Puzzles in low-energy QCD

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Pions

- Many of the quantities of interest at the precision frontier of particle physics require a good understanding of the strong interaction at low energies.
- In this context, the lightest hadrons are the most important

$$\pi^+ \quad \pi^0 \quad \pi^-$$

- It is essential that we know why the pions are so light. This understanding relies on symmetry.

Isospin symmetry

- Heisenberg 1932: strong interaction is invariant under isospin rotations – this is why $M_p \simeq M_n$.

$\Rightarrow$ Mass difference must be due to the e.m. interaction.

- Puzzle: e.m. field around the proton is stronger, makes the proton heavier than the neutron.
- Numerous unsuccessful attempts at solving this puzzle.
- If QCD describes the strong interaction correctly, then m_u must be very different from m_d. Gasser & L. 1975

  $m_u/m_d \simeq 0.67$, $m_s/m_d \simeq 22.5$ first crude estimate

Isospin symmetry

- $m_u/m_d \simeq 0.67$, $m_s/m_d \simeq 22.5$ first crude estimate 1975
- Masses of the pseudoscalar mesons confirm the picture: $M_{K^+} < M_{K^0}$ also requires a contribution due to $m_u < m_d$ that is larger than the e.m. self-energy difference
  $m_u/m_d \simeq 0.56$, $m_s/m_d \simeq 20.1$ Weinberg 1977
- Current lattice estimates
  $m_u/m_d = 0.46 \pm 0.03$, $m_s/m_d = 20.0 \pm 0.4$
  FLAG 2016, to be published very soon

Chiral symmetry

- Since m_u is very different from m_d: how come that isospin is a nearly perfect symmetry of the strong interaction ?
- QCD explains this very neatly: for yet unknown reasons, it so happens that m_u and m_d are very small.
- If m_u and m_d are set equal to zero $\Rightarrow$ QCD becomes invariant under independent flavour rotations of the right- and left-handed u, d-fields.
- Symmetry group: $\mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{L}$
- This symmetry was discovered before QCD: Nambu 1960.
  - strong interaction has an approximate chiral symmetry
  - chiral symmetry is hidden, spontaneously broken
  - spontaneous symmetry breakdown generates massless bosons
  - the pions are the massless bosons of chiral symmetry
  - are not exactly massless, because the symmetry is not exact

Mass of the pion

- For $m_u = m_d = 0$ the pions are massless (Nambu-Goldstone bosons of an exact, spontaneously broken symmetry).
- For small values of m_u, m_d: M_π^2 is proportional to $m_u + m_d$:

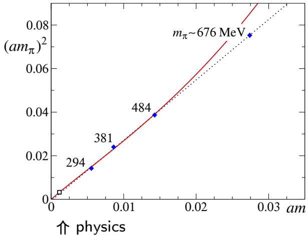
$$M_\pi^2 = (m_u \underset{\text{explicit}}{\overset{\Uparrow}{+}} m_d) \times |\langle 0|\, \underset{\text{spontaneous}}{\overset{\Uparrow}{\bar{u}u}}\, |0\rangle| \times \frac{1}{F_\pi^2}$$

symmetry breaking

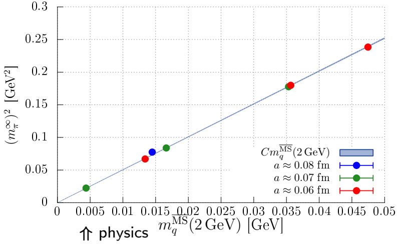
Gell-Mann, Oakes, Renner 1968

- Only $m_u + m_d$ counts.
- F_π is known from $\pi^+ \to \mu^+\nu$, but $|\langle 0|\, \bar{u}u\, |0\rangle| = ?$
  Non-perturbative method required to calculate $|\langle 0|\, \bar{u}u\, |0\rangle|$.

- GMOR formula is beautifully confirmed on the lattice: can determine M_π as a function of $m_u = m_d = m$.



