Nonequilibrium Photon Production in the Quark-Gluon Plasma

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with Kai Zhou, Florian Senzel, Heiner Kremer, Hendrik v. Hees, Carsten Greiner, Zhe Xu
Photon sources in heavy-ion collisions

+ photons from jets:
Our QGP transport approach: BAMPS

**BAMPS: Boltzmann Approach to Multi-Parton Scatterings**

\[ p^\mu \partial_\mu f(x, p) = C_{22}[f] + C_{23}[f] \]

- Boltzmann Approach to Multi-Parton Scatterings

- Spacetime-grid, stoch. collision probabs.
- Tot. cross sections from pQCD: \( \sigma_{22}(s), \sigma_{23}(s) \)
- Fully Lorentz-invariant formulation

\[
P_{22} = v_{rel} \frac{\sigma_{22}}{N_{\text{test}}} \frac{\Delta t}{\Delta^3 x}
\]
\[
P_{23} = v_{rel} \frac{\sigma_{23}}{N_{\text{test}}} \frac{\Delta t}{\Delta^3 x}, \quad P_{32} = \ldots
\]

- Massless onshell particles (ideal eos)
- This talk: only classical statistics
- See e.g.: PRD88,014018 / PRL102,202301 / PRL114,112301 / PRD90,094014 / PRL103,032301

Zhe Xu & Carsten Greiner, 2005
*Phys. Rev. C 71, 064901*
BAMPS: 3+1D expanding heavy-ion collisions

- Smooth Glauber initial **positions**
- Pythia 6 initial **momenta**: $pp \times N_{\text{binary}}$
- Reproduce $dE_T/dy$ distribution for RHIC/LHC data
- $v_2$ & $R_{AA}$ in common framework (PRL 114, 112301, 2015)

- Particle species: $q, \bar{q}, g, \gamma, e^+ e^-$
- LO-$\gamma$-production, incl. bremsstrahlung $qq \rightarrow qq\gamma$
- Born dilepton production
- Radiative gluons/photons: LPM-suppression modelled

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Photon rates and their implementation in BAMPS

Photon emission rate

\[ E_k \frac{d^3 R}{d^3 k} = -\frac{g^{\mu\nu}}{(2\pi)^3} \text{Im} \Pi^R_{\mu\nu}(E_k, \vec{k}) \leftrightarrow E_k \frac{d^3 R}{d^3 k} = \mathcal{N} \int \delta^{(4)}(...)|M|^2 f_p f_{p'} (1 \pm f_{k'}) \]

with \( \Pi^R_{\mu\nu} \): retarded photon self-energy, \( |M|^2 \): scattering matrix element

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Elastic photon production rate in BAMPs

**Microscopic** implementation: *Two* sources of lacking precision:

**A) Compton and annihilation matrix elements in BAMPs**

Instead of HTL resummed loop for soft momentum transfer:

\[
|M|^2_{\text{Compton}} = \frac{16}{3} \pi^2 \alpha \alpha_s \left( \frac{s^2 + st}{(s + m^2_{D,q})^2} + \frac{s^2 + st}{(u - m^2_{D,q})^2} \right), \quad |M|^2_{\text{annihilation}} = \ldots
\]

only naively screened propagators. **Correct** by \( m^2_{D,q} \rightarrow \kappa m^2_{D,q} \).

