Spectroscopy and Structure of Baryons

Willibald Plessas

Theoretical Physics / Institute of Physics
University of Graz, Austria

Humboldt Kolleg

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Common methods for dealing with low-energy hadronic phenomena on the basis of QCD nowadays:

- ▶ <u>Lattice QCD</u> (works but faces computational limitations)

- ▶ <u>Chiral perturbation theory</u> (works at low energies, as a systematic expansion but is limited to a few terms)

- ▶ <u>Effective field theories</u> or <u>functional methods</u> (depend on assumptions and regularizations)

- ▶ <u>Effective models</u>, e.g. **constituent-quark models** (depend on assumptions and input parameters)

Here, results for the **spectroscopy** and **structure** (electromagnetic, weak, scalar, gravitational form factors) of **baryons** from the **relativistic constituent-quark model** (RCQM)

Search for a non-perturbative tool to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD
  such phenomena like

▶ **hadron spectra**: ground states & excitations

▶ **hadron structure**: r_E, μ, g_A ; G_E, G_M, G_A, G_P, ...
                        i.e. electroweak form factors etc.

▶ **resonance excitations**: $\gamma N \to N^*$, $e^- N \to N^*$, ...

▶ **resonance decays**:
  $\rho \to \pi\pi$, $\omega \to \pi\pi\pi$, $N^* \to N\pi$, $\Delta \to N\pi$, $\Lambda^* \to KN$, ...

▶ **meson-baryon interactions**: $\pi - N$, $K - N$, ...

▶ **hyperon-hyperon interactions**: $N - N$, $N - Y$, ...
  etc. etc.

Outline

Low-energy QCD of N_f flavors is characterized by:

- spontaneous breaking of chiral symmetry ($SB\chi S$):

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

  $\rightarrow$ appearance of ($N_f^2 - 1$) **Goldstone bosons** $\vec{\phi}$
  $\rightarrow$ generation of quasiparticles with dynamical mass,
     i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

$$\mathcal{L}_{\text{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^f \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189

E.V. Shuryak: Phys. Rep. **115**, 151 (1984)

L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)

see also:

S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

Baryons

Baryons are considered as colorless bound states of three constituent quarks.

Here the proton:
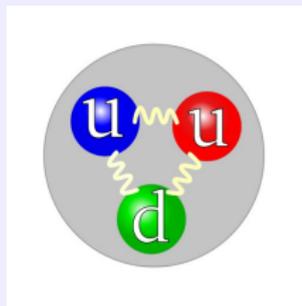
► 'Constituent' quarks are quasiparticles with dynamical mass, NOT the original QCD d.o.f. (i.e. 'current' quarks).

► 'Constituent' quarks are confined and interact via hyperfine interactions associated with $SB\chi S$, i.e. Goldstone-boson exchange.

Relativistic quantum mechanics (RQM)

i.e. **Hamiltonian quantum theory** respecting

Poincaré invariance

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M} \,|P, J, \Sigma\rangle = M \,|P, J, \Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu \,|P, J, \Sigma\rangle = P^\mu \,|P, J, \Sigma\rangle \quad , \qquad \hat{P}^\mu = \hat{M}\hat{V}^\mu$$

Interacting mass operator

$$
\begin{aligned}
\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\
\hat{M}_{free} &= \sqrt{\hat{H}_{free}^2 - \hat{\vec{P}}_{free}^2} \\
\hat{M}_{int}^{rest\ frame} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} [\hat{V}_{ij}^{conf} + \hat{V}_{ij}^{hf}]
\end{aligned}
$$

fulfilling the **Poincaré algebra**

$$
\begin{aligned}
[\hat{P}_i, \hat{P}_j] = 0, && [\hat{J}_i, \hat{H}] = 0, && [\hat{P}_i, \hat{H}] = 0, \\
[\hat{K}_i, \hat{H}] = -i\hat{P}_i && [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k && [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\
[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, && [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, && [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}
\end{aligned}
$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Universal GBE RCQM

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow \quad H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

▶ i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)

Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C\,r_{ij}$$

$$V_\beta(\vec{r}_{ij}) = \frac{g_\beta^2}{4\pi}\frac{1}{12m_im_j}\left\{\mu_\beta^2\frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\}$$

$$= \frac{g_\beta^2}{4\pi}\frac{1}{12m_im_j}\left\{\mu_\beta^2\frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2\frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right\}$$

$B = -402$ MeV, $C = 2.33$ fm^{-2}

$\beta = 24$: $\frac{g_{24}^2}{4\pi} = 0.7,$ $\mu_{24} = \mu_\pi = 139$ MeV, $\Lambda_{24} = 700.5$ MeV

$\beta = 0$: $\left(\frac{g_0}{g_{24}}\right)^2 = 1.5,$ $\mu_0 = \mu_{\eta'} = 958$ MeV, $\Lambda_0 = 1484$ MeV