Lüscher Lattice conference 2005



RQCD collaboration, arXiv:1603.00827

- Proportionality of M_π^2 to m_{ud} holds out to about $m_{ud} \simeq 10\times$ physical value of $\frac{1}{2}(m_u + m_d)$. Dürr, arXiv:1412.6434

Corrections to the GMOR relation

- Switch the electroweak interactions off, consider pure QCD.
  $M_\pi = M_\pi(\Lambda_{\rm QCD}, m_u, m_d, m_s, m_c, m_b, m_t)$
- Chiral expansion, chiral perturbation theory (χPT):
  expand M_π in powers of m_u, m_d.
  The formula of GMOR gives the leading term:
  $$M^2 \equiv (m_u + m_d)B \qquad B = \lim_{m_u, m_d \to 0} \frac{|\langle 0|\, \bar{u}u\, |0\rangle|}{F_\pi^2}$$
  B is independent of m_u, m_d.
- χPT shows that the next term in the expansion is given by
  $$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{2(4\pi F_\pi)^2} \bar{\ell}_3 + O(M^4) \right\}$$
  $$\bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M^2} \quad \text{depends logarithmically on } M$$

Corrections to the GMOR relation

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{2(4\pi F_\pi)^2} \bar{\ell}_3 + O(M^4) \right\} \qquad \bar{\ell}_3 = \ln \frac{\Lambda_3^2}{M^2}$$

- Chiral symmetry does not determine the scale Λ_3.
  Lattice calculations reduced the uncertainty very significantly.
  Review of Bijnens and Ecker: arXiv:1405.6488
  $\bar{\ell}_3 = 3.0 \pm 0.8 \leftrightarrow \Lambda_3 \simeq 600$ MeV.

$\Rightarrow$ Correction in M_π is tiny: $\dfrac{M_\pi^2}{2(4\pi F_\pi)^2} \bar{\ell}_3 \simeq 0.024$

- Not a surprize: m_u, m_d are small, of the order of a few MeV.
  SU(2)×SU(2) should be a nearly perfect symmetry !

Why is the strong interaction nearly isospin invariant ?

- m_u, m_d small $\Rightarrow$ SU(2)×SU(2) a nearly perfect symmetry.
- Isospin is a subgroup of SU(2)×SU(2).

$\Rightarrow$ Isospin is a nearly perfect symmetry.

$\Rightarrow$ The strong interaction is nearly invariant under isospin rotations because m_u, m_d are small.

- But: the fact that SU(2)×SU(2) symmetry is broken is clearly seen: $M_\pi \neq 0$
  Why is the breaking of isospin symmetry so well hidden ?
  Why is M_{π^0} nearly equal to M_{π^+} ?
- The Nambu-Goldstone bosons are shielded from isospin breaking: leading term in $\mathcal{L}_{eff}$ only knows about $m_u + m_d$.

$\Rightarrow$ Expansion of $M^2_{\pi^+} - M^2_{\pi^0}$ in powers of m_u, m_d does not contain a term $\propto m_u - m_d$. Leading contribution is of order $(m_u - m_d)^2 \Rightarrow$ in QCD, $M_{\pi^+} - M_{\pi^0}$ is tiny.

Mass of the kaon

- Kaons are not protected from isospin breaking,
  are also NG bosons, become massless if m_s is sent to zero
- π^+: $u\bar{d}$ K^+: $u\bar{s}$ K^0: $d\bar{s}$
  Leading terms in the expansion in powers of m_u, m_d, m_s:
  $M^2_{\pi^+} = (m_u + m_d)B$
  $M^2_{K^+} = (m_u + m_s)B$
  $M^2_{K^0} = (m_d + m_s)B \quad \Rightarrow M^2_{K^+} - M^2_{K^0} = (m_u - m_d)B$
- B drops out in the ratios

$$\Rightarrow \frac{M^2_{K^+}}{M^2_{\pi^+}} = \frac{m_u + m_s}{m_u + m_d} \qquad \text{up to higher order contributions}$$

- m_s happens to be much larger than m_u, m_d
  Explains why $M_K \gg M_\pi$
- Masses of the NG bosons are very sensitive to m_u, m_d, m_s
  m_u, m_d, m_s break chiral symmetry