**B) Classical statistics in BAMPs**

Correct by factor \( C_{\text{stat}} \) in collision probability.

\[
R \rightarrow R = C_{\text{stat}} N \int \int \int \int \ldots |M|^2 f(P)f(P')
\]

Parameters \( \kappa, C_{\text{stat}} \) must be tuned to analytic rates (e.g. AMY)!
Correction of the Debye mass: fixing $\kappa = 2.45$, $C_{\text{stat}} = 0.84$

\[
\frac{dR}{dE}/\alpha_{\text{EM}} \alpha_s T^3
\]

$\alpha_s = 0.3$, $N_f = 3$

Integrals equal 99.5%

<table>
<thead>
<tr>
<th>Moment</th>
<th>AMY/Born</th>
</tr>
</thead>
<tbody>
<tr>
<td>0th</td>
<td>99.5 %</td>
</tr>
<tr>
<td>1st</td>
<td>112.5 %</td>
</tr>
<tr>
<td>2nd</td>
<td>121.9 %</td>
</tr>
<tr>
<td>3rd</td>
<td>128.1 %</td>
</tr>
<tr>
<td>4th</td>
<td>132.1 %</td>
</tr>
</tbody>
</table>

Box-test in BAMPS: precise reproduction of $R_{\text{Born}}^\gamma$!

- $R_{\text{quantum, HTL}}^\gamma$ from AMY (JHEP. 0111, 057 (2001))
- $R_{\text{quantum, Born}}^\gamma$ using Shen et al. (Phys. Rev.C91, 014908, (2015))
- Compare $R_{\text{quantum, Born}}^\gamma$ with $R_{\text{classical, Born}}^\gamma$: $C_{\text{stat}} = 0.84$
Correction of the Debye mass: fixing $\kappa = 2.45$, $C_{stat} = 0.84$

Box-test in BAMPS: precise reproduction of $R^{\gamma}_{\text{Born}}$!

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- Compare $R^{\gamma}_{\text{quantum, Born}}$ with $R^{\gamma}_{\text{classical, Born}}$: $C_{stat} = 0.84$
Exact bremsstrahlung in BAMPs

- Exact matrix element $M_{23}$
- Debye screening
- Specific $\lambda_{mfp,\text{process}}^q$ from $2 \leftrightarrow 2$ scattering
- K-factor $= 1.42$

$|\mathcal{M}_{2 \rightarrow 3}|^2 \rightarrow |\mathcal{M}_{2 \rightarrow 3}|^2 \Theta(\lambda_{mfp,\text{process}}^q - \tau_f^\gamma)$

lower limits from LPM effect:
$\lambda_{mfp} = 5 \text{ fm}$
$\lambda_{mfp} = 10 \text{ fm}$
$\lambda_{mfp} = 20 \text{ fm}$

$E(dR/dE)/\alpha EM \alpha_s T^4$

$\alpha_s = 0.3, N_f = 3$

$\alpha_s = 0.3, N_f = 3$

$T = 0.4 \text{ GeV}$
$s = 1.44 \text{ GeV}^2$

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Fugacity dependence of rates

Elastic Rate: \[ R \sim \lambda_q \lambda_{\bar{q}}/g \cdot \ldots \cdot \frac{1}{(t-m_D(\lambda_q, \lambda_g))^2} \]

Inelastic Rate: \[ R \sim \lambda_q^2 \cdot \ldots \cdot \frac{1}{(t-m_D(\lambda_q, \lambda_g))^2} \cdot \ldots \cdot \Theta \left( \lambda_{mfp}(\lambda_q/g) - \tau_f \right) \]

- Fugacities \( t \lesssim 1 \) fm/c ill defined
- At RHIC: emission mainly at \( t \lesssim 3 \) fm/c
- Photon yield expected to be \( \sim 20\% \) of \( \lambda_q = 1 \) yield
Fugacity dependence of rates

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Results of realistic QGP simulations

- $p_T$-spectra from BAMPS
- Explanations: fugacities, initial condition
- Elliptic flow from BAMPS
- Explanations: jet-photon conversion and “leakeage” effect
$p_T$-spectra of photons in the QGP from BAMPS

- Higher effective temperature in BAMPS due to harder initial condition
- 80% lower yield than models with $\lambda_q = 1$

Hydro: Paquet et al, PRC 93 (2016), 044906

PHSD: Linnyk et al, PRC92 (2015), 054914
$p_T$-spectra of photons from BAMPS “added” to Hydro

