



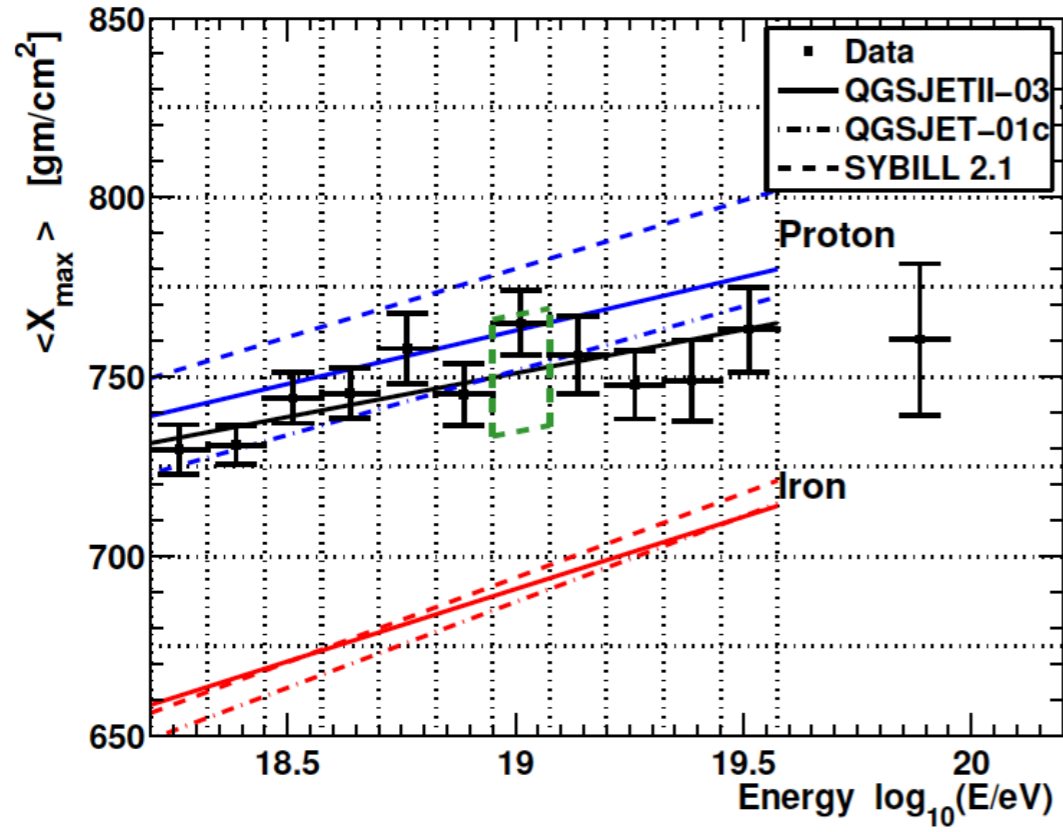
**X_{\max} Uncertainty from Extrapolation of
Cosmic Ray Air Shower Parameters**

UHECR2016

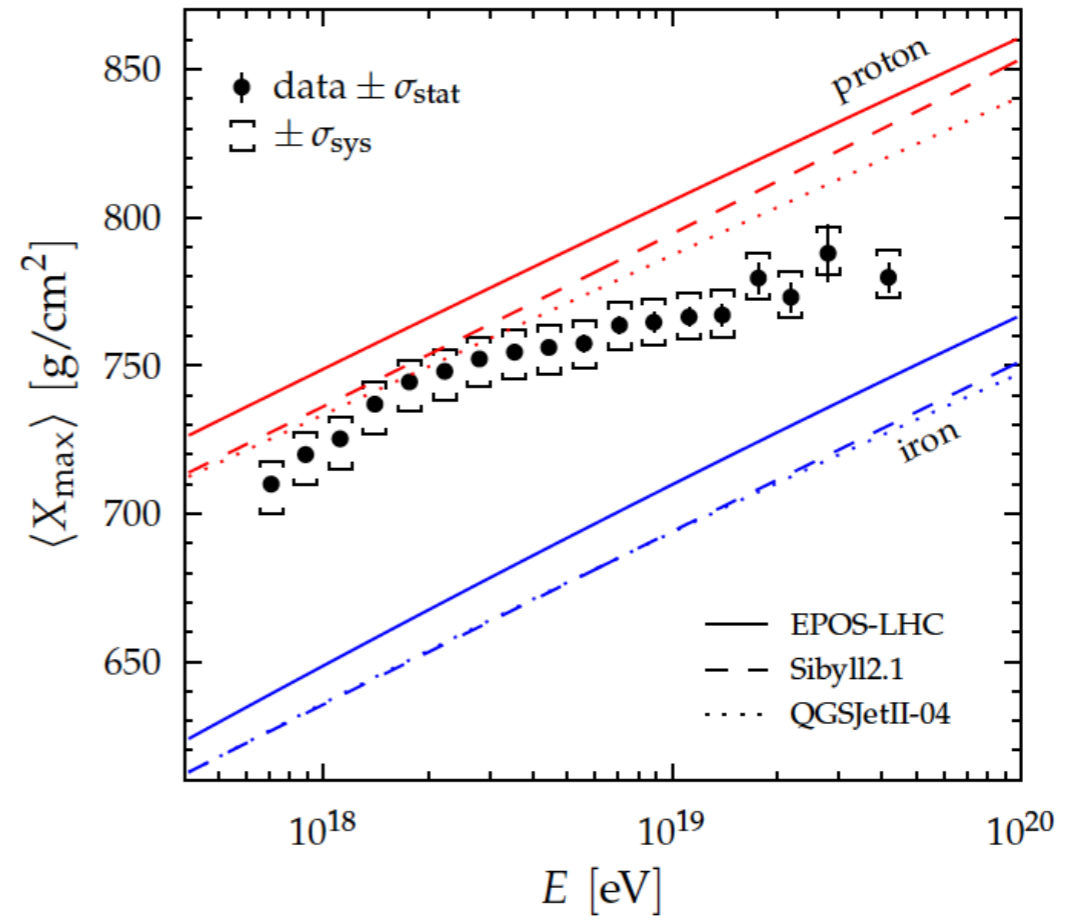
Rasha Abbasi, Gordon Thomson



MOTIVATION



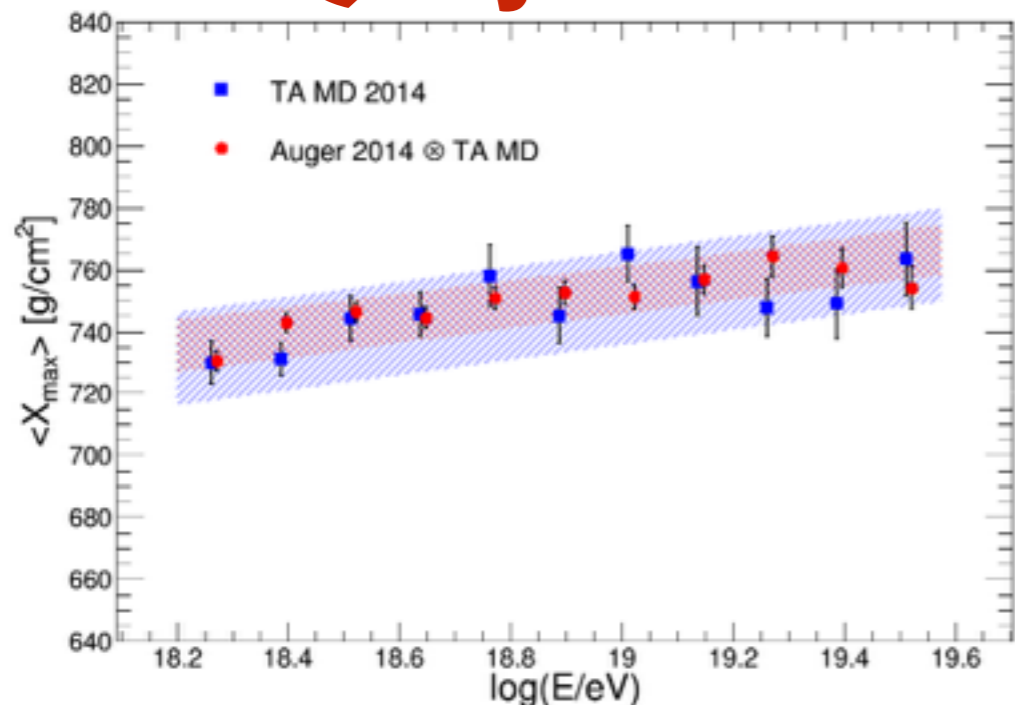
Telescope Array Collaboration, APP 64 (2014) 49



