Nonequilibrium Photon Production in the Quark-Gluon Plasma

Hard Probes 2016, 武汉

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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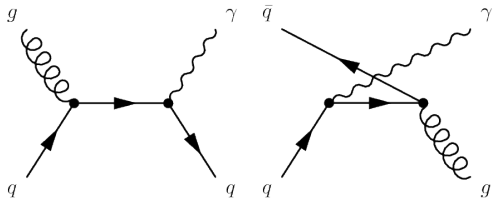
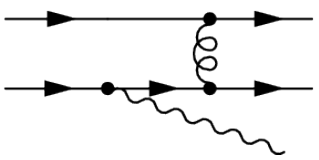
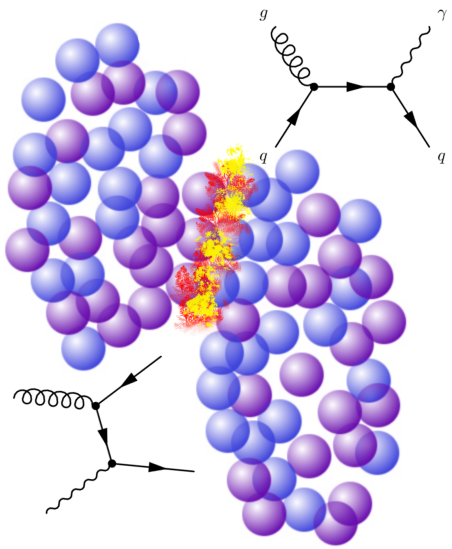
with Kai Zhou, Florian Senzel, Heiner Kremer, Hendrik v. Hees, Carsten Greiner, Zhe Xu

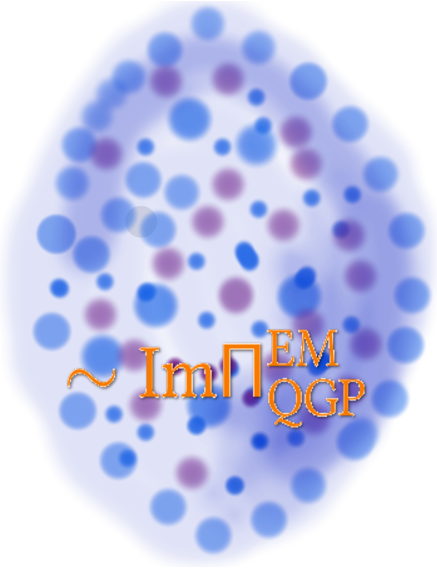


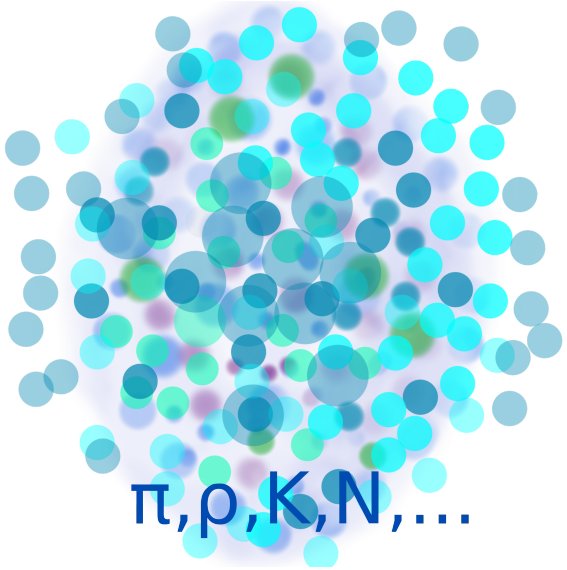
Wuhan HP2016



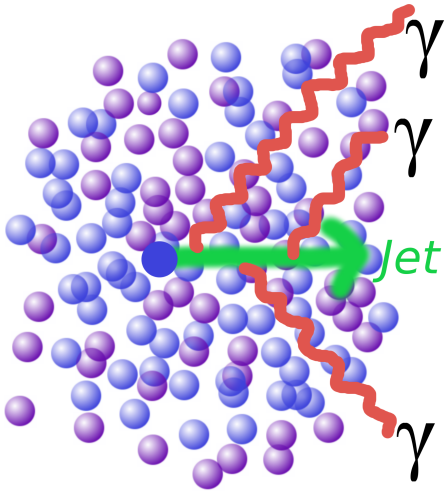
Photon sources in heavy-ion collisions

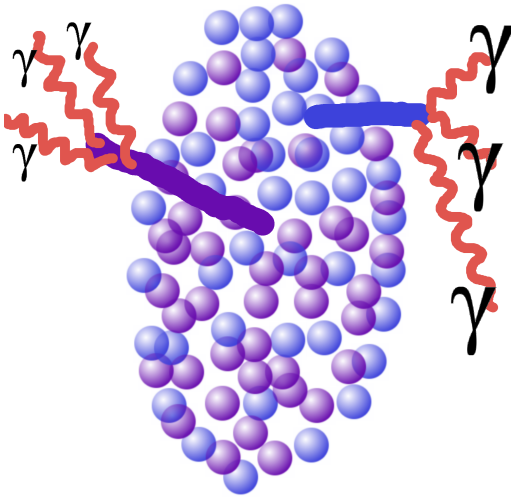






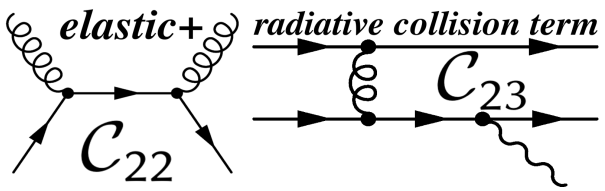
+ photons from jets:

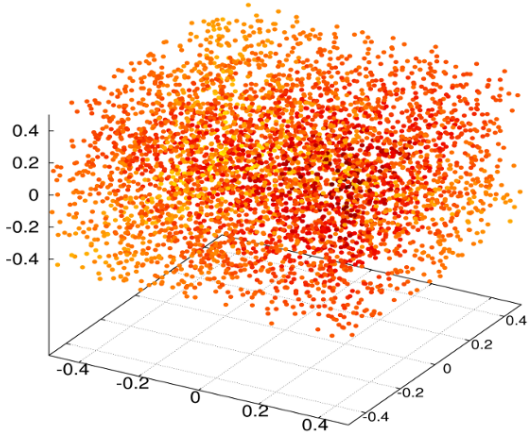


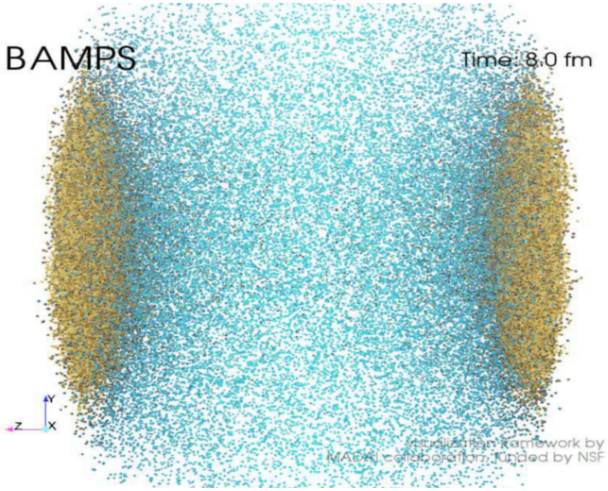


BAMPS: **B**oltzmann **A**pproach to **M**ulti-**P**arton **S**catterings

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{22}[f] + \mathcal{C}_{23}[f]$$







Box simulations & expanding QGP

Zhe Xu & Carsten Greiner, 2005

Phys. Rev. C 71, 064901

⋆ Spacetime-grid, stoch. collision probabs.

⋆ Tot. cross sections from **pQCD**: $\sigma_{22}(s)$, $\sigma_{23}(s)$

⋆ Fully Lorentz-invariant formulation

$$P_{22} = v_{rel} \frac{\sigma_{22}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

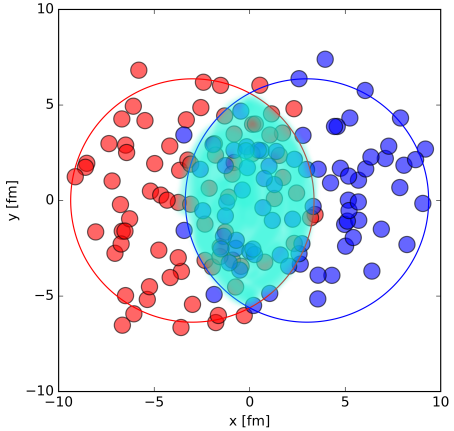
$$P_{23} = v_{\rm rel} \frac{\sigma_{23}}{N_{test}} \frac{\Delta t}{\Delta^3 x}, \quad P_{32} = \ldots$$

⋆ Massless onshell particles (ideal eos)

⋆ This talk: only classical statistics

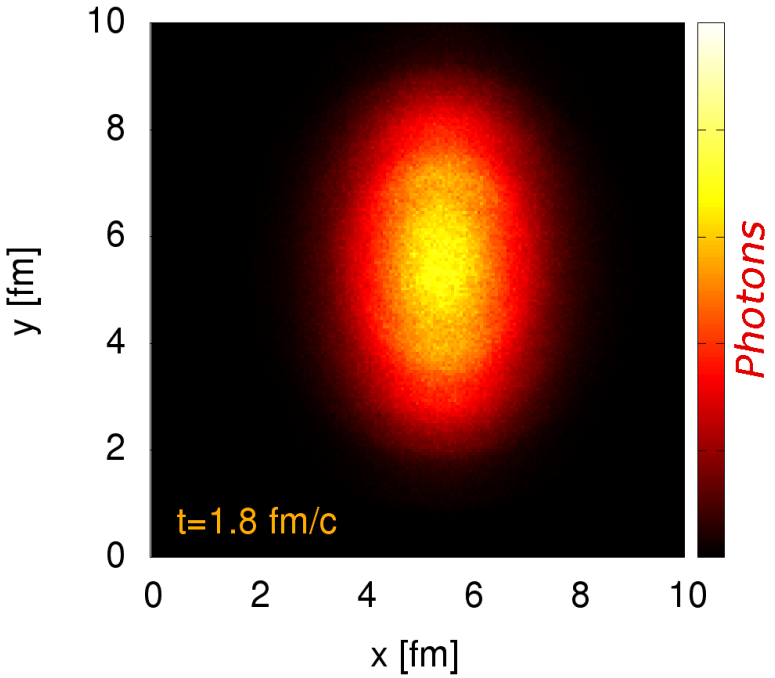
⋆ See e.g.: PRD88,014018 / PRL102,202301 / PRL114,112301 / PRD90,094014 / PRL103,032301

BAMPS: 3+1D expanding heavy-ion collisions



- Smooth Glauber initial **positions**
- Pythia 6 initial **momenta**: $pp \times N_{\text{binary}}$
- Reproduce $\mathrm{d}E_T/dy$ distribution for RHIC/LHC data
- v_2 & R_{AA} in common framework (PRL 114, 112301, 2015)

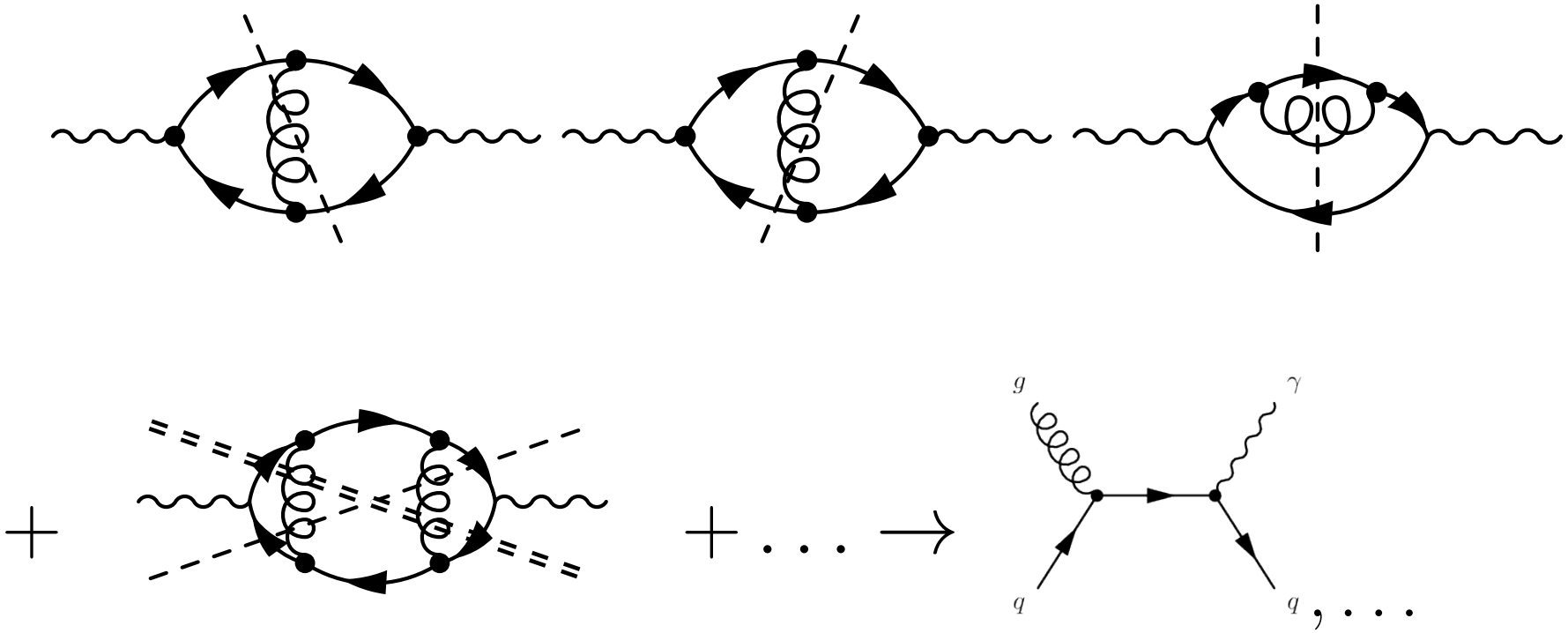
- Particle species: $q, \bar{q}, g, \gamma, e^+e^-$
- LO-γ-production, incl. bremsstrahlung $qq \longrightarrow qq\gamma$
- Born dilepton production
- Radiative gluons/photons: LPM-suppression modelled



Photon emission rate

$$E_k \frac{\mathrm{d}^3 R}{\mathrm{d}^3 k} = -\frac{g^{\mu\nu}}{(2\pi)^3} \mathrm{Im}\Pi^R_{\mu\nu}(E_k, \vec{k}) \leftrightarrow E_k \frac{\mathrm{d}^3 R}{\mathrm{d}^3 k} = \mathcal{N} \int_{pp'k'} \delta^{(4)}(\ldots) |\mathcal{M}|^2 f_p f_{p'} (1 \pm f_{k'})$$

with $\Pi^R_{\mu\nu}$: retarded photon self-energy, $|\mathcal{M}|^2$: scattering matrix element



Elastic photon production *rate* in BAMPS

Microscopic implementation: *Two* sources of lacking precision:

A) Compton and annihilation matrix elements in BAMPS

Instead of HTL resummed loop for soft momentum transfer:

$$|\mathcal{M}|^2_{\text{Compton}} = \frac{16}{3}\pi^2\alpha\alpha_s\left(\frac{s^2+st}{(s+m^2_{D,q})^2}+\frac{s^2+st}{(u-m^2_{D,q})^2}\right), \; |\mathcal{M}|^2_{\text{annihilation}} = \dots$$

only naively screened propagators. **Correct** by $m^2_{D,q} \longrightarrow \kappa m^2_{D,q}$.

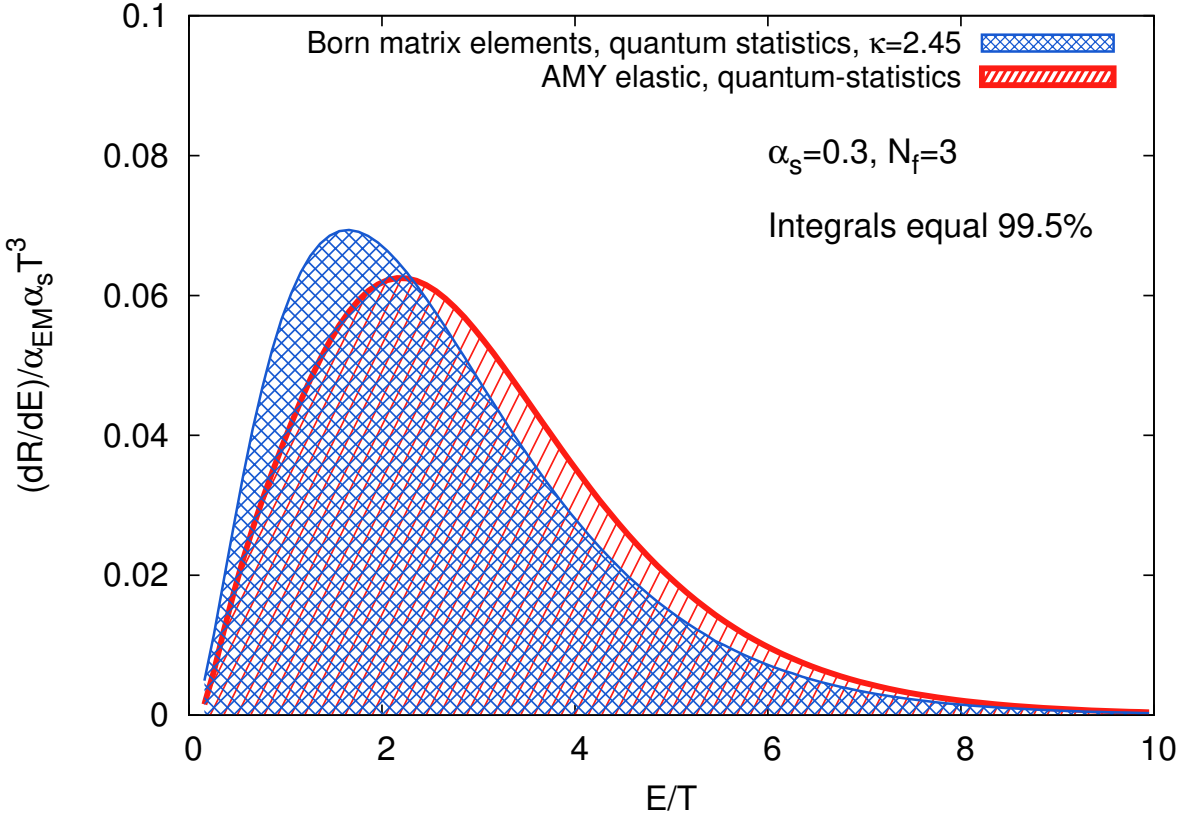
B) Classical statistics in BAMPS

Correct by factor C_{stat} in collision probability.

$$R \longrightarrow R = C_{\text{stat}}\mathcal{N}\int\int\int\int\dots|\mathcal{M}|^2 f(P)f(P')$$

Parameters κ, C_{stat} must be tuned to analytic rates (e.g. AMY)!