Toy plot: Naive “adding” of contributions - no phase transition

Preequilibrium-phase only small difference

BAMPS QGP underestimates data
Results from heavy-ion collisions

Time development of photon spectrum

Emission mainly between 2 – 3 fm/c

Only below $p_T \sim 1$ GeV: sensitivity to later times

Remember: fugacity still small at early times
Results from heavy-ion collisions

Photon spectra with different fugacity evolution

- Artificial chemical equilibration: increase $\sigma_{gg \to q\bar{q}}$ by $K = 10, 100$
- No large effect: photons emitted at *early times*, where $\lambda_q(K = 100) \sim \lambda_q(K = 1)$
- Initial condition should be improved

**Graphical Details:**

- **Axes:**
  - **x-axis:** $p_T$ [GeV]
  - **y-axis:** $dN/(2\pi p_T dp_T dy)$ [GeV$^{-2}$]

- **Data Points:**
  - **Red Line:** BAMPS, $K_{gg \to q\bar{q}} = 1$
  - **Green Line:** BAMPS, $K_{gg \to q\bar{q}} = 10$
  - **Orange Line:** BAMPS, $K_{gg \to q\bar{q}} = 100$

- **Legend:**
  - PHSD, QGP
  - Hydro, QGP

- **Additional Information:**
  - Au+Au, $\sqrt{s} = 200$ GeV
  - 20-40 % centrality, $|y| < 0.35$
Elliptic flow $v_2(p_T)$ of photons in the QGP

- low-$p_T$ $v_2$ from “hydro-push”
- $p_T$-cuts of parent-partons reveal: negative flow-photons from jet-photon conversion
- negative $v_2$ comparable to other jet-QGP studies

Toy-Plot: reweighting of photon sources not enough - $v_2$-problem even more severe
Jet-contribution: negative elliptic flow $v_2(p_T)$ of photons

Purely geometric effect: "leakage effect":

$E^\gamma \approx E_{\text{JET}} + \text{very collinear emission}$

$\rightarrow$ BAMPS is common framework: hydro-bahavior and conversion photons

$\rightarrow$ Leakage: verified in box-scenario
Conclusion and Outlook

- Successful implementation of LO photon production in a transport approach
- Smaller yield than hydro due to quark fugacities $< 1$
- Elliptic flow small, or negative due to jet-photon conversion

Outlook:
- Electric & Magnetic field influence?
- Closer look to dileptons
- Improvements for initial state

读万卷书，不如行万里路，行万里路不如阅览世人无数。
Effect of running coupling

\[ \frac{dN}{2\pi p_T dp_T dy} \] [GeV^{-2}]

- BAMPS $\alpha_s$ running
- BAMPS $\alpha_s = 0.3$
- PHSD, QGP
- Hydro, QGP

- $\alpha_s(s, t, u)$ at vertices
- $\alpha_s(T)$ within Debye-mass
- Roughly an increase by $\times 1.5$

Au+Au, $\sqrt{s} = 200$ GeV
20-40 % centrality, $|y| < 0.35$
Restrict parent-$p_T$: Jet-Photon Conversion

restrict $\max(p_T^1, p_T^2) < p_T^{\text{cut}}$:

- negative flow originates from high-$p_T$-particles
- exclusively from Jet-Photon Conversion
- low-$p_T$ from hydrodynamic push

⇒ leakage effect verified by box calculation

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Simpler processes: $q\bar{q} \rightarrow e^+e^-$, only s-channel

$dR/dM$ spectra allow redshift-free temperature fitting
PRELIMINARY: Dileptons

- Simpler processes: $q\bar{q} \rightarrow e^+ e^-$, only s-channel
- $dR/dM$ spectra allow redshift-free temperature fitting
Compare results from BAMPS to 2+1D viscous hydro code *MUSIC*

