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LHC Higgs Cross Section Working Group 2 (Higgs Properties)

# Pseudo-observables in Higgs physics

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# 1 Introduction

The idea of PO has been formalized the first time in the context of electroweak observables around the $Z$ pole [1]. A generalization of this concept to describe possible deformations from the SM in Higgs production and decay processes has been discussed in Refs. [2,3,4,5,6,7]. The basic idea is to identify a set of quantities that are

**I.** experimentally accessible,

**II.** well-defined from the point of view of QFT,

and capture all relevant New Physics (NP) effects (or all relevant deformations from the SM) without loosing information and with minimum theoretical bias. The last point implies that changes in the underlying NP model should not require any new processing of raw experimental data. In the same spirit, the PO should be independent from the theoretical precision (e.g. LO, NLO, ...) at which NP effects are computed. Finally, the PO are obtained after removing (via a proper deconvolution) the effect of the soft SM radiation (both QED and QCD radiation), that is assumed to be free from NP effects. In the case of observables around the $Z$ pole, the $\Gamma(Z \to f\bar{f})$ partial decay rates provide good examples of PO.

The independence from NP models can not be fulfilled in complete generality. However, it can be fulfilled under very general assumptions. As far as Higgs physics at LHC is concerned, the general requirement of Higgs PO is to

**III.** capture all relevant NP effects in the limit of no new (non-SM) particles below or close to the Higgs mass.

Under this additional hypothesis, the PO provide a bridge between the fiducial cross-section measurements and the determination of NP couplings in explicit NP frameworks.

On a more theoretical footing, the Higgs PO are defined from a general decomposition of on-shell amplitudes involving the Higgs boson –based on analyticity, unitarity, and crossing symmetry– and a momentum expansion following from the dynamical assumption of no new light particles (hence no unknown physical poles in the amplitudes) in the kinematical regime where the decomposition is assumed to be valid. These conditions ensure the generality of this approach and the possibility to match it to a wide class of explicit NP model, including the determination of Wilson coefficients in the context of Effective Field Theories.

The old $\kappa$ framework [8,9] satisfied the conditions I and II, but not the condition III, since the framework was not general enough to describe modifications in $(n > 2)$-body Higgs decays resulting in non-SM kinematics. Similarly, the old $\kappa$ framework could not describe modifications of the Higgs-cross sections that cannot be reabsorbed into a simple overall re-scaling with respect to the SM.

Similarly to the case of electroweak observables, it is convenient to introduce two complementary sets of Higgs PO:

- a set of *physical* PO, namely a set of (idealized) partial decay rates and asymmetries;
- a set of *effective-couplings* PO, parameterizing the on-shell production and decay amplitudes.

The two sets are in one-to-one correspondence: by construction, the effective-couplings PO are directly related to the physical PO after properly working out the decay kinematics. The effective-couplings PO are particularly useful to build tools to simulate data, taking into account the effect of soft QCD and QED radiation.[1] This is why, from the practical point of view, the effective-couplings PO are first extracted from data in the LHC Higgs analysis, and from these the physical PO are indirectly derived. As we discuss below, the latter provide a more intuitive and effective presentation of the measurements performed.

The note is organized as follows: the PO for Higgs decays are discussed in Section 2–4, separating two, three, and four-body decay modes. General aspects of PO in electroweak production processes are discussed in Section 5, whereas the specific implementation for VH and VBF is presented in Section 6. The total number of PO to discuss both production and decay processes

[1] A first public tool for Higgs PO is available in Ref. [38].

is summarized in Section 7, where we also address the reduction of the number of independent terms under specify symmetry assumptions (in particular CP conservation and flavor universality). Finally, a discussion about the matching between the PO approach and the SM Effective Field Theory (SMEFT) is presented in Sections 8. The latter section is not needed to discuss the PO implementation in data analyses, but it provides a bridge between this chapter of the YR (Measurements and Observables) and the one devoted to the EFT approaches.

## 2 Two-body decay modes

In the case of two-body Higgs decays into on-shell SM particles, namely $h \to f\bar{f}$ and $h \to \gamma\gamma$, the natural *physical PO* for each mode are the partial decay widths, and possibly the polarization asymmetry if the spin of the final state is accessible.

In the $h \to f\bar{f}$ case the main issue to be addressed is the optimal definition of the partial decay width taking into account the final state QED and QCD radiation.

In the $h \to \gamma\gamma$ case the point to be addressed is the extrapolation to real photons of electromagnetic showers with non-vanishing invariant mass.

### 2.1 $h \to f\bar{f}$

For each fermion species we can decompose the on-shell $h \to f\bar{f}$ amplitude in terms of two effective couplings ($y^f_{S,P}$), defined by

$$\mathscr{A}(h \to f\bar{f}) = -\frac{i}{\sqrt{2}}\left(y^f_S \bar{f} f + i y^f_P \bar{f}\gamma_5 f\right) . \qquad (1)$$

These couplings are real in the limit where we neglect re-scattering effects, that is an excellent approximation (also beyond the SM if we assume no new light states), for all the accessible $h \to f\bar{f}$ channels. If $h$ is a CP-even state (as in the SM), then $y^f_P$ is a CP-violating coupling.

In order to match our notation with the $\kappa$ framework [8], we define the two *effective couplings* PO of the $h \to f\bar{f}$ decays as follows:

$$\kappa_f = \frac{\mathrm{Re}(y^f_S)}{\mathrm{Re}(y^{f,\mathrm{SM}}_S)} , \qquad \lambda^{\mathrm{CP}}_f = \frac{\mathrm{Re}(y^f_P)}{\mathrm{Re}(y^{f,\mathrm{SM}}_S)} . \qquad (2)$$

Here $y^{f,\mathrm{SM}}_S$ is the SM effective coupling that provides the best SM prediction in the $\kappa_f \to 1$ and $\lambda^{\mathrm{CP}}_f \to 0$ limit.

The measurement of $\Gamma(h \to f\bar{f})_{(\text{incl})}$ determines the combination $|\kappa_f|^2 + |\lambda_f^{\text{CP}}|^2$, while the $\lambda_f^{\text{CP}}/\kappa_f$ ratio can be determined only if the lepton polarization is experimentally accessible. With this notation, the inclusive decay rates, computed assuming a pure bremsstrahlung spectrum can be written as

$$\Gamma(h \to f\bar{f})_{(\text{incl})} = \left[\kappa_f^2 + (\lambda_f^{\text{CP}})^2\right] \Gamma(h \to f\bar{f})^{(\text{SM})}_{(\text{incl})} , \tag{3}$$

where fermion-mass effects, of per-mil level even for the $b$ quark, have been neglected. In experiments $\Gamma(h \to f\bar{f})_{(\text{incl})}$ cannot be directly accessed, given tight cuts on the $f\bar{f}$ invariant mass to suppress the background: $\Gamma(h \to f\bar{f})_{(\text{incl})}$ is extrapolated from the experimentally accessible $\Gamma(h \to f\bar{f})_{(\text{cut})}$ assuming a pure bremsstrahlung spectrum, both as far as QED and as far as QCD (for the $q\bar{q}$ channels only) radiation is concerned.

The SM decay width is given by

$$\Gamma(h \to f\bar{f})^{(\text{SM})}_{(\text{incl})} = N_c^f \frac{|y_{\text{eff}}^{f,\text{SM}}|^2}{16\pi} m_H^2 , \tag{4}$$

where the color factor $N_c^f$ is 3 for quarks and 1 for leptons. Using the best SM prediction of the branching ratios in these channels [8], for $m_H = 125.0$ GeV and $\Gamma_H^{\text{tot}} = 4.07 \times 10^{-3}$ GeV, we extract the values of the $|y_{\text{eff}}^{f,\text{SM}}|$ couplings in Eq. (4):

| | $\bar{b}b$ | $\bar{\tau}\tau$ |
|---|---|---|
| $\mathscr{B}(h \to \bar{f}f)$ | $5.77 \times 10^{-1}$ | $6.32 \times 10^{-2}$ |
| $\lvert y_{\text{eff}}^{f,\text{SM}}\rvert$ | $1.77 \times 10^{-2}$ | $1.02 \times 10^{-2}$ |

,

| | $\bar{c}c$ | $\bar{\mu}\mu$ |
|---|---|---|
| $\mathscr{B}(h \to \bar{f}f)$ | $2.91 \times 10^{-2}$ | $2.19 \times 10^{-4}$ |
| $\lvert y_{\text{eff}}^{f,\text{SM}}\rvert$ | $3.98 \times 10^{-3}$ | $5.99 \times 10^{-4}$ |

,

As anticipated, the physical PO sensitive to $\lambda_f^{\text{CP}}/\kappa_f$ necessarily involve a determination (direct or indirect) of the fermion spins. Denoting by $\vec{k}_f$ the 3-momentum of the fermion $f$ in the Higgs center of mass frame, and with $\{\vec{s}_f, \vec{s}_{\bar{f}}\}$ the two fermion spins, we can define the following CP-odd asymmetry [10]

$$\mathscr{A}_f^{\text{CP}} = \frac{1}{|\vec{k}_f|} \langle \vec{k}_f \cdot (\vec{s}_f \times \vec{s}_{\bar{f}}) \rangle = -\frac{\lambda_f^{\text{CP}} \kappa_f}{\kappa_f^2 + (\lambda_f^{\text{CP}})^2} \tag{5}$$

As pointed out in Ref. [11], in the $h \to \tau^+\tau^- \to X_{\tau^+} X_{\tau^-}$ decay chains asymmetries proportional to $\mathscr{A}_f^{\text{CP}}$ are accessible through the measurement of the angular distribution of the $\tau^\pm$ decay products.

Note that, by construction, the effective couplings PO depend on the SM normalization. This imply an intrinsic theoretical uncertainty in their determination related to the theory error on the SM reference value. On the other hand, the physical PO are independent of any reference to the SM. Indeed the (conventional) SM normalization of $\kappa_f$ cancels in Eq. (3).

### 2.2 $h \to \gamma\gamma$

The general decomposition for the $h \to \gamma\gamma$ amplitude is

$$\mathscr{A}\left[h \to \gamma(q,\varepsilon)\gamma(q',\varepsilon')\right] = i\frac{2}{v_F}\varepsilon'_\mu \varepsilon_\nu \left[\varepsilon_{\gamma\gamma}(g^{\mu\nu}\; q\cdot q' - q^\mu q'^\nu) + \varepsilon^{\rm CP}_{\gamma\gamma}\varepsilon^{\mu\nu\rho\sigma} q_\rho q'_\sigma\right] , \tag{6}$$

from which we identify the two effective couplings $\varepsilon_{\gamma\gamma}$ and $\varepsilon^{\rm CP}_{\gamma\gamma}$ that, similarly to $y^f_{S,P}$, can be assumed to be real in the limit where we assume no new light states and small deviations from the SM limit. Here $v_F = (\sqrt{2}G_F)^{-1/2}$, and $G_F$ is the Fermi constant extracted from the muon decay. We define the effective couplings PO for this channels as

$$\kappa_{\gamma\gamma} = \frac{{\rm Re}(\varepsilon_{\gamma\gamma})}{{\rm Re}(\varepsilon^{\rm SM}_{\gamma\gamma})} , \qquad \lambda^{\rm CP}_{\gamma\gamma} = \frac{{\rm Re}(\varepsilon^{\rm CP}_{\gamma\gamma})}{{\rm Re}(\varepsilon^{\rm SM}_{\gamma\gamma})} , \tag{7}$$

where $\varepsilon^{\gamma\gamma}_{\rm SM}$ is the value of the PO which reproduces the best SM prediction of the decay width. By construction, the SM expectation for the two PO is $\kappa^{\rm SM}_{\gamma\gamma} = 1$ and $(\lambda^{\rm CP}_{\gamma\gamma})^{\rm SM} = 0$.

If the photon polarization is not accessible, the only physical PO for this channel is $\Gamma(h \to \gamma\gamma)$. Starting from realistic observables, where the electromagnetic showers have non-vanishing invariant mass, $\Gamma(h \to \gamma\gamma)$ is defined as the extrapolation to the limit of zero invariant mass for the electromagnetic showers. The relation between $\Gamma(h \to \gamma\gamma)$ and the two effective couplings PO is

$$\Gamma(h \to \gamma\gamma) = \left[\kappa^2_{\gamma\gamma} + (\lambda^{\rm CP}_{\gamma\gamma})^2\right]\Gamma(h \to \gamma\gamma)^{(\rm SM)} , \tag{8}$$

where

$$\Gamma(h \to \gamma\gamma)^{(\rm SM)} = \frac{|\varepsilon^{\rm SM,eff}_{\gamma\gamma}|^2}{16\pi}\frac{m_H^3}{v_F^2}. \tag{9}$$

Using the SM prediction for the branching ratios in two photons [8], for $v_F = 246.22$ GeV, $m_H = 125.0$ GeV and $\Gamma^{\rm tot}_H = 4.07 \times 10^{-3}$ GeV, we obtain

$$\mathscr{B}(h \to \gamma\gamma)^{\rm SM} = 2.28 \times 10^{-3} \quad \to \quad \varepsilon^{\gamma\gamma}_{\rm SM} = 3.8 \times 10^{-3} . \tag{10}$$

This value corresponds to the 1-loop contribution in the SM, which also fixes the relative sign. Similarly to the $f\bar{f}$ case, the SM normalization cancels in the definition of the physical PO.

The physical PO linear in the CP-violating coupling $\lambda^{\rm CP}_{\gamma\gamma}$ necessarily involves the measurement of the photon polarization and is therefore hardly accessible at the LHC (at least in a direct way, see for example []). Denoting by $\vec{q}_{1,2}$ the 3-momenta of the two photons in the centre of mass frame, and with $\vec{\varepsilon}_{1,2}$ the corresponding polarization vectors, we can define [to be checked]:

$$\mathscr{A}_{\gamma\gamma}^{\rm CP} = \frac{1}{m_h}\langle(\vec{q}_1-\vec{q}_2)\cdot(\vec{\varepsilon}_1\times\vec{\varepsilon}_2)\rangle = \frac{\lambda_{\gamma\gamma}^{\rm CP}\kappa_{\gamma\gamma}}{\kappa_{\gamma\gamma}^2+(\lambda_{\gamma\gamma}^{\rm CP})^2}\ . \tag{11}$$

## 3 Three-body decay modes

The guiding principle for the definition of PO in multi-body channels is the decomposition of the decay amplitudes in terms of contributions associated to a specific single-particle pole structure. In the absence of new light states, such poles are generated only by the exchange of the SM electroweak bosons ($\gamma$, $Z$, and $W$) or by hadronic resonances (whose contribution appears only beyond the tree level and is largely suppressed). Since positions and residues on the poles are gauge-invariant quantities, this decomposition satisfies the general requirements for the definitions of PO.

### 3.1 $h\to f\bar{f}\gamma$

The general form factor decomposition for these channels is

$$\mathscr{A}\left[h\to f(p_1)\bar{f}(p_2)\gamma(q,\varepsilon)\right] = i\frac{2}{v_F}\sum_{f=f_L,f_R}(\bar{f}\gamma_\mu f)\varepsilon_\nu\times$$
$$\times\left[F_T^{f\gamma}(p^2)(p{\cdot}q\, g^{\mu\nu}-q^\mu p^\nu)+F_{CP}^{f\gamma}(p^2)\varepsilon^{\mu\nu\rho\sigma}q_\rho p_\sigma\right], \tag{12}$$

where $p=p_1+p_2$. The form factors can be further decomposed as

$$F_T^{f\gamma}(p^2) = \varepsilon_{Z\gamma}\frac{g_Z^f}{P_Z(p^2)}+\varepsilon_{\gamma\gamma}\frac{eQ_f}{p^2}+\Delta_{f\gamma}^{\rm SM}(p^2)\ , \tag{13}$$

$$F_{CP}^{f\gamma}(p^2) = \varepsilon_{Z\gamma}^{\rm CP}\frac{g_Z^f}{P_Z(p^2)}+\varepsilon_{\gamma\gamma}^{\rm CP}\frac{eQ_f}{p^2}\ . \tag{14}$$

Here $g_Z^f$ are the effective PO describing on-shell $Z\to f\bar{f}$ decays[2] and $P_Z(q^2)=q^2-m_Z^2+im_Z\Gamma_Z$. In other words, we decompose the form factors identifying the physical poles associated to the $Z$ and $\gamma$ propagators. As in the $h\to\gamma\gamma$ case, we define $v_F=(\sqrt{2}G_F)^{-1/2}$, where $G_F$ is the Fermi constant extracted from the muon decay.

[2] We have absorbed a factor $g/\cos(\theta_W)$ with respect to the definition of the effective $Z$ couplings adopted at LEP-1, see Eq. (24).

The term $\Delta^{\rm SM}_{f\gamma}(p^2)$ denotes the remnant of the SM $h\to f\bar{f}\gamma$ loop function that is regular both in the limit $p^2\to 0$ and in the limit $p^2\to m_Z^2$. This part of the amplitude is largely subdominant (being not enhanced by a physical single-particle pole) and cannot receive non-standard contributions from operators of dimension up to 6 in the EFT approach to Higgs physics. For this reason it is fixed to its SM value.

In this channel we thus have four effective couplings PO, related to the four $\varepsilon_X$ terms in Eqs. (13) and (14), two of which are accessible also in $h\to 2\gamma$. Similarly to the $h\to 2\gamma$ case, it is convenient to define the PO normalizing them the corresponding reference SM values of the amplitudes. We thus define

$$\kappa_{Z\gamma}=\frac{{\rm Re}(\varepsilon_{Z\gamma})}{{\rm Re}(\varepsilon^{\rm SM}_{Z\gamma})}\,, \qquad \lambda^{\rm CP}_{Z\gamma}=\frac{{\rm Re}(\varepsilon^{\rm CP}_{Z\gamma})}{{\rm Re}(\varepsilon^{\rm SM}_{Z\gamma})}\,, \tag{15}$$

where the numerical value of the SM contribution $\varepsilon^{\rm SM}_{Z\gamma}$ is obtained from the best SM prediction for the $h\to Z\gamma$ decay width.

The simplest physical PO that can be extracted from this channel is $\Gamma(h\to Z\gamma)$, where both the $Z$ boson and the photon are on-shell. By construction, this can be written as

$$\Gamma(h\to Z\gamma)=\left[\kappa^2_{Z\gamma}+(\lambda^{\rm CP}_{Z\gamma})^2\right]\Gamma(h\to Z\gamma)^{\rm (SM)}\,, \tag{16}$$

where

$$\Gamma(h\to Z\gamma)^{\rm (SM)}=\frac{|\varepsilon^{\rm SM,eff}_{Z\gamma}|^2}{8\pi}\frac{m_H^3}{v^2}\left(1-\frac{m_Z^2}{m_H^2}\right)^3\,. \tag{17}$$

The SM prediction for this decay rate [8] provides the value of $\varepsilon^{Z\gamma}_{\rm SM}$:

$$\mathscr{B}(h\to Z\gamma)^{\rm (SM)}=1.54\times 10^{-3} \quad \to \quad \varepsilon^{\rm SM}_{Z\gamma}=6.9\times 10^{-3}\,. \tag{18}$$

The independent physical PO linear in the coupling $\lambda^{\rm CP}_{Z\gamma}$ is the following CP-odd asymmetry at the $Z$ peak [to be checked]:

$$\mathscr{A}^{\rm CP}_{Z\gamma}=\frac{1}{|\vec{p}||\vec{q}|}\langle\vec{p}\cdot(\vec{q}\times\vec{\varepsilon}_\gamma)\rangle\Big|_{(p^2=m_Z^2)}=\frac{\lambda^{\rm CP}_{Z\gamma}\kappa_{Z\gamma}}{\kappa^2_{Z\gamma}+(\lambda^{\rm CP}_{Z\gamma})^2}\,, \tag{19}$$

where all 3-momenta are defined in the Higgs center of mass frame.

This channel is also sensitive to $\Gamma(h\to\gamma\gamma)$ and $\mathscr{A}^{\rm CP}_{\gamma\gamma}$ via the effective couplings $\kappa_{\gamma\gamma}$ (or $\varepsilon_{\gamma\gamma}$) and $\lambda^{\rm CP}_{\gamma\gamma}$ (or $\varepsilon^{\rm CP}_{\gamma\gamma}$). Determining such couplings from a fit to the from factors in the low $p^2$ region, one can indirectly determine $\Gamma(h\to\gamma\gamma)$ and $\mathscr{A}^{\rm CP}_{\gamma\gamma}$ by means of Eq. (8) and Eq. (11), respectively.

## 4 Four-fermion decay modes

Similarly to the three-body modes, also in this case the guiding principle for the definition of PO is the decomposition of the decay amplitudes in terms of contributions associated to a specific pole structure. Such decomposition for the $h \to 4f$ channels has been presented in Ref. [2]. The effective coupling PO that appear in these channels consist of four sets:

- 3 flavor-universal charged-current PO: $\{\kappa_{WW}, \varepsilon_{WW}, \varepsilon_{WW}^{\rm CP}\}$;
- 7 flavor-universal neutral-current PO, 4 of which are appearing already in $h \to \gamma\gamma$ and $h \to f\bar{f}\gamma$: $\{\kappa_{\gamma\gamma}, \lambda_{\gamma\gamma}^{\rm CP}, \kappa_{Z\gamma}, \lambda_{Z\gamma}^{\rm CP}\}$, and another 3 which are specific for $h \to 4f$: $\{\kappa_{ZZ}, \varepsilon_{ZZ}, \varepsilon_{ZZ}^{\rm CP}\}$;
- the set of flavor non-universal charged-current PO: $\{\varepsilon_{Wf}\}$;
- the set of flavor non-universal neutral-current PO: $\{\varepsilon_{Zf}\}$.

While the number of flavor-universal PO is fixed, the number of flavor non-universal PO depend on the fermion species we are interested in. For instance, looking only at light leptons ($\ell = e, \mu$), we have 4 flavor non-universal PO contributing to $h \to 4\ell$ modes ($\varepsilon_{Zf}$, with $f = e_L, e_R, \mu_L, \mu_R$) and 4 PO contributing to $h \to 2\ell 2\nu$ modes ($\varepsilon_{We_L}, \varepsilon_{W\mu_L}, \varepsilon_{Z\nu_e}, \varepsilon_{Z\nu_\mu}$). The definition of these PO is done at the amplitude level, separating neutral-current and charged-current contributions to the $h \to 4f$ processes, as discussed below.

Starting from each of the effective couplings PO we can define a corresponding physical PO. In particular, $\Gamma(h \to ZZ)$ is defined as the (ideal) rate extracted from the full $\Gamma(h \to 4f)$, extrapolating the result in the limit $\kappa_{ZZ} \neq 0$ and all the other effective couplings set to zero. Similarly $\Gamma(h \to Zf\bar{f})$ is defined from the extrapolation in the limit $\varepsilon_{Zf} \neq 0$ and all the other effective couplings set to zero (see extended discussion below).

