



U.S. DEPARTMENT OF  
**ENERGY**

Office of Science



# The $x$ and scale dependence of $\hat{q}$

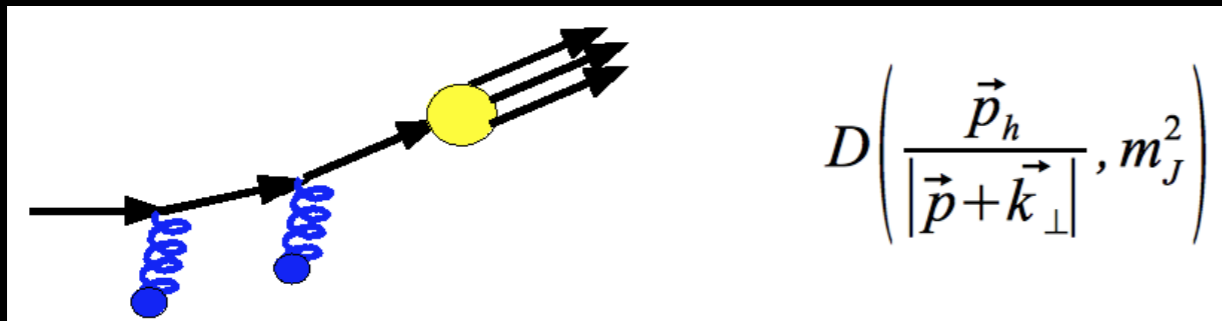
Abhijit Majumder  
Wayne State University

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Hard Probes 2016, Wuhan, China Sept 23-28

Validity at high resolution,  
 transport coefficients for near on-shell partons

$$p_z^2 \simeq E^2 - p_\perp^2 \quad p^+ \simeq p_\perp^2 / 2p^-$$



$$D\left(\frac{\vec{p}_h}{|\vec{p} + \vec{k}_\perp|}, m_J^2\right)$$

$$\hat{q} = \frac{\langle p_\perp^2 \rangle L}{L}$$

Transverse momentum  
 diffusion rate

Notion of transport coefficient valid in the regime of

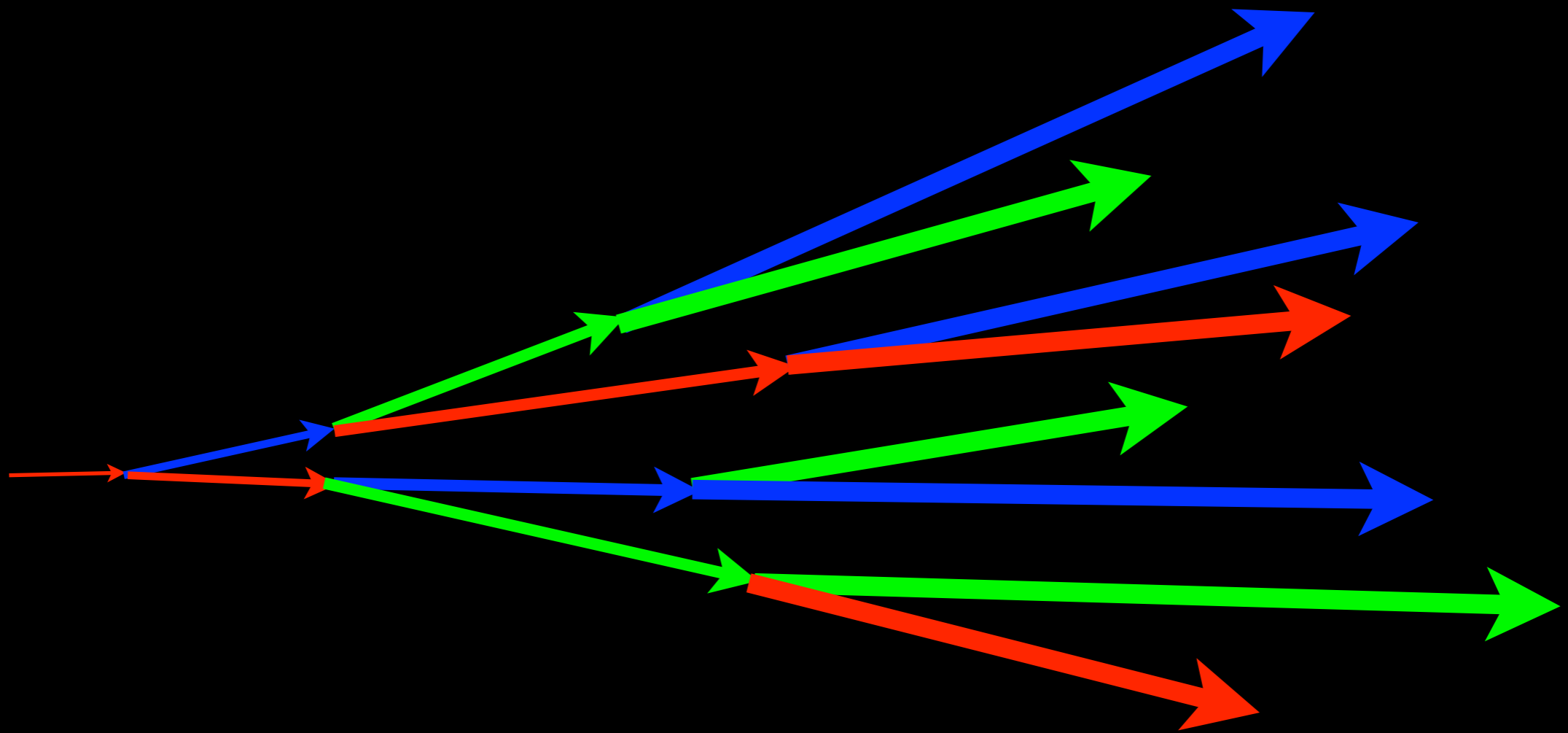
$$\mu \gg \Lambda_{\text{QCD}}$$

A hierarchy of scales:  $Q \gg \mu \gg \Lambda_{\text{QCD}}$

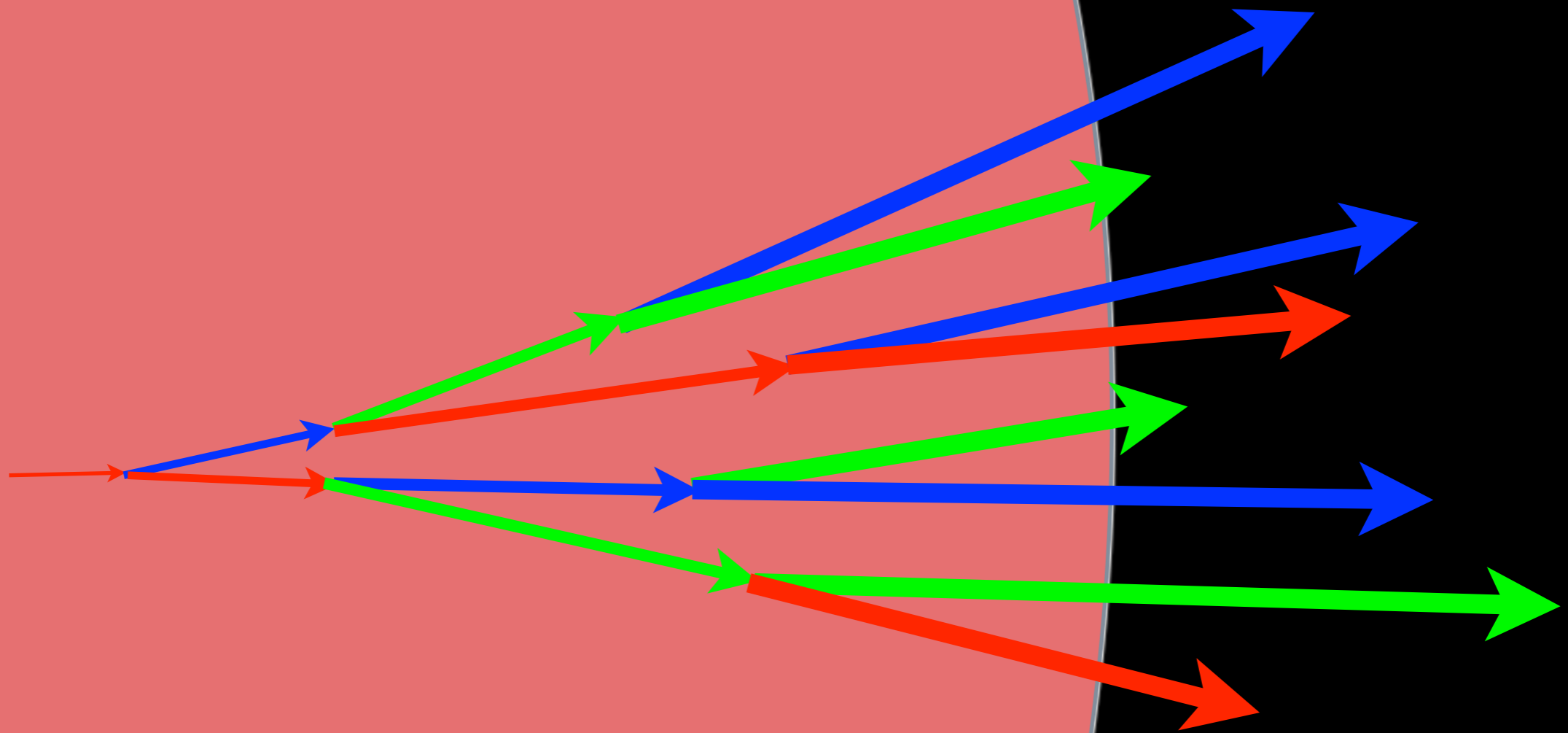


Full jets depend on many things

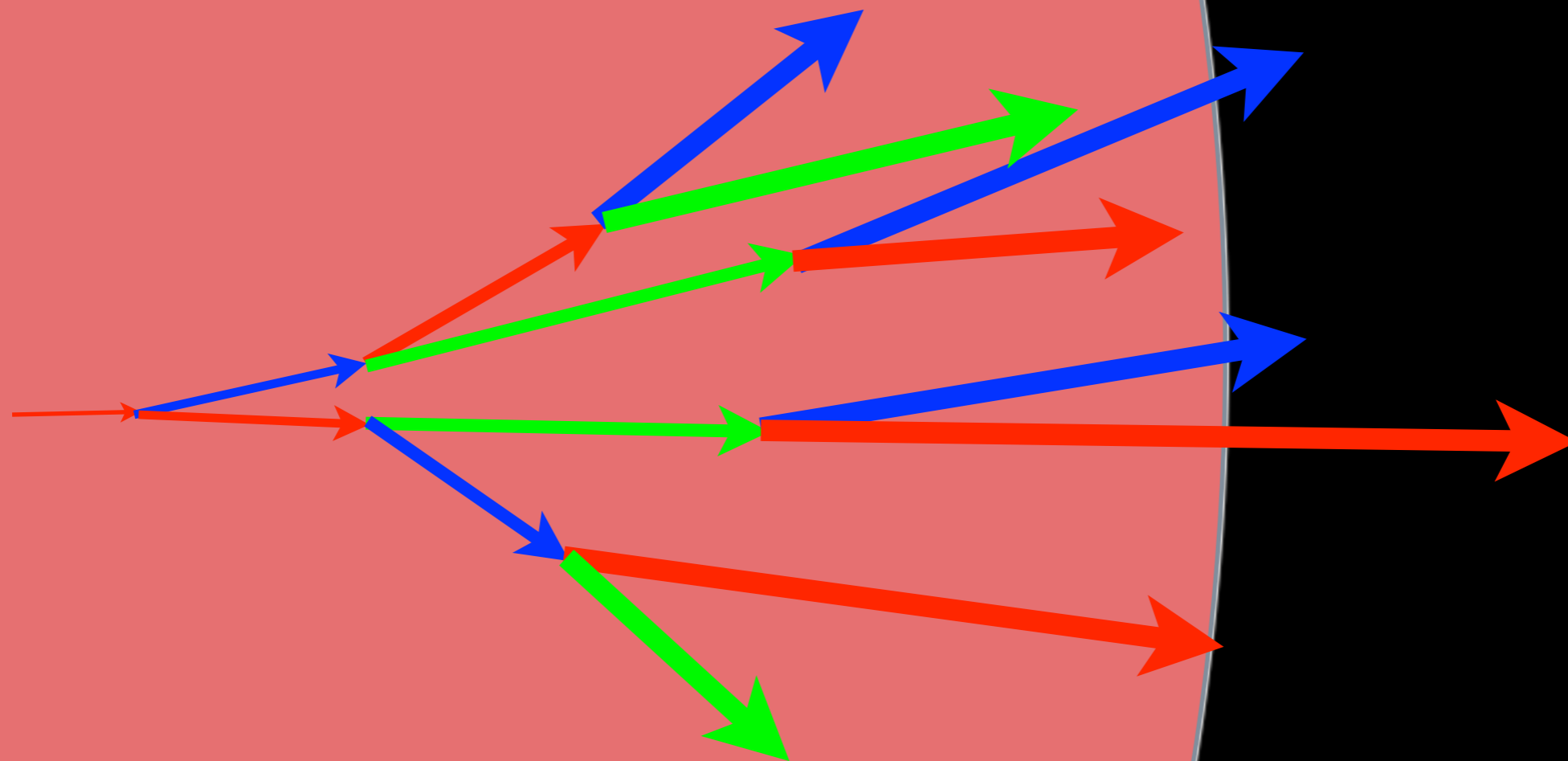
Full jets depend on many things



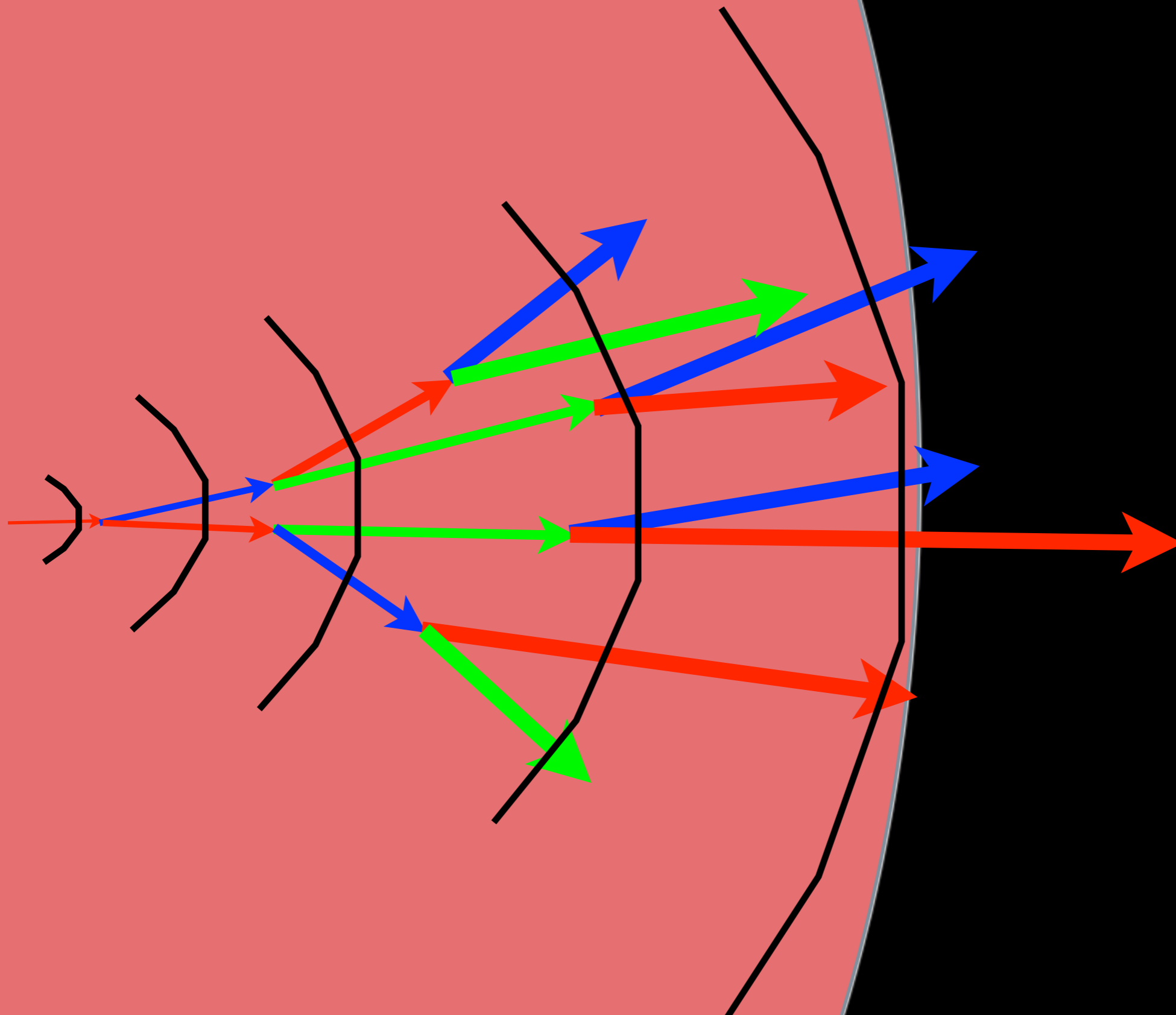
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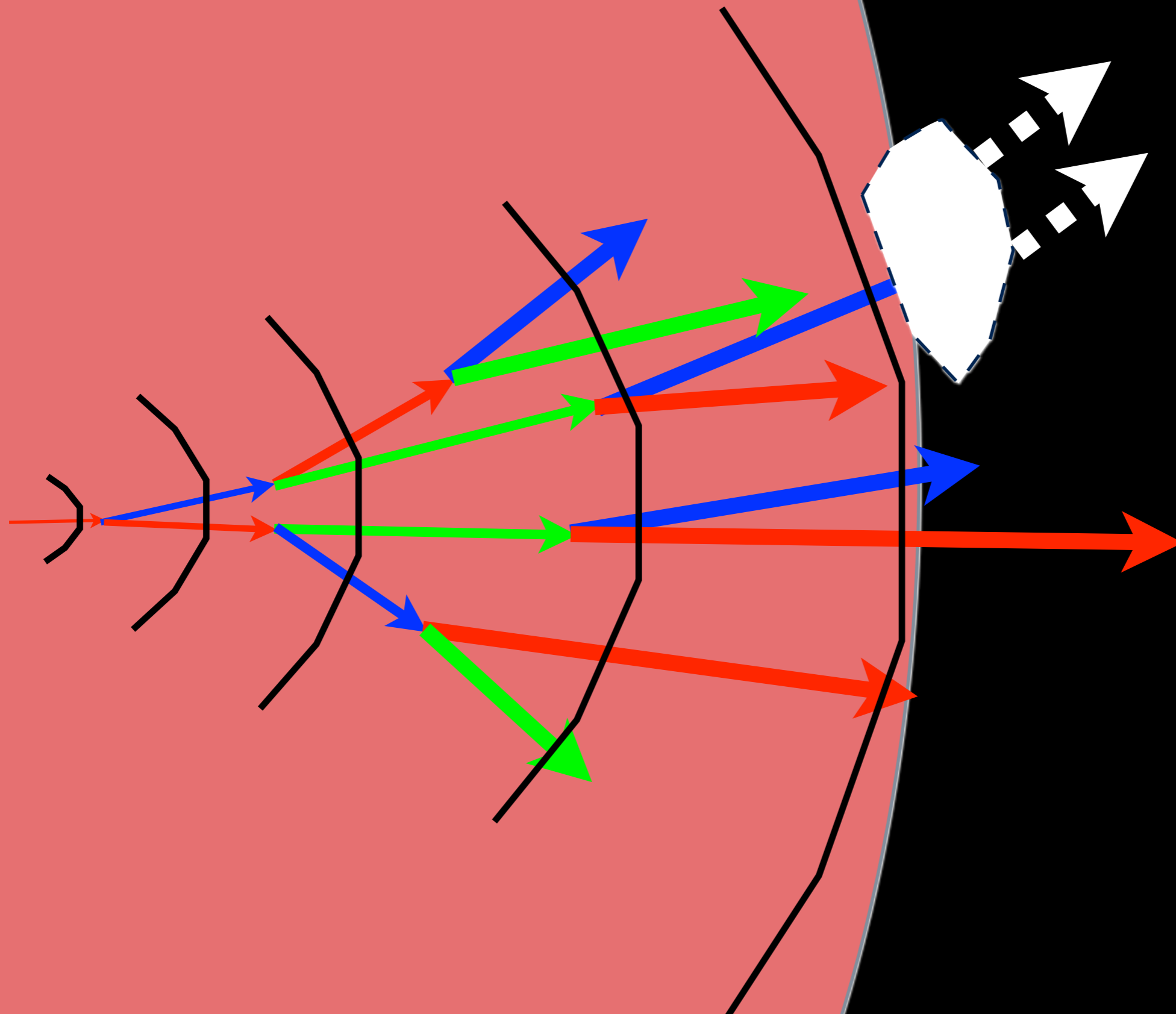


