Factorization of in-medium parton branching beyond the eikonal approximation

Liliana Apolinário
(CENTRA-IST & LIP)

in collaboration with:
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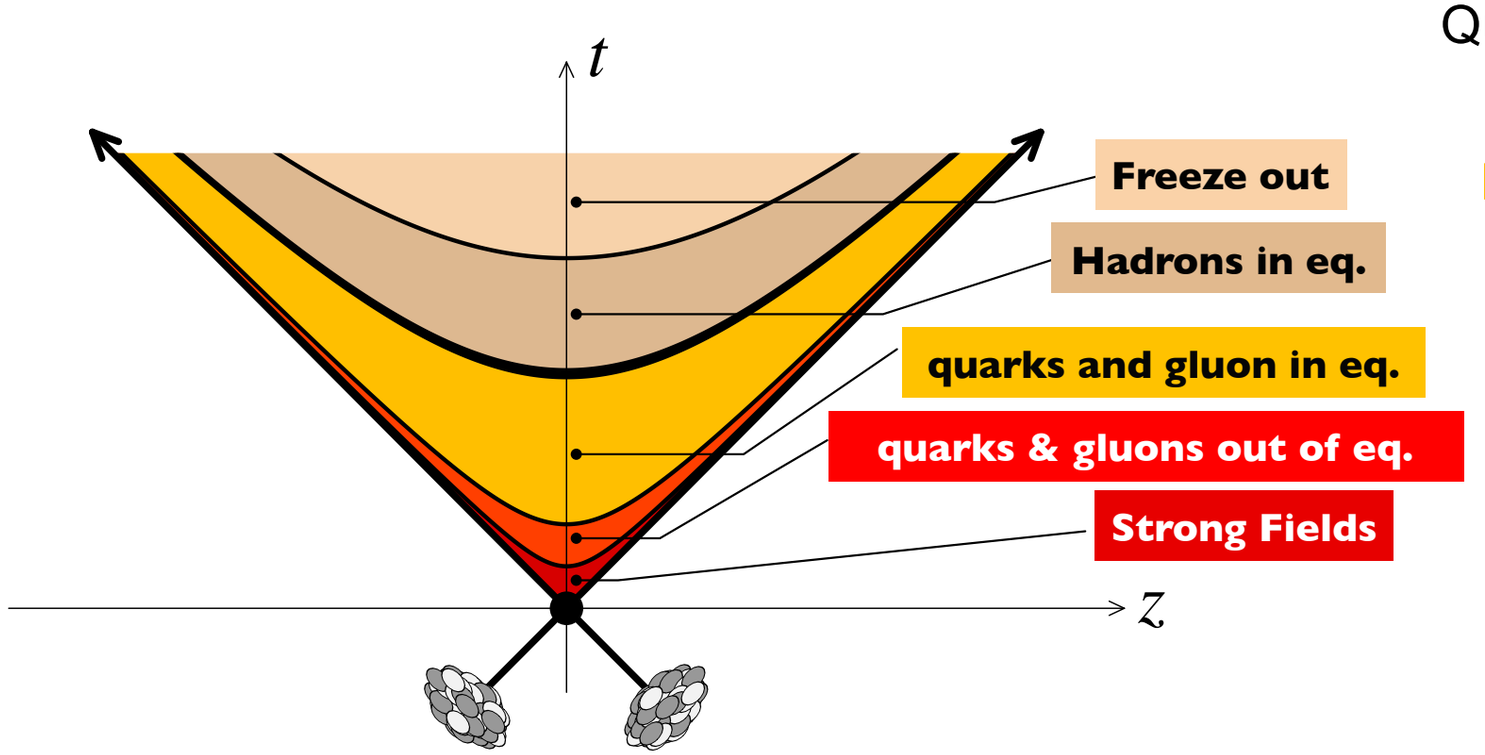
(CENTRA-IST & LIP) and Carlos A. Salgado (USC)

(manuscript under preparation…)

Introduction

- Heavy-Ion Collisions:
  - Perfect laboratory to evaluate QCD behaviour at high temperatures and densities



Quark-Gluon Plasma

Confinement/Deconfinement
Asymptotic freedom
QCD Phase transitions

Bulk of the collision

L. Apolinário et al, Hard Probes 2016

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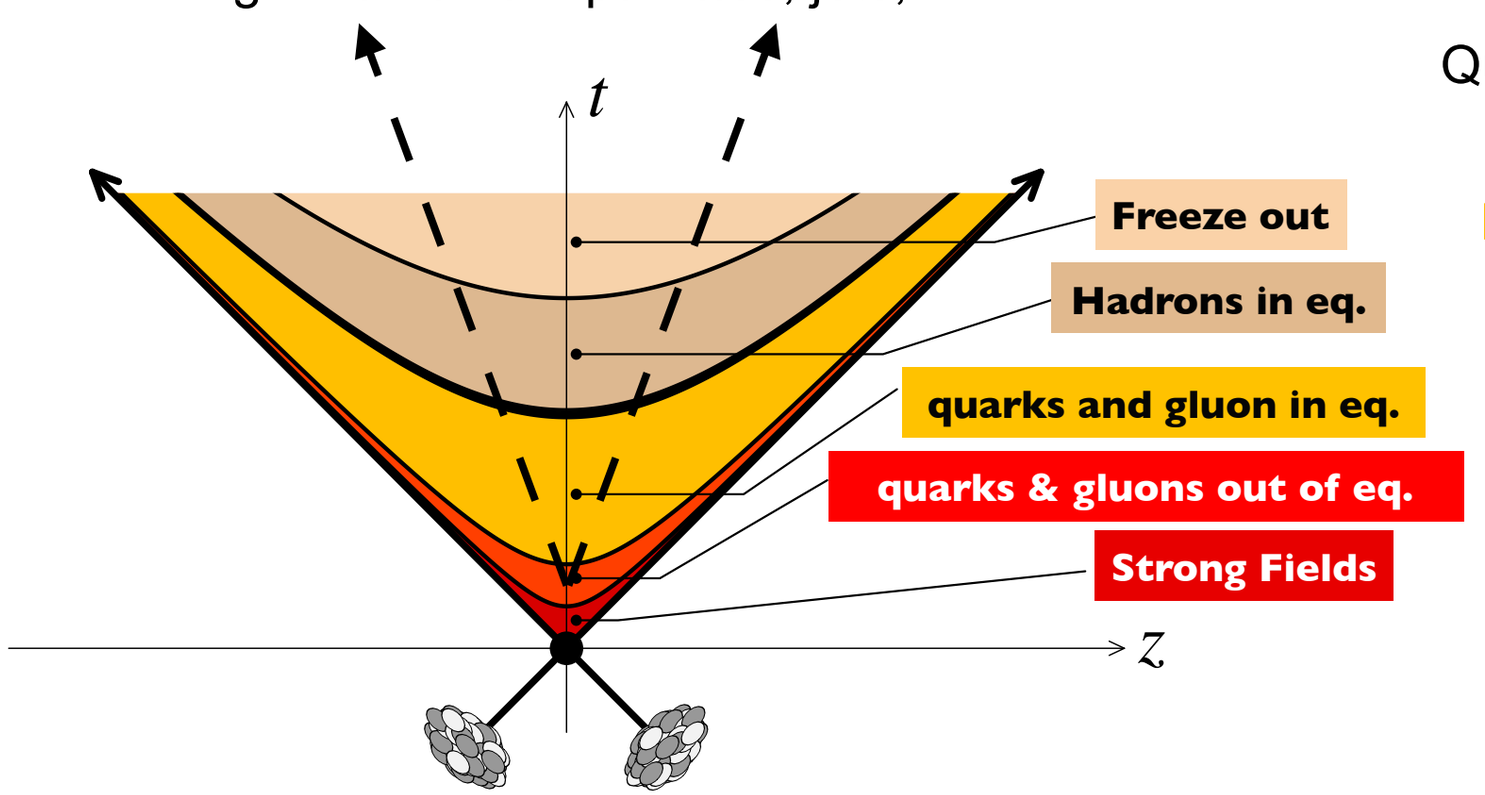
Modified probes (energy loss, broadening, …)

→ Macroscopic properties of the created medium
QCD dynamics in dense regime

High momentum particles, jets, …



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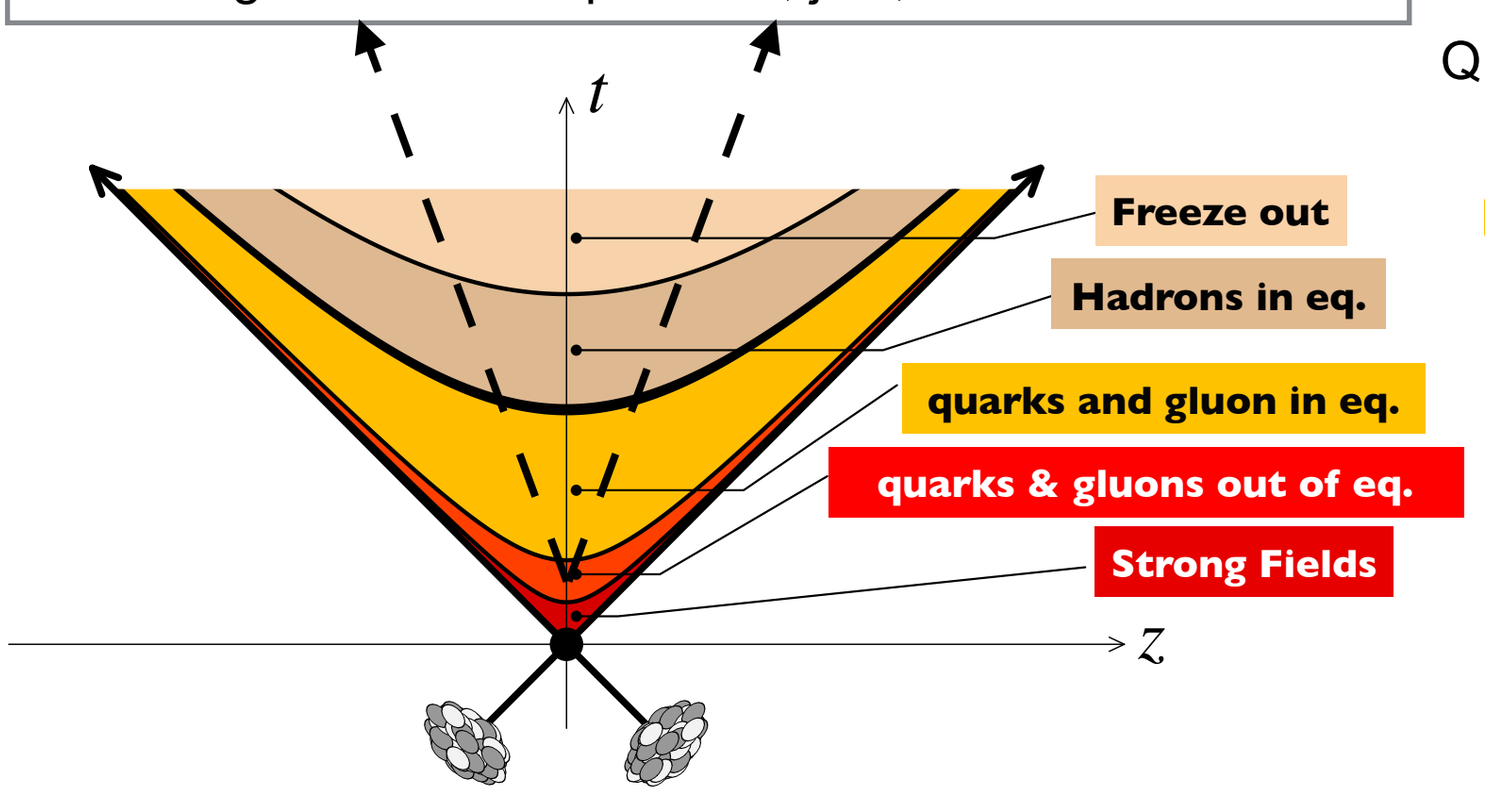

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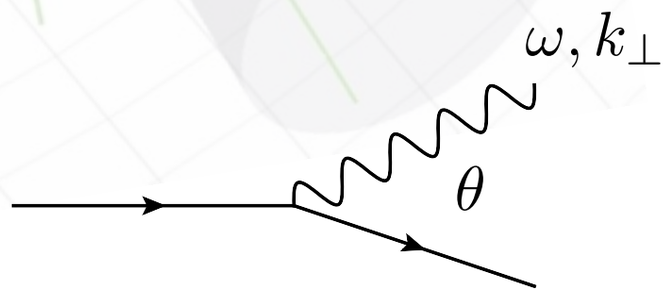
Vacuum Parton Shower

- Starting by what we know:
- pQCD description of a high virtuality parton
  - Phase space limited to: $\sqrt{Q^2} \geq \omega \geq k_\perp >> \Lambda_{QCD}$



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Double Log enhancement

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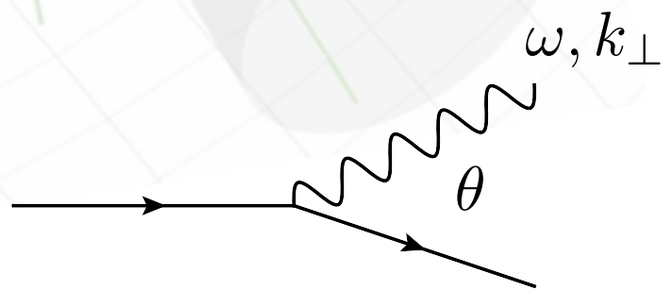

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Double Log enhancement

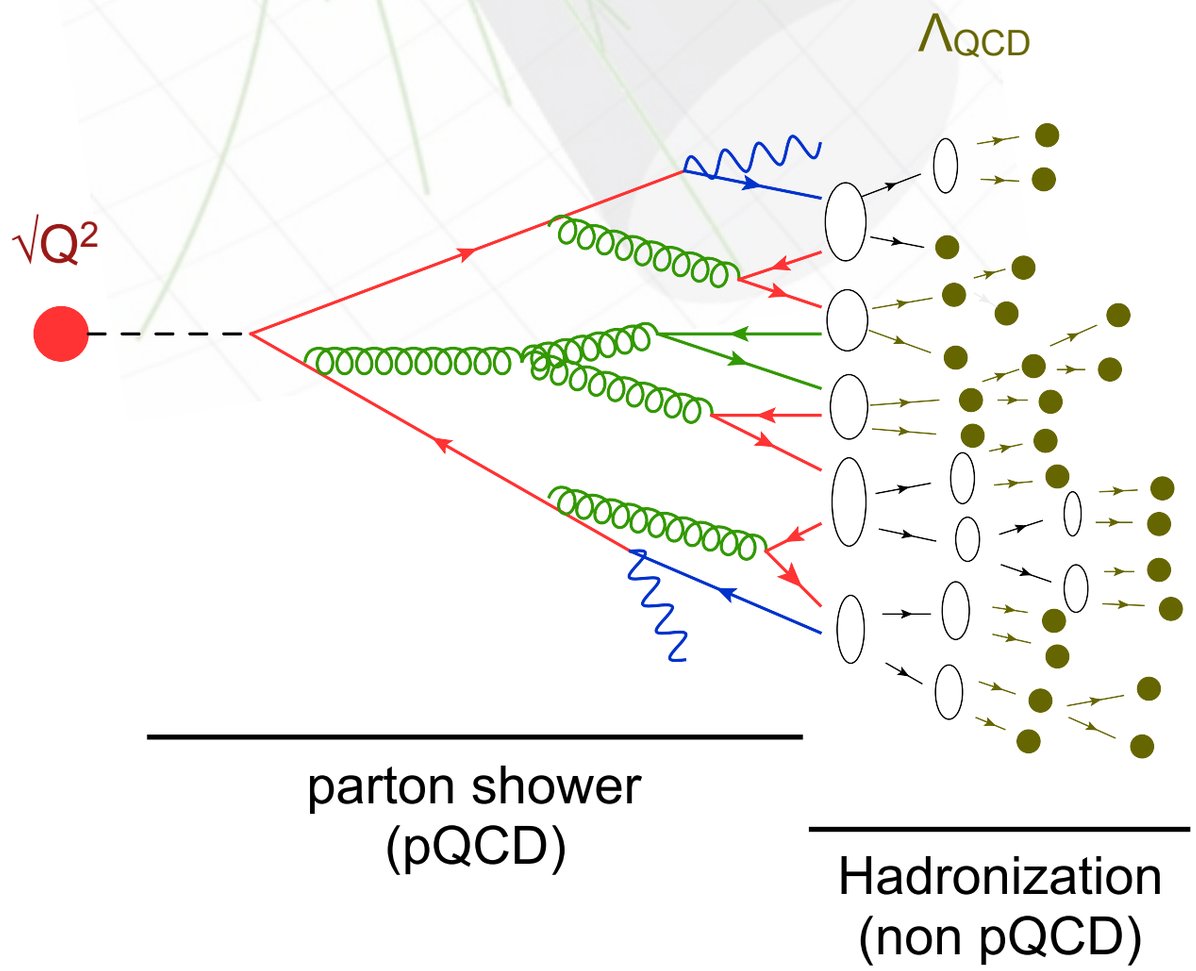
- Ordering variable t ~ ln Q
  - DGLAP evolution equation

