

Two Trends in Modern High Energy Physics

Major achievements coming from:

- 1. Energy Frontier:
 - LHC and Higgs discovery
 - (before that) Tevatron studied t quark physics and QCD
 - LEP and Standard Model success
- 2. Intensity/Precision Frontier:
 - E821 a_{μ} ; a_e and α
 - ϕ and B factories R studies, CP violation, new $c\bar{c}$ and $b\bar{b}$ states, exotics (tetraquarks, pentaquarks)
 - Proton radius



What Can We Learn from Low Energy e^+e^- Cross Sections?

1. Detailed study of exclusive processes $e^+e^- \rightarrow (2-7)h, h = \pi, K, \eta, p, \dots$

- Test of models and input to theory (ChPT, Vector Dominance, QCD, ...)
- Properties of vector mesons $(\rho', \omega', \phi', \ldots)$
- Search for exotic states (tetraquarks, hybrids, glueballs)
- Test of CVC relations between e^+e^- and τ -lepton
- Interactions of light (u, d, s) quarks
- 2. High precision determination of $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ at low energies and fundamental quantitites
 - $(g_{\mu}-2)/2$
 - $\alpha(M_Z^2)$
 - QCD sum rules (α_s , quark and gluon condensates)

Muon Anomalous Magnetic Moment

$$\vec{\mu} = g \frac{e}{2m} \vec{s}, \qquad a = (g - 2)/2.$$

In Dirac theory for pointlike particles g = 2, higher-order effects or new physics $\Rightarrow g \neq 2$

Any significant difference of a_{μ}^{exp} from a_{μ}^{th} indicates New Physics beyond the Standard Model.

 a_{μ} is much more sensitive to new physics effects than a_e : the gain is usually $\sim (m_{\mu}/m_{\rm e})^2 \approx 4.3 \cdot 10^4$.

$$a_{\mu}^{\rm th} = a_{\mu}^{\rm SM} + a_{\mu}^{\rm NP}, \qquad a_{\mu}^{\rm SM} = a_{\mu}^{\rm QED} + a_{\mu}^{\rm EW} + a_{\mu}^{\rm had}.$$





 $a_e = 1159652180.73(28) \times 10^{-12} \quad 0.24 \times 10^{-9}$

D. Hanneke et al., PRL 100, 120801 (2008) QED test or α determination

 $a_{\mu} = 116592091(63) \times 10^{-11} \quad 0.54 \times 10^{-6}$

G.W. Bennett et al. (E821), PRD 73, 072003 (2006) Sensitive test of the Standard Model

 $a_{\tau} = -0.018(17)$ or $-0.052 < a_{\tau} < 0.013$ 95%CL

J. Abdallah et al. (DELPHI), EPJ C 35, 159 (2004) Theory: $117721(5) \times 10^{-8}$, SE, M. Passera, MPL A 22, 159 (2007)



 $\hat{K}(s)$ grows from 0.63 at $s = 4m_{\pi}^2$ to 1 at $s \to \infty$, $1/s^2$ emphasizes low energies, particularly $e^+e^- \to \pi^+\pi^-$. $a_{\mu}^{\text{had,LO}} \sim 700 \cdot 10^{-10} \Rightarrow \text{accuracy better than 1\% needed}$



Experiment vs. Theory – I

$a_{\mu} = (g_{\mu} - 2)/2, \ 10^{-10}$	
Experiment	$11659209.1 \pm 5.4 \pm 3.3$
QED	11658471.895 ± 0.008
EW	15.4 ± 0.1
Had LO	692.3 ± 4.2
Had HO	-9.8 ± 0.1
Had LbL	10.5 ± 2.6
Theory	11659180.3 ± 4.9
ExpTh.	28.8 ± 8.0

Experiment is higher than theory by 3.6 standard deviations

Experiment vs. Theory – II





but may suffer from more complicated radiative effects,

a broad range of collision energies



and KLOE (ISR at $\sqrt{s} < 1.0 \text{ GeV}$)

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BaBar used ISR to study the energy range $\sqrt{s} < 3.0$ GeV, Belle/BelleII and BESIII can contribute as well to ISR measurements



The systematic error near the ρ is 0.5% J.P. Lees et al., Phys. Rev. D86 (2012) 032013