$\Rightarrow$ M_π, M_K measure the strength of the symmetry breaking

Isospin limit

- If u and d are given the same mass m_{ud} and $e = 0$, there are three degenerate isospin multiplets: M_π, M_K, M_η
- At leading order of the chiral expansion, the relative size of the three masses is determined by the relative size of m_{ud} and m_s

  $$\frac{M_K^2}{M_\pi^2} = \frac{m_{ud} + m_s}{2m_{ud}} \qquad \frac{M_\eta^2}{M_\pi^2} = \frac{m_{ud} + 2m_s}{3m_{ud}}$$

  How large are the contributions from the higher orders ?
- Denote the higher order contributions by Δ_M:

  $$\frac{M_K^2}{M_\pi^2} = \frac{m_{ud} + m_s}{2m_{ud}}(1 + \Delta_M)$$

- Lattice result for quark masses: $m_s / m_{ud} = 27.3(3)$ FLAG

$\Rightarrow$ $\Delta_M = \text{-}0.05(1)$

$\Rightarrow$ Higher order contributions are remarkably small.

Compare $\dfrac{F_K}{F_\pi} = 1 + \Delta_F$ $\qquad \Delta_F = 0.193(3)$ FLAG

Mass pattern of the NG bosons

- Relative size of M_η and M_K:
  $$\frac{M_\eta^2}{M_K^2} = \frac{2(m_{ud} + 2m_s)}{3(m_{ud} + m_s)}(1 + \Delta'_M)$$
  The higher order contributions are remarkably small also here:
  Δ'_M = -0.062(2)
- Not a surprise: the Gell-Mann-Okubo formula
  $4M_K^2 \simeq 3M_\eta^2 + M_\pi^2$
  is known to receive only small corrections from higher orders.
  If it was exact, Δ'_M would be determined by Δ_M.

Isospin breaking

- Two sources of isospin breaking: e^2 and $m_u \neq m_d$.
- Dashen theorem: at LO of the expansion in m_u, m_d, m_s, $M^2_{K^+}$ gets the same contribution from the e.m. interaction as $M^2_{\pi^+}$, while $M^2_{\pi^0}, M^2_{K^0}, M^2_{\bar{K}^0}, M^2_{\eta}$, do not get anything at all.

⇒ $(M^2_{K^+} - M^2_{K^0})_{\text{QED}} = (M^2_{\pi^+} - M^2_{\pi^0})_{\text{QED}} \times (1+\epsilon)$

Dashen theorem only holds at leading order of χPT.
ϵ collects the contributions of higher order in m_u, m_d, m_s.

- Oven fresh lattice determinations:

$\epsilon = 0.73(18)$ BMW arXiv:1604.07112
$\epsilon = 0.73(14)$ MILC arXiv:1606.01228

⇒ In the self-energies, the higher order effects are large.

Electromagnetic self-energies

- As pointed out long ago, the e.m. self-energy of the pion obeys a low energy theorem which neatly explains its magnitude.

  Das, Guralnik, Low, Mathur & Young 1967

  - This theorem does not rely on the expansion in powers of m_s

  $\Rightarrow$ Holds up to corrections of order $e^2 M_\pi^2$.
- For the kaon, the theoretical constraints are much weaker.
  - The corrections to the Dashen theorem are of order $e^2 M_K^2$.
  - The strange quark in the K^+ is heavier than the down quark in the π^+ $\rightarrow$ wave function narrower $\rightarrow$ self-energy larger.
  - An evaluation of the Cottingham formula for π^+ and K^+ is needed to understand the difference quantitatively.

Effects from $m_u \neq m_d$

- As mentioned already, the vacuum shields the pions from isospin breaking within QCD.
- For the kaons, there is a low-energy theorem Gasser & L. 1985

$$\left.\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2}\cdot\frac{M_\pi^2}{M_K^2}\right|_{\mathrm{QCD}} = \frac{m_d^2 - m_u^2}{m_s^2 - m_{ud}^2}(1+\delta_M)$$

  Similar to the one for M_K^2/M_π^2, but there is a difference: δ_M is of NNLO, hence expected to be very small.
- For small quantities like δ_M, details matter. Identify M_π^2, M_K^2 with the mean squared masses of the two multiplets and evaluate the e.m. self-energies of the neutral particles with the numbers quoted by FLAG.