Pierre Auger Collaboration, PRD 90 (2014) 12, 122005

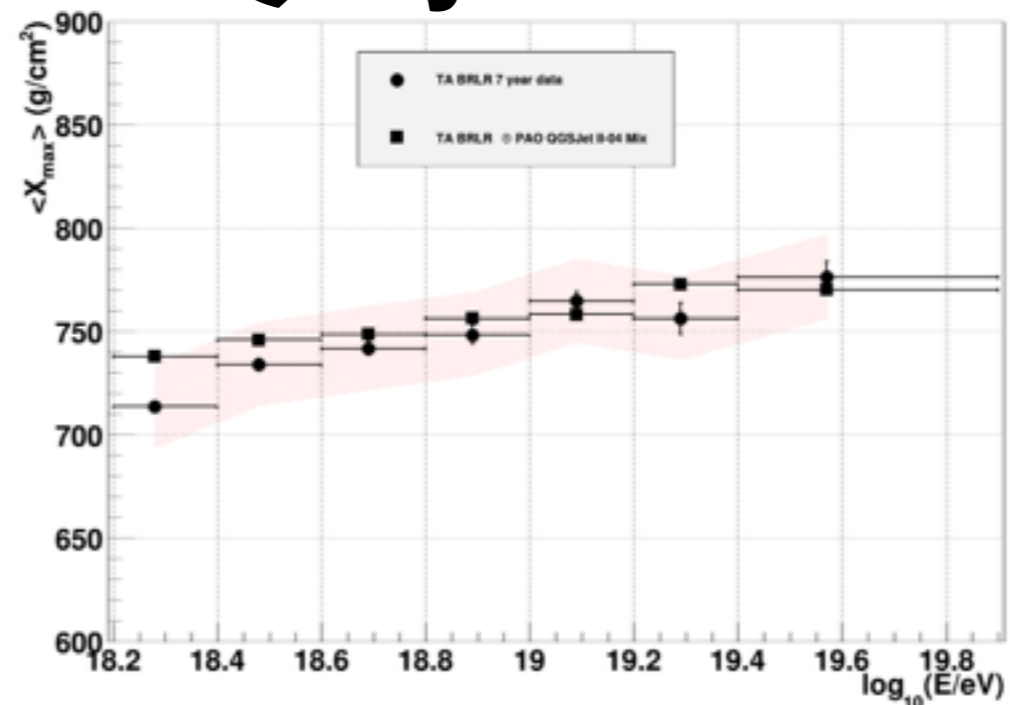
MOTIVATION

QGSJETII3



arXiv:1503.07540

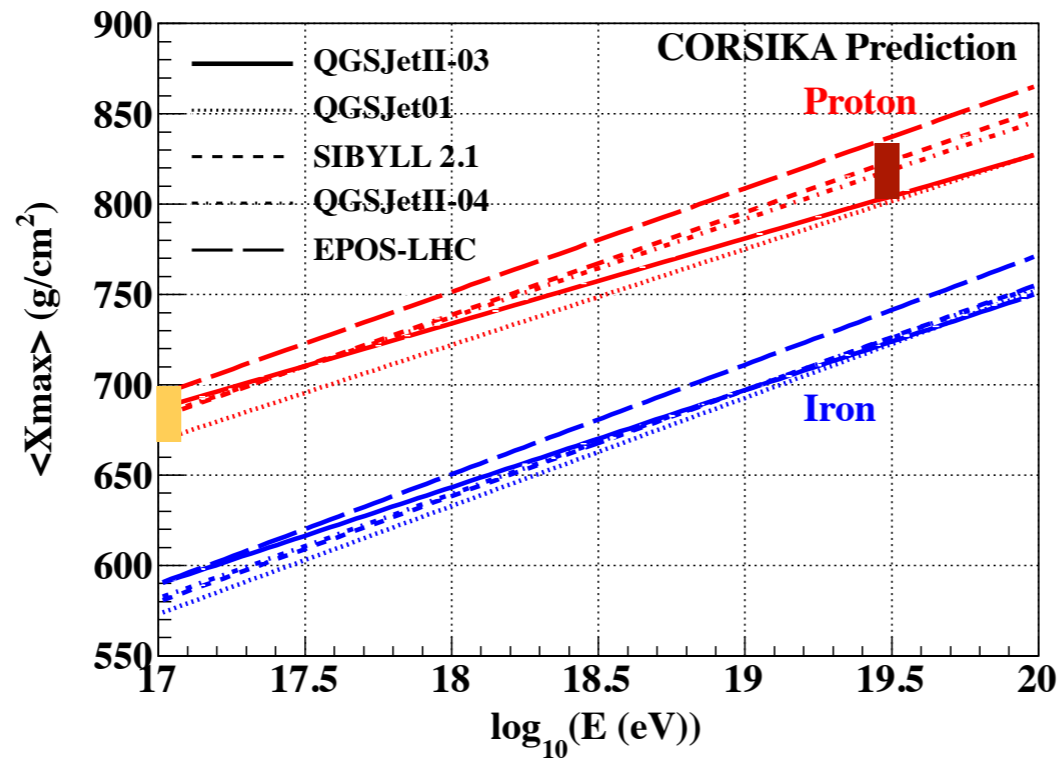
QGSJETII4



William Hanlon- Comp WG

The $\langle X_{\max} \rangle$ TA is in good agreement with Auger mix within systematics

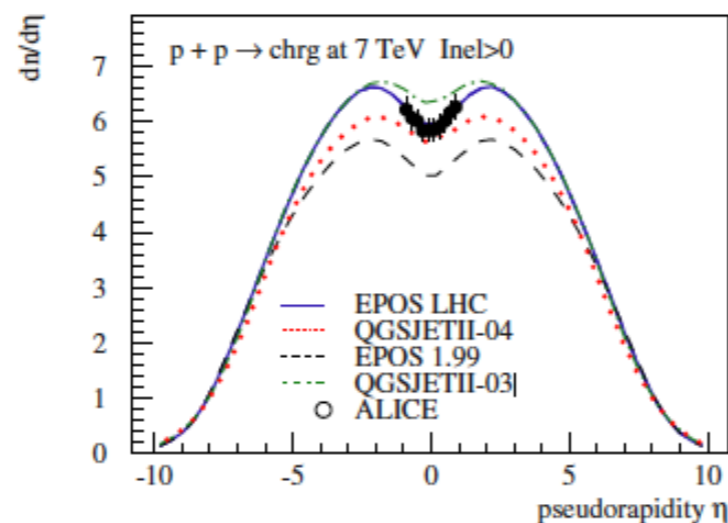
INTERPRETATIONS OF DATA IS MODEL DEPENDENT



- Difference between the models at 10^{17} eV is ~ 25 g/cm²
- Difference between the models at $10^{19.5}$ eV is ~ 40 g/cm²

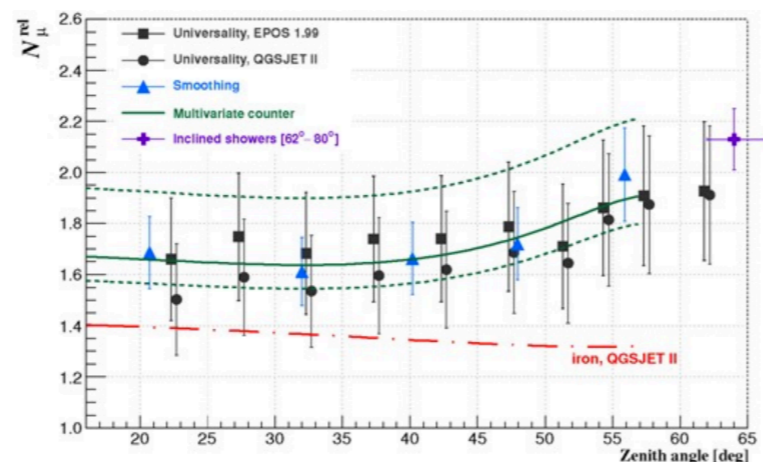
HADRON MODEL UNCERTAINTY

- LHC Maximum energy @ 7 TeV (p-p C.M) shows model deficiencies.
- None of the models is able to consistently describe the data
- Full phase-space measurements are only up to ~ 1 TeV (p-p C.M) ($10^{15.3}$ eV)