Correction of the Debye mass: fixing $\kappa = 2.45$, $C_{\text{stat}} = 0.84$



Moment	AMY/Born
0th	99.5 %
1st	112.5 %
2nd	121.9 %
3rd	128.1 %
4th	132.1 %

Box-test in BAMPS: precise reproduction of R^{γ}_{Born}!

- $R^{\gamma}_{\text{quantum, HTL}}$ from AMY (JHEP. 0111, 057 (2001))
- $R^{\gamma}_{\text{quantum, Born}}$ using Shen et al. (Phys. Rev.C91, 014908, (2015))
- Compare $R^{\gamma}_{\text{quantum, Born}}$ with $R^{\gamma}_{\text{classical, Born}}$: $C_{\text{stat}} = 0.84$

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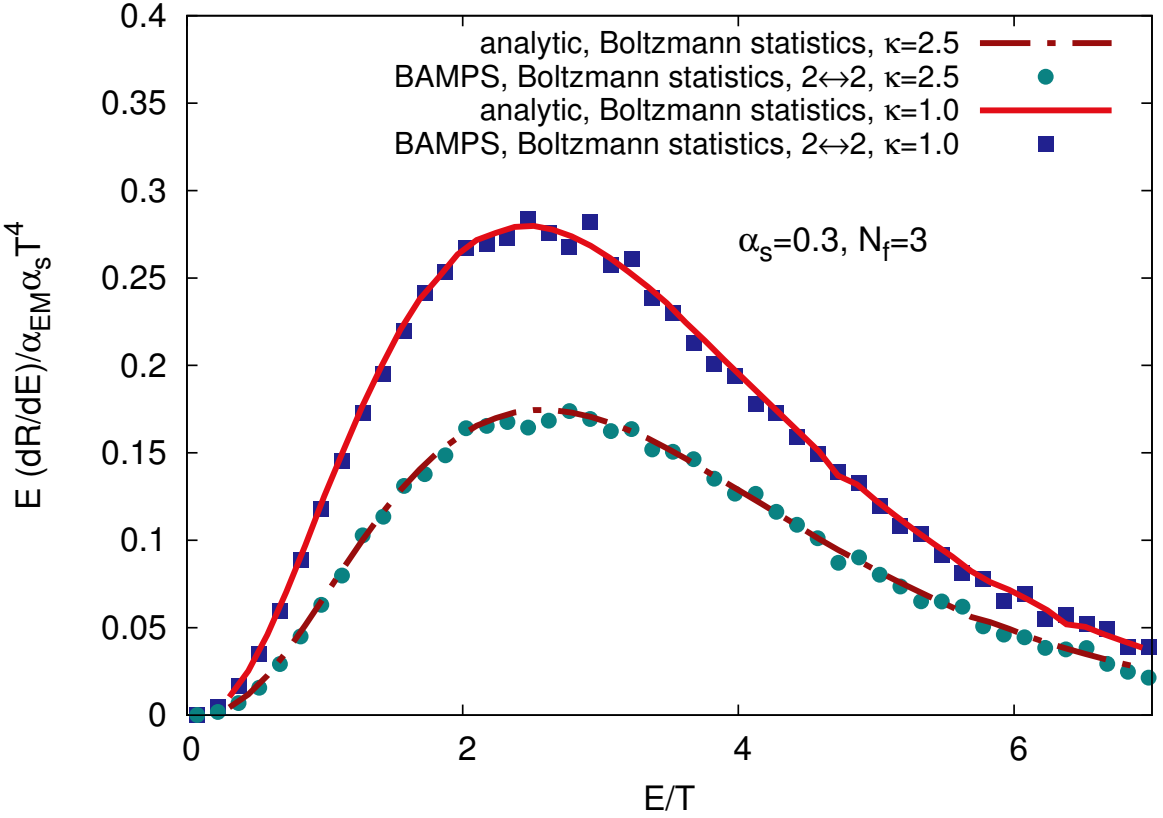

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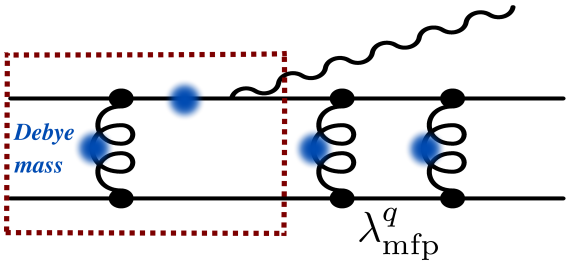
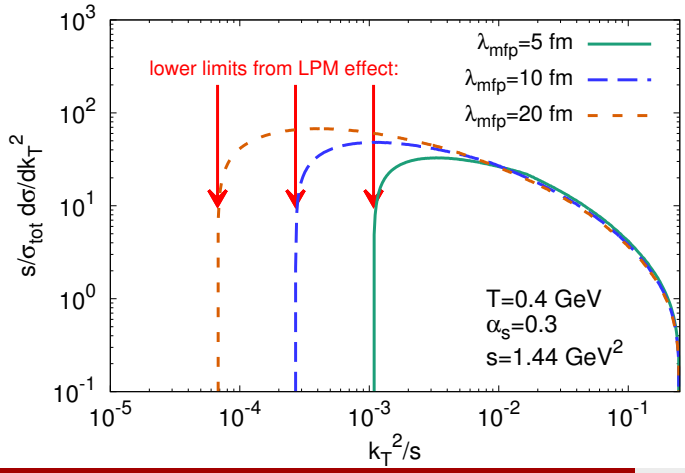
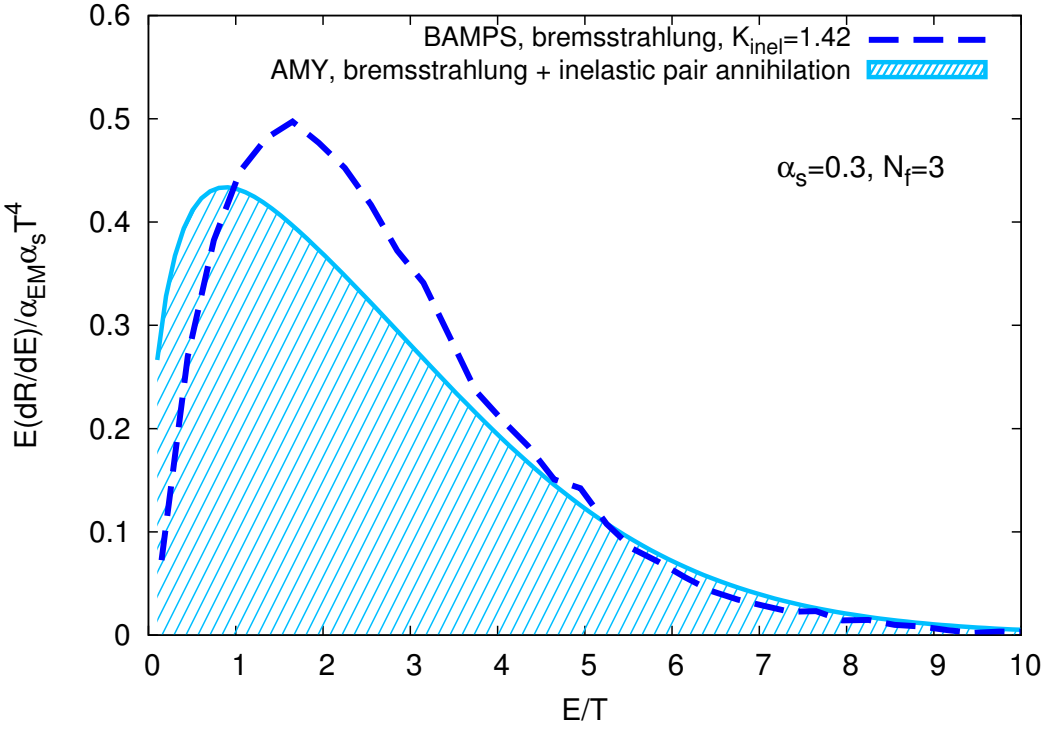
Exact bremsstrahlung in BAMPS



$$|\mathcal{M}_{2\to3}|^2 \to |\mathcal{M}_{2\to3}|^2 \Theta(\lambda^q_{\mathrm{mfp,process}} - \tau_f^\gamma)$$

- Exact matrix element $\mathcal{M}_{23}$
- Debye screening
- Specific $\lambda^q_{\mathrm{mfp,process}}$ from $2 \leftrightarrow 2$ scattering
- K-factor= 1.42

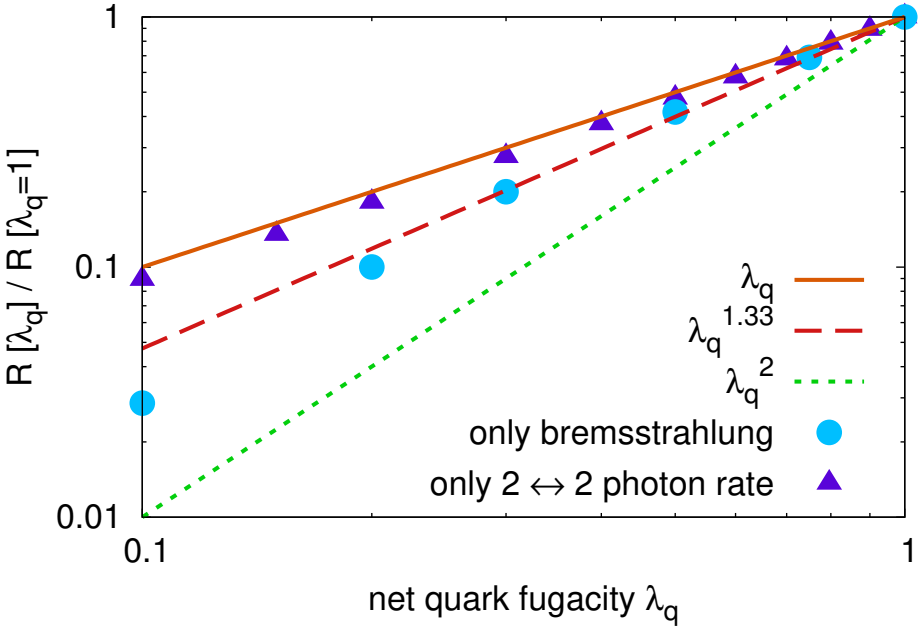


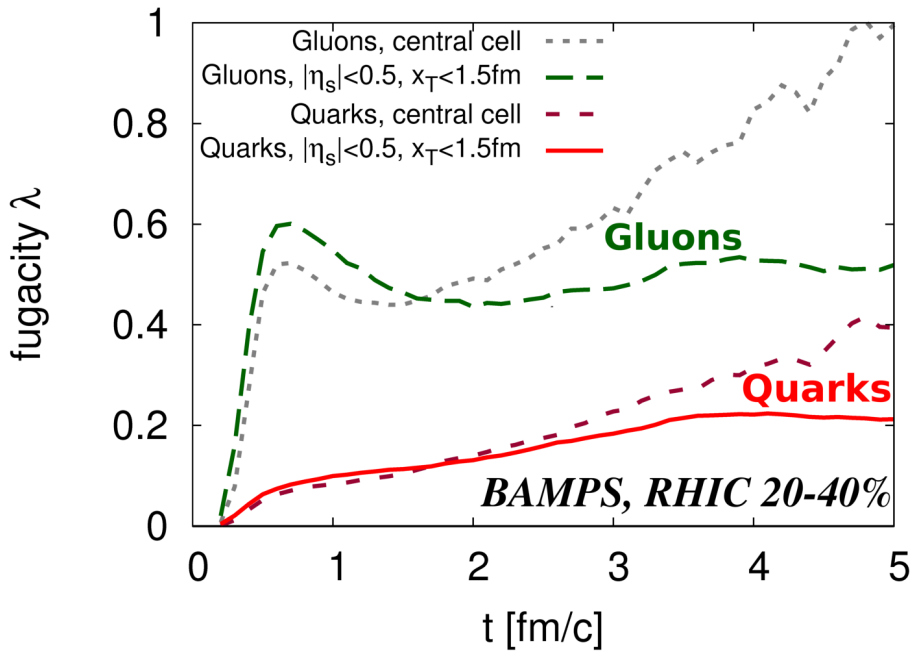


Fugacity dependence of rates

Elastic Rate: $R \sim \lambda_q \lambda_{\bar{q}/g} \cdot \ldots \cdot \frac{1}{(t - m_D(\lambda_q, \lambda_g))^2}$

Inelastic Rate: $R \sim \lambda_q^2 \cdot \ldots \cdot \frac{1}{(t - m_D(\lambda_q, \lambda_g))^2} \cdot \ldots \Theta\left(\lambda_{\mathrm{mfp}}(\lambda_{q/g}) - \tau_f\right)$





- Fugacities $t \lesssim 1$ fm/c ill defined
- At RHIC: emission mainly at $t \lesssim 3$ fm/c
- Photon yield expected to be $\sim 20\%$ of $\lambda_q = 1$ yield

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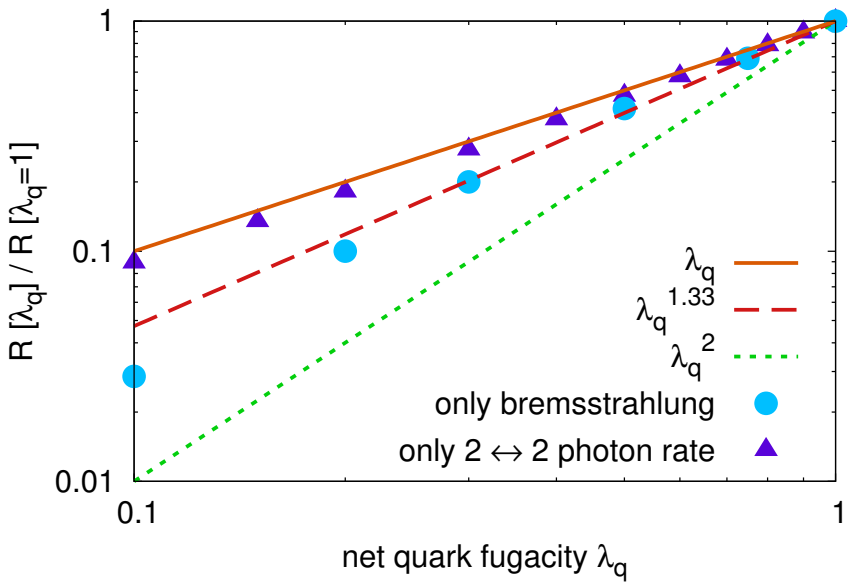

