}^{2}}{\hat{q}\Delta L_{+}} \right\} \]

Momentum broadening:

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

\[ x_{q(\bar{q})}/x_{g(\bar{g})} \]

Spectrum proportional to:

\[ \Delta_{coh} = 1 + \int_{x_{+}}^{L_{+}} d\tau \hat{q} (x_{q} - x_{\bar{q}}) \cdot (x_{g} - x_{\bar{g}}) \left| e^{-\hat{q} \int_{x_{+}}^{\tau} d\xi (x_{q} - x_{\bar{q}}) \cdot (x_{q} - x_{g})} \right| \]
**In-Medium Radiation**

- Single medium-induced gluon radiation:

\[
P(p_{\perp}) \rightarrow P(k_{\perp})
\]

Spectrum proportional to:

\[
\Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau \hat{q} (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}}) |_{\tau} e^{-\hat{q} \int_{x_+}^{\tau} d\xi (x_q - x_{\bar{g}}) \cdot (x_q - x_g)}
\]

\[
P(k_{\perp}) = \frac{4\pi}{\hat{q} \Delta L_+} \exp \left\{-\frac{\Delta k_{\perp}^2}{\hat{q} \Delta L_+}\right\}
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[Blaziot, Dominguez, Iancu, Mehtar-Tani (2013-2014)]
[LA, Armesto, Milhano, Salgado, (2015)]
In-Medium Radiation

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Momentum broadening:

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

\[ x_{q(\bar{q})}/x_{\bar{g}(g)} \]

Spectrum proportional to:

\[ \Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau \hat{q} \left( x_q - x_{\bar{q}} \right) \cdot \left( x_g - x_{\bar{g}} \right) e^{-\hat{q} \int_{x_+}^{\tau} d\xi (x_q - x_{\bar{g}}) \cdot (x_q - x_g)} \]
In-Medium Radiation

✦ Single medium-induced gluon radiation:

\[ \mathcal{P}(p_\perp) \]
\[ \mathcal{P}(k_\perp) \]
\[ \mathcal{P}(q_\perp) \]

Spectrum proportional to:

\[ \Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau \hat{q} \left( x_q - x_{\bar{q}} \right) \cdot \left( x_g - x_{\bar{g}} \right) e^{-\hat{q} \int_{x_+}^{\tau} d\xi (x_q - x_{\bar{g}}) \cdot (x_q - x_g)} \]

Complete factorisation
Independent radiation
Propagate coherently
Coherent gluon radiation

Momentum broadening:

\[ \mathcal{P}(k_\perp) = \frac{4\pi}{\hat{q}\Delta L_+} \exp \left\{ -\frac{\Delta k_\perp^2}{\hat{q}\Delta L_+} \right\} \]

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

\[ x_q(\bar{q})/x_{\bar{g}}(g) \]
Single medium-induced gluon radiation:

\[ \mathcal{P}(k_{\perp}) \]

\[ \mathcal{P}(p_{\perp}) \]

\[ \mathcal{P}(q_{\perp}) \]

Spectrum proportional to:

\[
\Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau \hat{q} \cdot (x_q - x_{\bar{q}}) \cdot (x_g - x_{\bar{g}}) \bigg|_{\tau} e^{-\hat{q} \int_{x_+}^{\tau} d\xi (x_{\bar{q}} - x_g) \cdot (x_q - x_{\bar{g}})}
\]

Complete factorisation
Independent radiation

Propagate coherently
Coherent gluon radiation

\[
\mathcal{P}(k_{\perp}) = \frac{4\pi}{\hat{q} \Delta L_+} \exp \left\{ -\frac{\Delta k_{\perp}^2}{\hat{q} \Delta L_+} \right\}
\]

Momentum broadening:

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

\[
X_{q(\bar{q})} / X_{\bar{g}(g)}
\]

Probabilistic picture still holds???
Subsequent emissions investigated in a quark-antiquark antenna setup:

\[
\frac{dI}{d\omega, k} = R_q + R_{\bar{q}} - 2J(1 - \Delta_{med}) = R_{coh} + 2J\Delta_{med}
\]