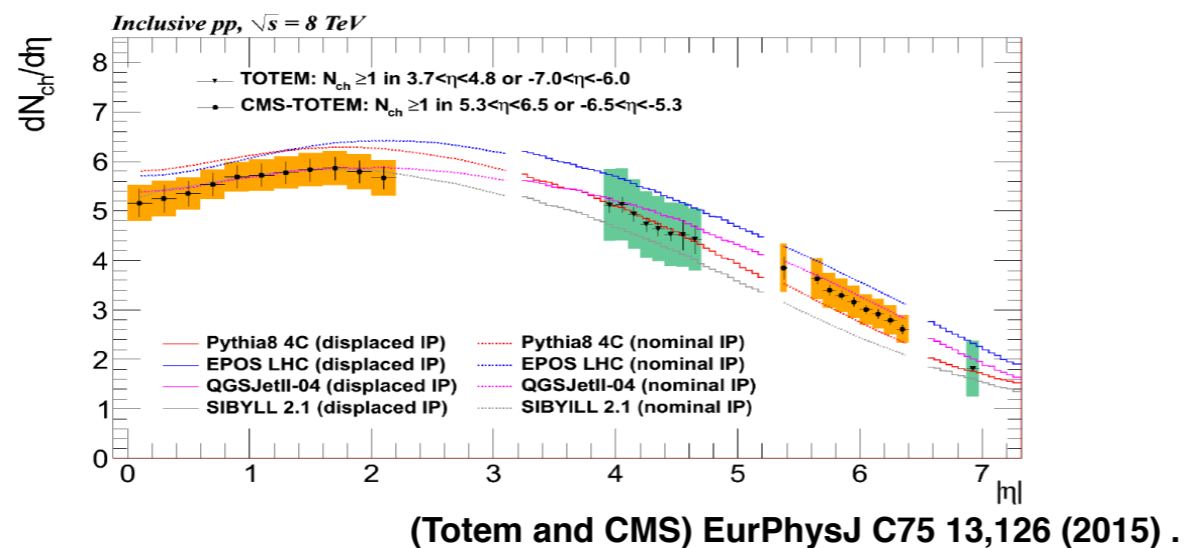
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HADRON MODEL UNCERTAINTY

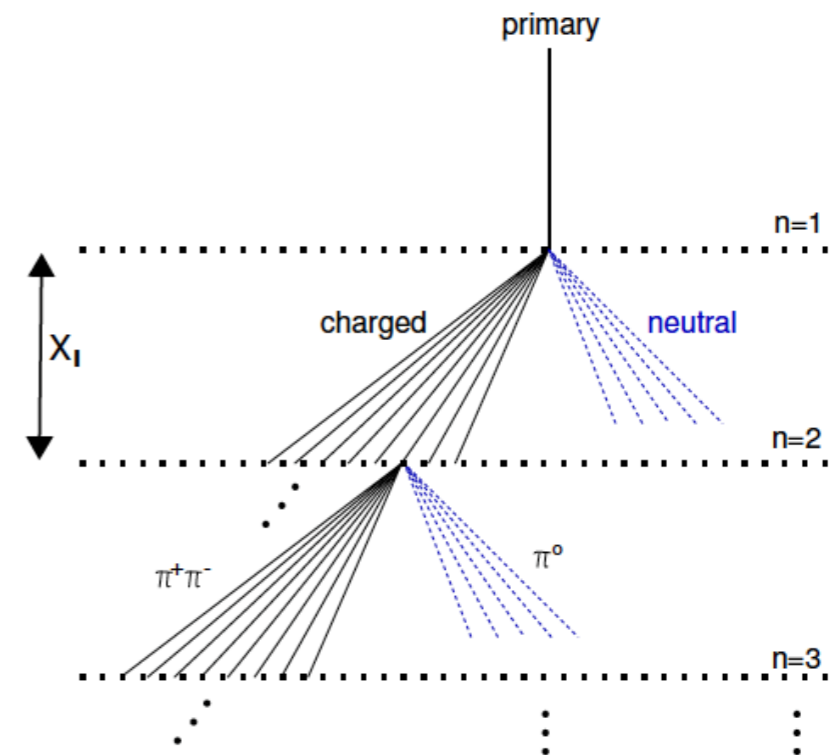
- Fluctuation of the model $\langle X_{\max} \rangle$ in relation to considered parameter.
- Estimate the uncertainty in the extrapolation of the model parameter at high energy

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HADRONIC MODELS INTERACTION FEATURES

- Proton-air cross section
- Multiplicity/Number of charged hadron particles
- Elasticity/Fraction of energy carried away by the leading particle



$$X_{\max} \approx \lambda_I + X_0 \ln \frac{E_0}{N_{\text{mult}} E_{\text{crit}}^{\text{e.m.}}}$$

Extended Heitler model

J. Matthews 2005

MODEL DEPENDENCE ON THE DIFFERENT MODEL PARAMETERS

- Ulrich *et al* (PRD 83 054026) *ad hoc* modification of existing hadronic interaction models during MC simulation.
- Allow us to modify the model freely
- Study the effect of each parameter independent of other parameters

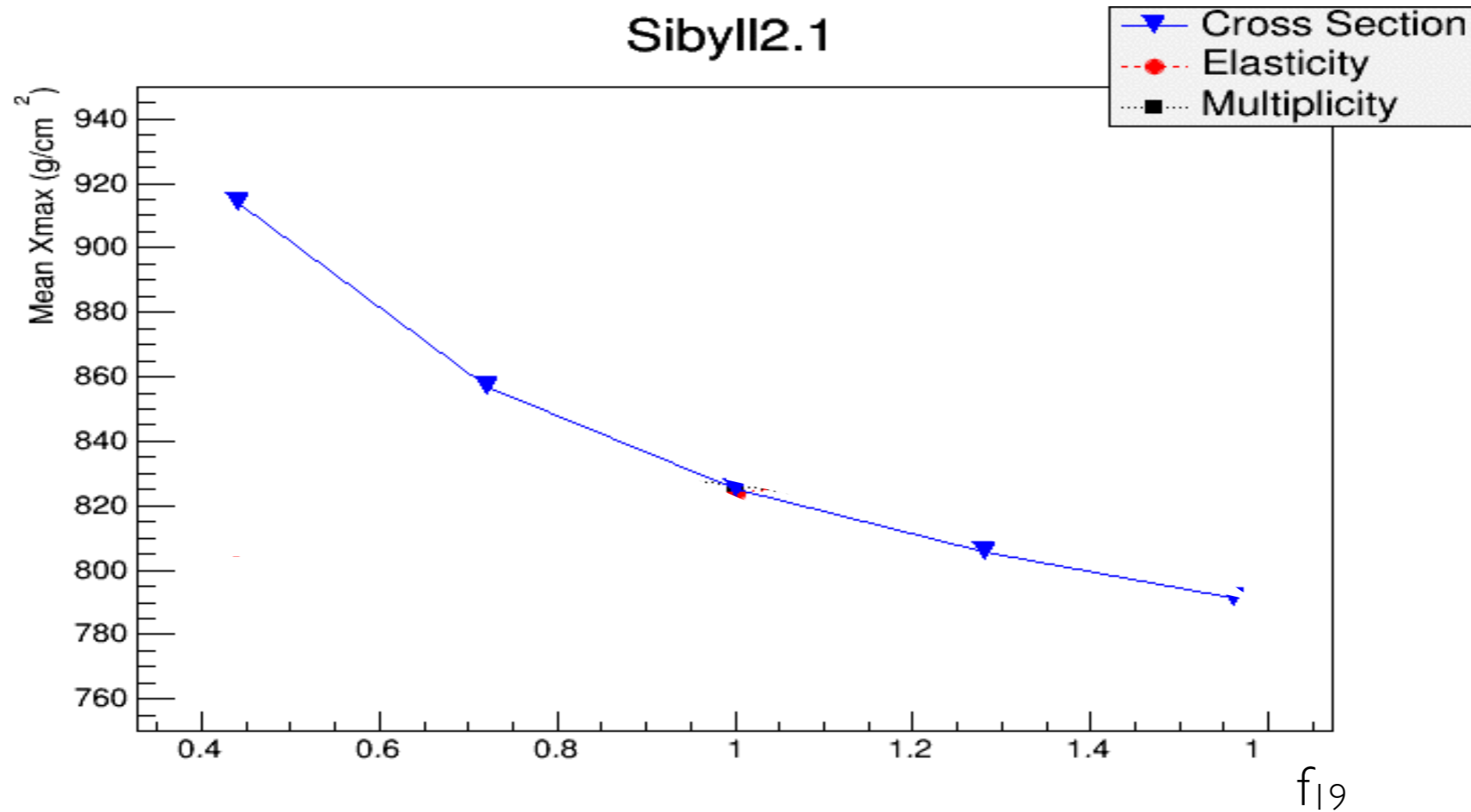
$$f(E, f_{19}) = 1 + (f_{19} - 1) F(E)$$

$$F(E) = \frac{\lg(E/10^{15} \text{ eV})}{\lg(10^{19} \text{ eV}/10^{15} \text{ eV})}$$

- $E < \text{PeV} / E > \text{PeV}$
- $f(E)$ rescales parameter
- Change only individual parameter while conserving
 - total energy
 - total charge
 - energy fraction in different particle type