### 4.1 $h \to 4f$ neutral currents

Let us consider the case of two different (light) fermion species: $h \to f\bar{f} + f'\bar{f}'$. Neglecting helicity-violating terms (yielding contributions suppressed by light fermion masses in the rates), we can decompose the neutral-current contribution to the amplitude in the following way

$$\mathscr{A}_{n.c.}\left[h \to f(p_1)\bar{f}(p_2)f'(p_3)\bar{f}'(p_4)\right] = i\frac{2m_Z^2}{v_F}\sum_{f=f_L,f_R}\sum_{f'=f'_L,f'_R}(\bar{f}\gamma_\mu f)(\bar{f}'\gamma_\nu f')\mathscr{T}^{\mu\nu}_{n.c.}(q_1,q_2)$$

$$\mathscr{T}^{\mu\nu}_{n.c.}(q_1,q_2) = \left[F_L^{ff'}(q_1^2,q_2^2)g^{\mu\nu} + F_T^{ff'}(q_1^2,q_2^2)\frac{q_1\cdot q_2\, g^{\mu\nu} - q_2{}^\mu q_1{}^\nu}{m_Z^2}\right.$$
$$\left. + F_{CP}^{ff'}(q_1^2,q_2^2)\frac{\varepsilon^{\mu\nu\rho\sigma}q_{2\rho}q_{1\sigma}}{m_Z^2}\right], \qquad (20)$$

where $q_1 = p_1 + p_2$ and $q_2 = p_3 + p_4$. The form factor $F_L$ describes the interaction with the longitudinal part of the current, as in the SM, the $F_T$ term describes the interaction with the transverse part, while $F_{CP}$ describes the CP-violating part of the interaction (if the Higgs is assumed to be a CP-even state).

We can further expand the form factors in full generality around the poles, providing the definition of the neutral-current PO [2]:

$$F_L^{ff'}(q_1^2,q_2^2) = \kappa_{ZZ}\frac{g_Z^f g_Z^{f'}}{P_Z(q_1^2)P_Z(q_2^2)} + \frac{\varepsilon_{Zf}}{m_Z^2}\frac{g_Z^{f'}}{P_Z(q_2^2)} + \frac{\varepsilon_{Zf'}}{m_Z^2}\frac{g_Z^{f}}{P_Z(q_1^2)} + \Delta_L^{\rm SM}(q_1^2,q_2^2)\ , \tag{21}$$

$$F_T^{ff'}(q_1^2,q_2^2) = \varepsilon_{ZZ}\frac{g_Z^f g_Z^{f'}}{P_Z(q_1^2)P_Z(q_2^2)} + \varepsilon_{Z\gamma}\left(\frac{eQ_{f'}g_Z^f}{q_2^2 P_Z(q_1^2)} + \frac{eQ_f g_Z^{f'}}{q_1^2 P_Z(q_2^2)}\right) + \varepsilon_{\gamma\gamma}\frac{e^2 Q_f Q_{f'}}{q_1^2 q_2^2} + \Delta_T^{\rm SM}(q_1^2,q_2^2), \tag{22}$$

$$F_{CP}^{ff'}(q_1^2,q_2^2) = \varepsilon_{ZZ}^{\rm CP}\frac{g_Z^f g_Z^{f'}}{P_Z(q_1^2)P_Z(q_2^2)} + \varepsilon_{Z\gamma}^{\rm CP}\left(\frac{eQ_{f'}g_Z^f}{q_2^2 P_Z(q_1^2)} + \frac{eQ_f g_Z^{f'}}{q_1^2 P_Z(q_2^2)}\right) + \varepsilon_{\gamma\gamma}^{\rm CP}\frac{e^2 Q_f Q_{f'}}{q_1^2 q_2^2}\ . \tag{23}$$

Here $g_Z^f$ are $Z$-pole PO extracted from $Z$ decays at LEP-I, the translation to the notation used at LEP being very simple

$$g_Z^f = \frac{2m_Z}{v_F} g_f^{\rm LEP}\ , \qquad \textit{and} \qquad (g_Z^f)_{\rm SM} = \tfrac{2m_Z}{v_F}(T_3^f - Q_f s_{\theta_W}^2)\ . \tag{24}$$

As anticipated, all the parameters but $\varepsilon_{Zf}$ and $g_Z^f$ are flavor universal, i.e. they do not depend on the fermion species. In fact, flavor non-universal effects in $g_Z^f$ have been very tightly constrained at LEP, however, sizeable effects in $\varepsilon_{Zf}$ are possible and should be tested at the LHC. In the limit where we neglect re-scattering effects, both $\kappa_{ZZ}$ and $\varepsilon_X$ are real. The functions $\Delta_{L,T}^{\rm SM}(q_1^2,q_2^2)$ denote subleading non-local contributions that are regular both in the limit $q_{1,2}^2 \to 0$ and in the limit $q_{1,2}^2 \to m_Z^2$. As in the 3-body decay case, this part of the amplitude is largely subdominant and not affected by operators with dimension up to 6, therefore it is fixed it to its SM value.

### 4.2 $h \to 4f$ charged currents

Let us consider the $h \to \ell\bar{\nu}_\ell \bar{\ell}'\nu_{\ell'}$ process.[3] Employing the same assumptions used in the neutral current case, we can decompose the amplitude in the following way:

$$\mathscr{A}_{c.c.}\left[h \to \ell(p_1)\bar{\nu}_\ell(p_2)\nu_{\ell'}(p_3)\bar{\ell}'(p_4)\right] = i\frac{2m_W^2}{v_F}(\bar{\ell}_L\gamma_\mu \nu_{\ell L})(\bar{\nu}_{\ell' L}\gamma_\nu \ell'_L)\mathscr{T}_{c.c.}^{\mu\nu}(q_1,q_2)$$

[3] The analysis of a process involving quarks is equivalent, with the only difference that the $\varepsilon_{Wf}$ coefficients are in this case non-diagonal matrices in flavor space, as the $g_{ud}^W$ effective couplings.

$$\mathcal{T}^{\mu\nu}_{c.c.}(q_1,q_2) = \left[ G_L^{\ell\ell'}(q_1^2,q_2^2) g^{\mu\nu} + G_T^{\ell\ell'}(q_1^2,q_2^2) \frac{q_1{\cdot}q_2\, g^{\mu\nu} - {q_2}^{\mu} {q_1}^{\nu}}{m_W^2} \right.$$
$$\left. + G_{CP}^{\ell\ell'}(q_1^2,q_2^2) \frac{\varepsilon^{\mu\nu\rho\sigma} q_{2\rho} q_{1\sigma}}{m_W^2} \right] , \tag{25}$$

where $q_1 = p_1 + p_2$ and $q_2 = p_3 + p_4$. The decomposition of the form factors, that allows us to define the charged-current PO, is [2]

$$G_L^{\ell\ell'}(q_1^2,q_2^2) = \kappa_{WW} \frac{(g_W^{\ell})^* g_W^{\ell'}}{P_W(q_1^2) P_W(q_2^2)} + \frac{(\varepsilon_{W\ell})^*}{m_W^2} \frac{g_W^{\ell'}}{P_W(q_2^2)} + \frac{\varepsilon_{W\ell'}}{m_W^2} \frac{(g_W^{\ell})^*}{P_W(q_1^2)} , \tag{26}$$

$$G_T^{\ell\ell'}(q_1^2,q_2^2) = \varepsilon_{WW} \frac{(g_W^{\ell})^* g_W^{\ell'}}{P_W(q_1^2) P_W(q_2^2)} , \tag{27}$$

$$G_{CP}^{\ell\ell'}(q_1^2,q_2^2) = \varepsilon_{WW}^{\rm CP} \frac{(g_W^{\ell})^* g_W^{\ell'}}{P_W(q_1^2) P_W(q_2^2)} , \tag{28}$$

where $P_W(q^2)$ is the $W$ propagator defined analogously to $P_Z(q^2)$ and $g_W^f$ are the effective couplings describing on-shell $W$ decays (we have absorbed a factor of $g$ compared to standard notations). In the SM,

$$(g_W^{ik})_{\rm SM} = \frac{g}{\sqrt{2}} V_{ik} , \tag{29}$$

where $V$ is the CKM mixing matrix.[4] In absence of rescattering effects, the Hermiticity of the underlying effective Lagrangian implies that $\kappa_{WW}$, $\varepsilon_{WW}$ and $\varepsilon_{WW}^{\rm CP}$ are real couplings, while $\varepsilon_{W\ell}$ can be complex.

### 4.3 $h \to 4f$ complete decomposition

The complete decomposition of a generic $h \to 4f$ amplitude is obtained combining neutral- and charged-current contributions depending on the nature of the fermions involved. For instance $h \to 2e2\mu$ and $h \to \ell\bar{\ell}q\bar{q}$ decays are determined by a single neutral current amplitude, while the case of two identical lepton pairs is obtained from Eq. (20) taking into account the proper (anti-)symmetrization of the amplitude:

$$\mathcal{A}\left[h \to \ell(p_1)\bar{\ell}(p_2)\ell(p_3)\bar{\ell}(p_4)\right] = \mathcal{A}_{n.c.}\left[h \to f(p_1)\bar{f}(p_2)f'(p_3)\bar{f}'(p_4)\right]_{f=f'=\ell}$$
$$- \mathcal{A}_{n.c.}\left[h \to f(p_1)\bar{f}(p_4)f'(p_3)\bar{f}'(p_2)\right]_{f=f'=\ell} . \tag{30}$$

The $h \to e^{\pm}\mu^{\mp}\nu\bar{\nu}$ decays receive contributions from a single charged-current amplitude, while in the $h \to \ell\bar{\ell}\nu\bar{\nu}$ case we have to sum charged and neutral-current contributions:

$$\mathcal{A}\left[h \to \ell(p_1)\bar{\ell}(p_2)\nu(p_3)\bar{\nu}(p_4)\right] = \mathcal{A}_{n.c.}\left[h \to \ell(p_1)\bar{\ell}(p_2)\nu(p_3)\bar{\nu}(p_4)\right]$$
$$- \mathcal{A}_{c.c.}\left[h \to \ell(p_1)\bar{\nu}(p_4)\nu(p_3)\bar{\ell}(p_2)\right] . \tag{31}$$

[4] More precisely, $(g_W^{ik})_{\rm SM} = \frac{g}{\sqrt{2}} V_{ik}$ if $i$ and $k$ refers to left-handed quarks, otherwise $(g_W^{ik})_{\rm SM} = 0$.

### 4.4 Physical PO for $h \to 4\ell$

To define the idealised physical PO we start with the quadratic terms for each of the form factors in Eqs. (21-23), and compute their contribution to the double differential decay rate for $h \to e^+e^-\mu^+\mu^-$ (for $\kappa_{ZZ}$, $\varepsilon_{ZZ}$ and $\varepsilon_{ZZ}^{\rm CP}$) and for $h \to Z\ell^+\ell^-$ (for the contact terms $\varepsilon_{Z\ell}$).

*Decay channel $h \to e^+e^-\mu^+\mu^-$*

We choose this particular decay channel for the (conventional) definition of the physical PO because it depends on all the PO relevant for $h \to 4\ell$ and because it does not contain interference between the two fermion currents as in Eq. (30). The independent contributions of the three form factors to the decay rate are:

$$\begin{aligned}
\frac{d\Gamma^{\rm LL}}{dm_1 dm_2} &= \frac{\lambda_p \beta_{10}}{2304\pi^5}\frac{m_Z^4 m_h^3}{v_F^2} m_1 m_2 \sum_{f,f'} \left| F_{\rm L}^{ff'} \right|^2 , \\
\frac{d\Gamma^{\rm TT}}{dm_1 dm_2} &= \frac{\lambda_p \beta_4}{1152\pi^5}\frac{m_h^3}{v_F^2} m_1^3 m_2^3 \sum_{f,f'} \left| F_{\rm T}^{ff'} \right|^2 , \\
\frac{d\Gamma^{\rm CP}}{dm_1 dm_2} &= \frac{\lambda_p \beta_2}{1152\pi^5}\frac{m_h^3}{v_F^2} m_1^3 m_2^3 \sum_{f,f'} \left| F_{\rm CP}^{ff'} \right|^2 ,
\end{aligned} \tag{32}$$

where $f = e_L, e_R$, $f' = \mu_L, \mu_R$, $m_{1(2)} \equiv \sqrt{q_{1(2)}^2}$ and

$$\lambda_p = \sqrt{1 + \left(\frac{m_1^2 - m_2^2}{m_h^2}\right)^2 - 2\frac{m_1^2 + m_2^2}{m_h^2}} , \qquad \beta_N = 1 + \frac{m_1^4 + N m_1^2 m_2^2 + m_2^4}{m_h^4} - 2\frac{m_1^2 + m_2^2}{m_h^2} . \tag{33}$$

Inside each term of the type $\sum_{f,f'} \left| F_i^{ff'} \right|^2$, we extract only the quadratic terms in each PO. By integrating in $m_1$ and $m_2$ we obtain the partial decay rates as given by each PO separately (in the limit where the others are negligible):

$$\begin{aligned}
\Gamma(h \to 2e2\mu)[\kappa_{ZZ}] &= 4.929 \times 10^{-2} (|g_{Ze_L}|^2 + |g_{Ze_R}|^2)(|g_{Z\mu_L}|^2 + |g_{Z\mu_R}|^2)\ |\kappa_{ZZ}|^2 \text{ MeV} \\
\Gamma(h \to 2e2\mu)[\varepsilon_{ZZ}] &= 4.458 \times 10^{-3} (|g_{Ze_L}|^2 + |g_{Ze_R}|^2)(|g_{Z\mu_L}|^2 + |g_{Z\mu_R}|^2)\ |\varepsilon_{ZZ}|^2 \text{ MeV} \\
\Gamma(h \to 2e2\mu)[\varepsilon_{ZZ}^{\rm CP}] &= 1.884 \times 10^{-3} (|g_{Ze_L}|^2 + |g_{Ze_R}|^2)(|g_{Z\mu_L}|^2 + |g_{Z\mu_R}|^2)\ |\varepsilon_{ZZ}^{\rm CP}|^2 \text{ MeV}
\end{aligned} \tag{34}$$

The numerical coefficients in Eq. (34) have been obtained neglecting QED corrections. The latter must be included at the simulation level by appropriate QED showering programs, such as PHOTOS [12]. As shown in Ref. [13]: the impact of such corrections is negligible after integrating over the full phase space, hence in the overall normalization of the partial rates in Eq. (34), while they can provide sizable distortions of the spectra in specific phase-space regions.

Since each effective coupling PO correspond to a well-defined pole contribution to the amplitude (with one or two poles of the $Z$ boson), and a well-defined Lorentz and flavor structure, we can associate to the those partial rates a well-defined physical meaning. In particular, we define the following *physical PO* for the $h \to 4\ell$ decays:

$$\begin{aligned} \Gamma(h \to Z_L Z_L) &\equiv \frac{\Gamma(h \to 2e2\mu)[\kappa_{ZZ}]}{\mathscr{B}(Z \to 2e)\mathscr{B}(Z \to 2\mu)} = 0.209\ |\kappa_{ZZ}|^2 \text{ MeV} \\ \Gamma(h \to Z_T Z_T) &\equiv \frac{\Gamma(h \to 2e2\mu)[\varepsilon_{ZZ}]}{\mathscr{B}(Z \to 2e)\mathscr{B}(Z \to 2\mu)} = 0.0189\ |\varepsilon_{ZZ}|^2 \text{ MeV} \\ \Gamma^{\text{CPV}}(h \to Z_T Z_T) &\equiv \frac{\Gamma(h \to 2e2\mu)[\varepsilon_{ZZ}^{\text{CP}}]}{\mathscr{B}(Z \to 2e)\mathscr{B}(Z \to 2\mu)} = 0.00799\ |\varepsilon_{ZZ}^{\text{CP}}|^2 \text{ MeV} \end{aligned} \tag{35}$$

where, due to the double pole structure of the amplitude, we have removed the (physical) branching ratios of the $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ decays. Here

$$\mathscr{B}(Z \to 2\ell) = \frac{\Gamma_0}{\Gamma_Z} R^\ell \left( (g_Z^{\ell_L})^2 + (g_Z^{\ell_R})^2 \right) \simeq 0.4856 \left( (g_Z^{\ell_L})^2 + (g_Z^{\ell_R})^2 \right) , \tag{36}$$

where $\Gamma_0 = \frac{m_Z}{24\pi}$, $\Gamma_Z$ is the total decay width and $R^\ell = \left(1 + \frac{3}{4\pi}\alpha(m_Z)\right)$ describes final state QED radiation.

*Decay channel $h \to Z\ell^+\ell^-$*

The idealised physical PO related to the contact terms can be defined directly from the on-shell decay $h \to Z\ell^+\ell^-$, where $\ell = e_L, e_R, \mu_L, \mu_R$ and the $Z$ boson is assumed to be on-shell (narrow width approximation). We compute this decay rate, neglecting QED corrections and light lepton masses, in presence of the contact terms $\varepsilon_{Z\ell}$ only. The Dalitz double differential rate in $s_{12} \equiv (p_{\ell^+} + p_{\ell^-})^2$ and $s_{23} \equiv (p_{\ell^-} + p_Z)^2$ is

$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32 m_h^2} \frac{4|\varepsilon_{Z\ell}|^2}{v^2} \left( s_{12} + \frac{(s_{23} - m_Z^2)(m_h^2 - s_{12} - s_{23})}{m_Z^2} \right) , \tag{37}$$

The allowed kinematical region is $0 < s_{12} < (m_h - m_Z)^2$ and, for any given value of $s_{12}$, $s_{23}^{\text{min}} < s_{23} < s_{23}^{\text{Max}}$ with

$$s_{23}^{\text{min(Max)}} = (E_2^* + E_Z^*)^2 - \left( E_2^* \pm \sqrt{(E_Z^*)^2 - m_Z^2} \right)^2 , \tag{38}$$

where $E_2^* = \sqrt{s_{12}}/2$ and $E_Z^* = \frac{m_h^2 - s_{12} - m_Z^2}{2\sqrt{s_{12}}}$. The total decay width defines the relation between the physical PO and the effective couplings PO as:

$$\Gamma(h \to Z\ell^+\ell^-) = 0.0366|\varepsilon_{Z\ell}|^2 \text{ MeV} . \tag{39}$$

Together with the physical PO already defined for $h \to \gamma\gamma$ and $h \to Z\gamma$, we have thus established a complete mapping between the effective couplings PO and the physical PO appearing in $h \to 4\ell$ decays.

### 4.5 Physical PO for $h \to 2\ell 2\nu$

Physical observables for charged-current processes can be defined in a very similar way as the neutral-current ones. In particular, we use the $h \to e^+\nu_e\mu^-\bar{\nu}_\mu$ process for the physical PO corresponding to $k_{WW}$, $\varepsilon_{WW}$, and $\varepsilon_{WW}^{\rm CP}$, and $h \to W^+\ell\bar{\nu}_\ell$ for the contact terms.

*Decay channel* $h \to e^+\nu_e\mu^-\bar{\nu}_\mu$

Integrating the differential distributions analogous to Eq. (32) we obtain the expression of the total decay rate in this channel, in the limit where only one PO is turned on:

$$\begin{aligned}
\Gamma(h \to e\mu 2\nu)[\kappa_{WW}] &= 2.20 \times 10^{-4}|g_{We_L}|^2|g_{W\mu_L}|^2\ |\kappa_{WW}|^2 \text{ MeV} \\
\Gamma(h \to e\mu 2\nu)[\varepsilon_{WW}] &= 4.27 \times 10^{-5}|g_{We_L}|^2|g_{W\mu_L}|^2\ |\varepsilon_{WW}|^2 \text{ MeV} \\
\Gamma(h \to e\mu 2\nu)[\varepsilon_{WW}^{\rm CP}] &= 1.77 \times 10^{-5}|g_{We_L}|^2|g_{W\mu_L}|^2\ |\varepsilon_{WW}^{\rm CP}|^2 \text{ MeV}
\end{aligned} \tag{40}$$

As in the neutral channel, the *physical PO* are defined from these quantities by factorizing the $W$ branching ratios:

$$\begin{aligned}
\Gamma(h \to W_L W_L) &\equiv \frac{\Gamma(h \to e\mu 2\nu)[\kappa_{WW}]}{\mathscr{B}(W \to e\bar{\nu}_e)\mathscr{B}(W \to \mu\bar{\nu}_\mu)} = 0.841\ |\kappa_{WW}|^2 \text{ MeV} \\
\Gamma(h \to W_T W_T) &\equiv \frac{\Gamma(h \to e\mu 2\nu)[\varepsilon_{WW}]}{\mathscr{B}(W \to e\bar{\nu}_e)\mathscr{B}(W \to \mu\bar{\nu}_\mu)} = 0.163\ |\varepsilon_{WW}|^2 \text{ MeV} \\
\Gamma^{\rm CPV}(h \to W_T W_T) &\equiv \frac{\Gamma(h \to e\mu 2\nu)[\varepsilon_{WW}^{\rm CP}]}{\mathscr{B}(W \to e\bar{\nu}_e)\mathscr{B}(W \to \mu\bar{\nu}_\mu)} = 0.0677\ |\varepsilon_{WW}^{\rm CP}|^2 \text{ MeV} .
\end{aligned} \tag{41}$$

The $W$ branching ratios are given by

$$\mathscr{B}(W \to \ell\bar{\nu}_\ell) = \frac{\Gamma_0}{\Gamma_W}(g_{W\ell_L})^2 \simeq 0.511(g_{W\ell_L})^2 , \tag{42}$$

where $\Gamma_0 = \frac{m_W}{24\pi}$, $\Gamma_W$ is the total decay width.

| PO | Physical PO | Relation to the eff. coupl. |
|---|---|---|
| $\kappa_f,\ \lambda_f^{\rm CP}$ | $\Gamma(h\to f\bar f)$ | $= \Gamma(h\to f\bar f)^{\rm (SM)}[(\kappa_f)^2+(\lambda_f^{\rm CP})^2]$ |
| $\kappa_{\gamma\gamma},\ \lambda_{\gamma\gamma}^{\rm CP}$ | $\Gamma(h\to\gamma\gamma)$ | $= \Gamma(h\to\gamma\gamma)^{\rm (SM)}[(\kappa_{\gamma\gamma})^2+(\lambda_{\gamma\gamma}^{\rm CP})^2]$ |
| $\kappa_{Z\gamma},\ \lambda_{Z\gamma}^{\rm CP}$ | $\Gamma(h\to Z\gamma)$ | $= \Gamma(h\to Z\gamma)^{\rm (SM)}[(\kappa_{Z\gamma})^2+(\lambda_{Z\gamma}^{\rm CP})^2]$ |
| $\kappa_{ZZ}$ | $\Gamma(h\to Z_LZ_L)$ | $= (0.209\ {\rm MeV})\times\vert\kappa_{ZZ}\vert^2$ |
| $\epsilon_{ZZ}$ | $\Gamma(h\to Z_TZ_T)$ | $= (1.9\times10^{-2}\ {\rm MeV})\times\vert\epsilon_{ZZ}\vert^2$ |
| $\epsilon_{ZZ}^{\rm CP}$ | $\Gamma^{\rm CPV}(h\to Z_TZ_T)$ | $= (8.0\times10^{-3}\ {\rm MeV})\times\vert\epsilon_{ZZ}^{\rm CP}\vert^2$ |
| $\epsilon_{Zf}$ | $\Gamma(h\to Zf\bar f)$ | $= (3.7\times10^{-2}\ {\rm MeV})\times N_c^f\ \vert\epsilon_{Zf}\vert^2$ |
| $\kappa_{WW}$ | $\Gamma(h\to W_LW_L)$ | $= (0.84\ {\rm MeV})\times\vert\kappa_{WW}\vert^2$ |
| $\epsilon_{WW}$ | $\Gamma(h\to W_TW_T)$ | $= (0.16\ {\rm MeV})\times\vert\epsilon_{WW}\vert^2$ |
| $\epsilon_{WW}^{\rm CP}$ | $\Gamma^{\rm CPV}(h\to W_TW_T)$ | $= (6.8\times10^{-2}\ {\rm MeV})\times\vert\epsilon_{WW}^{\rm CP}\vert^2$ |
| $\epsilon_{Wf}$ | $\Gamma(h\to Wf\bar f')$ | $= (0.14\ {\rm MeV})\times N_c^f\ \vert\epsilon_{Wf}\vert^2$ |

Table 1: Summary of the *effective coupling* PO and the corresponding *physical* PO. The parameter $N_c^f$ is 1 for leptons and 3 for quarks. In the case of the charged-current contact term, $f'$ is the $SU(2)_L$ partner of the fermion $f$.

