

# Photon from the Color Glass Condensate in the pA collision

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arXiv:1602.01989

Hard Probes 2016, Wuhan, China, 22 September - 27  
September 2016



# Motivation

- photon  $\rightarrow$  clean probes
- in pA  $\rightarrow$  initial state
- **goal of this work**  
saturation effects in photon spectrum

# Color Glass Condensate

- universal form of matter at

$$x \ll 1, \quad Q^2 = \text{fixed}$$

→ **saturation scale**  $Q_s^2$

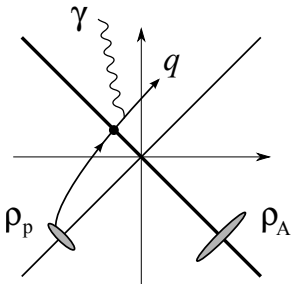
$$\frac{\alpha_s}{Q_s^2} \frac{x f_g(x, Q_s^2)}{\pi R^2} \sim 1$$

→ large gluon occupation number

→ classical color fields

# Photon in pA

- valence quark bremsstrahlung  $O(\alpha)$



- Gelis – Jalilian-Marian formula

$$\frac{1}{\pi R_A^2} \frac{d\sigma^{q \rightarrow q\gamma}}{d^2 k_\perp} = \frac{\alpha}{\pi} \frac{1}{k_\perp^2} \int_0^1 dz \frac{1 + (1-z)^2}{z} \int_{l_\perp} \boxed{C(l_\perp)} \frac{l_\perp^2}{(l_\perp - \mathbf{k}_\perp/z)^2}$$

color dipole:  $\int_{x_\perp} e^{i\mathbf{l}_\perp \cdot \mathbf{x}_\perp} \langle U(0) U^\dagger(\mathbf{x}_\perp) \rangle$

Gelis, Jalilian-Marian, Phys. Rev. D **66** (2002) 014021

# Photon in pA

- but: high energy (small  $x$ )
  - **gluon** component of the proton wave function becomes **dominant**
  - **new emission processes**

# Power counting

- proton: gluons more abundant than quarks

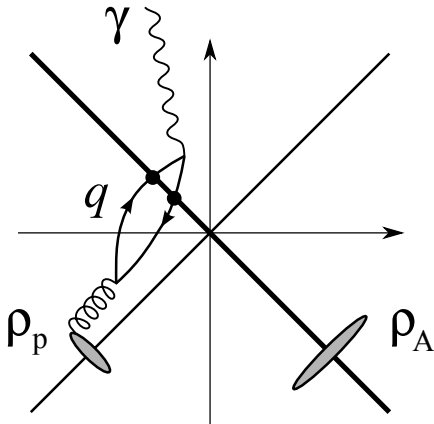
$$f_q \ll f_g$$

- nucleus dense, proton dilute

$$\rho_p \ll \rho_A$$

# Photon in pA

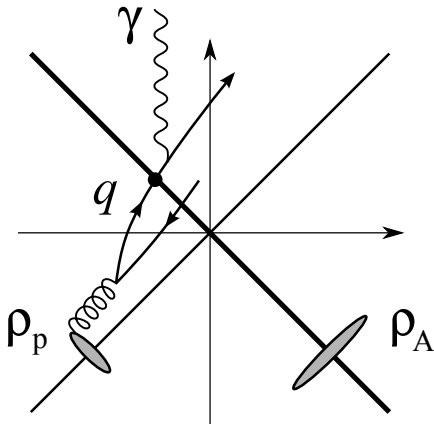
- annihilation  $O(\alpha\alpha_s)$



SB, Fukushima, arXiv:1602.01989

# Photon in pA

- bremsstrahlung from produced  $\bar{q}q$   $O(\alpha\alpha_s)$



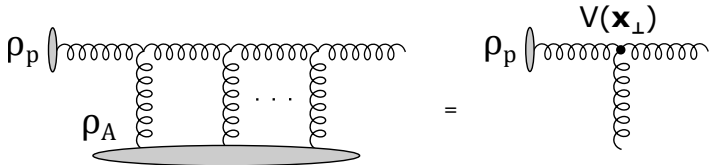
SB, Fukushima, arXiv:1602.01989



# pA CGC Feynman rules

- **light-cone gauge:**  $n^\mu \mathcal{A}_\mu = 0$ ,  $n^\mu \equiv \delta^{\mu-}$