Eikonal approximation:

\[
R_q \sim \alpha_s C_F \frac{q_{1+}}{(k \cdot q_1)}
\]

\[
R_{\bar{q}} \sim \alpha_s C_F \frac{q_{2+}}{(k \cdot q_2)}
\]

\[
2J \sim \alpha_s C_F \left[ \frac{q_{1+}}{(k \cdot q_1)} + \frac{q_{2+}}{(k \cdot q_2)} - \frac{k+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right]
\]

\[
R_{coh} \sim \alpha_s C_F \frac{k+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)}
\]

\[
1 - \Delta_{med} = \frac{1}{N_c^2} \text{Tr} \left\langle W_A(x_q)W_A^\dagger(x_{\bar{q}}) \right\rangle
\]
In-Medium Antenna

Subsequent emissions investigated in a quark-antiquark antenna setup:

Integrating over azimuthal angle (soft limit):

\[
dN_q^{\omega \to 0} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} \left[ \Theta(\cos \theta_1 - \cos \theta) + \Delta_{med} \Theta(\cos \theta - \cos \theta_1) \right]
\]

\[
\Delta_{med} \approx 1 - e^{-\frac{1}{12} Q_s^2 r_{\perp}^2}
\]

Antenna Transverse resolution: \( r_{\perp} = \theta L \)

Medium Transverse Scale: \( Q_s^{-1} = (\bar{q} L)^{-1/2} \)
In-Medium Antenna

Subsequent emissions investigated in a quark-antiquark antenna setup:

Integrating over azimuthal angle (soft limit):

\[
\begin{align*}
\frac{dN_q^{\omega \rightarrow 0}}{d\omega} & \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} \\
& \times \left[ \Theta(\cos \theta_1 - \cos \theta) + \Delta_{med} \Theta(\cos \theta - \cos \theta_1) \right]
\end{align*}
\]

\[
\Delta_{med} \approx 1 - e^{-\frac{1}{12} Q_s^2 r_\perp^2}
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Medium Transverse Scale: \( Q_s^{-1} = (\hat{Q} L)^{-1/2} \)
In-Medium Antenna

Subsequent emissions investigated in a quark-antiquark antenna setup:

\[
\begin{align*}
\sum_{\omega, k_\perp}^{2} q_2 & + \sum_{\omega, k_\perp}^{2} q_1 \\
& \Rightarrow \sum_{\omega, k_\perp}^{2} \theta_1 \\
& \Rightarrow \sum_{\omega, k_\perp}^{2} \theta_2
\end{align*}
\]

Integrating over azimuthal angle (soft limit):

\[
dN_q^{\omega \rightarrow 0} \sim \alpha_s C_R \left(\frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} \left[\Theta(\cos \theta_1 - \cos \theta) + \Delta_{med} \Theta(\cos \theta - \cos \theta_1)\right] + 1 - e^{-\frac{1}{12} Q_s^2 r_\perp^2}\right)
\]

Antenna Transverse resolution: \( r_\perp = \theta L \)
Medium Transverse Scale: \( Q_s^{-1} = \langle \hat{Q} L \rangle^{-1/2} \)
In-Medium Antenna

Subsequent emissions investigated in a quark-antiquark antenna setup:

\[ dN_{q \rightarrow 0} \sim \alpha_s C_{R} \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} \left[ \Theta(\cos \theta_1 - \cos \theta) + \Delta_{med} \Theta(\cos \theta - \cos \theta_1) \right] \]

\[ \Delta_{med} \approx 1 - e^{-\frac{1}{12}Q_s^2 r_{\perp}^2} \]

Antenna Transverse resolution: \( r_{\perp} = \theta L \)
Medium Transverse Scale: \( Q_s^{-1} = \langle \hat{Q} L \rangle^{-1/2} \)

Angular ordering

Anti-Angular ordering
In-Medium Antenna

✦ Subsequent emissions investigated in a quark-antiquark antenna setup:

\[
\begin{align*}
q_1 & \sim \omega, k, \\
\theta_1 & \sim Q_{s}^{-1} \quad q_2 \\
\end{align*}
\]

Integrating over azimuthal angle (soft limit):

\[
dN_q^{\omega \rightarrow 0} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} \left[ \Theta(\cos \theta_1 - \cos \theta) + \Delta_{med} \Theta(\cos \theta - \cos \theta_1) \right]
\]

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\Delta_{med} \approx 1 - e^{-\frac{1}{12} Q_s^2 r_{\perp}^2}
\]

Antenna Transverse resolution: \( r_{\perp} = \theta L \)
Medium Transverse Scale: \( Q_s^{-1} = (\hat{q} L)^{-1/2} \)
In-Medium Antenna

