Viscous corrections to photon production channels in QGP

Sigtryggur Hauksson
McGill University

In collaboration with C. Shen, S. Jeon, C. Gale

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Introduction

- Need both shear and bulk viscosity to fit hadronic data.

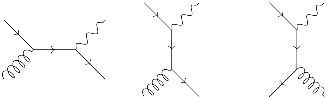
$$f_{\text{eq}} \longrightarrow f_{\text{eq}} + \delta f_{\text{shear}} + \delta f_{\text{bulk}}$$

Ryu, Paquet et al. arXiv: 1502.01675

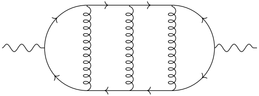
- For accurate modelling of photon production at RHIC and LHC we need to include δf.

- Might hope to get information about the viscosity of QGP through photons.

Leading order photon production for QGP in equilibrium:



2 to 2 scattering

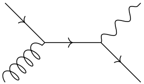


Bremsstrahlung
Pair annihilation
LPM effect

Calculations so far:	Equilibrium	Shear correction	Bulk correction
2 to 2 scattering	Yes [1]	Yes [2]	Forward scattering [3]
Inelastic channels	Yes [4]	No	No

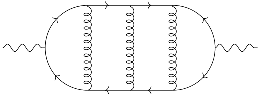
[1] Baier, Nakkagawa, Niegawa, Redlich '92; Kapusta, Lichard, Seibert '91
[2] Schenke, Strickland hep-ph/0611332; Shen, Paquet, Heinz, Gale arXiv:1410.3404
[3] Paquet, Shen et al. arXiv:1509.06738
[4] Arnold, Moore, Yaffe hep-ph/0109064

Bulk viscous correction to 2 to 2 scattering

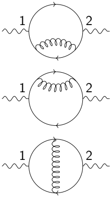


- Rate calculation
- Folding with hydro simulations

Viscous correction to inelastic channels



- The same diagrams contribute out of equilibrium as in equilibrium

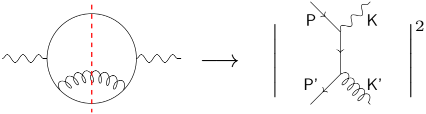


Rate of photon production is related to the 12 photon polarization tensor.

$$k\frac{dR}{d^3k} \sim (i\Pi_{12})^{\mu}{}_{\mu}$$

Use cutting rules to get

$$k\frac{dR}{d^3k} \quad \sim \sum_{\text{channels}} \int_P \int_{P'} \int_{K'} \delta^{(4)}(P+P'-K-K')\; |\mathcal{M}|^2\, f(P)\, f(P')\, (1\pm f(K'))$$



The integrals are evaluated numerically.

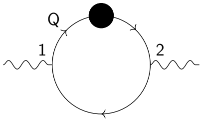
$$\delta f_{\text{bulk}}(P) = f_{\text{eq}}(1 \pm f_{\text{eq}})\; (E - \tfrac{m_{\text{th}}^2}{E})\; \frac{\Pi}{15(\epsilon+P)(\frac{1}{3}-c_s^2)}, \quad E = \sqrt{p^2 + m_{\text{th}}^2}$$

Paquet, Shen et al. arXiv:1509.06738

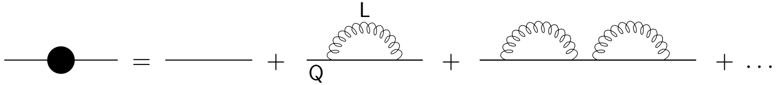
Infrared divergences

Run into infrared divergences because of massless particles.

Fix by including thermal masses for $|\mathbf{q}| \leq q_{\rm cut}$.



Use the method of hard thermal loops (HTL), $L \gg Q$, including bulk viscous correction.