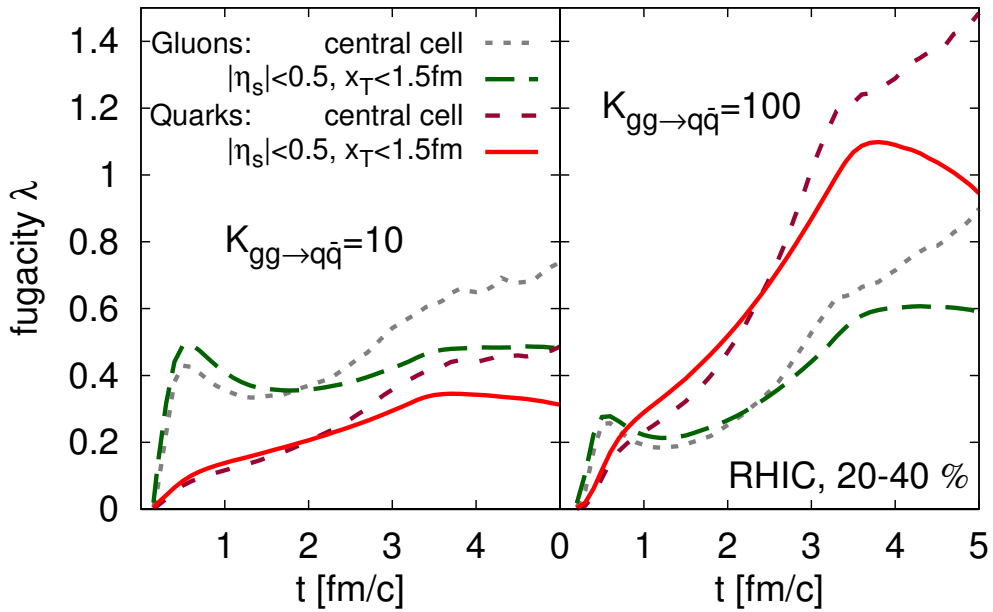

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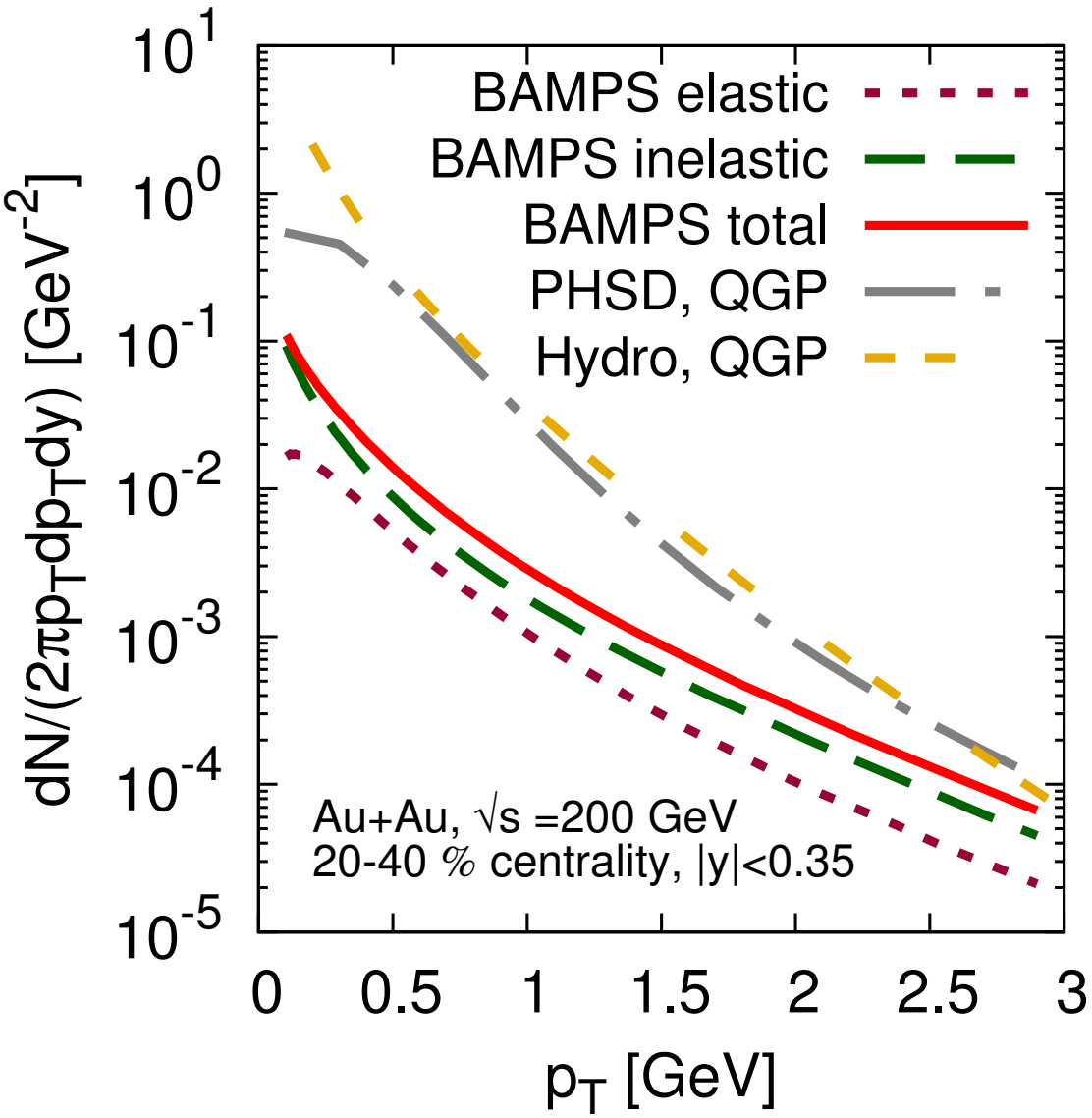
Results of realistic QGP simulations

- p_T-spectra from BAMPS
- Explanations: fugacities, initial condition
- Elliptic flow from BAMPS
- Explanations: jet-photon conversion and "leakeage" effect

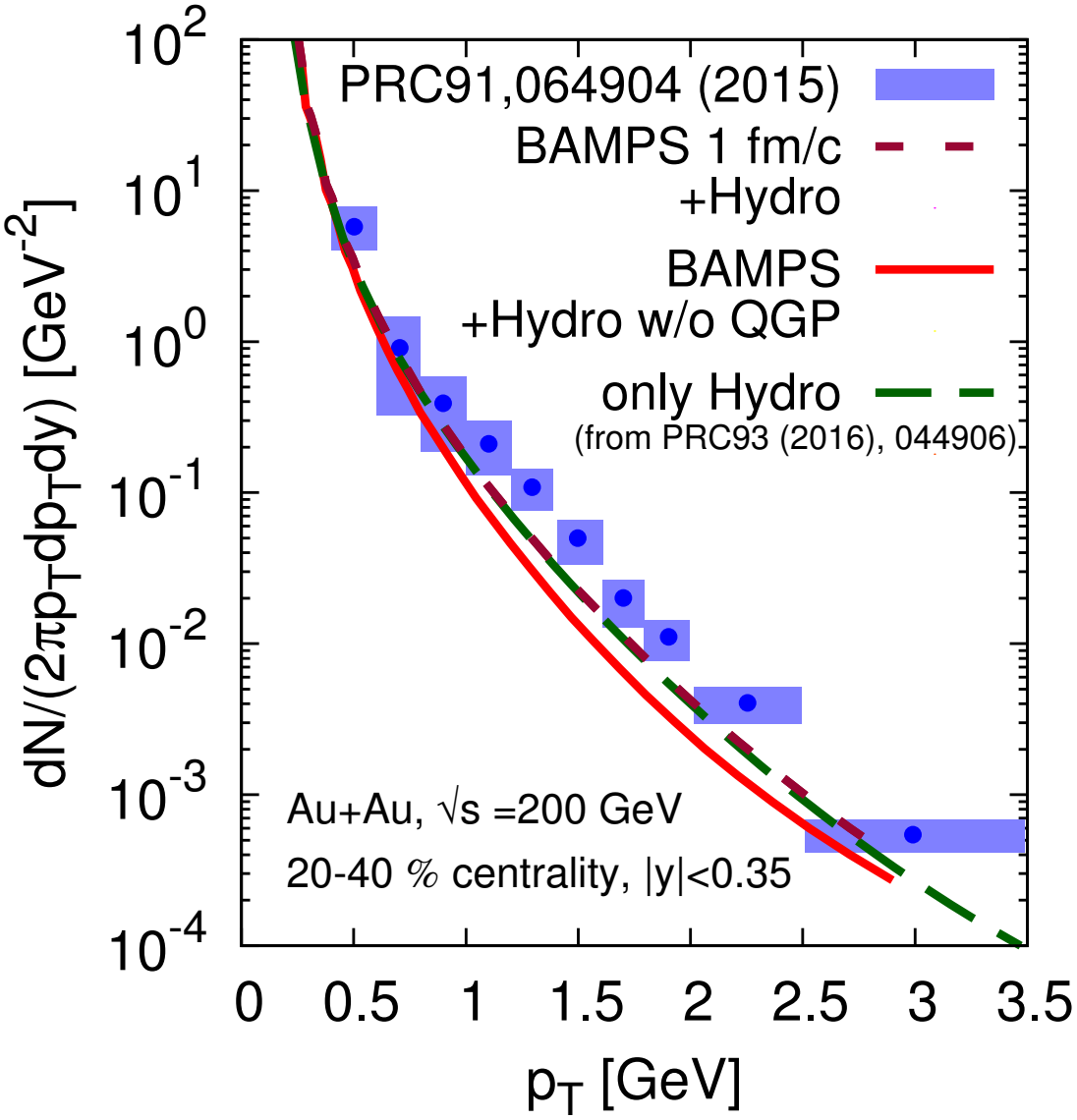
p_T-spectra of photons in the QGP from BAMPS



- Higher effective temperature in BAMPS due to harder initial condition
- 80% lower yield than models with $\lambda_q = 1$

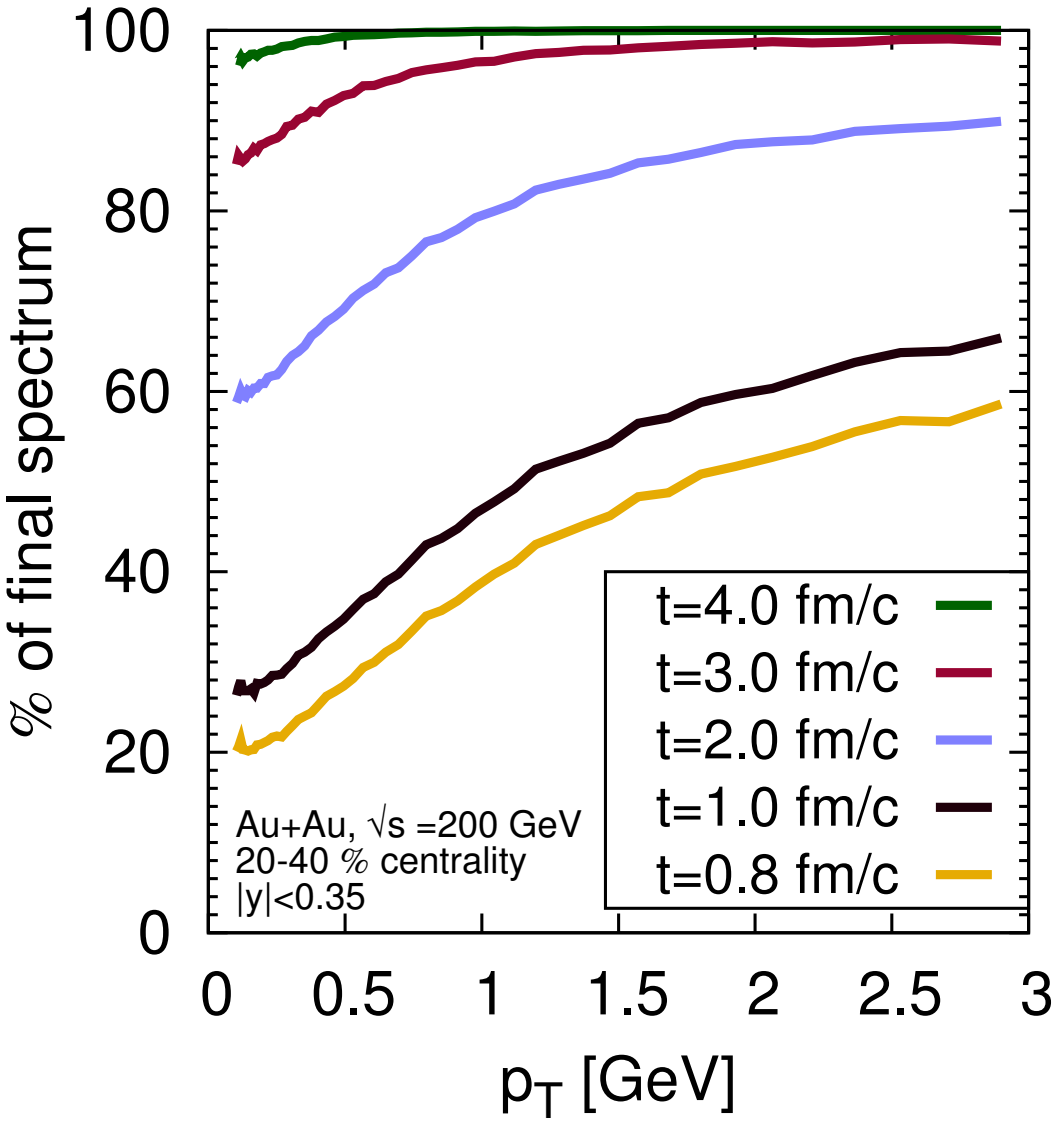
Hydro: Paquet et al, PRC 93 (2016), 044906
PHSD: Linnyk et al, PRC92 (2015), 054914

p_T-spectra of photons from BAMPS "added" to Hydro



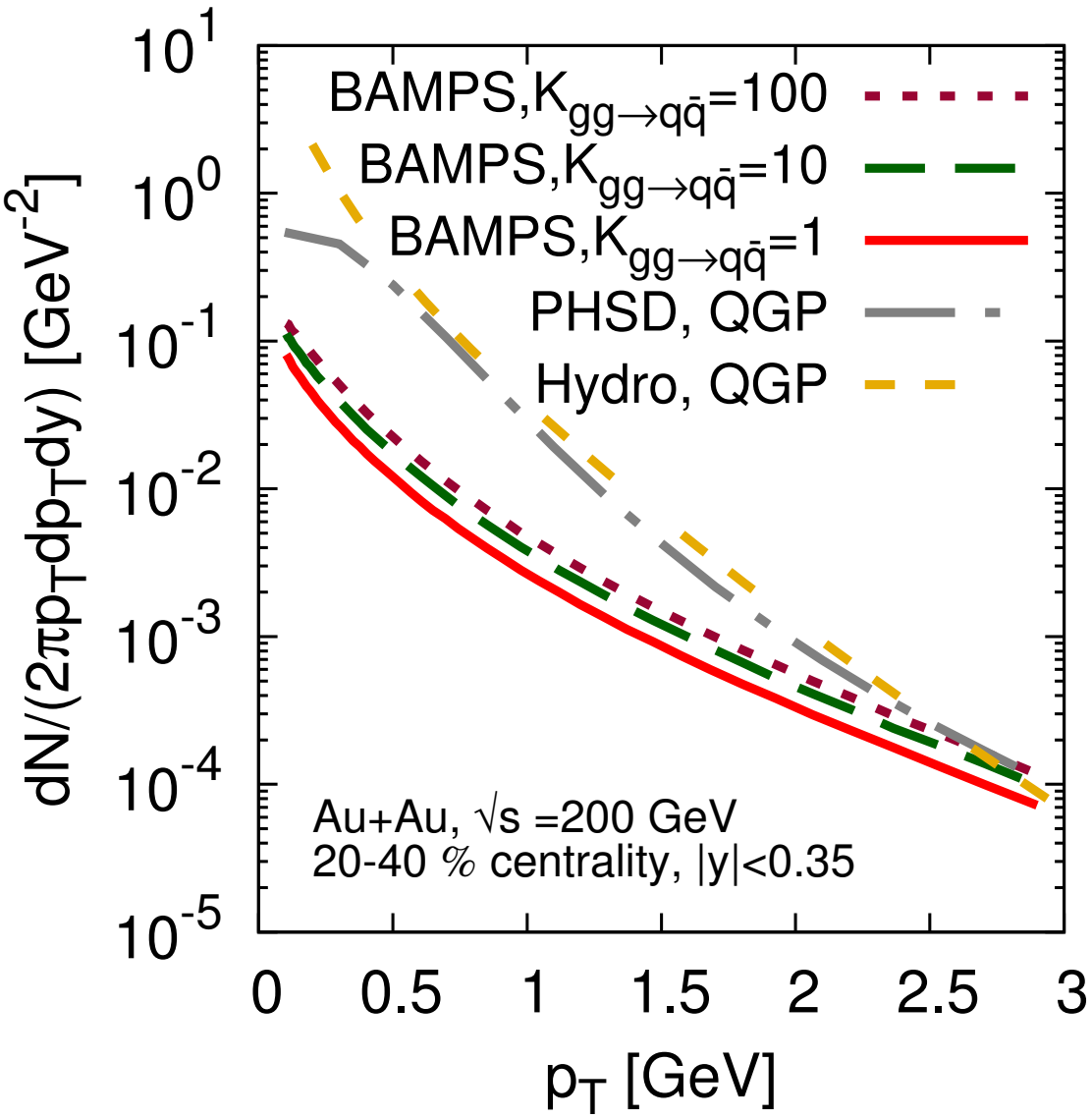
- *Toy plot*: Naive "adding" of contributions - no phase transition
- **Preequilibrium-phase** only small difference
- BAMPS QGP underestimates data