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M. Ablikim et al., Phys. Lett. B 753 (2016) 629

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M. Ablikim et al., Phys. Lett. B 753 (2016) 629



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New optics with round beams \Rightarrow higher luminosity, precise beam energy measurement using LCBS

VEPP-2000 - III







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VEPP-2000 and Detectors







High-resolution NaI calorimeter with excellent tracking and PID

Performance of VEPP-2000 and Detectors

- The maximum luminosity is $2 \cdot 10^{31}$ cm⁻¹s⁻¹ at 1.7-1.8 GeV, falling much slower with decreasing energy than before the round beams
- The integrated luminosity is about 60 pb⁻¹ per detector, a factor of 6 higher than before from ϕ to 2 GeV, the number of multihadronic events per 1 pb⁻¹ ~ 50k
- In 2013 we reached 2 × 160 MeV, the smallest \sqrt{s} ever
- Both detectors perform reasonably well with reconstruction of both tracks and photons and redundancy $(\eta \to 2\gamma, \pi^+\pi^-\pi^0, 3\pi^0, \pi^+\pi^-\gamma, \omega \to \pi^+\pi^-\pi^0, \pi^0\gamma)$
- At high energies lumi is limited by a deficit of positrons and maximum energy of the booster (825 MeV now)
- During the shutdown (2015) to upgrade the booster (energy increase to 1 GeV) and commission the new injection complex to reach 10^{32} cm⁻¹s⁻¹



At high energy - by energy deposition in calorimeters





Statistical precision better than that of BaBar Systematic error: goal 0.35% at the ρ (BaBar achieved 0.5%)



BaBar claims aggressive systematics of 0.72% at the ϕ , increasing to 7% at 2 GeV CMD-3 hopes to reach (1-2)% at the ϕ and not much worse at higher energy



based on 6.5×10^5 events, 1.8% systematic uncertainty



At each \sqrt{s} full information on invariant masses $\pi^+\pi^-\pi^0$: V. Aulchenko et al., JETP 121 (2015) 34, $\pi^+\pi^-\eta$: V. Aulchenko et al., Phys. Rev. D 91 (2015) 052013

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A ρ^0 is always present, $a_1^{\pm}(1260)\pi^{\mp}(a_2^{\pm}(1320)\pi^{\mp})$ significant, at higher \sqrt{s} other mechanisms like $\rho^0 f_0$, $\rho^0 f_2(1270)$ appear

$e^+e^- \rightarrow 3\pi^+3\pi^-$ at CMD-3 – I

- 1. $\int Ldt = 22 \text{ pb}^{-1}$ from 1.5 to 2.0 GeV, 25 MeV step
- 2. About 8k five- (5069) and six-track (2887) events selected
- 3. We study dynamics, pure phase space doesn't work, three models with $J^{PC} = 1^{--}$, each with one ρ^0 /event:
 - $\rho(1450)(\pi^+\pi^-)_{\mathrm{S-wave}} \to a_1(1260)^{\pm}\pi^{\mp}\pi^+\pi^- \to \rho^0 2(\pi^+\pi^-) \to 3(\pi^+\pi^-)$
 - $\rho(770)(2\pi^+2\pi^-)_{\text{S-wave}} \to 3(\pi^+\pi^-)$ 3 options for $2\pi^+2\pi^-$: phase space, $f_0(1370), f_0(1500)$
 - $\rho(770)f_2(1270) \to 3(\pi^+\pi^-)$
 - The best description is with one $\rho(770)$ and 4 pions in S-wave
- 4. Full analysis of dynamics common for $3\pi^+3\pi^-$, $2\pi^+2\pi^-2\pi^0$, $\pi^+\pi^-4\pi^0$
- 5. The systematic uncertainty is 6%, to be improved to 3%



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In addition to cross sections, first attempts of measuring f/f made R.R. Akhmetshin et al., 1507.08013, Phys. Lett. B







Ionization losses in DC (dE/dx) provide good K/π separation



From more than 10000 events many different mechanisms seen: $K_1(1270)\bar{K} \rightarrow K\bar{K}\rho, \ K^*(892)\bar{K}\pi, \ K_1(1400)\bar{K} \rightarrow K^*(892)\bar{K}\pi, \ \phi\pi^+\pi^-$ R.R. Akhmetshin et al., Phys. Lett. B 756 (2016) 153