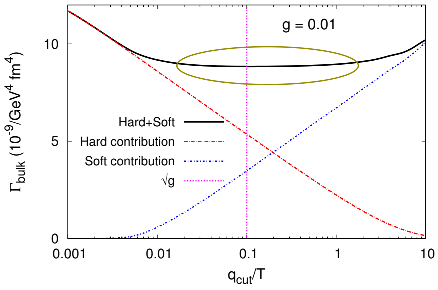


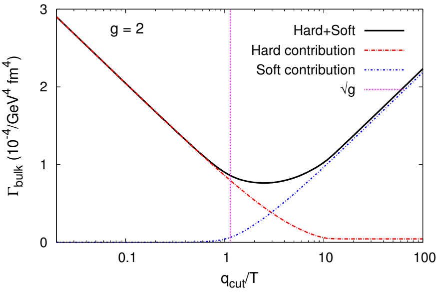
The cut between the hard and the soft part should obey

$$gT \ll q_{\rm cut} \ll T$$

Results

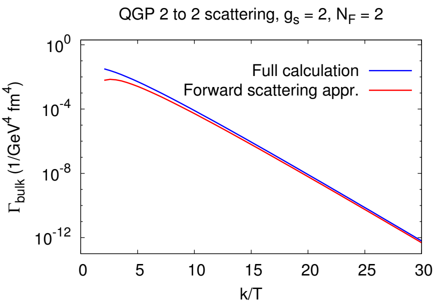
$$k\frac{dR}{d^3k} = T^2\left(\Gamma_{\rm eq} + \frac{\Pi}{15(\epsilon+P)(\frac{1}{3}-c_s^2)}\Gamma_{\rm bulk}\right)$$

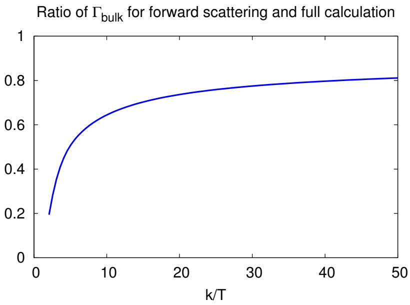




At low g a wide range of $q_{\rm cut}$ gives the same answer.
At high g we choose the minimum.

Results



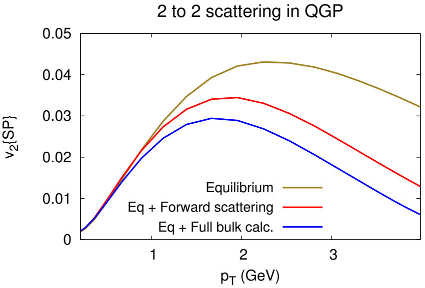


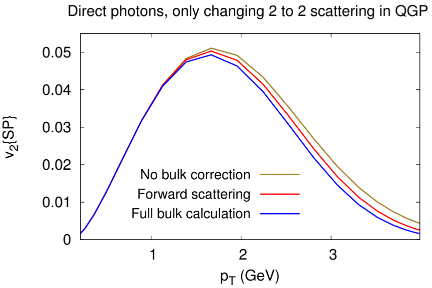
The full calculation and the forward approximation are very different at low k/T but similar at high k/T.

Fold the rates with a hydro evolution:

$$k\frac{dN^{\gamma}_{\text{thermal}}}{d^3k} = \int_{T>T_{\text{freezeout}}} d^4x \left[k\frac{dR^{\gamma}}{d^3k}(T(x), E_k)\Big|_{E_k = k\cdot u(x)} \right]$$

Elliptic flow for Au-Au collisions at 200 GeV, 0 - 40%:

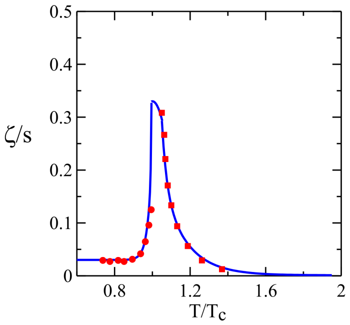




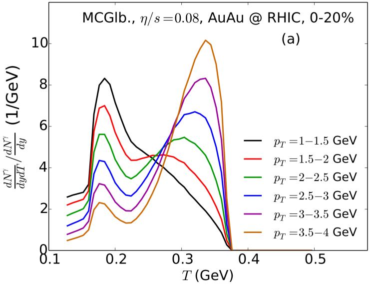
The bulk correction is big at higher p_T.