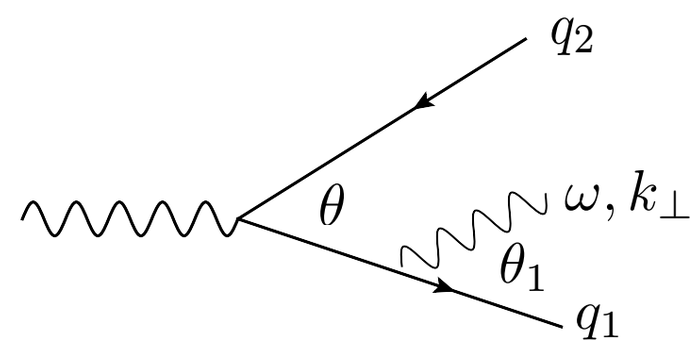
$f(z,Q^2)$ = p.d.f
P(x) = splitting function

$$\frac{\partial f_i(z,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 dx \sum_j P_{i\leftarrow j}\left(\frac{z}{x}\right) f_j(x,Q^2)$$

Vacuum Parton Shower

- Generalising for multiple gluon emissions:
  - Need to check interferences between subsequent emissions





$$dN_q^{\omega\to 0} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{sin\theta\, d\theta}{1-\cos\theta} \Theta(\cos\theta_1 - \cos\theta)$$

$$\Rightarrow \theta > \theta_1 > \theta_2 > \cdots$$

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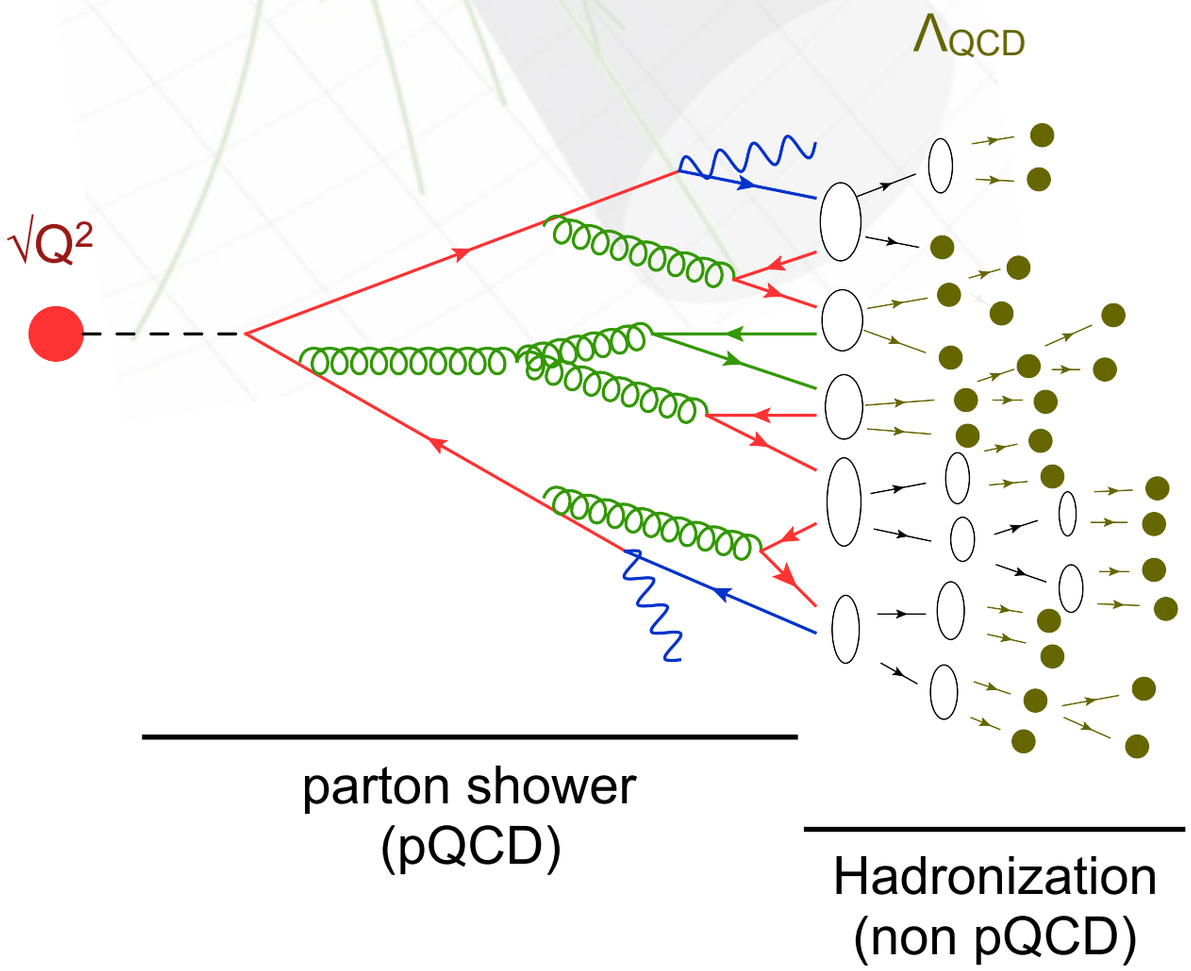

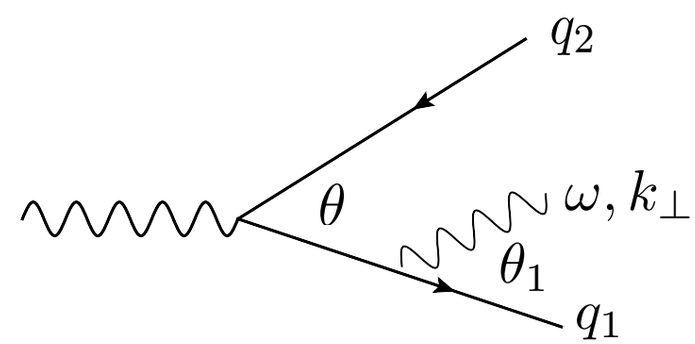

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Ordering variable: t ~ ln θ²
(MLLA Approximation)

Includes Single Logs

In-Medium Interactions

- Medium = Quark-Gluon Plasma: strongly coupled fluid (non pQCD)
- Hard probe: $\sqrt{Q^2} >> \Lambda_{QCD}$, T (pQCD evolution)

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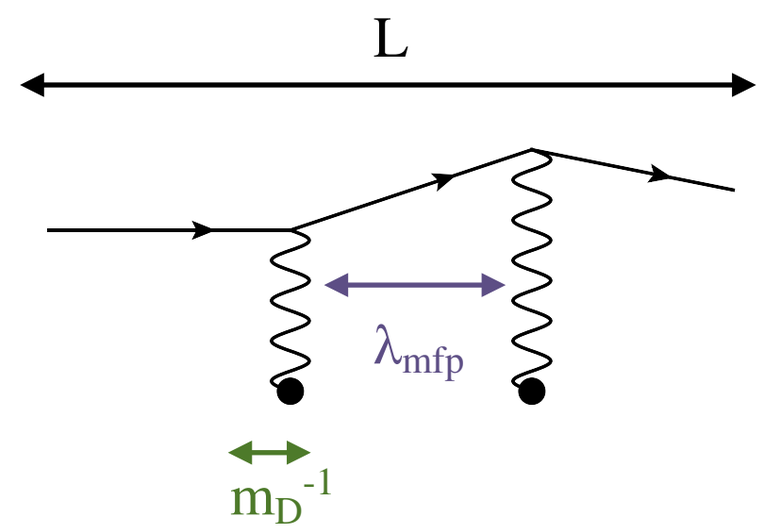
- Hard probe: $\sqrt{Q^2} >> \Lambda_{QCD}$, T (pQCD evolution)
- Medium - hard probe interaction: ???
  - Approximation: medium as a collection of independent and static scattering centres:

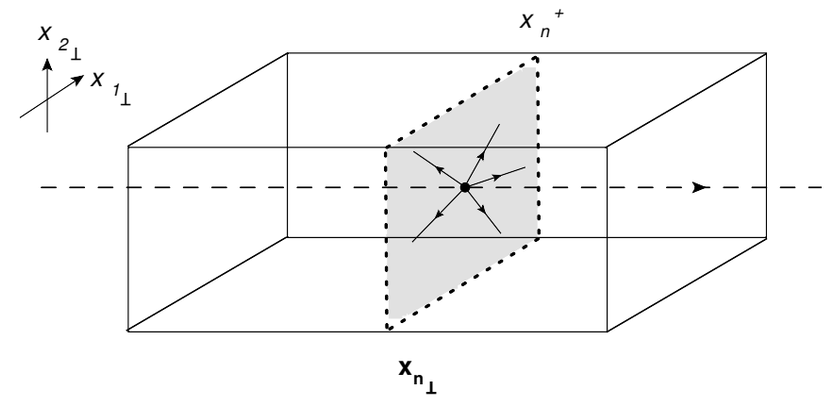
Medium Length: L
Mean free path: λ_{mfp}
Debye Mass: m_D^{-1}

$L >> \lambda_{mfp} >> m_D^{-1}$



Transport coefficient: $\hat{q} = \frac{< k_\perp^2 >}{\lambda_{mfp}}$

In-Medium Propagation



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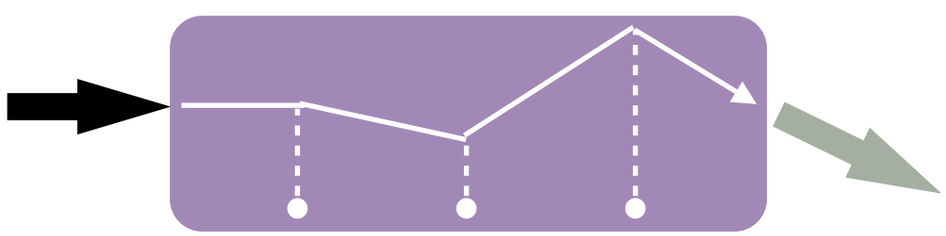
  - No energy loss in the longitudinal direction;
  - Transverse momentum “kick”: Brownian motion in the transverse direction
  - Color phase rotation:



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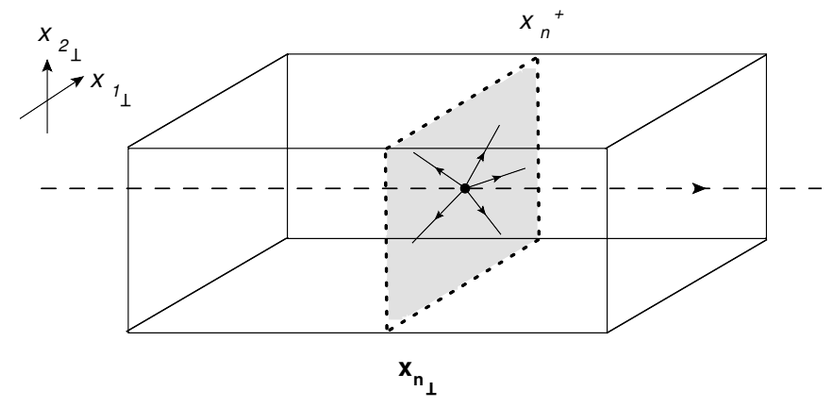

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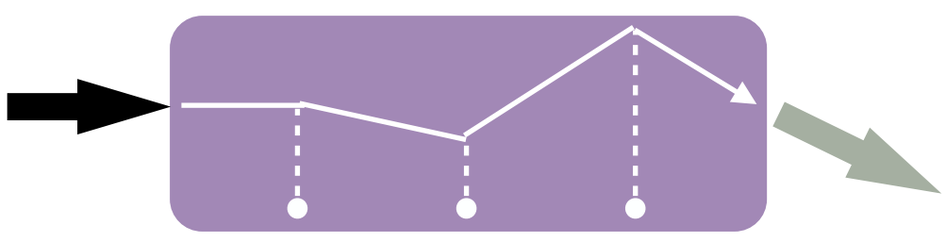
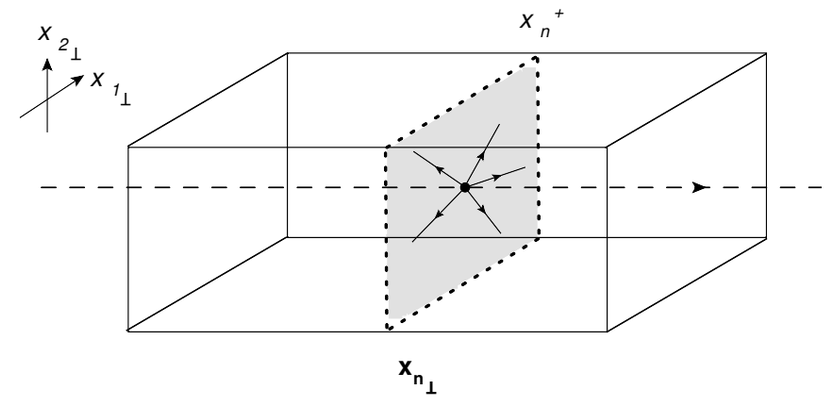


$$G(x_{0+}, \mathbf{x_0}_\perp; L_+, \mathbf{x}_\perp | p_+) = \int_{\mathbf{r}_\perp(x_{0+})=\mathbf{x_0}_\perp}^{\mathbf{r}_\perp(L_+)=\mathbf{x}_\perp} \mathcal{D}\mathbf{r}_\perp(\xi) \exp\left\{ \frac{ip_+}{2} \int_{x_{0+}}^{L_+} d\xi \left(\frac{d\mathbf{r}_\perp}{d\xi} \right)^2 \right\} \times W(x_{0+}, L_+; \mathbf{r}_\perp(\xi)),$$