Full jets depend on many things

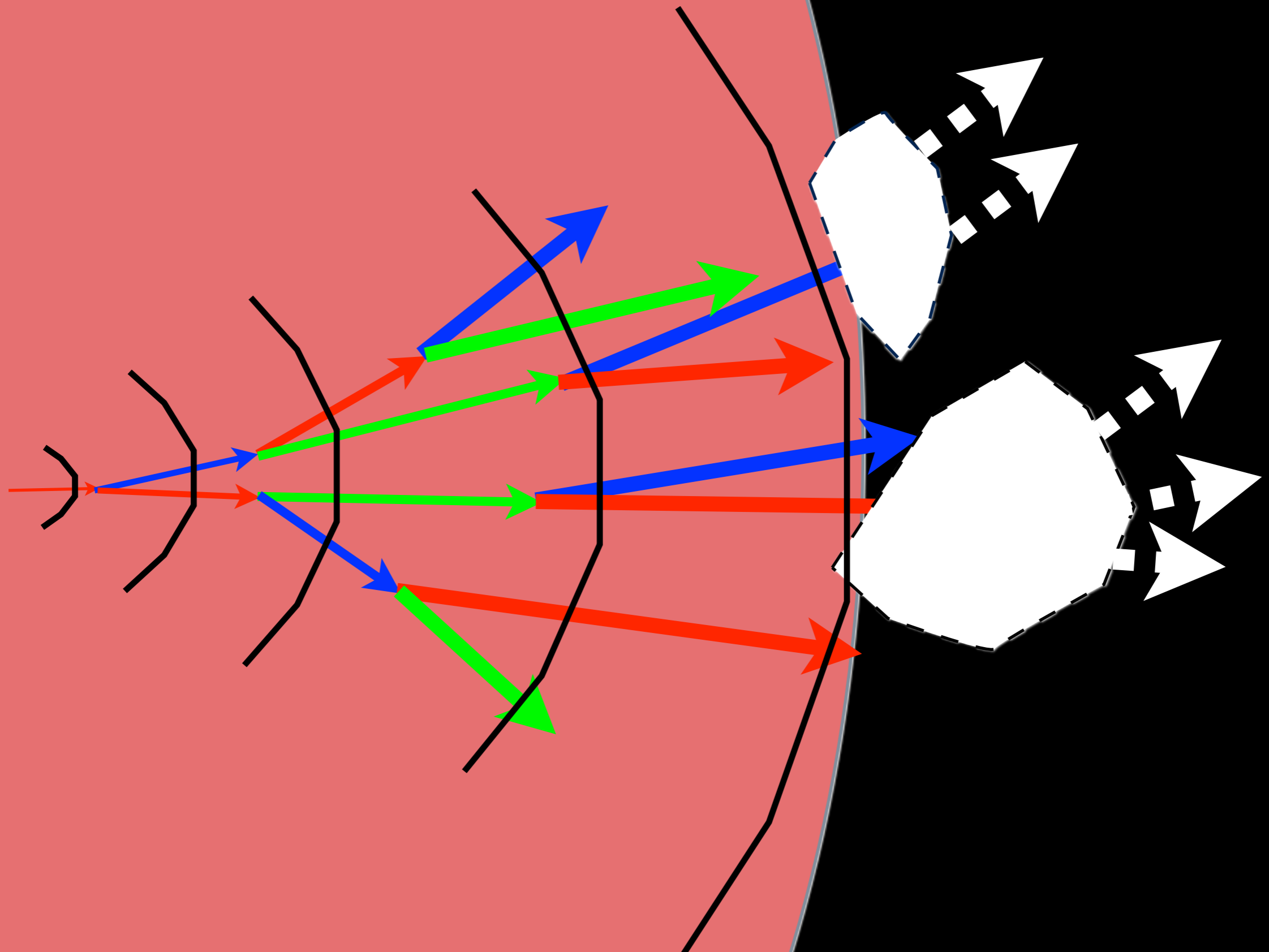




Full jets depend on many things

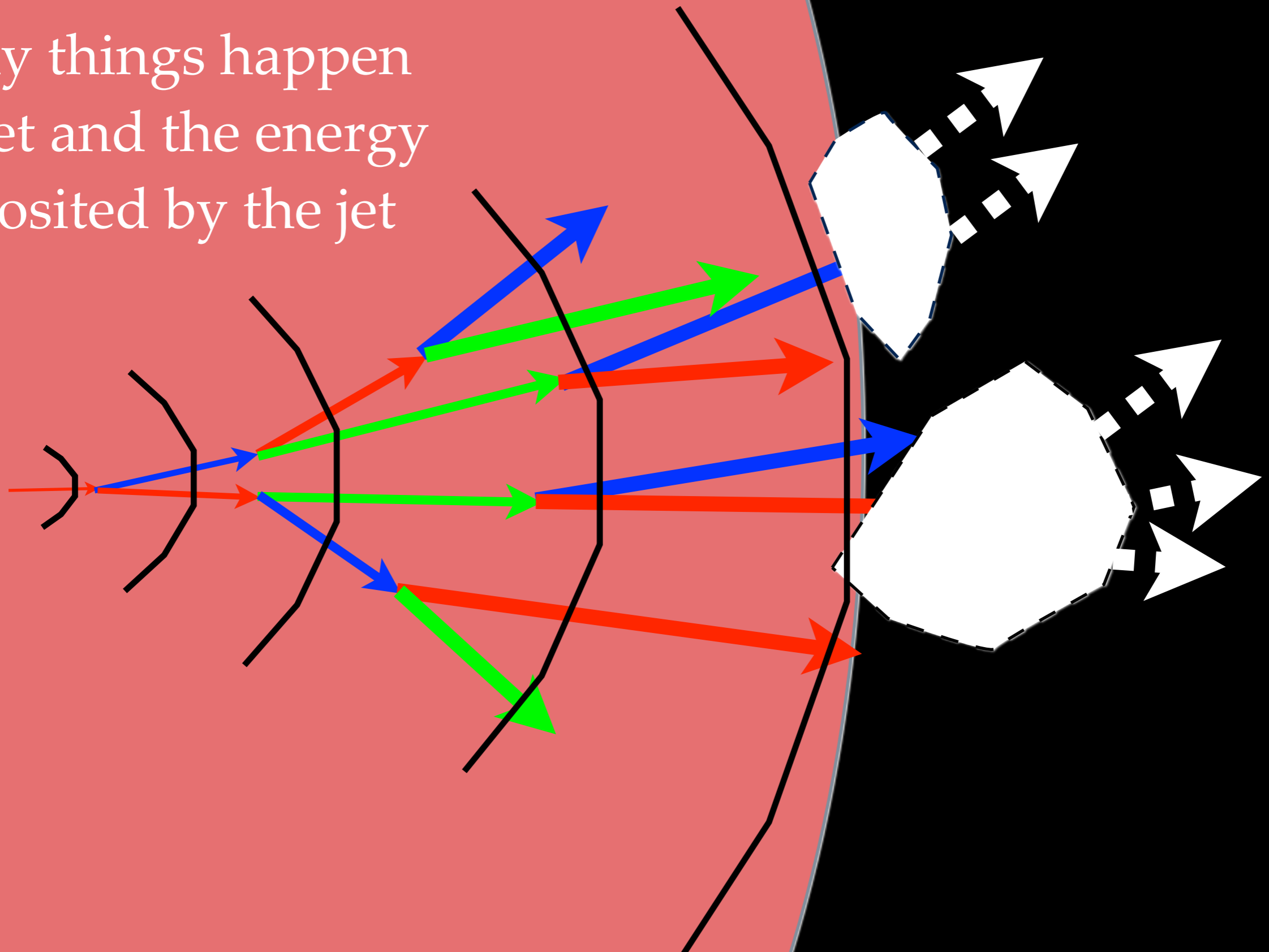


Full jets depend on many things



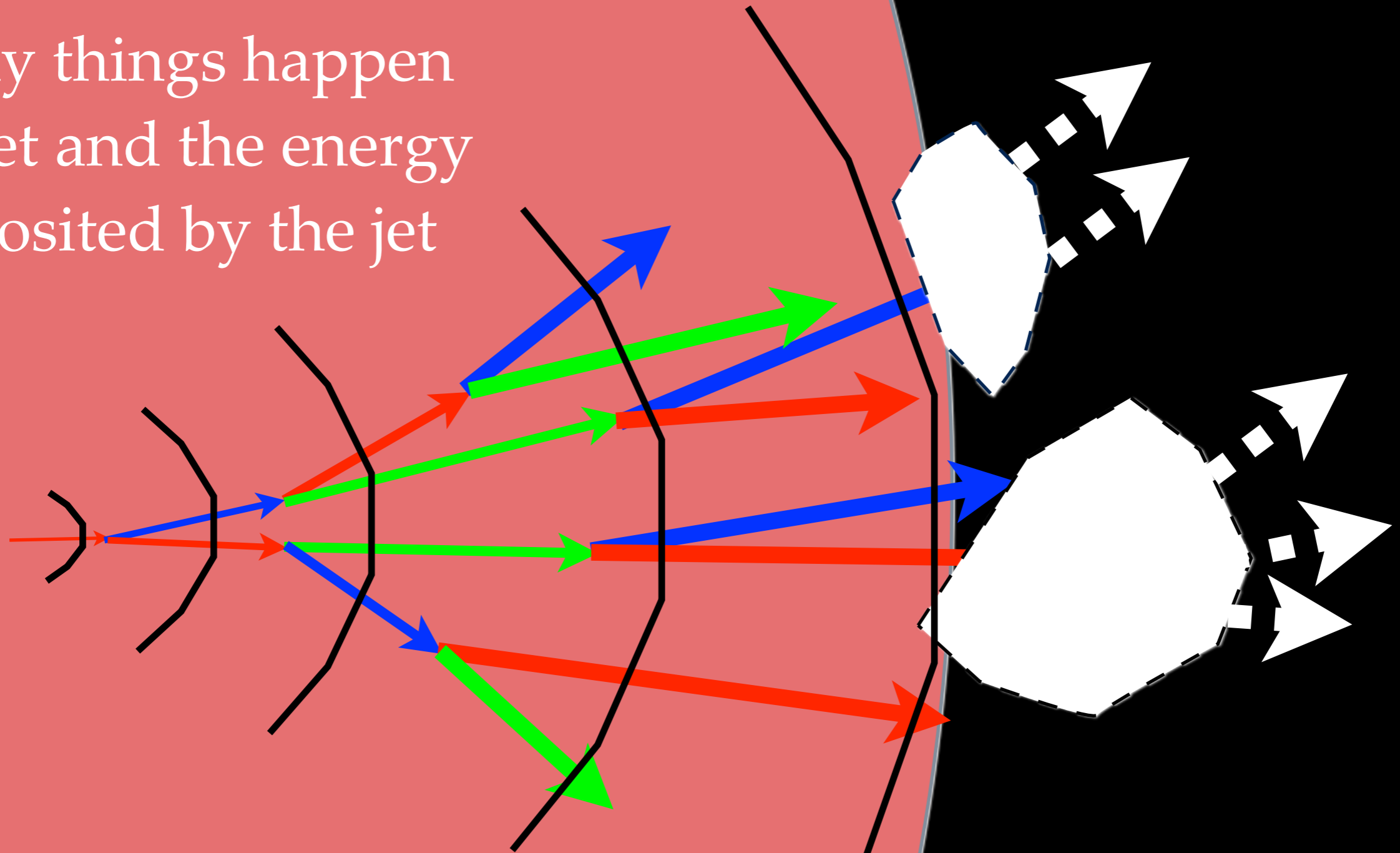
# Full jets depend on many things

Many things happen  
to a jet and the energy  
deposited by the jet



# Full jets depend on many things

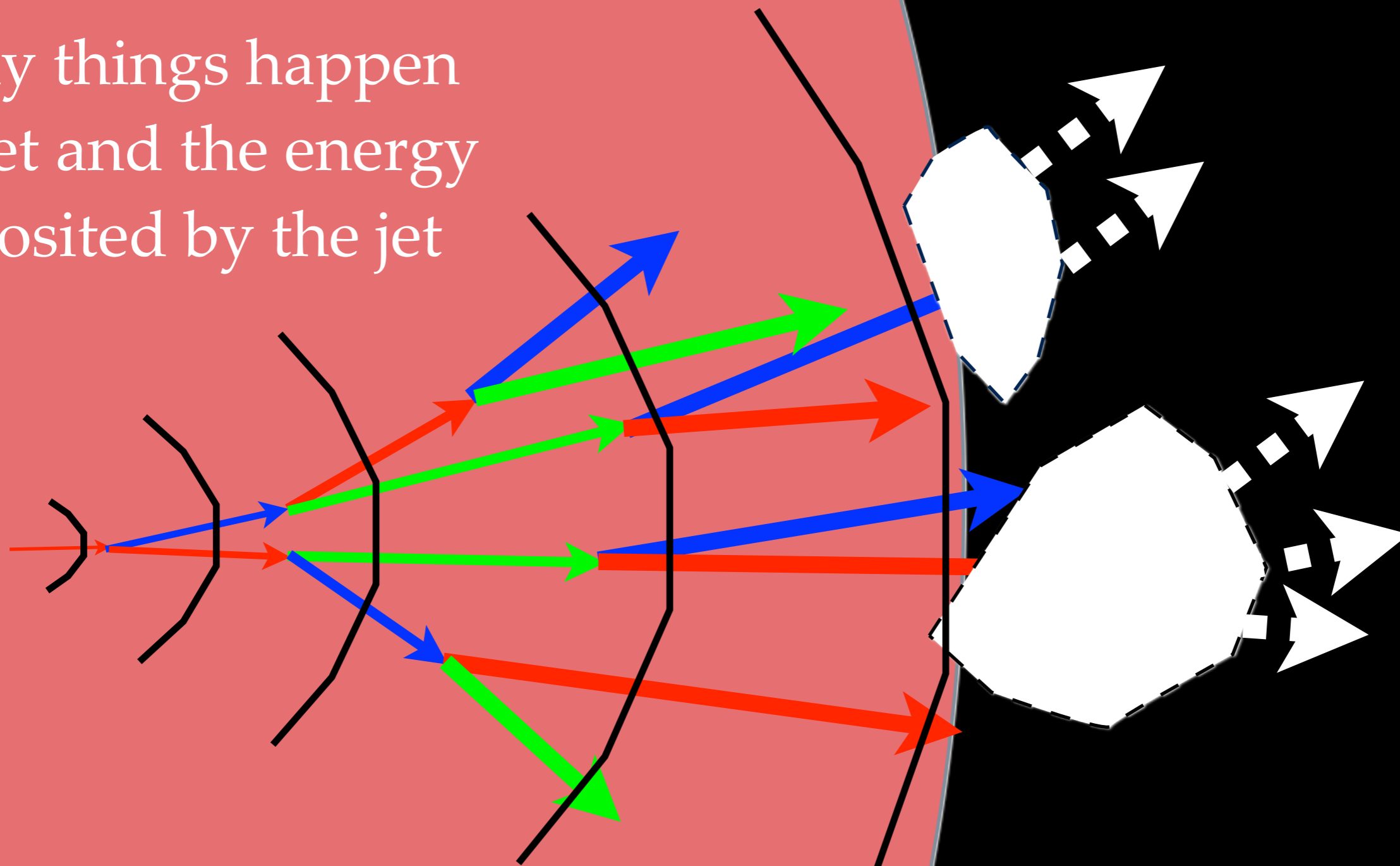
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See talk by M. Kordell  
other talks in MC session

# Full jets depend on many things

Many things happen  
to a jet and the energy  
deposited by the jet

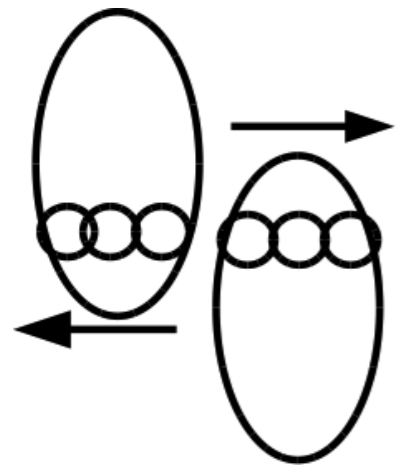
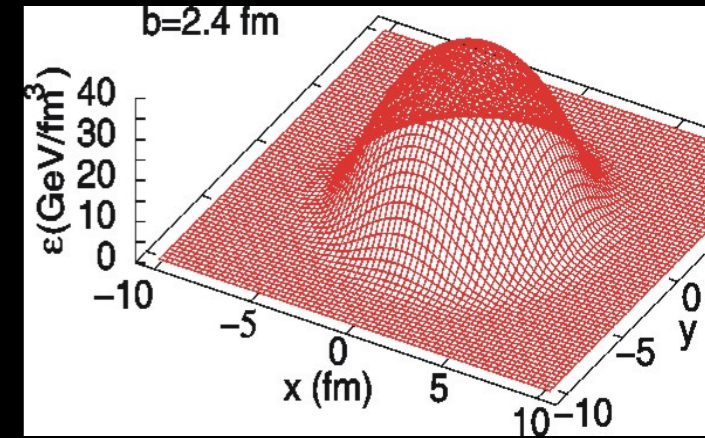