Low energy theorem for $M^2_{K^0} - M^2_{K^+}$

- Numerical result obtained from the most recent lattice data:

	BMW	MILC
m_u/m_d	0.485(20)	0.455(13)
m_s/m_{ud}	27.53(22)	27.36(10)
ϵ	0.73(18)	0.73(14)
δ_M	0.08(7)	-0.01(5)

Results agree within about 1 σ.

Treating these as independent determinations

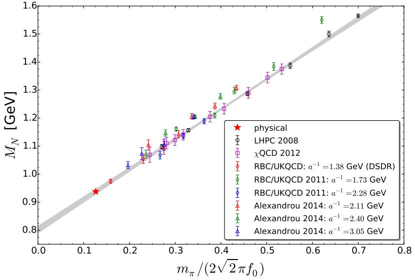
$\Rightarrow$ $\delta_M = 0.02(4)$ lattice results do obey the low-energy theorem.

Mass of the nucleon

- M_N, M_π are determined by $\Lambda_{\rm QCD}$, $m_u, m_d, \ldots, m_t$.
- Set m_u and m_d equal, common mass m_{ud}.
  Keep all other parameters fixed.

⇒ Values of M_N, M_π only depend on m_{ud}.
Conversely, m_{ud} is determined by M_π.

⇒ Value of M_N determined by value of M_π.



'Ruler plot' of
André Walker-Loud
I thank Claude Bernard
for providing this update
(see PoS(CD15)004)

- Lattice results shown are roughly on a straight line:
  $M_N = M_0 + c\,M_\pi$

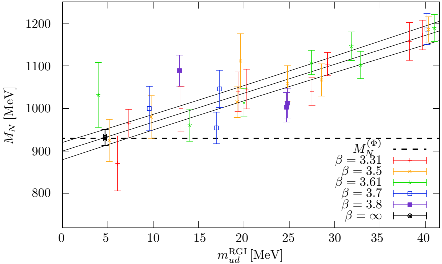
Mass of the nucleon

- Lattice results shown are roughly on a straight line:
  $M_N = M_0 + c\,M_\pi$
- In QCD, the Taylor series starts with
  $M_N = M_0 + c_1 M_\pi^2 + c_2 M_\pi^3 + c_3 M_\pi^4 \ell n(c_4 M_\pi) + O(M_\pi^5)$
  A term proportional to M_π does not occur.
  $M_\pi^2 \propto m_{ud} \Rightarrow M_\pi \propto \sqrt{m_{ud}}$

$\Rightarrow$ ruler fit is puzzling.

Mass of the nucleon

- New data from BMW



I thank Stephan Dürr for this plot (see arXiv:1510.08013)

- In the range shown, the data are consistent with

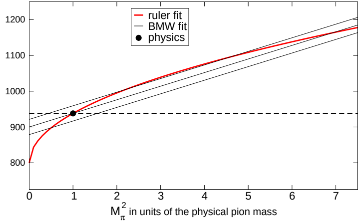
$$M_N = M_0 + k_1\, m_{ud}$$
$$M_\pi^2 = k_2\, m_{ud}$$

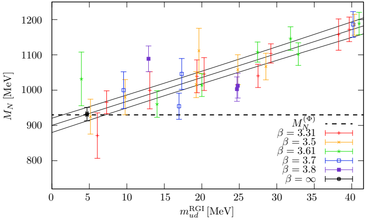
$\Rightarrow$ BMW data are well described by

$$M_N = M_0 + c\, M_\pi^2$$

Mass of the nucleon

- Comparison of ruler fit with BMW fit





⇒ No evidence for a term linear in M_π.