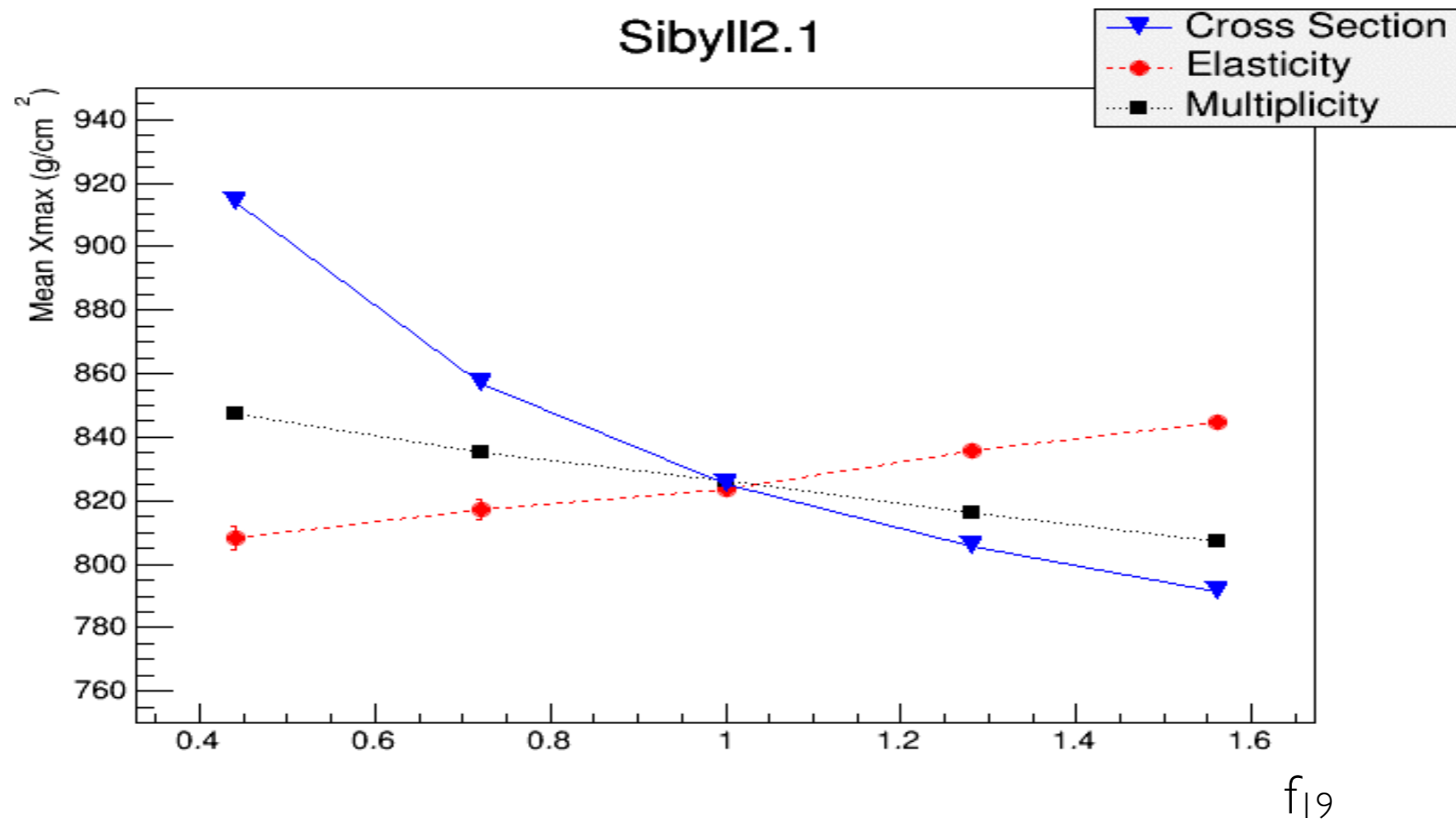
MODEL DEPENDENCE ON THE MODEL PARAMETERS

CONEX4.44 to produce high statistics data sets at $10^{19.5}$ eV



$$\left| \frac{\Delta X_{max}}{\Delta f_{19}} \right|_{\sigma_{p-air}} \approx 100 \text{ g/cm}^2$$

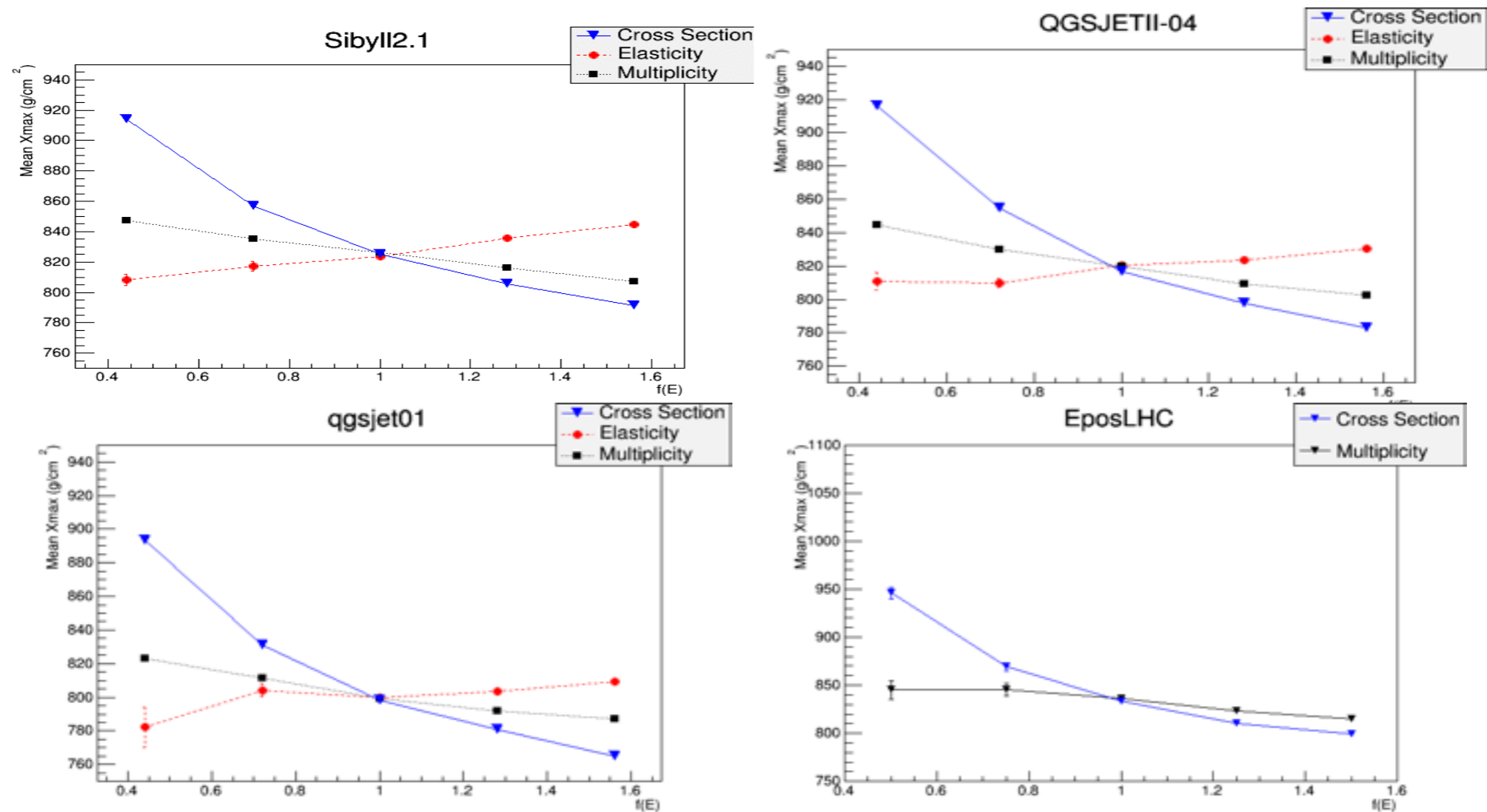
MODEL DEPENDENCE ON THE MODEL PARAMETERS



$$\left| \frac{\Delta X_{max}}{\Delta f_{19}} \right|_{\sigma_{p-air}} \approx 100 \text{ g/cm}^2$$

$$\left| \frac{\Delta X_{max}}{\Delta f_{19}} \right|_{\text{Multiplicity, Elasticity}} \approx 30 - 40 \text{ g/cm}^2$$