Results

Why does bulk viscosity give a bigger correction at high p_T than at low p_T?



Ryu et al. arXiv:1502.01675



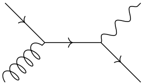
Shen et al. arXiv:1308.2440

At low p_T equilibrium rate and bulk correction come from the same cells.

At high p_T bulk correction comes from cells with higher flow.

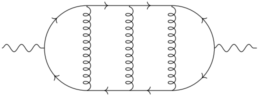
Bulk viscous correction to 2 to 2 scattering

- Rate calculation
- Folding with hydro simulations



Viscous correction to inelastic channels

The same diagrams contribute out of equilibrium as in equilibrium

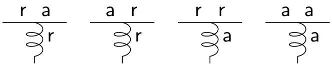


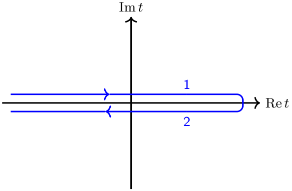
Equilibrium case

Arnold, Moore, Yaffe hep-ph/0109064

Arnold, Moore, Yaffe hep-ph/0111107

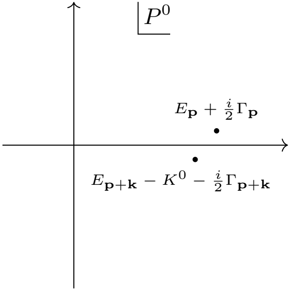
ra basis: $\phi_r = \frac{1}{2}(\phi_1 + \phi_2\ ),\quad \phi_a = \phi_1 - \phi_2$



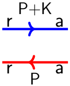


$$\begin{cases} G_{aa} = 0 \\ G_{ar} = G_{\mathrm{Adv}} \\ G_{ra} = G_{\mathrm{Ret}} \\ G_{rr} = (\frac{1}{2} + n_B(P^0))\ (G_{\mathrm{Ret}} - G_{\mathrm{Adv}}) \end{cases}$$

For gluon momentum $P \sim gT$ we get
$n_B(P^0) = \frac{1}{e^{P^0/T} - 1} \sim g^{-1}$ so $G_{rr} \sim g^{-3}$



Pinching poles:

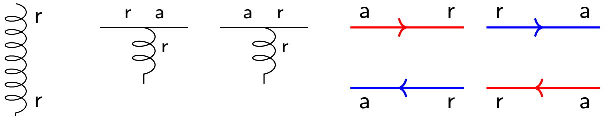


$$\sim \int dP^0\, G_{\mathrm{Ret}}(P+K) G_{\mathrm{Adv}}(P) \sim g^{-2}$$

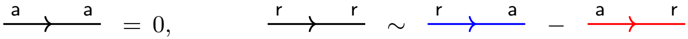
Equilibrium case

Need soft gluons.
Need quarks that are almost collinear to photons and almost on shell.

Our building blocks are



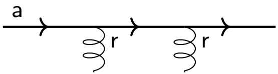
Also have



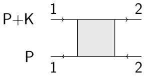
Get infinitely many diagrams that we need to sum up.
They better have a simple structure!

Equilibrium case

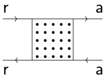
aa quark lines vanish and ra and ar lines are simple.