Time development of photon spectrum



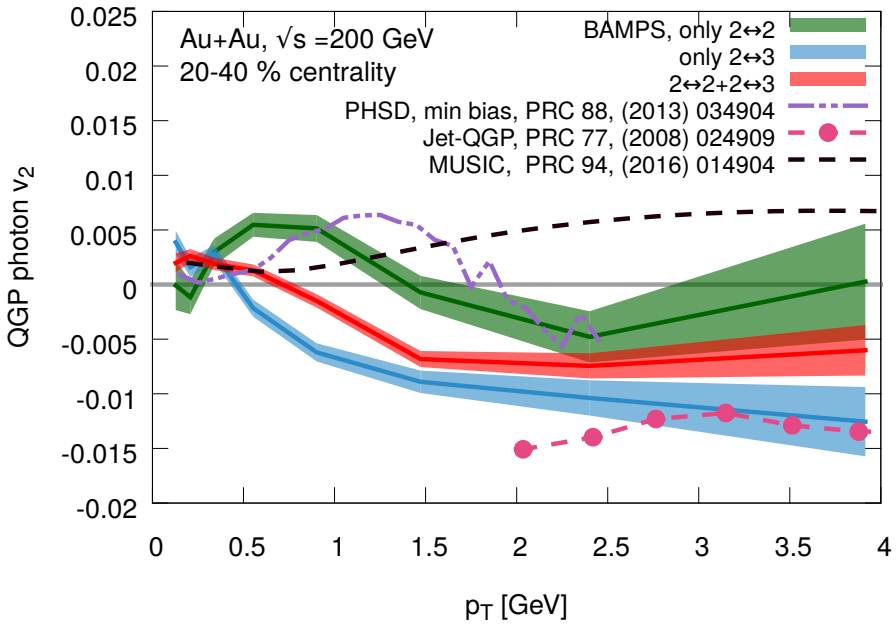
- Emission mainly between $2 - 3\ \mathrm{fm/c}$
- Only below $p_T \sim 1$ GeV: sensitivity to later times
- Remember: fugacity still small at early times

Photon spectra with different fugacity evolution

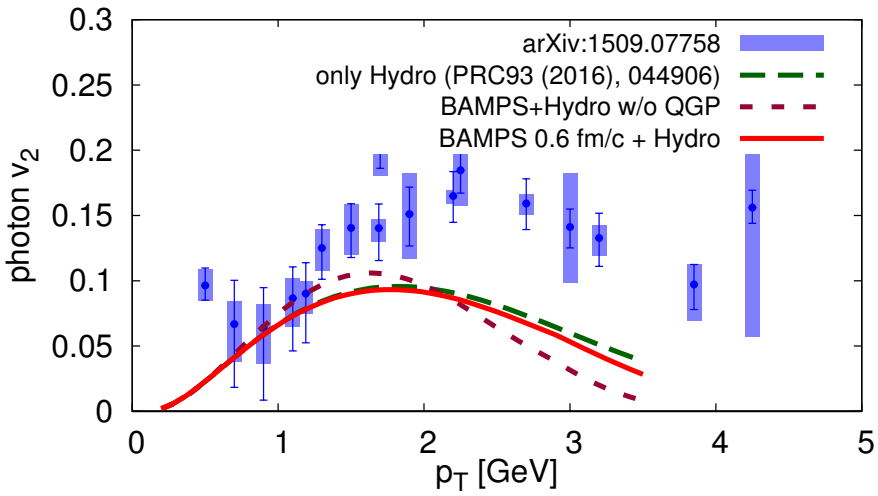


- *Artificial* chemical equilibration: increase $\sigma_{gg\to q\bar{q}}$ by $K = 10, 100$
- No large effect: photons emitted at *early times*, where $\lambda_q(K = 100) \sim \lambda_q(K = 1)$
- Initial condition should be improved

Elliptic flow $v_2(p_T)$ of photons in the QGP



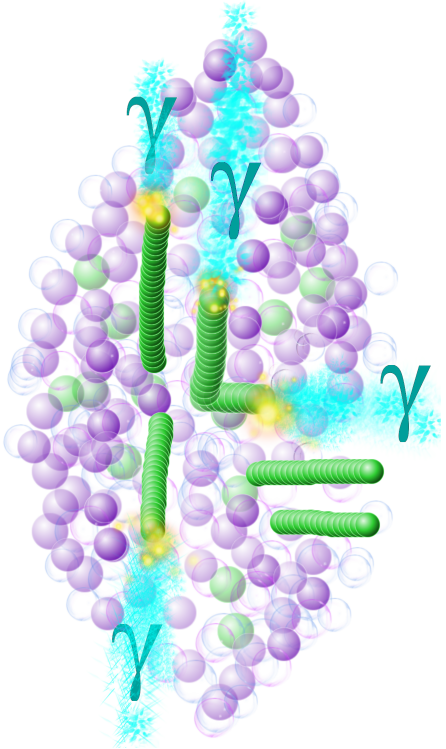
- low-p_T v_2 from "hydro-push"
- p_T-cuts of *parent-partons* reveal: negative flow-photons from **jet-photon conversion**
- negative v_2 **comparable** to other jet-QGP studies



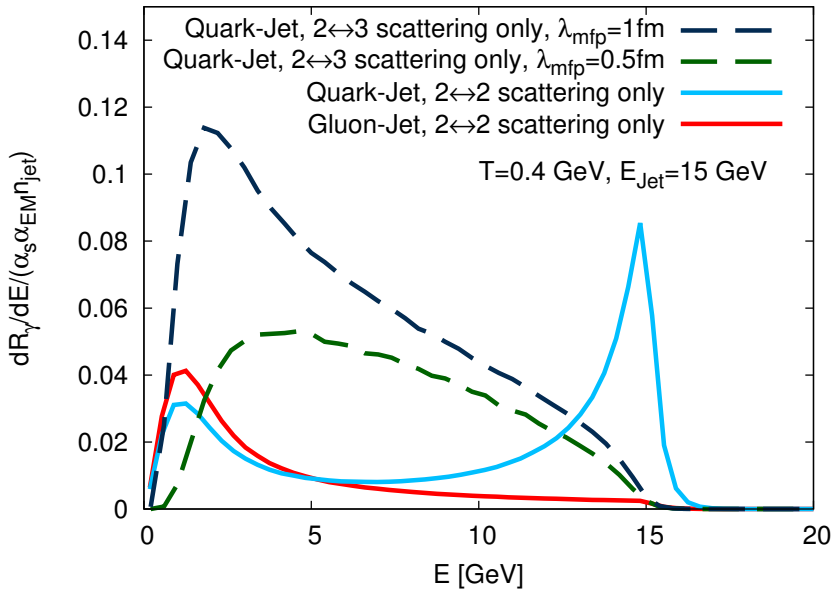
- *Toy-Plot:* **reweighting** of photon sources not enough - v_2-problem even more severe

Jet-contribution: negative elliptic flow $v_2(p_T)$ of photons

Purely geometric effect: "leakage effect":



Jet-Photon Conversion:



- $\longrightarrow$ $E^\gamma \approx E_{\mathrm{JET}}$ + very **collinear** emission
- $\longrightarrow$ BAMPS is **common** framework: hydro-bahavior and conversion photons
- $\longrightarrow$ **Leakeage:** verified in box-scenario

Conclusion and Outlook

- Successful implementation of LO photon production in a transport approach
- Smaller yield than hydro due to quark fugacities < 1
- Elliptic flow small, or negative due to jet-photon conversion

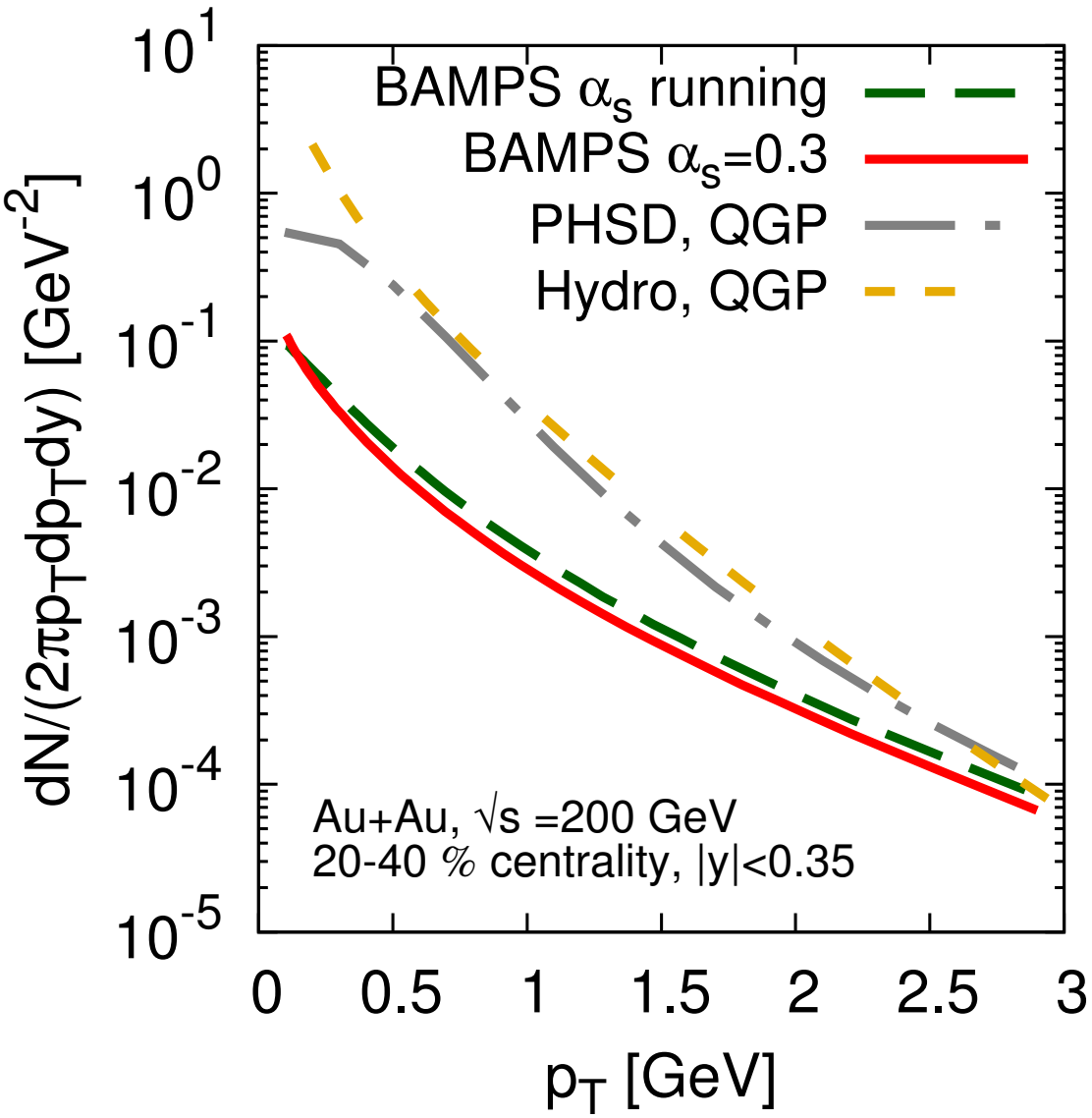
Outlook:

- Electric & Magentic field influence?
- Closer look to dileptons
- Improvements for initial state

读万卷书，不如行万里路，行万里路不如阅览世人无数。

APPENDIX

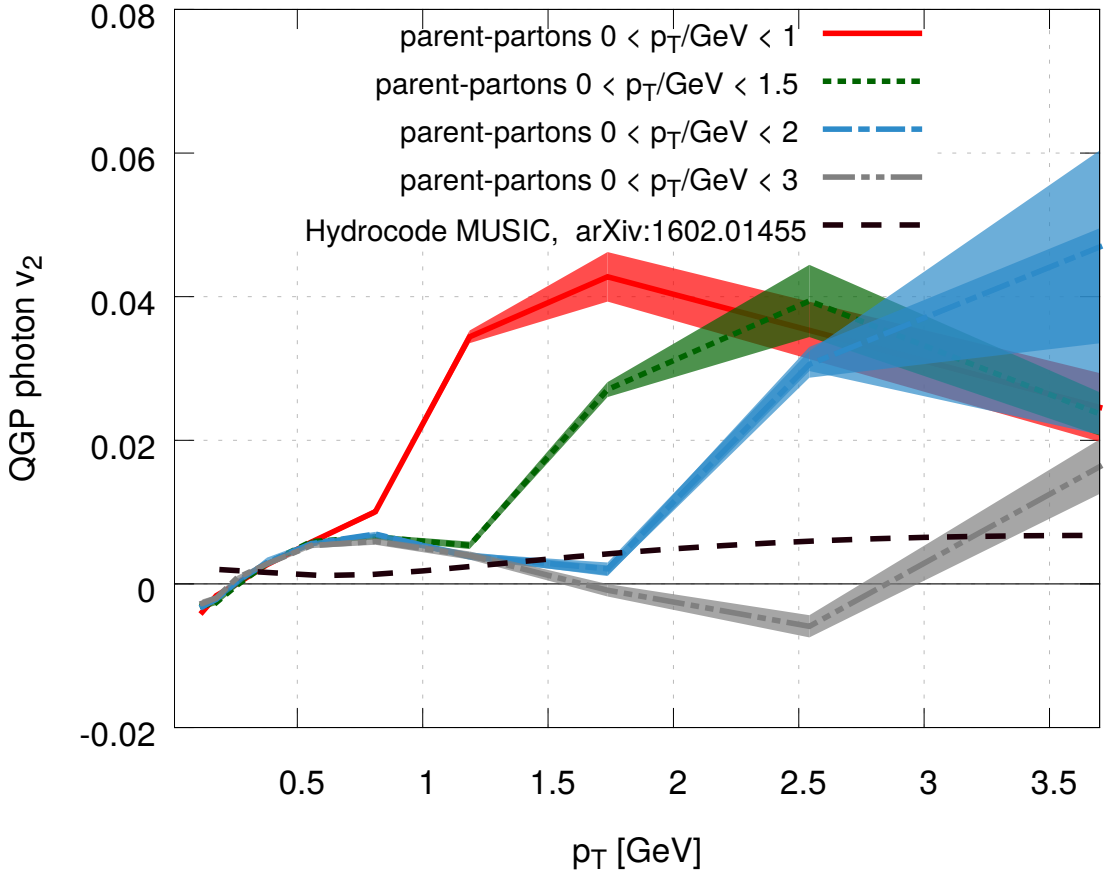
Effect of running coupling

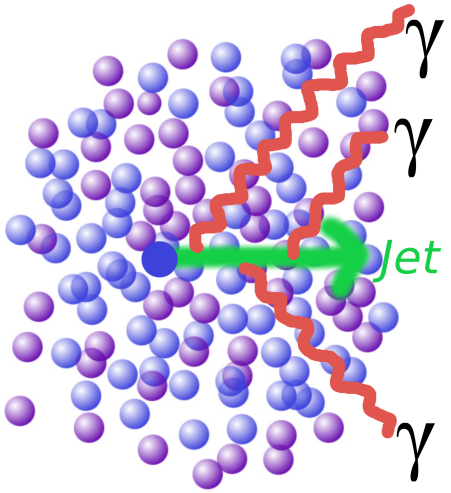


- $\alpha_s(s,t,u)$ at vertices
- $\alpha_s(T)$ within Debye-mass
- Roughly an increase by $\times 1.5$

Restrict parent-p_T: Jet-Photon Conversion

restrict $\max(p_T^1, p_T^2) < p_T^{\text{cut}}$:



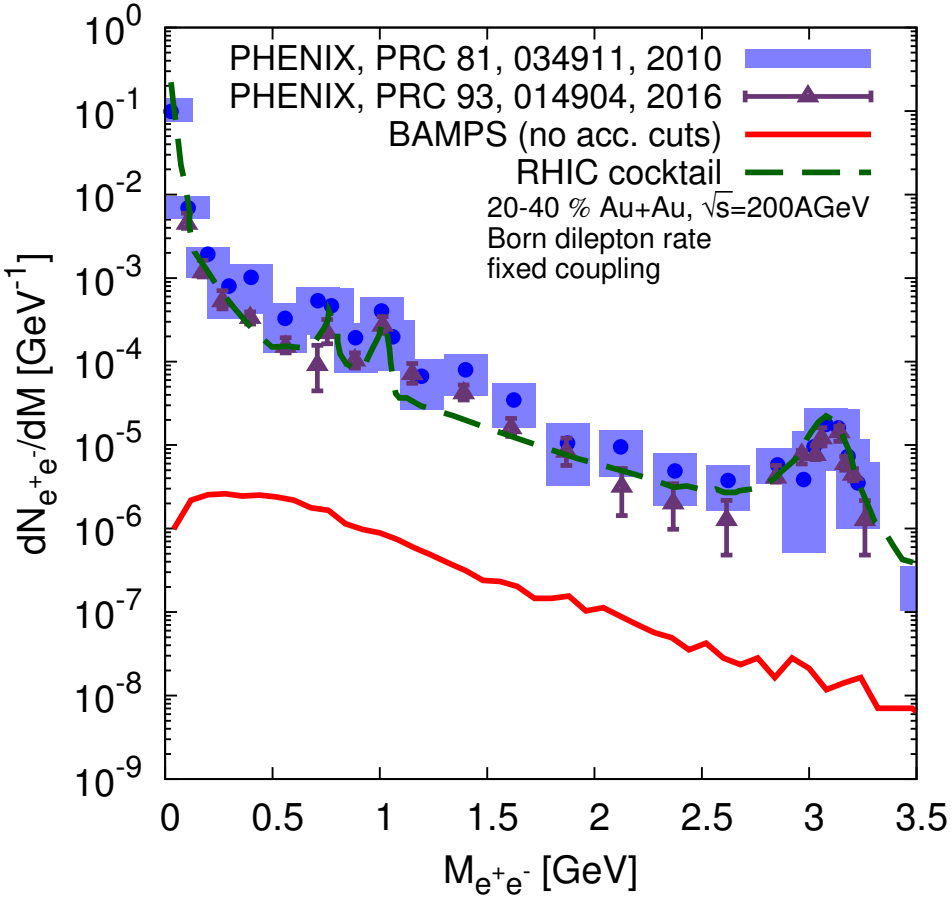


- negative flow originates from high-p_T-particles
- exclusively from Jet-Photon Conversion
- low-p_T from hydrodynamic push