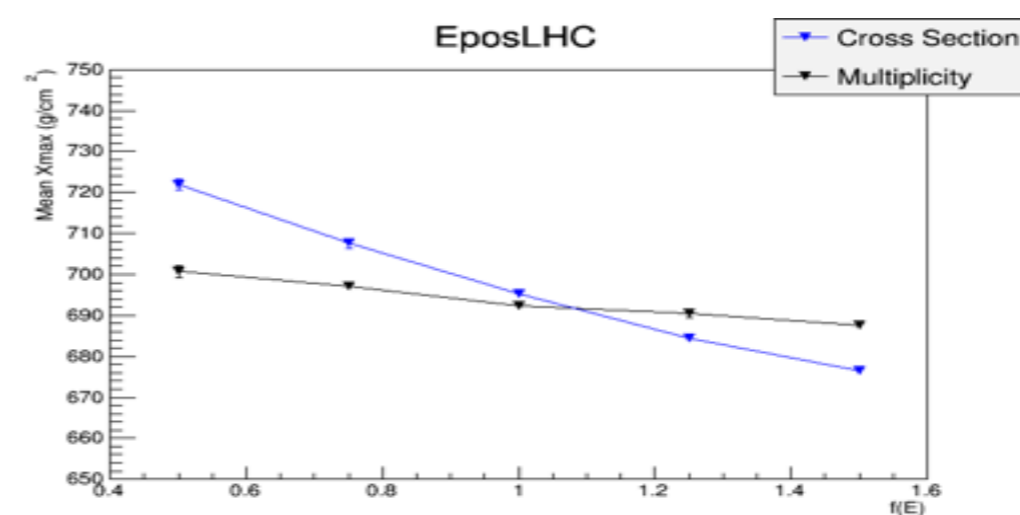
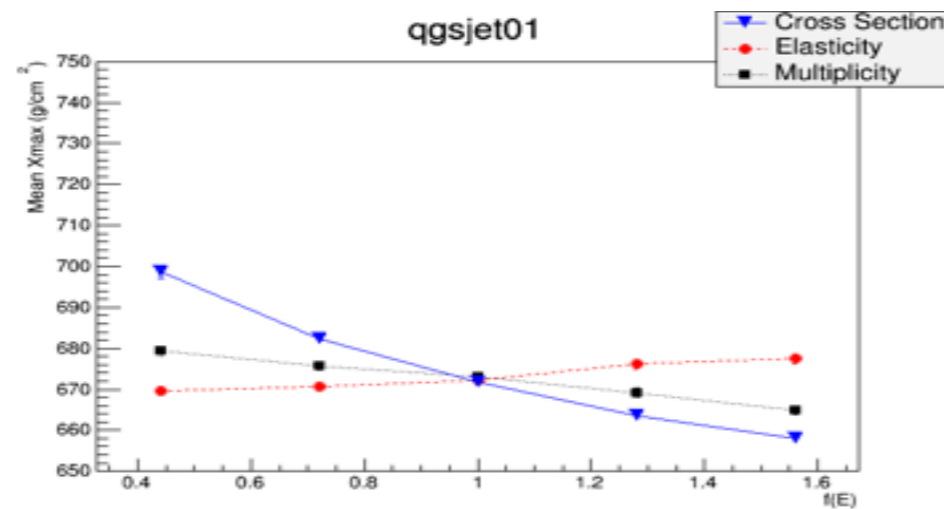
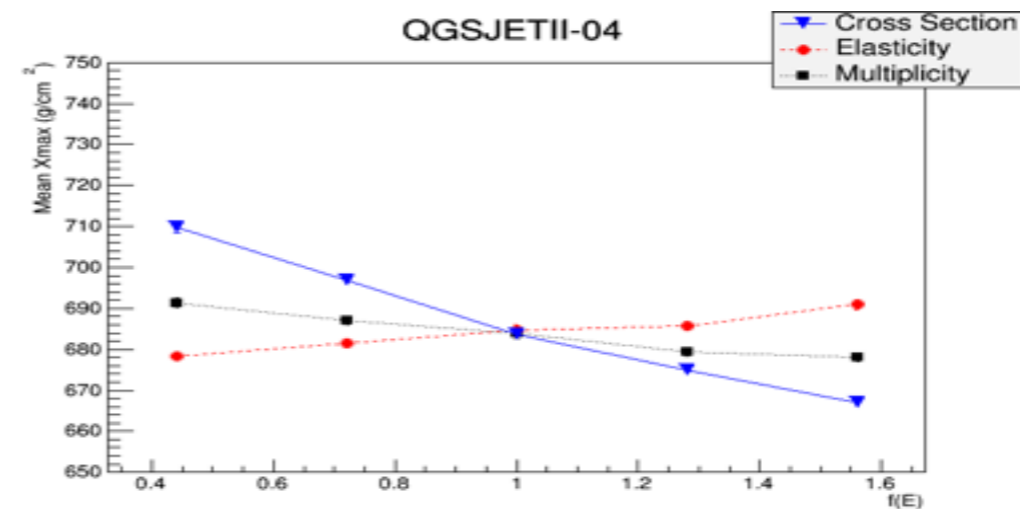
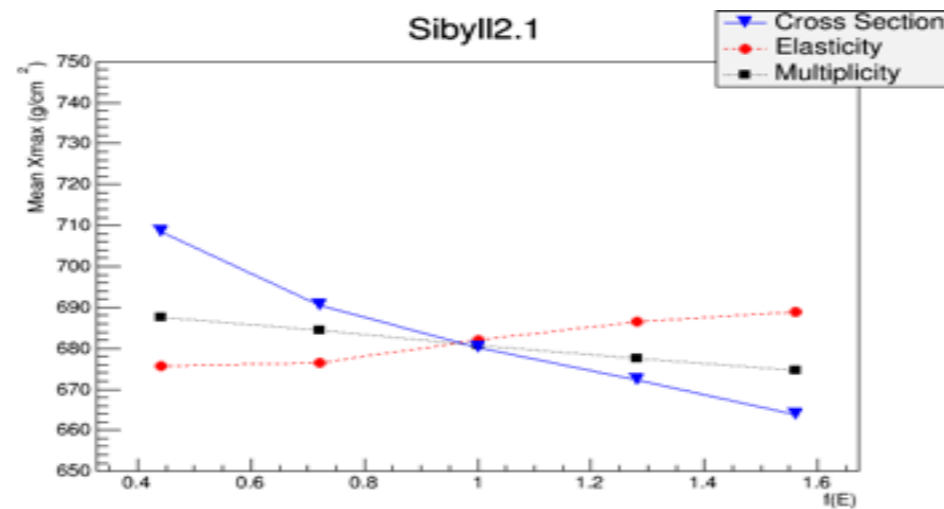
MODELS DEPENDENCE ON THE MODEL PARAMETERS @ $10^{19.5}$ EV



$$\left| \frac{\Delta X_{max}}{\Delta f} \right|_{19} \Big|_{\sigma_{p-air}} \approx 100 \text{ g/cm}^2$$

$$\left| \frac{\Delta X_{max}}{\Delta f} \right|_{19} \Big|_{\text{Multiplicity, Elasticity}} \approx 30 - 40 \text{ g/cm}^2$$

MODELS DEPENDENCE ON THE MODEL PARAMETERS @ 10¹⁷ EV



$$\left| \frac{\Delta X_{max}}{\Delta f} \right|_{19} \Big|_{\sigma_{p-air}} \approx 40 \text{ g/cm}^2$$

$$\left| \frac{\Delta X_{max}}{\Delta f} \right|_{19} \Big|_{\text{Multiplicity, Elasticity}} \approx 10 - 15 \text{ g/cm}^2$$