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Missing mass to K^+K^- clearly shows the dominant signal and BGs





Light-by-Light Scattering – I



Various approaches used:

- Vector Dominance and Chiral models
- Data on $\gamma\gamma^* \to \pi^0, \eta, \eta'$ (single-tag)
- Effective field theory
- Dyson-Schwinger equations

M. Knecht and A. Nyffeler, 2002: the correct sign!

Light-by-Light Scattering – II

Authors	Year	$a_{\mu}^{\rm lbl}, 10^{-10}$
J. Bijnens et al.	1996~(2002)	8.3 ± 3.2
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Hadronic LbL - New Approach

New dispersive approach relates data on transition f/f to the HLbL, G. Colangelo et al., JHEP 09 (2014) 091, Phys. Lett. B638 (2014) 6



Measurements of various processes are in order followed by calculations of many integrals with different kernels The largest contribution to a_{μ}^{LBL} is expected from the pseudoscalars (π^0, η, η') Search for C-even resonances in e^+e^-

Direct production of C-even resonances in e^+e^- collisions is possible via a $\gamma\gamma$ intermediate state.



The unitarity bound assuming 2 real photons is

$$\mathcal{B}_{P \to l^+ l^-} = \mathcal{B}_{P \to \gamma \gamma} \frac{\alpha^2}{2\beta} \left(\frac{m_e}{m_P}\right)^2 \left[\ln\left(\frac{1+\beta}{1-\beta}\right)\right]^2, \beta = \sqrt{1 - 4\left(\frac{m_e}{m_P}\right)^2}.$$

For η' the unitarity bound is $\mathcal{B} = 3.75 \cdot 10^{-11}$

"Standard" mechanism via $e^+e^- \rightarrow e^+e^-P$ involves two almost cd real photons and provides $\Gamma(P \rightarrow \gamma \gamma)$ only

Search for $e^+e^- \rightarrow \eta'(958)$ at VEPP-2000

• CMD-3 used 2.69 pb⁻¹ at $\sqrt{s} \sim m_{\eta'}$ to look for $e^+e^- \to \eta'(958), \ \eta' \to \eta \pi^+ \pi^-, \ \eta \to 2\gamma,$ $\Gamma(\eta' \to e^+e^-) < 0.0024 \text{ eV at } 90\% \text{CL},$ R.R. Akhmetshin et al., Phys. Lett. B740 (2015) 273

• SND used 2.9 pb⁻¹ to look for
$$e^+e^- \to \eta'(958)$$
:
 $\eta' \to \eta \pi^+ \pi^-, \ \eta \to 2\gamma, \ 3\pi^0, \ \eta' \to \eta \pi^0 \pi^0, \ \eta \to 2\gamma, \ 3\pi^0, \ \pi^+ \pi^- \pi^0, \ \Gamma(\eta' \to e^+e^-) < 0.0020 \text{ eV} \text{ at } 90\% \text{CL},$
M.N. Achasov et al., Phys. Rev. D91 (2015) 092010

- SND combines their data with CMD-3: $\Gamma(\eta' \to e^+e^-) < 0.0011 \text{ eV at } 90\% \text{CL},$ $\mathcal{B}(\eta' \to e^+e^-) < 5.6 \cdot 10^{-9} \text{ at } 90\% \text{CL}$
- The unitarity limit $\mathcal{B}(\eta' \to e^+e^-) > 3.75 \cdot 10^{-11}$

Search for $e^+e^- \rightarrow \eta$ at VEPP-2000

- A feasibility study of SND used 108 nb⁻¹ collected in the c.m. energy range 520-580 MeV to look for e⁺e⁻ → η, η → 3π⁰; η → 2γ, π⁺π⁻π⁰ dominated by QED background, a sensitivity of B(η → e⁺e⁻) ≈ 10⁻⁶ can be reached in 2 weeks, M.N. Achasov et al., JETP Lett. 102 (2015) 266
- The best limit has been set by HADES in G. Agakishiev et al., Phys. Lett. B731 (2014) 265, $\mathcal{B}(\eta \rightarrow e^+e^-) < 2.3 \cdot 10^{-6}$ at 90%CL
- The unitarity limit is $\mathcal{B}(\eta \to e^+e^-) > 1.8 \cdot 10^{-9}$