**This talk will only  
focus on leading hadrons**

See talk by M. Kordell  
other talks in MC session

# In all calculations presented bulk medium described by viscous fluid dynamics

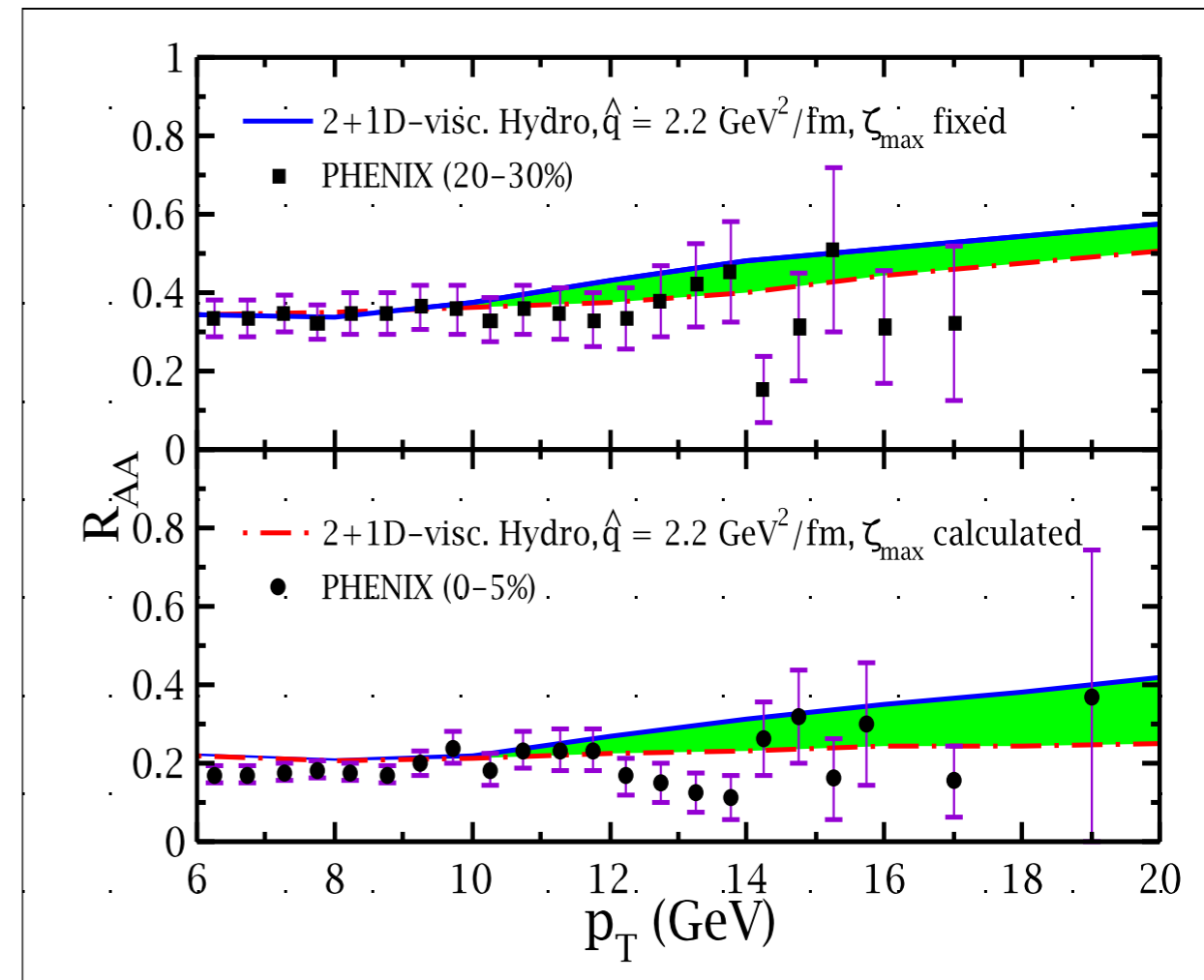
Medium evolves hydro-dynamically as the jet moves through it  
Fit the  $\hat{q}$  for the initial T in the hydro in central coll.



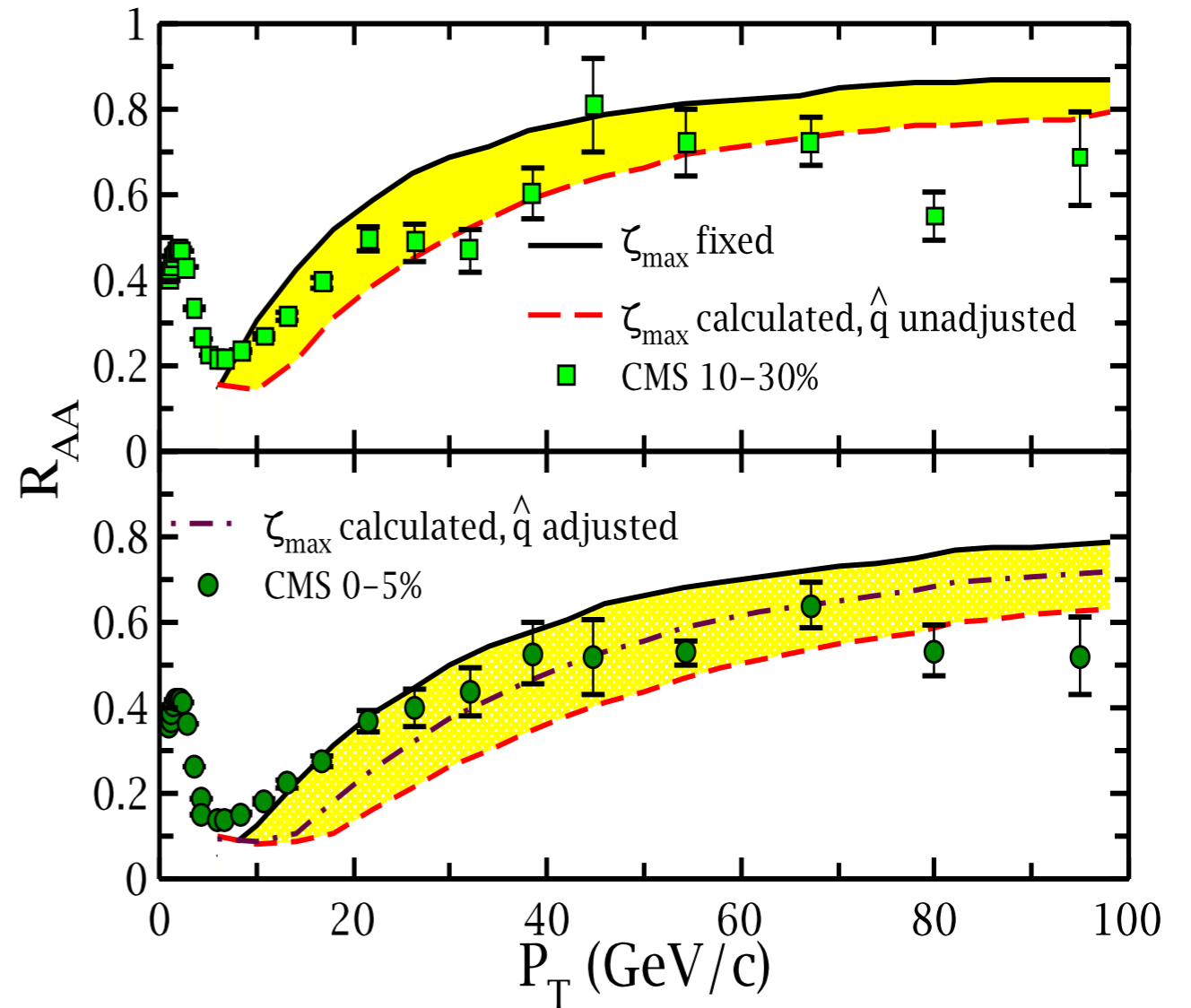
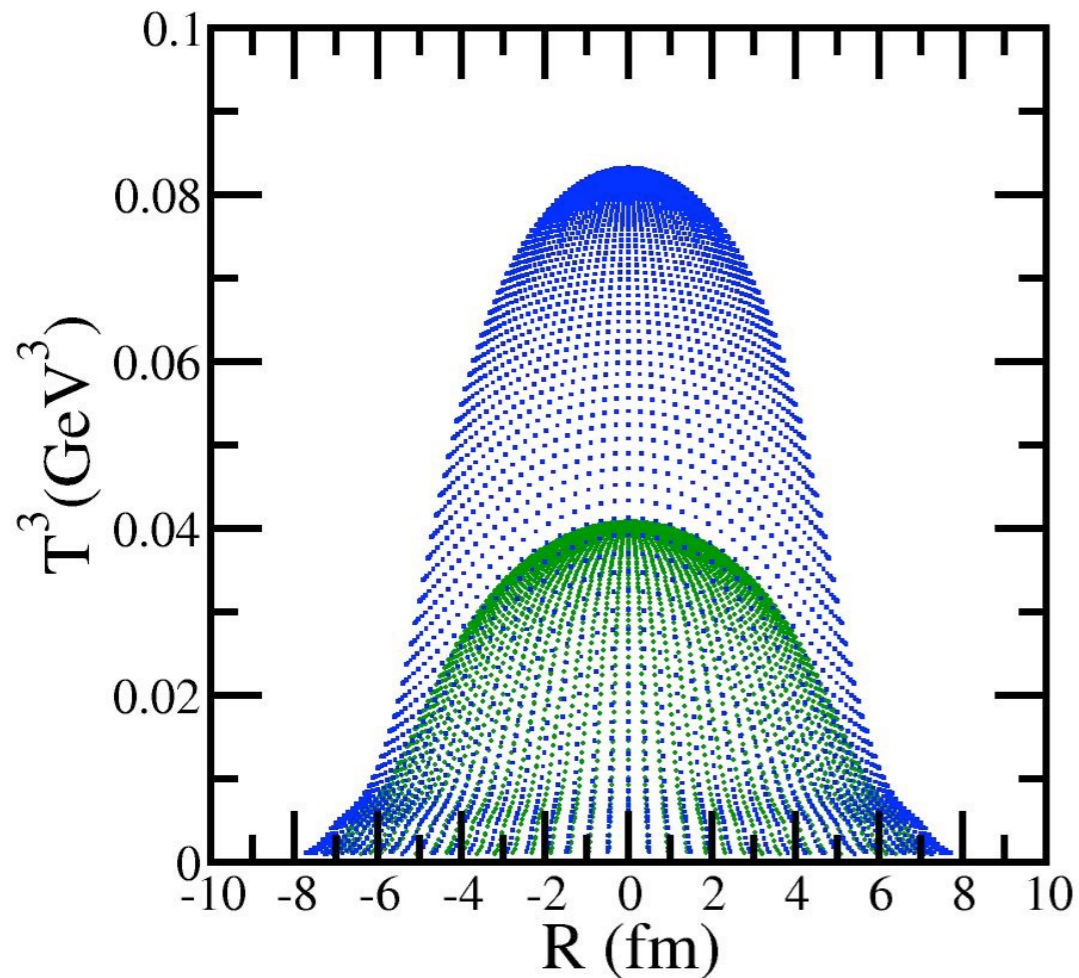
$$\hat{q}(\vec{r}, t) = \hat{q}_0 \frac{s(\vec{r}, t)}{s_0}$$

$$s_0 = s(T_0)$$

$$R_{AA} \sim \frac{\frac{dN_{AA}}{dp_T dy}}{N_{bin} \frac{dN_{pp}}{dp_T dy}}$$



# From RHIC to LHC circa 2012

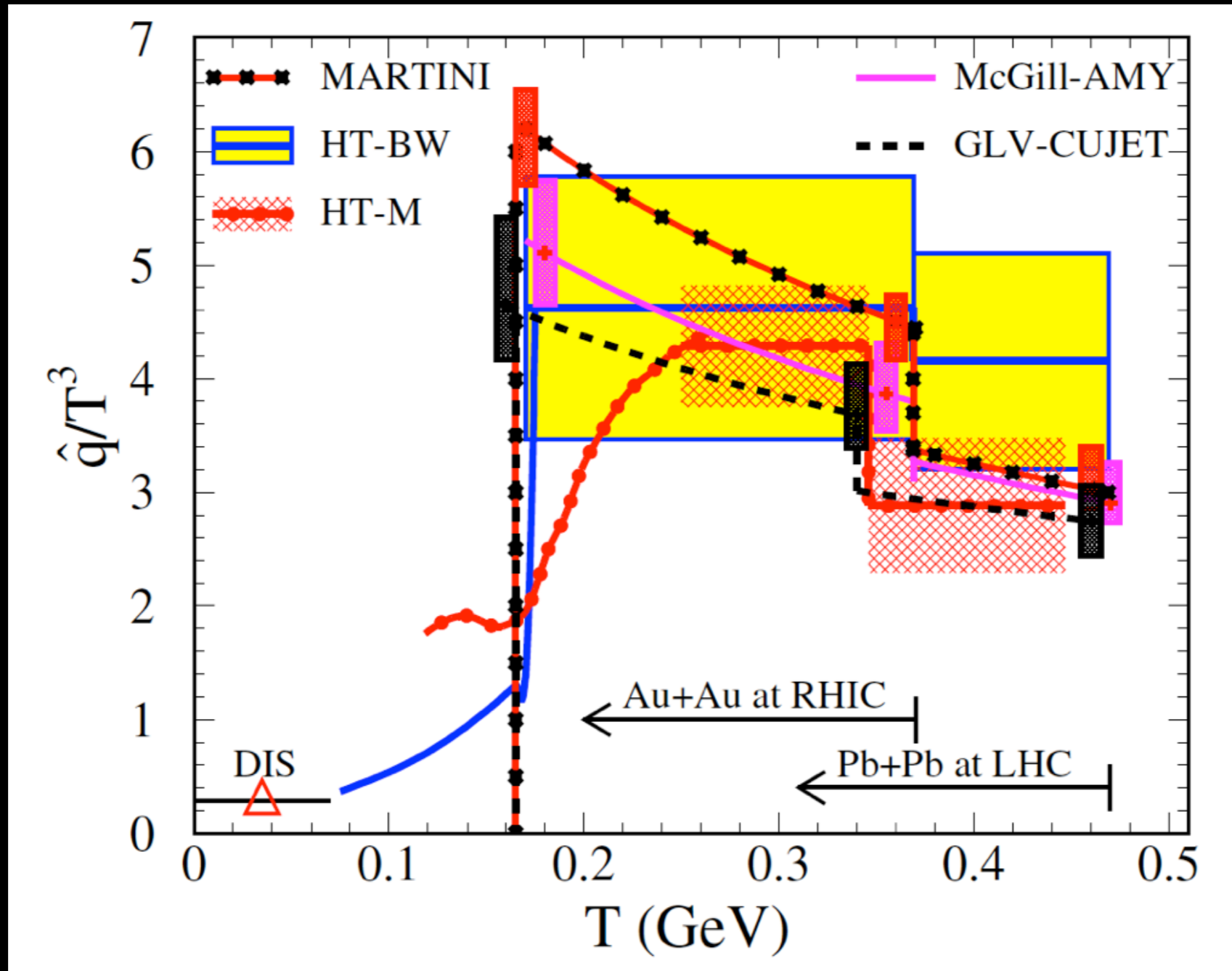


Reasonable agreement with data,  
no separate normalization at LHC

Without any non-trivial x-dependence (E dependence)

# Results from the JET collaboration

K. Burke et al.



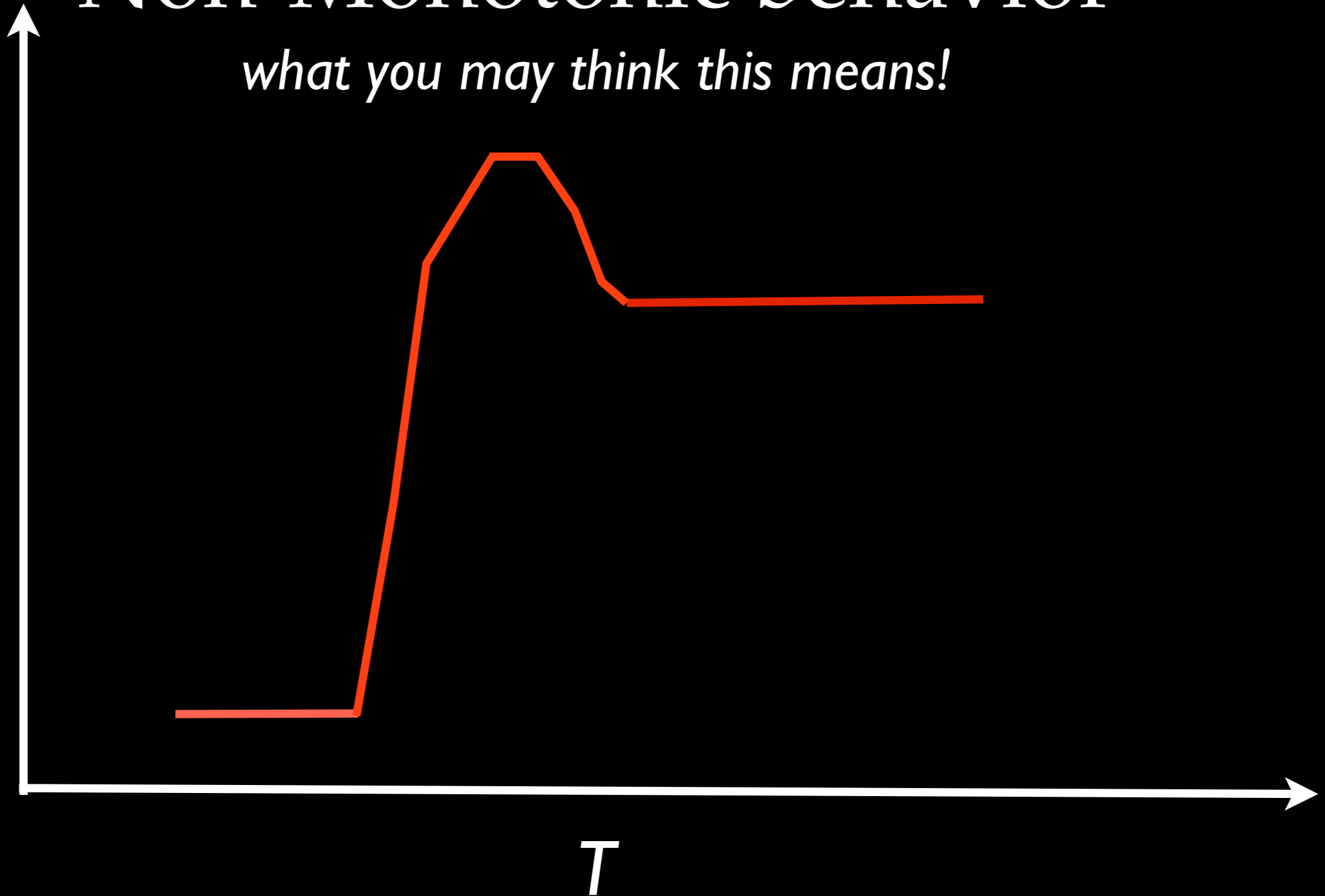
Do separate fits to the RHIC and LHC data for maximal  $\hat{q}$  without assuming any kink in the  $\hat{q}$  vs  $T^3$  curve