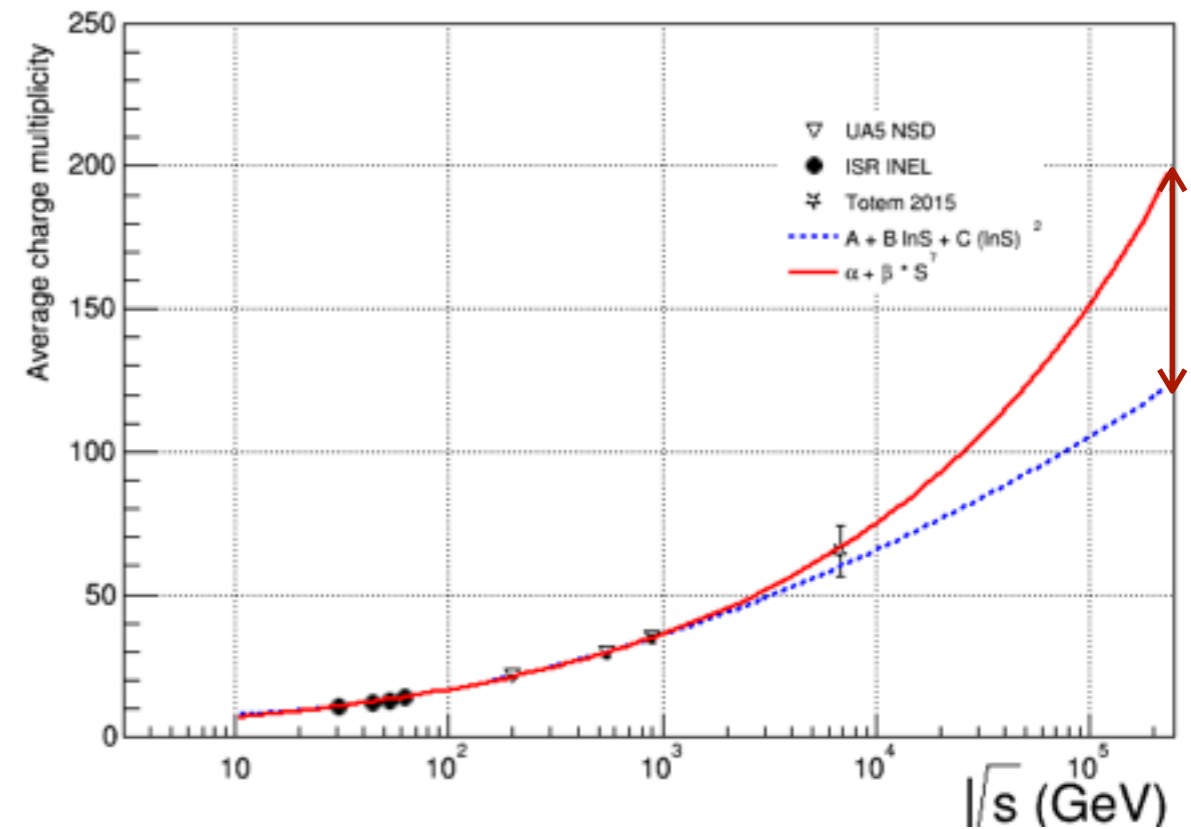
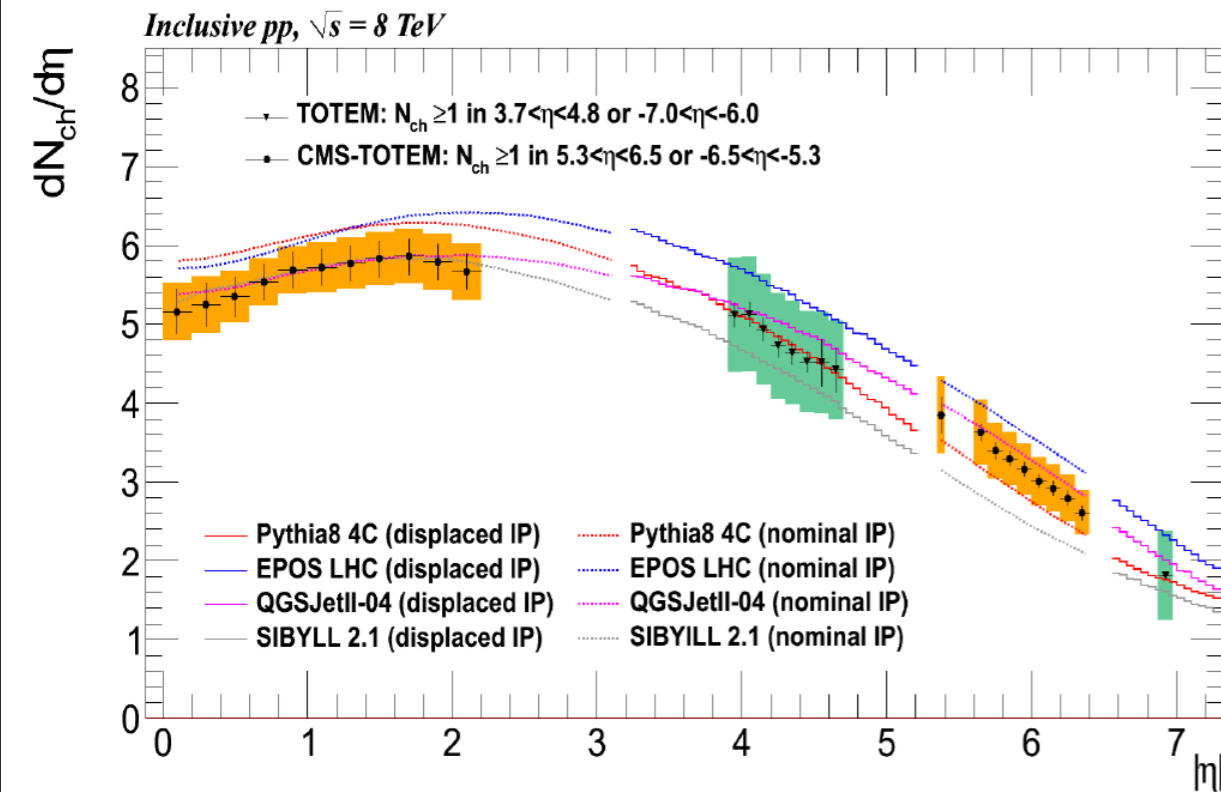
HADRON MODEL UNCERTAINTY

- Fluctuation of the model $\langle X_{\max} \rangle$ in relation to considered parameter.
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MULTIPLICITY

(Totem and CMS) EurPhysJ C75 13,126 (2015) .

G. J. Alner et al. (UA5), Phys. Lett. B167, 476 (1986).



$$\langle N_{ch} \rangle = \alpha + \beta S^\gamma$$

$$\langle N_{ch} \rangle = A + B(\ln S) + C(\ln S)^2$$

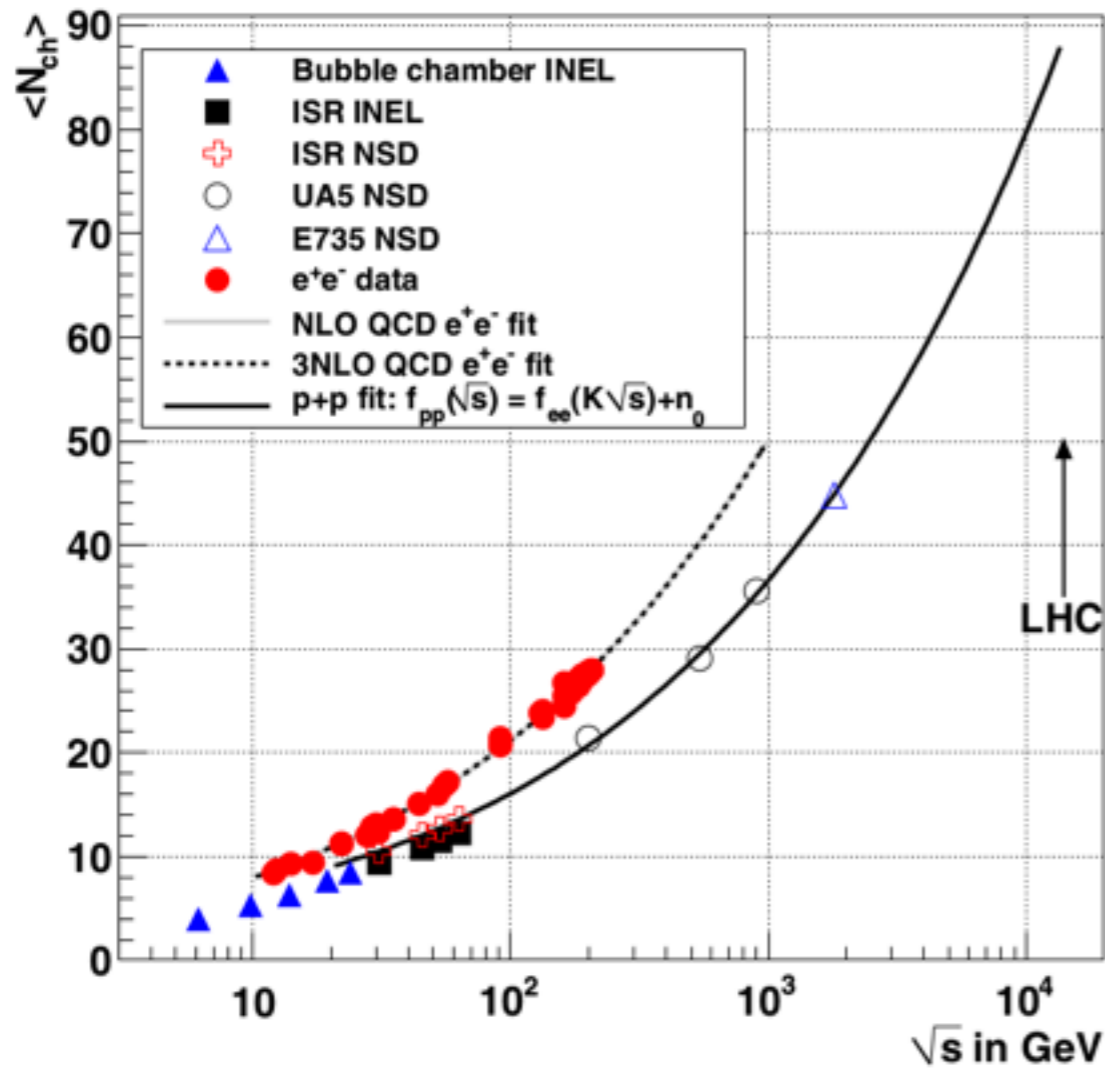
$$\Delta_{multiplicity} = \left| \frac{\Delta X_{max}}{\Delta f} \right|_{19} \times \text{Multiplicity} \times \text{Extrapolation Differences (E)}$$

