Viscous corrections to photon production channels in QGP

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Need both shear and bulk viscosity to fit hadronic data.

\[ f_{eq} \rightarrow f_{eq} + \delta f_{\text{shear}} + \delta f_{\text{bulk}} \]


For accurate modelling of photon production at RHIC and LHC we need to include \( \delta f \).

Might hope to get information about the viscosity of QGP through photons.
Leading order photon production for QGP in equilibrium:

2 to 2 scattering
Bremsstrahlung
Pair annihilation
LPM effect

Calculations so far:

<table>
<thead>
<tr>
<th></th>
<th>Equilibrium</th>
<th>Shear correction</th>
<th>Bulk correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 to 2 scattering</td>
<td>Yes [1]</td>
<td>Yes [2]</td>
<td></td>
</tr>
<tr>
<td>Inelastic channels</td>
<td>Yes [4]</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

[1] Baier, Nakkagawa, Niegawa, Redlich '92; Kapusta, Lichard, Seibert '91
Bulk viscous correction to 2 to 2 scattering
  Rate calculation
  Folding with hydro simulations

Viscous correction to inelastic channels
  The same diagrams contribute out of equilibrium as in equilibrium
Rate calculation

Rate of photon production is related to the 12 photon polarization tensor.

\[ k \frac{dR}{d^3k} \sim (i\Pi_{12})^\mu_{\mu} \]

Use cutting rules to get

\[ k \frac{dR}{d^3k} \sim \sum_{\text{channels}} \int_P \int_{P'} \int_{K'} \delta^{(4)}(P+P'-K-K') |\mathcal{M}|^2 f(P)f(P')(1\pm f(K')) \]

The integrals are evaluated numerically.

\[ \delta f_{\text{bulk}}(P) = f_{eq}(1 \pm f_{eq}) \left( E - \frac{m_{\text{th}}^2}{E} \right) \frac{\Pi}{15(\epsilon+P)(\frac{1}{3}-c_s^2)}, \quad E = \sqrt{p^2 + m_{\text{th}}^2} \]

Paquet, Shen et al. arXiv:1509.06738
Run into infrared divergences because of massless particles.

Fix by including thermal masses for $|q| \leq q_{\text{cut}}$.

Use the method of hard thermal loops (HTL), $L \gg Q$, including bulk viscous correction.

The cut between the hard and the soft part should obey

$$gT \ll q_{\text{cut}} \ll T$$
\[ k \frac{dR}{d^3k} = T^2 \left( \Gamma_{\text{eq}} + \frac{\Pi}{15(\epsilon + P)(\frac{1}{3} - c_s^2)} \Gamma_{\text{bulk}} \right) \]

At low \( g \) a wide range of \( q_{\text{cut}} \) gives the same answer. At high \( g \) we choose the minimum.
The full calculation and the forward approximation are very different at low $k/T$ but similar at high $k/T$. 
Results

Fold the rates with a hydro evolution:

\[ k \frac{dN_{\text{thermal}}^\gamma}{d^3 k} = \int_{T>T_{\text{freezeout}}} d^4 x \left[ \frac{k}{d^3 k} \frac{dR^\gamma}{d^3 k} (T(x), E_k) \bigg|_{E_k = k \cdot u(x)} \right] \]

Elliptic flow for Au-Au collisions at 200 GeV, 0 - 40%:

The bulk correction is big at higher \( p_T \).
Why does bulk viscosity give a bigger correction at high $p_T$ than at low $p_T$?

At low $p_T$ equilibrium rate and bulk correction come from the same cells.

At high $p_T$ bulk correction comes from cells with higher flow.
Outline

Bulk viscous correction to 2 to 2 scattering
  Rate calculation
  Folding with hydro simulations

Viscous correction to inelastic channels
  The same diagrams contribute out of equilibrium as in equilibrium
Equilibrium case

Arnold, Moore, Yaffe hep-ph/0109064
Arnold, Moore, Yaffe hep-ph/0111107

ra basis: \( \phi_r = \frac{1}{2}(\phi_1 + \phi_2) \), \( \phi_a = \phi_1 - \phi_2 \)

\[
\begin{align*}
G_{aa} &= 0 \\
G_{ar} &= G_{\text{Adv}} \\
G_{ra} &= G_{\text{Ret}} \\
G_{rr} &= \left(\frac{1}{2} + n_B(P^0)\right) (G_{\text{Ret}} - G_{\text{Adv}})
\end{align*}
\]

For gluon momentum \( P \sim gT \) we get
\( n_B(P^0) = \frac{1}{e^{P^0/T} - 1} \sim g^{-1} \) so \( G_{rr} \sim g^{-3} \)

Pinching poles:
\[
\int dP^0 G_{\text{Ret}}(P + K)G_{\text{Adv}}(P) \sim g^{-2}
\]
Equilibrium case

Need soft gluons.
Need quarks that are almost collinear to photons and almost on shell.