*Decay channel $h\to W^+\ell\bar\nu_\ell$*

Also in this case the physical PO corresponding to the charged-current contact terms are defined in complete analogy to the neutral-current case, starting from the 3-body decay $h\to W^+\ell\bar\nu_\ell$. The total decay width computed in the limit where only the contact term PO is switched on defines the relation between the physical PO and the effective couplings PO as:

$$\Gamma(h\to W^+\ell\bar\nu_\ell) = 0.143|\epsilon_{W\ell}|^2\ {\rm MeV}\ . \tag{43}$$

## 5 PO in Higgs electroweak production: generalities

The PO decomposition of $h\to 4f$ amplitude discussed above can naturally be generalized to describe electroweak Higgs-production processes, namely Higgs-production via vector-boson fusion (VBF) and Higgs-production in association with a massive SM gauge boson (VH).

The interest of such production processes is twofold. On the one hand, they are closely connected to the $h\to 4\ell, 2\ell 2\nu$ decay processes by crossing symmetry, and by the exchange of lepton currents into quark currents. As a result, some of the Higgs PO necessary to describe the

$h \to 4\ell, 2\ell 2\nu$ decay kinematics appear also in the description of the VBF and VH cross sections (independently of the Higgs decay mode). This facts opens the possibility of combined analyses of production cross sections and differential decay distributions, with a significant reduction on the experimental error on the extraction of the PO. On the other hand, the production cross sections allow to explore different kinematical regimes compared to the decays. By construction, the momentum transfer appearing in the Higgs decay amplitudes is limited by the Higgs mass, while such limitation is not present in the production amplitudes. The higher energies probed in the production processes provide an increased sensitivity to new physics effects. This fact also allows to test the momentum expansion that is intrinsic in the PO decomposition, as well as in any effective field theory approach to physics beyond the SM.

Despite the similarities at the fundamental level, the phenomenological description of VBF and VH in terms of PO is significantly more challenging compared to that of Higgs decays. On the one hand, QCD corrections plays a non-negligible role in the production processes. Although technically challenging, this fact does not represent a conceptual problem for the PO approach: the leading QCD corrections factorize in VBF and VH, similarly to the factorization of QED corrections in $h \to 4\ell$. This implies that NLO QCD corrections can be incorporated in general terms with suitable modifications of the existing Montecarlo tools. On the other hand, the relation between the kinematical variables at the basis of the PO decomposition (i.e. the momentum transfer of the partonic currents, $q^2$) and the kinematical variables accessible in $pp$ collisions is not straightforward, especially in the VBF case. This problem finds a natural solution in the VBF case due to strong correlation between $q^2$ and the $p_T$ of the VBF tagged jets, while in the VH case invariant mass of the VH system is correlated to the vector $p_T$.

### 5.1 Amplitude decomposition

Neglecting the light fermion masses, the electroweak production processes VH and VBF or, more precisely, the electroweak partonic amplitudes $f_1 f_2 \to h + f_3 f_4$, can be completely described by the three-point correlation function of the Higgs boson and two (color-less) fermion currents

$$\langle 0 | \mathscr{T} \left\{ J_f^\mu(x), J_{f'}^\nu(y), h(0) \right\} | 0 \rangle \ , \tag{44}$$

where all the states involved are on-shell. The same correlation function controls also the four-fermion Higgs decays discussed above. In the $h \to 4\ell, 2\ell 2\nu$ case both currents are leptonic and all fermions are in the final state. In case of VH associate production one of the currents describes the initial state quarks, while the other describes the decay products of the (nearly on-shell) vector boson. Finally, in VBF production the currents are not in the $s$-channel as in the previous cases, but in the $t$-channel. Strictly speaking, in VH and VBF the quark states are not on-shell; however, their off-shellness of order $\Lambda_{QCD}$ can be safely neglected compared to the electroweak scale characterizing the process (both within and beyond the SM).

As in the $h \to 4f$ case, we can expand the correlation function in Eq. (44) around the known physical poles due to the propagation of intermediate SM electroweak gauge bosons. The PO are then defined by the residues on the poles and by the non-resonant terms in this expansion. By construction, terms corresponding to a double pole structure are independent from the nature of the fermion current involved. As a result, the corresponding PO are universal and can be extracted from any of the above mention processes, both in production and in decays [7].

### 5.1.1 Vector boson fusion Higgs production

Higgs production via vector boson fusion (VBF) receives contribution both from neutral- and charged-current channels. Also, depending on the specific partonic process, there could be two different ways to construct the two currents, and these two terms interfere with each other. For example, for $uu \to uuh$ one has the interference between two neutral-current processes, while in $ud \to udh$ the interference is between neutral and charged currents. In this case it is clear that one should sum the two amplitudes with the proper symmetrization, as done in the case of $h \to 4e$.

We now proceed describing how each of these amplitudes can be parametrized in terms of PO. Let us start with the neutral-current one. The amplitude for the on-shell process $q_i(p_1)q_j(p_2) \to q_i(p_3)q_j(p_4)h(k)$ can be parametrized by

$$\mathscr{A}_{n.c}(q_i(p_1)q_j(p_2) \to q_i(p_3)q_j(p_4)h(k)) = i\frac{2m_Z^2}{v}\bar{q}_i(p_3)\gamma_\mu q_i(p_1)\bar{q}_j(p_4)\gamma_\nu q_j(p_2)\mathscr{T}^{\mu\nu}_{n.c.}(q_1,q_2), \tag{45}$$

where $q_1 = p_1 - p_3$, $q_2 = p_2 - p_4$ and $\mathscr{T}^{\mu\nu}_{n.c.}(q_1,q_2)$ is the same tensor structure appearing in $h \to 4f$ decays. Indeed, proceeding as in Eq. (20), using Lorentz invariance we decompose this tensor structure in term of three from factors:

$$\mathscr{T}^{\mu\nu}_{n.c.}(q_1,q_2) = \Bigg[F_L^{q_iq_j}(q_1^2,q_2^2)g^{\mu\nu} + F_T^{q_iq_j}(q_1^2,q_2^2)\frac{q_1\cdot q_2\, g^{\mu\nu} - q_2{}^\mu q_1{}^\nu}{m_Z^2} + F_{CP}^{q_iq_j}(q_1^2,q_2^2)\frac{\varepsilon^{\mu\nu\rho\sigma}q_{2\rho}q_{1\sigma}}{m_Z^2}\Bigg]\ . \tag{46}$$

Similarly, the charged-current contribution to the amplitude for the on-shell process $u_i(p_1)d_j(p_2) \to d_k(p_3)u_l(p_4)h(k)$ can be parametrized by

$$\mathscr{A}_{c.c}(u_i(p_1)d_j(p_2) \to d_k(p_3)u_l(p_4)h(k)) = i\frac{2m_W^2}{v}\bar{d}_k(p_3)\gamma_\mu u_i(p_1)\bar{u}_l(p_4)\gamma_\nu d_j(p_2)\mathscr{T}^{\mu\nu}_{c.c.}(q_1,q_2), \tag{47}$$

where, again, $\mathscr{T}^{\mu\nu}_{c.c.}(q_1,q_2)$ is the same tensor structure appearing in the charged-current $h \to 4f$ decays:

$$\mathscr{T}^{\mu\nu}_{c.c.}(q_1,q_2) = \Bigg[G_L^{ijkl}(q_1^2,q_2^2)g^{\mu\nu} + G_T^{ijkl}(q_1^2,q_2^2)\frac{q_1\cdot q_2\, g^{\mu\nu} - q_2{}^\mu q_1{}^\nu}{m_W^2}$$

$$+G_{CP}^{ijkl}(q_1^2,q_2^2)\frac{\varepsilon^{\mu\nu\rho\sigma}q_{2\rho}q_{1\sigma}}{m_W^2}\Bigg] \tag{48}$$

The amplitudes for the processes with initial anti-quarks can easily be obtained from the above ones.

The next step is to perform a momentum expansion of the form factors around the physical poles due to the propagation of SM electroweak gauge bosons ($\gamma$, $Z$ and $W^{\pm}$), and to define the PO (i.e. the set $\{\kappa_i,\varepsilon_i\}$) from the residues of such poles. We stop this expansion neglecting terms which can be generated only by local operators with dimension higher than six. A discussion about limitations and consistency checks of this procedure will be presented later on. The decomposition of the form factors closely follows the procedure already introduced for the decay amplitudes and will not be repeated here. We report explicitly only expression of the longitudinal form factors, where the contact terms not accessible in the leptonic decays appear:

$$\begin{aligned} F_L^{q_iq_j}(q_1^2,q_2^2) &= \kappa_{ZZ}\frac{g_Z^{q_i}g_Z^{q_j}}{P_Z(q_1^2)P_Z(q_2^2)}+\frac{\varepsilon_{Zq_i}}{m_Z^2}\frac{g_Z^{q_j}}{P_Z(q_2^2)}+\frac{\varepsilon_{Zq_j}}{m_Z^2}\frac{g_Z^{q_i}}{P_Z(q_1^2)}+\Delta_{L,n.c.}^{\rm SM}(q_1^2,q_2^2)\ , \\ G_L^{ijkl}(q_1^2,q_2^2) &= \kappa_{WW}\frac{g_W^{ik}g_W^{jl}}{P_W(q_1^2)P_W(q_2^2)}+\frac{\varepsilon_{Wik}}{m_W^2}\frac{g_W^{jl}}{P_W(q_2^2)}+\frac{\varepsilon_{Wjl}}{m_W^2}\frac{g_W^{ik}}{P_W(q_1^2)}+\Delta_{L,c.c}^{\rm SM}(q_1^2,q_2^2)\ . \end{aligned} \tag{49}$$

Here $P_V(q^2)=q^2-m_V^2+im_V\Gamma_V$, while $g_Z^f$ and $g_W^{ik}$ are the PO characterizing the on-shell couplings of $Z$ and $W$ boson to a pair of fermions, see Eqs. (24) and (29). The functions $\Delta_{L,n.c.(c.c.)}^{\rm SM}(q_1^2,q_2^2)$ denote non-local contributions generated at the one-loop level (and encoding multi-particle cuts) that cannot be re-absorbed in the definition of $\kappa_i$ and $\varepsilon_i$. At the level of precision we are working, taking into account also the high-luminosity phase of the LHC, these contributions can be safely fixed to their SM values.

As anticipated, the crossing symmetry between $h\to 4f$ and $2f\to h2f$ amplitudes ensures that the PO are the same in production and decay (if the same fermions species are involved). The amplitudes are explored in different kinematical regimes in the two type of processes (in particular the momentum-transfers, $q_{1,2}^2$, are space-like in VBF and time-like in $h\to 4f$). However, this does not affect the definition of the PO. This implies that the fermion-independent PO associated to a double pole structure, such as $\kappa_{ZZ}$ and $\kappa_{WW}$ in Eq. (49), are expected to be measured with higher accuracy in $h\to 4\ell$ and $h\to 2\ell 2\nu$ rather than in VBF. On the contrary, VBF is particularly useful to constrain the fermion-dependent contact terms $\varepsilon_{Zq_i}$ and $\varepsilon_{Wu_id_j}$, that appear only in the longitudinal form factors.

### *5.1.2 Associated vector boson plus Higgs production*

The VH production process denote the production of a Higgs boson with a nearly on-shell massive vector boson ($W$ or $Z$). For simplicity, in the following we will assume that the vector boson is on-shell and that the interference with the VBF amplitude can be neglected. However, we stress that the PO formalism clearly allow to describe both these effects (off-shell $V$ and interference with VBF in case of $V \to \bar{q}q$ decay) simply applying the general decomposition of neutral- and charged-current amplitudes as outlined above.

Similarly to VBF, Lorentz invariance allows us to decompose the amplitudes for the on-shell processes $q_i(p_1)\bar{q}_i(p_2) \to h(p)Z(k)$ and $u_i(p_1)\bar{d}_j(p_2) \to h(p)W^+(k)$ in three possible tensor structures: a longitudinal one, a transverse one, and a CP-odd one,

$$\begin{aligned} \mathscr{A}(q_i(p_1)\bar{q}_i(p_2) \to h(p)Z(k)) = i\frac{2m_Z^2}{v}\bar{q}_i(p_2)\gamma_\nu q_i(p_1)\varepsilon_\mu^{Z*}(k)\times \\ \times\left[F_L^{q_iZ}(q^2)g^{\mu\nu} + F_T^{q_iZ}(q^2)\frac{-(q\cdot k)g^{\mu\nu}+q^\mu k^\nu}{m_Z^2} + F_{CP}^{q_iZ}(q^2)\frac{\varepsilon^{\mu\nu\alpha\beta}q_\alpha k_\beta}{m_Z^2}\right], \end{aligned} \tag{50}$$

$$\begin{aligned} \mathscr{A}(u_i(p_1)\bar{d}_j(p_2) \to h(p)W^+(k)) = i\frac{2m_W^2}{v}\bar{d}_j(p_2)\gamma_\nu u_i(p_1)\varepsilon_\mu^{W*}(k)\times \\ \times\left[G_L^{q_{ij}W}(q^2)g^{\mu\nu} + G_T^{q_{ij}W}(q^2)\frac{-(q\cdot k)g^{\mu\nu}+q^\mu k^\nu}{m_W^2} + G_{CP}^{q_{ij}W}(q^2)\frac{\varepsilon^{\mu\nu\alpha\beta}q_\alpha k_\beta}{m_W^2}\right], \end{aligned} \tag{51}$$

where $q = p_1 + p_2 = k + p$. In the limit where we neglect the off-shellness of the final-state $V$, the form factors depend only on $q^2$. Already from this decomposition of the amplitude it is clear the importance of providing measurements of the differential cross-section as a function of $q^2$, as well as differential measurements in terms of the angular variables that allow to disentangle the different tensor structures.

Performing the momentum expansion of the form factors around the physical poles, and defining the PO as in Higgs decays and VBF, we find

$$\begin{aligned} F_L^{q_iZ}(q^2) &= \kappa_{ZZ}\frac{g_{Zq_i}}{P_Z(q^2)} + \frac{\varepsilon_{Zq_i}}{m_Z^2} \qquad & G_L^{q_{ij}W}(q^2) &= \kappa_{WW}\frac{(g_W^{u_id_j})^*}{P_W(q^2)} + \frac{\varepsilon^*_{Wu_id_j}}{m_W^2} \\ F_T^{q_iZ}(q^2) &= \varepsilon_{ZZ}\frac{g_{Zq_i}}{P_Z(q^2)} + \varepsilon_{Z\gamma}\frac{eQ_q}{q^2} \qquad & G_T^{q_{ij}W}(q^2) &= \varepsilon_{WW}\frac{(g_W^{u_id_j})^*}{P_W(q^2)} \\ F_{CP}^{q_iZ}(q^2) &= \varepsilon_{ZZ}^{\mathrm{CP}}\frac{g_{Zq_i}}{P_Z(q^2)} - \varepsilon_{Z\gamma}^{\mathrm{CP}}\frac{eQ_q}{q^2} \qquad & G_{CP}^{q_{ij}W}(q^2) &= \varepsilon_{WW}^{\mathrm{CP}}\frac{(g_W^{u_id_j})^*}{P_W(q^2)} \end{aligned} \tag{52}$$

where we have omitted the indication of the (tiny) non-local terms, fixed to their corresponding SM values. As in the VBF case, only the longitudinal form factors $F_L$ and $G_L$ contain PO not accessible in the leptonic decays, namely the quark contact terms $\varepsilon_{Zq_i}$ and $\varepsilon_{Wu_id_j}$.

## 6 PO in Higgs electroweak production: phenomenology

### 6.1 Vector Boson Fusion

At the parton level (i.e. in the $qq \to hqq$ hard scattering) the ideal observable relevant to extract the momentum dependence of the factor factors would be the double differential cross section $d^2\sigma/dq_1^2 dq_2^2$, where $q_1 = p_1 - p_3$ and $q_2 = p_2 - p_4$ are the momenta of the two fermion currents entering the process (here $p_1$, $p_2$ ($p_3$, $p_4$) are the momenta of the initial (final) state quarks). The $q_i^2$ are also the key variables to test and control the momentum expansion at the basis of the PO decomposition.

A first nontrivial task is to choose the proper pairing of the incoming and outgoing quarks, given we are experimentally blind to their flavor. For partonic processes receiving two interfering contributions when the final-state quarks are exchanged, such as $uu \to huu$ or $ud \to hud$, the definition of $q_{1,2}$ is even less transparent since a univocal pairing of the momenta can not be assigned, in general, even if one knew the flavor of all partons. This problem can be simply overcome at a practical level by making use of the VBF kinematics, in particular the fact that the two jets are always very forward. This implies one can always pair the momenta of the jet going, for example, on the $+z$ direction with the initial parton going in the same direction, and viceversa. The same argument can be used to argue that the interference between different amplitudes (e.g. neutral current and charged current) is negligible in VBF. In order to check this, we have performed a leading order parton level simulation of the VBF Higgs production ($pp \to hjj$) using MADGRAPH5_AMC@NLO [14] (version 2.2.3) at 13 TeV c.m. energy. We have imposed the basic set of cuts,

$$p_{\mathrm{T,j_{1,2}}} > 30\ \mathrm{GeV}, \quad |\eta_{\mathrm{j_{1,2}}}| < 4.5, \quad \text{and} \quad \mathrm{m_{j_1 j_2}} > 500\ \mathrm{GeV}. \tag{53}$$

In Fig. 1, we show the distribution in the opening angle of the incoming and outgoing quark momenta for the two different pairings. The left plot is for the SM, while the right plot is for a specific NP benchmark point. Shown in blue is the pairing based on the leading color connection using the color flow variable while in red is the opposite pairing. The plot shows that the momenta of the color connected quarks tend to form a small opening angle and the overlap between the two curves, i.e. where the interference effects might be sizable, is negligible. This implies that in the experimental analysis the pairing should be done based on this variable. Importantly, the same conclusions can be drawn in the presence of new physics contributions to the contact terms.

There is a potential caveat to the above argument: the color flow approximation ignores the interference terms that are higher order in $1/N_C$, where $N_C$ is number of colors. Let us consider a process with two interfering amplitudes with the final state quarks exchanged, for example in $uu \to uuh$. The differential cross section receives three contributions proportional to $|F_L^{ff'}(t_{13},t_{24})|^2$, $|F_L^{ff'}(t_{13},t_{24})F_L^{ff'}(t_{14},t_{23})|$ and $|F_L^{ff'}(t_{14},t_{23})|^2$, where $t_{ij} = (p_i - p_j)^2 =$

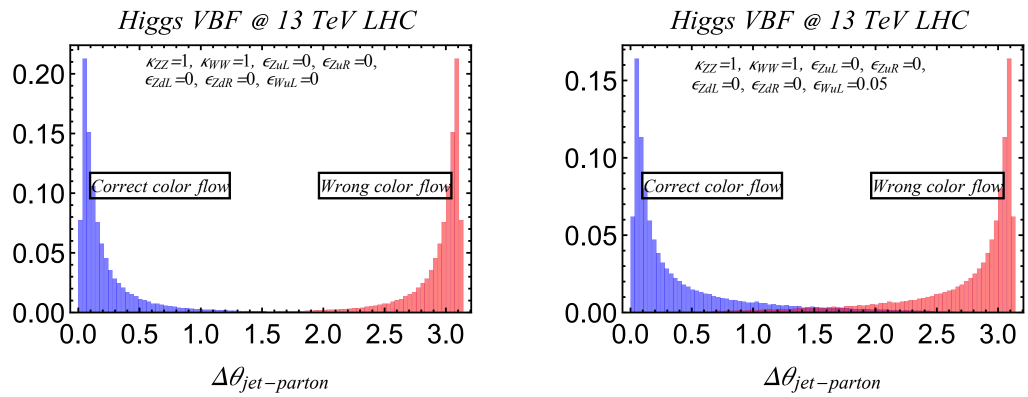


Figure 1: Leading order parton level simulation of the Higgs VBF production at 13 TeV pp c.m. energy (from Ref. [7]). Show in blue is the distribution in the opening angle of the color connected incoming and outgoing quarks $\measuredangle(\vec{p}_3,\vec{p}_1)$, while in red is the distribution for the opposite pairing, $\angle(\vec{p}_3,\vec{p}_2)$. The left plot is for the SM, while the plot on the right is for a specific NP benchmark.

$-2E_iE_j(1-\cos\theta_{ij})$. For the validity of the momentum expansion it is important that the momentum transfers ($t_{ij}$) remain smaller than the hypothesized scale of new physics. On the other hand, imposing the VBF cuts, the interference terms turns out to depend on one small and one large momentum transfer. However, thanks to the pole structure of the form factors, these interference effects turns out to give a very small contribution. Therefore, we can safely state that the momentum transfers marked with the leading color flow are reliable control variables of the momentum expansion validity.