**Reason for shallower slope: different effective temperature in BAMPS:**

![Graph showing temperature evolution](image)
Elastic photon production rate

Hard momentum transfers \( t > t^\star \):

\[
R = \mathcal{N} \int \frac{d^3 p}{2E_p} \int \frac{d^3 p'}{2E_{p'}} \int \frac{d^3 k}{2E_k} \int \frac{d^3 k'}{2E_{k'}} (2\pi)^4 \delta^4(P + P' - K - K') \\
\times |\mathcal{M}|^2 f(P)f(P') \left( 1 \pm f(K') \right)
\]

+ Soft momentum transfers \( t < t^\star \) (HTL propagators and vertices):

\[
E_k \frac{d^3 R}{d^3 k} = - \frac{g^{\mu\nu}}{(2\pi)^3} \text{Im} \Pi^R_{\mu\nu}(E_k, \vec{k})
\]

The sum gives total (tree-level) HTL resummed rate:

approximately... (see A.M.Y. JHEP. 0111, 057 (2001) for correct \( 2 \leftrightarrow 2 \) treatment)

\[
E \frac{dR}{d^3 p} \bigg|_{\text{Compton+Annihilation}} = \left( \sum_i q_i^2 \right) \frac{\alpha_{\text{EM}} \alpha_{\text{strong}}}{2\pi^2} T^2 e^{-E/T} \ln \left( \frac{2.912 \frac{E}{T}}{g_s^2} \right)
\]
Correction for statistics of elastic photon production rate

B) Statistics in BAMPS

Correct by overall factor $C_{\text{stat}}$ in collision probability. $C_{\text{stat}} = 0.84$. 

\[ \frac{dR}{dE}/\alpha EM \alpha_s T^3 \]

$E/T$ 

$\alpha_s=0.3$, $N_f=3$ 

$C_{\text{stat}}=0.84$
Final elastic matrix element in BAMPS:

\[ |\mathcal{M}|_{\text{effective}}^2 = C_{\text{stat}} \left[ 128 \cdot \frac{16}{3} \pi^2 \alpha_s \left( \frac{s^2 + st}{(s + \kappa m^2_{D,q})^2} + \frac{s^2 + st}{(u - \kappa m^2_{D,q})^2} \right) + 24 \cdot \frac{128}{9} \pi^2 \alpha_s \left( \frac{tu}{(t - \kappa m^2_{D,q})^2} + \frac{tu}{(u - \kappa m^2_{D,q})^2} \right) \right] \]
Photon production: higher order loops

Photon rate at order $\mathcal{O}(e^2 g_s^2 T^4)$ obtained via $\gamma$-self energy.

Figure: The diagrams we use in BAMPS.

\begin{enumerate}
\item[(d)] $iM_a$
\item[(e)] $iM_b$
\end{enumerate}
Photon Bremsstrahlung processes: The exact matrix element

Using the Mathematica package FeynCalc 8.2.0:

\[ A \equiv 2(25) + m^2_{D,q} \]
\[ B \equiv 2(45) + m^2_{D,q} \]
\[ C \equiv 4(45) + m^2_{D,q} \]
\[ D \equiv (35)B^2 - 2(34)A(2(25) - B - m^2_{D,q}) \]
\[ E \equiv (23)A((25)C + (45)(-B - m^2_{D,q})) + (24)A(2(34)A + (35)(A + B)) + (25)D \]
\[ F \equiv (24)A(A + B) + (25)B^2 \]
\[ G \equiv (23)A((24)B + (45)A) + (34)F \]
\[ H \equiv -2(23)B + (34)(-B - m^2_{D,q}) + (35)m^2_{D,q} \]
\[ J \equiv (45)H + (24)(35)B + (25)((34)C + 2(35)(45)) \]
\[ |\mathcal{M}|^2 = \frac{1}{4} \frac{2}{9} Q_{EM}^2 A(12)J - 2(13)(24)(45)A + (14)E + (15)G \]
\[ \frac{A^2 B^2 (2(24) + 2(25) - 2(45) + m^2_{D,q})^2}{128} \]