$\Rightarrow$ leakage effect verified by box calculation

PRELIMINARY: Dileptons

- Simpler processes: $q\bar{q} \to e^+e^-$, only s-channel
- dR/dM spectra allow redshift-free temperature fitting



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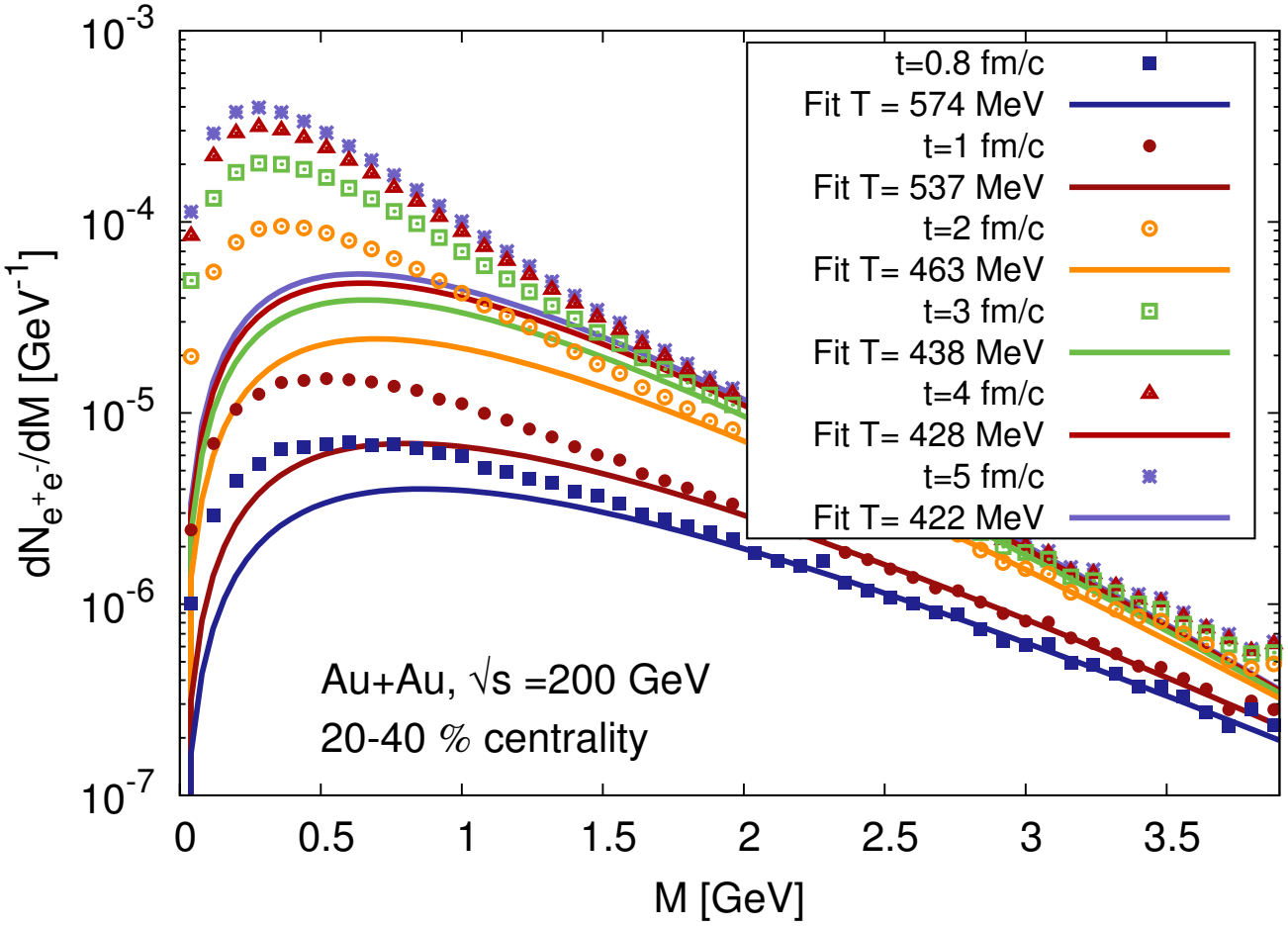
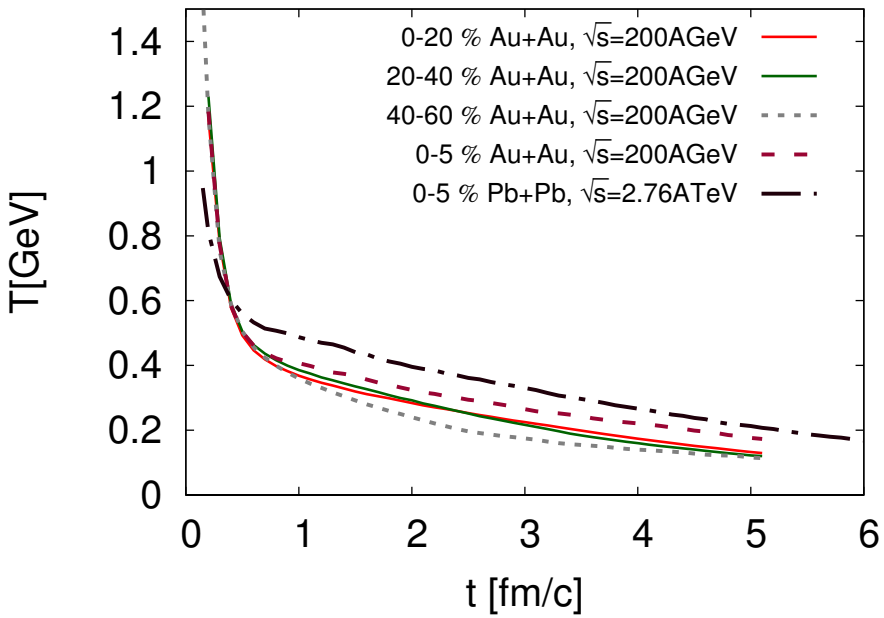
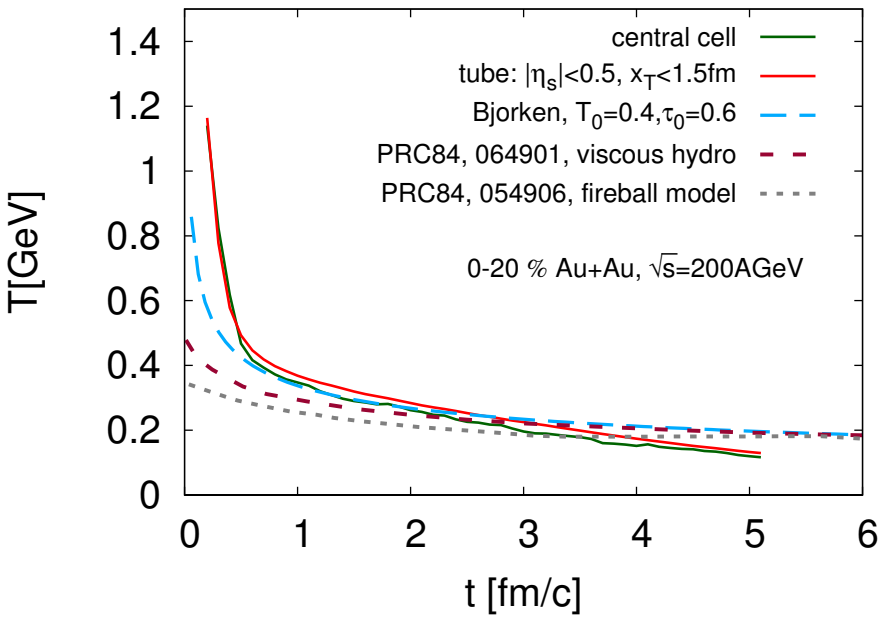


BAMPS - temperature evolution

Compare results from BAMPS to 2+1D viscous hydro code *MUSIC*

Reason for shallower slope: different effective temperature in BAMPS:





Elastic photon production *rate*

Hard momentum transfers $t > t^\star$:

$$R = \mathcal{N} \int \frac{\mathrm{d}^3 p}{2E_p} \int \frac{\mathrm{d}^3 p'}{2E_{p'}} \int \frac{\mathrm{d}^3 k}{2E_k} \int \frac{\mathrm{d}^3 k'}{2E_{k'}} (2\pi)^4 \delta^{(4)}(P + P' - K - K')$$
$$\times |\mathcal{M}|^2 f(P) f(P') \left(1 \pm f(K')\right)$$

+ Soft momentum transfers $t < t^\star$ (HTL propagators and vertices):

$$E_k \frac{\mathrm{d}^3 R}{\mathrm{d}^3 k} = -\frac{g^{\mu\nu}}{(2\pi)^3} \mathrm{Im}\Pi^R_{\mu\nu}(E_k, \vec{k})$$

The sum gives total (tree-level) HTL resummed rate:

approximately... (see A.M.Y. JHEP. 0111, 057 (2001) for correct $2 \leftrightarrow 2$ treatment)

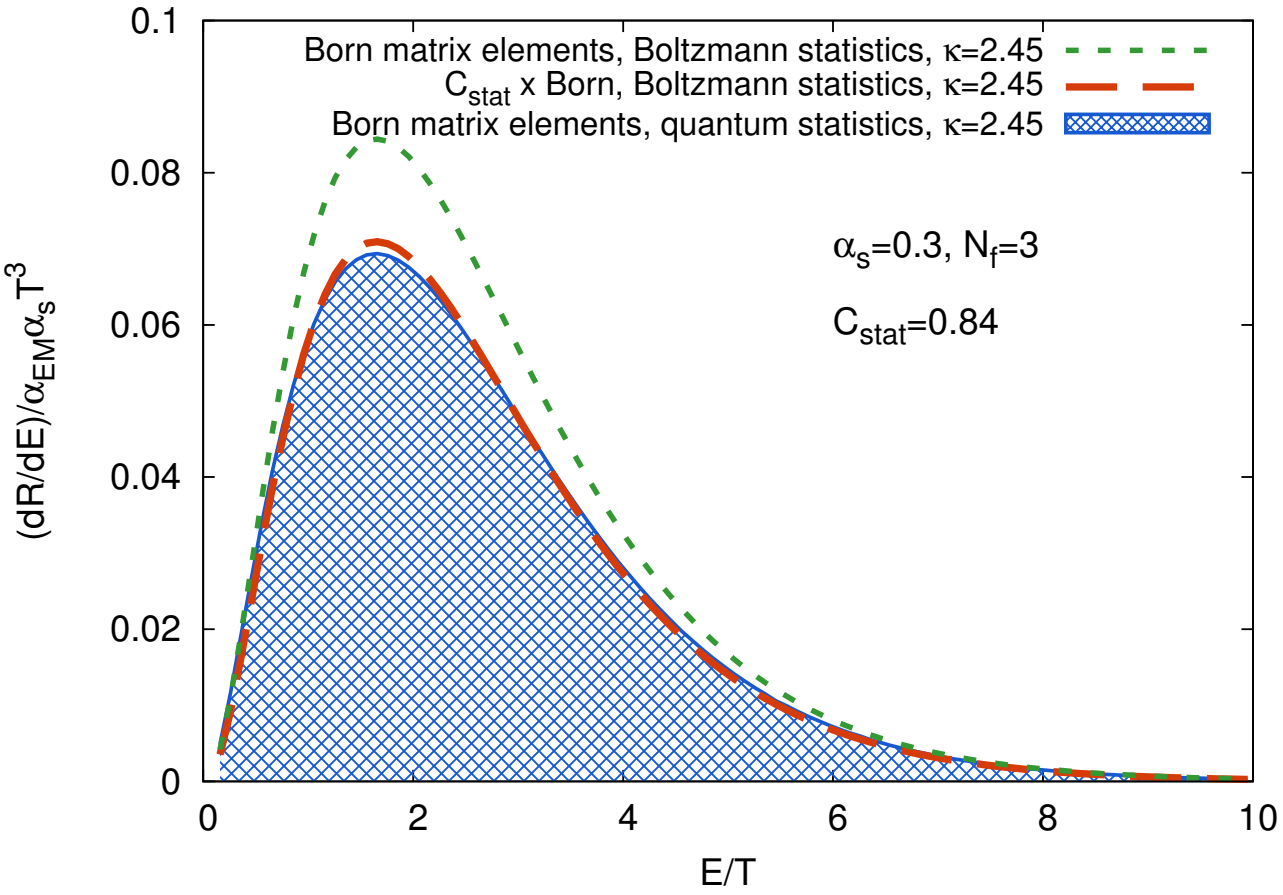
$$E \left.\frac{dR}{d^3 p}\right|_{\text{Compton+Annihilation}} = \left(\sum_i q_i^2\right) \frac{\alpha_{\text{EM}} \alpha_{\text{strong}}}{2\pi^2} T^2 e^{-E/T} \ln\left(\frac{2.912}{g_s^2} \frac{E}{T}\right)$$

Kapusta, Lichard, Seibert, Phys. Rev. D 44, 2774 (1991)

Correction for statistics of elastic photon production *rate*

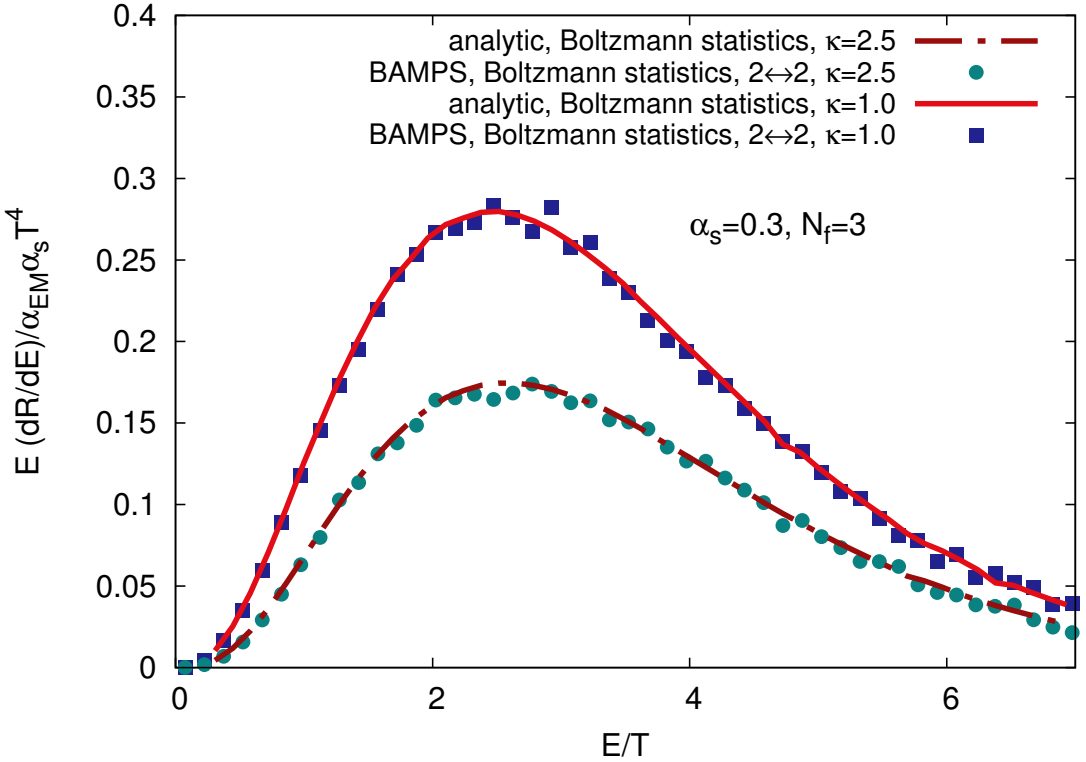
B) Statistics in BAMPS

Correct by overall factor C_{stat} in collision probability. $C_{\text{stat}} = 0.84$.