In some realistic experimental analyses, after reconstructing the momenta of the two VBF tagged jets and the Higgs boson, one can compute the relevant momentum transfers $q_1$ and $q_2$, adopting the pairing based on the opening angle. However, for some interesting Higgs decays modes, such as $h\to 2\ell 2\nu$, it is not possible to reconstruct the Higgs boson momentum. In this case, a good approximation of the momentum transfer is the jet $p_T$. This can be understood by explicitly computing the momentum transfer $q^2_{1,2}$ in the limit $|p_T| 1^{+}1^{-} E_{jet}$ and for a Higgs produced close to threshold. Let us consider the partonic momenta in c.o.m. frame for the process: $p_1=(E,\vec{0},E)$, $p_2=(E,\vec{0},-E)$, $p_3=(E'_1,\vec{p}_{T1},\sqrt{E'^2_1-p^2_{T1}})$ and $p_4=(E'_2,\vec{p}_{T2},\sqrt{E'^2_2-p^2_{T2}})$. Conservation of energy for the whole process dictates $2E=E'_1+E'_2+E_h$, where $E^2_h$ is the Higgs energy, usually of order $m_h$ if the Higgs is not strongly boosted. In this case $E-E'_i=\Delta E_i 1^{+}1^{-} E$ since the process is symmetric for $1\leftrightarrow 2$. For each leg, energy and momentum conservation (along the $z$ axis) give

$$\begin{cases} q^z_i = E-\sqrt{E'^2_i-p^2_{Ti}} \\ q^0_i = E-E'_i \end{cases} \quad\to\quad \begin{cases} q^0_i-q^z_i=\sqrt{E'^2_i-p^2_{Ti}}-E'_i\approx-\frac{p^2_{Ti}}{2E'_i} \\ q^0_i+q^z_i\approx 2\Delta E_i+\frac{p^2_{Ti}}{2E'_i} \end{cases} . \tag{54}$$

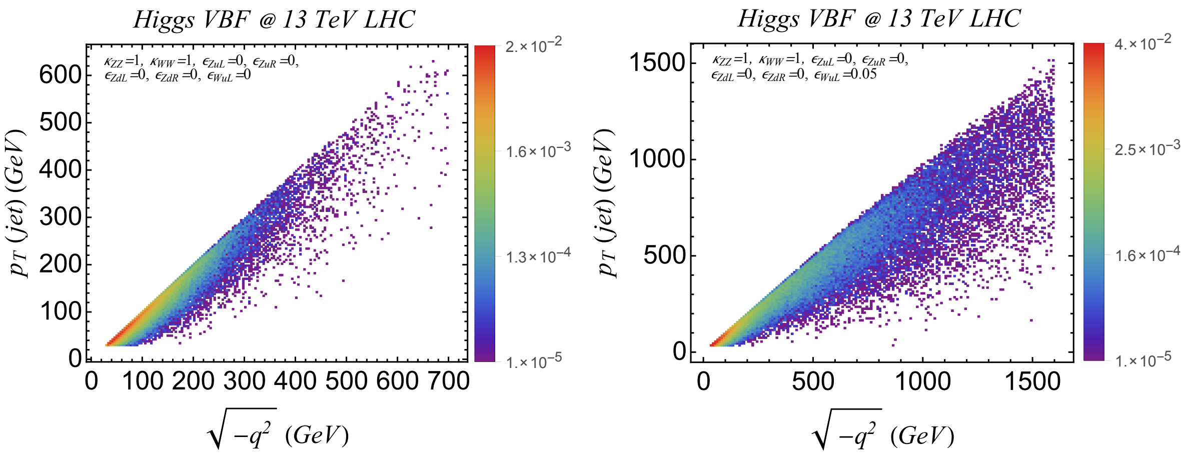


Figure 2: Leading order parton level simulation of the Higgs VBF production at 13 TeV pp c.m. energy [7]. Shown here is the density histogram in two variables; the outgoing quark $p_T$ and the momentum transfer $\sqrt{-q^2}$ with the initial "color-connected" quark. The left plot is for the SM, while the plot on the right is for a specific NP benchmark.

Putting together these two relations one gets

$$q_i^2 = (q_i^0)^2 - p_{Ti}^2 - (q_i^z)^2 = -p_{Ti}^2 + (q_i^0 - q_i^z)(q_i^0 + q_i^z) \approx -p_{Ti}^2 - \frac{p_{Ti}^2 \Delta E_i}{2E_i'} + \mathscr{O}(p_{Ti}^4/E'^2)\ . \quad (55)$$

We can thus conclude that, for a Higgs produced near threshold $(\Delta E_i 1^+ 1^- E')$, $q^2 \approx -p_T^2$.

To illustrate the above conclusion, in Fig. 2 we show a density histogram in two variables: the outgoing quark $p_T$ and the momentum transfer $\sqrt{-q^2}$ obtained from the correct color flow pairing (the left and the right plots are for the SM and for a specific NP benchmark, respectively). The plots indicate the strong correlation of the jet $p_T$ with the momentum transfer $\sqrt{-q^2}$ associated with the correct color pairing. We stress that this conclusion holds both within and beyond the SM.

Given the strong $q^2 \leftrightarrow p_T^2$ correlation, we strongly encourage the experimental collaborations to report the unfolded measurement of the double differential distributions in the two VBF tagged jet $p_T$'s: $\tilde{F}(p_{Tj_1}, p_{Tj_2})$. This measurable distribution is closely related to the form factor entering the amplitude decomposition, $F_L(q_1^2, q_2^2)$, and encode (in a model-independent way) the dynamical information about the high-energy behavior of the process. Moreover, the extraction of the PO in VBF must be done preserving the validity of the momentum expansion: the latter can be checked and enforced setting appropriate upper cuts on the $p_T$ distribution. As an example, in Fig. 3, we show the prediction in the SM (left plot) and in the specific NP benchmark (right plot) of the normalized $p_T$-ordered double differential distribution.

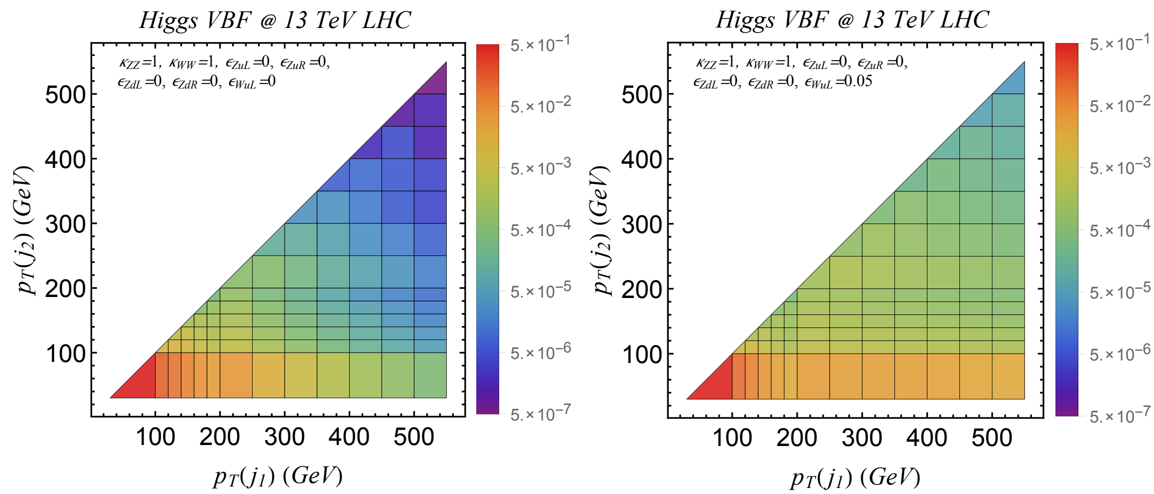


Figure 3: Double differential distribution in the two VBF-tagged jet $p_T$ for VBF Higgs production at 13 TeV LHC [7]. The distribution is normalized such that the total sum of events in all bins is 1. (Left) Prediction in the SM. (Right) Prediction for NP in $\varepsilon_{Wu_L} = 0.05$.

### 6.2 Associated vector boson plus Higgs production

Higgs production in association with a $W$ or $Z$ boson are respectively the third and fourth Higgs production processes in the SM, by total cross section. Combined with VBF studies, they offer other important handles to disentangle the various Higgs PO. Due to the lower cross section, this process is mainly studied in the highest-rate Higgs decay channels, such as $h \to b\bar{b}$ and $WW^*$. The drawback of these channels is the background, which is overwhelming in the $b\bar{b}$ case and of the same order as the signal in the $WW^*$ channels. Nonetheless, kinematical cuts, such as the Higgs $p_T$ in the $b\bar{b}$ case, and the use of multivariate analysis allow the experiments to precisely extract the the signal rates from these measurements.

An important improvement for future studies of these channels with the much higher luminosity which will be available, is to study differential distributions in some specific kinematical variables. In Section 5.1.2 we showed that the invariant mass of the $Vh$ system is the most important observable in this process, since the form factors directly depend on it. In those channels where the $Vh$ invariant mass can not be reconstructed due to the presence of neutrinos, another observable which shows some correlation with the $q^2$ is the $p_T$ of the vector boson, or equivalently of the Higgs, as can be seen in the Fig. 4. Even though this correlation is not as good as the one between the jet $p_T$ and the momentum transfer in the VBF channel, a measurement of the vector boson (or Higgs) $p_T$ spectrum, i.e. of some form factor $\tilde{F}^{Vh}(p_{TV})$ would still offer important information on the underlying structure of the form factors appearing in Eq. (52), $F_L^{q_i Z}(q^2)$ or $G_L^{q_{ij} W}(q^2)$. The invariant mass of the $Vh$ system is given by $m_{Vh}^2 = q^2 = (p_V + p_h)^2 = m_V^2 + m_h^2 + 2 p_V \cdot p_h$. Going in the center of mass frame, we have

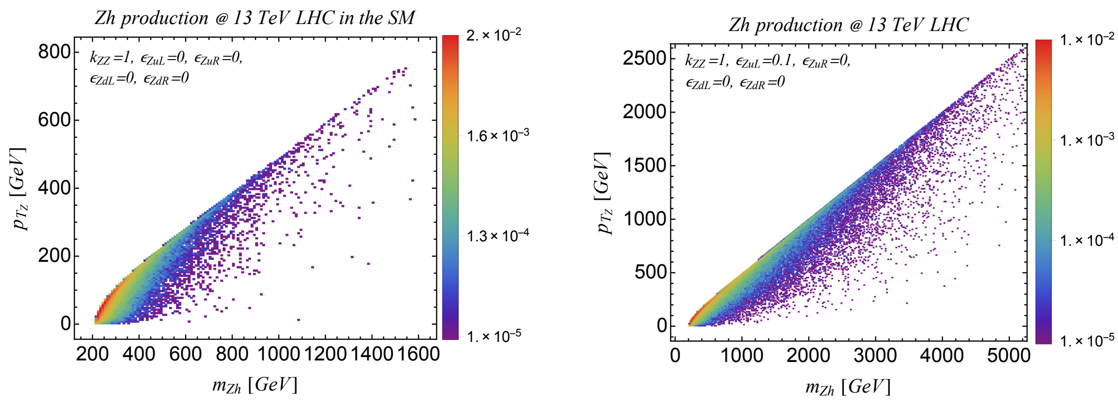


Figure 4: The correlation between the $Zh$ invariant mass and the $p_T$ of the $Z$ boson in $Zh$ associate production at the 13TeV LHC in the SM (left plot) and for a BSM point $\kappa_{ZZ} = 1$, $\varepsilon_{Zu_L} = 0.1$ (right plot) [7]. A very similar correlation is present in the $Wh$ channel.

$p_V = (E_V, \vec{p}_T, p_z)$ and $p_h = (E_h, -\vec{p}_T, -p_z)$, where $E_i = \sqrt{m_i^2 + p_T^2 + p_z^2}$ $(i = V, h)$. Computing $m_{Vh}^2$ explicitly:

$$m_{Vh}^2 = m_V^2 + m_h^2 + 2p_T^2 + 2p_z^2 + 2\sqrt{m_V^2 + p_T^2 + p_z^2}\sqrt{m_h^2 + p_T^2 + p_z^2} \overset{|p_T|\to\infty}{\longrightarrow} 4p_T^2 \ . \tag{56}$$

For $p_z = 0$ this equation gives the minimum $q^2$ for a given $p_T$, which can be seen as the left edge of the distributions in the Fig. 4. This is already a valuable information, for example the boosted Higgs regime used in some $b\bar{b}$ analysis implies a lower cut on the $q^2$: a bin with $p_T > 300$ GeV implies $\sqrt{q^2} \gtrsim 630$ GeV, which could be a problem for the validity of the momentum expansion.

In the $Wh$ process, if the $W$ decays leptonically its $p_T$ can not be reconstructed independently of the Higgs decay channel. One could think that the $p_T$ of the charged lepton from the $W$ decay would be correlated with the $Wh$ invariant mass, but we checked that there is no significant correlation between the two observables.

### 6.3 Validity of the momentum expansion

In order to control the momentum expansion at the basis of the PO composition, it is necessary to set an upper cut on appropriate kinematical variables. These are the $p_T$ of the leading VBF-tagged jet in VBF, and the $Vh$ invariant mass (or the $p_T$ of the massive gauge boson) in VH.

The momentum expansion of the form factors in Eq. (49) makes sense only if the higher order terms in $q_{1,2}^2$ are suppressed. This leads to a consistency condition,

$$\varepsilon_{X_f}\, q_{\max}^2 \lesssim m_Z^2\, g_X^f \ , \tag{57}$$

where $q^2_{\max}$ is the largest momentum transfer in the process. A priori we don't know which is the size of the $\varepsilon_{X_f}$ or, equivalently, the effective scale of new physics. However, a posteriori we can verify by means of Eq. (57) if we are allowed to truncate the momentum expansion to the first non-trivial terms. In VBF, setting a cut-off on $p_T$ we implicitly define a value of $q_{\max}$. Extracting the $\varepsilon_{X_f}$ for $p^j_T < (p^j_T)^{\max} \approx q_{\max}$ we can check if Eq. (57) is satisfied. Ideally, the experimental collaborations should perform the extraction of the $\varepsilon_{X_f}$ for different values of $(p^j_T)^{\max}$ optimizing the range according to the results obtained. The issue is completely analog in VH, where the $q^2_{\max}$ is controlled by $m^2_{Vh}$.

Even more important is matching the PO approach with a differential measurement of the cross-section as function of $p^j_T$, that could be achieved via the so-called *template-cross-section* method. In such distribution a possible break-down of the momentum expansion at the basis of the PO decomposition could indeed be seen (or excluded) directly by data.

A further check to assess the validity of the momentum expansion is obtained comparing the fit performed including the full quadratic dependence of the distributions, as function on the PO, with the fit in which such distributions are linearized in $\delta\kappa_X \equiv \kappa_X - \kappa_X^{\rm SM}$ and $\varepsilon_X$. The idea behind this procedure is that the quadratic corrections to physical observable in $\delta\kappa_X$ and $\varepsilon_X$ are formally of the same order as the interference of the first neglected term in Eq. (49) with the leading SM contribution. If the two fits yields significantly different results, the difference can be used as an estimate of the uncertainty due to the neglected higher-order terms in the momentum expansion. However, as discussed in detail in Ref. [7], such procedure naturally leads to a large overestimate of the uncertainty. This is because in the linearized fit only a few linear combinations of the PO enter the observables, and thus the number of independent constraints derived from data is effectively reduced. On general grounds, the fit obtained with the full quadratic dependence should be considered as the most reliable result, provided that the obtained PO satisfy the consistency condition in Eq. (57).

## 7 Parameter counting and symmetry limits

We are now ready to identify the number of independent pseudo-observables necessary to describe various sets of Higgs decay amplitudes and productions cross sections. We list them below separating four set of observables:

i) the Yukawa decay modes ($h \to f\bar{f}$);

ii) the EW decays ($h \to \gamma\gamma, f\bar{f}\gamma, 4f$);

iii) the EW production production cross sections (VBF and VH);

iv) the non-EW production cross sections (gluon fusion and $t\bar{t}H$) and the total Higgs decay width.

We list the PO needed for a completely general analysis, and the reduction of the number of independent PO obtained under well-defined symmetry hypotheses, such as CP invariance or flavor universality. The latter can be more efficiently tested considering specific sub-sets of observables.

### 7.1 Yukawa modes

As discussed in Sec. 2.1 the $h \to f\bar{f}$ amplitudes are characterized by two independent PO ($\kappa_f$ and $\lambda_f^{\rm CP}$) for each fermion species. Considering only the decay channels relevant for LHC, the full set of 8 parameters is:

$$\kappa_b, \kappa_c, \kappa_\tau, \kappa_\mu, \lambda_b^{\rm CP}, \lambda_c^{\rm CP}, \lambda_\tau^{\rm CP}, \lambda_\mu^{\rm CP} \,. \tag{58}$$

Assuming $CP$ conservation (that implies $\lambda_f^{\rm CP} = 0$ for each $f$) the number of PO is reduced to 4. This is also the number of independent PO effectively measurable if the spin polarization of the final-state fermions is not accessible. The corresponding physical PO are the $\Gamma(h \to f\bar{f})$ partial widths (see Table 1).

### 7.2 Higgs EW decays

The category of EW decays includes a long list of channels; however, not all of them are accessible at the LHC. The clean neutral current processes $h \to e^+e^-\mu^+\mu^-$, $h \to e^+e^-e^+e^-$ and $h \to \mu^+\mu^-\mu^+\mu^-$, together with the photon channels $h \to \gamma\gamma$ and $h \to \ell^+\ell^-\gamma$, can be described in terms of 11 real parameters:

$$\kappa_{ZZ}, \kappa_{Z\gamma}, \kappa_{\gamma\gamma}, \varepsilon_{ZZ}, \varepsilon_{ZZ}^{CP}, \lambda_{Z\gamma}^{CP}, \lambda_{\gamma\gamma}^{CP}, \varepsilon_{Ze_L}, \varepsilon_{Ze_R}, \varepsilon_{Z\mu_L}, \varepsilon_{Z\mu_R} \tag{59}$$

(of which only the subset $\{\kappa_{\gamma\gamma}, \kappa_{Z\gamma}, \lambda_{\gamma\gamma}^{CP}, \lambda_{Z\gamma}^{CP}\}$ is necessary to describe $h \to \gamma\gamma$ and $h \to \ell^+\ell^-\gamma$). The charged-current process $h \to \bar{\nu}_e e \bar{\mu} \nu_\mu$ needs 7 further independent real parameters to be completely specified:

$$\kappa_{WW}, \varepsilon_{WW}, \varepsilon_{WW}^{CP}\ (\text{real}) \quad + \quad \varepsilon_{We_L}, \varepsilon_{W\mu_L}\ (\text{complex}) \,. \tag{60}$$

Finally, the mixed processes $h \to e^+e^-\nu\bar{\nu}$ and $h \to \mu^+\mu^-\nu\bar{\nu}$ can be described by a subset of the coefficients already introduced plus 2 further real contact interactions coefficients:

$$\varepsilon_{Z\nu_e}\,, \varepsilon_{Z\nu_\mu} \,. \tag{61}$$

This brings the total number of (real) parameters to 20 for all the (EW) decays involving muons, electrons, and photons.

The extension to discuss $h \to 4f$ or $h \to f\bar{f}\gamma$ decays with one or two pairs of tau leptons is straightforward: it requires the introduction of the corresponding set of contact terms ($\varepsilon_{Z\tau_L}, \varepsilon_{Z\tau_R}$,

$\varepsilon_{W\tau_L}, \varepsilon_{Z\nu_\tau}$). Similarly, quark contact terms need to be introduced if one or two lepton pairs are replaced by a quark pair.

A first simple restriction in the number of parameters is obtained by assuming flavor universality. This hypothesis imply that the contact terms are the same for all flavors. In particular, for muon and electron modes, this implies

$$\varepsilon_{Ze_L} = \varepsilon_{Z\mu_L} \,, \qquad \varepsilon_{Ze_R} = \varepsilon_{Z\mu_R} \,, \qquad \varepsilon_{Z\nu_e} = \varepsilon_{Z\nu_\mu} \,, \qquad \varepsilon_{We_L} = \varepsilon_{W\mu_L} \,. \tag{62}$$

Technically, this correspond to assume an underlying $U(N_\ell)^2$ flavor symmetry, for the $N_\ell$ generations of leptons considered (namely the maximal flavor symmetry compatible with the SM gauge group).

Since the $\varepsilon_{W\ell_L}$ parameters are complex in general, the relations (62) allow to reduce the total number of parameters to 15. This assumption can be tested directly from data by comparing the extraction of the contact terms from $h \to 2e2\mu$, $h \to 4e$ and $h \to 4\mu$ modes.

The assumption that CP is a good approximate symmetry of the BSM sector and that the Higgs is a CP-even state, allows us to set to zero six independent (real) coefficients:

$$\varepsilon^{CP}_{ZZ} = \lambda^{CP}_{Z\gamma} = \lambda^{CP}_{\gamma\gamma} = \varepsilon^{CP}_{WW} = Im\varepsilon_{We_L} = Im\varepsilon_{W\mu_L} = 0 \,. \tag{63}$$

Assuming, at the same time, flavor universality, the number of free real parameters reduces to 10.

The various cases are prorated in the upper panel of Table 2: in the second column we list the 10 PO needed assuming both CP invariance and flavor universality, while in the third and fourth column we list the additional PO needed if these hypotheses are relaxed (for the clean modes involving only muons and electrons). The corresponding physical PO are the partial widths reported in Table 1.