Beyond eikonal limit:

\[ 2J \sim \alpha_s C_F \left[ \frac{q_1^+}{(k \cdot q_1)} + \frac{q_2^+}{(k \cdot q_2)} - \frac{k^+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right] \]

\[ R_q \sim \alpha_s C_F \frac{q_1^+}{(k \cdot q_1)} \quad R_{\bar{q}} \sim \alpha_s C_F \frac{q_2^+}{(k \cdot q_2)} \]

[LA, Armesto, Milhano and Salgado (in preparation)]
In-Medium Antenna

Beyond eikonal limit:

\[ 2J \sim \alpha_s C_F \left[ \frac{q_1^+}{(k \cdot q_1)} + \frac{q_2^+}{(k \cdot q_2)} - \frac{k^+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right] \]

\[ 1 - \Delta'_{med} = \frac{1}{N_c^2} \text{Tr} \left\langle G_1 G_2^\dagger \right\rangle \text{Tr} \left\langle G_1^\dagger G_2 \right\rangle \]

\[ R_q \sim \alpha_s C_F \frac{q_1^+}{(k \cdot q_1)} \quad \text{and} \quad R_\bar{q} \sim \alpha_s C_F \frac{q_2^+}{(k \cdot q_2)} \]
In-Medium Antenna

Beyond eikonal limit:

\[ 2J \sim \alpha_s C_F \left( \frac{q_1^+}{(k \cdot q_1)} + \frac{q_2^+}{(k \cdot q_2)} - \frac{k^+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right) \]

\[ 1 - \Delta'_{med} = \frac{1}{N_c^2} \text{Tr} \left( G_1 G_2^\dagger \right) \text{Tr} \left( G_1^\dagger G_2 \right) \]

\[ R_q \sim \alpha_s C_F \frac{q_1^+}{(k \cdot q_1)} \quad R=q \sim \alpha_s C_F \frac{q_2^+}{(k \cdot q_2)} \]

\[ \Delta'_{coh} = \frac{1}{N_c^2} \text{Tr} \left( G_1 G_2^\dagger \right) \text{Tr} \left( G_1^\dagger G_2 \right) \Delta_{coh} \]

\[ \Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau \hat{q} (x_1 - x_1) \cdot (x_2 - x_2) e^{-\hat{q} \int_{x_+}^{\tau} d\xi (x_1 - x_2) \cdot (x_1 - x_2)} \]

[LA, Armesto, Milhano and Salgado (in preparation)]
In-Medium Antenna

Beyond eikonal limit:

\[ 2J \sim \alpha_s C_F \left[ \frac{q_1^+}{(k \cdot q_1)} + \frac{q_2^+}{(k \cdot q_2)} - \frac{k^+ (q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right] \]

\[ 1 - \Delta'_\text{med} = \frac{1}{N_c^2} \text{Tr} \left\langle G_1 G_2^\dagger \right\rangle \text{Tr} \left\langle G_1^\dagger G_2 \right\rangle \]

\[ R_q \sim \alpha_s C_F \frac{q_1^+}{(k \cdot q_1)} \quad R_{\bar{q}} \sim \alpha_s C_F \frac{q_2^+}{(k \cdot q_2)} \]

\[ \Delta'_\text{coh} = \frac{1}{N_c^2} \text{Tr} \left\langle G_1 G_2^\dagger \right\rangle \text{Tr} \left\langle G_1^\dagger G_2 \right\rangle \Delta_{\text{coh}} \]

\[ \Delta_{\text{coh}} = 1 + \int_{x_+}^{L_+} d\tau \hat{q} (x_1 - x_\perp) \cdot (x_2 - x_\perp) |_\tau e^{-\hat{q} \int_{x_+}^{\tau} \text{d} \xi (x_1 - x_\perp) \cdot (x_1 - x_\perp)} \]

\[ \sim p_0 \rightarrow 0 \]
In-Medium Antenna

Beyond eikonal limit

Spectrum: \[ |S_{tot}|^2 = \Delta'_{coh} \otimes (R_q + R_{\bar{q}}) - 2(1 - \Delta'_m)J \]
In-Medium Antenna

Beyond eikonal limit

\[
|S_{\text{tot}}|^2 = \Delta'_{\text{coh}} \otimes (R_q + R_{\bar{q}}) - 2(1 - \Delta'_{\text{med}})J
\]