Future

- Two new measurements of a_µ are expected in 3-5 years:
 E989 at Fermilab plans to improve the uncertainty from 0.5ppm to 0.14 ppm, they plan to start running in 2017
 J-PARC has the same precision goal, data taking planned in 2019-2021
- What is expected for the theoretical prediction? Progress in low energy e⁺e⁻ annihilation expected, improving the LO error from 4.2 to 2.0, so 2.6 from the LbL dominates, in the new approach 1.0 may be achieved (?) giving 2.2 in total First principles (lattice) give promising results, but far from final C.M. Carloni Calame et al., Phys. Lett. B 746 (2015) 325, a^{had,LO} from α(t) in the spacelike region of Bhabha
- With the same central values of a_{μ}^{exp} and a_{μ}^{th} the today difference will correspond to about $10\sigma!!$

Conclusions

- VEPP-2000 ran smoothly with CMD-3 and SND at $0.32 < \sqrt{s} < 2.00$ GeV, the achieved accuracy is comparable or better than in ISR measurements
- The goals are 0.35%(0.5%) for $\pi^+\pi^-$ and 3% for multibody modes
- Below 2 GeV progress (a factor of 2-3) expected in exclusive σ 's due to scans in Novosibirsk and ISR from KLOE2, BaBar, Belle, BESIII and Belle2
- Various high-statistics experiments will substantially improve the accuracy of vacuum polarization calculations for $(g_{\mu} - 2)/2$
- Higher statistics (~ 1fb⁻¹) ⇒ a detailed study of dynamics, thus a study of mesons with various quantum numbers, can ChT provide low energy predictions?
- Good prospects for a study of transition form factors and hLbL
- Meanwhile a ~ 3.5σ deviation of a_{μ}^{SM} from a_{μ}^{exp} persists: New Physics or various experimental and interpretation errors?



$$a_{\mu}^{\text{had,LO}}$$
 and $\alpha(t)$

$$a_{\mu}^{\rm HLO} = \frac{\alpha}{\pi} \int_0^1 dx \left(1 - x\right) \Delta \alpha_{\rm had}[t(x)] \,.$$

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QED Contribution a_{μ}^{QED}

 $a_{\mu}^{\text{QED}} \cdot 10^{10} = \Sigma C_i (\frac{\alpha}{\pi})^i = 0.5(\alpha/\pi)^1 \quad 1 \text{ diagram}$

- + $0.765857426(16)(\alpha/\pi)^2$ 9
- + 24.05050988(28) $(\alpha/\pi)^3$ > 100

+
$$130.8796(63)(\alpha/\pi)^4 > 1000$$

+
$$753.29(1.04)(\alpha/\pi)^5 > 20000$$

Analytically – S.Laporta and E.Remiddi, Karlsruhe group, ..., Numerically – group of T. Kinoshita

With $\alpha^{-1} = 137.035999049(90)$ $a_{\mu}^{\text{QED}} = (116584718.951 \pm 0.022 \pm 0.077) \cdot 10^{-11}.$ The errors are due to $\mathcal{O}(\alpha^4)$ and α The $(\alpha/\pi)^4$ term \approx the experimental error!



One-loop electroweak contributions

Authors	Year	$a_{\mu}^{\rm EW},10^{-10}$
\ldots, \ldots, \ldots	1972	19.5
A. Czarnecki et al.	1996	15.2 ± 0.4
A. Czarnecki et al.	2002	15.36 ± 0.01

The errors are due to hadr. loops and 3-loop NLO logs



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Experiment is higher than theory by 3.6 standard deviations

Experiment vs. Theory – II



$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ at CMD-3 – I

- CMD-3 studied $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ with 22 pb⁻¹ between 1.5 and 2 GeV
- More than 10000 4-track and 3-track events observed
- Analysis of $\pi^+\pi^-$, $K^{\pm}\pi^{\mp}$, K^+K^- invariant masses shows clear ρ^0 , $K^{*0}(892)$, ϕ signals
- Many different mechanisms seen: $K_1(1270)\bar{K} \to K\bar{K}\rho, \ K^*(892)\bar{K}\pi, K_1(1400)\bar{K} \to K^*(892)\bar{K}\pi, \ \phi\pi^+\pi^-$