AMY used the Kubo-Martin-Schwinger relation for four-point functions.



$$= -n_{\mathrm{F}}(K+P)\left[1-n_{\mathrm{F}}(P)\right] \quad \mathrm{Re}$$



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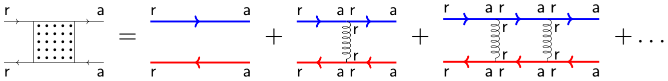

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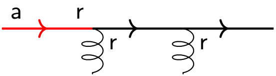

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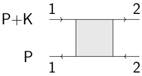

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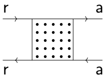

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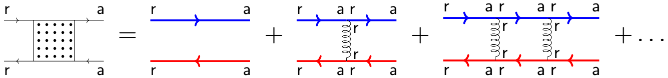

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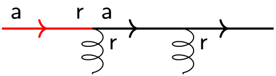

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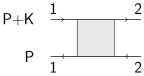

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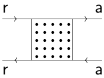

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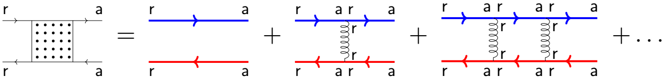

Equilibrium case

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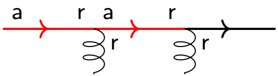

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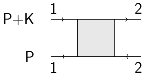

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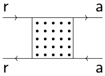

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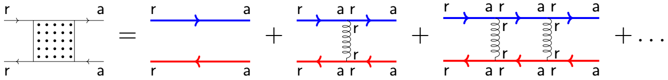

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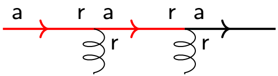

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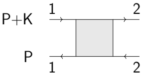

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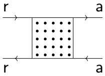

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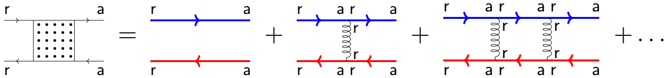

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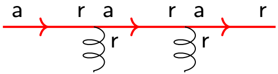

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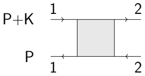

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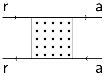

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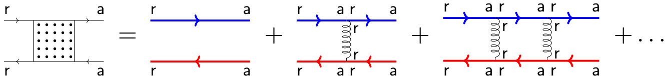
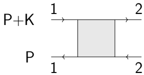
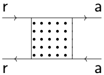


Out-of-equilibrium case

Without using the KMS condition we derived that out of equilibrium



$$= \left.\frac{\Sigma^<}{\Sigma^> - \Sigma^<}\right|_{P+K} \left(1 + \left.\frac{\Sigma^<}{\Sigma^> - \Sigma^<}\right|_{P}\right) \ \mathrm{Re}$$



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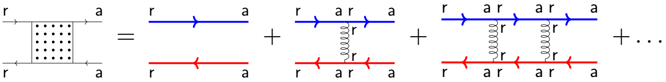


Exactly the same diagrams contribute in and out of equilibrium.

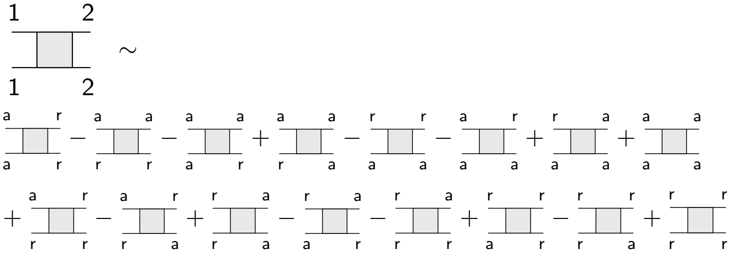
Need the quark and gluon self energies.

The diagrams can be summed up giving an integral equation that needs to be solved numerically.

Need to check that we have the same power counting scheme out of equilibrium.

Calculations so far show that the power counting scheme is still valid for $\delta f \sim f_{\text{eq}}$.