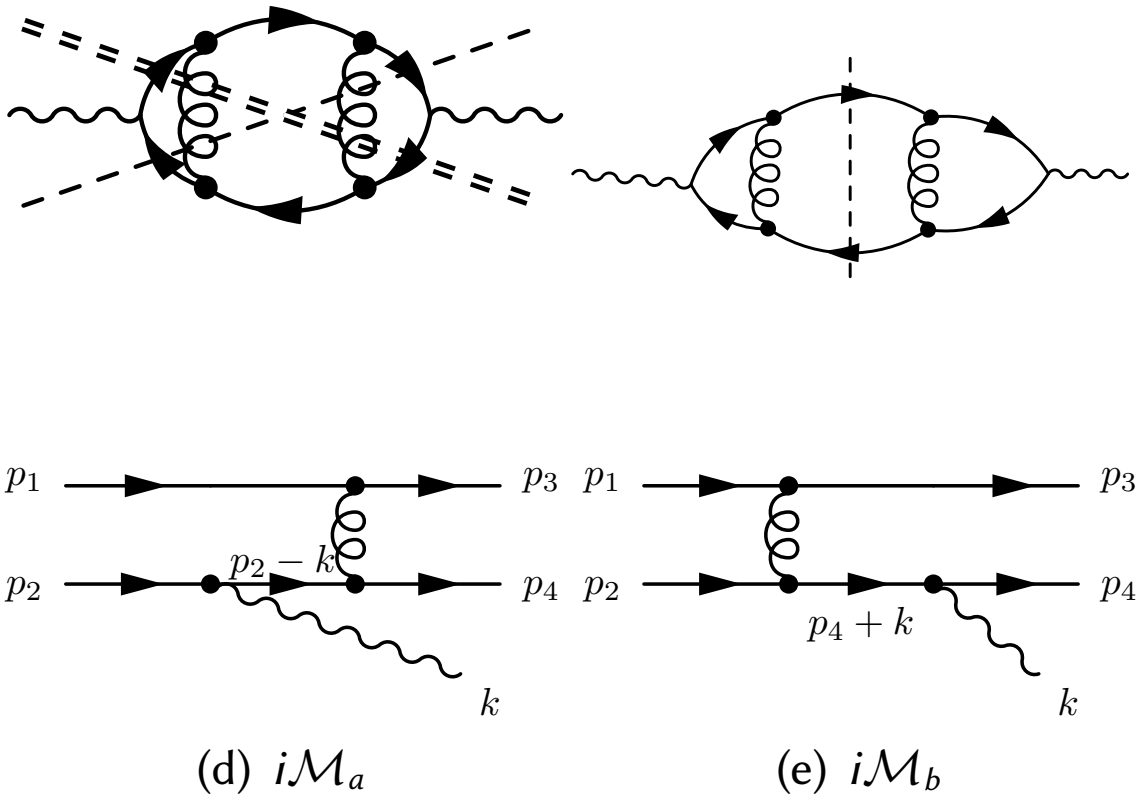
Final elastic matrix element in BAMPS:

$$|\mathcal{M}|^2_{\text{effective}} = C_{\text{stat}} \left[128 \cdot \frac{16}{3}\pi^2 \alpha\alpha_s \left(\frac{s^2+st}{(s+\kappa m^2_{D,q})^2} + \frac{s^2+st}{(u-\kappa m^2_{D,q})^2} \right) + 24 \cdot \frac{128}{9}\pi^2 \alpha\alpha_s \left(\frac{tu}{(t-\kappa m^2_{D,q})^2} + \frac{tu}{(u-\kappa m^2_{D,q})^2} \right) \right]$$



Photon production: higher order loops

Photon rate at order $\mathcal{O}(e^2 g_s^2 T^4)$ obtained via γ-self energy.



(d) $i\mathcal{M}_a$ (e) $i\mathcal{M}_b$

Figure: The diagrams we use in BAMPS.

e.g. P. Arnold, G. D. Moore, and L. G. Yaffe, JHEP. 0111, 057 (2001)

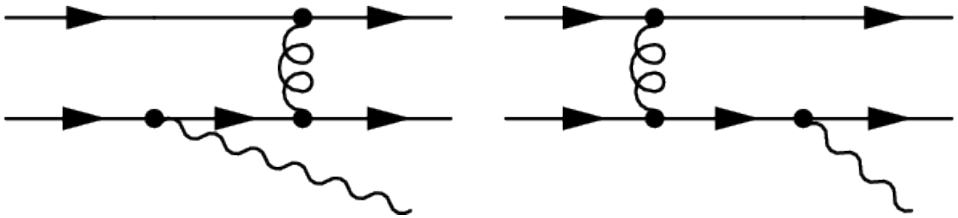
Photon Bremsstrahlung processes: The exact matrix element

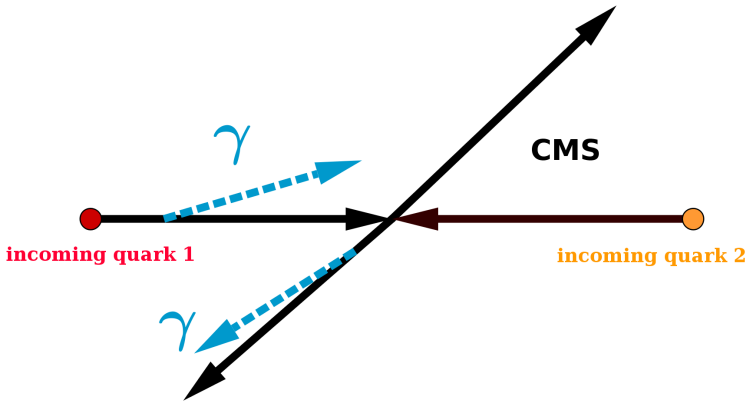
Using the MATHEMATICA package FEYNCALC 8.2.0:

$$A \equiv 2(25) + m_{D,q}^2$$
$$B \equiv 2(45) + m_{D,q}^2$$
$$C \equiv 4(45) + m_{D,q}^2$$
$$D \equiv (35)B^2 - 2(34)A(2(25) - B - m_{D,q}^2)$$
$$E \equiv (23)A((25)C + (45)(-B - m_{D,q}^2)) + (24)A(2(34)A + (35)(A+B)) + (25)D$$
$$F \equiv (24)A(A+B) + (25)B^2$$
$$G \equiv (23)A((24)B + (45)A) + (34)F$$
$$H \equiv -2(23)B + (34)(-B - m_{D,q}^2) + (35)m_{D,q}^2$$
$$J \equiv (45)H + (24)(35)B + (25)((34)C + 2(35)(45))$$
$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\frac{2}{9}Q_{EM}^2 g^4 128 \frac{A((12)J - 2(13)(24)(45)A) + (14)E + (15)G}{A^2B^2(2(24) + 2(25) - 2(45) + m_{D,g}^2)^2}$$

(we defined the scalar product of 4-vectors $(ij) \equiv p_i \cdot p_j$)

Photon Bremsstrahlung processes: Some details





Useful coordinates for radiated photon:

Reference: incoming quark 1 $p_z > 0$

y: rapidity wrt incoming quark 1, $y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$

$k_\perp$: transverse momentum of photon wrt to p_z

$q_\perp$: gluon momentum transfer

φ: $\measuredangle(\vec{q}_\perp, \vec{k}_\perp)$

- We use the **exact** pQCD computation of $|\mathcal{M}_{\text{brems}}|^2$ with screened quark and gluon propagators
- Inelastic pair annihilation neglected

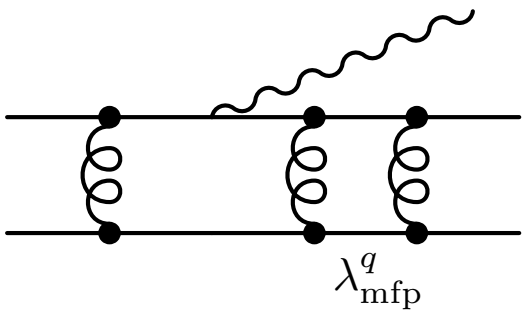
Inelastic cross section for radiated photons

Exact matrix element computed, coordinate transformation from $P_{\text{in }1}, P_{\text{in }2}, P_{\text{out }1}, P_{\text{out }2}, K \rightarrow$ integrate cross section:

$$\sigma_{23} = \frac{1}{2s} \int_{p_1'} \int_{p_2'} \int_{p_3'} \int_{p_1} \int_{p_2} |\mathcal{M}_{12\to 1'2'3'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2' - p_3')$$
$$= \frac{1}{256\pi^4 s} \int \mathrm{d}^2 q_\perp \int \mathrm{d}^2 k_\perp \int \mathrm{d}y \int \mathrm{d}\phi \, |\mathcal{M}_{12\to 1'2'3'}|^2 \, \mathcal{J}(k_\perp, q_\perp, y, \phi)$$

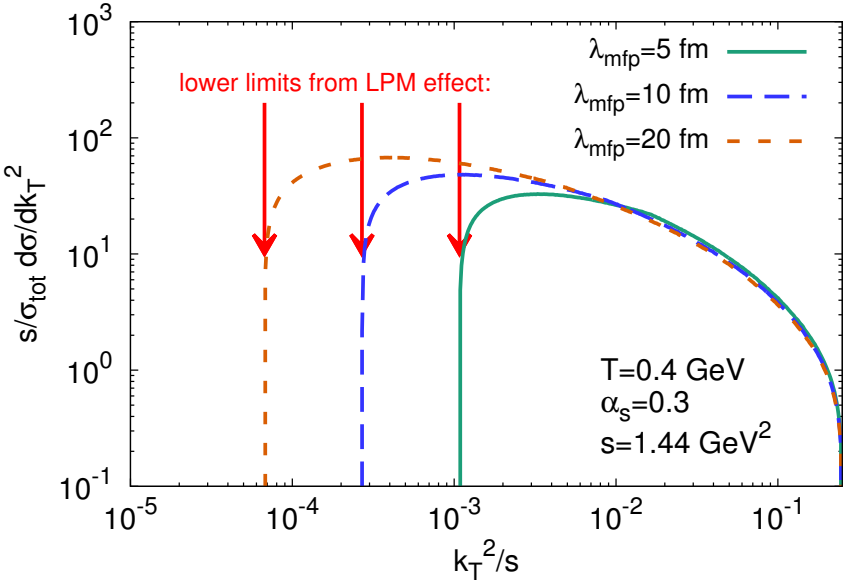
- For each particle pair in cell: compute σ_{23}
- $|\mathcal{M}_{12\to 1'2'3'}|^2 (P_{\text{in }1}, P_{\text{in }2}, k_\perp, q_\perp, y, \phi)$
- *VEGAS* integration algorithm
- If collision happens: sample outgoing momenta with *Metropolis*-algorithm according to $|\mathcal{M}|^2$
- Numerically very demanding, needs Lookup-Tables.

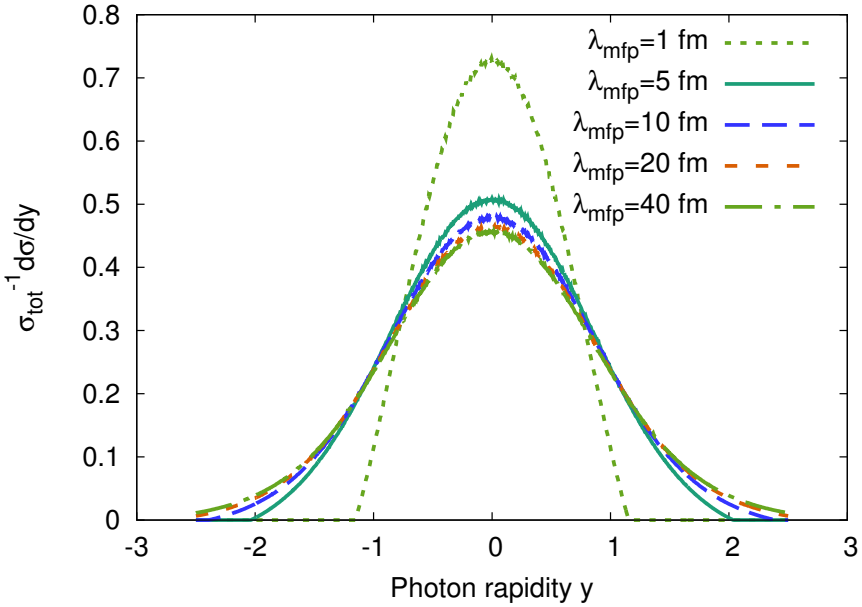
Interference effect: LPM



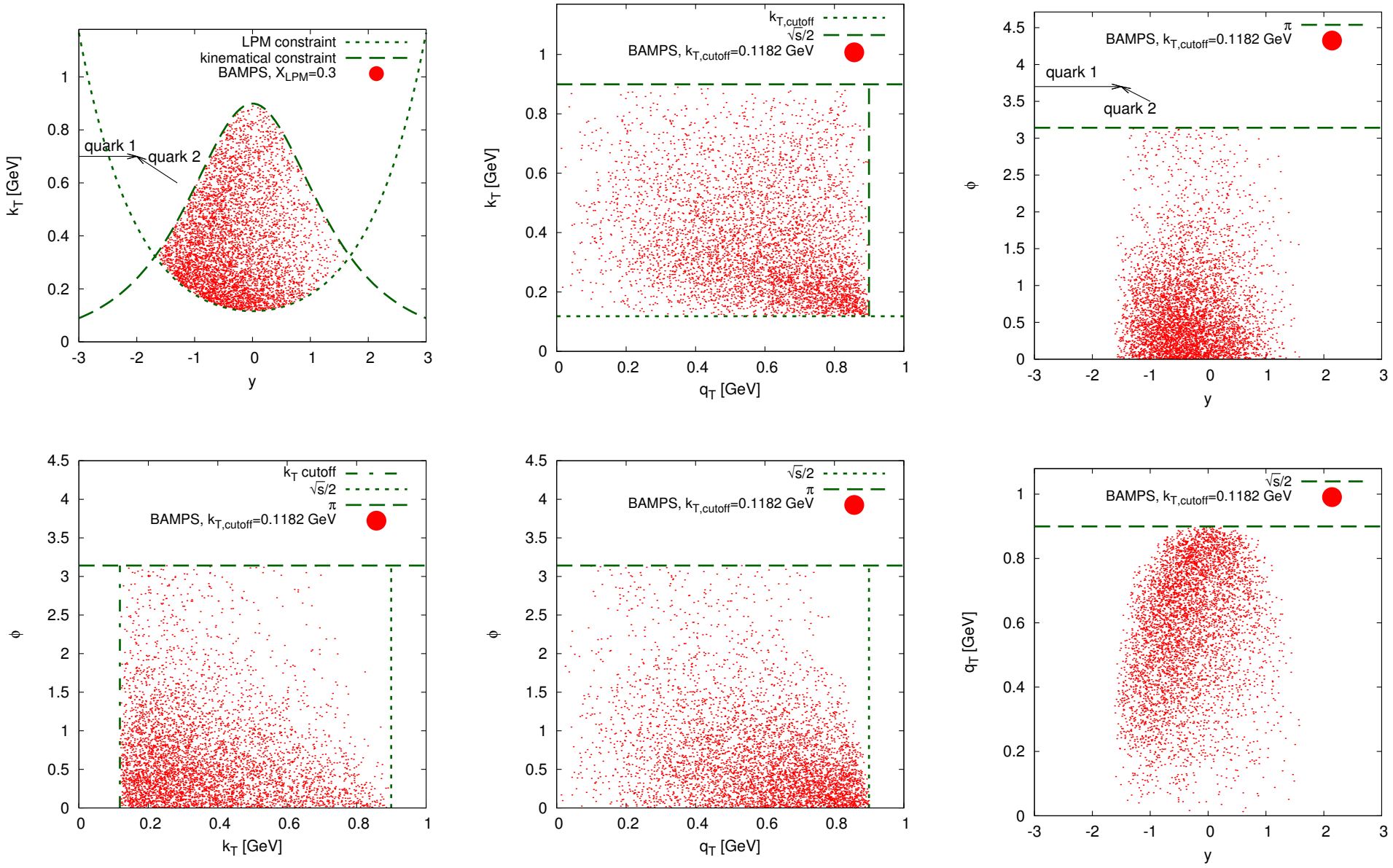
$$\lambda^q_{\mathrm{mfp},qq\to qq} = \left(n_q \left\langle \sigma(s) v_{\mathrm{rel}} \right\rangle_{\mathrm{therm}}\right)^{-1}$$

$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \Theta(\lambda_{\mathrm{mfp},qq\to qq} - \text{formation time})$$





Numerical sampling of the outgoing photon momenta



Comparison of bremsstrahlung in BAMPS with full rate

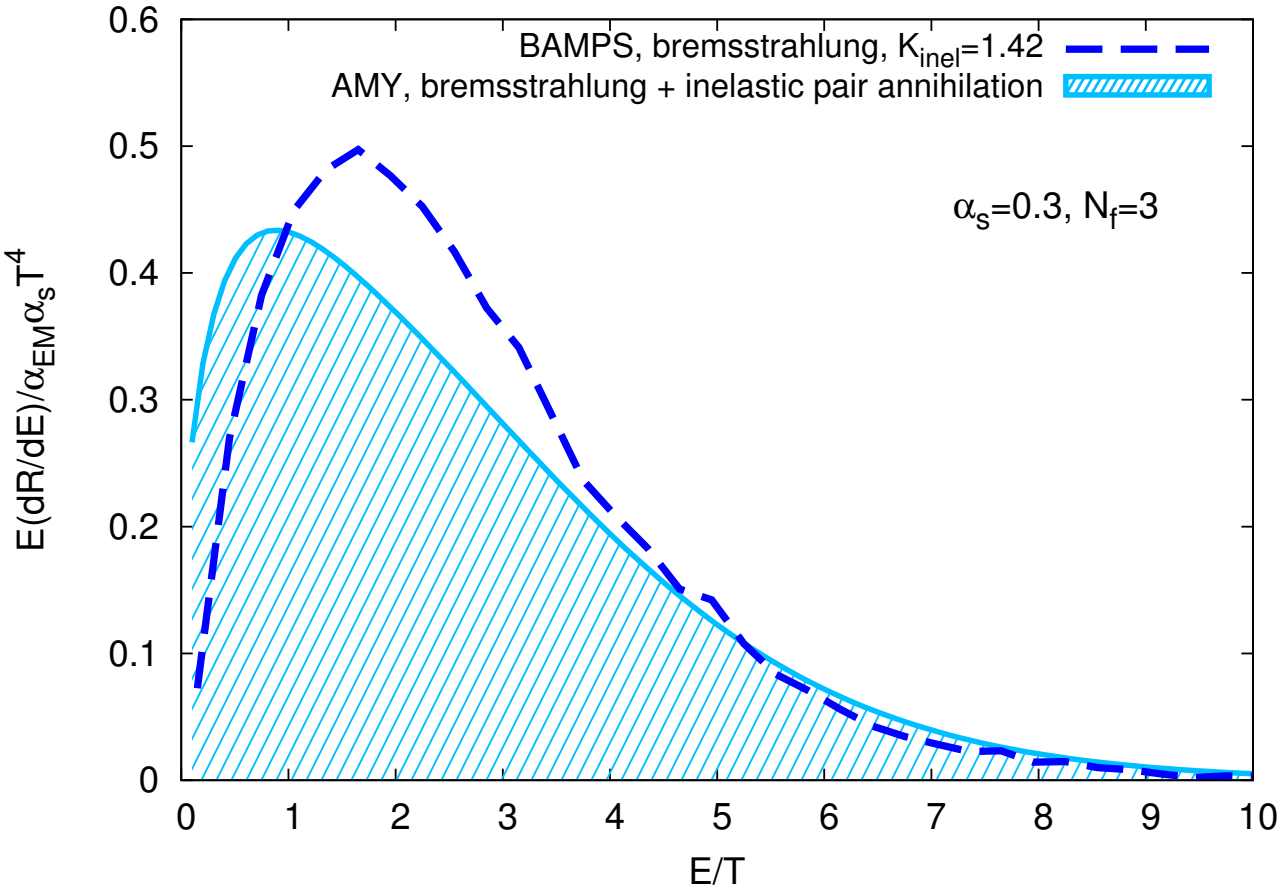
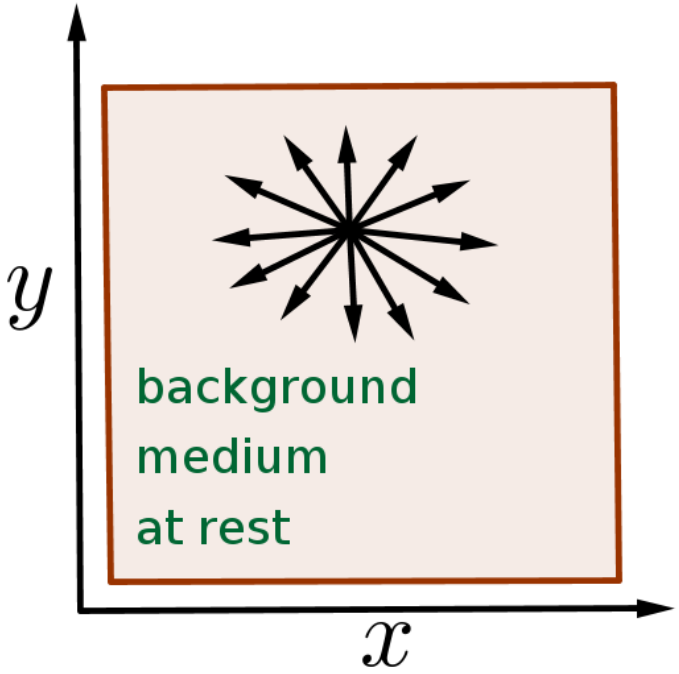