# Non-Monotonic behavior

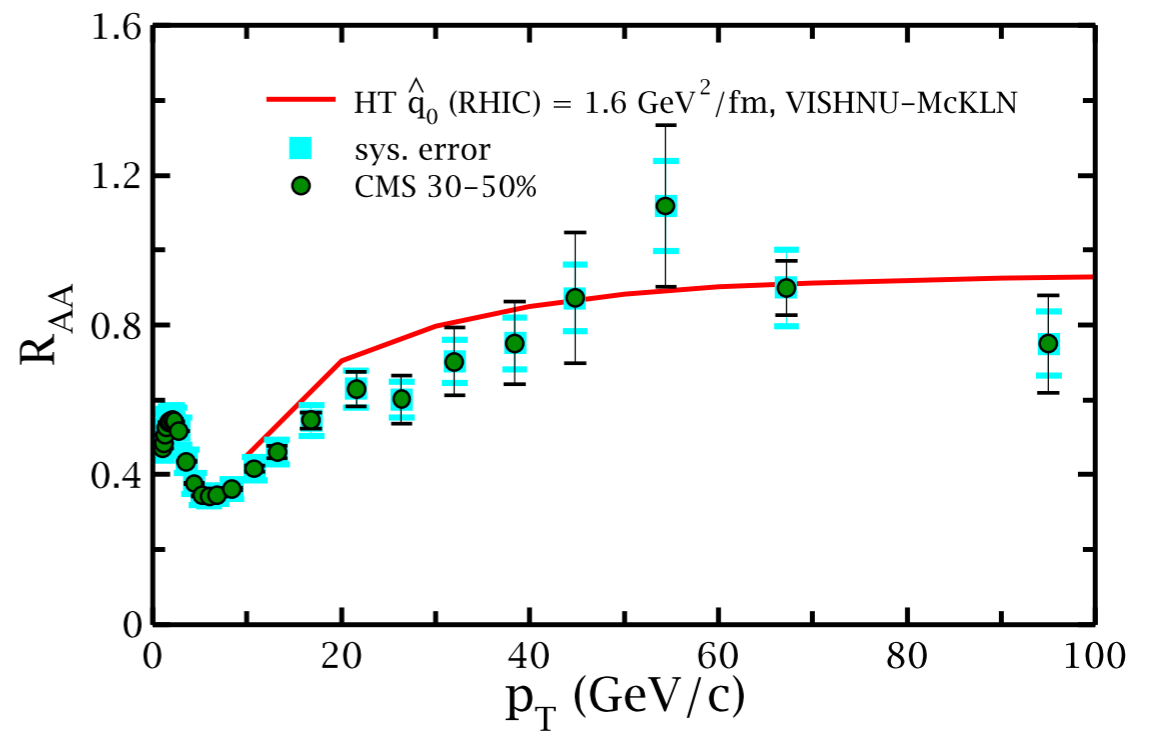
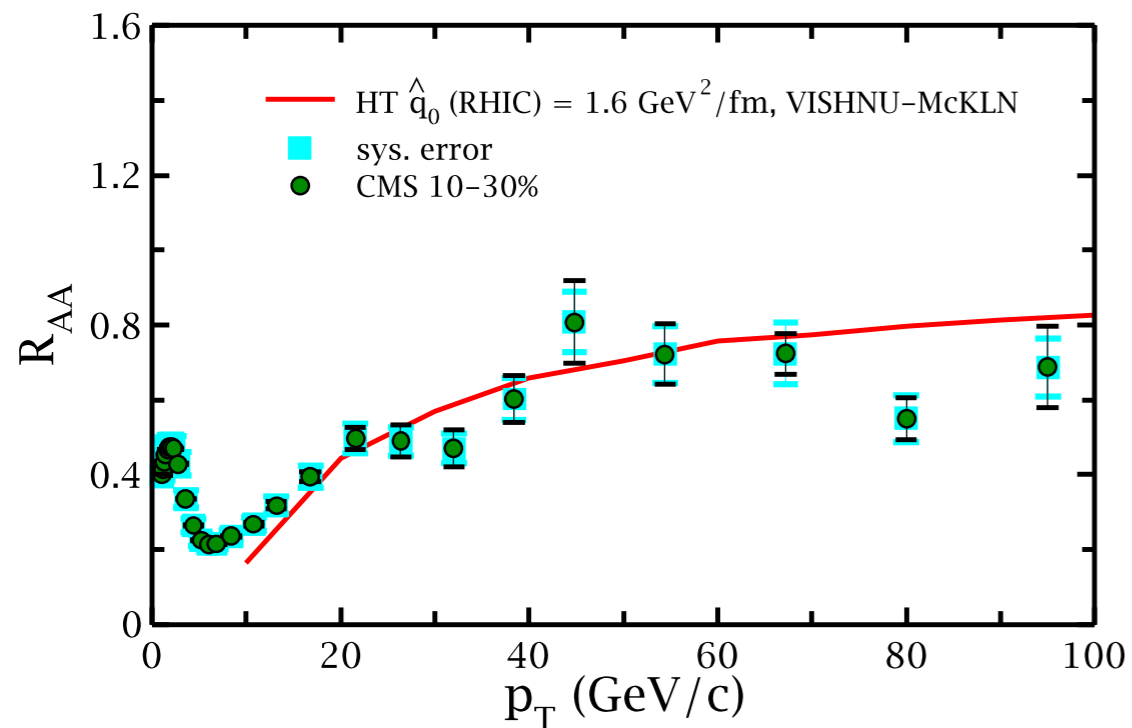
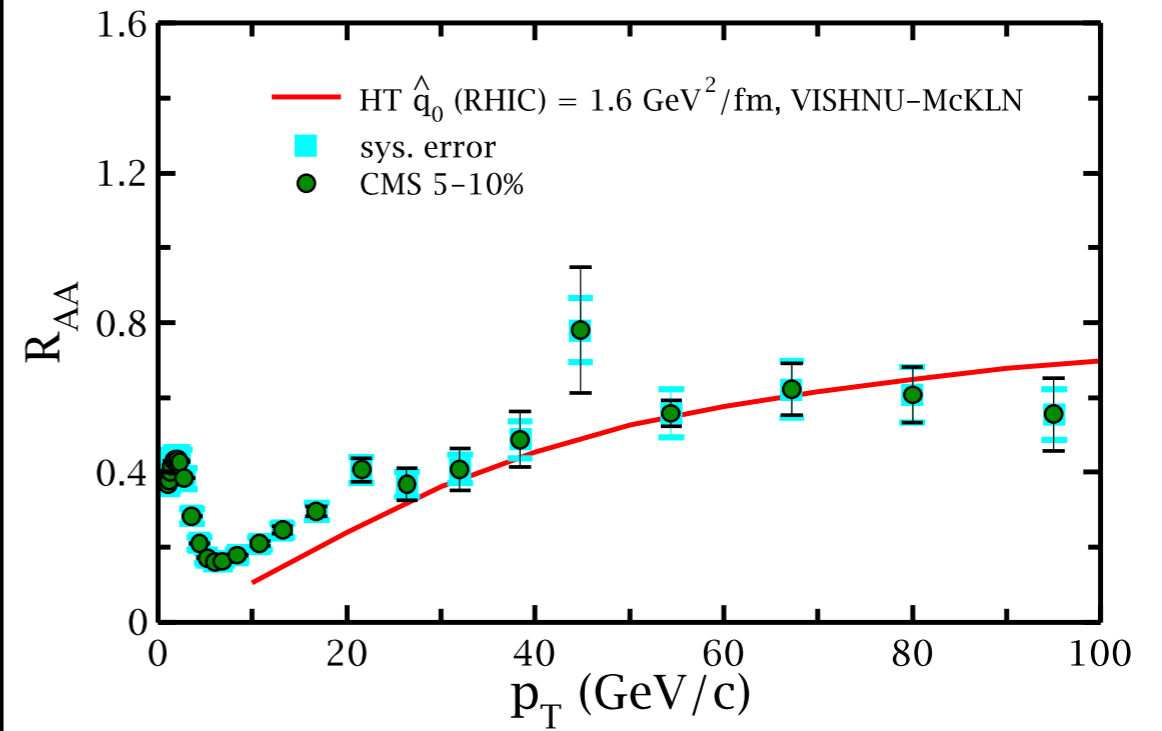
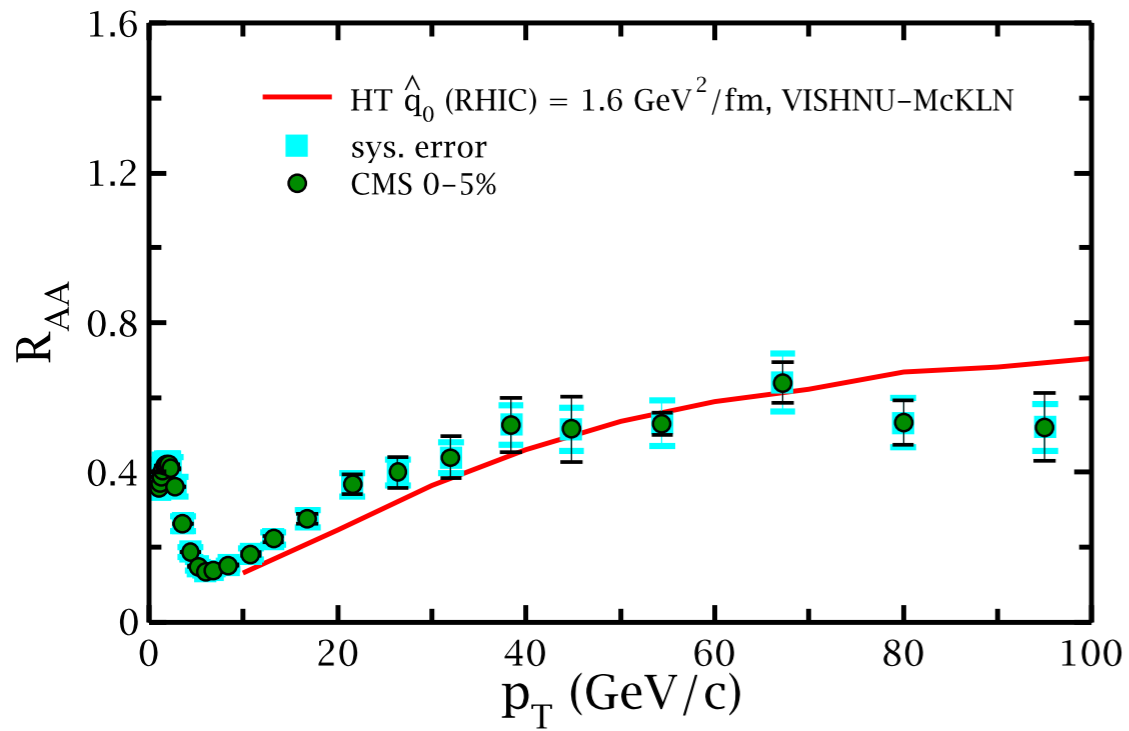
*what you may think this means!*

$$\frac{\hat{q}(T)}{T^3}$$

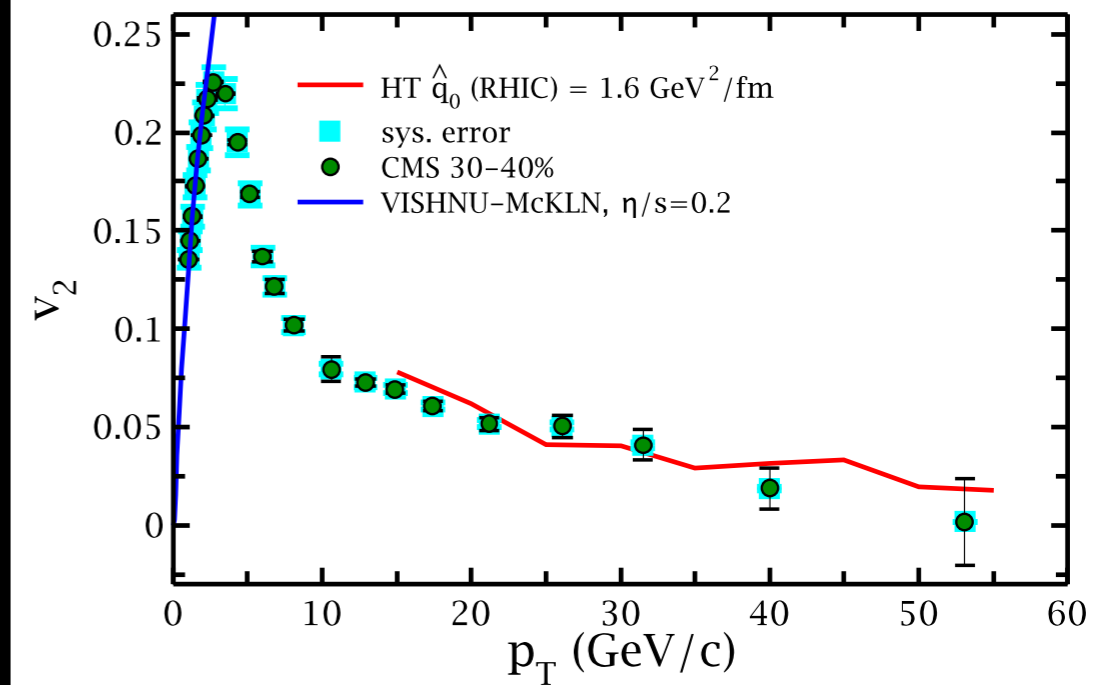
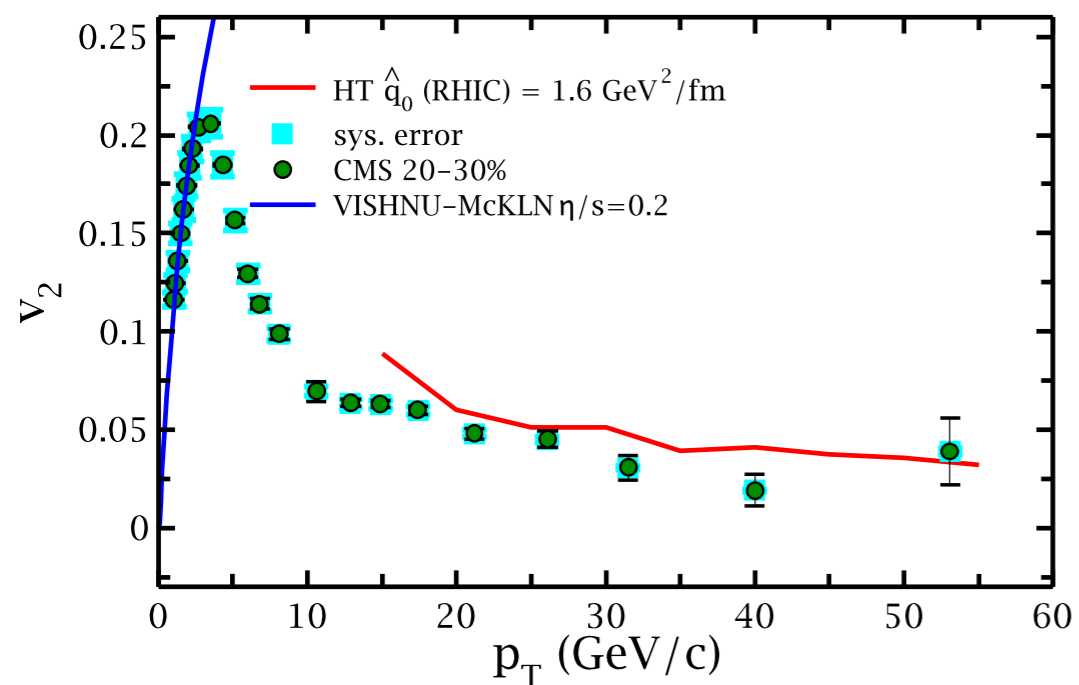
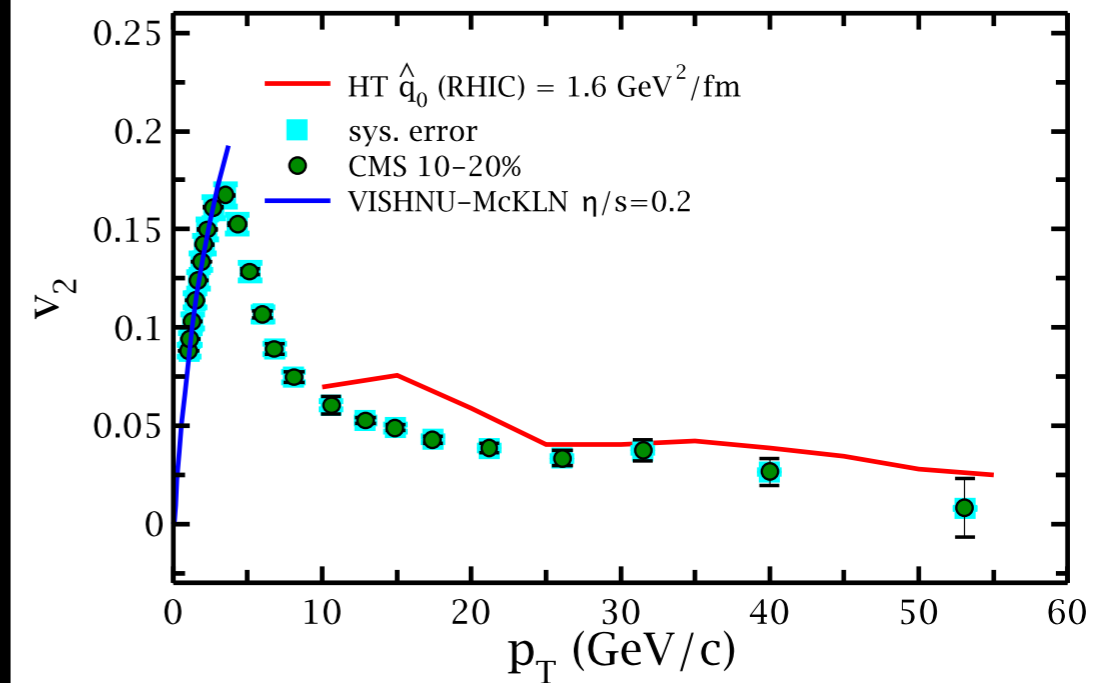
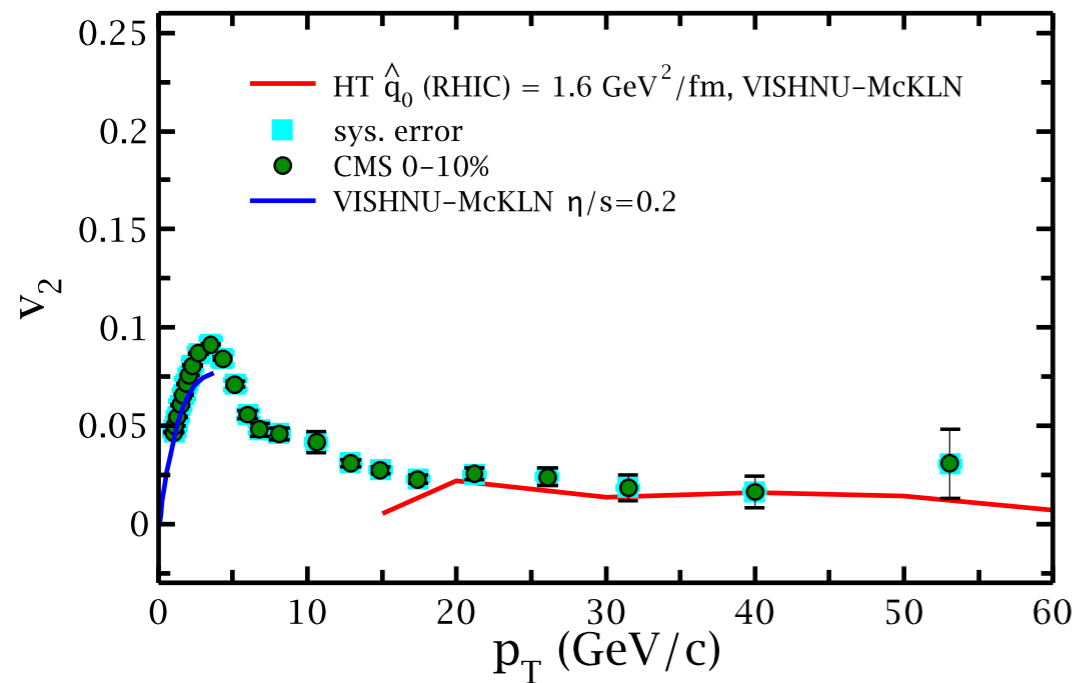


