<Xmax> Uncertainty from Extrapolation of Cosmic Ray Air Shower Parameters

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MOTIVATION
The $<X_{\text{max}}>$ TA is in good agreement with Auger mix within systematics.
• Difference between the models at $10^{17}$ eV is $\sim 25$ g/cm$^2$

• Difference between the models at $10^{19.5}$ eV is $\sim 40$ g/cm$^2$
HADRON MODEL UNCERTAINTY

• LHC Maximum energy @ 7 TeV (p-p C.M) shows model deficiencies.

• None of the models is able to consistently describe the data.

• Full phase-space measurements are only up to ~ 1 TeV (p-p C.M) ($10^{15.3}$ eV).
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![Graph showing inclusive pp, \(\sqrt{s} = 8\) TeV](image_url)

HADRON MODEL UNCERTAINTY

• Fluctuation of the model $<X_{\text{max}}>\text{ in relation to considered parameter.}$

• Estimate the uncertainty in the extrapolation of the model parameter at high energy
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HADRONIC MODELS INTERACTION FEATURES

- Proton-air cross section
- Multiplicity/Number of charged hadron particles
- Elasticity/Fraction of energy carried away by the leading particle

Extended Heitler model
J. Matthews 2005
MODEL DEPENDENCE ON THE DIFFERENT MODEL PARAMETERS

- Ulrich et al (PRD 83 054026) *ad hoc* modification of existing hadronic interaction models during MC simulation.

- Allow us to modify the model freely

- Study the effect of each parameter independent of other parameters

\[
\begin{align*}
f(E, f_{19}) &= 1 + (f_{19} - 1) F(E) \\
F(E) &= \frac{\lg \left( \frac{E}{10^{15}\text{eV}} \right)}{\lg \left( \frac{10^{19}\text{eV}}{10^{15}\text{eV}} \right)}
\end{align*}
\]

- \( E < \text{PeV} / E > \text{PeV} \)

- \( f(E) \) rescales parameter

- Change only individual parameter while conserving
  - total energy
  - total charge
  - energy fraction in different particle type
MODEL DEPENDENCE ON THE MODEL PARAMETERS

CONEX4.44 to produce high statistics data sets at $10^{19.5} \text{ eV}$

\[ \frac{\Delta X_{\text{max}}}{\Delta f} \bigg|_{19} \sigma_{p-\text{air}} \approx 100 \text{ g/cm}^2 \]
MODEL DEPENDENCE ON THE MODEL PARAMETERS

\[ \frac{\Delta X_{max}}{\Delta f} |_{19} \approx 100 \text{ g/cm}^2 \]

\[ \frac{\Delta X_{max}}{\Delta f} |_{19} \text{ Multiplicity, Elasticity} \approx 30 - 40 \text{ g/cm}^2 \]
MODELS DEPENDENCE ON THE MODEL PARAMETERS @ $10^{19.5}$ EV

\[ \frac{\Delta X_{\text{max}}}{\Delta f} \bigg|_{19} \sigma_{p-\text{air}} \approx 100 \ g/cm^2 \]

\[ \frac{\Delta X_{\text{max}}}{\Delta f} \bigg|_{19} \text{Multiplicity, Elasticity} \approx 30 - 40 \ g/cm^2 \]
MODELS DEPENDENCE ON THE MODEL PARAMETERS @ $10^{17}$ EV

\[ \frac{\Delta X_{\text{max}}}{\Delta f} \approx 40 \text{ g/cm}^2 \]

\[ \frac{\Delta X_{\text{max}}}{\Delta f} \approx 10 - 15 \text{ g/cm}^2 \]
HADRON MODEL UNCERTAINTY

- Fluctuation of the model \(<X_{\text{max}}\) in relation to considered parameter.
- Estimate the uncertainty in the extrapolation of the model parameter at high energy.
\[ \langle N_{ch} \rangle = \alpha + \beta S^\gamma \]

\[ \langle N_{ch} \rangle = A + B(\ln S) + C(\ln S)^2 \]

\[ \Delta_{\text{multiplicity}} = \left| \frac{\Delta X_{\text{max}}}{\Delta f} \right| \text{ Multiplicity} \times \text{Extrapolation Differences (E)} \]

\[ \Delta_{\text{multiplicity}} = 40 \times \pm 0.32 = \pm 13 \text{ g/cm}^2 \]
Elasticity

\[ f_{ee}(K \sqrt{S_{pp}}) = <N_{ch}>_{pp} - n_0 \]

\[ f_{ee}(K \sqrt{S_{pp}} - \Delta m) = <N_{ch}>_{pp} - n_0 \]

Elasticity = \( \left| \frac{\Delta X_{max}}{\Delta f} \right| \) Elasticity \times Extrapolation Differences (E)

\[ \Delta \text{Elasticity} = \pm 2 \text{ g/cm}^2 \]
**P-AIR CROSS SECTION**


**M. Block and F. Halzen, Phys.Rev. D72, 036006 (2005).**

**Block, Phys.Rev. D84, 091501 (2011).**

\[
\begin{align*}
BHS &= c_o + c_1 \ln\left(\frac{v}{m}\right) + c_2 \ln^2\left(\frac{v}{m}\right) + \beta\left(\frac{v}{m}\right)^{\mu - 1} \\
COMPETE &= \alpha + \beta \ln\left(\frac{S}{S_H}\right)^2 \\
L_\gamma &= \alpha + \beta \ln\left(\frac{S}{S_H}\right)^\gamma
\end{align*}
\]

\[
\Delta \sigma_{p\text{-}air} = \left| \frac{\Delta X_{max}}{\Delta f} \right| \sigma_{p\text{-}air} \times \text{Extrapolation Differences (E)}
\]

\[
\Delta \sigma_{p\text{-}air} = \pm 6.5 \text{ g/cm}^2
\]
ESTIMATING UNCERTAINTIES

<table>
<thead>
<tr>
<th>Model</th>
<th>$&lt;X_{\text{max}}&gt;$ uncertainty $10^{17}$eV</th>
<th>$&lt;X_{\text{max}}&gt;$ uncertainty $10^{19.5}$eV</th>
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<tbody>
<tr>
<td>SIBYLL2.1</td>
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<td>±18 g/cm²</td>
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<tr>
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<tr>
<td>EPOS-LHC</td>
<td>±3 g/cm²</td>
<td>±18 g/cm²</td>
</tr>
</tbody>
</table>

$\rightarrow$ Adding in quadrature?
SUMMARY

• Estimated the uncertainty in $<X_{\text{max}}>$ in Cross Section, Multiplicity, and Elasticity.

• The extrapolation uncertainty at $10^{19.5} \text{ eV}$ approximately matched the difference among the five models.