### 7.3 EW production processes

The fermion-independent PO present in Higgs decays appear also in EW production processes. The additional PO appearing only in production (assuming Higgs decays to quark are not detected) are the contact terms for the light quarks. In a four-flavor scheme, in absence of any symmetry assumption, the number of independent parameters for the neutral currents contact terms is 16 ($\varepsilon_{Zq^{ij}}$, where $q = u_L, u_R, d_L, d_R$, and $i, j = 1,2$): 8 real parameters for flavor diagonal terms and 4 complex flavour-violating parameters. Similarly, there are 16 independent parameters in charged currents, namely the 8 complex terms $\varepsilon_{Wu^i_L d^j_L}$ and $\varepsilon_{Wu^i_R d^j_R}$. However, we can safely reduce the number of independent PO under neglecting the terms that violates the $U(1)_f$ flavour symmetry acting on each of the light fermion species, $u_R$, $d_R$, $s_R$, $c_R$, $q^{(d)}_L$, and $q^{(s)}_L$, where $q^{(d,s)}_L$ denotes the two quark doublets in the basis where down quarks are diagonal. This symmetry is

<table>
<caption>Higgs (EW) decay amplitudes</caption>
<tr><th>Amplitudes</th><th>Flavor + CP</th><th>Flavor Non Univ.</th><th>CPV</th></tr>
<tr><td rowspan="2">$h \to \gamma\gamma, 2e\gamma, 2\mu\gamma$<br>$4e, 4\mu, 2e2\mu$</td><td rowspan="2">$\kappa_{ZZ}, \kappa_{Z\gamma}, \kappa_{\gamma\gamma}, \varepsilon_{ZZ}$<br>$\varepsilon_{Ze_L}, \varepsilon_{Ze_R}$</td><td rowspan="2">$\varepsilon_{Z\mu_L}, \varepsilon_{Z\mu_R}$</td><td rowspan="2">$\varepsilon^{CP}_{ZZ}, \lambda^{CP}_{Z\gamma}, \lambda^{CP}_{\gamma\gamma}$</td></tr>
<tr></tr>
<tr><td rowspan="2">$h \to 2e2\nu, 2\mu 2\nu, e\nu\mu\nu$</td><td rowspan="2">$\kappa_{WW}, \varepsilon_{WW}$<br>$\varepsilon_{Z\nu_e}$, Re($\varepsilon_{We_L}$)</td><td>$\varepsilon_{Z\nu_\mu}$, Re($\varepsilon_{W\mu_L}$)</td><td>$\varepsilon^{CP}_{WW}$, Im($\varepsilon_{We_L}$)</td></tr>
<tr><td colspan="2">Im($\varepsilon_{W\mu_L}$)</td></tr>
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<table>
<caption>Higgs (EW) production amplitudes</caption>
<tr><th>Amplitudes</th><th>Flavor + CP</th><th>Flavor Non Univ.</th><th>CPV</th></tr>
<tr><td>VBF neutral curr.<br>and $Zh$</td><td>[ $\kappa_{ZZ}, \kappa_{Z\gamma}, \varepsilon_{ZZ}$ ]<br>$\varepsilon_{Zu_L}, \varepsilon_{Zu_R}, \varepsilon_{Zd_L}, \varepsilon_{Zd_R}$</td><td>$\varepsilon_{Zc_L}, \varepsilon_{Zc_R}$<br>$\varepsilon_{Zs_L}, \varepsilon_{Zs_R}$</td><td>[ $\varepsilon^{CP}_{ZZ}, \lambda^{CP}_{Z\gamma}$ ]</td></tr>
<tr><td rowspan="2">VBF charged curr.<br>and $Wh$</td><td rowspan="2">[ $\kappa_{WW}, \varepsilon_{WW}$ ]<br>Re($\varepsilon_{Wu_L}$)</td><td>Re($\varepsilon_{Wc_L}$)</td><td>Im($\varepsilon_{Wu_L}$)</td></tr>
<tr><td colspan="2">Im($\varepsilon_{Wc_L}$)</td></tr>
</table>

<table>
<caption>EW production and decay modes, with custodial symmetry</caption>
<tr><th>Amplitudes</th><th>Flavor + CP</th><th>Flavor Non Univ.</th><th>CPV</th></tr>
<tr><td>production & decays</td><td>$\kappa_{ZZ}, \kappa_{Z\gamma}, \varepsilon_{ZZ}$</td><td></td><td>$\varepsilon^{CP}_{ZZ}, \lambda^{CP}_{Z\gamma}$</td></tr>
<tr><td>VBF and VH only</td><td>$\varepsilon_{Zu_L}, \varepsilon_{Zu_R}, \varepsilon_{Zd_L}, \varepsilon_{Zd_R}$</td><td>$\varepsilon_{Zc_L}, \varepsilon_{Zc_R}$<br>$\varepsilon_{Zs_L}, \varepsilon_{Zs_R}$</td><td></td></tr>
<tr><td>decays only</td><td>$\kappa_{\gamma\gamma}, \varepsilon_{Ze_L}, \varepsilon_{Ze_R}$, Re($\varepsilon_{We_L}$)</td><td>$\varepsilon_{Z\mu_L}, \varepsilon_{Z\mu_R}$</td><td>$\lambda^{CP}_{\gamma\gamma}$</td></tr>
</table>

Table 2: Summary of the effective couplings PO appearing in EW Higgs decays and in the VBF and VH production cross-sections (see main text). The terms between square brakes in the middle table are the PO present both in production and decays. The last table denote the PO needed to describe both production and decays under the assumption of custodial symmetry.

an exact symmetry of the SM in the limit where we neglect light quark masses. Enforcing it at the PO level is equivalent to neglecting terms that do not interfere with SM amplitudes in the limit of vanishing light quark masses. Under this (rather conservative) assumption, the number of independent neutral currents contact terms reduces to 8 real parameters,

$$\varepsilon_{Zu_R},\ \varepsilon_{Zc_R},\ \varepsilon_{Zd_R},\ \varepsilon_{Zs_R},\ \varepsilon_{Zd_L},\ \varepsilon_{Zs_L},\ \ \varepsilon_{Zu_L},\ \varepsilon_{Zc_L}, \tag{64}$$

and only 2 complex parameters appear in the charged-current case:

$$\varepsilon_{Wu^i_L d^j_L} \equiv V_{ij}\varepsilon_{Wu^j_L}, \qquad \varepsilon_{Wu^i_R d^j_R} = 0\ . \tag{65}$$

Similarly to the decays, a further interesting reduction of the number of parameters is obtained assuming flavor universality or, more precisely, under the assumption of an $U(2)^3$ symmetry acting on the first two generations of quarks. The latter is the the maximal flavor symmetry for the light quarks compatible with the SM gauge group. In this case the independent parameters in this case reduces to six:

$$\varepsilon_{Zu_L},\ \varepsilon_{Zu_R},\ \varepsilon_{Zd_L},\ \varepsilon_{Zd_R},\ \varepsilon_{Wu_L}\ , \tag{66}$$

where $\varepsilon_{Wu_L}$ is complex, or five if we further neglect CP-violating contributions (in such case $\varepsilon_{Wu_L}$ is real). This case is listed in the second column of Table 2 (middle panel), where the terms between brackets denote the PO appearing also in decays.

### *Custodial symmetry and the combination of EW production and decay modes*

Assuming flavor universality and CP conservation, the number of independent PO necessary to describe all EW decays and production cross sections is 15. These are the terms listed in the second column of the first two panels of Table 2.

A further reduction of the number of independent PO is obtained under the hypothesis of custodial symmetry, that relates charged and neutral current modes. The complete list of custodial symmetry relations can be found in Refs. [2,7]. Here we only mention the one between $\kappa_{WW}$ and $\kappa_{ZZ}$, noting that the presence of contact terms modify it with respect to the one known in the context of the kappa-framework:

$$\kappa_{WW} - \kappa_{ZZ} = -\frac{2}{g}\left(\sqrt{2}\varepsilon_{W\ell_L} + 2\frac{m_W}{m_Z}\varepsilon_{Z\ell_L}\right)\ , \tag{67}$$

where $\ell = e, \mu$. After imposing flavor and CP conservation, custodial symmetry allow a reduction of the number of independent PO from 15 down to 11, as shown in the lower panel of Table 2.

### 7.4 Additional PO

The remaining PO needed for a complete description of Higgs physics at the LHC are those related to the non-EW production processes (gluon fusion and $t\bar{t}H$) and to the total Higgs decay width (i.e. NP effects in invisible or undetected decay modes).

A detailed formalism, similar to the one developed for EW production and decay process, has not been developed yet for gluon fusion and $t\bar{t}H$ production process. However, it should be stressed that the latter are on a very different footing compared to EW processes since they involve a significantly smaller number of observables. Moreover, a smaller degrees of modelization is required in order to analyze the corresponding data in generic NP frameworks. As a result, the combination of PO for the total cross sections, and template-cross-section analyses of the kinematical distributions, provide an efficient way to report data in a sufficiently general and unbiased way.

More precisely, for the time being we suggest to introduce the following two PO

$$\kappa_{gg}^2 = \left.\frac{\sigma(pp \to h)}{\sigma^{\rm SM}(pp \to h)}\right|_{gg-{\rm fusion}} , \qquad \kappa_t^2 = \left.\frac{\sigma(pp \to t\bar{t}h)}{\sigma^{\rm SM}(pp \to t\bar{t}h)}\right|_{\rm Yukawa} , \tag{68}$$

in close analogy to what it is presently done within the $\kappa$ formalism. As far as the gluon fusion is concerned, it is well known that the Higgs $p_T$ distribution carries additional dynamical information about the underlying process. However, such distribution can be efficiently reported via the template-cross-section method. Moreover, the steep fall of the $p_T$ spectrum (that is a general consequence of the infrared structure of QCD) implies that the determination of $\kappa_{gg}$ is practically unaffected by possible NP effects in this distribution.

Finally, as far as the Higgs width is concerned, we need to introduce a single effective physical PO to account for all the invisible or undetected Higgs decay modes. This additional partial width must be added to the various visible partial widths in order to determine the total Higgs width. Alternatively, it is possible to define an effective coupling PO as the ratio

$$\kappa_\Gamma^2 = \frac{\Gamma_{\rm tot}(h)}{\Gamma_{\rm tot}^{\rm SM}(h)} . \tag{69}$$

## 8 PO meet SMEFT

One of the main goals of the LHC is to perform high-precision studies of possible deviations from the SM. Ideally, this would require the following four steps: i) for each process write down some (QFT-compatible) amplitude allowing for SM-deviations, both for the main signal analyzed (e.g. a given Higgs cross-section, close to the resonance) as well as for the background (non-resonant signal); ii) compute fiducial observables; iii) fit the signal (SM+NP) via an appropriate set of conventionally-defined PO, without subtracting the SM background; iv) using the PO thus obtained to derive information on the Wilson coefficients of an appropriate Lagrangian allowing for deviations from the SM.

In the previous sections we have discussed a convenient choice for the definition of the PO relevant to resonant Higgs physics (steps i and iii). In this Section we outline how to address the last step in the case of the so-called SM Effective Field Theory (SMEFT), i.e. how to extract the Wilson coefficients of the SMEFT from the measured PO.

Before starting, it is worth stressing that PO are *not* Wilson coefficients, despite one can derive a linear relation between the two sets of parameters when working at the lowest-order (LO) in a given Lagrangian framework. The distinction between PO and Wilson coefficients is quite clear from their different "status" in QFT: the PO provide a general parameterization a given set of on-shell scattering amplitudes and are not Lagrangian parameters. Once a PO is observed to deviate from its SM value we cannot, without further theoretical assumptions, to predict deviations in other amplitudes. The latter can be obtained only using a given Lagrangian and after extracting from data (or better from PO) the corresponding set of Wilson coefficients. Conversely, Wilson coefficients are scale and scheme dependent parameters that require specific theoretical prescriptions to be extracted from physical observables. This is why the PO can be measured including only the SM THU[5], while the extraction of SMEFT Wilson coefficients require also an estimate of the corresponding SMEFT THU[6].

There is a line of thought where the Wilson coefficients in any LO EFT approach to physics beyond the SM are not actual Wilson coefficients, but parameters encoding deformation possibilities. According to this line of though, PO and and Wilson coefficients are somehow the same object. But this way of proceeding has a limited applicability, especially if a deviations from the SM is found. Proceeding along this line one could write an ad-hoc effective Lagrangian, do some calculations at LO (deviation parameters at tree-level), interpret the data, and limit the considerations to answer the question "are there deviations from the SM?". If we want to go a step further, viz. answering the question "What do the deviations from the SM mean?" then it is important to separate the role of PO and Wilson coefficients. Indeed after extracting the PO, two possibilities appear: i) *top-down*, namely employ a specific UV model, compute the PO and try to figure out if it matches or not with the observed deviation; in such case there will be an uncertainty in projecting down the UV model to the parameters and in the choice of the input parameter set (IPS); ii) *bottom-up*, namely do a SMEFT analysis to extract from the PO conclusions on the actual Wilson coefficients; here there will be an uncertainty from the order at which the calculation is done, as well as a parametric uncertainty. In the following we illustrate

[5]By THU we mean theoretical uncertainty which has two components, parametric (PU) and missing higher order uncertainties (MHOU)

[6]Although SMEFT converges to SM in the limit of zero Wilson coefficients, SMEFT and SM are different theories in the UV.

the basic strategy of for the latter (bottom-up) approach.

### 8.1 SMEFT summary

To establish our notations we observe that in the SMEFT a (lepton number preserving) amplitude can be written as

$$\mathscr{A} = \sum_{n=\mathrm{N}}^{\infty} g^n \mathscr{A}_n^{(4)} + \sum_{n=\mathrm{N}_6}^{\infty} \sum_{l=1}^{n} \sum_{k=1}^{\infty} g^n \left[ \frac{1}{(\sqrt{2}\, G_{\mathrm{F}} \Lambda^2)^k} \right]^l \mathscr{A}_{nlk}^{(4+2k)}, \tag{70}$$

where $g$ is a SM coupling. $G_{\mathrm{F}}$ is the Fermi coupling constant and $\Lambda$ is the cut off scale. $l$ is an index that indicates the number of SMEFT operator insertions leading to the amplitude, and $k$ indicates the inverse mass dimension of the Lagrangian terms inserted. $N$ is a label for each individual process, that indicates the order of the coupling dependence for the leading non vanishing term in the SM (e.g. $N = 1$ for $\mathrm{H} \to \mathrm{VV}$ etc. but $N = 3$ for $\mathrm{H} \to \gamma\gamma$). $N_6 = N$ for tree initiated processes in the SM. For processes that first occur at loop level in the SM, $N_6 = N - 2$ when operators in the SMEFT can mediate such decays directly thought a contact operator, for example, through a $\mathrm{dim} = 6$ operator for $\mathrm{H} \to \gamma\gamma$. For instance, the $\mathrm{H}\gamma\gamma$ (tree) vertex is generated by $O_{HB} = \Phi^\dagger \Phi B^{\mu\nu} B_{\mu\nu}$, by $O^8_{HW} = \Phi^\dagger B^{\mu\nu} B_{\mu\rho} \mathrm{D}^\rho \mathrm{D}_\nu \Phi$ etc. Therefore, SMEFT is a double expansion: in $g$ and $g_6 = \mathrm{v}_{\mathrm{F}}^2/\Lambda^2$ for pole observables and in $g, g_6\, E^2/\mathrm{v}_{\mathrm{F}}^2$ for off-shell ones; furthermore, the combination of parameters $g\, g_6\, \mathscr{A}^{(6)}_{1,1,1}$ defines the LO SMEFT expression for the process while $g^3\, g_6\, \mathscr{A}^{(6)}_{3,1,1}$ defines the NLO SMEFT amplitude in the perturbative expansion.

To summarize, LO SMEFT refers to $\mathrm{dim} = 6$ operators in tree diagrams, sometimes called "contact terms" while NLO SMEFT refers to one loop diagrams with a single insertion of $\mathrm{dim} = 6$ operators. One can make additional assumptions by introducing classification schemes in SMEFT. One example of a classification scheme is the Artz-Einhorn-Wudka "potentially-tree-generated" (PTG) scenario [15,16]. In this scheme, it is argued that classes of Wilson coefficients for operators of $\mathrm{dim} = 6$ can be argued to be tree level, or loop level (suppressed by $g^2/16\pi^2$)[7]. In these cases the expansion in Eq.(70) is reorganized in terms of TG (we assume a BSM model where PTG is actually TG) and LG insertions, i.e. LG contact terms and one loop TG insertions, one loop LG insertions and two loop SM etc. It is clear that LG contact terms alone do not suffice.

Strictly speaking we are considering here the virtual part of SMEFT, under the assumptions that LHC PO are defined à la LEP, i.e. when QED and QCD corrections are deconvoluted. Otherwise, the real (emission) part of SMEFT should be included and it can be shown that the infrared/collinear part of the one-loop virtual corrections and of the real ones respect factorization: the total = virtual + real is IR/collinear finite at $\mathscr{O}(g^4\, g_6)$.

[7] This classification scheme corresponds only to a subset of weakly coupled and renormalizable UV physics cases.

It is worth nothing that SMEFT has limitations, obviously the scale should be such that $E \ll \Lambda$. Understanding SM deviations in tails of distributions requires using SMEFT, but only up to the point where it stops to be valid, or using the kappa–BSM-parameters connection, i.e. replace SMEFT with BSM models, optimally matching to SMEFT at lower scales.

In any process, the residues of the poles corresponding to unstable particles (starting from maximal degree) are numbers while the non-resonant part is a multivariate function that requires some basis, i.e. a less model independent, underlying, theory of SM deviations. That is to say, residue of the poles can be PO by themselves, expressing them in terms of Wilson coefficients is an operation the can be eventually postponed. The very end of the chain, the non-resonant part, may require model dependent BSM interpretation. Numerically speaking, it depends on the impact of the non-resonant part wich is small in gluon-fusion but not in Vector Boson Scattering. Therefore, the focus for data reporting should always be on real observables, fiducial cross sections and pseudo-observables.

To explain SMEFT in a nutshell consider a process described by some SM amplitude

$$\mathscr{A}_{\mathrm{SM}} = \sum_{i=1,n} \mathscr{A}^{(i)}_{\mathrm{SM}}\,, \tag{71}$$

where $i$ labels gauge-invariant sub-amplitudes. In general, the same process is given by a contact term or a collection of contact terms of $\dim = 6$; for instance, direct coupling of H to $\mathrm{VV}(\mathrm{V} = \gamma, \mathrm{Z}, \mathrm{W})$. In order to construct the theory one has to select a set of higher-dimensional operators and to start the complete procedure of renormalization. Of course, different sets of operators can be interchangeable as long as they are closed under renormalization. It is a matter of fact that renormalization is best performed when using the so-called Warsaw basis, see Ref. [17]. Moving from SM to SMEFT we obtain

$$\mathscr{A}^{\mathrm{LO}}_{\mathrm{SMEFT}} = \sum_{i=1,n} \mathscr{A}^{(i)}_{\mathrm{SM}} + i\, g_{_6}\, \hat{\kappa}_{\mathrm{c}}\,, \qquad \mathscr{A}^{\mathrm{NLO}}_{\mathrm{SMEFT}} = \sum_{i=1,n} \hat{\kappa}_i\, \mathscr{A}^{(i)}_{\mathrm{SM}} + i\, g_{_6}\, \hat{\kappa}_{\mathrm{c}} + g_{_6} \sum_{i=1,\mathrm{N}} a_i\, \mathscr{A}^{(i)}_{\mathrm{nfc}}\,, \tag{72}$$

where $g_{_6}^{-1} = \sqrt{2}\, G_{\mathrm{F}}\, \Lambda^2$. The last term in Eq.(72) collects all loop contributions that do not factorize and the coefficients $a_i$ are Wilson coefficients. The $\hat{\kappa}_i$ are linear combinations of Wislon coefficients.[8] We conclude that Eq.(72) gives a consistent and convenient generalization of the original $\kappa$-framework at the price of introducing additional, non-factorizable, terms in the amplitude.

There are several reasons why loops should not be neglected in SMEFT, one is as follows: consider the "off-shell"$\mathrm{gg} \to \mathrm{H}$ fusion [18,19,20,21], the "contact" term is real while the SM amplitude crosses normal thresholds, e.g. at $s = 4\, m_{\mathrm{t}}^2$, where $s$ is the Higgs virtuality. Therefore, in the interference one misses the large effect induced by the SM imaginary part while this effect

[8] We denote these combinations of Wilson coefficients $\hat{\kappa}_i$, rather than $\kappa_i$, in order to distinguish them from the PO defined in the previous sections.

(of the order of 5% above the $\bar{t}t$-threshold) is properly taken into account by the inclusion of SMEFT loops, also developing an imaginary part after crossing the same normal threshold. To summarize, the LO part (contact term) alone shows large deviations from the SM around the $\bar{t}t$-threshold while the one-loop part reproduces, with the due rescaling, the SM lineshape in a case where there is no reason to neglect the insertion of PTG operators in loops. Only the formulation including loops gives an accurate result, with deviations of $\mathscr{O}(5\%)$ wrt tree (uncritical as long as experimental precision is $\gg 10\%$ but experiments are getting close).

### 8.2 Theoretical uncertainty

A theoretical uncertainty arises when the value of the Wilson coefficients in the PO scenario is inferred. A fit defined in a perturbative expansion must always include an estimate of the missing higher order terms (MHOU) [1]. Various ways exist to estimate this uncertainty, at any order in perturbation theory. One can compute the same observable with different "options", e.g. linearization or quadratization of the squared matrix element, resummation or expansion of the (gauge invariant) fermion part in the wave function factor for the external legs (does not apply at tree level), variation of the renormalization scale, $G_{\mathrm{F}}$ renormalization scheme or $\alpha$-scheme, etc.

A conservative estimate of the associated theoretical uncertainty is obtained by taking the envelope over all "options"; the interpretation of the envelope is a log-normal distribution (this is the solution preferred in the experimental community) or a flat Bayesian prior [22,23] (a solution preferred in a large part of the theoretical community). It is clear that MHOU for the SM should always be included.

The notion of MHOU has a long history but it is worth noting that there is no statistical foundation and that it cannot be derived from a set of consistent (incomplete) principles. Ideally, calculations should be repeated using a well defined (and definable) set of options, results from different calculations should be compared and their MHOU assumptions subjectable to falsification. Therefore, no estimate of the theoretical errors is general enough and it is clear that there are several ways to approach the problem with conceptual differences between the bottom-up and the top-down scenarios.

### 8.3 Examples

In this Section we provide a number of examples connecting PO to Wilson coefficients; results are based on the work of Refs. [4,24], and of Refs. [25,26,27,28]. For simplicity we will confine the presentation to CP-even couplings.