$m_u = m_d = 340$ MeV, $m_s = 480$ MeV,

$m_c = 1675$ MeV, $m_b = 5055$ MeV

Baryon **Excitation Spectra**

and

Mass-Operator **Eigenstates**

Light Baryon Spectra
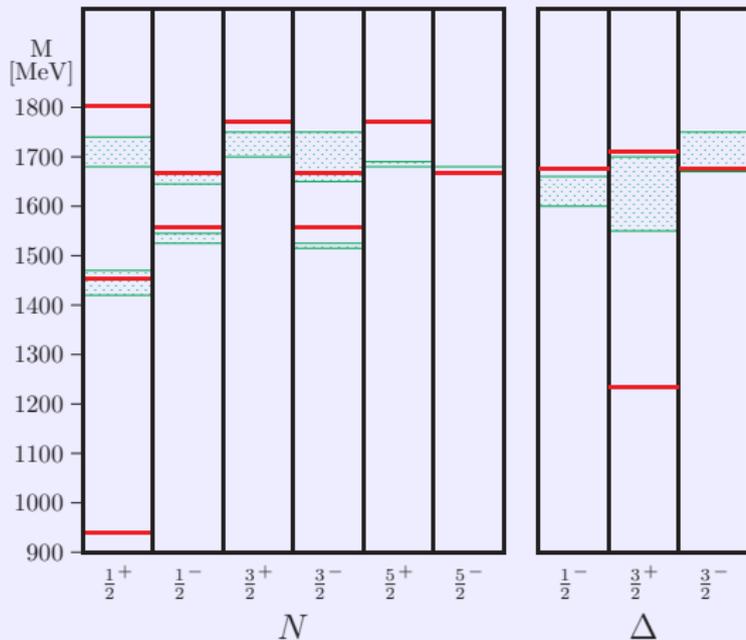
red   Universal GBE RCQM
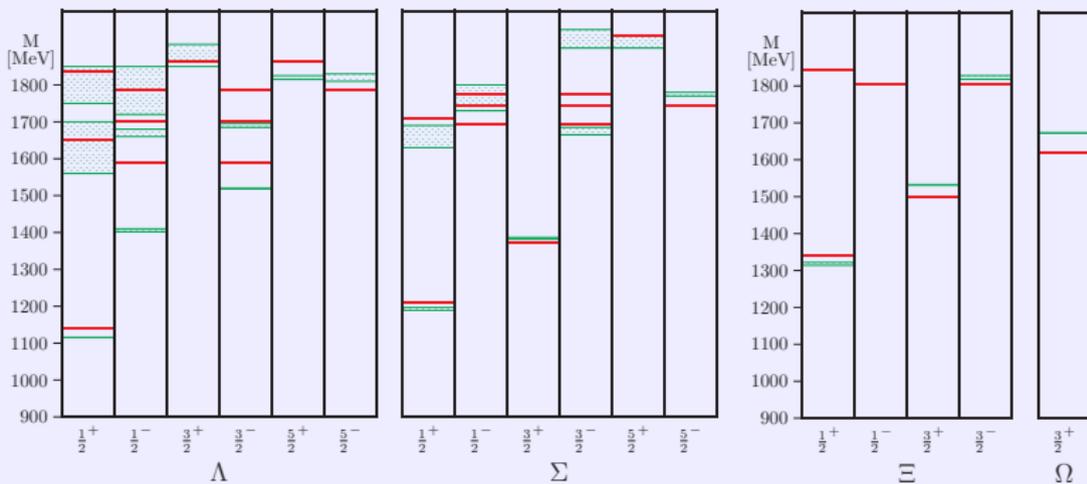
green   PDG 2013 (experiment)

Strange Baryon Spectra

red   Universal GBE RCQM

green   PDG 2013 (experiment)

Charm Baryon Spectra
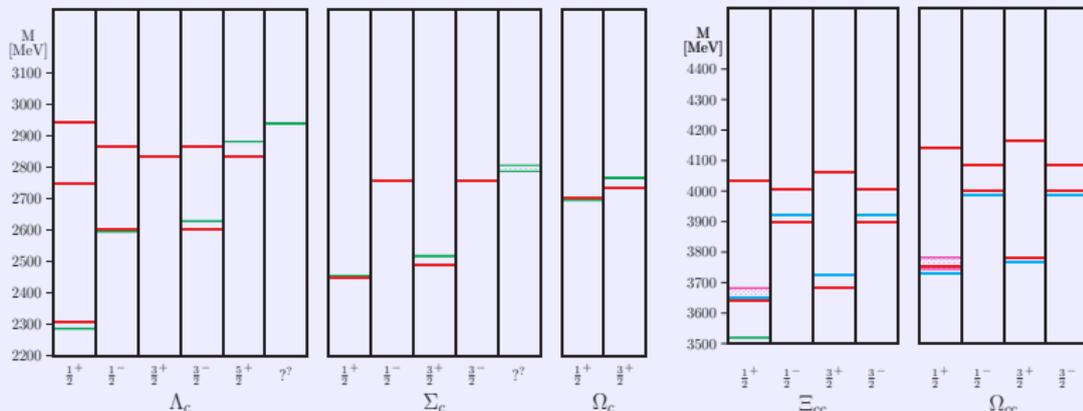
Left panel – single charm:

red — Universal GBE RCQM prediction

green — PDG 2013 (experiment)