If this is true, must effect the centrality dependence of  $R_{AA}$ ,  $v_2$ , and its centrality dependence at a given collision energy

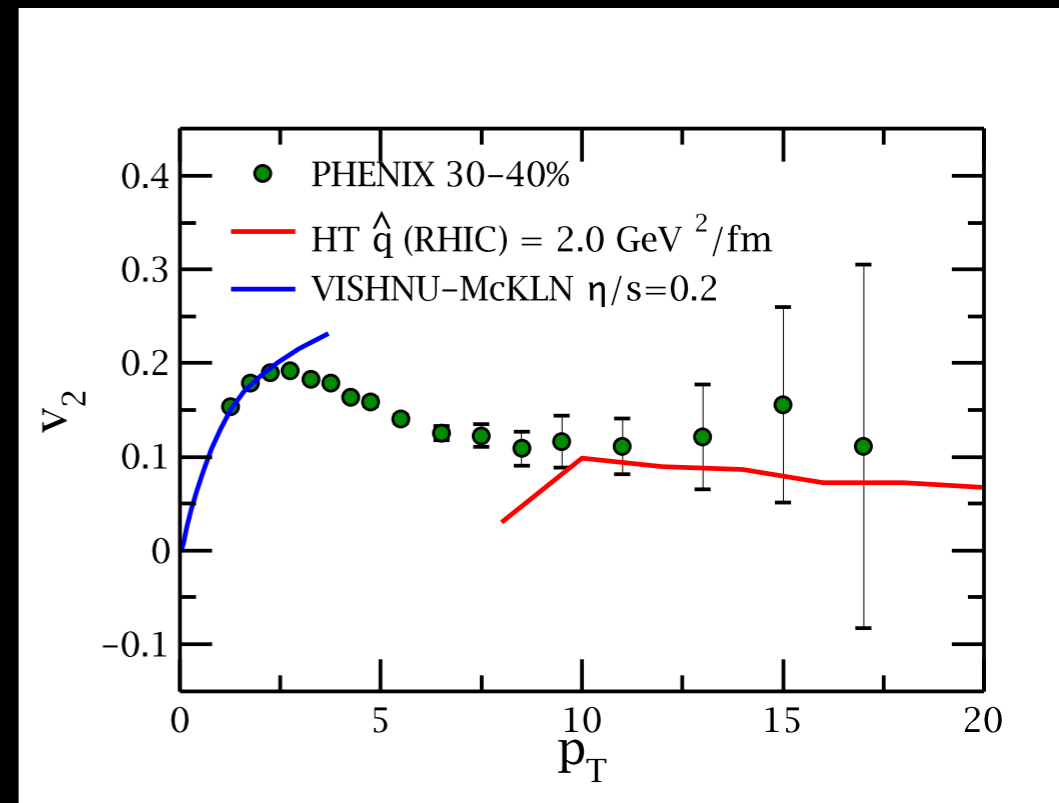
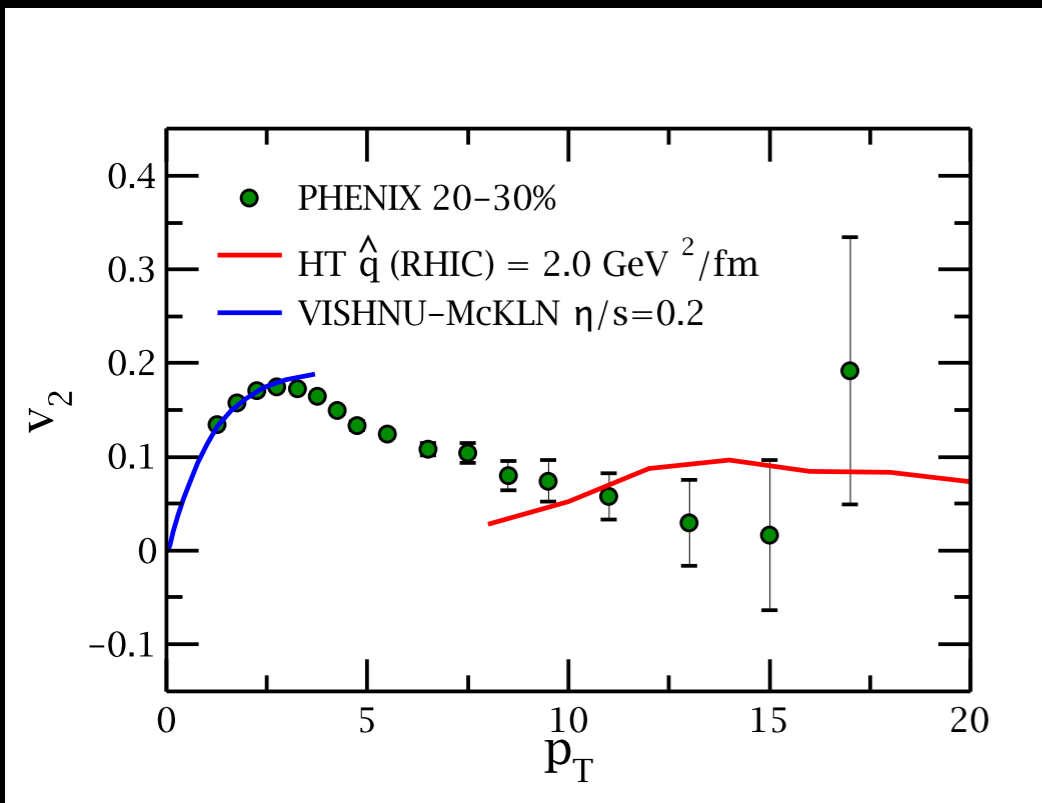
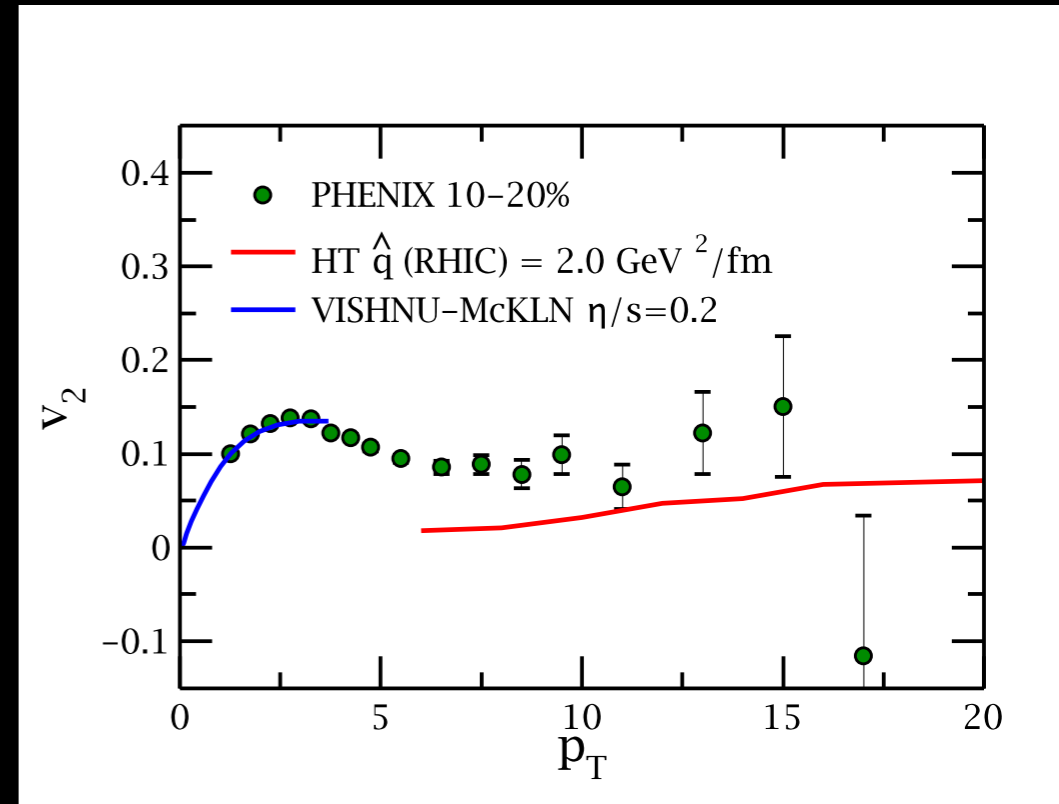
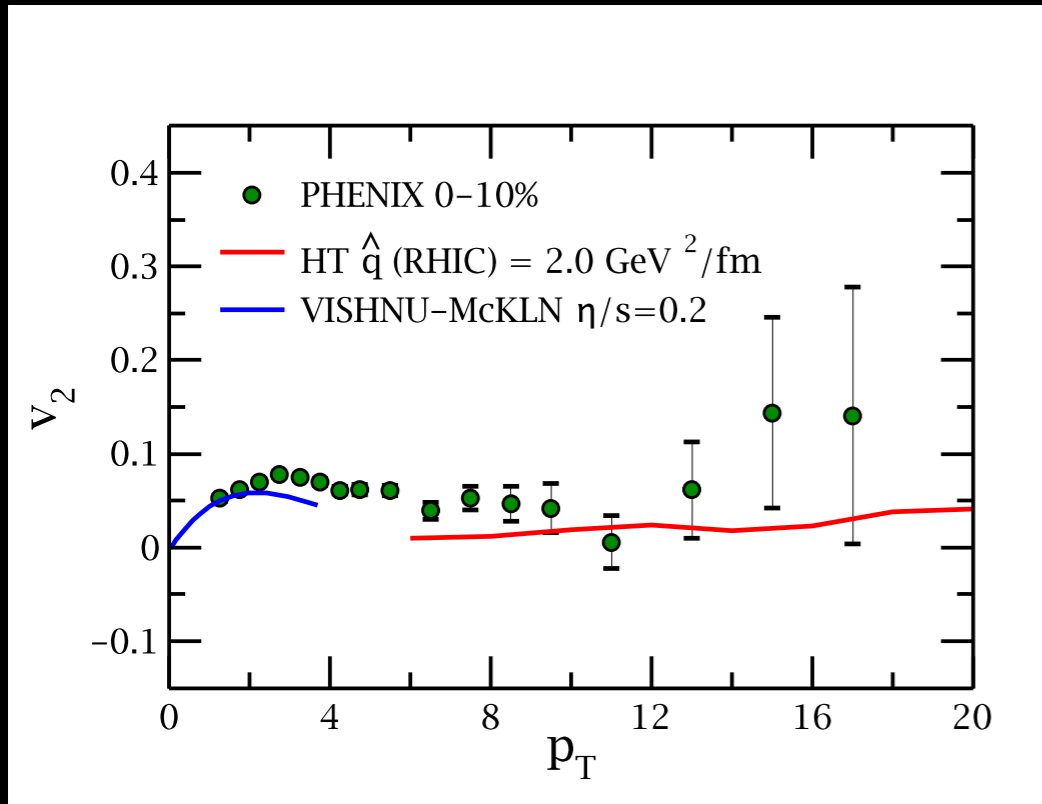
# LHC $R_{AA}$ without a bump in $\hat{q}/T^3$



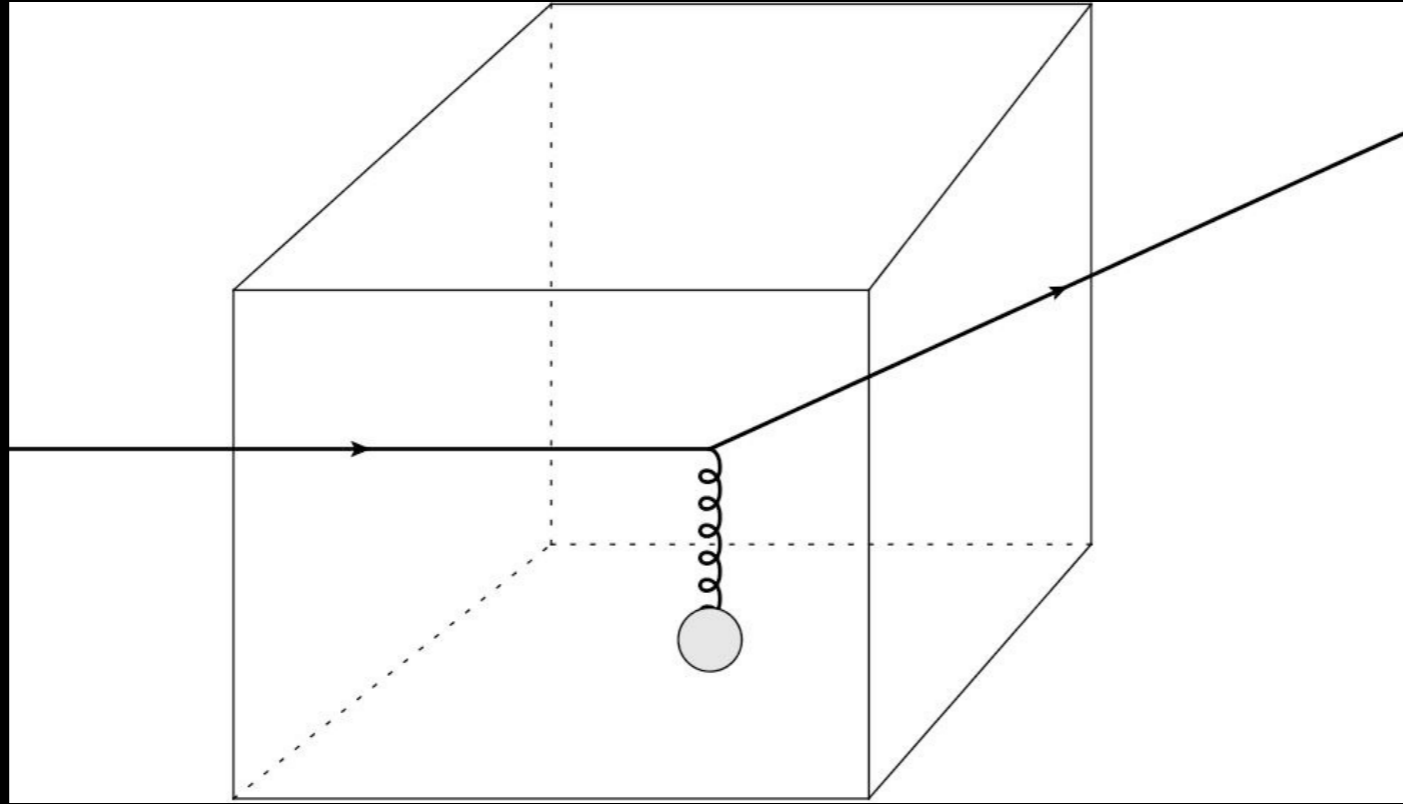
# $v_2$ at LHC without a bump in $\hat{q}/T^3$



# $v_2$ at RHIC without a bump in $\hat{q}/T^3$



# Calculating $\hat{q}$ with more care



$$\begin{aligned}
 W(k) &= \frac{g^2}{2N_c} \langle q^-; M | \int d^4x d^4y \bar{\psi}(y) A(y) \psi(y) \\
 &\times |q^- + k_\perp; X \rangle \langle q^- + k_\perp; X | \\
 &\times \bar{\psi}(x) A(x) \psi(x) |q^-; M \rangle
 \end{aligned}$$

in terms of  $W$ , we get

$$\hat{q} = \sum_k k_\perp^2 \frac{W(k)}{t},$$

Final state is close to ``on-shell''

$$\delta[(q+k)^2] \simeq \frac{1}{2q^-} \delta\left(k^+ - \frac{k_\perp^2}{2q^-}\right).$$

Also we are calculating in a finite temperature heat bath

$$\hat{q} = \frac{4\pi^2 \alpha_s}{N_c} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} d^2 k_\perp e^{-i \frac{k_\perp^2}{2q^-} y^- + i \vec{k}_\perp \cdot \vec{y}_\perp} \langle n | F_{\perp}^{+, \perp}(y^-, \vec{y}_\perp) F_{\perp}^+(0) | n \rangle$$

$$\hat{q}(q^+, q^-) \quad 2q^- q^+ = Q^2, \quad \frac{k_\perp^2}{2q^-} = xP^+$$

Can evaluate on Lattice, see talk by C. Nonaka

# What one usually does at this point

- Take the  $q^-$  to be infinity

$$\hat{q} \sim \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} d^2 k_\perp e^{i\vec{k}_\perp \cdot \vec{y}_\perp} \langle n | F^{+, \perp}(y^-, \vec{y}_\perp) F_\perp^+(0) | n \rangle$$

$$= \int \frac{dy^-}{2\pi} \langle n | F^{+, \perp}(y^-) F_\perp^+(0) | n \rangle$$

This makes  $\hat{q}$  into a one dimensional quantity  
an assumption of small  $x$  or high  $E$ .

$\hat{q}$  at vanishing  $x$  has been taken to NLO

Z. Kang, E. Wang, X.-N. Wang, H. Xing, PRL 112 (2014) 102001

T. Liou, A. Mueller, B. Wu, Nucl.Phys. A916 (2013) 102-125

J. Blaizot, Y. Mehtar-tani, arXiv:1403.2323 [hep-ph]

E. Iancu, arXiv:1403.1996 [hep-ph]

None of these NLO corrections have been tested in phenomenology.



# What is $x$ for a QGP

- Bjorken  $x$  in DIS on a proton  $x_B = \frac{Q^2}{2p \cdot Q}$

- In rest frame of proton  $x_B = \frac{Q^2}{2E \cdot M} = \frac{\eta}{M}$

- In the PDF  $f(x_B) = \int \frac{dy^-}{2\pi} e^{ix_B P^+ y^-} \langle P | \bar{\psi}(y^-) \frac{\gamma^+}{2} \psi | P \rangle$

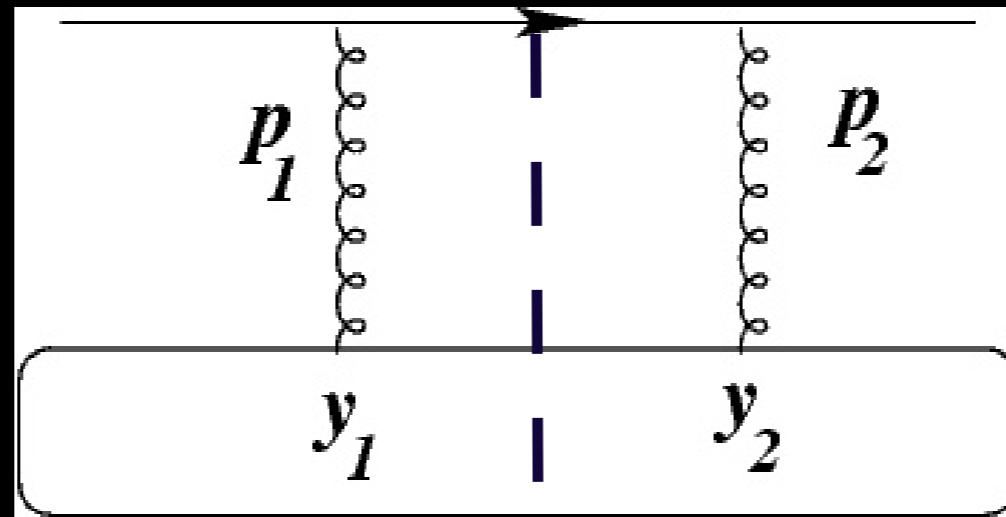
$$g(\eta) = \int \frac{dy^-}{2\pi} e^{i\eta y^-} \langle P | \bar{\psi}(y^-) \frac{\gamma^+}{2} \psi | P \rangle$$

In the rest frame of the proton,  $x \sim \eta$

We can compare  $\eta$  values between DIS and heavy-ions

# How about $x$ or $\eta$ dependence of $\hat{q}$

- The Glauber condition prevents a direct application of this established procedure.