σ-term

- Recent lattice data allow a determination of the σ-term matrix elements (throughout, I consider the isospin limit)

$$\sigma_{\pi N} = \frac{m_{ud}}{2M_N}\langle p|\bar{u}u + \bar{d}d|p\rangle$$

$$y = \frac{2\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}$$

- Numerical results:

	$\sigma_{\pi N}$ (MeV)	y	archiv
BMW	38(3)(3)	0.20(8)(8)	1510.08013
XQCD	44.4(3.2)(5.5)	0.058(6)(8)	1511.09089
ETM	$37.22(2.57)(^{+0.99}_{-0.63})$	0.075(16)	1601.01624
RQCD	35(6)	0.104(51)	1603.00827
blind average	38.2(2.0)	0.064(8)	

- The four results are consistent with one another.

$\Rightarrow$ Data indicate a σ-term around 38 MeV and a small value of y.

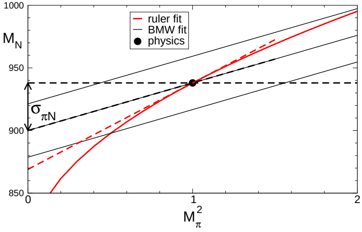
Significance of $\sigma_{\pi N}$

- Feynman-Hellman theorem: $\sigma_{\pi N} = m_{ud} \left.\frac{\partial M_N}{\partial m_{ud}}\right|_{\text{at physical } m_{ud}}$
- Since the physical value of m_{ud} is small, it is in the region where $M_\pi^2 = k_2\, m_{ud}$ holds to high accuracy.

$\Rightarrow$ $\sigma_{\pi N} = M_\pi^2 \left.\frac{\partial M_N}{\partial M_\pi^2}\right|_{\text{at physical } M_\pi}$

$\Rightarrow$ In the plot of M_N versus M_π^2, the σ-term measures the slope at the physical point.

$\Rightarrow$ $\sigma_{\pi N} \simeq M_N - M_0$



BMW: $\sigma_{\pi N} \simeq 38$ MeV
ruler fit: $\sigma_{\pi N} \simeq 70$ MeV

Significance of y

- y measures the size of $\langle p|\bar{s}s|p\rangle$.
  Violates the Okubo-Zweig-Iizuka-rule, vanishes for $N_c \to \infty$.
- y is relevant for matrix element of the octet operator:
  $$\sigma_0 = \frac{m_{ud}}{2M_N}\langle p|\bar{u}u + \bar{d}d - 2\,\bar{s}s|p\rangle = \sigma_{\pi N}\,(1-y)$$
- Blind average over the four lattice results yields
  $\sigma_0 = 35.7(1.9)$ MeV

$\Rightarrow$ σ_0 smaller than $\sigma_{\pi N} = 38.2(2.0)$, but only slightly.

Low energy theorem for σ_0

- For $m_u = m_d = m_s$, SU(3) is an exact symmetry of QCD.

$\Rightarrow$ N, Σ, Λ, Ξ have the same mass.

- $m_s - m_{ud}$ removes the degeneracy, breaks SU(3).
  (disregard from isospin breaking, take $m_u = m_d$)
  Expand in powers of $m_s - m_{ud}$.
- $2M_N + 2M_\Xi = 3M_\Lambda + M_\Sigma$ Gell-Mann-Okubo-formula
  valid to $O(m_s - m_{ud})$. Works very well, also for the baryons.
- Mass splitting is determined by the matrix element

$$M_\Sigma + M_\Xi - 2M_N = \frac{m_s - m_{ud}}{2M_N} \langle p | \bar{u}u + \bar{d}d - 2\bar{s}s | p \rangle$$

$\Rightarrow$ This leads to a low-energy theorem for σ_0:

$$\sigma_0 = \frac{m_{ud}}{m_s - m_{ud}} (M_\Sigma + M_\Xi - 2M_N) \left\{ 1 + O(m_s - m_{ud}) \right\}$$

Low energy theorem for σ_0

- $\sigma_0 = \frac{m_{ud}}{m_s - m_{ud}}(M_\Sigma + M_\Xi - 2M_N)\left\{1 + O(m_s - m_{ud})\right\}$
  Numerically, the leading term amounts to $\sigma_0 \simeq 25$ MeV.
- The NLO corrections were analyzed long ago. The perturbations generated by the quark mass term in the Lagrangian of QCD have infrared singularities (the unperturbed system contains massless mesons). These amplify the corrections, increasing the value of σ_0 by about 10 MeV:

  $\sigma_0 = 35 \pm 5$ MeV Gasser 1981

$\Rightarrow$ The quoted lattice results beautifully confirm this prediction.