**Right panel – double charm:**

green — M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan — S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta — L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Bottom Baryon Spectra
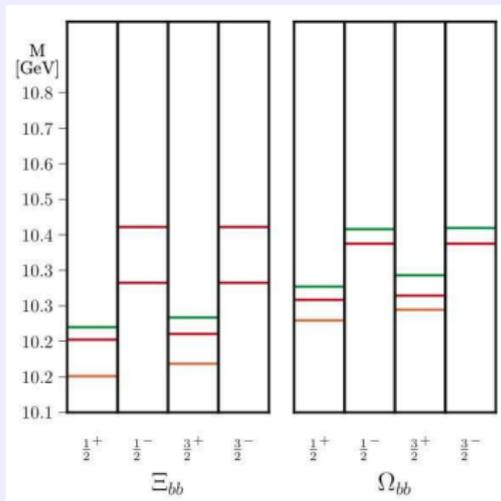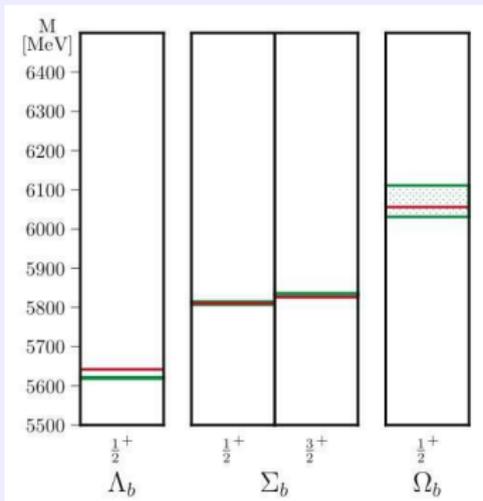
Left panel – single bottom:

red   Universal GBE RCQM prediction

green   PDG 2013 (experiment)

Right panel – double bottom:

green   W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange   D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra

Low-energy
QCD

RCQM
GBE RCQM

Spectroscopy

Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
π NN, π NΔ
N Scalar FF
N Gravitational FF

Summary
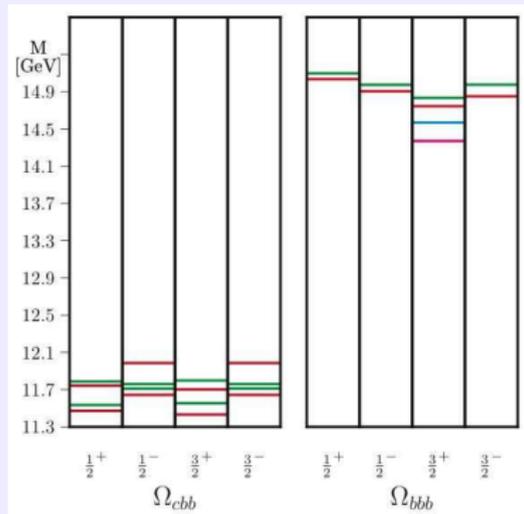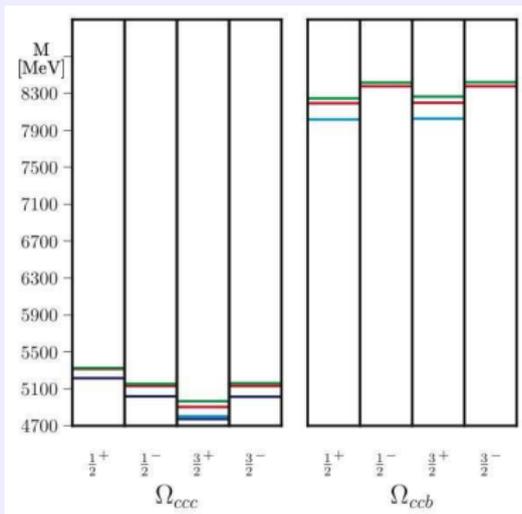
red	Universal GBE RCQM
green	W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrelativistic one-gluon-exchange CQM)
blue	S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)
cyan	A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)
magenta	S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Influence of Light-Heavy Q-Q Interaction
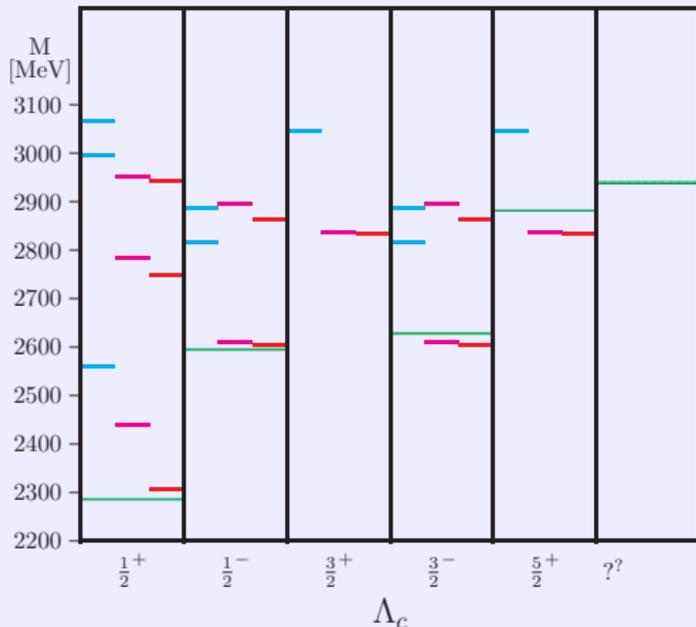
leftmost cyan levels	confinement only
middle magenta levels	including only light-light GBE
rightmost red levels	including full GBE RCQM

Mass operator eigenstates

$$\hat{M}\,|P, J, \Sigma, T, M_T\rangle = M\,|P, J, \Sigma, T, M_T\rangle$$

represented in configuration space

$$\left\langle \vec{\xi}, \vec{\eta}\,\Big|\,P, J, \Sigma, T, M_T \right\rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi}, \vec{\eta})$$

with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

Picture the baryon wave functions through
spatial probability density distributions