$$\Delta_{multiplicity} = 40 \times \pm 0.32 = \pm 13 \text{ g/cm}^2$$

ELASTICITY

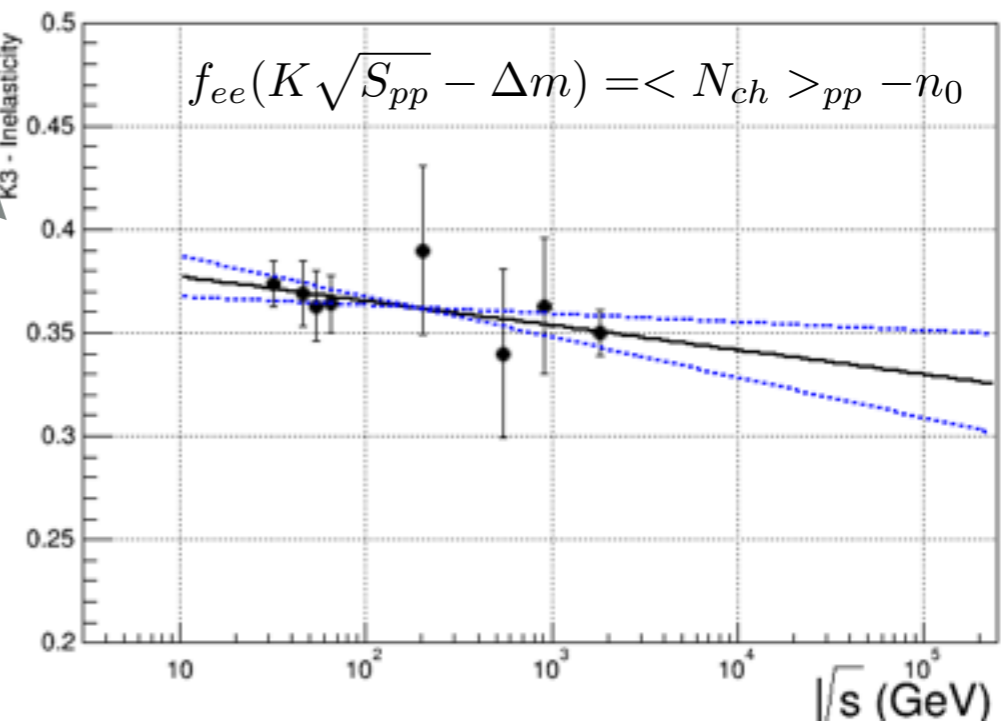
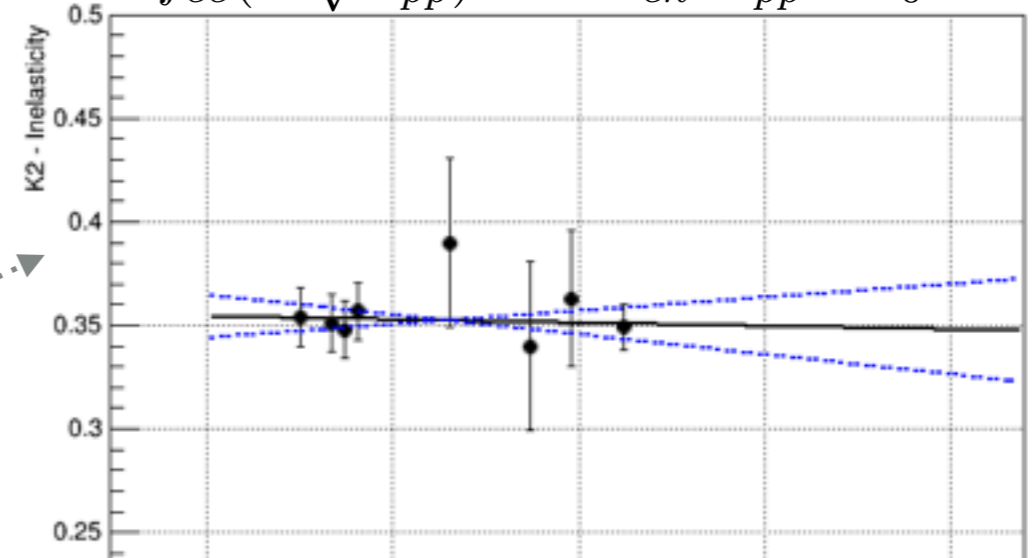
Grosse-Oetringhaus and K. Reygers

J. Phys. G: Nucl. Part. Phys **37** 083001 (2010)



e^+e^- - pp similar if
 $(p+p)$ plotted at
 half their energy collision

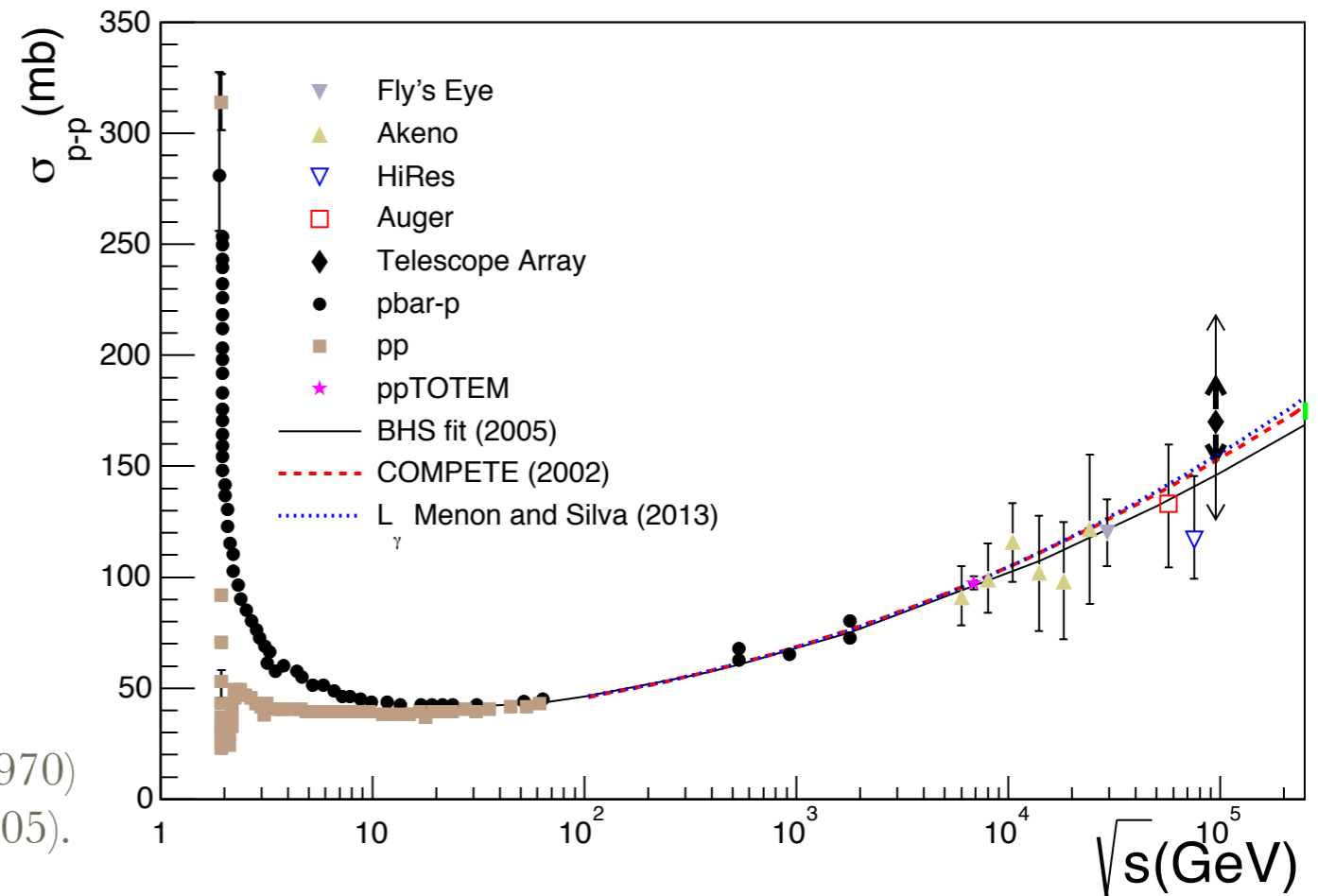
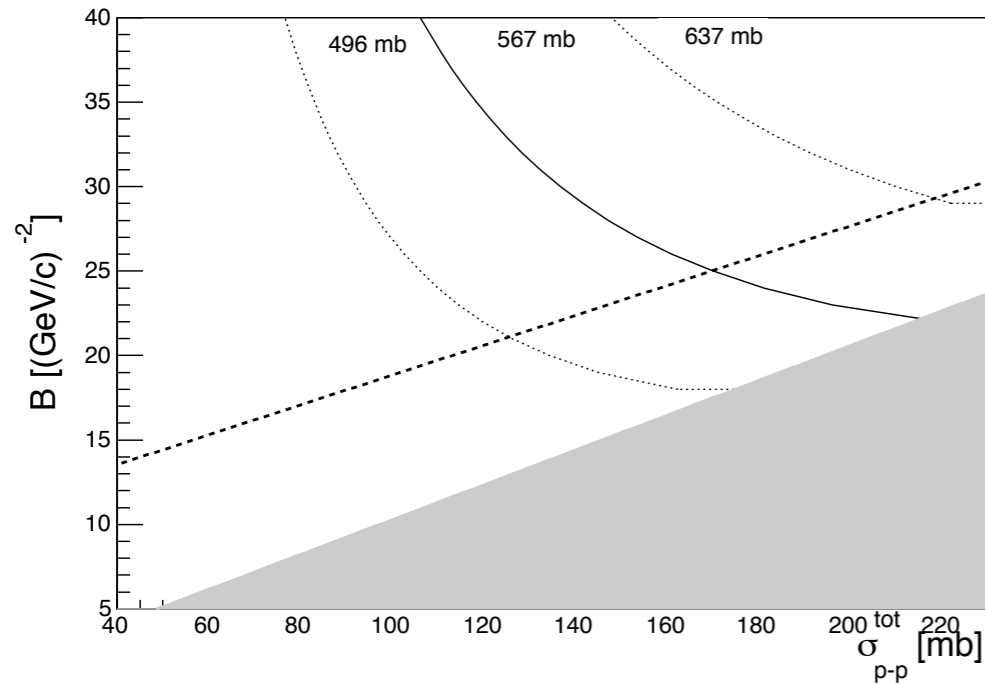
$$f_{ee}(K\sqrt{S_{pp}}) = \langle N_{ch} \rangle_{pp} - n_0$$