Our building blocks are

Also have

Get infinitely many diagrams that we need to sum up.
They better have a simple structure!
aa quark lines vanish and ra and ar lines are simple.

AMY used the Kubo-Martin-Schwinger relation for four-point functions.

\[
P + K = -n_F(K + P) \left[ 1 - n_F(P) \right] \text{ Re } \]

where

\[
\begin{align*}
&= r a + r a + r a r a + r a r a + \ldots
\end{align*}
\]
Equilibrium case

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\[ \text{AMY used the Kubo-Martin-Schwinger relation for four-point functions.} \]

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where

\[ \text{Re } \]
Equilibrium case

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\[ P + K \]

\[ = -n_F(K + P) [1 - n_F(P)] \quad \text{Re} \]

where

\[ = \quad + \quad + \quad + \ldots \]
Equilibrium case

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\end{align*} \]

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where

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AMY used the Kubo-Martin-Schwinger relation for four-point functions.

where

\[ r \quad a \quad r \quad a \quad r \quad a \quad r \quad a \quad + \ldots \]
Equilibrium case

aa quark lines vanish and ra and ar lines are simple.

\[
\begin{align*}
& a \quad r \quad a \\
\end{align*}
\]

AMY used the Kubo-Martin-Schwinger relation for four-point functions.

\[
P + K = -n_F(K + P) \left[ 1 - n_F(P) \right] \quad \text{Re}
\]

where

\[
\begin{align*}
& r \quad a \\
\end{align*}
\]
Without using the KMS condition we derived that out of equilibrium

\[ P + K \begin{array}{c} 1 \\ P \end{array} 2 = \frac{\sum <}{\sum > - \sum <} \left| P + K \left( 1 + \frac{\sum <}{\sum > - \sum <} \right) \right| \text{Re} \begin{array}{c} r \\ a \end{array} \]

where

\[ \begin{array}{c} r \\ a \end{array} = \begin{array}{c} r \\ a \end{array} + \begin{array}{c} r \\ a \end{array} + \begin{array}{c} r \\ a \end{array} + \begin{array}{c} r \\ a \end{array} + \begin{array}{c} r \\ a \end{array} + \ldots \]

Exactly the same diagrams contribute in and out of equilibrium.

Need the quark and gluon self energies.

The diagrams can be summed up giving an integral equation that needs to be solved numerically.
Out-of-equilibrium case

Need to check that we have the same power counting scheme out of equilibrium.

Calculations so far show that the power counting scheme is still valid for \( \delta f \sim f_{eq} \).

We can always write

\[
\begin{array}{c}
1 & 2 \\
\hline
1 & 2 \\
\end{array}
\sim
\begin{array}{c}
a & r \\
r & a \\
\hline
r & a \\
a & r \\
\end{array}
\]

aa quark lines vanish and ra and ar lines are simple.
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We can always write

$$\begin{array}{c}
1 \\
\hline
2 \\
1 \\
\hline
2
\end{array} \sim \begin{array}{c}
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\end{array} - \begin{array}{c}
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\end{array} + \begin{array}{c}
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\end{array} + \begin{array}{c}
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\hline
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\end{array}
\end{array}$$

aa quark lines vanish and ra and ar lines are simple
Out-of-equilibrium case

\[
\begin{aligned}
\begin{array}{c}
\text{r} \\
\text{r}
\end{array}
= & \left( \frac{1}{2} + \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \left( \\
& \begin{array}{c}
\text{r} \\
\text{a} \\
\text{r}
\end{array} \\
& - \\
& \begin{array}{c}
\text{a} \\
\text{r}
\end{array}
\end{array}
\right)
\end{aligned}
\]

Thus the \text{rr} quark lines are a telescoping series. In the end we only get one \text{ra} and one \text{ar} quark line.