$\delta\left(k^+ - \frac{k_\perp^2}{2q^-}\right)$  forces the incoming lines off-shell

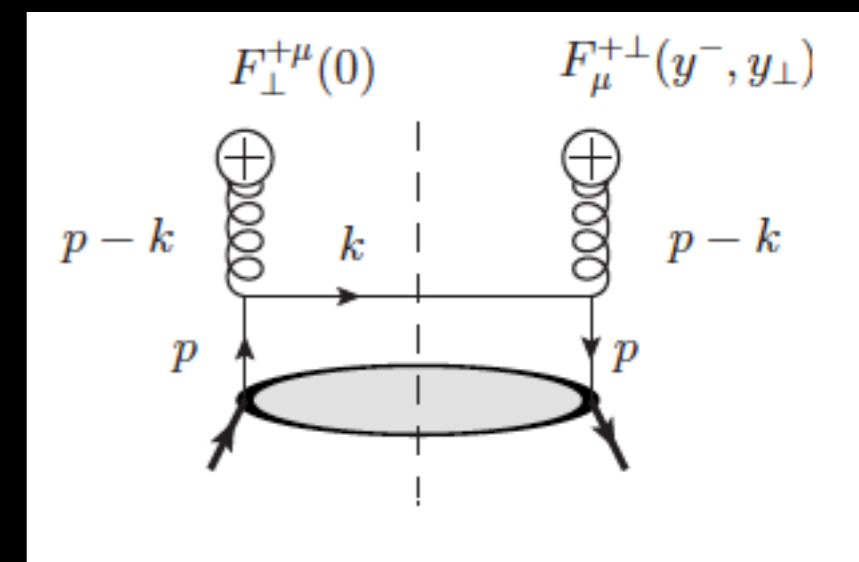
$q$  is a 3-D object depending on  $x$ ,  $\underline{k}_T$

Like a TMDPDF,

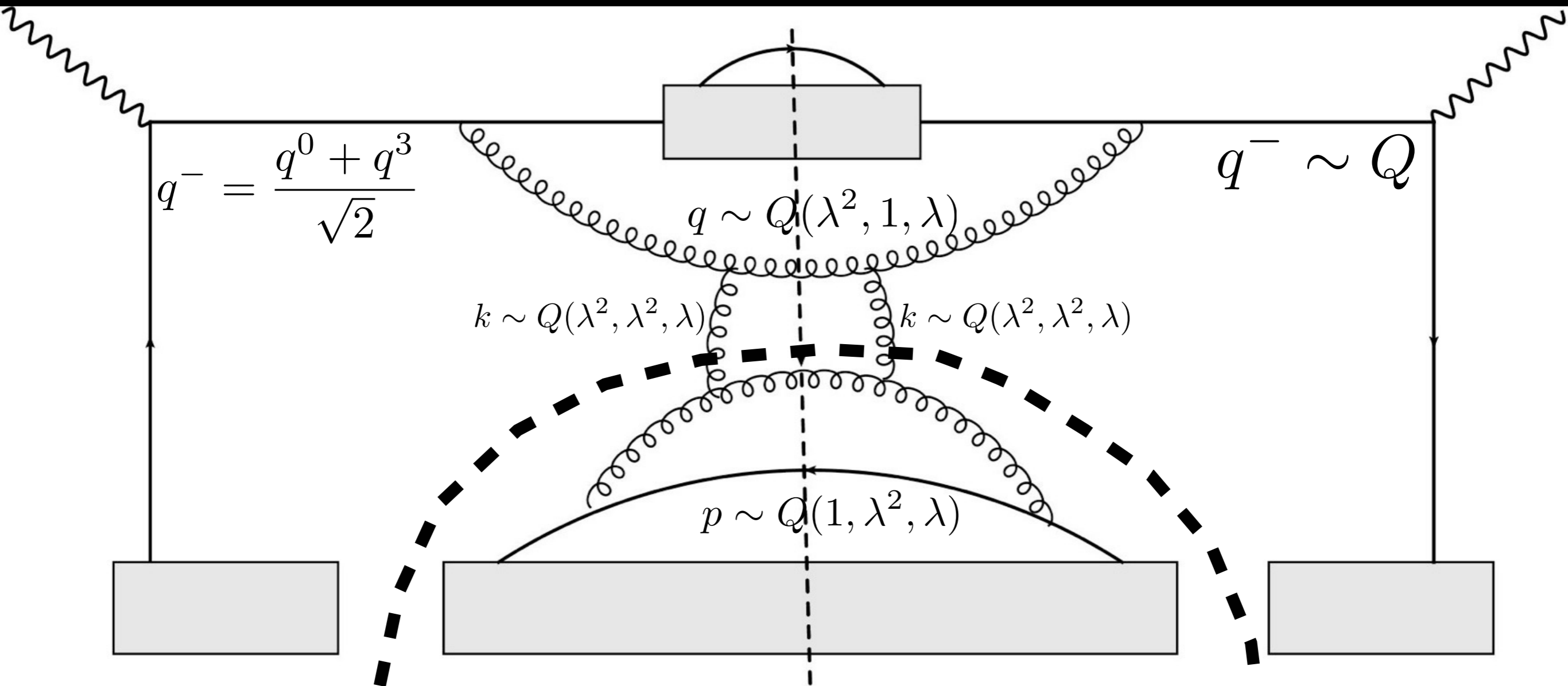
at large  $\underline{k}_T$  can *refactorize* to

regular PDF  $\times$  radiated gluon

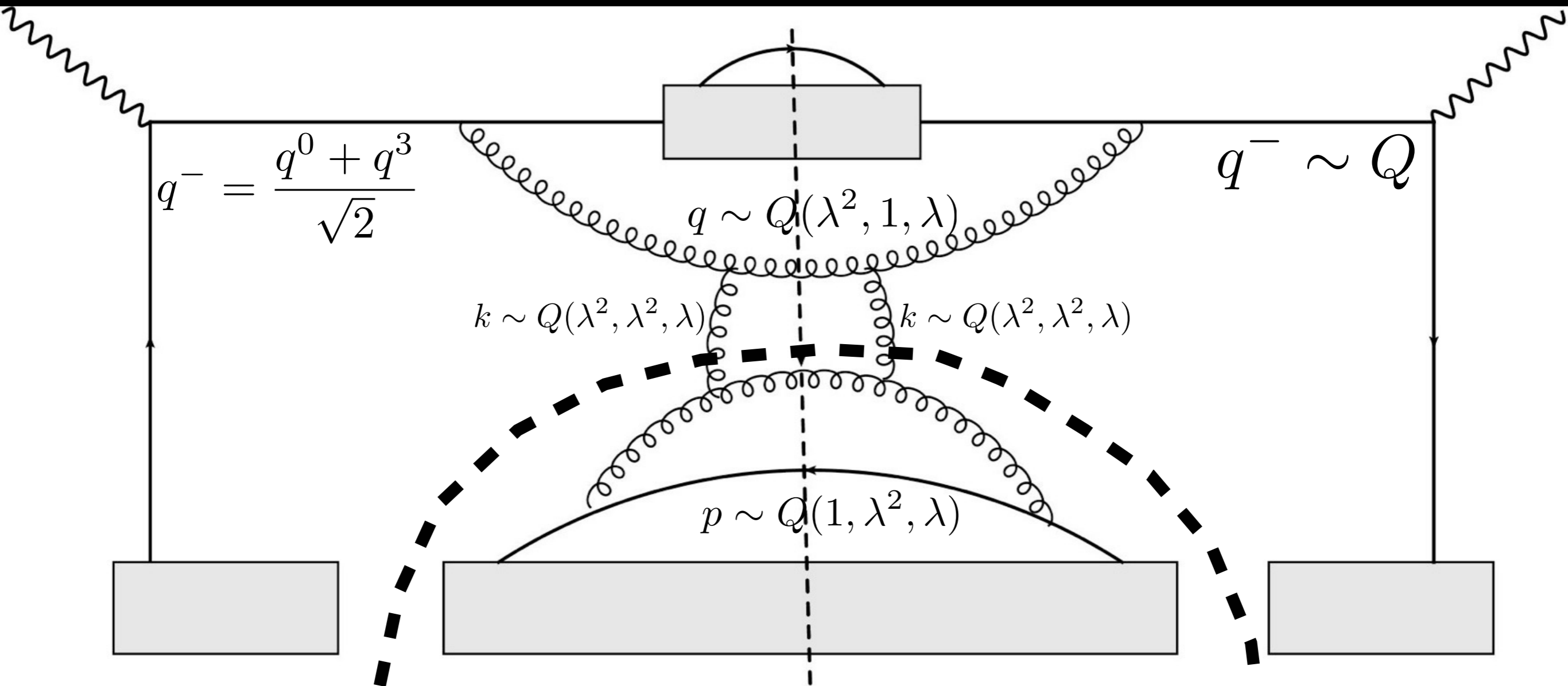
Contributions start at order  $\alpha_s$ ,



# A factorized picture



# A factorized picture

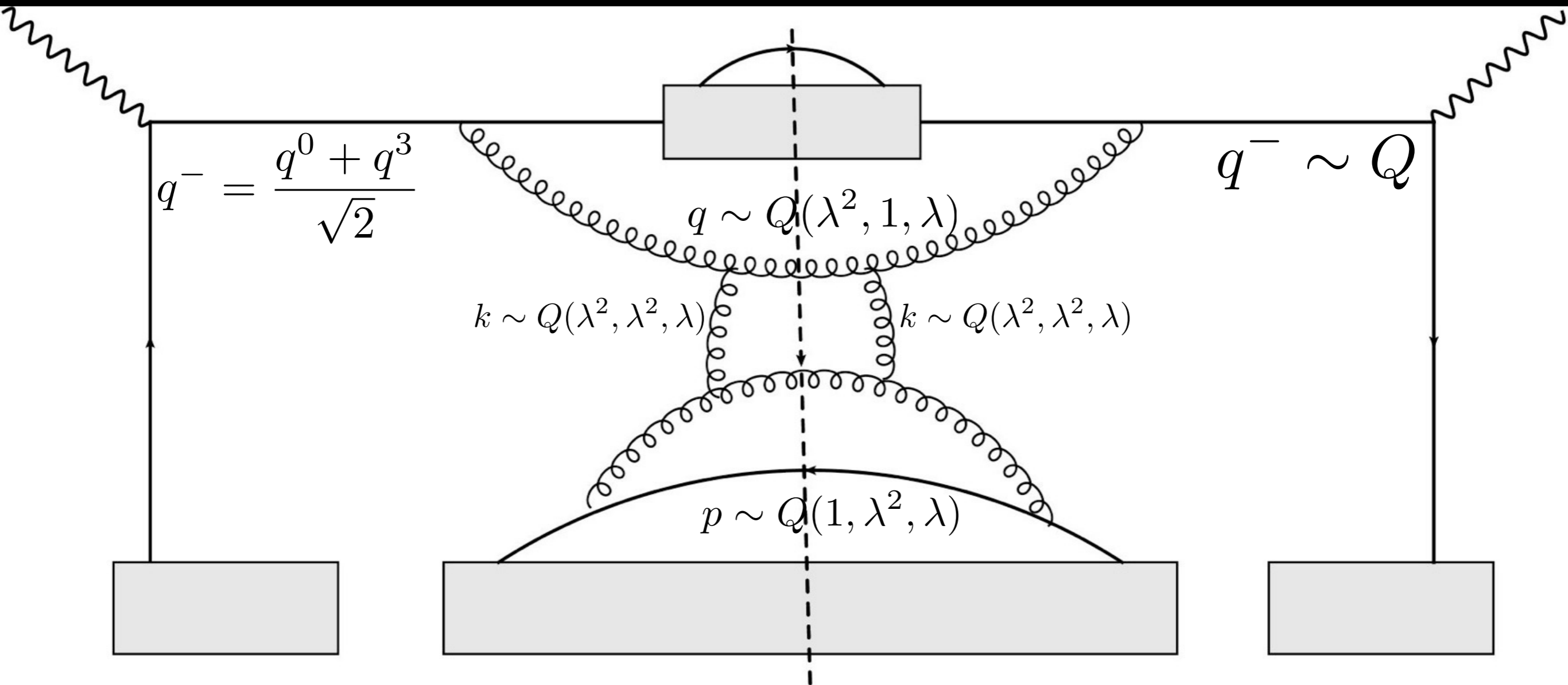


$Q$  is the hard scale of the jet  $\sim E$

$Q\lambda$  is a semi-hard scale  $\sim (ET)^{1/2}, \lambda \rightarrow 0$

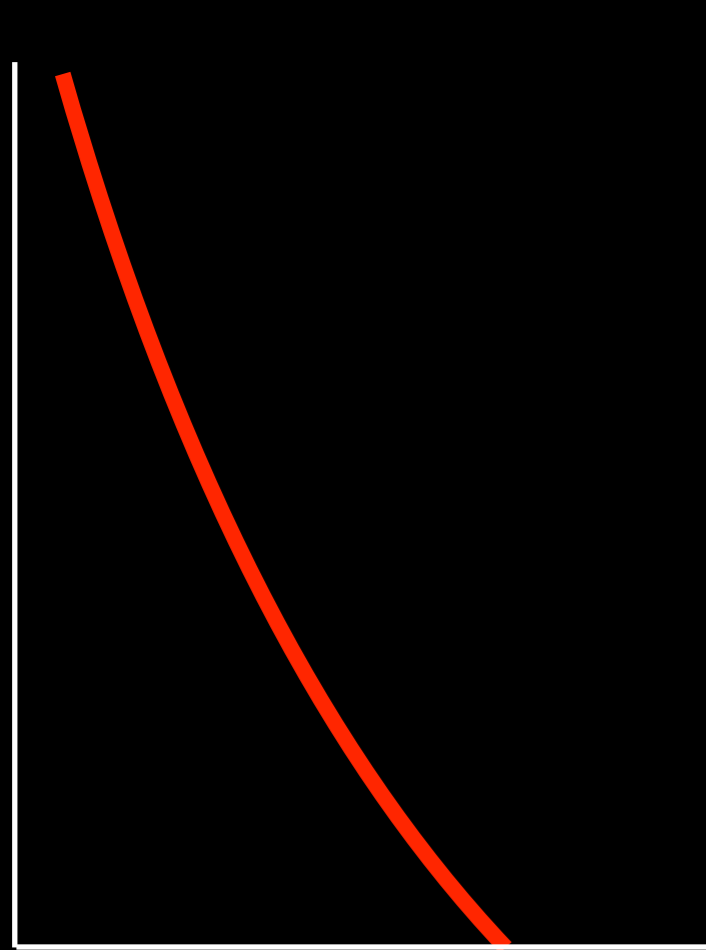
$\hat{q}$  contains all dynamics below  $Q\lambda$

# A factorized picture



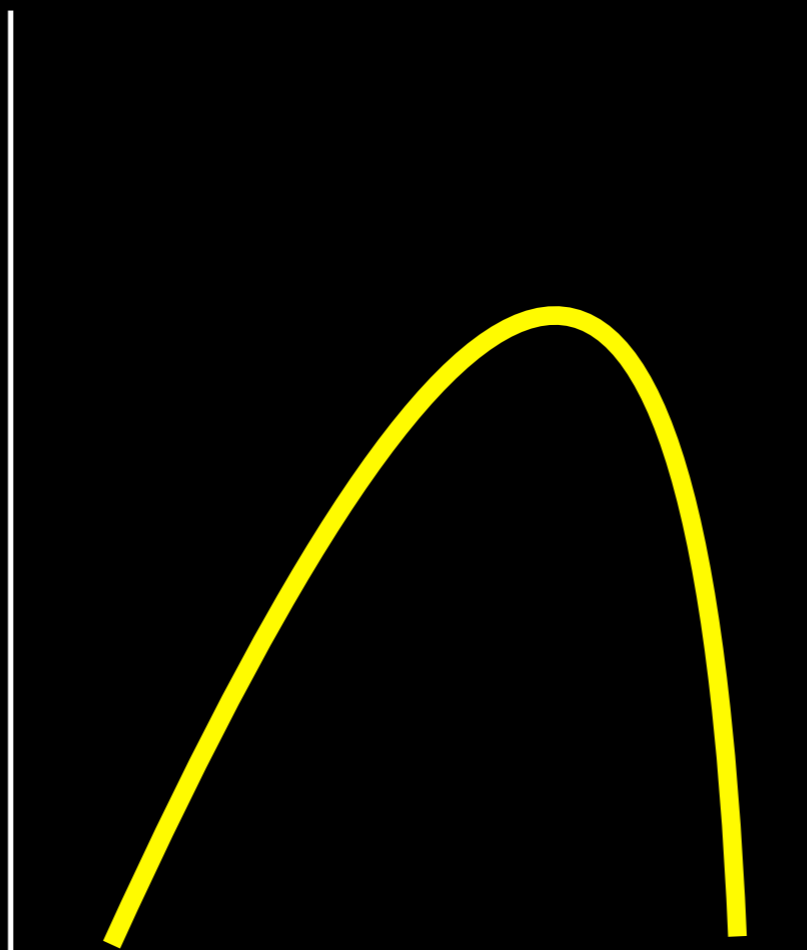
$Q$  is the hard scale of the jet  $\sim E$   
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 $\hat{q}$  contains all dynamics below  $Q\lambda$