- $\underline{\mathrm{H} \to \overline{\mathrm{b}}\mathrm{b}}$ At tree level this amplitude is given by

$$\mathscr{A}_{\mathrm{H\bar{b}b}} = -\frac{1}{2}\, g\, \frac{m_{\mathrm{b}}}{M_{\mathrm{W}}} \left[\mathrm{A}^{\mathrm{SM}}_{\mathrm{H\bar{b}b}} + g_6 \left(a_{\phi\,\mathrm{W}} - \frac{1}{4}\, a_{\phi\,\mathrm{D}} + a_{\phi\,\square} - a_{\mathrm{b}\,\phi}\right)\right], \tag{73}$$

giving the following connection with Eq. (1) (note that all deviations are real):

$$y^{\mathrm{b}}_{\mathrm{S}} = -\frac{i}{\sqrt{2}}\, g\, \frac{m_{\mathrm{b}}}{M_{\mathrm{W}}} \left[\mathrm{A}^{\mathrm{SM}}_{\mathrm{H\bar{b}b}} + g_6 \left(a_{\phi\,\mathrm{W}} - \frac{1}{4}\, a_{\phi\,\mathrm{D}} + a_{\phi\,\square} - a_{\mathrm{b}\,\phi}\right)\right]. \tag{74}$$

- $\underline{\mathrm{H} \to \gamma\gamma}$ The amplitude for the process $\mathrm{H}(P) \to \gamma_\mu(p_1)\gamma_\nu(p_2)$ can be written as

$$\mathrm{A}^{\mu\nu}_{\mathrm{HAA}} = i\, \mathscr{T}_{\mathrm{HAA}}\, \mathrm{T}^{\mu\nu}, \quad M^2_{\mathrm{H}}\, \mathrm{T}^{\mu\nu} = p^{\mu}_2\, p^{\nu}_1 - p_1 \cdot p_2\, g^{\mu\nu}. \tag{75}$$

The $S$-matrix element follows from Eq.(75) when we multiply the amplitude by the photon polarizations $e_\mu(p_1)\, e_\nu(p_2)$; in writing Eq.(75) we have used $p \cdot e(p) = 0$. A convenient way for writing the amplitude is the following: after renormalization we neglect all fermion masses but $m_{\mathrm{t}}, m_{\mathrm{b}}$ and write

$$\mathscr{T}_{\mathrm{HAA}} = \frac{g^3_{\mathrm{F}}\, \mathrm{s}^2_{\mathrm{W}}}{8\,\pi^2} \sum_{\mathrm{I=W,t,b}} \rho^{\mathrm{HAA}}_{\mathrm{I}}\, \mathscr{T}^{\mathrm{I}}_{\mathrm{HAA\,;LO}} + g_{\mathrm{F}}\, g_6\, \frac{M^2_{\mathrm{H}}}{M_{\mathrm{W}}}\, a_{\mathrm{AA}} + \frac{g^3_{\mathrm{F}}\, g_6}{\pi^2}\, \mathscr{T}^{\mathrm{nfc}}_{\mathrm{HAA}}, \tag{76}$$

where $g^2_{\mathrm{F}} = 4\sqrt{2}\, G_{\mathrm{F}}\, M^2_{\mathrm{W}}$ and $\mathrm{c_W} = M_{\mathrm{W}}/M_{\mathrm{Z}}$. Note that, at this point we have selected the $\{G_{\mathrm{F}}\,, M_{\mathrm{Z}}\,, M_{\mathrm{W}}\}$ IPS, alternatively one could use the $\{\alpha\,, G_{\mathrm{F}}\,, M_{\mathrm{Z}}\}$ IPS where

$$g^2_{\mathrm{A}} = \frac{4\,\pi\,\alpha}{\hat{\mathrm{s}}^2_\theta} \qquad \hat{\mathrm{s}}^2_\theta = \frac{1}{2}\left[1 - \sqrt{1 - 4\, \frac{\pi\,\alpha}{\sqrt{2}\, G_{\mathrm{F}}\, M^2_{\mathrm{Z}}}}\right], \tag{77}$$

with a numerical difference that enters the MHOU. Referring to Eq.(72) we have

$$\hat{\kappa}^{\mathrm{HAA}}_{\mathrm{I}} = \frac{g^3_{\mathrm{F}}\, \mathrm{s}^2_{\mathrm{W}}}{8\,\pi^2}\, \rho^{\mathrm{HAA}}_{\mathrm{I}}, \qquad \hat{\kappa}^{\mathrm{HAA}}_{\mathrm{c}} = g_{\mathrm{F}}\, \frac{M^2_{\mathrm{H}}}{M_{\mathrm{W}}}\, a_{\mathrm{AA}}. \tag{78}$$

In writing deviations in terms of Wilson coefficients we introduce the following combinations:

$$a_{\mathrm{ZZ}} = \mathrm{s}^2_{\mathrm{W}}\, a_{\phi\,\mathrm{B}} + \mathrm{c}^2_{\mathrm{W}}\, a_{\phi\,\mathrm{W}} - \mathrm{s_W}\, \mathrm{c_W}\, a_{\phi\,\mathrm{WB}}, \qquad a_{\mathrm{AA}} = \mathrm{c}^2_{\mathrm{W}}\, a_{\phi\,\mathrm{B}} + \mathrm{s}^2_{\mathrm{W}}\, a_{\phi\,\mathrm{W}} + \mathrm{s_W}\, \mathrm{c_W}\, a_{\phi\,\mathrm{WB}},$$
$$a_{\mathrm{AZ}} = 2\,\mathrm{c_W}\, \mathrm{s_W}\, \left(a_{\phi\,\mathrm{W}} - a_{\phi\,\mathrm{B}}\right) + \left(2\,\mathrm{c}^2_{\mathrm{W}} - 1\right)\, a_{\phi\,\mathrm{WB}} \quad . \tag{79}$$

The process dependent $\rho$-factors are given by

$$\rho^{\mathrm{proc}}_{\mathrm{I}} = 1 + g_6\, \Delta\rho^{\mathrm{proc}}_{\mathrm{I}}, \tag{80}$$

and there are additional, non-factorizable, contributions. For $\mathrm{H} \to \gamma\gamma$ the $\Delta\rho$ factors are as follows:

$$
\begin{aligned}
\Delta\rho_{\mathrm{t}}^{\mathrm{HAA}} &= \frac{3}{16}\,\frac{M_{\mathrm{H}}^2}{\mathrm{s_W}\,M_{\mathrm{W}}^2}\,a_{\mathrm{tWB}} + (2-\mathrm{s_W^2})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\,a_{\mathrm{AZ}} + (6-\mathrm{s_W^2})\,a_{\mathrm{AA}} \\
&\quad - \frac{1}{2}\left[a_{\phi\mathrm{D}} + 2\,\mathrm{s_W^2}\,(\mathrm{c_W^2}\,a_{\mathrm{ZZ}} - a_{\mathrm{t}\phi} - 2\,a_{\phi\Box})\right]\frac{1}{\mathrm{s_W^2}}\,, \\
\Delta\rho_{\mathrm{b}}^{\mathrm{HAA}} &= -\frac{3}{8}\,\frac{M_{\mathrm{H}}^2}{\mathrm{s_W}\,M_{\mathrm{W}}^2}\,a_{\mathrm{bWB}} + (2-\mathrm{s_W^2})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\,a_{\mathrm{AZ}} + (6-\mathrm{s_W^2})\,a_{\mathrm{AA}} \\
&\quad - \frac{1}{2}\left[a_{\phi\mathrm{D}} + 2\,\mathrm{s_W^2}\,(\mathrm{c_W^2}\,a_{\mathrm{ZZ}} + a_{\mathrm{b}\phi} - 2\,a_{\phi\Box})\right]\frac{1}{\mathrm{s_W^2}}\,, \\
\Delta\rho_{\mathrm{W}}^{\mathrm{HAA}} &= (2+\mathrm{s_W^2})\,\frac{\mathrm{c_W}}{\mathrm{s_W}}\,a_{\mathrm{AZ}} + (6+\mathrm{s_W^2})\,a_{\mathrm{AA}} - \frac{1}{2}\left[a_{\phi\mathrm{D}} - 2\,\mathrm{s_W^2}\,(2\,a_{\phi\Box} + \mathrm{c_W^2}\,a_{\mathrm{ZZ}})\right]\frac{1}{\mathrm{s_W^2}}\,. \qquad (81)
\end{aligned}
$$

In the PTG scenario we only keep $a_{\mathrm{t}\phi}, a_{\mathrm{b}\phi}, a_{\phi\mathrm{D}}$ and $a_{\phi\Box}$ in Eq.(81). The advantage of Eq.(76) is to establish a link between the EFT and the $\kappa$-framework, which has a validity restricted to LO. As a matter of fact Eq.(76) tells us that appropriate $\hat{\kappa}$-factors can be introduced also at the loop level; they are combinations of Wilson coefficients but we have to extend the scheme with the inclusion of process dependent, non-factorizable, contributions.

We also derive the following result for the non-factorizable part of the amplitude:

$$
\mathscr{T}_{\mathrm{HAA}}^{\mathrm{nfc}} = M_{\mathrm{W}} \sum_{a\in\{A\}} \mathscr{T}_{\mathrm{HAA}}^{\mathrm{nfc}}(a)\,a\,, \quad \{A\} = \{a_{\mathrm{tWB}}, a_{\mathrm{bWB}}, a_{\mathrm{AA}}, a_{\mathrm{AZ}}, a_{\mathrm{ZZ}}\}\,. \qquad (82)
$$

In the PTG scenario all non-factorizable amplitudes for $\mathrm{H} \to \gamma\gamma$ vanish. Comparing with Eq. (6) we obtain

$$
\varepsilon_{\gamma\gamma} = -\frac{1}{2}\,\frac{\mathrm{v_F}}{M_{\mathrm{H}}^2}\,\mathscr{T}_{\mathrm{HAA}}\,, \quad \mathscr{T}_{\mathrm{HAA}}^{\mathrm{LO}} = \mathscr{T}_{\mathrm{HAA}}^{\mathrm{SM}} + g_{\mathrm{F}}\,g_6\,\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}\,a_{\mathrm{AA}}\,. \qquad (83)
$$

- $\underline{\mathrm{H} \to 4\,\mathrm{f}}$

Few additional definitions are needed: by on-shell S-matrix for an arbitrary process (involving external unstable particles) we mean the corresponding (amputated) Green's function supplied with LSZ factors and sources, computed at the (complex) poles of the external lines [29,30]. For processes that involve stable particles this can be straightforwardly transformed into a physical PO.

The connection of the $\mathrm{HVV}, \mathrm{V} = \mathrm{Z}, \mathrm{W}$ (on-shell) S-matrix with the off shell vertex $\mathrm{H} \to \mathrm{VV}$ and the full process $\mathrm{pp} \to 4\,\psi$ is more complicated and is discussed in some detail in Sect. 3 of Ref. [6]. The "on-shell" S-matrix for HVV, being built with the the residue of the $\mathrm{H}-\mathrm{V}-\mathrm{V}$ poles in $\mathrm{pp} \to 4\,\psi$ is gauge invariant by construction (it can be proved by using Nielsen identities) and represents one of the building blocks for the full process: in other words, it is a PO.

Technically speaking the "on-shell" limit for external legs should be understood "to the complex poles" (for a modification of the LSZ reduction formulas for unstable particles see Ref. [31]) but, as well known, at one loop we can use on-shell masses (for unstable particles) without breaking the gauge parameter independence of the result. In order to understand the connection with Eqs. (21)–(23), defining neutral current PO we consider the process

$$\mathrm{H}(P) \to \mathrm{e}^-(p_1) + \mathrm{e}^+(p_2) + \mathrm{f}(p_3) + \bar{\mathrm{f}}(p_4) \tag{84}$$

where $\mathrm{f} \neq \mathrm{e}, \nu_{\mathrm{e}}$, and introduce the following invariants: $s_{\mathrm{H}} = P^2$, $s_1 = q_1^2 = (p_1 + p_2)^2$ and $s_2 = q_2^2 = (p_3 + p_4)^2$, while $s_i, i = 3, \dots, 5$ denote the remaining invariants describing the process. We also introduce $s_{\mathrm{Z}}$, the Z complex pole. Propagators are

$$\Delta_{\mathrm{A}}(i) = \frac{1}{s_i}, \qquad \Delta_{\mathrm{Z}}(i) = \mathrm{P}_{\mathrm{Z}}^{-1}(s_i) = \frac{1}{s_i - s_{\mathrm{Z}}}. \tag{85}$$

The total amplitude for process Eq.(84) is given by the sum of different contributions, doubly Z resonant etc.

$$\begin{aligned}
\mathscr{A}\left(\mathrm{H} \to \mathrm{e}^-\mathrm{e}^+\mathrm{f}\bar{\mathrm{f}}\right) = {} & \mathrm{A}_{\mathrm{ZZ}}(s_{\mathrm{H}}, s_1, s_2)\, \Delta_{\mathrm{Z}}(s_1)\, \Delta_{\mathrm{Z}}(s_2) \\
& + \mathrm{A}_{\mathrm{AA}}(s_{\mathrm{H}}, s_1, s_2)\, \Delta_{\mathrm{A}}(s_1)\, \Delta_{\mathrm{A}}(s_2) + \mathrm{A}_{\mathrm{AZ}}(s_{\mathrm{H}}, s_1, s_2)\, \Delta_{\mathrm{A}}(s_1)\, \Delta_{\mathrm{Z}}(s_2) \\
& + \mathrm{A}_{\mathrm{AZ}}(s_{\mathrm{H}}, s_2, s_1)\, \Delta_{\mathrm{Z}}(s_1)\, \Delta_{\mathrm{A}}(s_2) + \mathrm{A}_{\mathrm{Z}}(s_{\mathrm{H}}, s_1, s_2)\, \Delta_{\mathrm{Z}}(s_1) \\
& + \mathrm{A}_{\mathrm{Z}}(s_{\mathrm{H}}, s_2, s_1)\, \Delta_{\mathrm{Z}}(s_2) + \mathrm{A}_{\mathrm{A}}(s_{\mathrm{H}}, s_1, s_2)\, \Delta_{\mathrm{A}}(s_1) \\
& + \mathrm{A}_{\mathrm{A}}(s_{\mathrm{H}}, s_2, s_1)\, \Delta_{\mathrm{A}}(s_2) + \mathrm{A}_{\mathrm{NR}}.
\end{aligned} \tag{86}$$

To describe in details the various terms in Eq.(86) we introduce fermion currents defined by

$$\begin{aligned}
J^{\mu}_{\mathrm{Z\,f}}(p\,; q, k) &= \bar{\mathrm{u}}_{\mathrm{f}}(q)\, \gamma^{\mu} \left[\mathscr{V}_{\mathrm{f}}(p^2) + \mathscr{A}_{\mathrm{f}}(p^2)\, \gamma^5\right] \mathrm{v}_{\mathrm{f}}(k) \\
&= \mathscr{V}_{\mathrm{f}}^{+}(p^2)\, \bar{\mathrm{u}}_{\mathrm{f\,L}}(q)\, \gamma^{\mu}\, \mathrm{v}_{\mathrm{f\,L}}(k) + \mathscr{V}_{\mathrm{f}}^{-}(p^2)\, \bar{\mathrm{u}}_{\mathrm{f\,R}}(q)\, \gamma^{\mu}\, \mathrm{v}_{\mathrm{f\,R}}(k), \\
J^{\mu}_{\mathrm{A\,f}}(p\,; q, k) &= \mathscr{Q}_{\mathrm{f}}(p^2)\, \bar{\mathrm{u}}_{\mathrm{f}}(q)\, \gamma^{\mu}\, \mathrm{v}_{\mathrm{f}}(k),
\end{aligned} \tag{87}$$

where $p = q + k$. At tree level we have

$$\mathscr{V}_{\mathrm{f}}^{+} = \frac{g}{\mathrm{c_W}} \left(\mathrm{I}_{\mathrm{f}}^{(3)} - \mathrm{Q_f}\, \mathrm{s_W^2}\right), \qquad \mathscr{V}_{\mathrm{f}}^{-} = -g\, \mathrm{Q_f}\, \frac{\mathrm{s_W^2}}{\mathrm{c_W}}, \qquad \mathscr{Q}_{\mathrm{f}} = g\, \mathrm{Q_f}\, \mathrm{s_W}. \tag{88}$$

The amplitude for $\mathrm{H}(P) \to \gamma(q_1) + \gamma(q_2)$ is

$$\mathrm{A}_{\mathrm{AA}}(s_{\mathrm{H}}, s_1, s_2) = \mathscr{T}_{\mathrm{HAA}}(s_{\mathrm{H}}, s_1, s_2)\, \mathrm{T}_{\mu\nu}(q_1, q_2)\, J^{\mu}_{\mathrm{A\,e}}(q_1\,; p_1, p_2)\, J^{\nu}_{\mathrm{A\,f}}(q_2\,; p_3, p_4), \tag{89}$$

with $q_1^2 = s_1$ and $q_2^2 = s_2$. Similarly, the amplitude for $\mathrm{H}(P) \to \mathrm{Z}(q_1) + \mathrm{Z}(q_2)$ is

$$\mathrm{A}_{\mathrm{ZZ}}(s_{\mathrm{H}}, s_1, s_2) = \left[\mathscr{P}_{\mathrm{HZZ}}(s_{\mathrm{H}}, s_1, s_2)\, q_{2\mu}\, q_{1\nu} - \mathscr{D}_{\mathrm{HZZ}}(s_{\mathrm{H}}, s_1, s_2)\, g_{\mu\nu}\right]$$

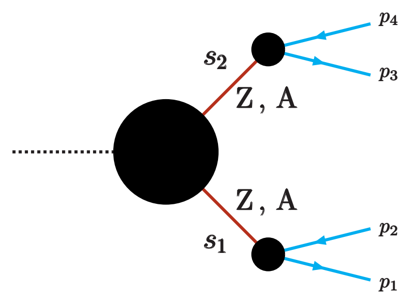


Figure 5: Doubly-resonant (ZZ, AA or AZ) part of the amplitude for the process of Eq.(84).

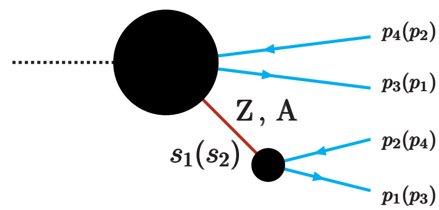


Figure 6: Singly-resonant (Z or A) part of the amplitude for the process of Eq.(84).

$$\times\, J^{\mu}_{\mathrm{Z e}}(q_1\,;p_1,p_2)\, J^{\nu}_{\mathrm{Z f}}(q_2\,;p_3,p_4)\;. \tag{90}$$

The ZZ, AA or AZ, doubly-resonant parts of the amplitude are shown in Fig. 5 while the singly, Z or A, parts are shown in Fig. 6. For the singly-resonant amplitudes we write

$$\mathrm{A_Z}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{_2}) = \bar{\mathrm{u}}_{\mathrm{f}}(p_3)\,\mathscr{F}_{\mathrm{HZ}\,\mu}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{_2})\,\mathrm{v_f}(p_4)\,J^{\mu}_{\mathrm{Z,e}}\,(q_1\,;p_1,p_2)\;, \tag{91}$$

where the form factor $\mathscr{F}$ is again decomposed as follows:

$$\bar{\mathrm{u}}_{\mathrm{f}}(p_3)\,\mathscr{F}_{\mathrm{HZ}\,\mu}\,\mathrm{v_f}(p_4) = \sum_i \mathscr{F}^{\,i}_{\mathrm{HZ}}\,\mathscr{C}_{i\,\mu}\;,$$
$$\mathscr{C}_{i\,\mu} = \{\,\bar{\mathrm{u}}_{\mathrm{f}}(p_3)\,\gamma_\mu\,\mathrm{v_f}(p_4)\;,\;\bar{\mathrm{u}}_{\mathrm{f}}(p_3)\,\gamma_\mu\,\gamma^5\,\mathrm{v_f}(p_4)\;,\;\dots\}\;. \tag{92}$$

Having the full amplitude we start expanding, e.g.

$$\mathrm{A_{ZZ}}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{_2}) = \mathrm{A_{ZZ}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}\,,s_{_2}) + (s_{_1}-s_{\mathrm{Z}})\;\mathrm{A^{(1)}_{ZZ}}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{_2})\;,$$
$$\mathrm{A^{(1)}_{ZZ}}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{_2}) = \mathrm{A^{(1)}_{ZZ}}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{\mathrm{Z}}) + (s_{_2}-s_{\mathrm{Z}})\;\mathrm{A^{(12)}_{ZZ}}\,(s_{\mathrm{H}}\,,s_{_1}\,,s_{_2})\;, \tag{93}$$

etc. The total amplitude of Eq.(86) can be split into several components, Z doubly resonant (DR) ... Z singly resonant (SR) ... non resonant (NR). Note that NR includes multi-leg functions, up to pentagons:

$$\mathscr{A}_{\mathrm{DR\,;ZZ}} \;=\; \mathrm{A_{ZZ}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}\,,s_{\mathrm{Z}})\,\Delta_{\mathrm{Z}}(s_{_1})\,\Delta_{\mathrm{Z}}(s_{_2})\,,$$

$$
\begin{aligned}
\mathscr{A}_{\mathrm{DR}\,;\,\mathrm{AA}} &= \mathrm{A}_{\mathrm{AA}}\,(s_{\mathrm{H}}\,,0\,,0)\,\Delta_{\mathrm{A}}(s_1)\,\Delta_{\mathrm{A}}(s_2)\,,\\
\mathscr{A}_{\mathrm{SR}\,;\,\mathrm{Z}} &= \Big[\mathrm{A}_{\mathrm{Z}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}\,,s_2)+\mathrm{A}^{(2)}_{\mathrm{AZ}}\,(s_{\mathrm{H}}\,,s_2\,,s_{\mathrm{Z}})+\mathrm{A}^{(2)}_{\mathrm{ZZ}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}\,,s_2)\Big]\,\Delta_{\mathrm{Z}}(s_1)\,,\\
&+ \Big[\mathrm{A}_{\mathrm{Z}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}\,,s_1)+\mathrm{A}^{(1)}_{\mathrm{AZ}}\,(s_{\mathrm{H}}\,,s_1\,,s_{\mathrm{Z}})+\mathrm{A}^{(1)}_{\mathrm{ZZ}}\,(s_{\mathrm{H}}\,,s_1\,,s_{\mathrm{Z}})\Big]\,\Delta_{\mathrm{Z}}(s_2)\,,\\
\mathscr{A}_{\mathrm{SR}\,;\,\mathrm{A}} &= \Big[\mathrm{A}_{\mathrm{A}}\,(s_{\mathrm{H}}\,,0\,,s_2)+\mathrm{A}^{(2)}_{\mathrm{AA}}\,(s_{\mathrm{H}}\,,0\,,s_2)+\mathrm{A}^{(2)}_{\mathrm{AZ}}\,(s_{\mathrm{H}}\,,0\,,s_2)]\,\Delta_{\mathrm{A}}(s_1)\\
&+ \Big[\mathrm{A}_{\mathrm{A}}\,(s_{\mathrm{H}}\,,0\,,s_1)+\mathrm{A}^{(1)}_{\mathrm{AA}}\,(s_{\mathrm{H}}\,,s_1\,,0)+\mathrm{A}^{(1)}_{\mathrm{AZ}}\,(s_{\mathrm{H}}\,,0\,,s_1)]\,\Delta_{\mathrm{A}}(s_2)\\
\mathscr{A}_{\mathrm{DR}\,;\,\mathrm{AZ}} &= \mathrm{A}_{\mathrm{AZ}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}\,,s_1)\,\Big[\Delta_{\mathrm{Z}}(s_1)\,\Delta_{\mathrm{A}}(s_2)+\Delta_{\mathrm{A}}(s_1)\,\Delta_{\mathrm{Z}}(s_2)\Big]\\
\mathscr{A}_{\mathrm{NR}} &= \mathrm{A}^{(1,2)}_{\mathrm{DR}\,;\,\mathrm{ZZ}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)+\mathrm{A}^{(2,1)}_{\mathrm{DR}\,;\,\mathrm{ZZ}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)+\mathrm{A}^{(1,2)}_{\mathrm{DR}\,;\,\mathrm{AZ}}\,(s_{\mathrm{H}}\,,s_2\,,s_1)\\
&+ \mathrm{A}^{(2,1)}_{\mathrm{DR}\,;\,\mathrm{AZ}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)+\mathrm{A}^{(1,2)}_{\mathrm{DR}\,;\,\mathrm{AA}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)+\mathrm{A}^{(2,1)}_{\mathrm{DR}\,;\,\mathrm{AA}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)\\
&+ \mathrm{A}^{(1)}_{\mathrm{SR}\,;\,\mathrm{Z}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)+\mathrm{A}^{(2)}_{\mathrm{SR}\,;\,\mathrm{Z}}\,(s_{\mathrm{H}}\,,s_2\,,s_1)+\mathrm{A}^{(1)}_{\mathrm{SR}\,;\,\mathrm{A}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)\\
&+ \mathrm{A}^{(2)}_{\mathrm{SR}\,;\,\mathrm{A}}\,(s_{\mathrm{H}}\,,s_2\,,s_1)+\mathrm{A}_{\mathrm{NR}}\,. \qquad (94)
\end{aligned}
$$