(we defined the scalar product of 4-vectors \((ij) \equiv p_i \cdot p_j\))
Photon Bremsstrahlung processes: Some details

Useful coordinates for radiated photon:

Reference: incoming quark 1 \( p_z > 0 \)

\( y \): rapidity wrt incoming quark 1, \( y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z} \)

\( k_\perp \): transverse momentum of photon wrt to \( p_z \)

\( q_\perp \): gluon momentum transfer

\( \varphi \): \( \angle(\vec{q}_\perp, \vec{k}_\perp) \)

- We use the exact pQCD computation of \( |M_{\text{brems}}|^2 \) with screened quark and gluon propagators
- Inelastic pair annihilation neglected
Inelastic photon rates in BAMPS

Inelastic cross section for radiated photons

**Exact matrix element** computed, coordinate transformation from $P_{\text{in } 1}, P_{\text{in } 2}, P_{\text{out } 1}, P_{\text{out } 2}, K \rightarrow$ integrate cross section:

$$
\sigma_{23} = \frac{1}{2s} \int_{p_1'} \int_{p_2'} \int_{p_3'} \int_{p_1} \int_{p_2} |M_{12 \rightarrow 1'2'3'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2' - p_3')
$$

$$
= \frac{1}{256\pi^4 s} \int d^2 q_\perp \int d^2 k_\perp \int dy \int d\phi |M_{12 \rightarrow 1'2'3'}|^2 J(k_\perp, q_\perp, y, \phi)
$$

- For each particle pair in cell: compute $\sigma_{23}$
- $|M_{12 \rightarrow 1'2'3'}|^2 (P_{\text{in } 1}, P_{\text{in } 2}, k_\perp, q_\perp, y, \phi)$
- **VEGAS** integration algorithm
- If collision happens: sample outgoing momenta with **Metropolis**-algorithm according to $|M|^2$
- Numerically very demanding, needs Lookup-Tables.
Inelastic photon rates in BAMPS

Interference effect: LPM

\[ \lambda_{mfp,qq\rightarrow qq} = \left( n_q \langle \sigma(s)v_{rel} \rangle_{\text{therm}} \right)^{-1} \]

\[ |M|^2 \rightarrow |M|^2 \Theta(\lambda_{mfp,qq\rightarrow qq} - \text{formation time}) \]

Lower limits from LPM effect:
- \( \lambda_{mfp} = 5 \text{ fm} \)
- \( \lambda_{mfp} = 10 \text{ fm} \)
- \( \lambda_{mfp} = 20 \text{ fm} \).

\( s/\sigma_{tot} \text{ d} \sigma/\text{d}k_T^2 \) vs. \( k_T^2/s \):
- \( T = 0.4 \text{ GeV} \)
- \( \alpha_s = 0.3 \)
- \( s = 1.44 \text{ GeV}^2 \)

\( \sigma_{tot}^{-1} \text{ d}\sigma/\text{d}y \) vs. Photon rapidity \( y \):
- \( \lambda_{mfp} = 1 \text{ fm} \)
- \( \lambda_{mfp} = 5 \text{ fm} \)
- \( \lambda_{mfp} = 10 \text{ fm} \)
- \( \lambda_{mfp} = 20 \text{ fm} \)
- \( \lambda_{mfp} = 40 \text{ fm} \)
Numerical sampling of the outgoing photon momenta

Inelastic photon rates in BAMPS

\[ k_T, \phi \]

\[ q_T, \phi \]

\[ y \]

\[ k_T, y \]

\[ q_T, y \]

\[ Y, Quark 1, Quark 2 \]

\[ X_{\text{LPM}} = 0.3 \]

\[ k_{T, \text{cutoff}} = 0.1182 \text{ GeV} \]

\[ \sqrt{s}/2 \]

\[ \pi \]
Comparison of bremsstrahlung in BAMPS with full rate

Figure: Inelastic rate from AMY compared to BAMPS. The integrals are equal, if we use a factor $K_{\text{inel}} = 1.42$. Second moments agree within 1%. 