# Input PDF at $Q^2 = 1 \text{ GeV}^2$



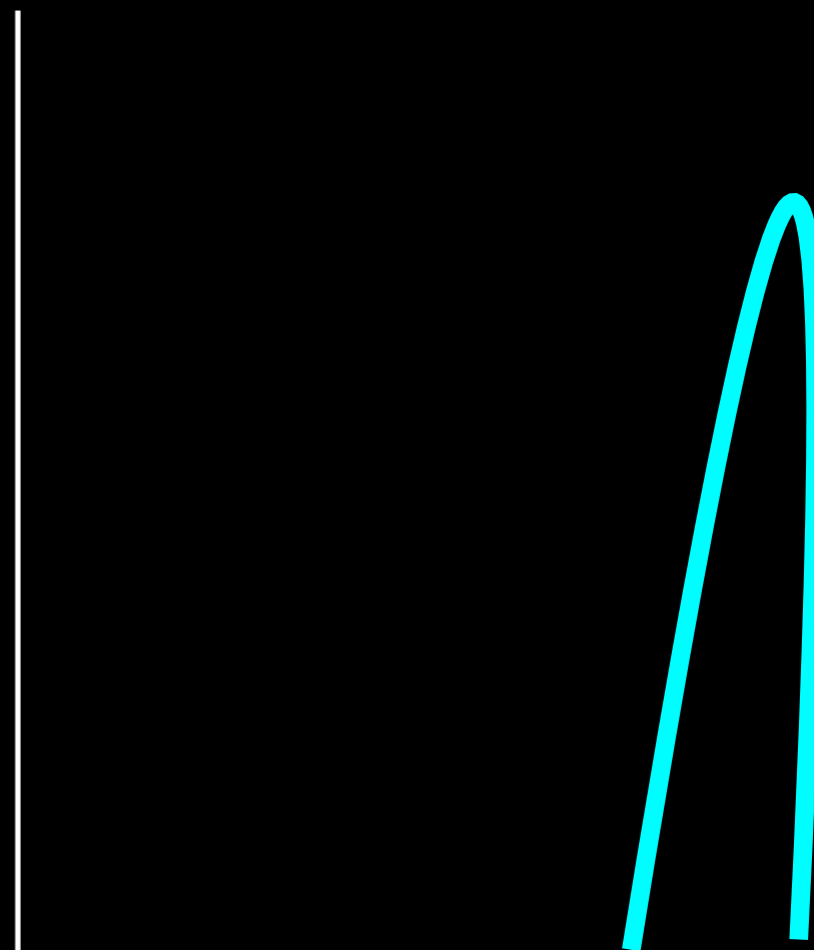
$x$

Sea like



$x$

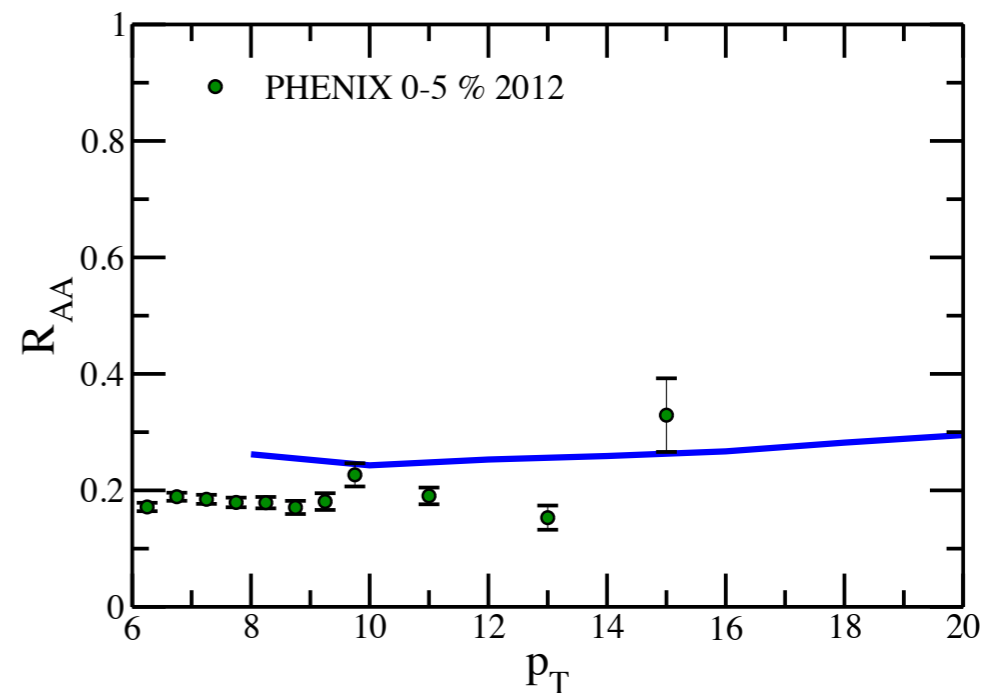
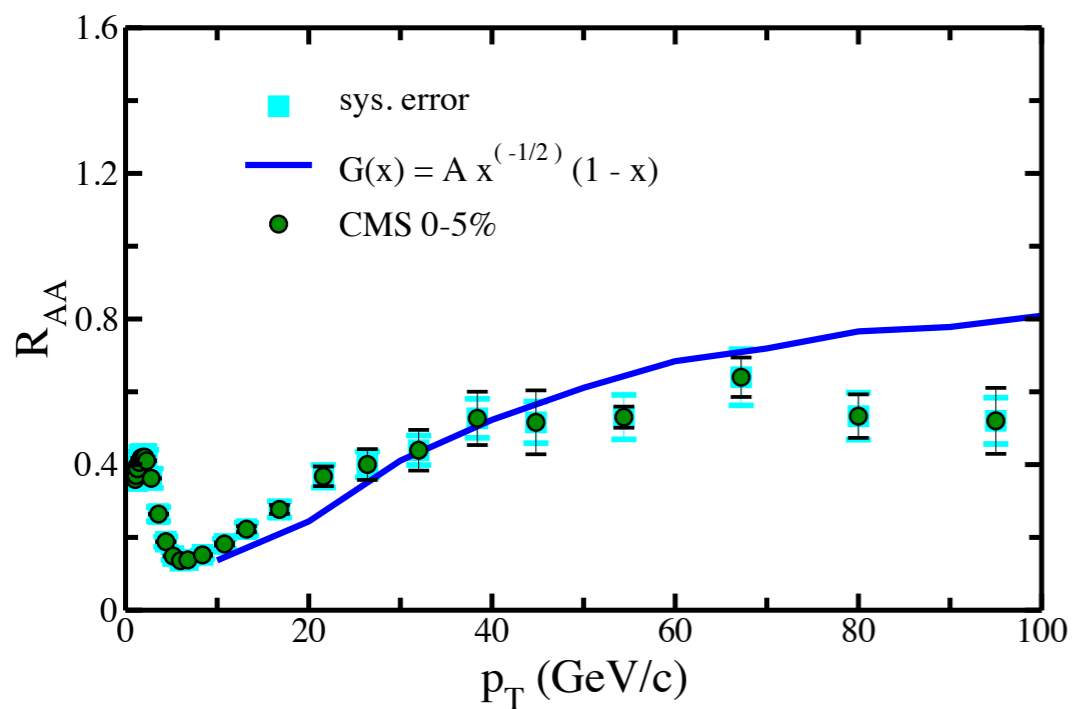
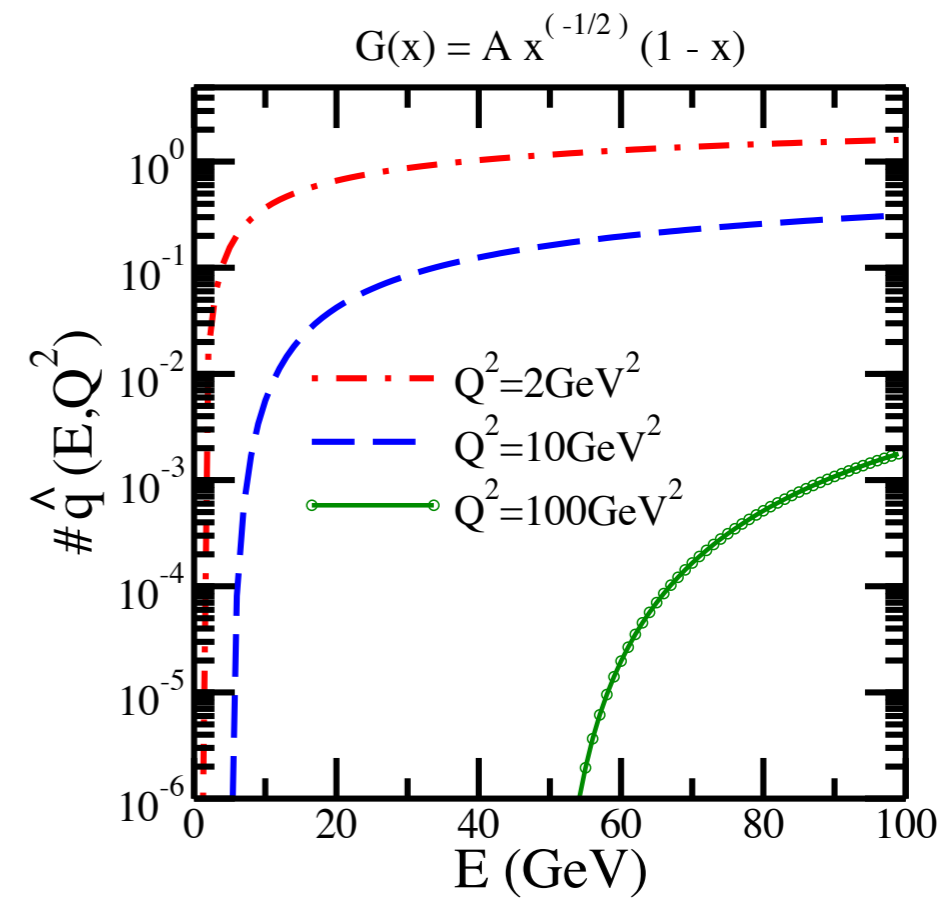
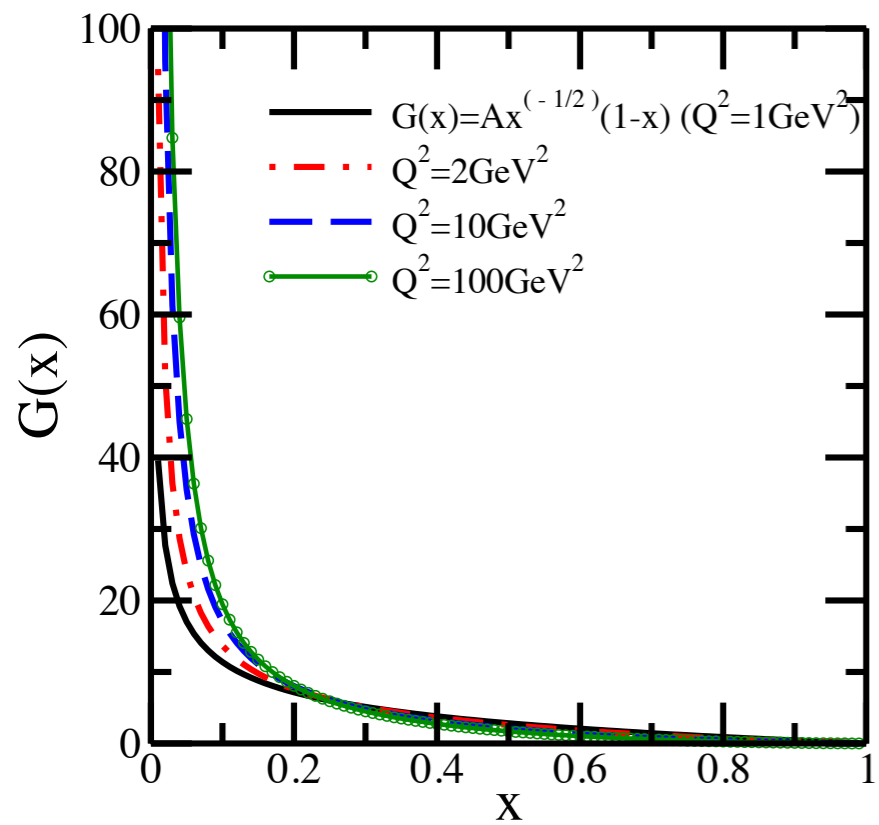
Wide Valence



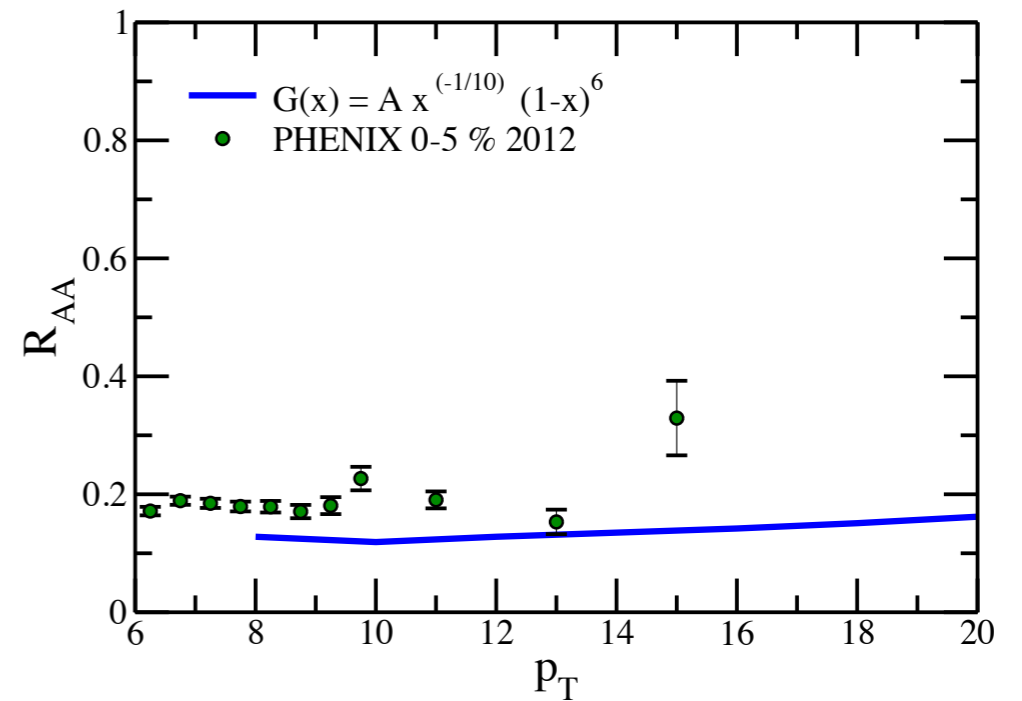
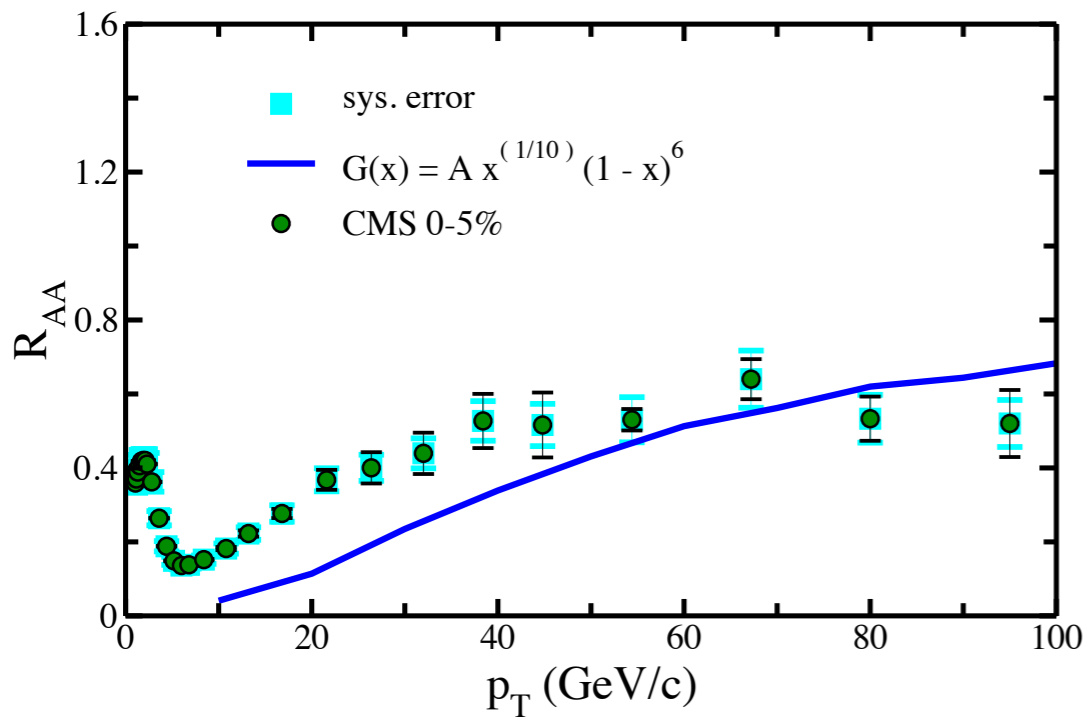
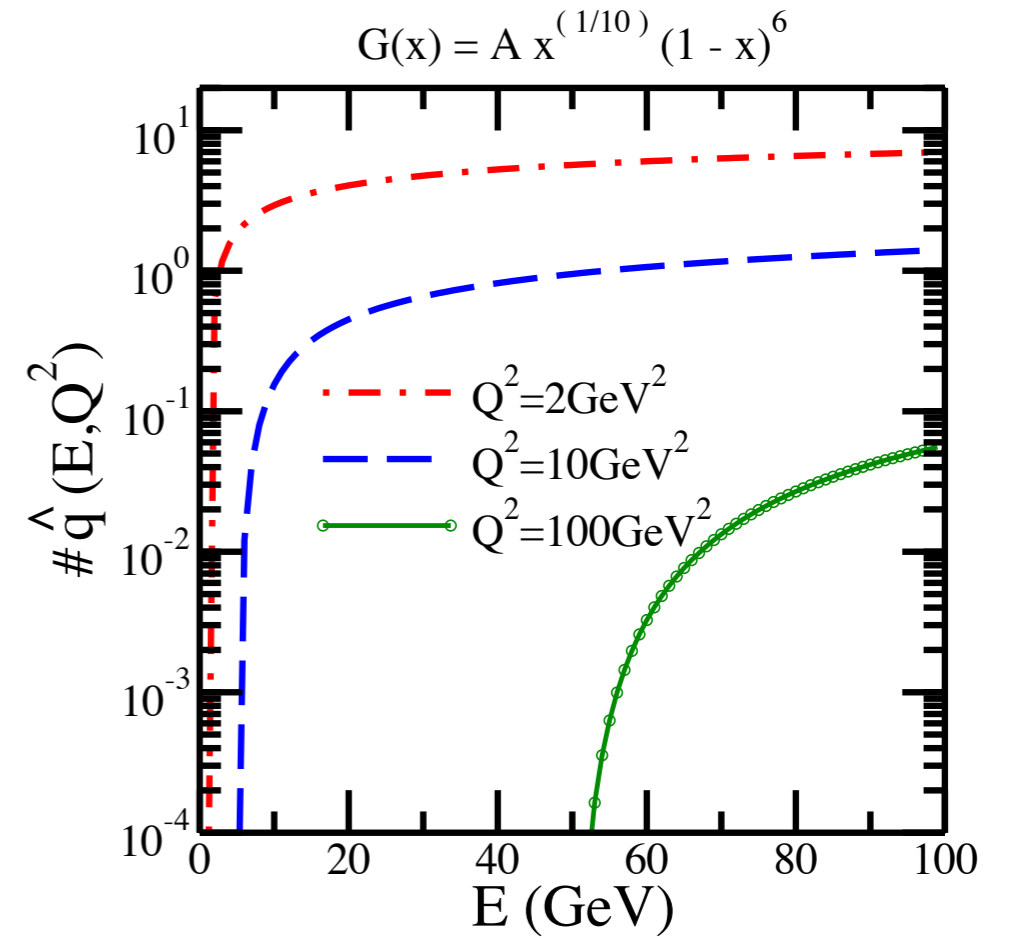
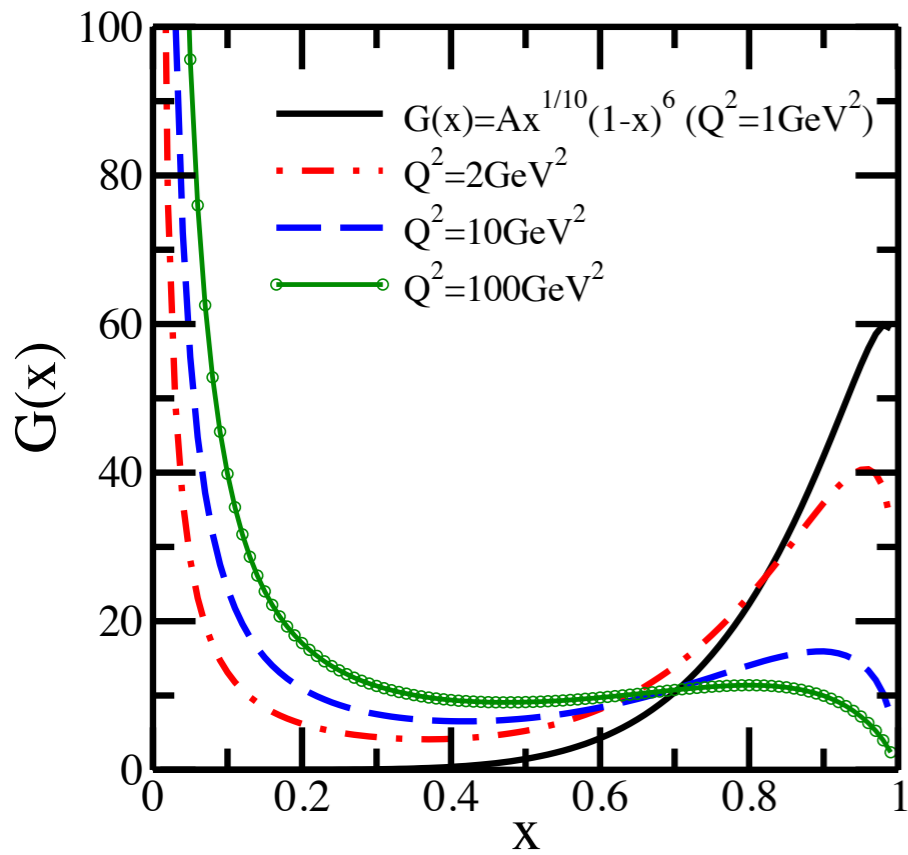
$x$