Each $\mathscr{A}$ amplitude is gauge parameter independent. Let us consider SMEFT at tree level, so that

$$
\mathrm{A}_{\mathrm{AA}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)=\mathrm{A}^{\mathrm{SM}}_{\mathrm{AA}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)+\Delta\mathrm{A}_{\mathrm{AA}}\,(s_{\mathrm{H}}\,,s_1\,,s_2)\,, \qquad (95)
$$

etc.. In $\Delta\mathrm{A}$ we do not include loops with $\mathrm{dim}=6$ insertions. Taking $\mathrm{f}=\mu$ and neglecting fermion masses we obtain the following result

$$
\begin{aligned}
\Delta\mathscr{A}\,\left(\mathrm{H}\to\mathrm{e}^-\mathrm{e}^+\mu^-\mu^+\right) &= -\,i\,g^3\,g_6\,J^{\mu}_{\mathrm{L}}\,(q_1\,;p_1,p_2)\,J_{\mu\,\mathrm{L}}\,(q_2\,;p_3,p_4)\,\Delta\mathscr{A}_{\mathrm{L}}\\
-\,i\,\frac{g^3\,g_6}{M_{\mathrm{W}}}\,\left(q_{2\mu}\,q_{1\nu}-q_1\cdot q_2\,g_{\mu\nu}\right)\,J^{\mu}_{\mathrm{L}}\,(q_1\,;p_1,p_2)\,J^{\nu}_{\mathrm{L}}\,(q_2\,;p_3,p_4)\Big]\,\Delta\mathscr{A}_{\mathrm{T}}\,, \qquad (96)
\end{aligned}
$$

where $J^{\mu}_{\mathrm{L}}$ is the left-handed fermion current (fermion masses are neglected) and $\Delta\mathscr{A}_{\mathrm{L}\,,\,\mathrm{T}}$ are the longitudinal and transverse parts of the LO SMEFT deviations. We obtain

$$
\begin{aligned}
\Delta\mathscr{A}_{\mathrm{T}} &= 2\,\mathrm{s}^2_{\mathrm{W}}\,a_{\mathrm{AA}}\,\Delta_{\mathrm{A}}(s_1)\,\Delta_{\mathrm{A}}(s_2)+\frac{1}{2}\,\mathrm{v_l}\,\frac{\mathrm{s_W}}{\mathrm{c_W}}\,a_{\mathrm{AZ}}\,\Big[\Delta_{\mathrm{A}}(s_1)\,\Delta_{\mathrm{Z}}(s_2)+\Delta_{\mathrm{A}}(s_2)\,\Delta_{\mathrm{Z}}(s_1)\Big]\\
&-\,\frac{1}{2}\,\mathrm{v}^2_{\mathrm{l}}\,\frac{a_{\mathrm{ZZ}}}{\mathrm{c}^2_{\mathrm{W}}}\,\Delta_{\mathrm{Z}}(s_1)\,\Delta_{\mathrm{Z}}(s_2)\,, \qquad (97)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\mathscr{A}_{\mathrm{L}} &= \frac{1}{4}\,\frac{\mathrm{v_l}}{\mathrm{c}^2_{\mathrm{W}}}\,\frac{1}{M_{\mathrm{W}}}\,\left(a_{\phi\mathrm{l\,V}}+a_{\phi\mathrm{l\,A}}\right)\,\Big[\Delta_{\mathrm{Z}}(s_1)+\Delta_{\mathrm{Z}}(s_2)\Big]\\
&-\,\frac{1}{16}\,\Big[\Big(7-20\,\mathrm{s}^2_{\mathrm{W}}+12\,\mathrm{s}^4_{\mathrm{W}}\Big)\,\frac{a_{\phi\mathrm{D}}}{\mathrm{c}^4_{\mathrm{W}}}+4\,\mathrm{v}^2_{\mathrm{l}}\,\frac{a_{\phi\Box}}{\mathrm{c}^4_{\mathrm{W}}}+8\,\frac{\mathrm{v_l}}{\mathrm{c}^4_{\mathrm{W}}}\,\left(a_{\phi\mathrm{l\,V}}+a_{\phi\mathrm{l\,A}}\right)
\end{aligned}
$$

$$
+\,4\,\left(3-7\,\mathrm{s}_{\mathrm{W}}^{2}+4\,\mathrm{s}_{\mathrm{W}}^{6}\right)\,\frac{a_{\mathrm{ZZ}}}{\mathrm{c}_{\mathrm{W}}^{4}}-4\,\left(5-12\,\mathrm{s}_{\mathrm{W}}^{2}+4\,\mathrm{s}_{\mathrm{W}}^{4}\right)\,\frac{\mathrm{s}_{\mathrm{W}}}{\mathrm{c}_{\mathrm{W}}^{4}}\,\left(\mathrm{s}_{\mathrm{W}}\,a_{\mathrm{AA}}+\mathrm{c}_{\mathrm{W}}\,a_{\mathrm{AZ}}\right)\Big]
$$

$$
\times\,M_{\mathrm{W}}\,\Delta_{\mathrm{Z}}(s_{1})\,\Delta_{\mathrm{Z}}(s_{2})\,, \tag{98}
$$

where $\mathrm{v}_{\mathrm{l}}=1-2\,\mathrm{s}_{\mathrm{W}}^{2}$. With the help of Eqs.(97)–(98) it is straightforward to establish the relation between the PO of Sect. 4 and the SMEFT Wilson coefficients (when the complex poles are identified with on-shell masses). It is worth noting that we have not included dipole operators.

It is worth noting that there are subtleties when the H is off-shell, they are described in Appendix C.1 of Ref. [30]. Briefly, there is a difference between performing an analytical continuation (H virtuality $\to$ H on-shell mass) in the off-shell decay width and using leading-pole approximation (LPA) of Ref. [32], i.e. the DR part, where the matrix element (squared) is projected but not the phase-space. Analytical continuation is a unique, gauge invariant procedure, the advantage of LPA is that it allows for a straightforward implementation of cuts.

In order to extend the SMEFT-PO connection to loop-level SMEFT we have to consider various ingredients separately.

- H $\to$ ZZ The amplitude for $\mathrm{H}(P)\to\mathrm{Z}_{\mu}(p_1)\mathrm{Z}_{\nu}(p_2)$ can be written as

$$
\mathrm{A}_{\mathrm{HZZ}}^{\mu\nu}=i\,\left(\mathscr{P}_{\mathrm{HZZ}}^{11}\,p_1^{\mu}\,p_1^{\nu}+\mathscr{P}_{\mathrm{HZZ}}^{12}\,p_1^{\mu}\,p_2^{\nu}+\mathscr{P}_{\mathrm{HZZ}}^{21}\,p_2^{\mu}\,p_1^{\nu}+\mathscr{P}_{\mathrm{HZZ}}^{22}\,p_2^{\mu}\,p_2^{\nu}-\mathscr{D}_{\mathrm{HZZ}}\,g^{\mu\nu}\right)\,. \tag{99}
$$

The result in Eq.(99) is fully general and can be used to prove Ward-Slavnov-Taylor identities (WSTI). As far as the partial decay width is concerned only $\mathscr{P}_{\mathrm{HZZ}}^{21}\equiv\mathscr{P}_{\mathrm{HZZ}}$ will be relevant, due to $p\cdot e(p)=0$ where $e$ is the polarization vector. Note that computing WSTI requires additional amplitudes, i.e. $\mathrm{H}\to\phi^0\gamma$ and $\mathrm{H}\to\phi^0\phi^0$. The result can be written as follows:

$$
\mathscr{D}_{\mathrm{HZZ}}=-g_{\mathrm{F}}\,\frac{M_{\mathrm{W}}}{\mathrm{c}_{\mathrm{W}}^{2}}\,\rho_{\mathrm{D;LO}}^{\mathrm{HZZ}}+\frac{g_{\mathrm{F}}^{3}}{\pi^2}\Big[\sum_{\mathrm{I=t,b,W}}\rho_{\mathrm{I;D;NLO}}^{\mathrm{HZZ}}\,\mathscr{D}_{\mathrm{HZZ;NLO}}^{\mathrm{I}}+\mathscr{D}_{\mathrm{HZZ}}^{(4)\,;\mathrm{nfc}}+g_6\sum_{\{a\}}\mathscr{D}_{\mathrm{HZZ}}^{(6)\,;\mathrm{nfc}}(a)\Big]\,,
$$

$$
\mathscr{P}_{\mathrm{HZZ}}=2\,g_{\mathrm{F}}\,g_6\,\frac{a_{\mathrm{ZZ}}}{M_{\mathrm{W}}}+\frac{g_{\mathrm{F}}^{3}}{\pi^2}\Big[\sum_{\mathrm{I=t,b,W}}\rho_{\mathrm{I;P;NLO}}^{\mathrm{HZZ}}\,\mathscr{P}_{\mathrm{HZZ;NLO}}^{\mathrm{I}}+g_6\sum_{\{a\}}\mathscr{P}_{\mathrm{HZZ}}^{(6)\,;\mathrm{nfc}}(a)\Big]\,. \tag{100}
$$

$$
\Delta\rho_{\mathrm{D;LO}}^{\mathrm{HZZ}}=\mathrm{s}_{\mathrm{W}}^{2}\,a_{\mathrm{AA}}+\left[4+\mathrm{c}_{\mathrm{W}}^{2}\,\left(1-\frac{M_{\mathrm{H}}^{2}}{M_{\mathrm{W}}^{2}}\right)\right]\,a_{\mathrm{ZZ}}+\mathrm{c}_{\mathrm{W}}\,\mathrm{s}_{\mathrm{W}}\,a_{\mathrm{AZ}}+2\,a_{\phi\Box}\,, \tag{101}
$$

$$
\Delta\rho_{\mathrm{q;D;NLO}}^{\mathrm{HZZ}}=\Delta\rho_{\mathrm{q;P;NLO}}^{\mathrm{HZZ}}=2\,\mathrm{I}_{\mathrm{q}}^{(3)}\,a_{\mathrm{q}\,\phi}+2\,a_{\phi\Box}-\frac{1}{2}\,a_{\phi\mathrm{D}}+2\,a_{\mathrm{ZZ}}+\mathrm{s}_{\mathrm{W}}^{2}\,a_{\mathrm{AA}}\,,
$$

$$
\Delta\rho_{\mathrm{W;D;NLO}}^{\mathrm{HZZ}}=\frac{1}{12}\,\left(4+\frac{1}{\mathrm{c}_{\mathrm{W}}^{2}}\right)\,a_{\phi\mathrm{D}}+2\,a_{\phi\Box}+\mathrm{s}_{\mathrm{W}}^{2}\,a_{\mathrm{AA}}
$$

$$
+\,\mathrm{s}_{\mathrm{W}}^{2}\,\left(3\,\mathrm{c}_{\mathrm{W}}+\frac{5}{3}\,\frac{1}{\mathrm{c}_{\mathrm{W}}}\right)\,a_{\mathrm{AZ}}+\left(4+\mathrm{c}_{\mathrm{W}}^{2}\right)\,a_{\mathrm{ZZ}}\,,
$$

$$\Delta\rho^{\mathrm{HZZ}}_{\mathrm{W\,;\,P\,;\,NLO}} = 4\,a_{\phi\Box} + \frac{5}{2}\,a_{\phi\mathrm{D}} + 12\,a_{\mathrm{ZZ}} + 3\,\mathrm{s}^2_{\mathrm{W}}\,a_{\mathrm{AA}} \tag{102}$$

It is convenient to define sub-amplitudes; however, to respect a factorization into t, b and bosonic components, we have to introduce the following quantities:

$$\begin{array}{ll}
\mathrm{W_H} = \mathrm{W_{H\;W}} + \mathrm{W_{H\;t}} + \mathrm{W_{H\;b}} & \mathrm{W_Z} = \mathrm{W_{Z\;W}} + \mathrm{W_{Z\;t}} + \mathrm{W_{Z\;b}} + \overline{\sum}_{\mathrm{gen}}\,\mathrm{W_{Z\,;\,f}} \\
\mathrm{d}\mathscr{Z}_g = \mathrm{d}\mathscr{Z}_{g\,;\mathrm{W}} + \sum_{\mathrm{gen}}\mathrm{d}\mathscr{Z}_{g\,;\,\mathrm{f}} & \mathrm{d}\mathscr{Z}_{\mathrm{c_W}} = \mathrm{d}\mathscr{Z}_{\mathrm{c_W}\,;\mathrm{W}} + \mathrm{d}\mathscr{Z}_{\mathrm{c_W}\,;\mathrm{t}} + \mathrm{d}\mathscr{Z}_{\mathrm{c_W}\,;\mathrm{b}} + \overline{\sum}_{\mathrm{gen}}\,\mathrm{d}\mathscr{Z}_{\mathrm{c_W}\,;\,\mathrm{f}} \\
\mathrm{d}\mathscr{Z}_{M_\mathrm{W}} = \mathrm{d}\mathscr{Z}_{M_\mathrm{W}\,;\mathrm{W}} + \sum_{\mathrm{gen}}\mathrm{d}\mathscr{Z}_{M_\mathrm{W}\,;\,\mathrm{f}} &
\end{array} \tag{103}$$

where $\mathrm{W}_{\Phi\,;\,\phi}$ denotes the $\phi$ component of the $\Phi$ (LSZ) wave-function factor etc. By $\mathrm{d}\mathscr{Z}_{\mathrm{par}}$ we denote the (UV finite) counterterm that is needed in connecting the renormalized parameters to an input parameter set (IPS). In the actual calculation we use IPS = $\{G_\mathrm{F}\,, M_\mathrm{Z}\,, M_\mathrm{W}\}$. Furthermore, $\sum_{\mathrm{gen}}$ implies summing over all fermions and all generations, while $\overline{\sum}_{\mathrm{gen}}$ excludes t and b from the sum.

$$\mathscr{D}^{(4)\,;\,\mathrm{nfc}}_{\mathrm{HZZ}} = \frac{1}{32}\,\frac{M_\mathrm{W}}{\mathrm{c}^2_{\mathrm{W}}}\,\left(2\overline{\sum}_{\mathrm{gen}}\mathrm{W_{Z\,;\,f}} - \sum_{\mathrm{gen}}\mathrm{dZ}_{M_\mathrm{W}\,;\,\mathrm{f}} + 4\overline{\sum}_{\mathrm{gen}}\mathrm{dZ}_{\mathrm{c_W}\,;\,\mathrm{f}} - 2\sum_{\mathrm{gen}}\mathrm{dZ}_{g\,;\,\mathrm{f}}\right)\,. \tag{104}$$

The connection to Eqs. (20)–(24) is given by

$$\mathrm{v_F}\,g^f_\mathrm{Z}\,g^{f'}_\mathrm{Z}\,\mathscr{P}_{\mathrm{HZZ}} = -2\,\varepsilon_{\mathrm{ZZ}}\,, \qquad \mathrm{v_F}\,g^f_\mathrm{Z}\,g^{f'}_\mathrm{Z}\,\left(p_1\cdot p_2\,\mathscr{P}_{\mathrm{HZZ}} - \mathscr{D}_{\mathrm{HZZ}}\right) = 2\,M^2_\mathrm{Z}\,\kappa_{\mathrm{ZZ}}\,. \tag{105}$$

- H → WW The derivation of the amplitude for H → WW follows closely the one for H → ZZ.

$$\mathscr{D}_{\mathrm{HWW}} = -g_\mathrm{F}\,M_\mathrm{W}\,\rho^{\mathrm{HWW}}_{\mathrm{D\,;\,LO}} + \frac{g^3_\mathrm{F}}{\pi^2}\left[\sum_{\mathrm{I=q,W}}\rho^{\mathrm{HWW}}_{\mathrm{I\,;\,D\,;\,NLO}}\,\mathscr{D}^{\mathrm{I}}_{\mathrm{HWW\,;\,NLO}} + \mathscr{D}^{(4)\,;\,\mathrm{nfc}}_{\mathrm{HWW}} + g_6\sum_{\{a\}}\mathscr{D}^{(6)\,;\,\mathrm{nfc}}_{\mathrm{HWW}}(a)\right],$$

$$\mathscr{P}_{\mathrm{HWW}} = 2\,g_\mathrm{F}\,g_6\,\frac{1}{M_\mathrm{W}}\,a_{\phi\mathrm{W}} + \frac{g^3_\mathrm{F}}{\pi^2}\left[\sum_{\mathrm{I=q}}\rho^{\mathrm{HWW}}_{\mathrm{I\,;\,P\,;\,NLO}}\,\mathscr{P}^{\mathrm{I}}_{\mathrm{HWW\,;\,NLO}} + g_6\sum_{\{a\}}\mathscr{P}^{(6)\,;\,\mathrm{nfc}}_{\mathrm{HWW}}(a)\right], \tag{106}$$

where we have introduced

$$\mathscr{D}^{(4)\,;\,\mathrm{nfc}}_{\mathrm{HWW}} = \frac{1}{32}\,M_\mathrm{W}\overline{\sum}_{\mathrm{gen}}\left(2\,\mathrm{W_{W\,;\,f}} - \mathrm{d}\mathscr{Z}_{M_\mathrm{W}\,;\,\mathrm{f}} - 2\,\mathrm{d}\mathscr{Z}_{g\,;\,\mathrm{f}}\right)\,. \tag{107}$$

$$\Delta\rho^{\mathrm{HWW}}_{\mathrm{D\,;\,LO}} = \mathrm{s}^2_{\mathrm{W}}\,\left(\frac{M^2_\mathrm{H}}{M_\mathrm{W}} - 5\,M_\mathrm{W}\right)\,(a_{\mathrm{AA}} + a_{\mathrm{AZ}} + a_{\mathrm{ZZ}}) + \frac{1}{2}\,M_\mathrm{W}\,a_{\phi\mathrm{D}} - 2\,M_\mathrm{W}\,a_{\phi\Box}\,, \tag{108}$$

$$\Delta\rho^{\mathrm{HWW}}_{\mathrm{q\,;\,D\,;\,NLO}} = \Delta\rho^{\mathrm{HWW}}_{\mathrm{q\,;\,P\,;\,NLO}} = a_{\phi\mathrm{t\,V}} + a_{\phi\mathrm{t\,A}} + a_{\phi\mathrm{b\,V}} + a_{\phi\mathrm{b\,A}} - a_{\mathrm{b}\phi} + 2\,a_{\phi\Box} - \frac{1}{2}\,a_{\phi\mathrm{D}}$$

$$+\, s_W\, a_{bWB} + c_W\, a_{bBW} + 5\, s_W^2\, a_{AA} + 5\, c_W\, s_W\, a_{AZ} + 5\, c_W^2\, a_{ZZ}\,,$$

$$\Delta\rho^{HWW}_{W;D;NLO} = \frac{1}{96}\,\frac{s_W^2}{c_W^2}\, a_{\phi D} + \frac{23}{12}\, a_{\phi\Box} - \frac{35}{96}\, a_{\phi D}$$

$$+\, 4\, s_W^2\, a_{AA} + \frac{1}{12}\, s_W \left(3\,\frac{1}{c_W} + 49\, c_W\right) a_{AZ} + \frac{1}{2}\left(9\, c_W^2 + s_W^2\right) a_{ZZ}\,,$$

$$\Delta\rho^{HWW}_{W;P;NLO} = 7\, a_{\phi\Box} - 2\, a_{\phi D} + 5\, s_W^2\, a_{AA} + 5\, c_W\, s_W\, a_{AZ} + 5\, c_W^2\, a_{ZZ}\,. \qquad (109)$$

These results allow us to write $\varepsilon_{WW}$ and $\kappa_{WW}$ of Eqs. (25)–(27) in terms of Wilson coefficients.

- $\underline{Z \to \bar{f} f}$ Let us consider the $Z\bar{f}F$ vertex, entering the process $H \to 4\, f$:

$$J^{\mu}_{Zf}(p\,;q,k) = \bar{u}_f(q)\left\{\gamma^{\mu}\left[\mathscr{V}_f(p^2) + \mathscr{A}_f(p^2)\,\gamma^5\right] + \mathscr{T}_f(p^2)\,\sigma^{\mu\nu}\, p_{\nu}\, v_f(k)\right\}. \qquad (110)$$

At lowest order we have deviations defined by

$$\mathscr{V}_f = i\, I_f^{3)}\,\frac{g}{c_W}\left(v_f + g_6\,\Delta\mathscr{V}_f\right), \quad \mathscr{A}_f = i\, I_f^{(3)}\,\frac{g}{c_W}\left(\frac{1}{2} + g_6\,\Delta\mathscr{A}_f\right), \quad \mathscr{T}_f = -\frac{g}{2}\, g_6\,\frac{m_f}{M_W}\, a_{fWB}\,, \qquad (111)$$

$$\Delta\mathscr{V}_f = a_{\phi f V} + \left[v_f + c_W^2\,\left(v_f - 1\right)\right]\left(a_{AA} + \frac{c_W}{s_W}\, a_{AZ} - \frac{1}{4\, s_W^2}\, a_{\phi D}\right) + \left(1 - v_f\right)\, c_W^2\, a_{ZZ}\,,$$

$$\Delta\mathscr{A}_f = a_{\phi f A} + \frac{1}{2}\left(a_{\phi W} - \frac{1}{4}\, a_{\phi D}\right). \qquad (112)$$

where the vector couplings are $v_u = 1/2 - 2\, Q_u\, s_W^2$, $v_d = 1/2 + 2\, Q_d\, s_W^2$ and $u, d$ are generic up, down fermions. When loops are included the decomposition in gauge invariant sub-amplitudes is not as simple as in the previous case, fermion loops and boson loops. Here the decomposition is given in terms of abelian and non-abelian (Z and W) parts, Q-components (those proportional to $\gamma^{\mu}$) and L-parts (those proportional to $\gamma^{\mu}\,\gamma_+$). Details can be found in Sect. 6.15 of Ref. [33]. The general expression in SMEFT will not be reported here. It is worth noting that

$$A_{Z\bar{f}f}(P^2) = A^{inv}_{Z\bar{f}f}(M_Z^2) + \left(P^2 - M_Z^2\right)\, A^{\xi}_{Z\bar{f}f}(P^2)\,, \qquad (113)$$

where $\xi$ denotes the collection of gauge parameters.