In the collinear antenna limit, for soft gluon radiation:

\[
\Delta'_{\text{coh}} = \left[ 1 + \frac{(q_{1,\perp} - q_{2,\perp})^2}{\hat{q}_F \Delta L_+} \right] e^{-\frac{(q_{1,\perp} - q_{2,\perp})^2}{2\hat{q}_F \Delta L_+}}
\]

\[
1 - \Delta'_{\text{med}} \sim e^{-\frac{i \tan(\Omega \Delta L_+)}{\Omega}} [(1 - z)(q_{1,\perp} + k_\perp) - zq_{2,\perp}]^2
\]

\[
\Omega^2 \sim \frac{\hat{q}}{z(1 - z)p_+}
\]

\[
\Omega'^2 = -\Omega^2
\]
In-Medium Antenna

Beyond eikonal limit

\begin{equation}
\left| S_{tot} \right|^2 = \Delta'_{coh} \otimes (R_q + R_{\bar{q}}) - 2(\Delta'_{coh} - \Delta'_{med})J
\end{equation}

In the collinear antenna limit, for soft gluon radiation:

\begin{equation}
\Delta'_{coh} = \left[ 1 + \frac{(q_1,_{\perp} - q_2,_{\perp})^2}{\hat{q} F \Delta L_+} \right] e^{-\frac{(q_1,_{\perp} - q_2,_{\perp})^2}{2\hat{q} F \Delta L_+}}
\end{equation}

\begin{equation}
\Delta'_{coh} - \Delta'_{med} \sim e^{-\frac{i \tan(\Omega \Delta L_+)}{\Omega} \left[ (1-z)(q_1,_{\perp} + k_{\perp}) - zq_2,_{\perp} \right]^2}
\end{equation}

\begin{equation}
\Omega^2 \sim \frac{\hat{q}}{z(1-z)p_+}
\end{equation}

\begin{equation}
\Omega'^2 = -\Omega^2
\end{equation}

[Spectrum: [LA, Armesto, Milhano and Salgado (in preparation)]

L. Apolinário et al, Hard Probes 2016
In-Medium Antenna

Beyond eikonal limit

\[ \left| S_{\text{tot}} \right|^2 = \Delta'_{\text{coh}} \otimes (R_q + R_{\bar{q}}) - 2(\Delta'_{\text{coh}} - \Delta'_{\text{med}})J \]

In the collinear antenna limit, for soft gluon radiation:

\[ \Delta'_{\text{coh}} = \left[ 1 + \frac{(q_1, \perp - q_2, \perp)^2}{\hat{q} F \Delta L_+} \right] e^{-\frac{(q_1, \perp - q_2, \perp)^2}{2\hat{q} F \Delta L_+}} \]

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~ 2 independent broadenings

Two independent harmonic oscillators

[LA, Armesto, Milhano and Salgado (in preparation)]
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\[ Q_s^{-1} \]

\[ r_\perp \]

\[ Q_s^{-1} \]

~ 2 independent broadenings

[LA, Armesto, Milhano and Salgado (in preparation)]
In-Medium Antenna

- Beyond eikonal limit
- Elastic Channel:

\[ q_1, q_2, \ldots, q_n \]

[LA, Armesto, Milhano and Salgado (in preparation)]
In-Medium Antenna

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\[
\Delta'_{coh} = \frac{1}{N_c^2} \text{Tr} \left\langle G_1 G_2^\dagger \right\rangle \text{Tr} \left\langle G_1^\dagger G_2 \right\rangle
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[LA, Armesto, Milhano and Salgado (in preparation)]
In-Medium Antenna

- Beyond eikonal limit
- Elastic Channel:

\[ \Delta'_{coh} = \frac{1}{N_c^2} \text{Tr} \left( G_1 G_2^\dagger \right) \text{Tr} \left( G_1^\dagger G_2 \right) \]

\[ |S_{tot}|^2 = |S_{el}|^2 \otimes \left[ (R_q + R_q - 2J) + 2\Delta_{med}J \right] \]

\[ = |S_{el}|^2 \otimes \left[ R_{coh} + 2\Delta_{med}J \right] \]

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\]

\[
\Delta_{med} = \frac{\Delta'_{med}}{\Delta'_{coh}}
\]

\[
|S_{tot}|^2 = |S_{el}|^2 \otimes [(R_q + R_q - 2J) + 2\Delta_{med}J]
\]

\[
= |S_{el}|^2 \otimes [R_{coh} + 2\Delta_{med}J]
\]