Figure: Inelastic rate from AMY compared to BAMPS. The integrals are equal, if we use a factor $K_{\text{inel}} = 1.42$. Second moments agree within 1%.

Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario A: square geometry, no flow, Jets



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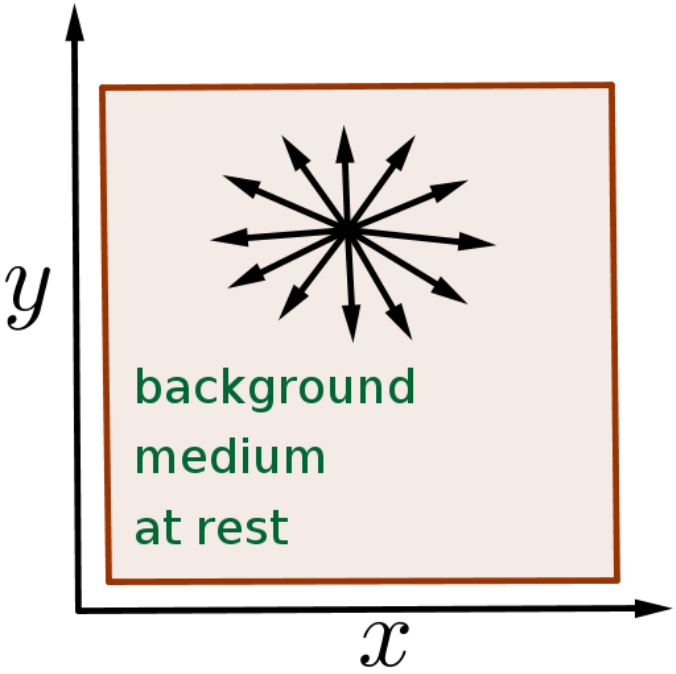
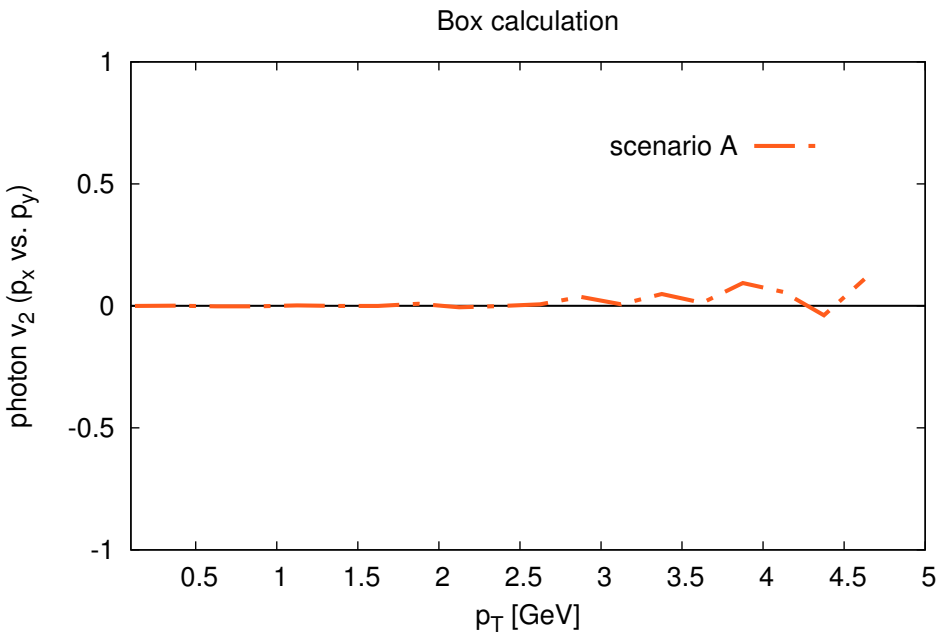
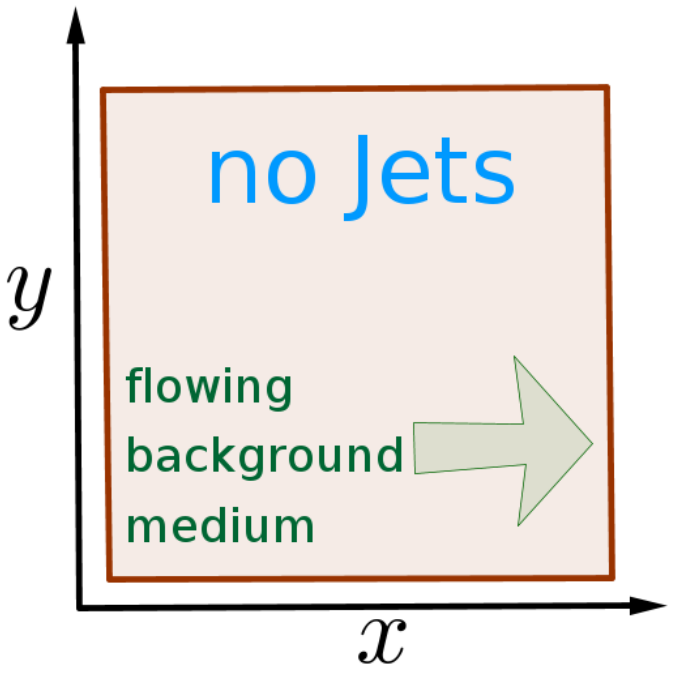




Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

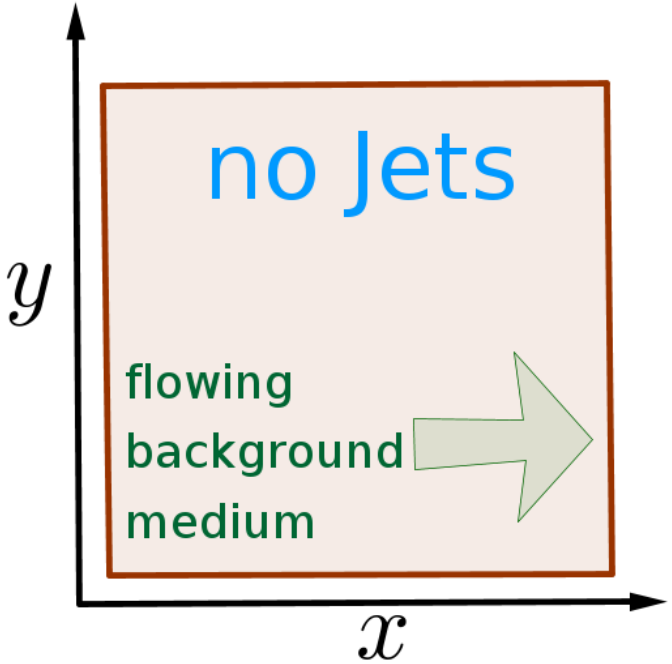
Scenario B: square geometry, flow in x-direction, no Jets

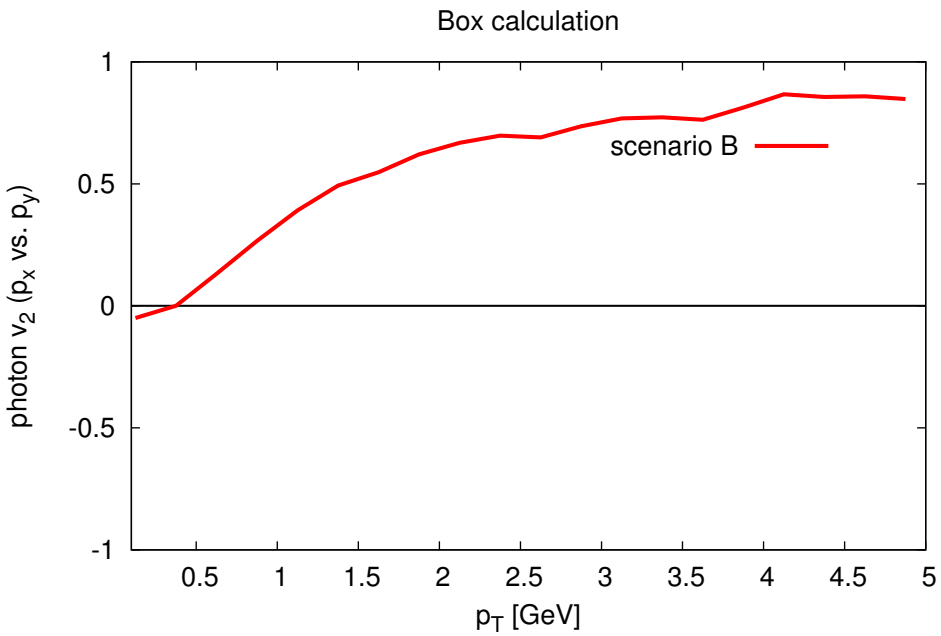


Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario B: square geometry, flow in x-direction, no Jets

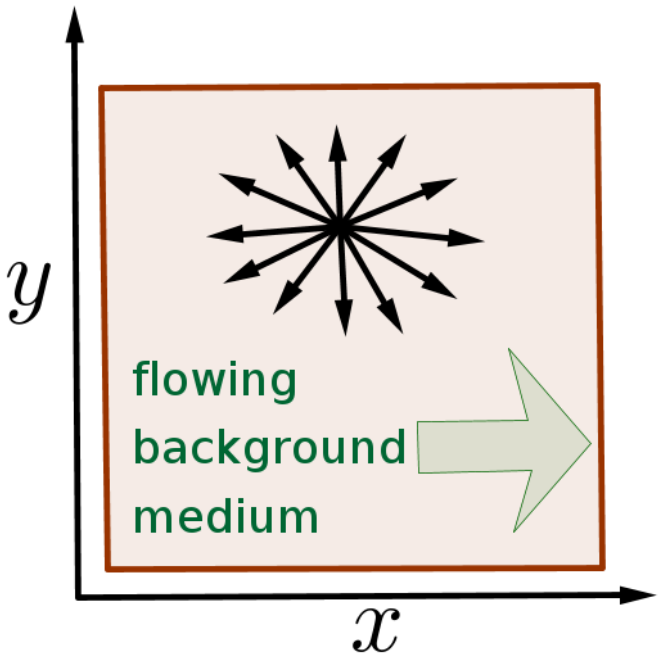




Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

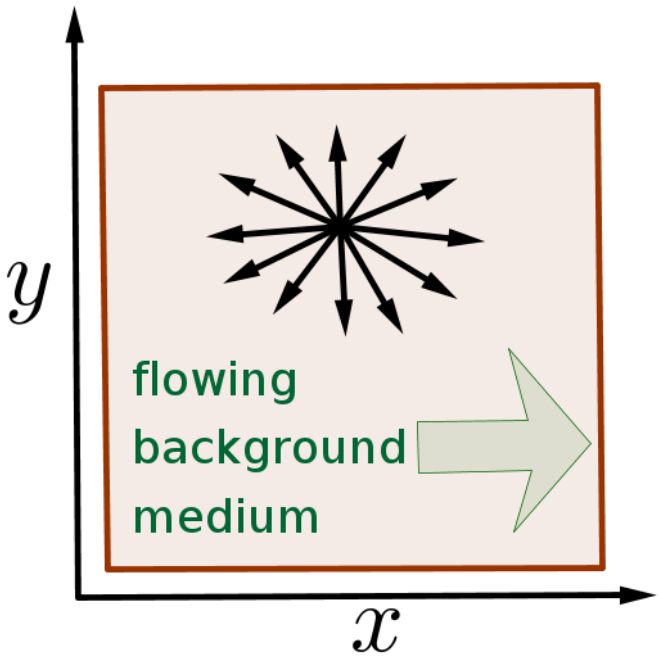
Scenario C: square geometry, flow in x-direction, Jets

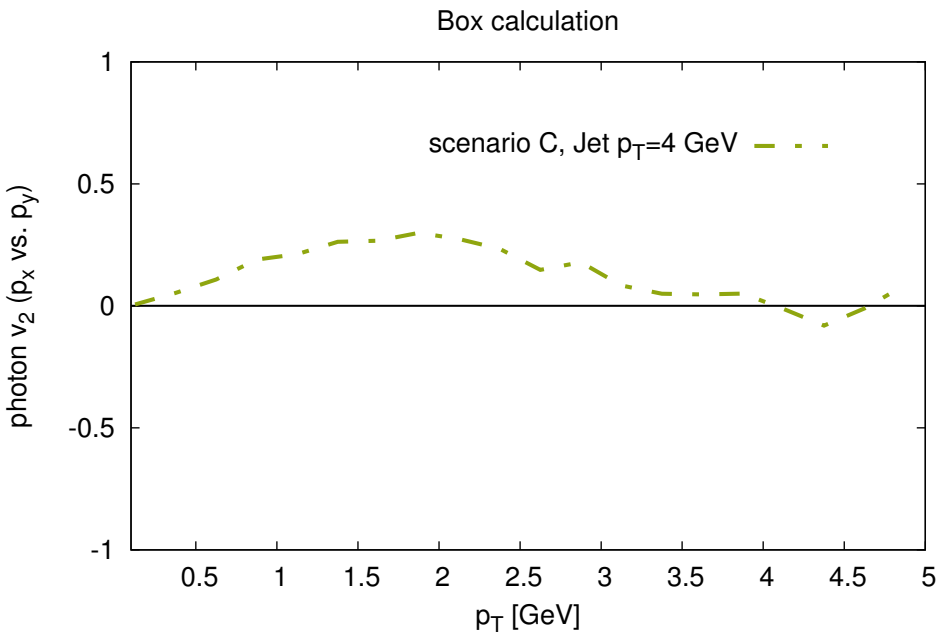


Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario C: square geometry, flow in x-direction, Jets

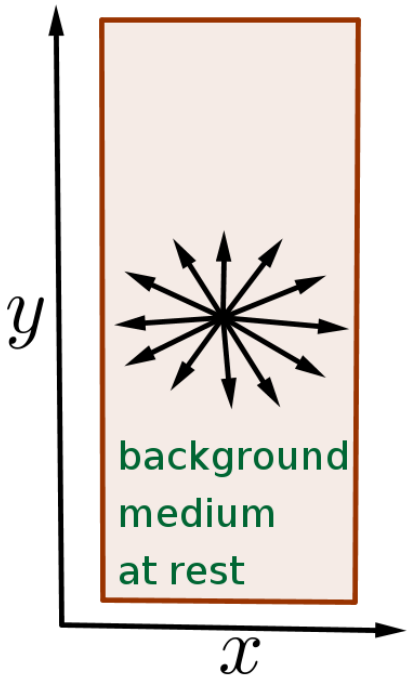




Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

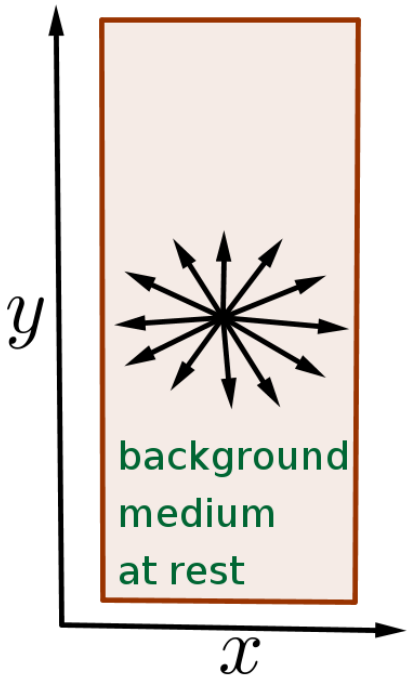
Scenario D: asymmetric geometry, no flow, Jets