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta$$

$$\Psi^\star_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$

Low-energy
QCD

RCQM
GBE RCQM

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Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
πNN, $\pi N\Delta$
N Scalar FF
N Gravitational FF

Summary

N GBE CQM

N(1440) GBE CQM

units are [fm]

units are [fm]

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002
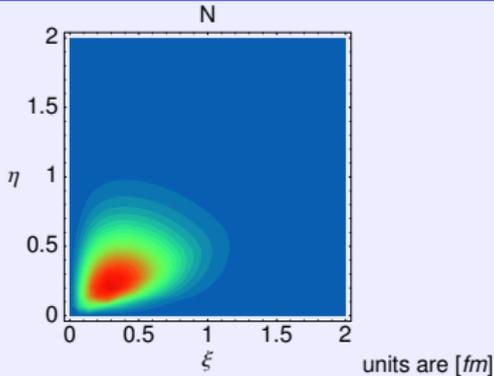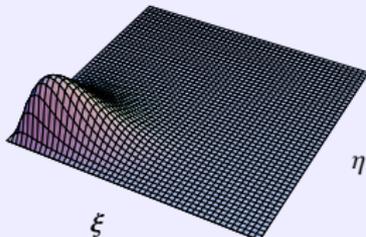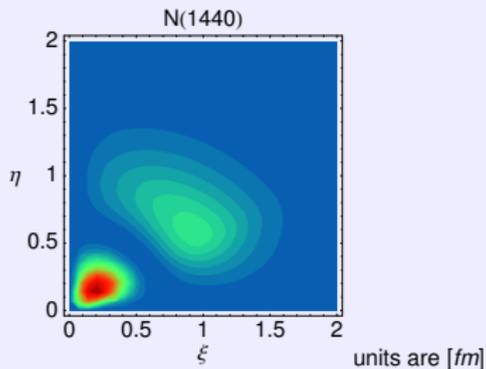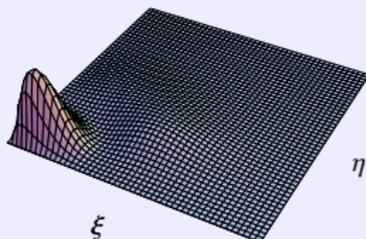
Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ **octet** baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:
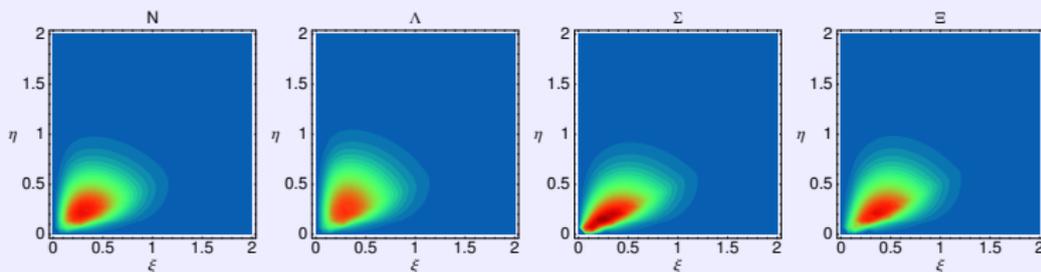
$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ **octet** baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:
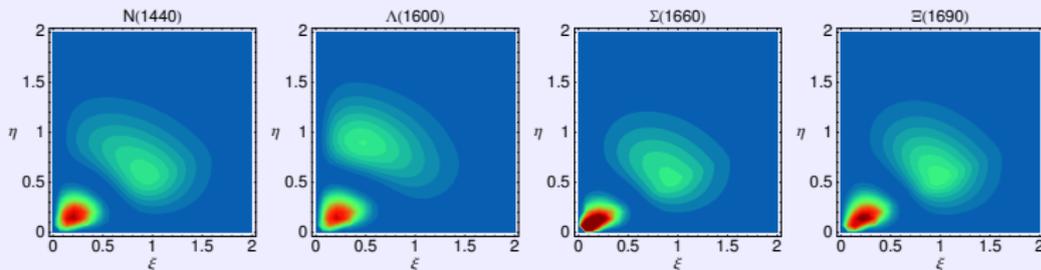
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states Δ**(1232)**, Σ**(1385)**, Ξ**(1530)**, Ω**(1672)**:

$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states Δ**(1600)**, Σ**(1690)**:

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

New Quark-Model Classification
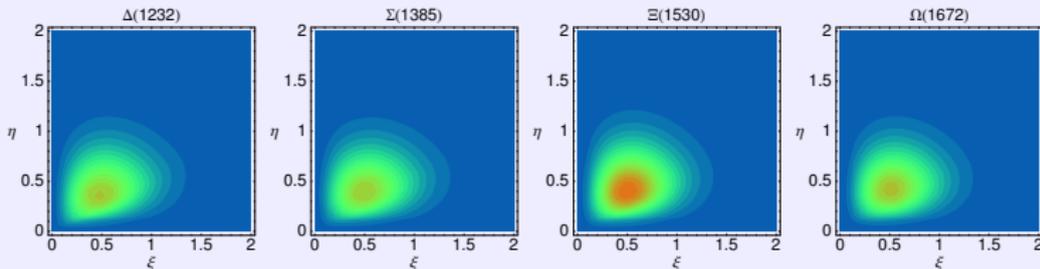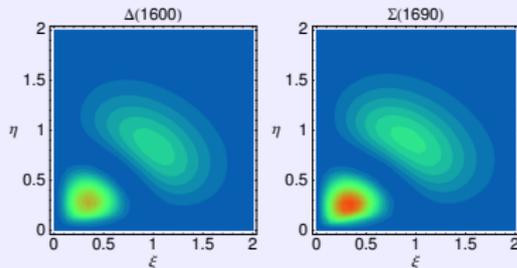
multiplet	$(LS)J^P$				
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
octet	$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
octet	$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
octet	$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
octet	$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
octet	$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
decuplet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
decuplet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			
singlet	$(1\frac{1}{2})\frac{1}{2}^-$		$\Lambda(1405)^{71}$		
singlet	$(1\frac{1}{2})\frac{3}{2}^-$		$\Lambda(1520)^{71}$		
singlet	$(0\frac{1}{2})\frac{1}{2}^+$		$\Lambda(1810)^{92}$		