Low-energy theorem for πN scattering

- The isospin even πN scattering amplitude obeys a low-energy theorem: the leading term in the expansion of the quantity
  $\Sigma = F_\pi^2 \bar{D}^+|_{s=u,\,t=2M_\pi^2}$ ←'Cheng-Dashen point'
  in powers of m_{ud} is given by $\sigma_{\pi N}$.

⇒ If the common mass of the two lightest quarks is turned off, both Σ and $\sigma_{\pi N}$ tend to 0 and the ratio $\Sigma/\sigma_{\pi N}$ tends to 1.

- The theorem can be used to measure $\sigma_{\pi N}$ in πN scattering. Relying on the dispersive analysis of Höhler et al. (Karlsruhe-Helsinki collaboration), we obtained
  $\sigma_{\pi N} = 45$ MeV   Gasser, L. & Sainio 1991
- This was compatible with $\sigma_0 = \sigma_{\pi N}(1-y) = 35(5)$ MeV, provided a modest violation of the OZI-rule was allowed for:
  $y = 0.2$ Gasser, L. & Sainio 1991
- The picture thus looked coherent, but the πN data showed serious inconsistencies. For this reason we were not able to attach meaningful uncertainties to the above estimates.

Roy-Steiner equations

- In the meantime, the lattice results have confirmed the value of σ_0, but indicate that the violation of the OZI-rule is smaller: the lattice values for $\sigma_{\pi N}$ are below 45 MeV.
- There is very significant progress in the dispersive analysis.
  Hoferichter, de Elvira, Kubis & Meissner, 2015, 2016
- Solutions of the Roy-Steiner equations for the πN scattering amplitude are now available. Extension from $\pi\pi$ to πN is a highly nontrivial achievement, because not all three channels involve the same physics: while the s- and u-channels carry the quantum numbers of πN, the t-channel concerns the transition $\pi\pi \leftrightarrow N\bar{N}$.
- Spin is a nontrivial complication: 4 amplitudes are needed. For $\pi\pi$ scattering a single amplitude suffices.

σ-term puzzle

- Outcome of the Roy-Steiner analysis:
  $\sigma_{\pi N} = 59.1(3.5)$ MeV　　Hoferichter et al., arXiv:1506.04142
- I find this result very puzzling because of two prejudices:
  1. SU(3) is a decent approximate symmetry, also for the matrix elements of the operator $\bar{q}\lambda^a q$ in the baryon octet.
  2. The rule of Okubo, Zweig and Iizuka is approximately valid.

  If $\sigma_{\pi N}$ is above 50 MeV $\Rightarrow$ at least one of these is wrong.
  The lattice results are consistent with both of them.
- Clash between two independent determinations of $\sigma_{\pi N}$:

baryon masses	πN scattering
Lattice	Roy-Steiner
38 MeV	59 MeV

σ-term puzzle

- Clash between two independent determinations of $\sigma_{\pi N}$:

baryon masses	πN scattering
Lattice	Roy-Steiner
38 MeV	59 MeV

- Lattice: two different methods
  - Feynman-Hellman-theorem.
  - Direct determinations of the σ-term matrix elements.
- Roy-Steiner
  - πN phase shifts.
  - S-wave πN scattering lengths from pionic atoms.
  - The analysis of the t-channel partial waves is new. Previously, the subtraction constant d_{01}^{+} was estimated from the available experimental information, now it is calculated by solving a Muskhelishvili-Omnès problem. It will be very interesting to compare the outcome with experiment.