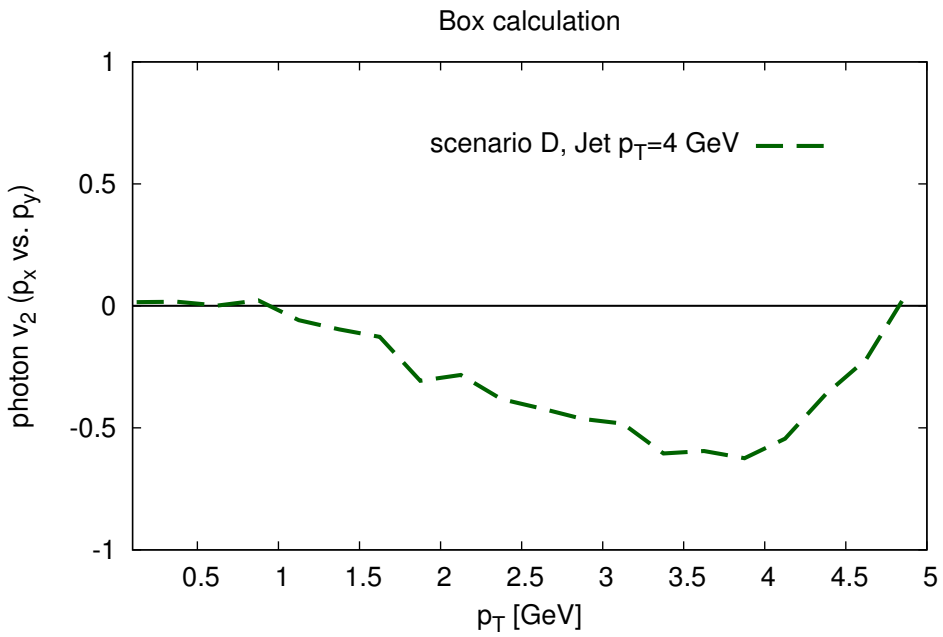


Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario D: asymmetric geometry, no flow, Jets





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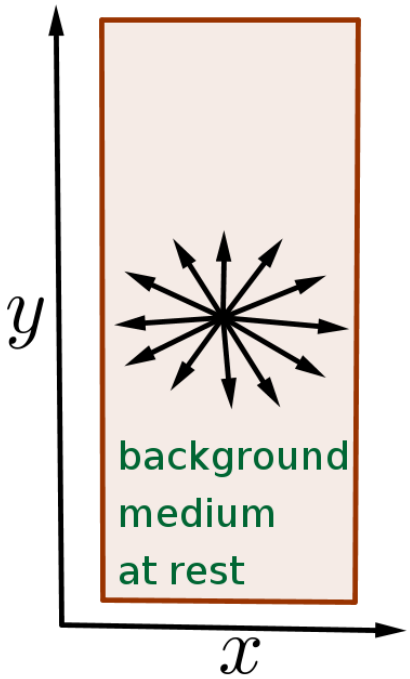

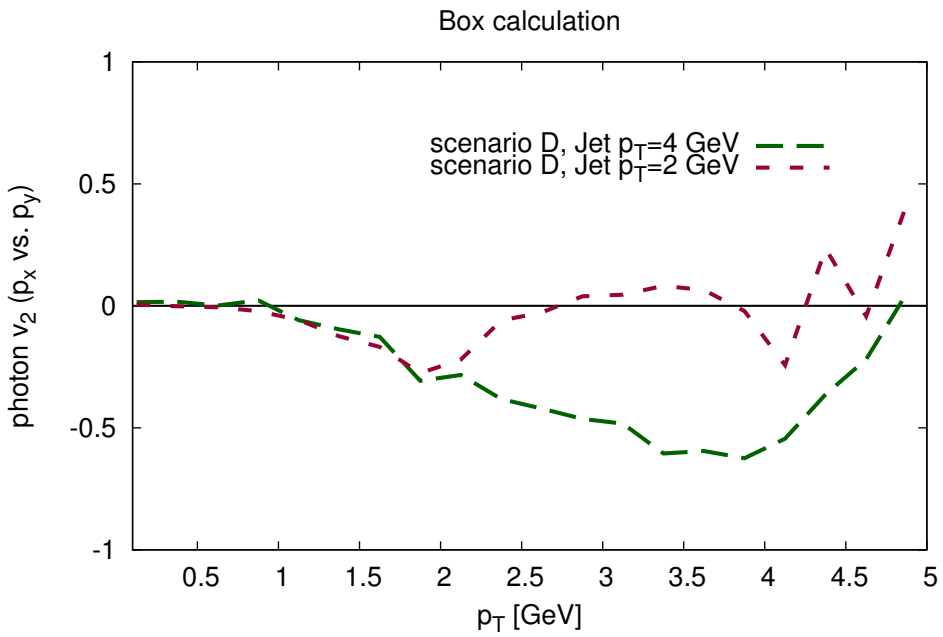

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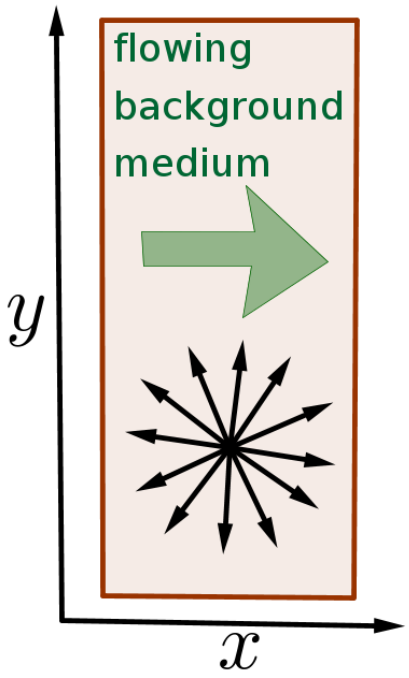
5 different scenarios A-E:

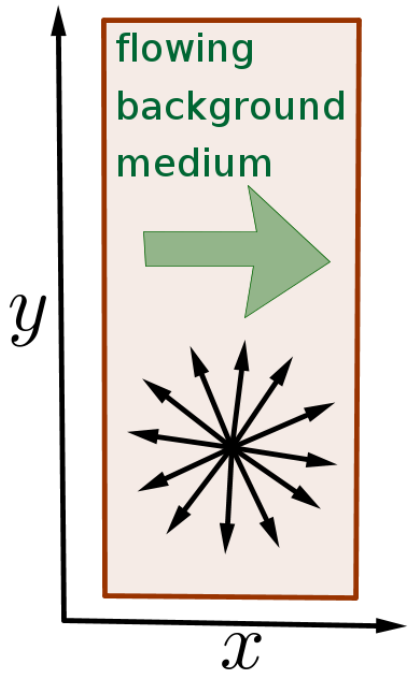
Scenario E: asymmetric geometry, flow in x-direction, Jets

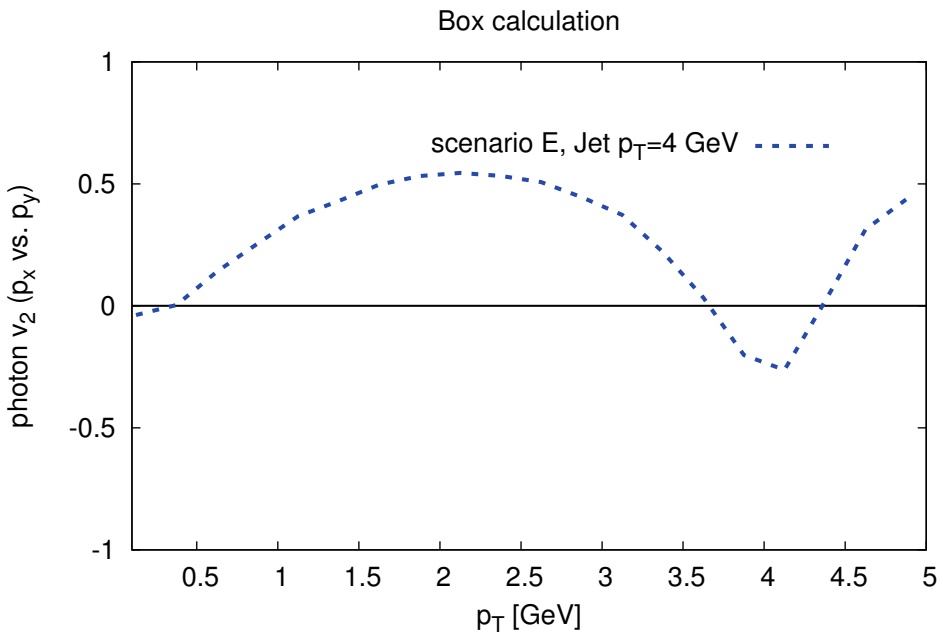


Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

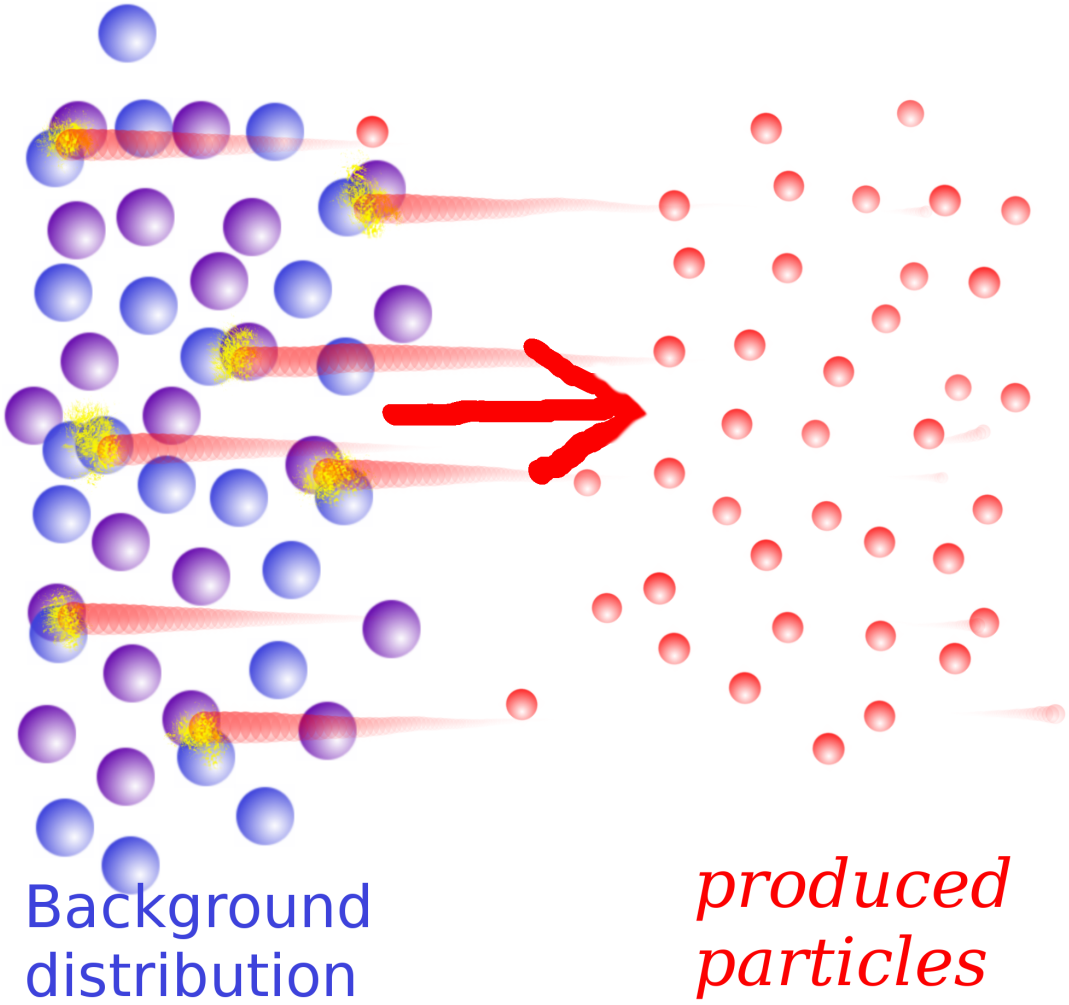
5 different scenarios A-E:

Scenario E: asymmetric geometry, flow in x-direction, Jets





Hydrodynamic push for produced particles



Transfer of momentum anisotropy in equilibrium

Anisotropy of quark/gluon distribution

Fix $\frac{\mathrm{d}N}{\mathrm{d}^3\vec{p}} \sim \exp(-p^\mu u_\mu / T)$ equilibrium with boost: $\gamma > 1$

Analytic: $v_2 = \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}N}{\mathrm{d}^3\vec{p}} \left(\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right) / \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}N}{\mathrm{d}^3\vec{p}}$

BAMPS: Average $\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}$ over all partons

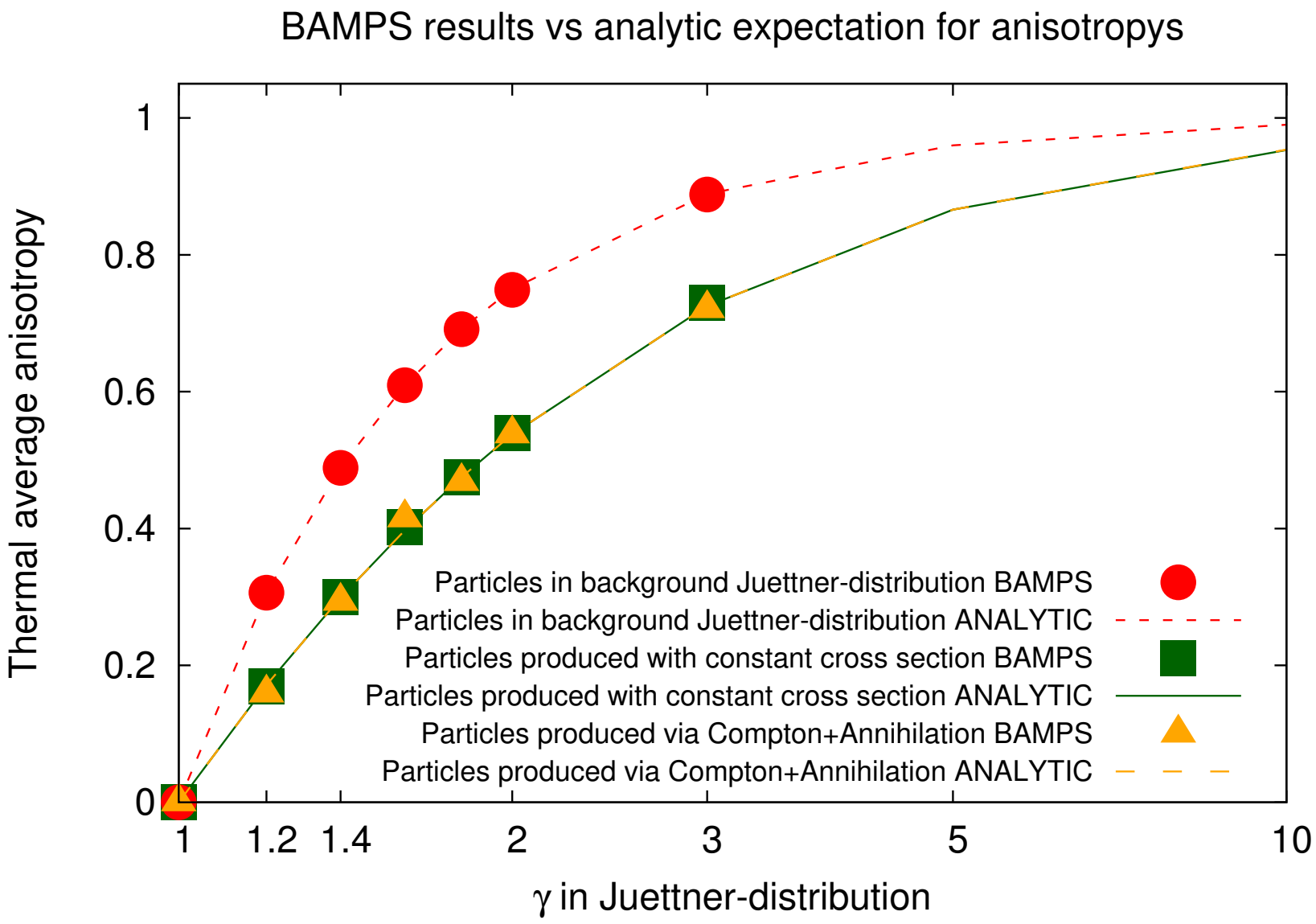
Anisotropy of produced photons

Photon production rate: $E \frac{\mathrm{d}R}{\mathrm{d}^3\vec{p}} = \text{function}(p^\mu u_\mu, T)$ same $\gamma > 1$

Analytic: $v_2 = \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}R}{\mathrm{d}^3\vec{p}} \left(\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right) / \int \mathrm{d}^3\vec{p}\, \frac{\mathrm{d}R}{\mathrm{d}^3\vec{p}}$

BAMPS: Average $\frac{p_x^2 - p_y^2}{p_x^2 + p_y^2}$ over all photons

Transfer of momentum anisotropy in equilibrium



Transfer of momentum anisotropy in equilibrium