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Chin. Phys. C **38**, 090001 (2014)

Classification of baryon resonances by the PDG since **2010**

(results from the GBE relativistic CQM marked by asterisks)

J^P	$(D, L_N^P) S$	Octet members				Singlets
$1/2^+$	$(56,0_0^+)\ 1/2$	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)\ 1/2$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)^\dagger$	
$1/2^-$	$(70,1_1^-)\ 1/2$	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$	$\Lambda(1405)$
				$\Sigma(1560)^\dagger$		
$3/2^-$	$(70,1_1^-)\ 1/2$	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)\ 3/2$	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$	
				$\Sigma(1620)^\dagger$		
$3/2^-$	$(70,1_1^-)\ 3/2$	$N(1700)$	$\Lambda(?)$	$\Sigma(1940)^\dagger$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)\ 3/2$	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(1950)^\dagger$	
$1/2^+$	$(70,0_2^+)\ 1/2$	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(1810)^\dagger$
$3/2^+$	$(56,2_2^+)\ 1/2$	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)\ 1/2$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)\ 1/2$	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)\ 3/2$	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)\ 1/2$	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	

PDG: J. Phys. G **37**, 075021 (2010); Phys. Rev. D **86**, 010001 (2012);

Chin. Phys. C **38**, 090001 (2014)

Low-energy QCD

RCQM
GBE RCQM

Spectroscopy
Light, strange, charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
πNN, $\pi N\Delta$
N Scalar FF
N Gravitational FF

Summary

N and Δ Rest-Frame Wave Functions

Rest-frame **spatial distribution of constituent quarks**
in terms of 3-body Jacobi coordinates $\vec{\xi}$ and $\vec{\eta}$:

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta$$

$$\Psi^\star_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$
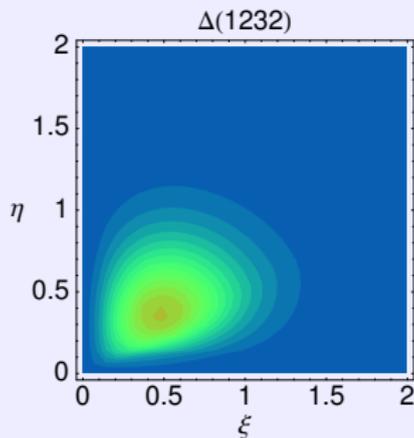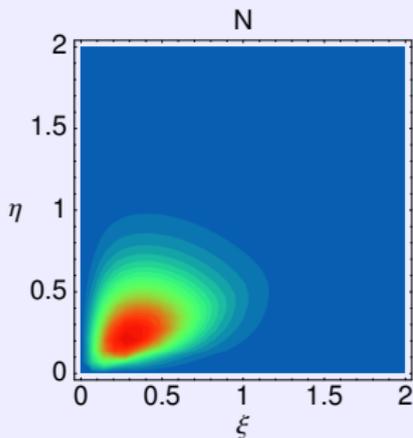

Units on abscissa and ordinates are [*fm*]

Root-Mean-Square Radii

The **root-mean-square radius** (in the rest frame):

$$r_{\text{rms}} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P = 0, J, \Sigma \left| \hat{r}_i^{\,2} \right| P = 0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

$\rightarrow$ Idea about the spatial distribution of constituent quarks.

$r_{\text{rms}}^N = 0.304$ fm

$r_{\text{rms}}^\Delta = 0.390$ fm

The **root-mean-square radius** (in the rest frame):

$$r_{\rm rms} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P = 0, J, \Sigma \left| \hat{r}_i^{\,2} \right| P = 0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

$\rightarrow$ Idea about the spatial distribution of constituent quarks.

N

$\Delta(1232)$
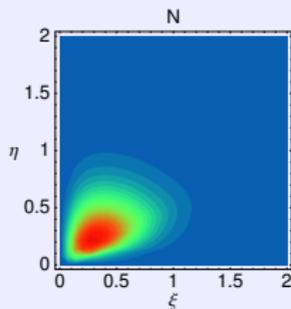
$r_{\rm rms}^N = 0.304$ fm
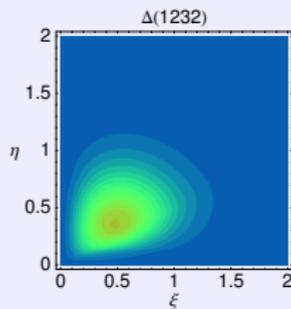
$r_{\rm rms}^\Delta = 0.390$ fm