Initial/Final coordinates

In-Medium Propagation



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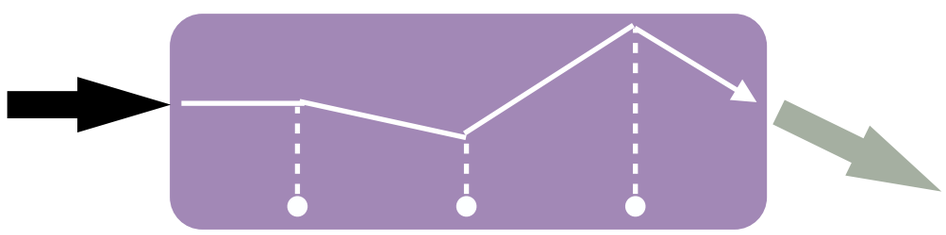
$$W(x_{0+}, L_+; \mathbf{x}_\perp) = \mathcal{P} \exp\left\{ ig \int_{x_{0+}}^{L_+} dx_+ A_-(x_+, \mathbf{x}_\perp) \right\}$$

Path-ordering

Medium colour field

Transverse coordinate



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In-Medium Energy Loss

- Medium interactions induces accumulation of momenta
  - Enhancement of gluon radiation

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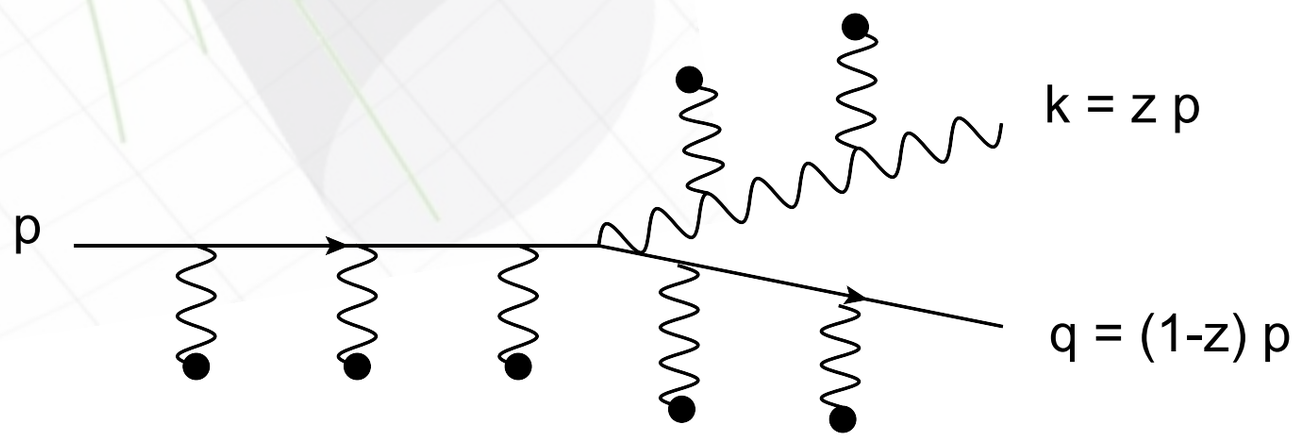

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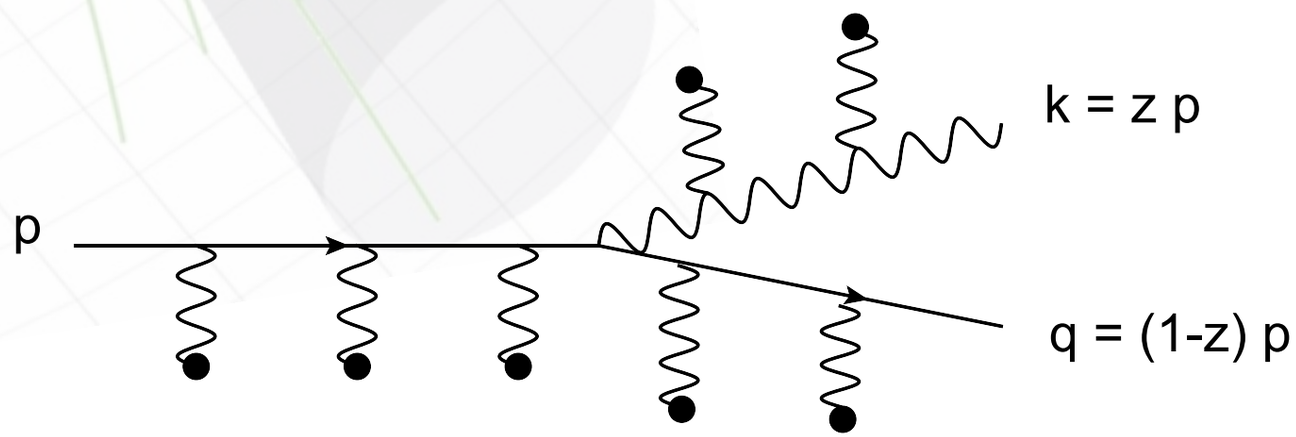
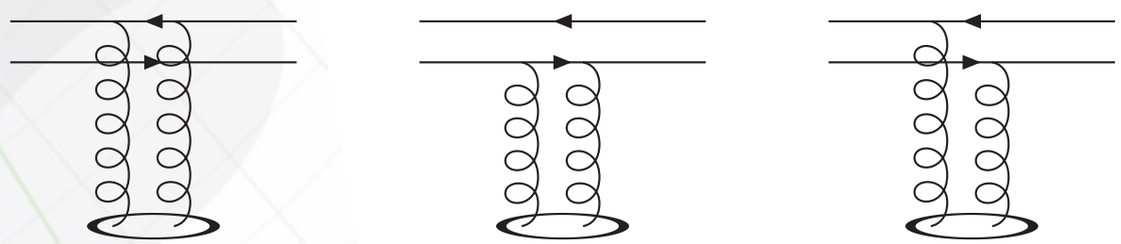


- Timescale ~ Q^{-1}:
  - Frozen medium color configuration during propagation
    - Average over the configurations ensemble

Medium Averages

- Color information in the W:
  - N-field correlators up to second order in fields: $< A(\mathbf{x}_\perp)A(\mathbf{y}_\perp) >$



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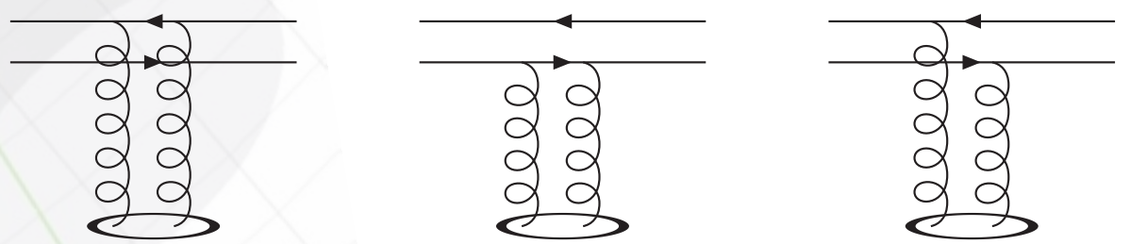

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  - Path-integral evaluation using a semi-classical approximation:

$$G_0(x_+, \mathbf{x}; y_+, \mathbf{y}) = \int_{\mathbf{r}(x_+)=\mathbf{x}}^{\mathbf{r}(y_+)=\mathbf{y}} \mathcal{D}\mathbf{r}(\xi) \exp\left\{ \frac{ip_+}{2} \int_{x_+}^{y_+} d\xi \left(\frac{d\mathbf{r}}{d\xi}\right)^2 \right\}$$

$$= \frac{1}{(2\pi i)^{D/2}} \left| \det\left(-\frac{\partial^2 R_{cl}}{\partial \mathbf{y}_i \partial \mathbf{x}_i} \right) \right|^{1/2} \mathrm{e}^{iR_{cl}(x_+, \mathbf{x}; y_+, \mathbf{y})}$$

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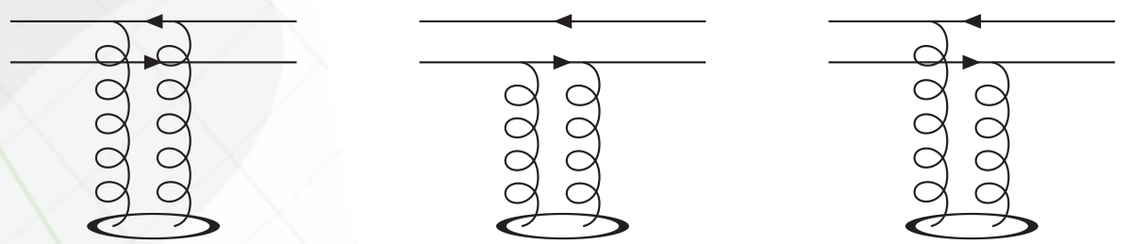

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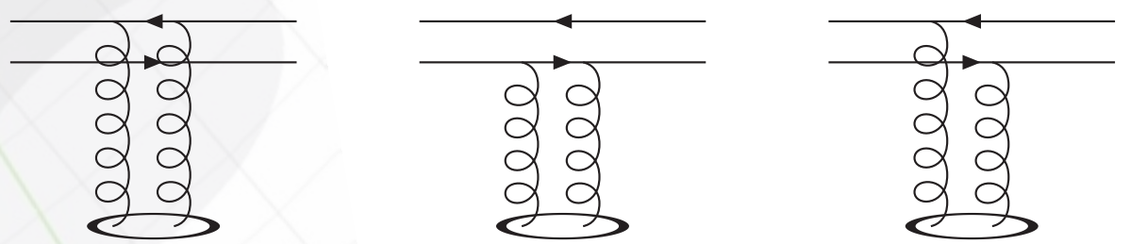

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+ Fluctuations of the classical action

Dominant contribution for the average trajectory given by the classical path

In-Medium Radiation

[Blaizot, Dominguez, Iancu, Mehtar-Tani (2013-2014)]
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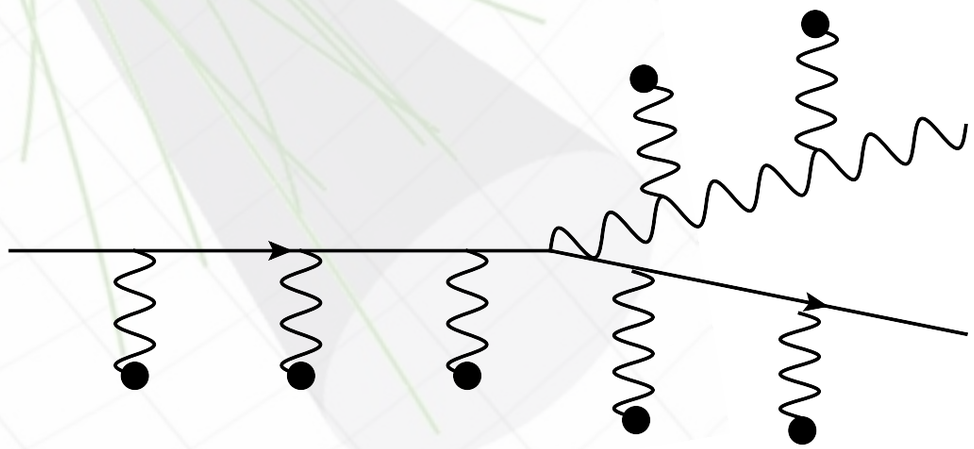
- Single medium-induced gluon radiation:



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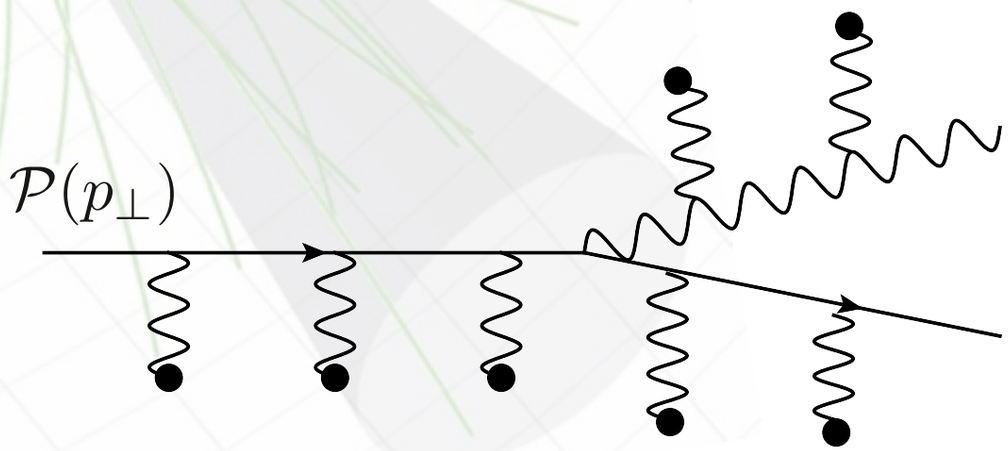

Momentum broadening:

$$\mathcal{P}(k_\perp) = \frac{4\pi}{\hat{q}\Delta L_+} \exp\left\{-\frac{\Delta k_\perp^2}{\hat{q}\Delta L_+}\right\}$$

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

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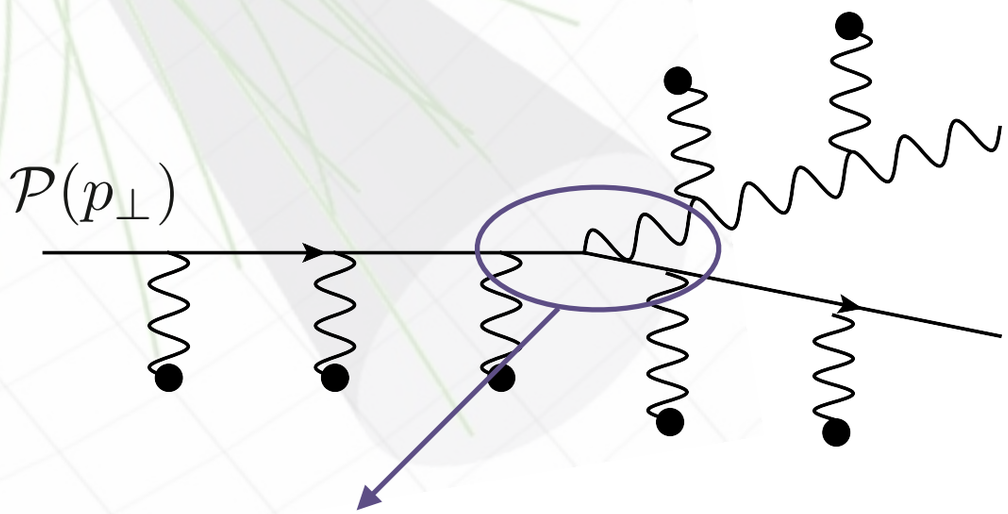

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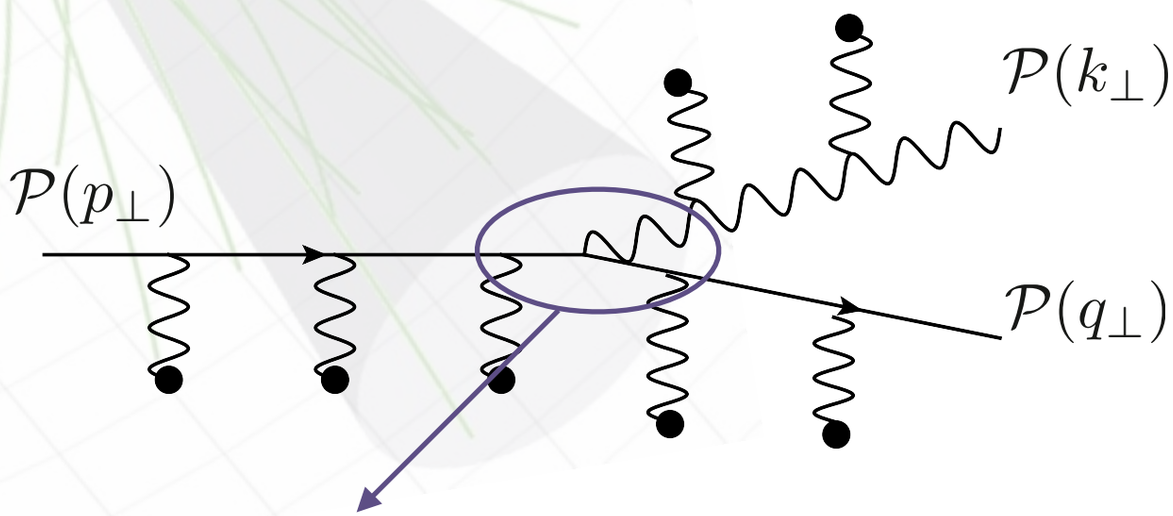