$$\Delta_{Elasticity} = \left| \frac{\Delta X_{max}}{\Delta f} \right|_{Elasticity} \times \text{Extrapolation Differences } (E)$$

$$\Delta_{Elasticity} = \pm 2 \text{ g/cm}^2$$

P-AIR CROSS SECTION



*R. Glauber and G. Matthiae, Nucl.Phys. B21, 135 (1970)

**M. Block and F. Halzen, Phys.Rev. D72, 036006 (2005).

** Block, Phys.Rev. D84, 091501 (2011).

$$BHS = c_0 + c_1 \ln\left(\frac{v}{m}\right) + c_2 \ln^2\left(\frac{v}{m}\right) + \beta \left(\frac{v}{m}\right)^{\mu-1}$$

$$COMPETE = \alpha + \beta \ln\left(\frac{S}{S_H}\right)^2$$

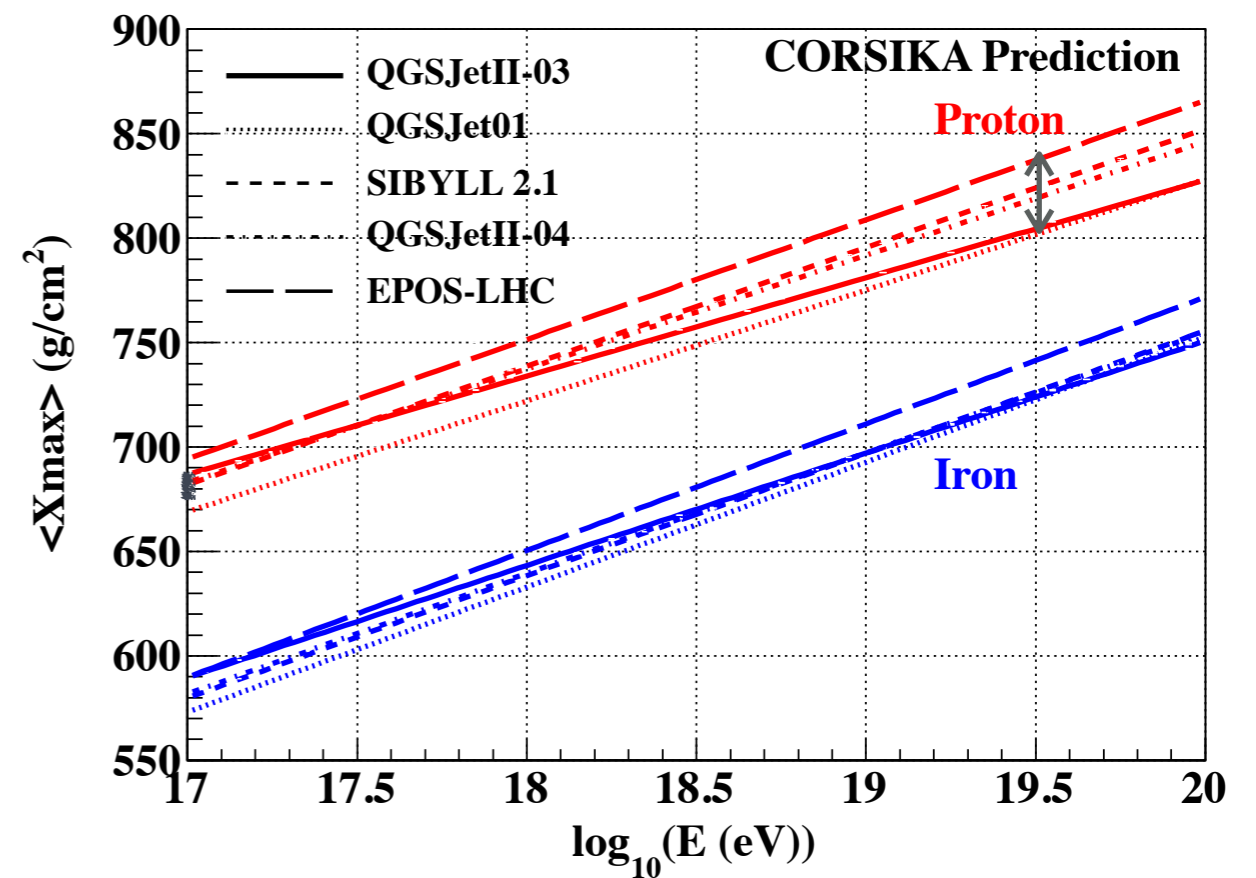
$$L_\gamma = \alpha + \beta \ln\left(\frac{S}{S_H}\right)^\gamma$$

$$\Delta\sigma_{p-air} = \left| \frac{\Delta X_{max}}{\Delta f_{19}} \right|_{\sigma_{p-air}} \times \text{Extrapolation Differences (E)}$$

$$\Delta\sigma_{p-air} = \pm 6.5 \text{ g/cm}^2$$

ESTIMATING UNCERTAINTIES

Model	$\langle X_{\max} \rangle$ uncertainty 10^{17}eV	$\langle X_{\max} \rangle$ uncertainty $10^{19.5}\text{eV}$
SIBYLL2.1	$\pm 3 \text{ g/cm}^2$	$\pm 18 \text{ g/cm}^2$
QGSJETII4	$\pm 3.5 \text{ g/cm}^2$	$\pm 16 \text{ g/cm}^2$
QGSJET01	$\pm 3 \text{ g/cm}^2$	$\pm 18 \text{ g/cm}^2$
EPOS-LHC	$\pm 3 \text{ g/cm}^2$	$\pm 18 \text{ g/cm}^2$



—> Adding in quadrature ?

SUMMARY

- *Estimated the uncertainty in $\langle X_{max} \rangle$ in Cross Section, Multiplicity, and Elasticity.*
- The extrapolation uncertainty at $10^{19.5}$ eV approximately matched the difference among the five models.