Exp.: $r_E^p \sim 0.88$ fm

$(r_E^n)^2 \sim -0.12$ fm^2

$r_E^{\Delta^{++}} = r_E^{\Delta^+} = r_E^{\Delta^-} = 0.656$ fm

$r_E^{\Delta^0} = 0$ fm

Baryon **Reactions**

&

Baryon **Structure**

Matrix elements of a transition operator $\hat{O}$ between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T_3', Y' | \hat{O} | P, J, \Sigma, T, T_3, Y \rangle$$

$\hat{O}$ ... $\hat{J}_{\mathrm{em}}^{\mu}$ $\rightarrow$ electromagnetic FF's

    ... $\hat{A}_{\mathrm{axial}}^{\mu}$ $\rightarrow$ axial FF's

    ... $\hat{\Theta}^{\mu\nu}$ $\rightarrow$ gravitational/tensor FF's

    ... $\hat{S}$ $\rightarrow$ scalar FF

    ... $\hat{D}_{\lambda}^{\mu}$ $\rightarrow$ strong FF's and hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'; f_{i_1'}, f_{i_2'}, f_{i_3'} | \hat{O} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3} \rangle$$

               $\Uparrow$ $\Uparrow$

boosted 3-body states   boosted 3-body states

Rel. Dynamics: Poincaré Transf. Generators

Invariant hypersurfaces in Minkowski space $\rightsquigarrow$ 3 forms

P.A.M. Dirac: Rev. Mod. Phys. 21 (1949) 392

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Summary

instant	point	front

$$x^0 = 0 \qquad\qquad x^2 - a^2 = 0 \qquad\qquad x^0 - x^3 = 0$$

J_i, P_i $\qquad\qquad\qquad$ J_i, K_i $\qquad\qquad\qquad$ $P^+, \vec{P}^\perp, E^1, E^2, J_z, K_z$
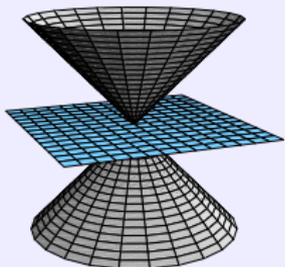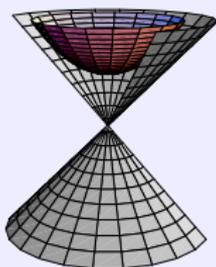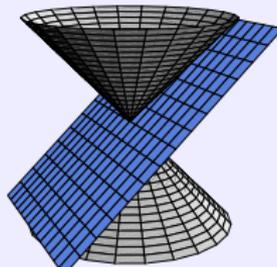
$P^0 = H$, K_1, K_2, K_3 $\qquad$ P^0, P^1, P^2, P^3 $\qquad$ P^-, F^1, F^2

interaction-free and **interaction-dependent** generators.

There are 2 and only 2 more forms discussed by Leutwyler & Stern with less, namely only three interaction-free generators !

H. Leutwyler and J. Stern: Ann. Phys. 112 (1978) 94

Covariant predictions for:

- **Electromagnetic** nucleon form factors
  $G_E^p(Q^2)$, $G_M^p(Q^2)$; $G_E^n(Q^2)$, $G_M^n(Q^2)$

- **Electric radii** and **magnetic moments**
  r_E^p, μ^p; r_E^n, μ^n

$\rightarrow$ Comparison to experiment

Elastic electron scattering:
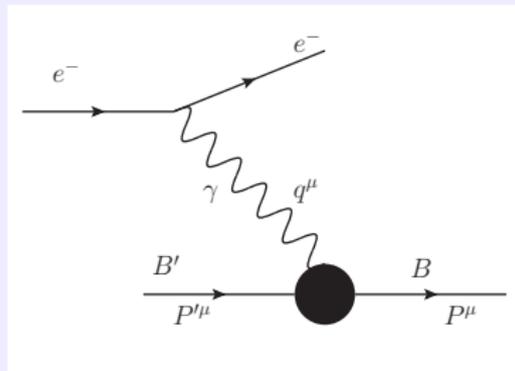
Invariant form factors:

$$F_{\Sigma'\Sigma}^{\nu}(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}_{\text{em}}^{\nu} | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2 ; \qquad q^{\mu} = P^{\mu} - P'^{\mu}$$

Low-energy
QCD

RCQM
GBE RCQM

Spectroscopy
Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
πNN, $\pi N\Delta$
N Scalar FF
N Gravitational FF

Summary

Incoming baryon state: $|V, M, J, \Sigma\rangle$ $\qquad \hat{=} |P, J, \Sigma\rangle$

Outgoing baryon state: $|V', M', J', \Sigma'\rangle$ $\qquad \hat{=} |P', J', \Sigma'\rangle$

Transition operator: $\qquad \hat{O} = \hat{\mathcal{J}}^{\mu}_{em}$

$$\left\langle V', M', J', \Sigma' \left| \hat{\mathcal{J}}^{\mu}_{em} \right| V, M, J, \Sigma \right\rangle =$$

$$= \frac{2}{MM'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3\vec{k}_2 \, d^3\vec{k}_3 \, d^3\vec{k}'_2 \, d^3\vec{k}'_3$$

$$\times \sqrt{\frac{\left(\sum_i \omega'_i\right)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D^{\star \frac{1}{2}}_{\sigma'_i \mu'_i} \left\{ R_W \left[k'_i ; B\left(V'\right) \right] \right\} \Psi^{\star}_{M'J'\Sigma'} \left(\vec{k}'_1, \vec{k}'_2, \vec{k}'_3 ; \mu'_1, \mu'_2, \mu'_3 \right)$$