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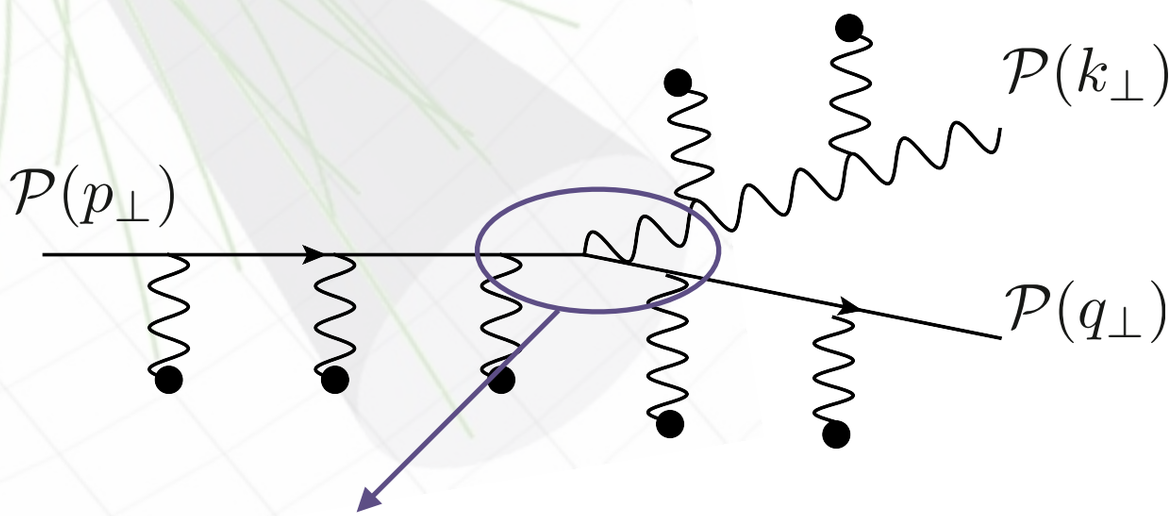

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Complete factorisation
Independent radiation

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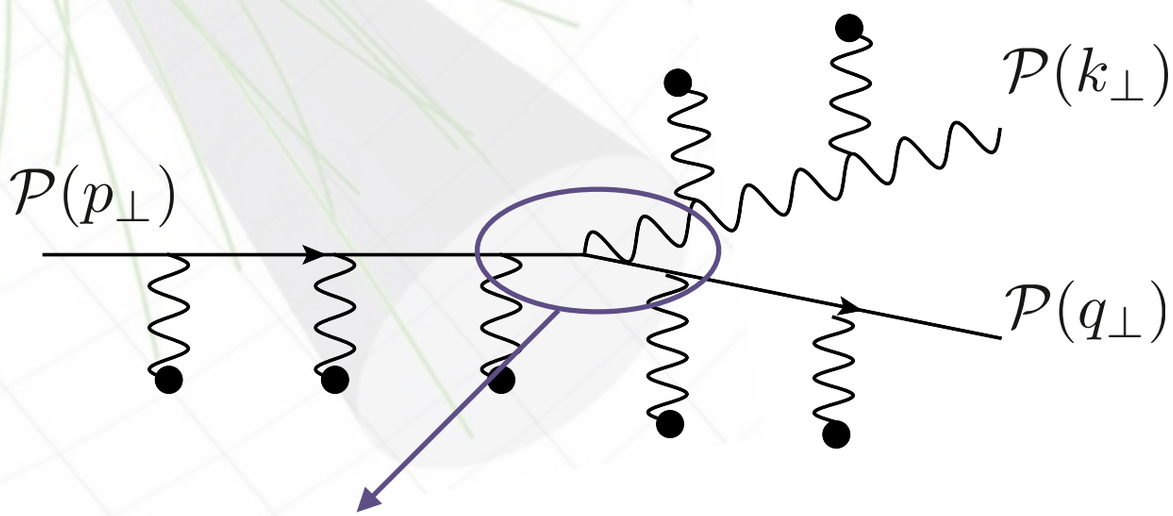

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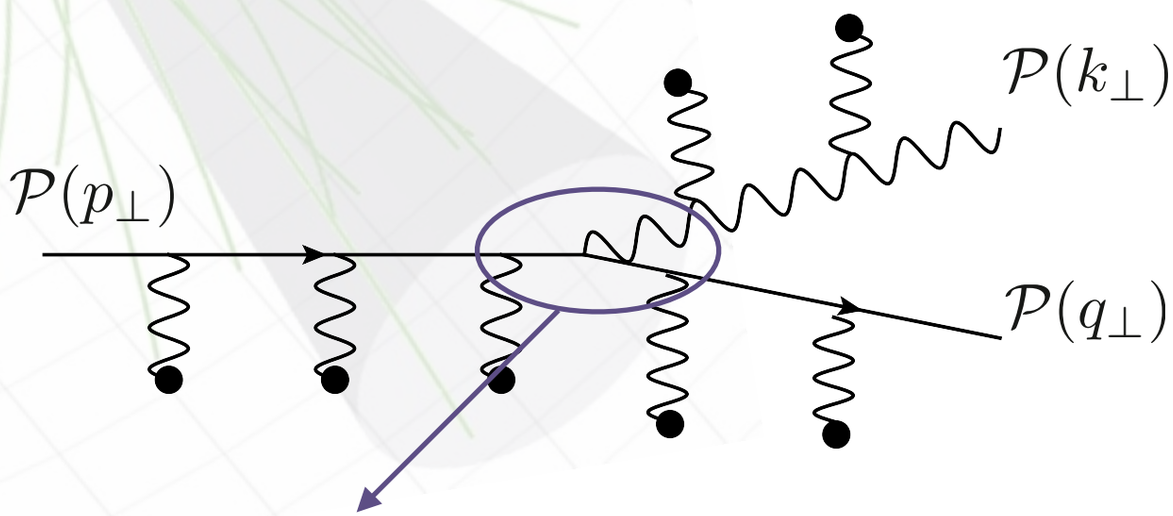
Complete factorisation
Independent radiation

Propagate coherently
Coherent gluon radiation

In-Medium Radiation

[Blaizot, Dominguez, Iancu, Mehtar-Tani (2013-2014)]
[LA, Armesto, Milhano, Salgado, (2015)]

- Single medium-induced gluon radiation:



Momentum broadening:

$$\mathcal{P}(k_\perp) = \frac{4\pi}{\hat{q}\Delta L_+} \exp\left\{-\frac{\Delta k_\perp^2}{\hat{q}\Delta L_+}\right\}$$

Transverse position of the quark/gluon in the (complex conjugate) amplitude:

$$\mathbf{x}_{q(\bar{q})} / \mathbf{x}_{\bar{g}(g)}$$

Spectrum proportional to:

$$\Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau\, \hat{q}\, (\mathbf{x}_q - \mathbf{x}_{\bar{q}}) \cdot (\mathbf{x}_g - \mathbf{x}_{\bar{g}})\big|_\tau\, \mathrm{e}^{-\hat{q}\int_{x_+}^{\tau} d\xi (\mathbf{x}_q - \mathbf{x}_{\bar{g}})\cdot(\mathbf{x}_{\bar{q}} - \mathbf{x}_g)}$$

Complete factorisation
Independent radiation

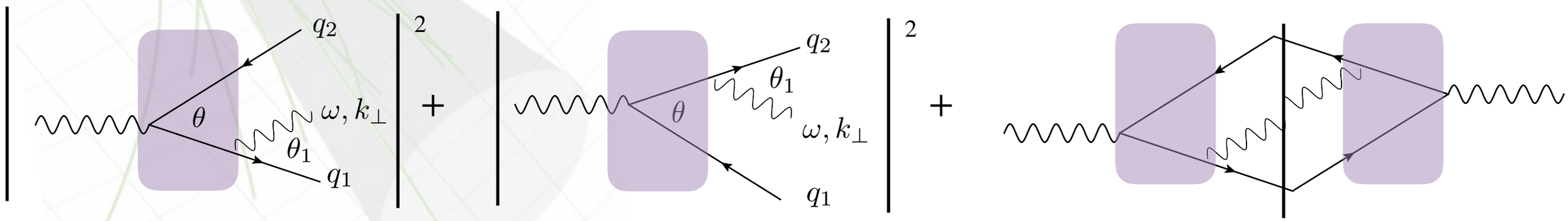
Propagate coherently
Coherent gluon radiation

Probabilistic picture still holds???

In-Medium Antenna

[Mehtar-Tani, Salgado, Tywoniuk (2010-2011)]
[Casalderrey-Solana, Iancu (2011)]

- Subsequent emissions investigated in a quark-antiquark antenna setup:



Eikonal approximation: $$\frac{dI}{d\Omega_k} = R_q + R_{\bar{q}} - 2J(1-\Delta_{med}) = R_{coh} + 2J\Delta_{med}$$

$$R_q \sim \alpha_s C_F \frac{q_{1+}}{(k\cdot q_1)}$$

$$R_{coh} \sim \alpha_s C_F \frac{k_+(q_1\cdot q_2)}{(k\cdot q_1)(k\cdot q_2)}$$

$$R_{\bar{q}} \sim \alpha_s C_F \frac{q_{2+}}{(k\cdot q_2)}$$

$$1-\Delta_{med} = \frac{1}{N_c^2}\mathrm{Tr}\left\langle W_A(\mathbf{x}_q)W_A^\dagger(\mathbf{x}_{\bar{q}})\right\rangle$$

$$2J \sim \alpha_s C_F\left[\frac{q_{1+}}{(k\cdot q_1)} + \frac{q_{2+}}{(k\cdot q_2)} - \frac{k_+(q_1\cdot q_2)}{(k\cdot q_1)(k\cdot q_2)}\right]$$

In-Medium Antenna

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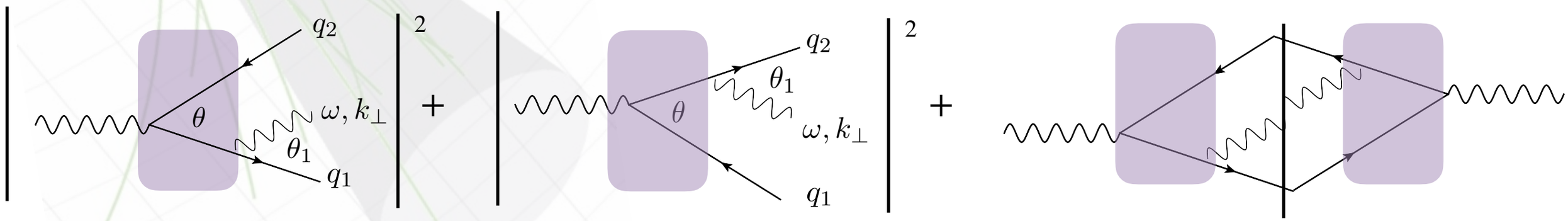

Integrating over azimuthal angle (soft limit):

$$dN_q^{\omega\to 0} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{\sin\theta d\theta}{1-\cos\theta} \left[\Theta(\cos\theta_1 - \cos\theta) + \Delta_{med}\Theta(\cos\theta - \cos\theta_1)\right] \qquad \Delta_{med} \approx 1 - \mathrm{e}^{-\frac{1}{12}Q_s^2 r_\perp^2}$$

Antenna Transverse resolution: $r_\perp = \theta L$

Medium Transverse Scale: $Q_s^{-1} = (\hat{q} L)^{-1/2}$

In-Medium Antenna

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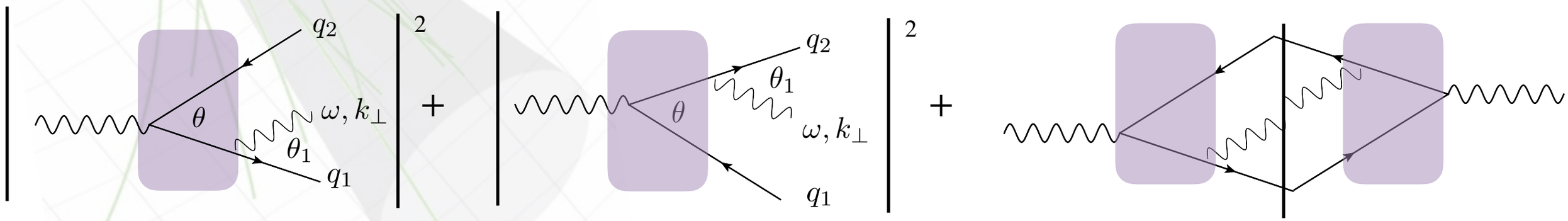

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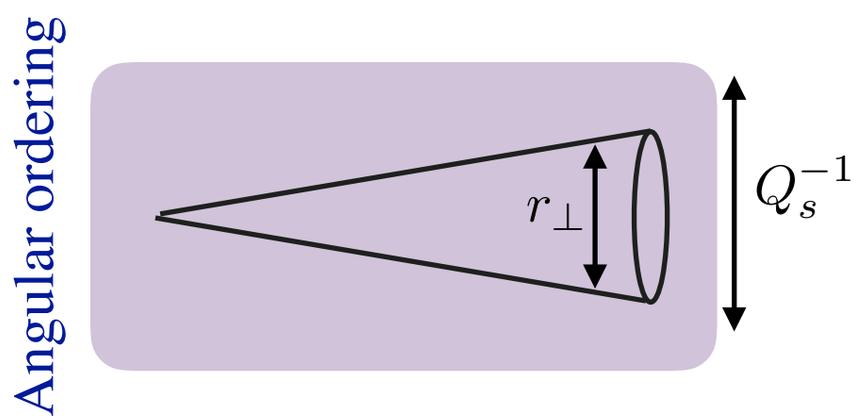

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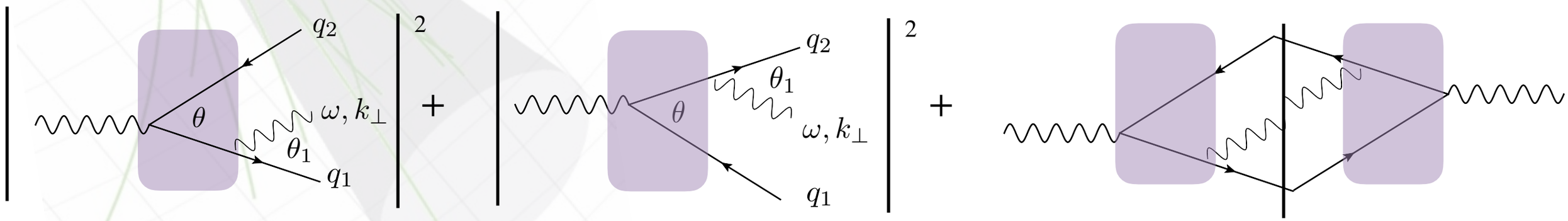