## 1. background gluon field $\mathcal{A}_{(1)}(x)$

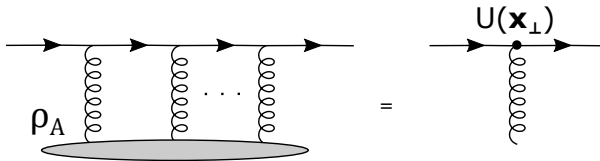


Gelis, Mehtar-Tani, Phys. Rev. D **73** (2006) 034019  
Fukushima and Hidaka, Nucl. Phys. A **813** (2008) 171

# pA CGC Feynman rules

- light-cone gauge:  $n^\mu \mathcal{A}_\mu = 0$ ,  $n^\mu \equiv \delta^{\mu-}$

## 2. quark propagator $S_{(0)}(x, y)$



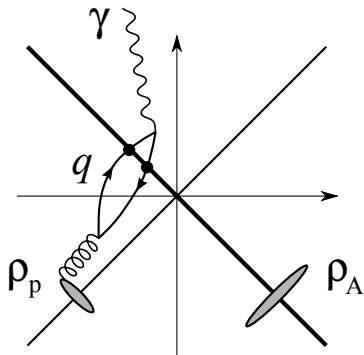
Baltz, McLerran, Phys. Rev. C **58** (1998) 1679

# Annihilation - amplitude

$$\mathcal{M}_\lambda(\mathbf{k}) = eg \int_{xy} e^{ik \cdot x} \text{Tr}[\not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, y) \mathcal{A}_{(1)}(y) S_{(0)}(y, x)]$$

# Annihilation - amplitude

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# Annihilation - rate

$$\begin{aligned} \frac{1}{\pi R_A^2} \frac{dN}{d^2\mathbf{k}_\perp dy} &= \frac{\alpha \alpha_s}{16\pi^8} \frac{N_c}{N_c^2 - 1} \int_{xx'} \int_{\mathbf{u}_\perp \mathbf{u}'_\perp \mathbf{w}_\perp} e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \\ &\times S\left(\mathbf{u}_\perp - \frac{\mathbf{v}_\perp}{2}, \mathbf{u}_\perp + \frac{\mathbf{v}_\perp}{2}, \mathbf{u}'_\perp - \frac{\mathbf{v}'_\perp}{2}, \mathbf{u}'_\perp + \frac{\mathbf{v}'_\perp}{2}\right) \\ &\times \int_{I_\perp} e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp} \varphi_p(I_\perp) (\hat{\mathbf{u}}_\perp \cdot \hat{\mathbf{u}}'_\perp \Psi_1 \Psi_1^* + \Psi_2 \Psi_2^* + 2\hat{\mathbf{u}}_\perp \cdot \hat{\mathbf{l}}_\perp \Psi_1 \Psi_2^*) \end{aligned}$$

- unintegrated gluon distribution

$$g^2 \langle \rho_p^a(\mathbf{l}_\perp) \rho_p^{a'}(\mathbf{l}_\perp) \rangle \equiv \frac{\delta^{aa'}}{\pi(N_c^2 - 1)} l_\perp^2 \varphi_p(I_\perp)$$

- inelastic quadrupole

$$S(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv \frac{1}{N_c} \left\langle \text{Tr}_c [U(\mathbf{y}_\perp) T_F^a U^\dagger(\mathbf{z}_\perp)] \text{Tr}_c [U(\mathbf{z}'_\perp) T_F^a U^\dagger(\mathbf{y}'_\perp)] \right\rangle$$

# Remarks

- ✓ photon Ward identity
- ✓ Furry theorem (vanishing of  $gg \rightarrow \gamma$ )
- ✓ UV finite (lowest order is  $ggg \rightarrow \gamma$ )
- ✓ collinear factorization on the proton side
  - chiral limit