$$\times \left\langle p'_1, p'_2, p'_3 ; \sigma'_1, \sigma'_2, \sigma'_3 \left| \hat{\mathcal{J}}^{\mu}_{rd} \right| p_1, p_2, p_3 ; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$

$$\times \sqrt{\frac{\left(\sum_i \omega_i\right)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i \mu_i} \left\{ R_W \left[k_i ; B\left(V\right) \right] \right\} \Psi_{MJ\Sigma} \left(\vec{k}_1, \vec{k}_2, \vec{k}_3 ; \mu_1, \mu_2, \mu_3 \right)$$

$$\times 2MV_0 \delta^3 \left(M\vec{V} - M'\vec{V}' - \vec{q} \right)$$

where $\quad p_i = B_c(V)k_i, \quad p'_i = B_c(V')k'_i, \quad$ and $\quad \omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

Electromagnetic Nucleon Form Factors
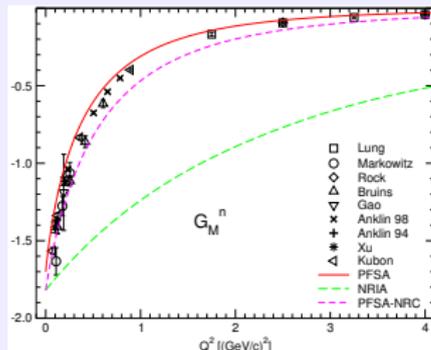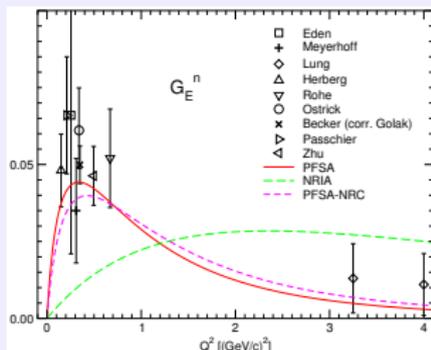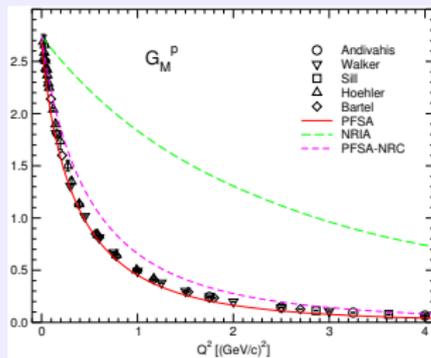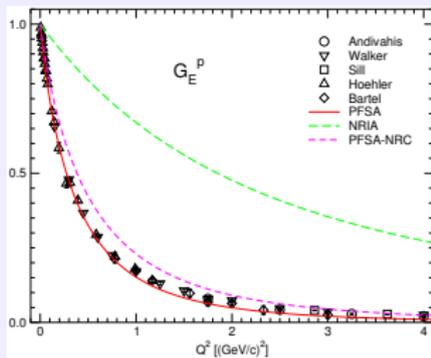
## Covariant predictions of the GBE CQM:

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R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:
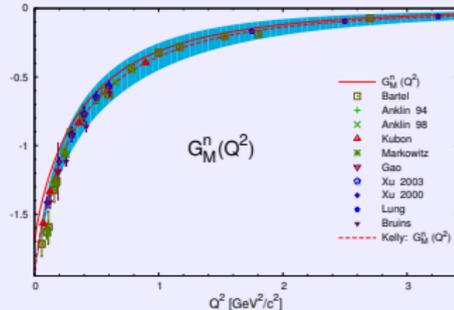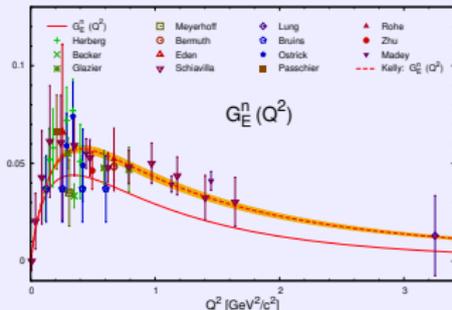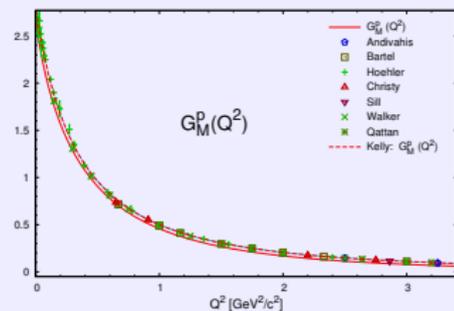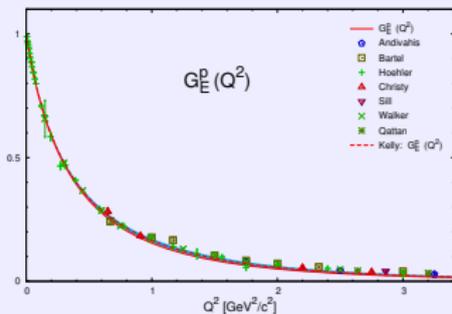
R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

M. Rohrmoser: Diploma Thesis, Univ. of Graz, 2013

Electric radii r_E^2 [fm^2]

Baryon	**GBE PFSM**	Experiment
p	0.82	0.7692 ± 0.0123[1]
		0.70870 ± 0.00113[2]
n	-0.13	-0.1161 ± 0.0022