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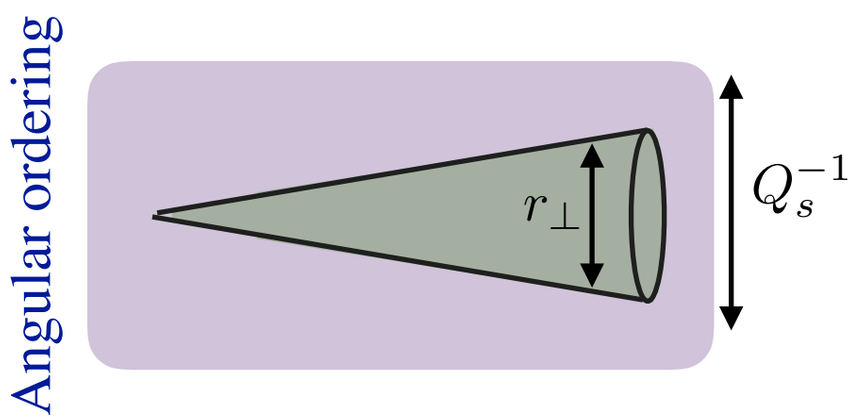

$\Delta_{med} \to 0$

In-Medium Antenna

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[Casalderrey-Solana, Iancu (2011)]

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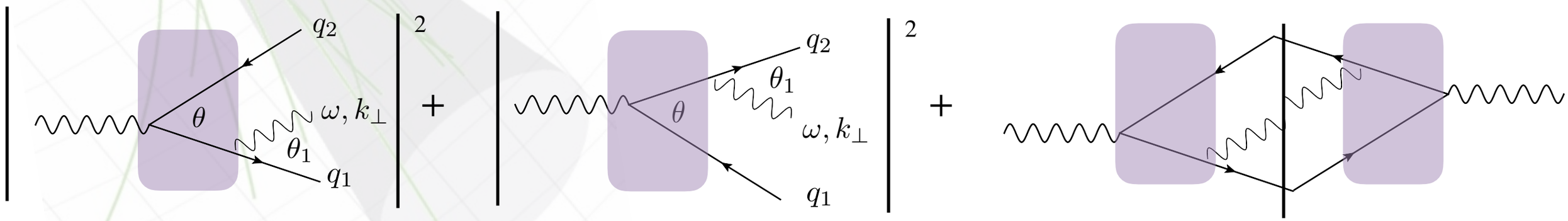

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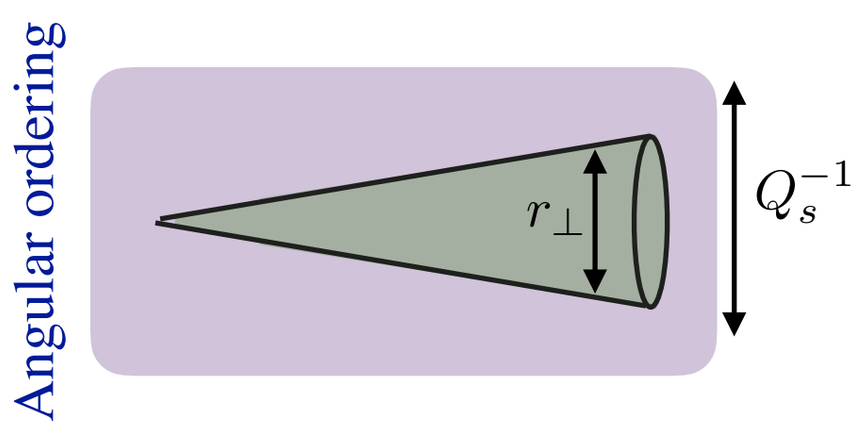
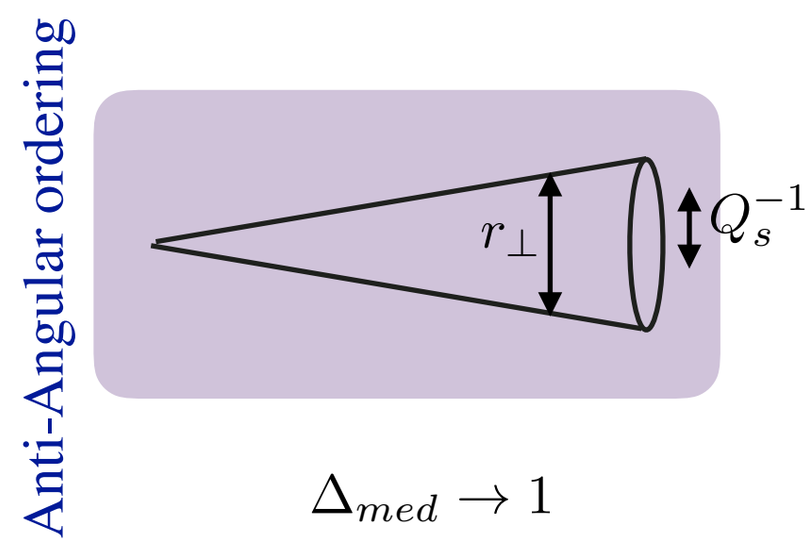


$\Delta_{med} \to 0$

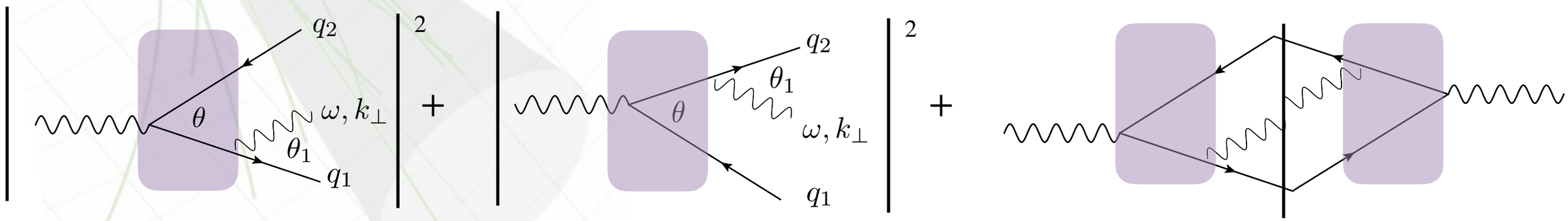


$\Delta_{med} \to 1$

In-Medium Antenna

[Mehtar-Tani, Salgado, Tywoniuk (2010-2011)]
[Casalderrey-Solana, Iancu (2011)]

- Subsequent emissions investigated in a quark-antiquark antenna setup:



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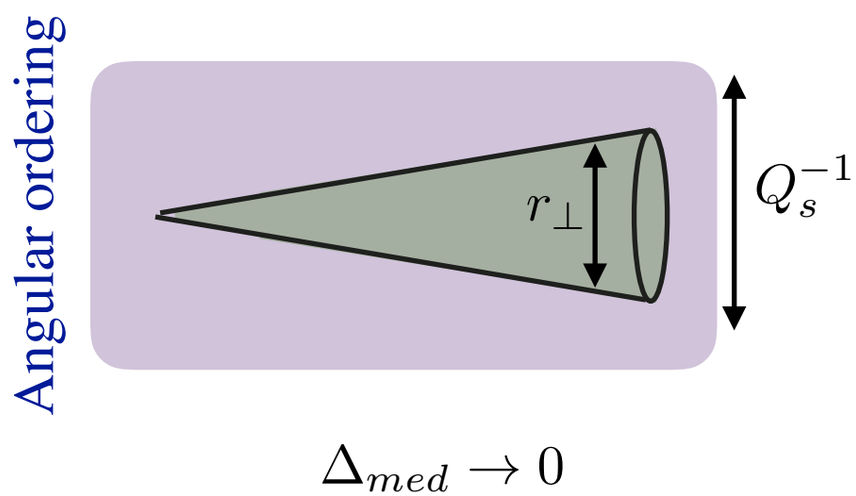
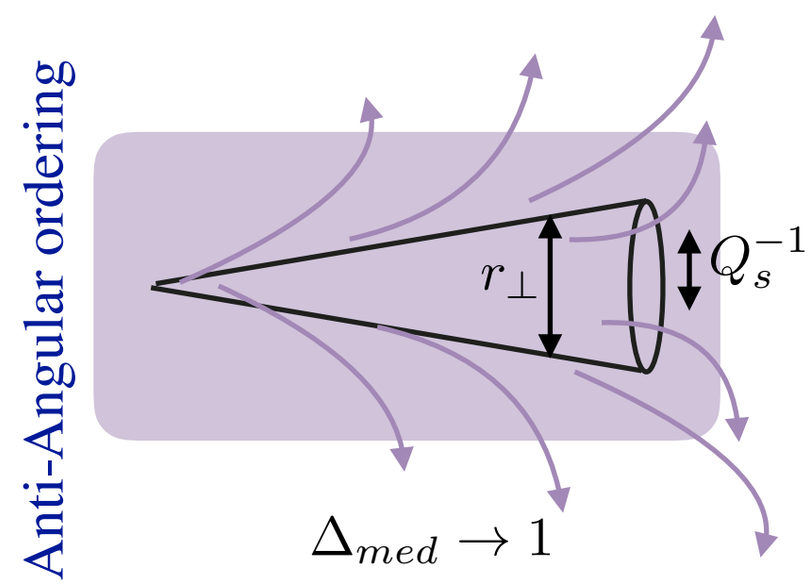
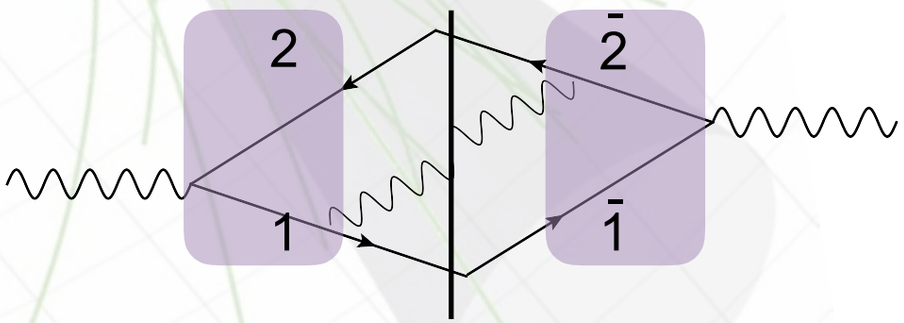




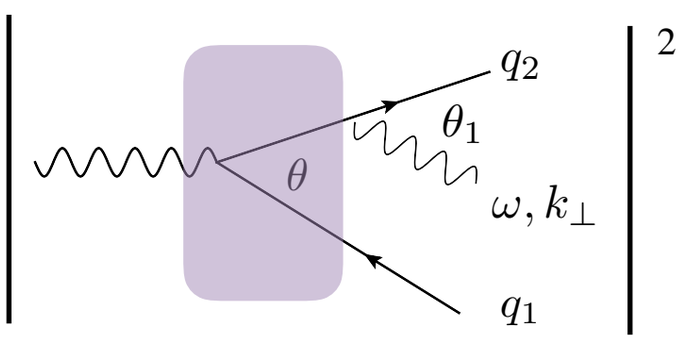
In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit:



$$2J \sim \alpha_s C_F \left[\frac{q_{1+}}{(k \cdot q_1)} + \frac{q_{2+}}{(k \cdot q_2)} - \frac{k_+ (q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right]$$



$$R_q \sim \alpha_s C_F \frac{q_{1+}}{(k \cdot q_1)} \qquad R_{\bar{q}} \sim \alpha_s C_F \frac{q_{2+}}{(k \cdot q_2)}$$

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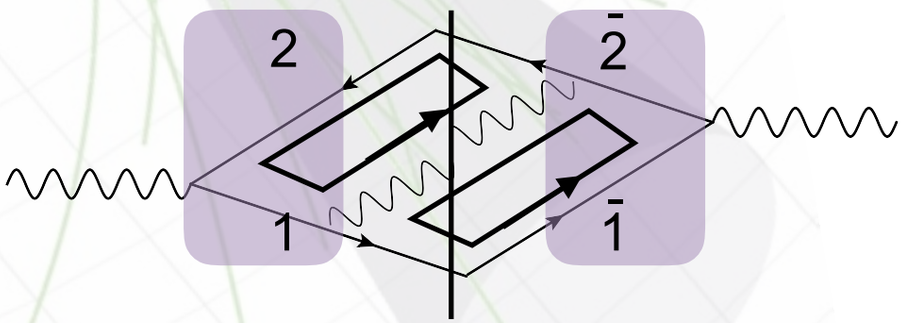

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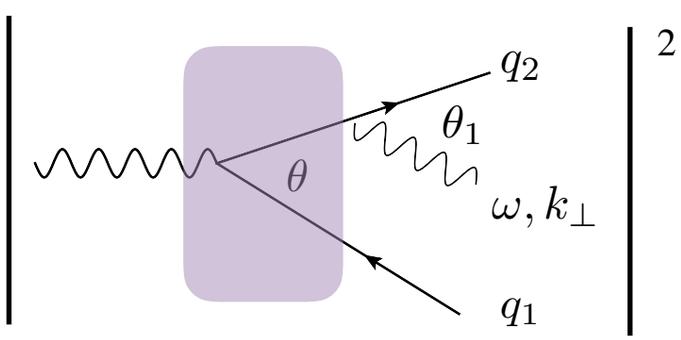
$$1 - \Delta'_{med} = \frac{1}{N_c^2} \mathrm{Tr} \left\langle G_1 G_2^\dagger \right\rangle \mathrm{Tr} \left\langle G_{\bar{1}}^\dagger G_{\bar{2}} \right\rangle$$



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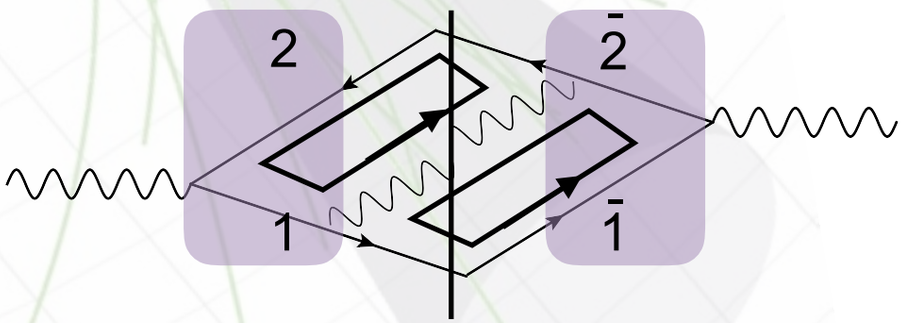

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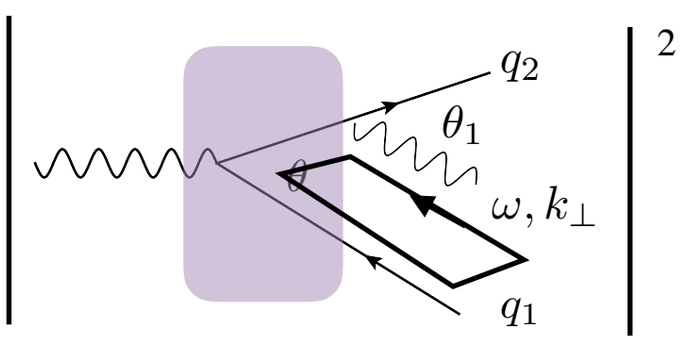


$$R_q \sim \alpha_s C_F \frac{q_{1+}}{(k \cdot q_1)} \qquad R_{\bar{q}} \sim \alpha_s C_F \frac{q_{2+}}{(k \cdot q_2)}$$