We can always write



aa quark lines vanish and ra and ar lines are simple

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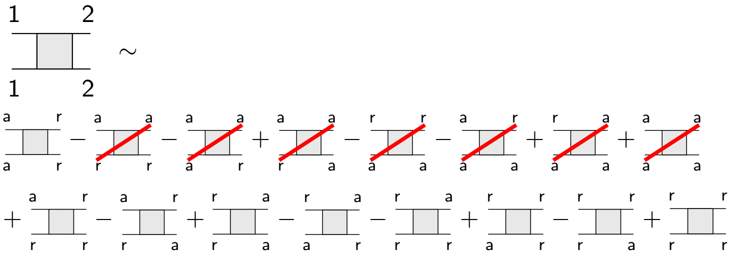

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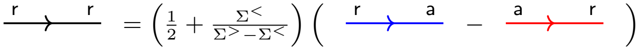
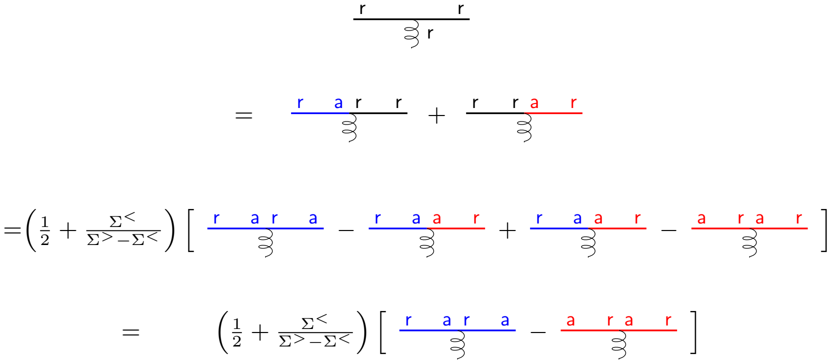


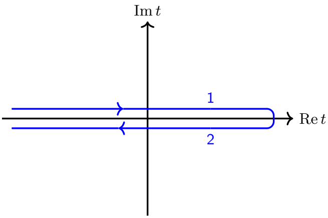
Thus the rr quark lines are a telescoping series. In the end we only get one ra and one ar quark line.



Conclusion

- Calculated bulk viscous correction to 2 to 2 scattering in QGP
- Big effect on photon v_2 at higher p_T
- Out of equilibrium we have the same power counting scheme as in the equilibrium AMY calculation.
  The same diagrams contribute.
- Viscous corrections take photon calculations to a new level of precision.
- Paves the way for viscous corrections to jet-medium interaction.

Backup slides



Perturbative QCD where $\alpha_{EM} \ll \alpha_s \ll 1$.

Infinite medium with constant $T, \Pi, \pi^{\mu\nu}$ ...

Real time formalism:

$$S_{21}(P) = \not{P}\, 2\pi i\, \delta(P^2) \left(-\theta\left(P^0\right) + f(P)\right)$$

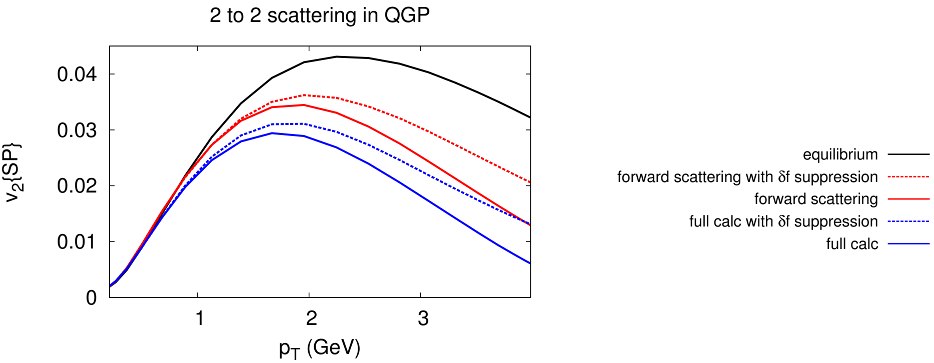
where $f = f_{\text{eq}} + \delta f_{\text{shear}} + \delta f_{\text{bulk}}$

Expand up to first order in δf.

$$\delta f_{\text{bulk}}(P) = f_{\text{eq}}(1 \pm f_{\text{eq}})\,(E - \frac{m_{\text{th}}^2}{E})\,\frac{\Pi}{15(\epsilon + P)(\frac{1}{3} - c_s^2)}, \quad E = \sqrt{p^2 + m_{\text{th}}^2}$$

Is δf too big in hydro?