Elastic channel that includes Non-eikonal corrections

Non-eikonal \(\Delta_{med}\)

“Vacuum Angular Ordering”

[LA, Armesto, Milhano and Salgado (in preparation)]
Summary

- Picture of what a quenched jet is based on a scale separation:
  - $\hat{q}_L > > \theta L$: Medium does not resolve object
  - $\hat{q}_L << \theta L$: Medium resolve object

<table>
<thead>
<tr>
<th>Two distinct showers</th>
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<td>Both described by evolution equations</td>
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AO “Vacuum” showers

$\theta L << \hat{q} L$
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- Anti-AO “Medium” shower
  - \( \theta L \gg \hat{q} L \)
  - \( t \sim L \)
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✦ Picture of what a quenched jet is based on a scale separation:

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Two distinct showers
Both described by evolution equations

Anti-AO “Medium” shower
\( \theta L >> \hat{q} L \)
\( t \sim L \)

AO “Vacuum” showers
\( \theta L \ll \hat{q} L \)
\( t \sim \ln Q^2 \)

Still need to understand how is the interplay between these two showers

Different ordering variables
Additional scales (\( Q_s \) and \( \theta L \)) on top of hard scale (\( Q \))

But different angular behaviour!
Summary

✦ Current picture seems to agree with experimental observations:

\[ z = \frac{p_{\perp, \text{part}}}{p_{\perp, \text{jet}}} \]

[CMS (2013)]

All jet is able to emit (Decoh limit)
Summary

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All jet is able to emit (Decoh limit)

\[ l = \log \left( \frac{1}{x_h} \right) \]

\[ x_h = \sqrt{z^2 + \left( \frac{m_h}{p_{\perp, \text{jet}}} \right)^2} \]

[CMS (2013)]

Q-PYTHIA 2.76 TeV

[Coh = Coherent Jet + Energy Loss]

[Coh + Decoh = Decoherent Jet + Energy Loss]

[Vacuum = MLLA fit]

[Mehtar-Tani, Tywoniuk (2014)]
Conclusions

✦ Significant progress been made to understand the pQCD evolution of the parton shower in the presence of a hot and dense medium:

✦ Antenna picture beyond eikonal approximation:

✦ In the limit of a collinear antenna and soft limit, possible factor out an elastic channel that includes non-eikonal effects;

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✦ On-going effort to improve current limitations of the qualitative picture obtained so far:

✦ Exp: new observables to constrain jet quenching mechanisms (sub-jets [Apolinário, Milhano, Ploskon and Zhang, 2016], Jet SF [Sunday, session IV], Soft Drop [Saturday, session III talk]

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Conclusions

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Thank you!!
Backup Slides
Vacuum Jets

- Starting by something more simple...
- pQCD scale:
- Hard parton decreasing virtuality: parton branching
  
  - Possible to calculate its evolution through an equation cascade that is derived from the fact that it is possible to factorize the process description
Medium Jets

- In a first approximation, if one sees the medium as a collection of scattering centres, and since the timescale are different, propagators are described by Green functions.
- Each interaction will enhance the WW field cloud inducing extra-gluon radiation.
- Results include additional energy loss and brownian motion.
  - Single gluon emission spectrum with Delta_coh (show correlators)
  - Delta_coh indicating loss of locality? Factorisation breaking?
- In vacuum, to see the interferences we need to go the antenna setup.
In-medium antenna

- Results of the eikonal antenna show that there are two kinds of shower, happening at the same time.

- It also shows that it is possible to separate two kinds of showers based on a scale separation.

- Inclusion of brownian motion.
  - Interference term: delta_med modification
  - Direct term: quadrupole (delta_coh)

- Momentum space (non-factorized piece identically zero)
  - Results not so easy to interpret, but dependence only relies on the previous emission.
Summary

- Current picture seems to agree with experimental observations:

\[ p_T^{\parallel} = p_{T,\text{trk}} \cos(\phi_{\text{trk}} - \phi_{\text{dijet}}) \]

Transverse momentum (energy) of the jets

\[ A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}} \]

[LA, N. Armesto, L. Cunqueiro (2013)]

[CMS JHEP 01 (2016) 006]

[CMS PRC 84 (2011) 024906]