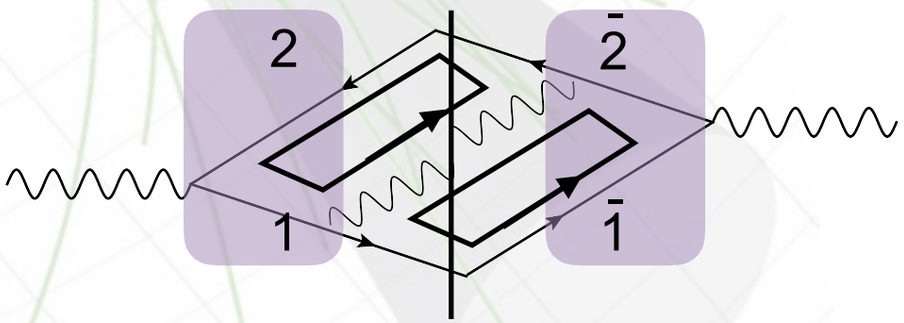
$$\Delta'_{coh} = \frac{1}{N_c^2} \mathrm{Tr} \left\langle G_1 G_2^\dagger \right\rangle \mathrm{Tr} \left\langle G_1^\dagger G_2 \right\rangle \Delta_{coh}$$

$$\Delta_{coh} = 1 + \int_{x_+}^{L_+} d\tau \, \hat{q} \, \left(\mathbf{x}_1 - \mathbf{x}_{\bar{1}} \right) \cdot \left(\mathbf{x}_{\bar{2}} - \mathbf{x}_2 \right) \big|_\tau \, \mathrm{e}^{-\hat{q} \int_{x_+}^{\tau} d\xi (\mathbf{x}_1 - \mathbf{x}_{\bar{2}}) \cdot (\mathbf{x}_{\bar{1}} - \mathbf{x}_2)}$$

In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit:



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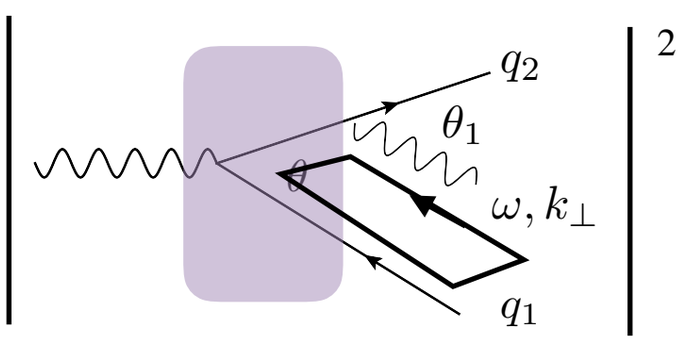

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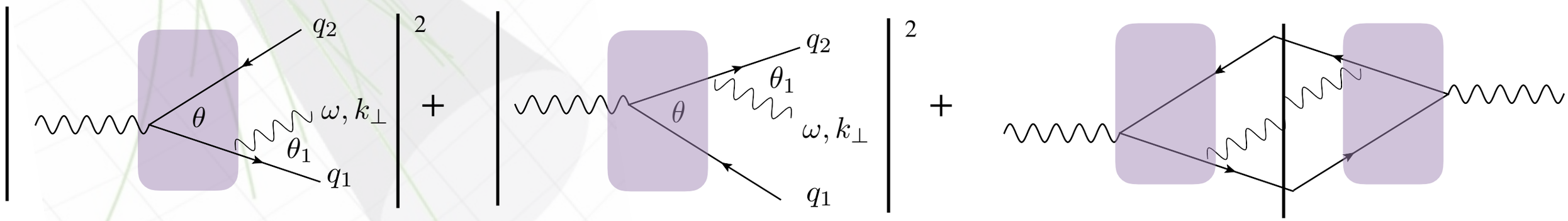
Momentum Space

$\sim p_{0\perp} \to 0$

In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit

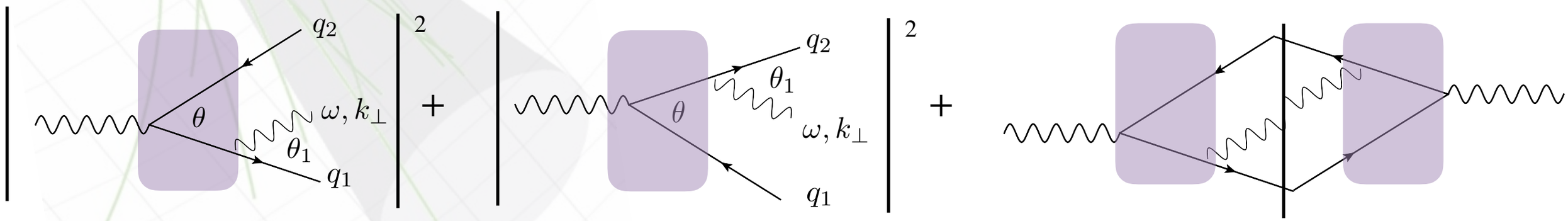


Spectrum: $|S_{tot}|^2 = \Delta'_{coh} \otimes (R_q + R_{\bar{q}}) - 2(1 - \Delta'_{med})J$

In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit



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In the collinear antenna limit, for soft gluon radiation:

$$\Delta'_{coh} = \left[1 + \frac{(q_{1,\perp} - q_{2,\perp})^2}{\hat{q}_F \Delta L_+}\right] \mathrm{e}^{-\frac{(q_{1,\perp} - q_{2,\perp})^2}{2\hat{q}_F \Delta L_+}}$$

$$1 - \Delta'_{med} \sim \mathrm{e}^{-\frac{i\tan(\Omega\Delta L_+)}{\Omega}[(1-z)(q_{1,\perp}+k_\perp)-zq_{2,\perp}]^2}$$
$$\mathrm{e}^{\frac{i\tan(\Omega'\Delta L_+)}{\Omega'}[(1-z)q_{1,\perp}-z(q_{2,\perp}+k_\perp)]^2}$$

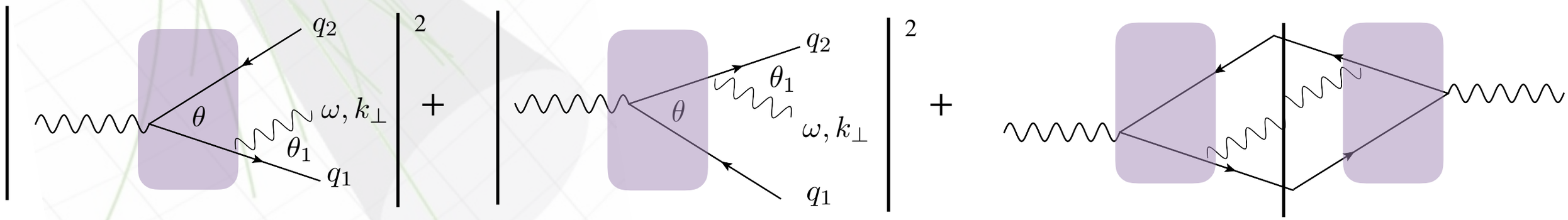
$$\Omega^2 \sim \frac{\hat{q}}{z(1-z)p_+}$$

$$\Omega'^2 = -\Omega^2$$

In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit



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$$\Delta'_{coh} - \Delta'_{med} \sim \mathrm{e}^{-\frac{i\tan(\Omega \Delta L_+)}{\Omega}[(1-z)(q_{1,\perp}+k_\perp) - z q_{2,\perp}]^2}$$

$$\mathrm{e}^{\frac{i\tan(\Omega' \Delta L_+)}{\Omega'}[(1-z) q_{1,\perp} - z(q_{2,\perp}+k_\perp)]^2}$$

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In-Medium Antenna

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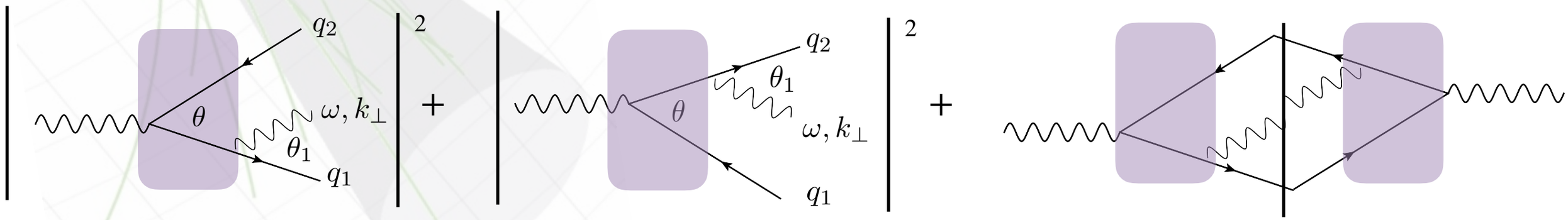

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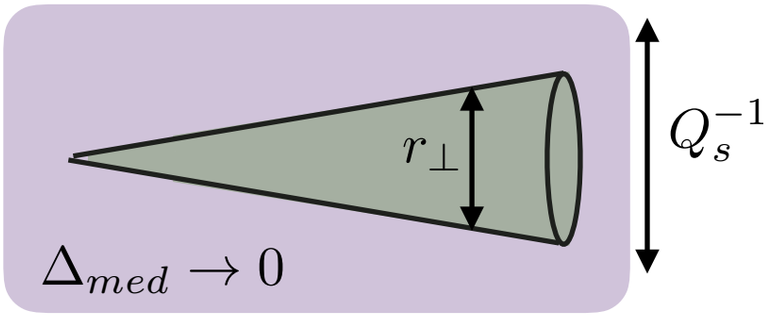
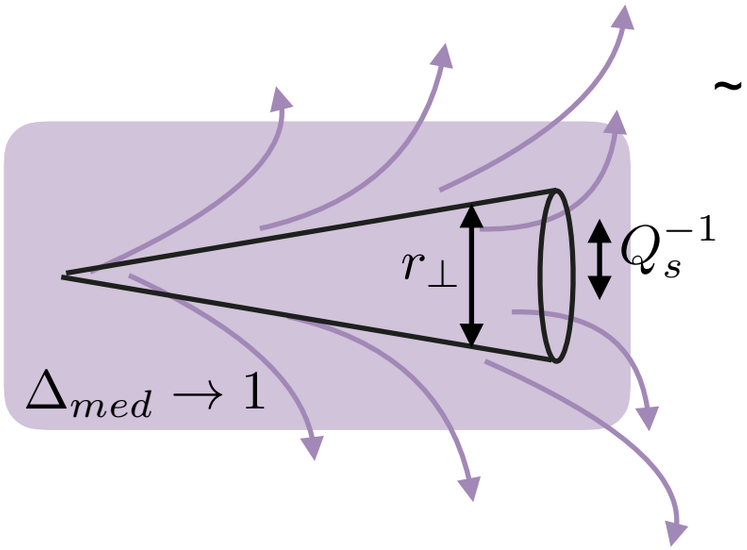
$$\mathrm{e}^{\frac{i\tan(\Omega'\Delta L_+)}{\Omega'}[(1-z)q_{1,\perp}-z(q_{2,\perp}+k_\perp)]^2}$$

$$\Omega^2 \sim \frac{\hat{q}}{z(1-z)p_+}$$

$$\Omega'^2 = -\Omega^2$$

Two independent harmonic oscillators



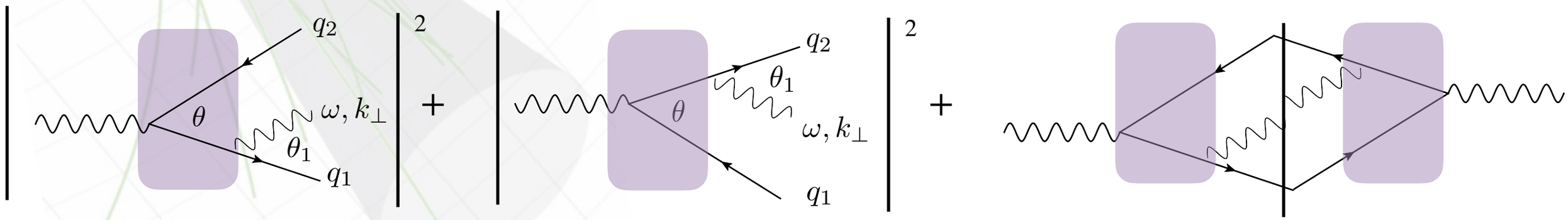


~ 2 independent broadenings

In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit



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In the collinear antenna limit, for soft gluon radiation:

$$\Delta'_{coh} = \left[1 + \frac{(q_{1,\perp} - q_{2,\perp})^2}{\hat{q}_F \Delta L_+}\right] \mathrm{e}^{-\frac{(q_{1,\perp}-q_{2,\perp})^2}{2\hat{q}_F \Delta L_+}}$$

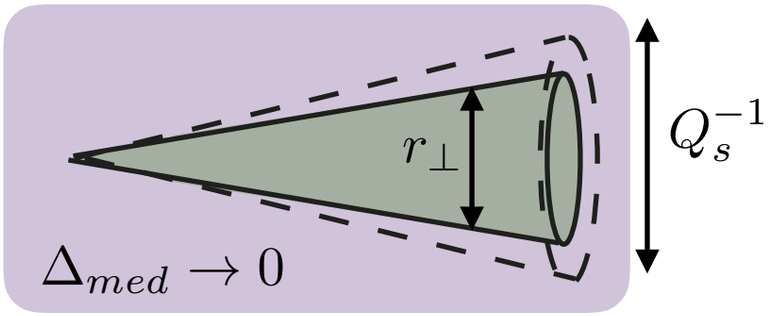
$$\Delta'_{coh} - \Delta'_{med} \sim \mathrm{e}^{-\frac{i\tan(\Omega\Delta L_+)}{\Omega}[(1-z)(q_{1,\perp}+k_\perp)-zq_{2,\perp}]^2}$$

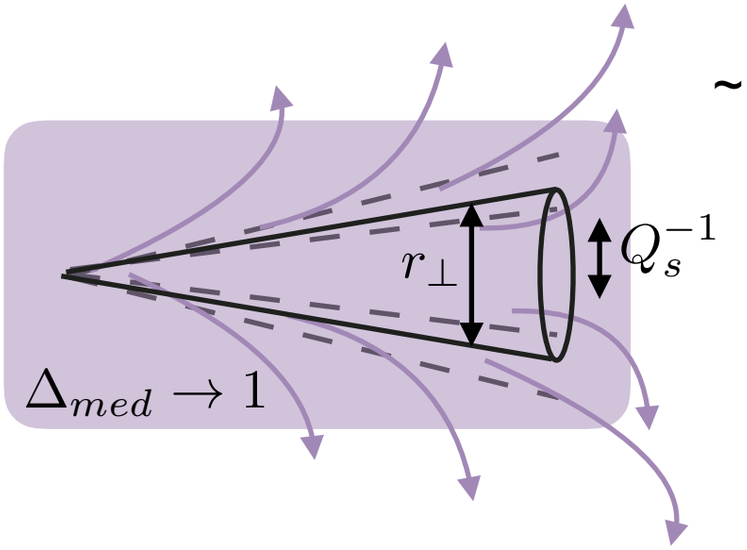
$$\mathrm{e}^{\frac{i\tan(\Omega'\Delta L_+)}{\Omega'}[(1-z)q_{1,\perp}-z(q_{2,\perp}+k_\perp)]^2}$$

$$\Omega^2 \sim \frac{\hat{q}}{z(1-z)p_+}$$

$$\Omega'^2 = -\Omega^2$$

Two independent harmonic oscillators



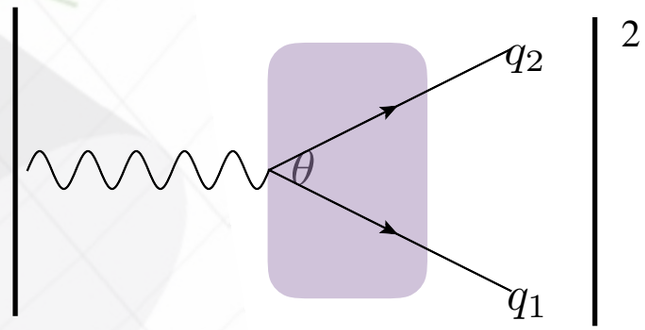


~ 2 independent broadenings

In-Medium Antenna

[LA, Armesto, Milhano and Salgado (in preparation)]