Narrow Valence

# Sea-like PDF of the QGP

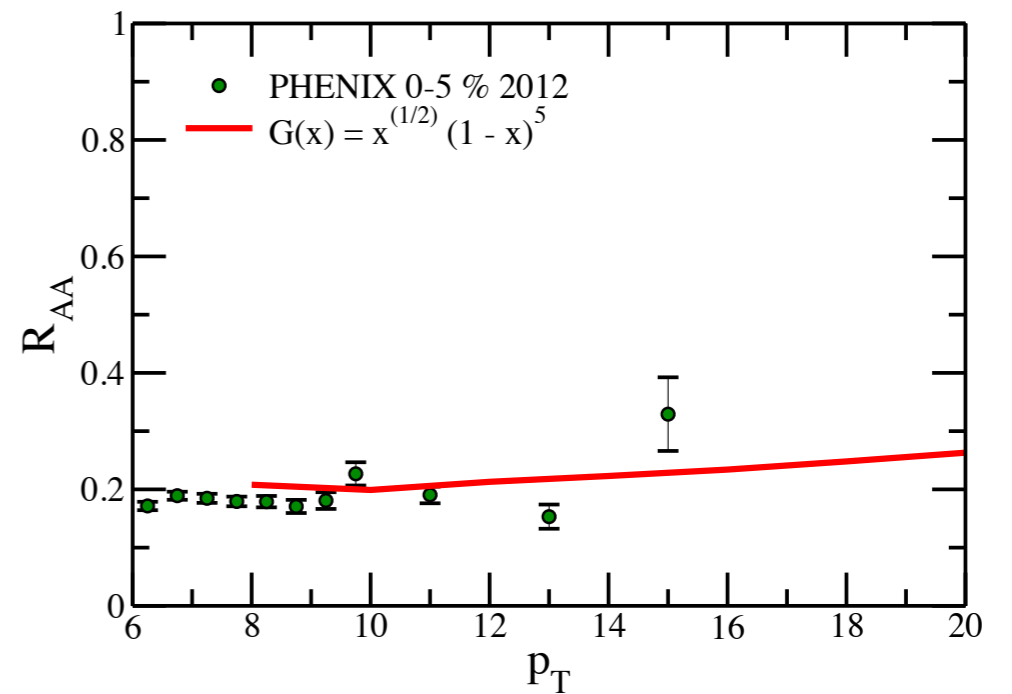
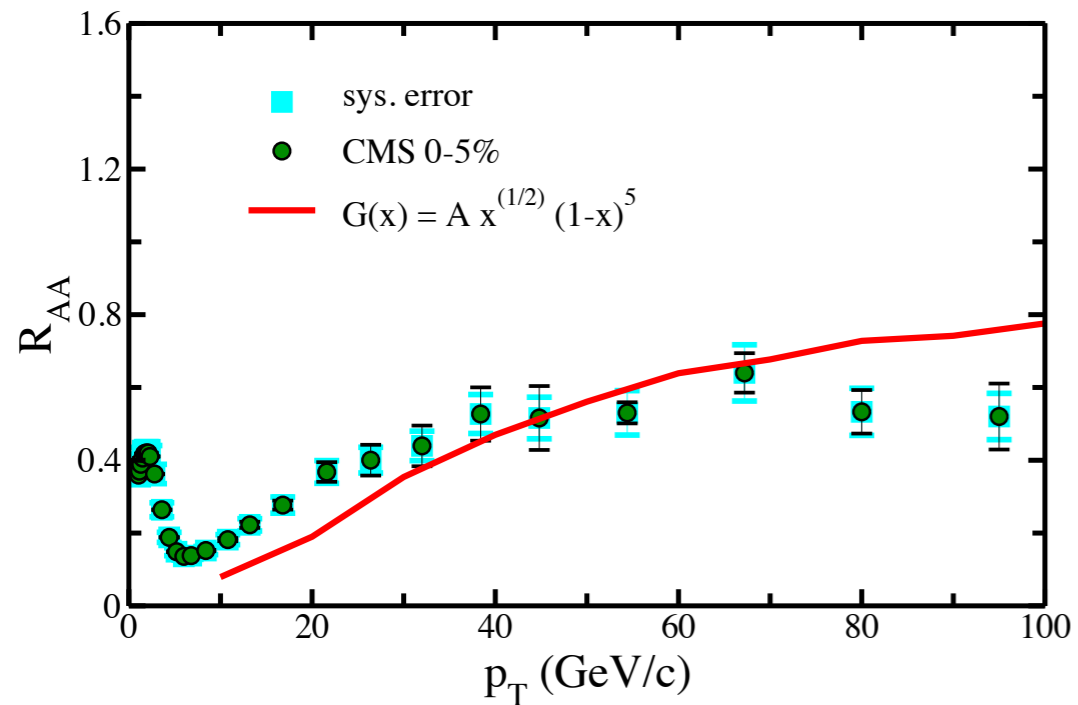
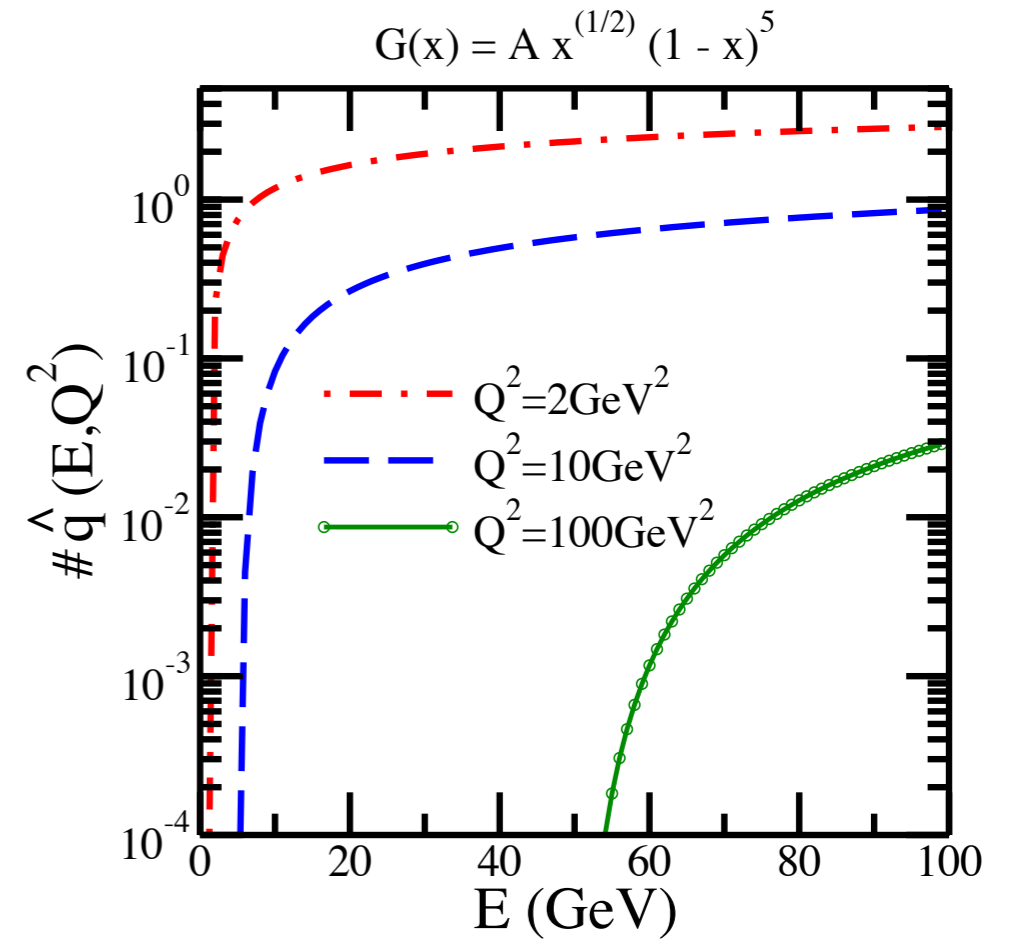
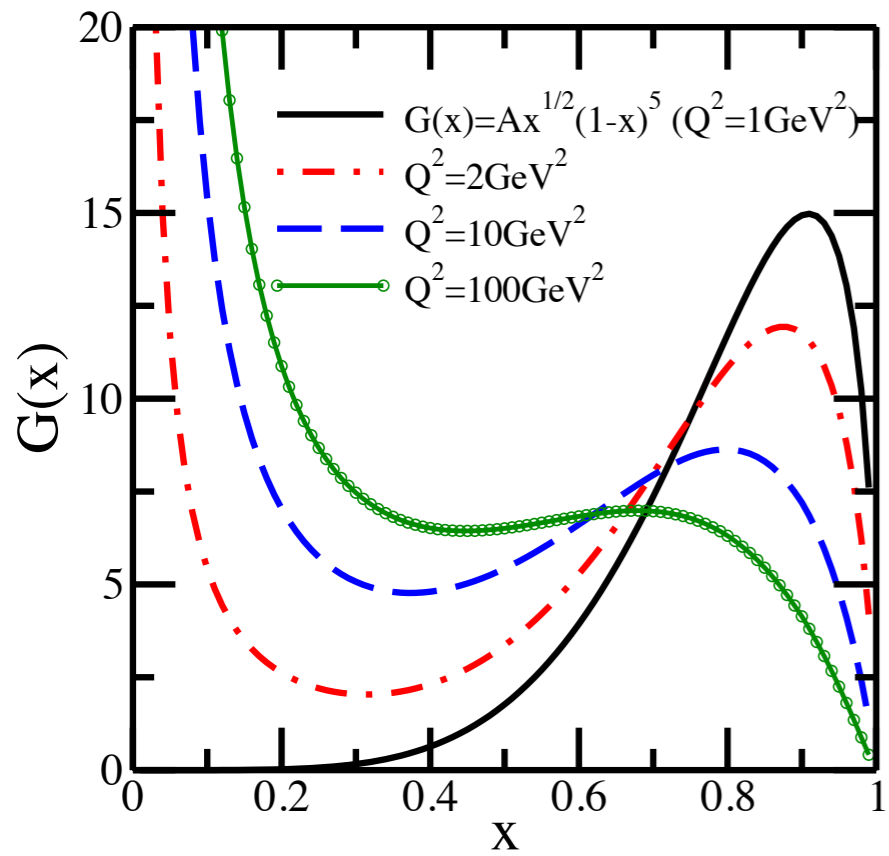


# Narrow valence like PDF of QGP





# Wide valence like PDF of the QGP

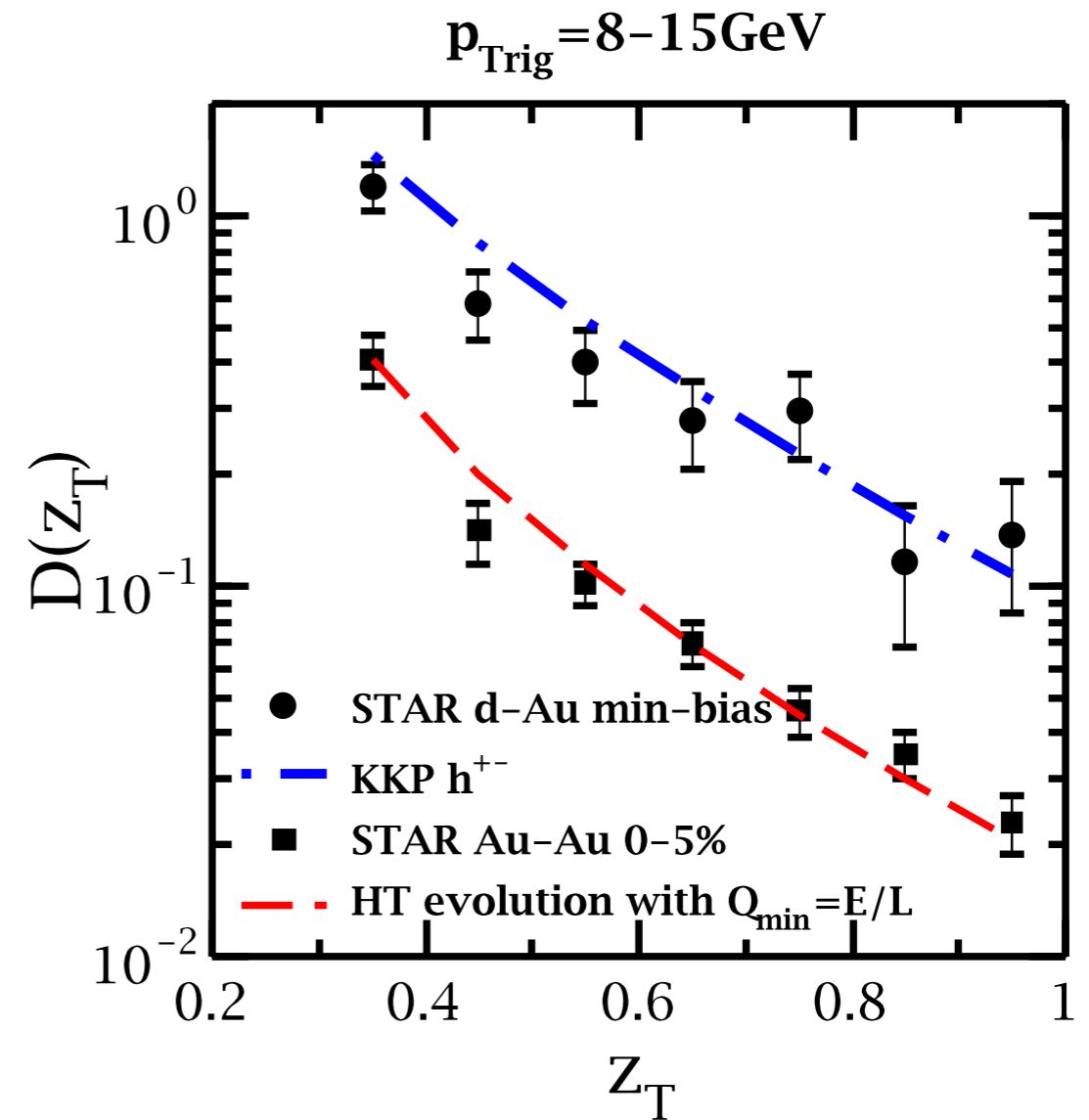
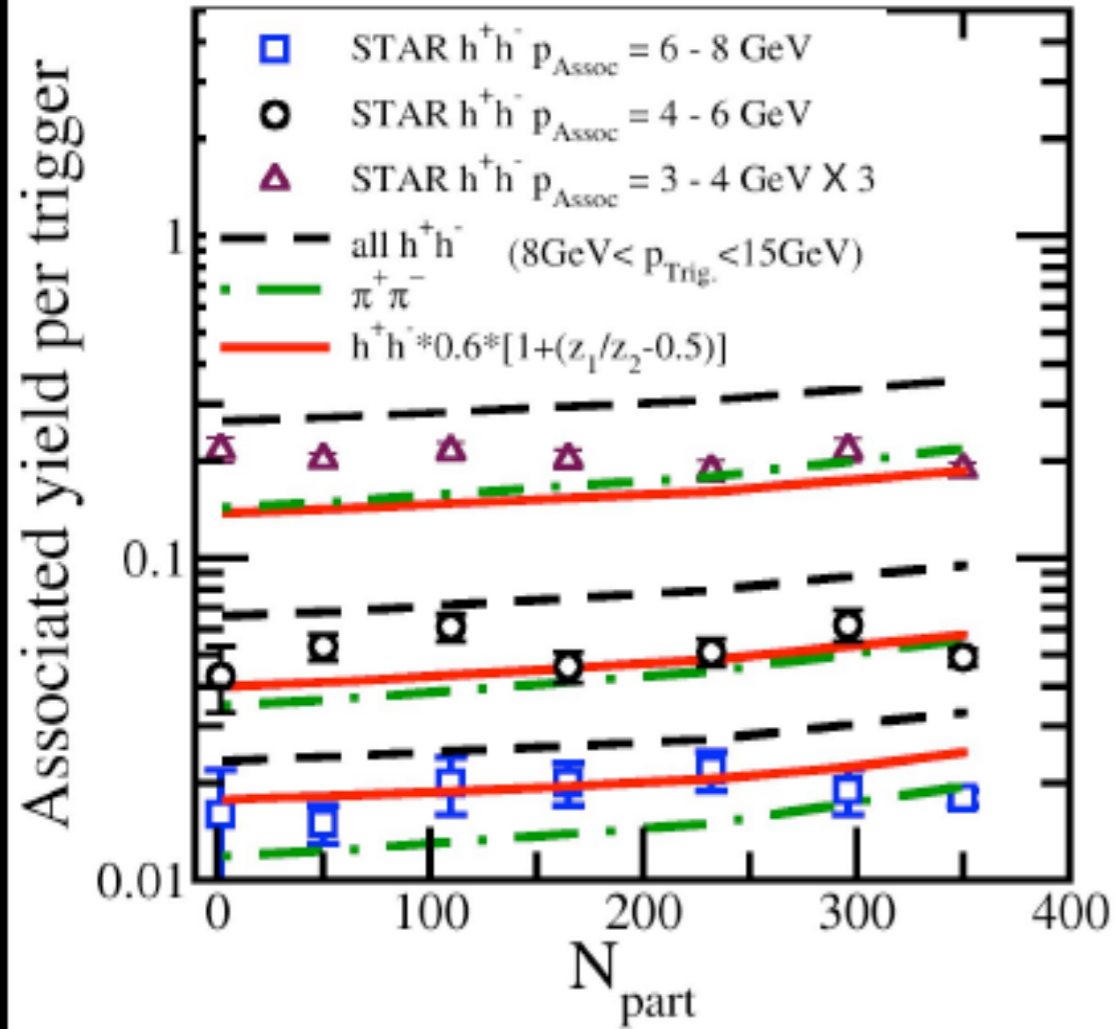


# What does this mean?

- Possible resolution of the JET puzzle
- Based on consistent  $Q^2$  evolution of  $q$
- Should have  $x$  evolution at high energy
- Will be done in reverse very soon, will get PDF's with bands (by Quark Matter !!!)
- Applying TMD systematics, may complicate this interpretation.

# Near side and away side correlations

A. Majumder, et. al., nucl-th/0412061



A wide range of single particle observables can be explained by a weak coupling formalism