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Jet-contribution: test elliptic flow $\nu_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario A: square geometry, no flow, Jets

Background medium at rest
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario A: square geometry, no flow, Jets
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario B: square geometry, flow in $x$-direction, no Jets
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario B: square geometry, flow in x-direction, no Jets

\[
\begin{align*}
\text{Box calculation scenario B}
\end{align*}
\]

photon $v^2(p_x \text{ vs. } p_y)$

$p_T [\text{GeV}]$

\[
\begin{align*}
\text{Box calculation scenario B}
\end{align*}
\]
Jet-contribution: test elliptic flow $\nu_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario C: square geometry, flow in x-direction, Jets
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

**Scenario C:** square geometry, flow in x-direction, Jets

![Diagram showing photon v2 (p_x vs. p_y) for scenario C, Jet $p_T = 4$ GeV.](image)

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**Box calculation**

scenario C, Jet $p_T = 4$ GeV
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario D: asymmetric geometry, no flow, Jets
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

**Scenario D:** asymmetric geometry, no flow, Jets
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

**Scenario D:** asymmetric geometry, no flow, Jets

![Diagram showing photon $v_2(p_x, p_y)$ vs. $p_T$ for scenarios D with Jet $p_T=4$ GeV and $p_T=2$ GeV.](image)
Jet-contribution: test elliptic flow $v_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario E: asymmetric geometry, flow in x-direction, Jets
Jet-contribution: test elliptic flow $\nu_2(p_T)$ of photons in box

5 different scenarios A-E:

Scenario E: asymmetric geometry, flow in x-direction, Jets
Hydrodynamic push for produced particles

Background distribution

produced particles
Hydrodynamic push for produced particles

Transfer of momentum anisotropy in equilibrium

### Anisotropy of quark/gluon distribution

Fix $\frac{dN}{d^3p} \sim \exp(-p^\mu u_\mu / T)$ equilibrium with boost: $\gamma > 1$

**Analytic:**
$$ v_2 = \int d^3p \frac{dN}{d^3p} \frac{p^2_x - p^2_y}{p^2_x + p^2_y} / \int d^3p \frac{dN}{d^3p} $$

**BAMPS:**
Average $\frac{p^2_x - p^2_y}{p^2_x + p^2_y}$ over all partons

### Anisotropy of produced photons

Photon production rate: $E \frac{dR}{d^3p} = \text{function}(p^\mu u_\mu, T)$ same $\gamma > 1$

**Analytic:**
$$ v_2 = \int d^3p \frac{dR}{d^3p} \frac{p^2_x - p^2_y}{p^2_x + p^2_y} / \int d^3p \frac{dR}{d^3p} $$

**BAMPS:**
Average $\frac{p^2_x - p^2_y}{p^2_x + p^2_y}$ over all photons
Transfer of momentum anisotropy in equilibrium

BAMPS results vs analytic expectation for anisotropys

Thermal average anisotropy

$\gamma$ in Juettner-distribution

Particles produced via Compton+Annihilation

Particles produced with constant cross section

Particles in background Juettner-distribution

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Transfer of momentum anisotropy in equilibrium

BAMPS results vs analytic expectation for anisotropys

- Bremsstrahlung BAMPS, $X_{LPM}=0.1$
- Particles in background Juettner-distribution BAMPS
- Particles in background Juettner-distribution ANALYTIC
- Particles produced with constant cross section BAMPS
- Particles produced with constant cross section ANALYTIC
- Particles produced via Compton+Annihilation BAMPS
- Particles produced via Compton+Annihilation ANALYTIC

Independence of photon production-process!

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