- Beyond eikonal limit
  - Elastic Channel:



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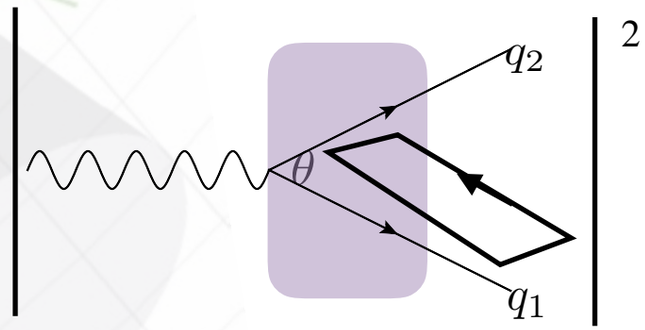

$$\Delta'_{coh} = \frac{1}{N_c^2} \mathrm{Tr} \left\langle G_1 G_2^\dagger \right\rangle \mathrm{Tr} \left\langle G_1^\dagger G_2 \right\rangle$$

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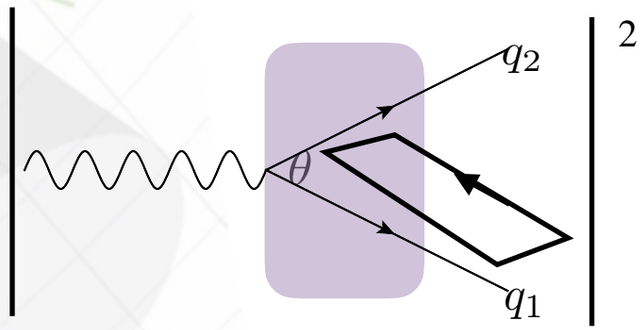

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$$\overline{\Delta_{med}} = \frac{\Delta'_{med}}{\Delta'_{coh}}$$

$$\begin{aligned} \overline{|S_{tot}|^2} &= \overline{|S_{el}|^2} \otimes \left[(R_q + R_{\bar{q}} - 2J) + 2\overline{\Delta_{med}} J \right] \\ &= \overline{|S_{el}|^2} \otimes \left[R_{coh} + 2\overline{\Delta_{med}} J \right] \end{aligned}$$

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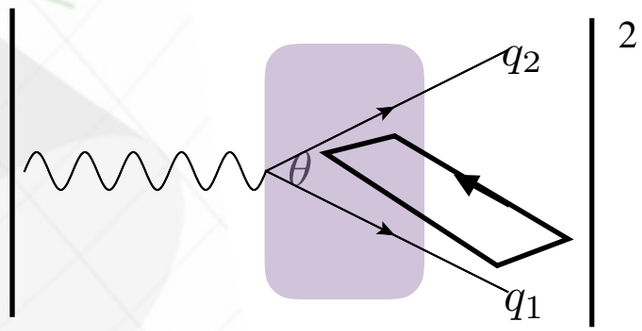

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Elastic channel that includes Non-eikonal corrections

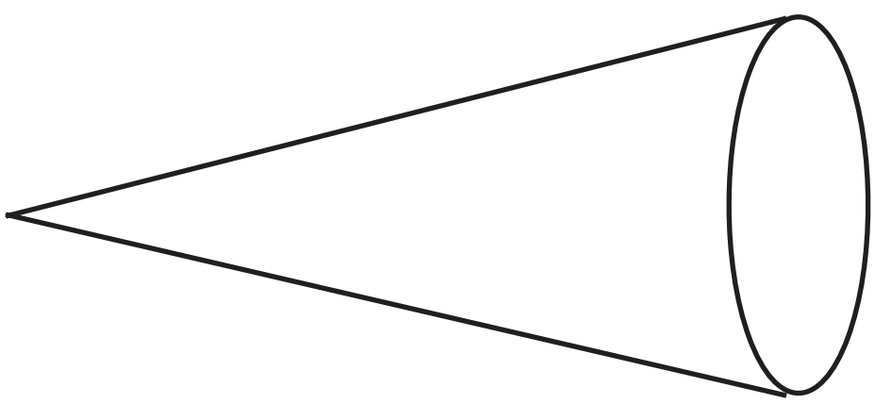
"Vacuum Angular Ordering"

Non-eikonal Δ_{med}

Summary

- Picture of what a quenched jet is based on a scale separation:
  - $\hat{q}L_+ >> \theta L_+$: Medium does not resolve object
  - $\hat{q}L_+ << \theta L_+$: Medium resolve object

Two distinct showers
Both described by evolution equations



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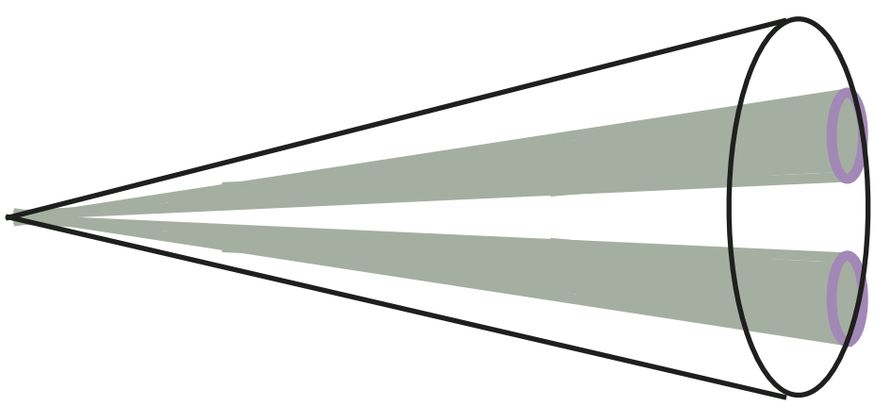

AO "Vacuum" showers

$\theta L << \hat{q}\, L$

$t \sim \ln Q^2$

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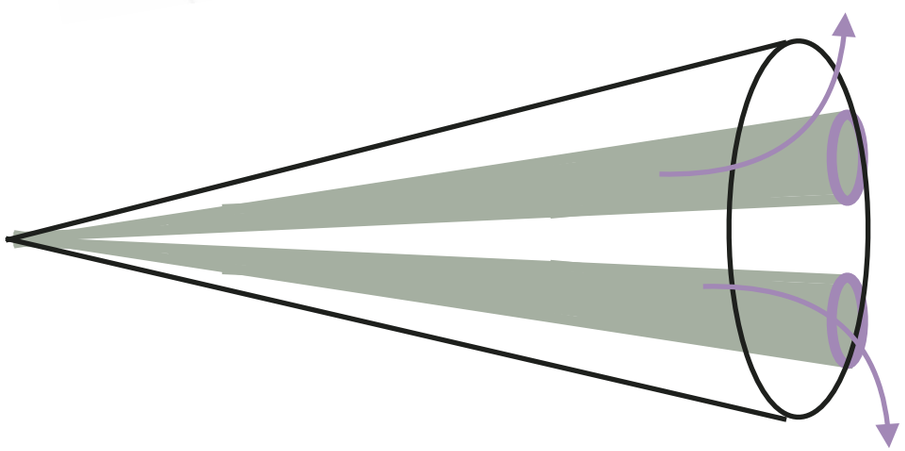

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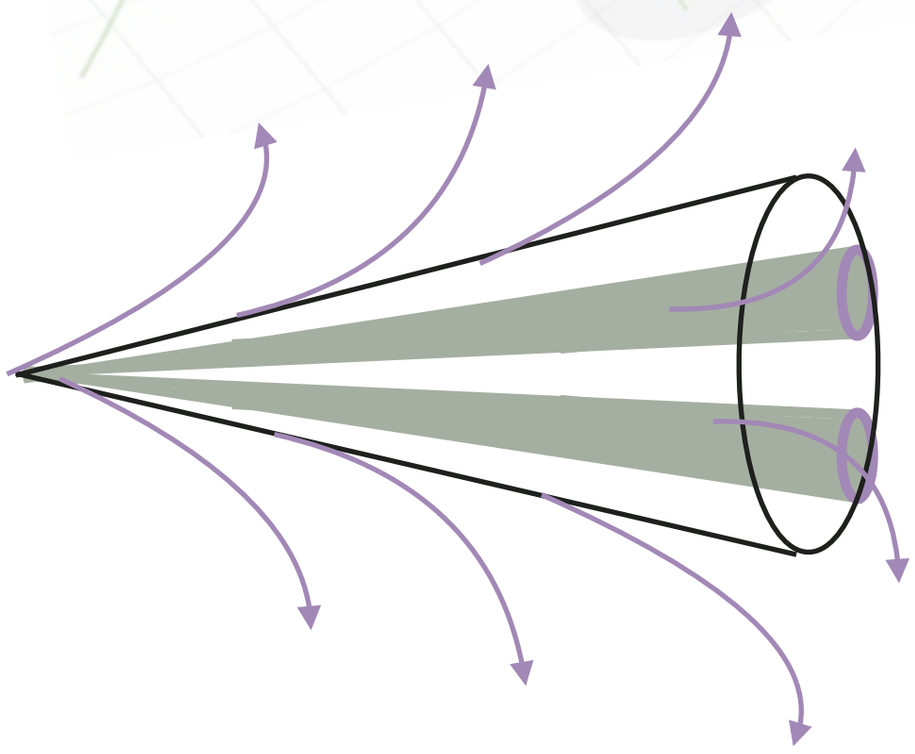

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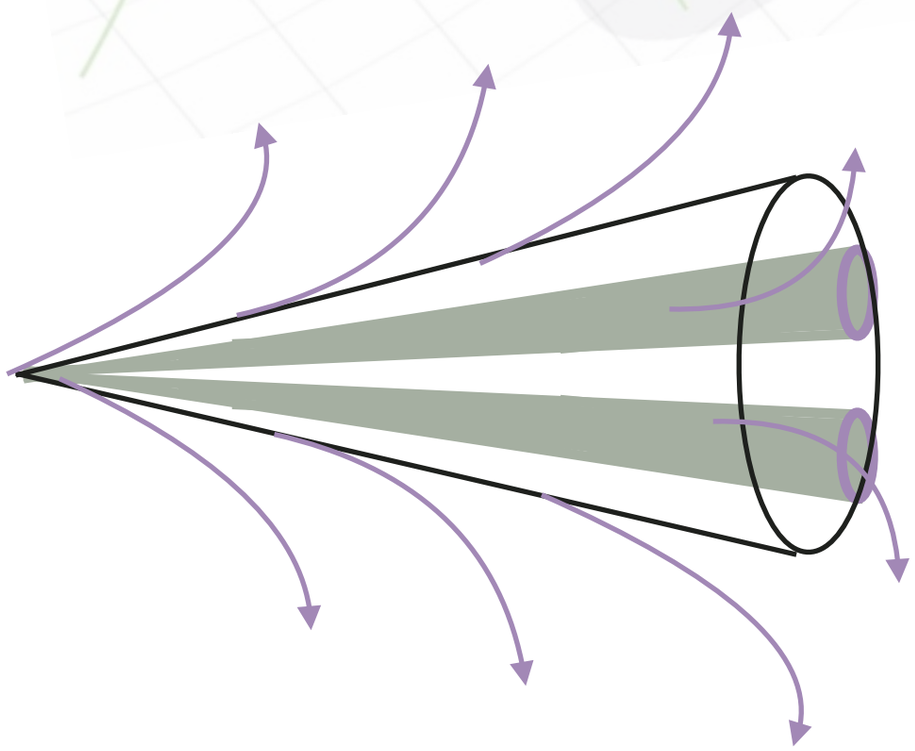

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AO "Vacuum" showers
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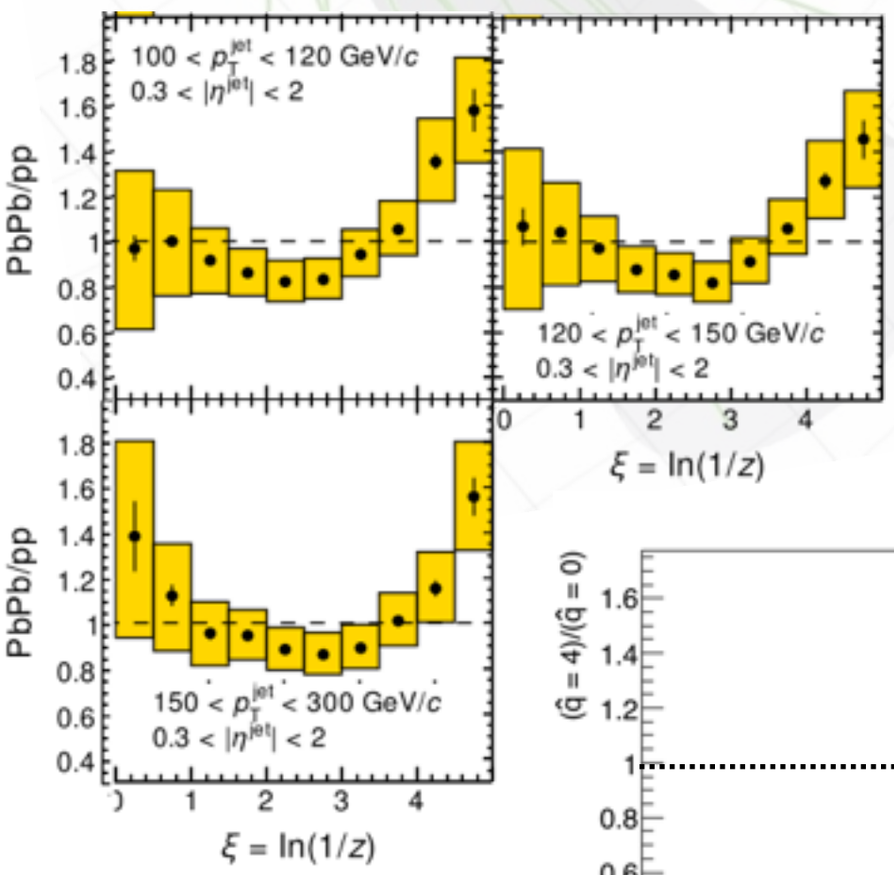
Still need to understand how is the interplay between these two showers

Different ordering variables
Additional scales (Q_s and θL) on top of hard scale (Q)

But different angular behaviour!