- $\underline{W \to ud}$ Similarly to the Z vertex we obtain

$$i\,\frac{g}{2\sqrt{2}}\,\gamma^{\mu}\left[V_F^{(+)}\,\gamma^+ + V_F^{(-)}\,\gamma^-\right] + \frac{g}{4}\, g_6\left(\frac{m_U^2}{M_W}\, a_{UW} - \frac{m_D^2}{M_W}\, a_{DW}\right)\sigma^{\nu\mu}\, p_{\nu}\,, \qquad (114)$$

$$\mathrm{V}_{\mathrm{F}}^{(+)} = 1 + \sqrt{2}\, g_6 \left( a^{(3)}_{\phi \mathrm{F}} + \frac{1}{2}\, a_{\phi \mathrm{W}} \right), \qquad \mathrm{V}_{\mathrm{F}}^{(-)} = \sqrt{2}\, g_6\, a_{\phi \mathrm{UD}}\,. \tag{115}$$

Here F is a generic doublet of components U = u or $\nu_{\mathrm{l}}$ and D = d or l. Note that $a_{\phi \mathrm{UD}} = 0$ for leptons. The general expression in SMEFT will not be reported here.

- Z → WW Triple gauge boson couplings are dscribed by the following deviations (all momenta flowing inwards):

$$\begin{aligned}
&\mathrm{V}^{\mu\nu\rho}_{\mathrm{ZWW}}(p_1\,,\,p_2\,,\,p_3) = g\,\mathrm{c_W}\,\mathrm{F}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3) + \frac{3}{2}\,g\,g_6\,\mathrm{H}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3)\,\frac{a_{\mathrm{QW}}}{M^2_{\mathrm{W}}} \\
&+ g\,g_6 \mathrm{c_W}\,\mathrm{F}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3)\,\left[\left(1 - 2\,\mathrm{s}^2_{\mathrm{W}}\right)\left(\frac{\mathrm{s_W}}{\mathrm{c_W}}\,a_{\phi \mathrm{WB}} - a_{\phi \mathrm{W}}\right) + 2\,\mathrm{s}^2_{\mathrm{W}}\,a_{\phi \mathrm{B}} + \frac{1}{4}\,a_{\phi \mathrm{D}}\right] \\
&+ g\,g_6 \mathrm{c_W}\,\mathrm{s_W}\,\mathrm{G}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3)\,\left[\left(1 - 2\,\mathrm{s}^2_{\mathrm{W}}\right)\,\frac{\mathrm{s_W}}{\mathrm{c_W}} + 2\,\mathrm{c_W}\right] a_{\phi \mathrm{WB}}\,,
\end{aligned} \tag{116}$$

$$\begin{aligned}
&\mathrm{F}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3) = p^{\rho}_1\,p^{\mu}_2\,p^{\nu}_3 - p^{\nu}_1\,p^{\rho}_2\,p^{\mu}_3 + g^{\nu\rho}\,\left(p^{\mu}_3\,p_1\cdot p_2 - p^{\mu}_2\,p_1\cdot p_3\right) \\
&+ g^{\nu\mu}\,\left(p^{\rho}_2\,p_1\cdot p_3 - p^{\rho}_1\,p_2\cdot p_3\right) + g^{\rho\mu}\,\left(p^{\nu}_1\,p_2\cdot p_3 - p^{\nu}_3\,p_1\cdot p_2\right), \\
&\mathrm{G}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3) = g^{\nu\rho}\,\left(p^{\mu}_2 - p^{\mu}_3\right) + g^{\nu\mu}\,\left(p^{\rho}_1 - p^{\rho}_2\right) + g^{\rho\mu}\,\left(p^{\nu}_3 - p^{\nu}_1\right), \\
&\mathrm{H}^{\mu\nu\rho}(p_1\,,\,p_2\,,\,p_3) = g^{\nu\rho}\,\left(p^{\mu}_3 - p^{\mu}_2\right) + g^{\nu\mu}\,\left(p^{\rho}_2 - p^{\nu}_3\right).
\end{aligned} \tag{117}$$

- VBF The process that we want to consider is

$$\mathrm{u}(p_1) + \mathrm{u}(p_2) \to \mathrm{u}(p_3) + \mathrm{e}^-(p_4) + \mathrm{e}^+(p_5) + \mu^-(p_6) + \mu^+(p_7) + \mathrm{u}(p_8)\,. \tag{118}$$

At LO SMEFT we introduce the triply-resonant (TR) part of the amplitude ($t$-channel propagators are never resonant):

$$J^{\mu}_{\pm}(p_i\,,\,p_j) = \bar{\mathrm{u}}_{p_i}\,\gamma^{\mu}\,\gamma_{\pm}\,\mathrm{u}_{p_j}\,, \qquad \Delta^{-1}_{\Phi}(p) = s - M^2_{\Phi}\,, \quad s = p^2\,, \tag{119}$$

$$\begin{aligned}
\mathscr{A}^{\mathrm{TR}}_{\mathrm{LO}} = &\left[J^{\mu}_{-}(p_4\,,\,p_5)\,(1 - v_{\mathrm{l}}) + J^{\mu}_{+}(p_4\,,\,p_5)\,(1 + v_{\mathrm{l}})\right]\left[J^{-}_{\mu}(p_6\,,\,p_7)\,(1 - v_{\mathrm{l}}) + J^{+}_{\mu}(p_6\,,\,p_7)\,(1 + v_{\mathrm{l}})\right] \\
\times &\left[J^{\nu}_{-}(p_3\,,\,p_2)\,(1 - v_{\mathrm{u}}) + J^{\nu}_{+}(p_3\,,\,p_2)\,(1 + v_{\mathrm{u}})\right]\left[J^{-}_{\nu}(p_8\,,\,p_1)\,(1 - v_{\mathrm{u}}) + J^{+}_{\nu}(p_8\,,\,p_1)\,(1 + v_{\mathrm{u}})\right],
\end{aligned} \tag{120}$$

$$\mathscr{A}^{\mathrm{TR}}_{\mathrm{SMEFT}} = -\,\frac{g^6}{4096}\,\Delta_{\mathrm{H}}(q_1 + q_2)\,\prod_{i=1,4}\Delta_{\mathrm{Z}}(q_i)\,\frac{M^2_{\mathrm{W}}}{c^8_{\theta}}\,\rho_{\mathrm{LO}}\,\mathscr{A}^{\mathrm{TR}}_{\mathrm{LO}} + g^6\,g_6\,\mathscr{A}^{\mathrm{TR}\,;\,\mathrm{nfc}}_{\mathrm{SMEFT}}$$

$$\Delta\rho_{\rm LO} = 2\,a_{\phi\Box} - \frac{2\,M_{\rm Z}^2 - 2\,M_{\rm H}^2 + q_1\cdot q_2 + q_2\cdot q_2}{M_{\rm W}^2}\,{\rm c}_{\rm W}^2\,a_{\rm ZZ}\,, \tag{121}$$

where $q_1 = p_8 - p_1$, $q_2 = p_3 - p_2$ are the incoming momenta in VBF and $q_3 = p_4 + p_5$, $q_4 = p_6 + p_7$ are the outgoing ones. Furthermore, $\gamma_\pm = 1 \pm \gamma^5$.

### 8.4 SMEFT and physical PO

In this Section we describe the connection between a possible realization of physical PO and SMEFT.

Multi pole expansion (MPE) has a dual role: as we mentioned, poles and their residues are intimately related to the gauge invariant splitting of the amplitude (Nielsen identities); residues of poles (after squaring the amplitude and after integration over residual variables) can be interpreted as physical PO, which requires factorization into subprocesses. However, gauge invariant splitting is not the same as "factorization" of the process into sub-processes, indeed phase space factorization requires the pole to be inside the physical region. For all technical details we refer to the work in Sect. 3 of Ref. [6] which is based on the following decomposition of the square of a propagator

$$\Delta = \frac{1}{\left(s - M^2\right)^2 + \Gamma^2\,M^2} = \frac{\pi}{M\,\Gamma}\,\delta\left(s - M^2\right) + {\rm PV}\left[\frac{1}{\left(s - M^2\right)^2}\right]. \tag{122}$$

and on the n-body decay phase space

$$d\Phi_n\left(P, p_1 \dots p_n\right) = \frac{1}{2\,\pi}\,dQ^2\,d\Phi_{n-j+1}\left(P, Q, p_{j+1} \dots p_n\right)\,d\Phi_j\left(Q, p_1 \dots p_j\right). \tag{123}$$

To "complete" the decay ($d\Phi_j$) we need the $\delta$-function in Eq.(123). We can say that the $\delta$-part of the resonant (squared) propagator opens the corresponding line allowing us to define physical PO ($t$-channel propagators cannot be cut). Consider the process ${\rm qq} \to \bar{\rm f}_1{\rm f}_1\bar{\rm f}_2{\rm f}_2\,j\,j$, according to the structure of the resonant poles we have different options in extracting physical PO, e.g.

$$\begin{aligned}
\sigma({\rm qq} \to \bar{\rm f}_1{\rm f}_1\bar{\rm f}_2{\rm f}_2\,j\,j) &\xmapsto{PO} \sigma({\rm qq} \to {\rm H}\,j\,j)\,{\rm Br}({\rm H} \to {\rm Z}\bar{\rm f}_1{\rm f}_1)\,{\rm Br}({\rm Z} \to \bar{\rm f}_2{\rm f}_2)\,,\\
\sigma({\rm qq} \to \bar{\rm f}_1{\rm f}_1\bar{\rm f}_2{\rm f}_2\,j\,j) &\xmapsto{PO} \sigma({\rm qq} \to {\rm ZZ}\,j\,j)\,{\rm Br}({\rm Z} \to \bar{\rm f}_1{\rm f}_1)\,{\rm Br}({\rm Z} \to \bar{\rm f}_2{\rm f}_2)\,.
\end{aligned} \tag{124}$$

There are fine points when factorizing a process into "physical" sub-processes (PO): extracting the $\delta$ from the (squared) propagator, Eq.(122), does not necessarily factorize the phase space; if cuts are not introduced, the interference terms among different helicities oscillate over the phase space and drop out, i.e. we achieve factorization, see Refs. [34]. Furthermore, MPE should be understood as "asymptotic expansion", see Refs. [35,36], not as Narrow-Width-Approximation

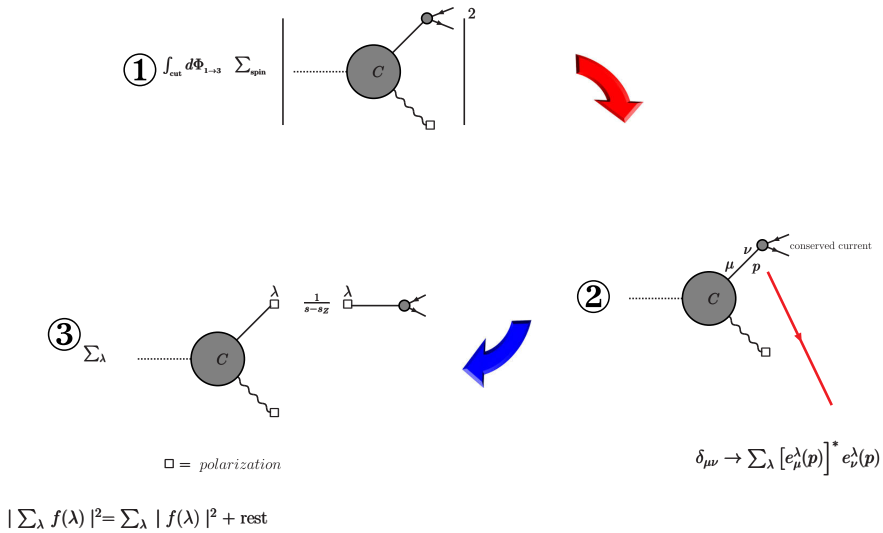


Figure 7: Building a physical PO.

(NWA). The phase space decomposition obtains by using the two parts in the propagator expansion of Eq.(123): the $\delta$-term is what we need to reconstruct PO, the PV-term (understood as a distribution [35]) gives the remainder and PO are extracted without making any approximation. It is worth noting that, in extracting PO, analytic continuation (on-shell masses into complex poles) is performed only after integrating over residual variables [30].

We can illustrate the SMEFT - MPE - PO connection by using a simple but non-trivial example: Dalitz decay of the Higgs boson, see Refs. [37,6]. Consider the process

$$\mathrm{H}(P) \to \bar{\mathrm{f}}(p_1) + \mathrm{f}(p_2) + \gamma(p_3)\,, \tag{125}$$

and introduce invariants $s_{\mathrm{H}} = -P^2$, $s = -(p_1+p_2)^2$ and propagators, $\Delta_{\mathrm{A}}(i) = 1/s_i$, $\Delta_{\mathrm{Z}}(i) = 1/(s_i - s_{\mathrm{Z}})$. With $s_{\mathrm{H}} = \mu_{\mathrm{H}}^2 - i\,\mu_{\mathrm{H}}\,\gamma_{\mathrm{H}}$ we denote the H complex pole etc. In the limit $m_{\mathrm{f}} \to 0$ the total amplitude for process Eq.(125) is given by the sum of three contributions, Z, A-resonant and non-resonant:

$$\mathscr{A}\left(\mathrm{H} \to \bar{\mathrm{f}}\mathrm{f}\gamma\right) = \Big[\mathrm{A}^{\mu}_{\mathrm{Z}}\,(s_{\mathrm{H}}\,,s)\,\Delta_{\mathrm{Z}}(s) + \mathrm{A}^{\mu}_{\mathrm{A}}\,(s_{\mathrm{H}}\,,s)\,\Delta_{\mathrm{A}}(s)\Big]\,e_{\mu}\,(p_3,l) + \mathrm{A}_{\mathrm{NR}}\,, \tag{126}$$

where $e_{\mu}$ is the photon polarization vector. The two resonant components are given by

$$\mathrm{A}^{\mu}_{\mathrm{V}}\,(s_{\mathrm{H}}\,,s) = \mathscr{T}_{\mathrm{HAV}}\,(s_{\mathrm{H}}\,,s)\;\mathrm{T}^{\mu}_{\nu}\,(q\,,p_3)\;J^{\nu}_{\mathrm{V\,f}}\,(q\,;p_1,p_2)\,, \tag{127}$$

where $J^{\mu}_{\mathrm{V\,f}}$ is the V fermion (f) current, $\mathrm{V} = \mathrm{A},\mathrm{Z}$, $q = p_1 + p_2$ and $\mathrm{T}^{\mu\nu}\,(k_1\,,k_2) = k_1\cdot k_2\,g^{\mu\nu} - k_1^{\nu}\,k_2^{\mu}$. Having the full amplitude we start expanding (MPE) according to

$$\mathscr{T}_{\mathrm{HAZ}}\,(s_{\mathrm{H}}\,,s) = \mathscr{T}_{\mathrm{HAZ}}\,(s_{\mathrm{H}}\,,s_{\mathrm{Z}}) + (s - s_{\mathrm{Z}})\;\mathscr{T}^{(1)}_{\mathrm{HAZ}}\,(s_{\mathrm{H}}\,,s) \qquad \text{etc.} \tag{128}$$

Derivation continues till we define physical PO:

$$\Gamma_{\rm PO}\left(\rm H \to Z\gamma\right) = \frac{1}{16\,\pi}\,\frac{1}{M_{\rm H}}\,\left(1 - \frac{\mu_{\rm Z}^2}{M_{\rm H}^2}\right)\,\rm F_{H\to Z\gamma}\left(s_{\rm Z}\,,\,\mu_{\rm Z}^2\right)\,,$$
$$\Gamma_{\rm PO}\left(\rm Z \to \bar{f}f\right) = \frac{1}{48\,\pi}\,\frac{1}{\mu_{\rm Z}}\,\rm F_{Z\to\bar{f}f}\left(s_{\rm Z}\,,\,\mu_{\rm Z}^2\right)\,.$$
$$\Gamma_{\rm SR}\left(\rm H \to \bar{f}f\gamma\right) = \frac{1}{2}\,\Gamma_{\rm PO}\left(\rm H \to Z\gamma\right)\,\frac{1}{\gamma_{\rm Z}}\,\Gamma_{\rm PO}\left(\rm Z \to \bar{f}f\right) + \text{ remainder}\,. \tag{129}$$

In the NWA the remainder is neglected while we keep it in our formulation where the goal is extracting PO without making approximations. The interpretation in terms of SMEFT is based on $\mathscr{T}_{\rm HAZ}\,(s_{\rm H}\,,\,s_{\rm Z})$. A convenient way for writing $\mathscr{T}_{\rm HAZ}$ is the following:

$$\mathscr{T}_{\rm HAZ} = \frac{g_{\rm F}^3}{\pi^2 M_{\rm Z}}\,\sum_{\rm I=W,t,b}\,\rho_{\rm I}^{\rm HAZ}\,\mathscr{T}^{\rm I}_{\rm HAZ\,;\,LO} + g_{\rm F}\,g_6\,\frac{M_{\rm H}^2}{M}\,a_{\rm AZ} + \frac{g_{\rm F}^3\,g_6}{\pi^2}\,\mathscr{T}^{\rm nfc}_{\rm HAZ}\,. \tag{130}$$

The factorizable part is defined in terms of $\rho$-factors

$$\Delta\rho_{\rm q}^{\rm HAZ} = \left(2\,{\rm I}_{\rm q}^{(3)}\,a_{\rm q\,\phi} + 2\,a_{\phi\,\Box} - \frac{1}{2}\,a_{\phi\,\rm D} + 3\,a_{\rm AA} + 2\,a_{\rm ZZ}\right)\,,$$
$$\Delta\rho_{\rm W}^{\rm HAZ} = \frac{1+6\,{\rm c}_{\rm W}^2}{{\rm c}_{\rm W}^2}\,a_{\phi\,\Box} - \frac{1}{4}\,\frac{1+4\,{\rm c}_{\rm W}^2}{{\rm c}_{\rm W}^2}\,a_{\phi\,\rm D} - \frac{1}{2}\,\frac{1+{\rm c}_{\rm W}^2-24\,{\rm c}_{\rm W}^4}{{\rm c}_{\rm W}^2}\,a_{\rm AA}\,,$$
$$+\,\frac{1}{4}\,\left(1+12\,{\rm c}_{\rm W}^2-48\,{\rm c}_{\rm W}^4\right)\,\frac{{\rm s}_{\rm W}}{{\rm c}_{\rm W}^3}\,a_{\rm AZ} + \frac{1}{2}\,\frac{1+15\,{\rm c}_{\rm W}^2-24\,{\rm c}_{\rm W}^4}{{\rm c}_{\rm W}^2}\,a_{\rm ZZ}\,. \tag{131}$$

In the PTG scenario we only keep $a_{\rm t\,\phi}, a_{\rm b\,\phi}, a_{\phi\,\rm D}$ and $a_{\phi\,\Box}$ in Eq.(131). We also derive the following result for the non-factorizable part of the amplitude:

$$\mathscr{T}^{\rm nfc}_{\rm HAZ} = \sum_{a\,\in\,\{A\}}\,\mathscr{T}^{\rm nfc}_{\rm HAZ}(a)\,a\,, \tag{132}$$

where $\{A\} = \{a_{\phi\,\rm t\,V}, a_{\rm t\,BW}, a_{\rm t\,WB}, a_{\phi\,\rm b\,V}, a_{\rm b\,WB}, a_{\rm b\,BW}, a_{\phi\,\rm D}, a_{\rm AZ}, a_{\rm AA}, a_{\rm ZZ}\}$. In the PTG scenario there are only 3 non-factorizable amplitudes for $\rm H \to \gamma Z$, those proportional to $a_{\phi\,\rm t\,V}, a_{\phi\,\rm b\,V}$ and $a_{\phi\,\rm D}$.

### 8.5 Summary on the PO-SMEFT matching

As we have shown, there are different layers of measurable parameters that can be extracted from LHC data. An external layer, where the kinematics is kept exact, is represented by *physical PO* such as $\Gamma_{\rm SR}\left(\rm H \to \bar{f}f\gamma\right)$ of Eq.(129): these are similar to the $\sigma_{\rm f}^{\rm peak}$ measured at LEP and, similarly to the LEP case, can be extracted from data via a non-trivial NWA. A first intermediate inner layer is represented by the *effective-couplings PO* introduced in Section 2-6 (and summarized in Section 7): these are similar to effective Z-boson couplings ($g^{\rm e}_{\rm V\,A}$) measured at LEP and control the parameterization of on-shell amplitudes. A further internal layer is represented by the $\hat{\kappa}_i$ introduced in this Section, that are appropriate combinations of Wilson coefficients in the SMEFT. Finally, the innermost layer is represented by the Wilson coefficients (or the Lagrangian couplings) of the specific EFT (or explicit NP model) employed to analyze the data. When moving to the innermost layer in the SMEFT context we still have the option of performing the tree-level translation, which is well defined and should be integrated with the corresponding estimate of MHOU, or we can go to SMEFT at the loop level, again with its own MHOU.

# 9 Conclusions

The experimental precision on the kinematical distributions of Higgs decays and production cross sections is expected to significantly improve in the next few years. This will allow us to investigate in depth a wide class of possible extensions of the SM. To reach this goal, an accurate and sufficiently general parameterization of possible NP effects in such distributions is needed.

The Higgs PO presented in this note are conceived exactly to fulfill this goal: they provide a general decomposition of on-shell amplitudes involving the Higgs boson, based on analyticity, unitarity, and crossing symmetry. A further key assumption is the absence of new light particles in the kinematical regime of interest, or better no unknown physical poles in theses amplitudes. These conditions ensure the generality of this approach and the possibility to match it to a wide class of explicit NP models, including the determination of Wilson coefficients in the context of Effective Field Theories.

As we have shown, the PO can be organized in two complementary sets: the so-called physical PO, that are nothing but a series of idealized Higgs partial decay widths, and the effective-couplings PO, that are particularly useful for the developments of simulation tools. The two sets are in one-to-one correspondence, and their relation is summarized in Table 1. The complete set of effective-couplings PO that can be realistically accessed in Higgs-related measurements at the LHC, both in production and in decays, is summarized in Section 7. The reduction of independent PO obtained under specific symmetry assumptions (in particular flavor universality and CP invariance) is also discussed in Section 7. In two-body processes the effective-couplings PO are in one-to-one correspondence with the parameters of the original $\kappa$ framework. A substantial difference arises in more complicated processes, such as $h \to 4f$ or VBF and VH production. Here, in order to take into account possible kinematical distortions in the decay distributions and/or in the production cross-sections, the PO framework requires the introduction of a series of additional terms. These terms encode generic NP effects in the $hVf\bar{f}$ amplitudes and their complete list is summarized in Table 2.

The PO framework can be systematically improved to include the effect of higher-order QCD and QED corrections, recovering the best up-to-date SM predictions in absence of new physics. The effective-couplings PO should not be confused with EFT Wilson coefficients. However, their measurement can can facilitate the extraction of Wilson coefficients in any EFT approach to Higgs physics, as briefly illustrated in Section 8 in the context of the so-called SMEFT. These physical and the effective-couplings PO can be considered as the most general and external layers in the characterization physics beyond the SM, whose innermost layer is represented by the couplings of some explicit NP model.

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