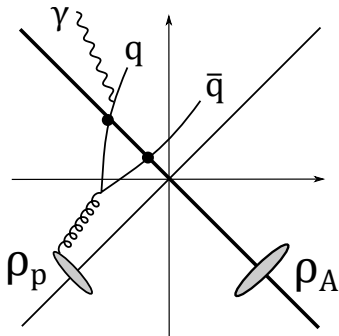
SB, Fukushima, arXiv:1602.01989

# Brems from produced $\bar{q}q$ - amplitude

$$\begin{aligned} \mathcal{M}_\lambda(\mathbf{k}, \mathbf{q}, \mathbf{p}) &= ieg \int_{xyz} e^{ik \cdot x + iq \cdot y + ip \cdot z} \bar{u}(\mathbf{q})(i\vec{\not{\partial}}_y - m) \\ &\times \left\{ S_{(0)}(y, w) \cancel{A}_{(1)}(w) S_{(0)}(w, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, z) \right. \\ &\left. + S_{(0)}(y, x) \not{\epsilon}_\lambda(\mathbf{k}) S_{(0)}(x, w) \cancel{A}_{(1)}(w) S_{(0)}(w, z) \right\} (i\vec{\not{\partial}}_z + m) v(\mathbf{p}) \end{aligned}$$

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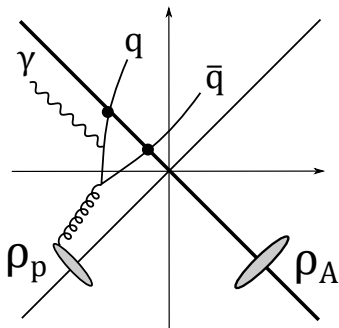


SB, Fukushima, Garcia-Montero, Venugopalan, in preparation



# Brems from produced $\bar{q}q$ - amplitude

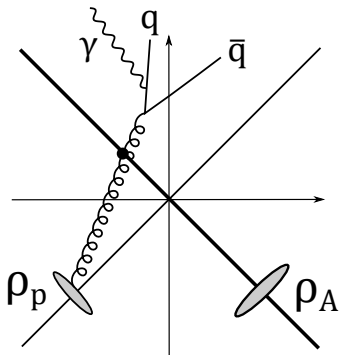
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SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

# Brems from produced $\bar{q}q$ - rate

$$\begin{aligned}
 & \frac{1}{\pi R_A^2} \frac{d\sigma}{d^2\mathbf{k}_\perp dy d^2\mathbf{q}_\perp dy_q d^2\mathbf{p}_\perp dy_p} = -\frac{\alpha\alpha_s N_c}{256\pi^8 (N_c^2 - 1)} \int_{\mathbf{l}_\perp \mathbf{s}_\perp \mathbf{s}'_\perp} \frac{\varphi_p(l_\perp)}{l_\perp^2} \\
 & \times \int_{\mathbf{u}_\perp \mathbf{u}'_\perp \mathbf{w}_\perp} e^{i(\mathbf{s}_\perp + \frac{\mathbf{l}_\perp - \mathbf{p}_\perp}{2}) \cdot \mathbf{u}_\perp} e^{-i(\mathbf{s}'_\perp + \frac{\mathbf{l}_\perp - \mathbf{p}_\perp}{2}) \cdot \mathbf{u}'_\perp} e^{i(\mathbf{l}_\perp - \mathbf{p}_\perp) \cdot \mathbf{w}_\perp} \\
 & \times C\left(\mathbf{u}_\perp - \frac{\mathbf{v}_\perp}{2}, \mathbf{u}_\perp + \frac{\mathbf{v}_\perp}{2}, \mathbf{u}'_\perp - \frac{\mathbf{v}'_\perp}{2}, \mathbf{u}'_\perp + \frac{\mathbf{v}'_\perp}{2}\right) \\
 & \times \left\{ C_\mu(P, \mathbf{l}_\perp, \mathbf{p}_\perp - \mathbf{l}_\perp) C_{\mu'}(P, \mathbf{l}_\perp, \mathbf{p}_\perp - \mathbf{l}_\perp) \text{Tr}_D [(\not{q} + m) R_g^{\mu\nu} (\not{p} - m) \bar{R}_{g\nu}^{\mu'}] \right. \\
 & + \frac{l_i}{P_+} \frac{l'_i}{P_+} \text{Tr}_D [(\not{q} + m) R_{\bar{q}q}^{i\nu}(\mathbf{l}_\perp, \mathbf{s}_\perp) (\not{p} - m) \bar{R}_{\bar{q}q\nu}^{i'}(\mathbf{l}_\perp, \mathbf{s}'_\perp)] \\
 & \left. + \frac{l'_i}{P_+} C_\mu(P, \mathbf{l}_\perp, \mathbf{p}_\perp - \mathbf{l}_\perp) \text{Tr}_D [(\not{q} + m) R_g^{\mu\nu} (\not{p} - m) \bar{R}_{\bar{q}q\nu}^{i'}(\mathbf{l}_\perp, \mathbf{s}'_\perp)] + \text{h. c.} \right\}
 \end{aligned}$$