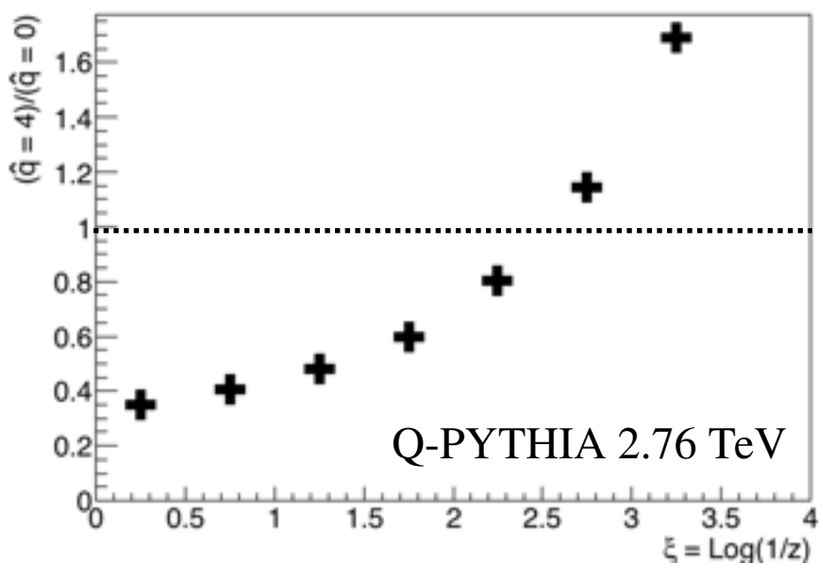
Summary

- Current picture seems to agree with experimental observations:



[CMS (2013)]

$$z = \frac{p_{\perp,part}}{p_{\perp,jet}}$$



All jet is able to emit (Decoh limit)

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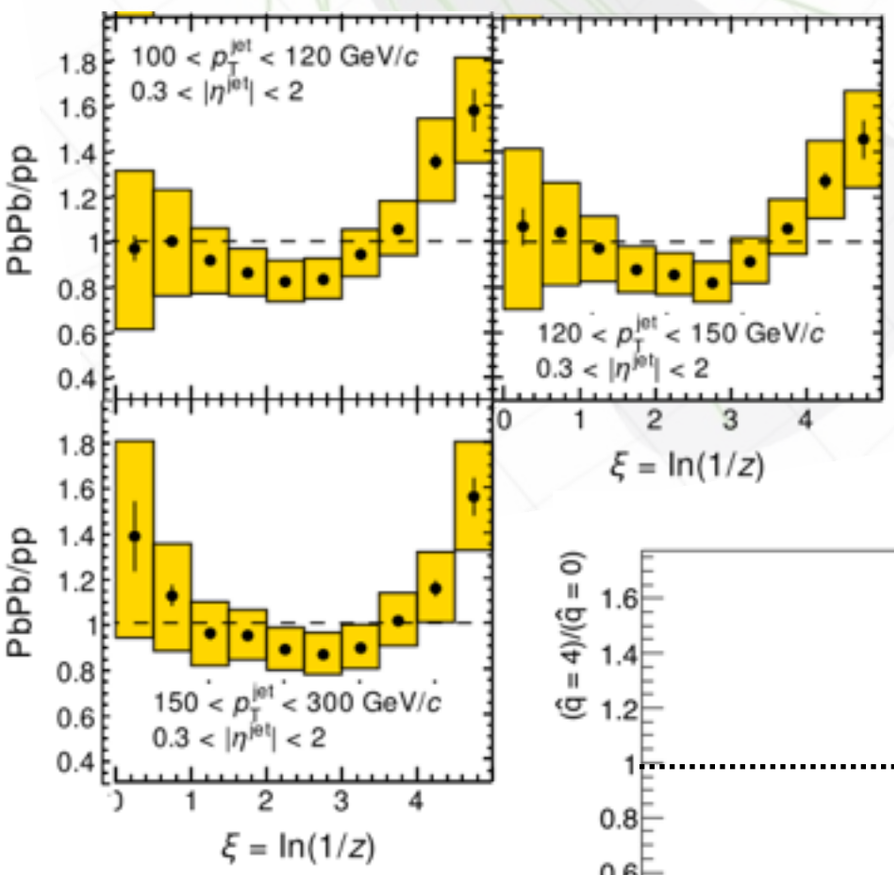

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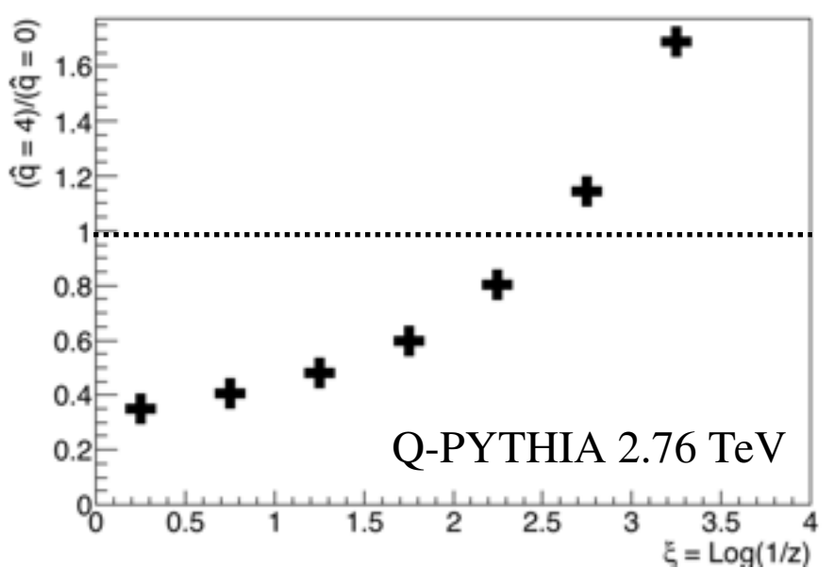

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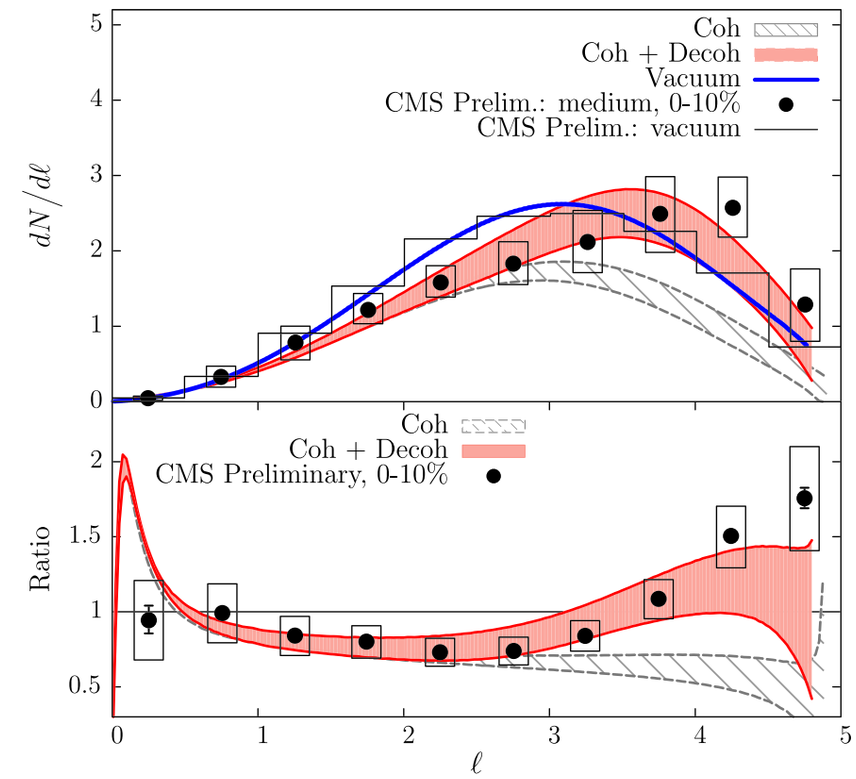

Coh = Coherent Jet + Energy Loss

Coh + Decoh = Decoherent Jet + Energy Loss

Vacuum = MLLA fit

$$l = \log(\frac{1}{x_h}) \quad x_h = \sqrt{z^2 + \left(\frac{m_h}{p_{\perp,jet}}\right)^2}$$

[Mehtar-Tani, Tywoniuk (2014)]

Conclusions

- Significant progress been made to understand the pQCD evolution of the parton shower in the presence of a hot and dense medium:
  - Antenna picture beyond eikonal approximation:
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Thank you!!

Backup Slides

Vacuum Jets

- Starting by something more simple…
  - pQCD scale:
  - Hard parton decreasing virtuality: parton branching
    - Possible to calculate its evolution trough an equation cascade that is derived from the fact that it is possible to factorize the process description

Medium Jets

- In a first approximation, if one sees the medium as a collection of scattering centres, and since the timescale are different, propagators are described by Green functions
- Each interaction will enhance the WW field cloud inducing extra-gluon radiation.
- Results include additional energy loss and brownian motion
  - Single gluon emission spectrum with Delta_coh (show correlators)
  - Delta_coh indicating loss of locality? Factorisation breaking?
- In vacuum, to see the interferences we need to go the antenna setup

In-medium antenna

- Results of the eikonal antenna show that there are two kinds of shower, happening at the same time
- It also shows that it is possible to separate two kinds of showers based on a scale separation
- Inclusion of brownian motion
  - Interference term: delta_med modification
  - Direct term: quadrupole (delta_coh)
- Momentum space (non-factorized piece identically zero)
  - Results not so easy to interpret, but dependence only relies on the previous emission

Summary

[LA, N. Armesto, L. Cunqueiro (2013)]

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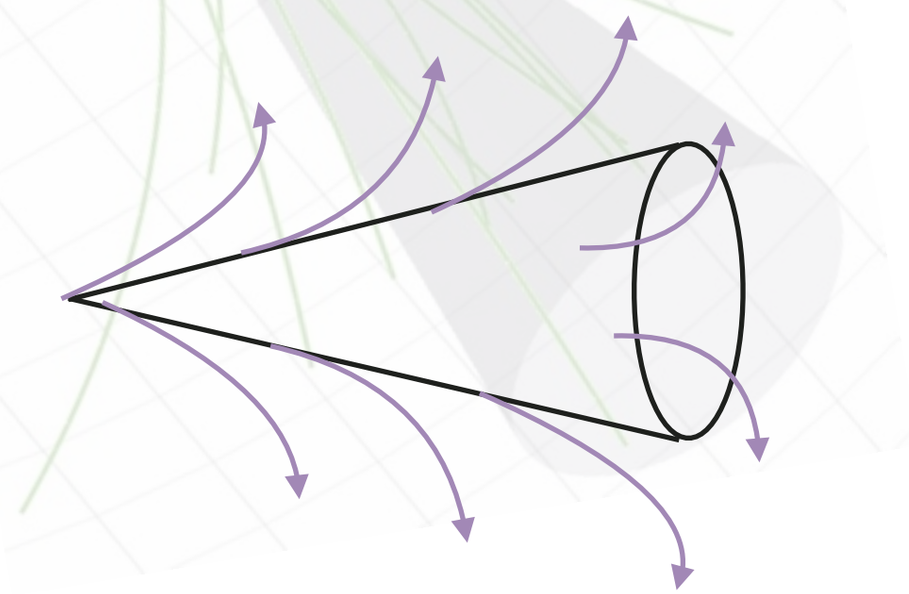
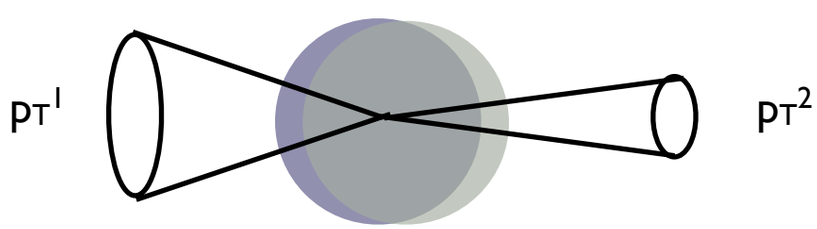
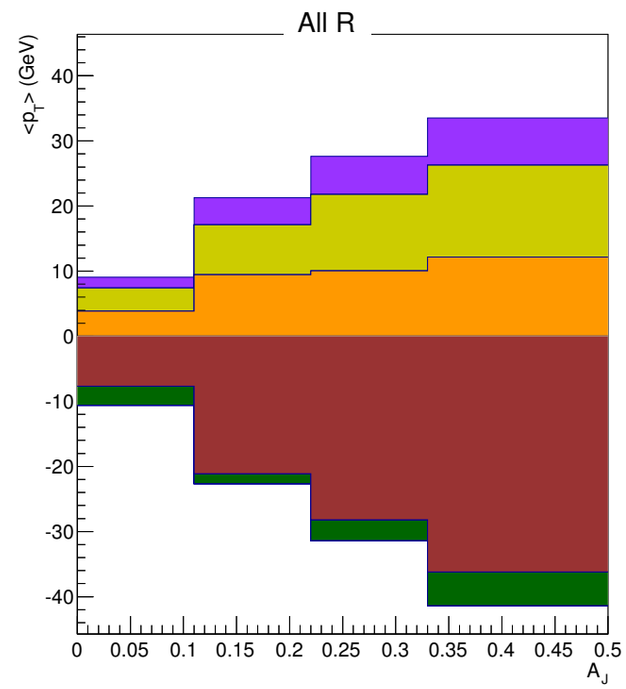


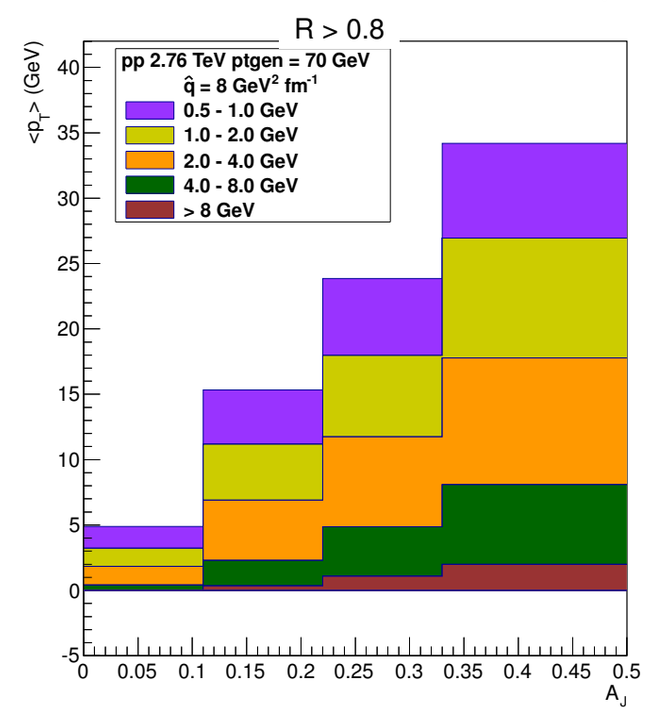
$p_T^{\parallel} = -p_{T,trk} \cos(\phi_{trk} - \phi_{dijet})$



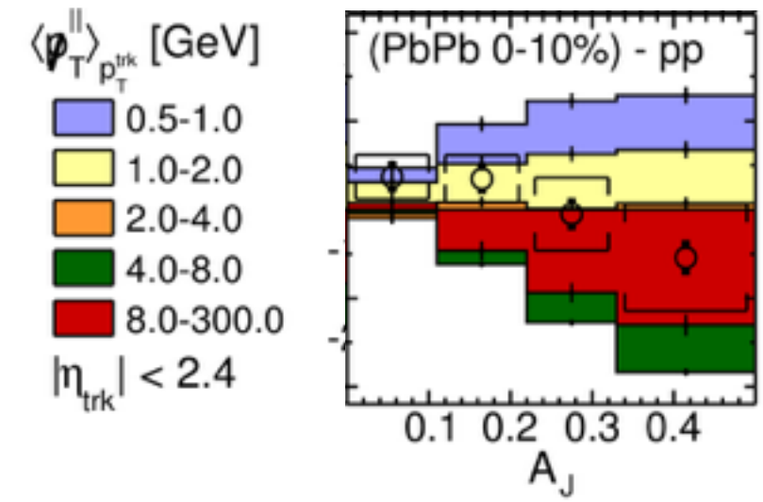
Transverse momentum (energy) of the jets

$$A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}} .$$





[CMS JHEP 01 (2016) 006]



[CMS PRC 84 (2011) 024906]