σ-term puzzle

- The clash is not new – many references deal with the subject.

  see for instance Pavan et al. 2002, Matsinos & Rasche 2015

- New results accentuate the problem:
  Model dependence of the analysis is reduced.
  Uncertainty estimates have become small.

- Can the discrepancy be resolved with χPT ?

  Alarcon, Alvarez-Ruso, V. Bernard, de Elvira, Epelbaum, Gasparyan, Gegelia Geng, Hoferichter, Krebs, Kubis, Ledwig, Martin Camalich, Meißner, Meng, Oller, Ren, Siemens, Vicente Vacas, Yao

- A reliable lattice determination of the LECs relevant for the masses of the meson and baryon octets would be most welcome, but is not easy to achieve.

- The lattice results depend on extra parameters related to the regularization used. This may be the reason why the values of $\sigma_{\pi N}$ obtained by analyzing lattice data with χPT differ from those found by the collaborations responsible for the data.

  (recall comparison of ruler fit with BMW fit.)

Conclusions

Mesons

- The quark masses are mysterious, but the mass spectrum of the lightest hadrons is well-understood in terms of these.
- Key point: m_u, m_d, m_s are small, can expand and retain only the first few terms, i.e. use χPT.
- χPT predictions for the dependence of M_π on m_{ud} confirmed.
- χPT predictions for $M_\pi : M_K : M_\eta$ confirmed.
- If isospin breaking is disregarded, the mass pattern of the lightest mesons is controlled by the quark mass ratio m_s/m_{ud}, which happens to be large.
- The mass difference between π^+ and π^0 is due almost exclusively to the e.m. interaction and is understood on the basis of a low-energy theorem that does not require an expansion in m_s.

Conclusions

- The mass difference between K^+ and K^0 is dominated by the contribution proportional to $m_u - m_d$.
- There is a low-energy theorem for this contribution, valid to NNLO of χPT. The lattice results confirm this prediction.
- The e.m. self-energy of the K^+ is small and strongly modified by non-leading orders of the expansion in powers of m_s. Their size is determined quite well on the lattice, but more work is needed to comprehend the numerical results.

Conclusions

Baryons

- Significant progress on the lattice.
  - The results are consistent with the chiral expansion.
  - In particular, the values obtained for σ_0 confirm the old estimate obtained from the expansion of the baryon masses.
  - Violations of the OZI-rule are found to be small.
- Significant progress in dispersive analysis of πN scattering.
  - New analysis of t-channel dispersion relations.
  - Outcome for $\sigma_{\pi N}$ is puzzling.
  - Disagrees with the lattice results and calls for exorbitant violations of SU(3)-symmetry in the matrix elements of $\bar{q}\lambda^a q$.

Conclusions

Not discussed and work to be done

- There is a wealth of data on πN scattering.
- Comparison with Roy-Steiner analysis will be most interesting.
  Can the experimental inconsistencies be resolved ?
  In particular: $\pi^- p \to \pi^0 n$, $\pi^0 p \to \pi^+ n$?
- Are the basic theoretical constraints obeyed ?
  Goldberger-Treiman relation (ties $g_{\pi N}$ to g_A)
  Adler-Weisberger sum rule (ties g_A to the total cross section)
- Are the predictions for $\pi\pi \to N\bar{N}$ consistent with experiment ?

- Determine the matrix elements of $\bar{u}u$, $\bar{d}d$, $\bar{s}s$ for other members of the meson and baryon octets.
- 'Scalar charge' $g_S = \dfrac{1}{2m}\langle p|\bar{u}d|n\rangle \overset{\text{isospin}}{\Downarrow}{=} \dfrac{1}{2m}\langle p|\bar{u}u - \bar{d}d|p\rangle$

  Relevant for the mass difference between p and n in QCD.

  González-Alonso & Martin Camalich, arXiv:1309.4434
  Bhattacharya, Cirigliano et al., arXiv:1606.07049
- Any evidence for strong violations of SU(3) in the scalar matrix elements ?

$\sigma_{\pi N}$ not the only puzzle worth thinking about ...

- Proton charge radius
- Standard Model prediction for magnetic moment of the muon

⋮