- same Wilson line product as in  $\bar{q}q$  production

$$C(\mathbf{y}_\perp, \mathbf{z}_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \equiv \frac{1}{N_c} \langle \text{Tr}_c [U(\mathbf{y}_\perp) T_F^a U^\dagger(\mathbf{z}_\perp) U(\mathbf{z}'_\perp) T_F^a U^\dagger(\mathbf{y}'_\perp)] \rangle$$

SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

# Remarks

- ✓ photon Ward identity
- ✓ soft photon theorem
- ✓ collinear factorization
- ✓ Lorenz gauge:  $\partial_\mu \mathcal{A}^\mu = 0$
- ✓ leading twist matches to  $gg \rightarrow \bar{q}q\gamma$  from  $k_\perp$ -factorization

Garcia-Montero, Master thesis (2016)

Baranov, Lipatov, Zotov, Phys. Rev. D **77** (2008) 074024

SB, Fukushima, Garcia-Montero, Venugopalan, in preparation

# Color average

$$\langle \mathcal{O}[\rho_p, \rho_A] \rangle = \int [d\rho_p][d\rho_A] W_p[x_p; \rho_p] W_A[x_A; \rho_A] \mathcal{O}[\rho_p, \rho_A]$$

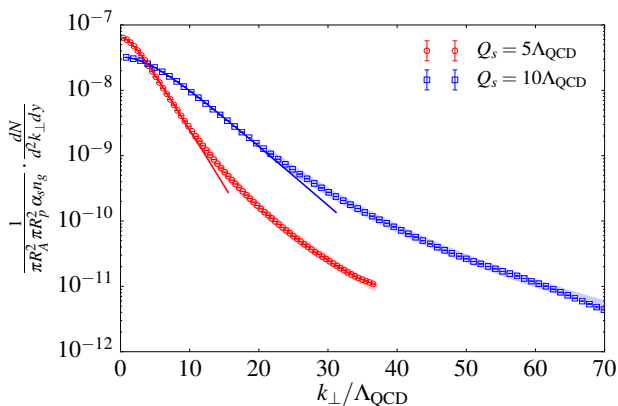
- McLerran-Venugopalan model

$$\langle \rho_A^a(\mathbf{x}_\perp) \rho_A^b(\mathbf{y}_\perp) \rangle = g^2 \delta^{ab} \mu_A^2 \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

$$Q_s^2 \equiv \frac{N_c^2 - 1}{4N_c} g^4 \mu_A^2$$

- reasonable for  $x \sim 10^{-2}$
- $x$  evolution  $\rightarrow$  JIMWLK

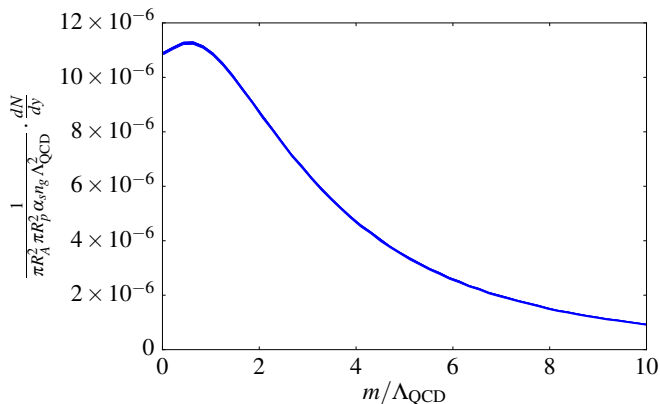
# Annihilation - photon spectrum



- single flavor, chiral limit
- thin lines:  $\exp(-\sqrt{k_{\perp}^2 + (0.5Q_s)^2}) / 0.5Q_s$
- thick lines:  $(\log(k_{\perp} / Q_s))^{1.5} / k_{\perp}^{5.6}$

SB, Fukushima, arXiv:1602.01989

# Annihilation - mass dependence



- fit  $(\log(m/\Lambda_{\text{QCD}}))^{1.8}/m^{2.6}$
- mass corrections important

# Conclusions and outlook

- ✓ complete analytical result at  $O(\alpha\alpha_s)$
- ✓ numerical evaluation of the annihilation diagram
- sensitivity to quadrupole gluon correlators
- phenomenological applications  
bremsstrahlung - numerical evaluation  
x evolution