δf suppression: If $\delta f > f_{\mathrm{eq}}$ in some cell we put $\delta f = f_{\mathrm{eq}}$.



At experimentally interesting p_T this doesn't make a big difference.

Is the power counting scheme the same?

Need to check that we have the same power counting scheme out of equilibrium.

The position of the pinching poles gets shifted because of viscous corrections.

In the gluon propagator

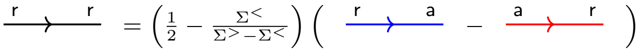
$$n_B \quad \longrightarrow \quad \frac{\Pi^<}{\Pi^> - \Pi^<}$$

where

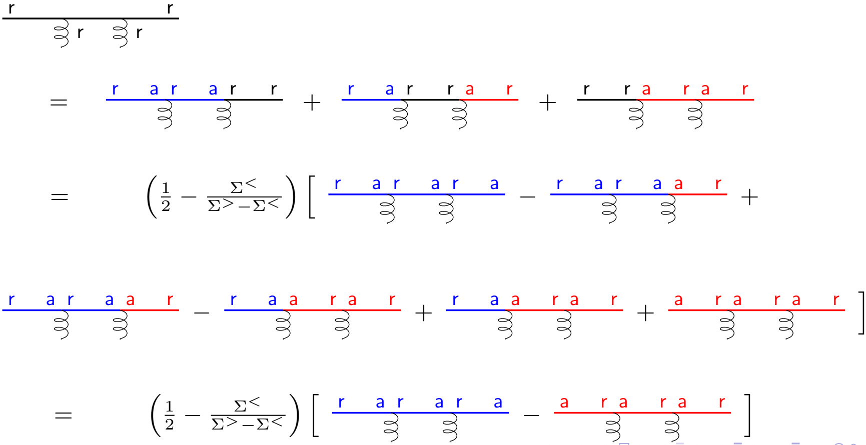
$$\Pi^< = \Pi^<_{\rm eq} + \delta\,\Pi^<$$

Calculations so far show that the power counting scheme is still valid for $\delta f \sim f_{\rm eq}$.

More complicated cancellation



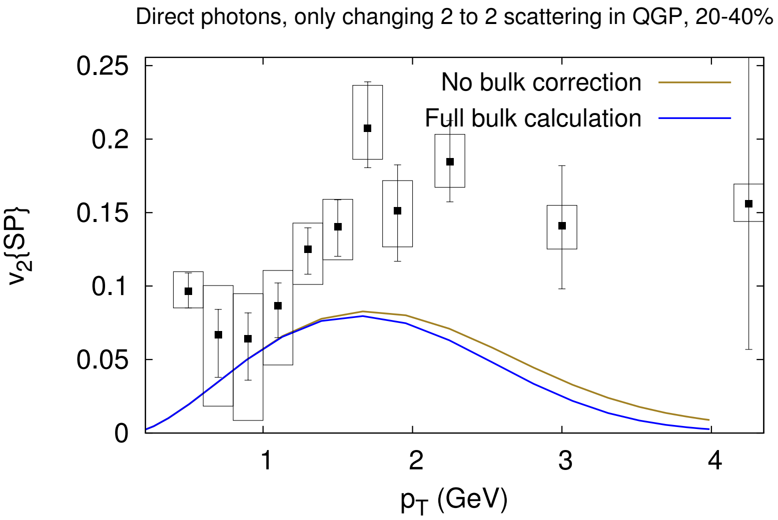
Thus the rr quark lines are a telescoping series.



Some formulas

$$v_n\{SP\} = \frac{\langle v_n^\gamma v_n^h \cos n\left(\Psi_n^\gamma - \Psi_n^h\right)\rangle}{\sqrt{\langle\left(v_n^h\right)^2\rangle}}$$

With experimental data



With experimental data