\[
\begin{aligned}
\begin{array}{c}
\text{r} \\
\text{r}
\end{array}
= & \left( \frac{1}{2} + \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \\
& \begin{array}{c}
\text{r} \\
\text{a} \\
\text{r} \\
\text{r}
\end{array} + \\
& \begin{array}{c}
\text{r} \\
\text{r} \\
\text{a} \\
\text{r}
\end{array}
\end{aligned}
\]

\[
\begin{aligned}
= & \left( \frac{1}{2} + \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \left[ \\
& \begin{array}{c}
\text{r} \\
\text{a} \\
\text{r} \\
\text{a}
\end{array} - \\
& \begin{array}{c}
\text{r} \\
\text{a} \\
\text{a} \\
\text{r}
\end{array} + \\
& \begin{array}{c}
\text{r} \\
\text{a} \\
\text{a} \\
\text{r}
\end{array} - \\
& \begin{array}{c}
\text{a} \\
\text{r} \\
\text{a} \\
\text{r}
\end{array}
\right]
\end{aligned}
\]

\[
= \left( \frac{1}{2} + \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \left[ \\
& \begin{array}{c}
\text{r} \\
\text{a} \\
\text{r} \\
\text{a}
\end{array} - \\
& \begin{array}{c}
\text{a} \\
\text{r} \\
\text{a} \\
\text{r}
\end{array}
\right]
\]
Conclusion

- Calculated bulk viscous correction to 2 to 2 scattering in QGP

- Big effect on photon $v_2$ at higher $p_T$

- Out of equilibrium we have the same power counting scheme as in the equilibrium AMY calculation. The same diagrams contribute.

- Viscous corrections take photon calculations to a new level of precision.

- Paves the way for viscous corrections to jet-medium interaction.
Approximations and formalism

Perturbative QCD where $\alpha_{EM} \ll \alpha_s \ll 1$.

Infinite medium with constant $T, \Pi, \pi^{\mu\nu} ...$

Real time formalism:

\[ S_{21}(P) = \not{P} 2\pi i \delta(P^2) \left(-\theta(P^0) + f(P)\right) \]

where $f = f_{eq} + \delta f_{\text{shear}} + \delta f_{\text{bulk}}$

Expand up to first order in $\delta f$.

\[ \delta f_{\text{bulk}}(P) = f_{eq}(1 \pm f_{eq}) \left(E - \frac{m_{th}^2}{E}\right) \frac{\Pi}{15(\epsilon + P)(\frac{1}{3} - c_s^2)}, \quad E = \sqrt{p^2 + m_{th}^2} \]
Is $\delta f$ too big in hydro?

$\delta f$ suppression: If $\delta f > f_{eq}$ in some cell we put $\delta f = f_{eq}$.

At experimentally interesting $p_T$ this doesn't make a big difference.
Is the power counting scheme the same?

Need to check that we have the same power counting scheme out of equilibrium.

The position of the pinching poles gets shifted because of viscous corrections.

In the gluon propagator

\[
  n_B \rightarrow \Pi^< \frac{\Pi^<}{\Pi^> - \Pi^<}
\]

where

\[
  \Pi^< = \Pi_{eq}^< + \delta \Pi^<
\]

Calculations so far show that the power counting scheme is still valid for \(\delta f \sim f_{eq}\).
More complicated cancellation

\[
\begin{align*}
\frac{r}{r} &= \left( \frac{1}{2} - \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \left( \frac{r}{a} - \frac{a}{r} \right) \\
\text{Thus the } rr \text{ quark lines are a telescoping series.}
\end{align*}
\]

\[
\begin{align*}
\frac{r}{r} &= \frac{r}{a} \frac{a}{r} + \frac{r}{a} \frac{a}{r} + \frac{r}{a} \frac{a}{r} \\
&= \left( \frac{1}{2} - \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \left[ \frac{r}{a} \frac{a}{r} \frac{a}{r} - \frac{r}{a} \frac{a}{r} \frac{a}{r} + \frac{r}{a} \frac{a}{r} \frac{a}{r} \frac{a}{r} \right] \\
&= \left( \frac{1}{2} - \frac{\Sigma^<}{\Sigma^+ - \Sigma^<} \right) \left[ \frac{r}{a} \frac{a}{r} \frac{a}{r} - \frac{a}{r} \frac{a}{r} \frac{a}{r} \frac{a}{r} + \frac{a}{r} \frac{a}{r} \frac{a}{r} \frac{a}{r} \right]
\end{align*}
\]
Some formulas

\[ v_n \{SP\} = \frac{\langle v_n^\gamma v_n^h \cos n (\Psi_n^\gamma - \Psi_n^h) \rangle}{\sqrt{\langle (v_n^h)^2 \rangle}} \]
Direct photons, only changing 2 to 2 scattering in QGP, 20-40%
Direct photons, only changing 2 to 2 scattering in QGP, 0-20%

No bulk correction

Full bulk calculation