$e^+e^- \rightarrow K^+K^-\eta$ at CMD-3 – I

- A data sample of 22 pb⁻¹ collected in 2011-2012 is used to study $e^+e^- \rightarrow K^+K^-\eta$
- 23 c.m. energy points between 1.57 and 2.0 GeV
- Analysis method emphasizes the dominant $\phi\eta$ signal, studies of non-resonant $K^+K^-\eta$ needed
- Rich background with numerous components seen
- The data sample includes 1600 events of the signal and \sim 600 background events



Dynamics dominated by the $\phi\eta, \phi \to K^+K^-$ channel



Missing mass to K^+K^- clearly shows the dominant signal and BGs



Cross section is consistent with and more precise than BaBar



the $\phi \pi^0$ and $K^{*\pm}(892)K^{\mp}$ mechanisms

How Real is a_{μ}^{had} Accuracy?

- Radiative corrections: ISR and HVP probably OK, FSR demands testing (charge asymmetry, $\pi^+\pi^-\gamma$)
- Scan vs. ISR method
- Missing states: neutrals, $\pi^+\pi^-n\pi^0$, $K\bar{K}n\pi$ isospin
- Correlations
- Averaging
- Light-by-light term
- Double counting (LO and HO)







Do we have completely correct ISR theory?

Possible Progress for
$$a_{\mu}^{\text{LO,had}}$$

Three upgraded e^+e^- colliders are running at low energy:

- VEPP-2000 (VEPP-2M upgrade) in Novosibirsk with 2 detectors (CMD-3 and SND), \sqrt{s} = from 0.3 to 2 GeV with $L_{\text{max}} = 10^{32} \text{ cm}^{-2} \text{s}^{-1}$, more than 60 pb⁻¹ per detector collected
- DA Φ NE in Frascati should resume operation with the KLOE-2 detector at 1.02 GeV and $L \sim (2-3) \cdot 10^{33} \text{ cm}^{-2} \text{s}^{-1}$
- BEPCII in Beijing with the BESIII detector from 2 to 4.6 GeV and $L = 7 \cdot 10^{32} \text{ cm}^{-2} \text{s}^{-1}$

BaBar and Belle are continuing ISR analysis



1. Experiment

The new projects at FermiLAB and JPARC expect 4 times better accuracy each

- 2. Theory (Experiment + Models)
 - Such accuracy for $a_{\mu}^{\text{had,LO}}$ corresponds to 0.2%, hardly ever achievable with absolute $\sigma(e^+e^- \rightarrow \text{hadrons})$
 - Additional limitation from $a_{\mu}^{had,LBL}$
- 3. Theory (First principles QCD, Lattice)
 - QCD instanton model (A. Dorokhov, 2003)
 - Lattice T. Blum et al., K. Jansen et al., M. Hayakawa et al.

CVC.
$$e^+e^- \to X^0$$
 and $\tau^- \to \nu_\tau X^-$

$$\frac{d\Gamma}{dq^2} = \frac{G_{\rm F}^2 |V_{\rm ud}|^2 S_{\rm EW}}{32\pi^2 m_\tau^3} f_{\rm kin} v_1(q^2) \text{ with}$$
$$v_1(q^2) = \frac{q^2 \sigma_{e^+e^-}^{\rm I=1}(q^2)}{4\pi\alpha^2}.$$



Allowed $I^G J^P = 1^+ 1^-$: $X^- = \pi^- \pi^0, (4\pi)^-, \omega \pi^-,$ $\eta \pi^- \pi^0, K^- K^0, (6\pi)^-, \dots$

CVC tests showed good agreement of the τ branchings predicted from e^+e^- with τ data (N. Kawamoto and A. Sanda, 1978, F. Gilman and D. Miller, 1978, S. Eidelman and V. Ivanchenko, 1991, 1997). The very first application of τ data to $a_{\mu}^{\text{had},\text{LO}}$ improved the accuracy by a factor of 1.5 (R. Alemany, M. Davier, A. Höcker, 1998)!