[1] CODATA value (PDG)   [2] Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	**GBE PFSM**	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Electric radii r_E^2 [fm^2]

Baryon	**GBE PFSM**	GBE NRIA	Experiment
p	0.82	0.10	0.7692 ± 0.0123[1]
			0.70870 ± 0.00113[2]
n	-0.13	-0.01	-0.1161 ± 0.0022

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K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleons N

Proton Electric Form Factor

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$

M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665
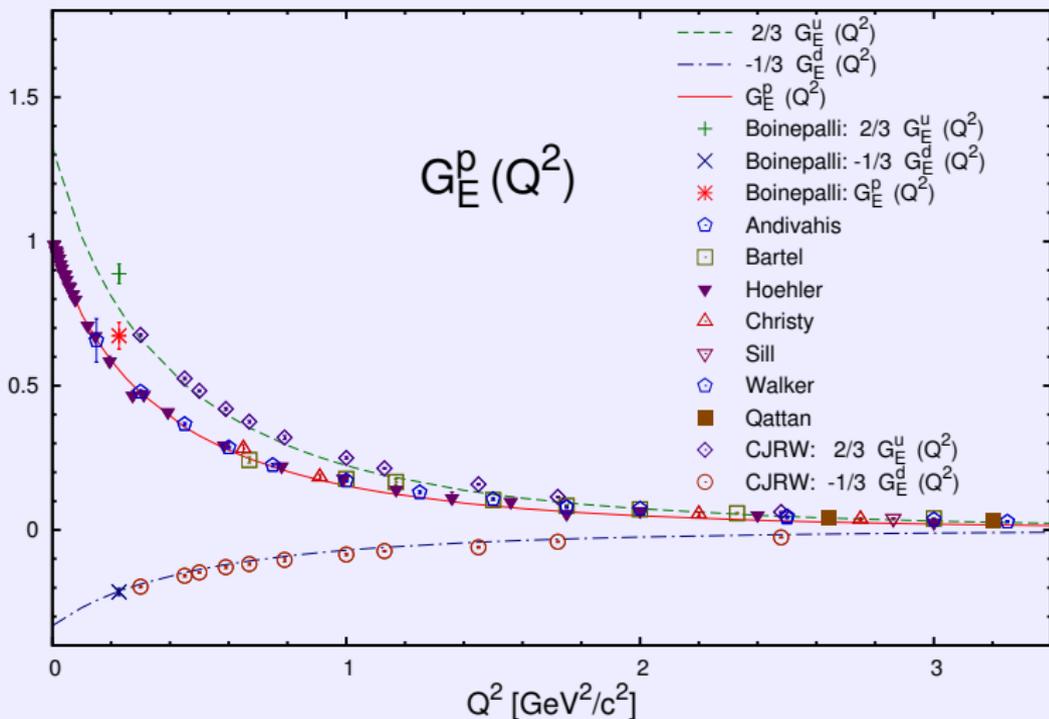
Neutron Electric Form Factor

$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$

M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665
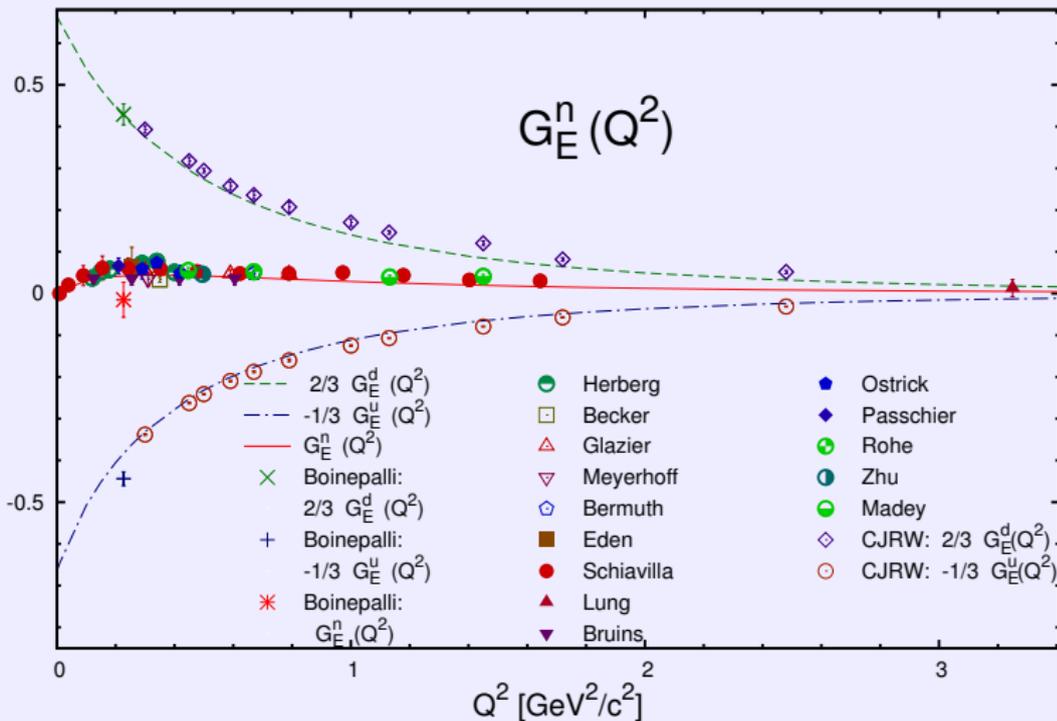
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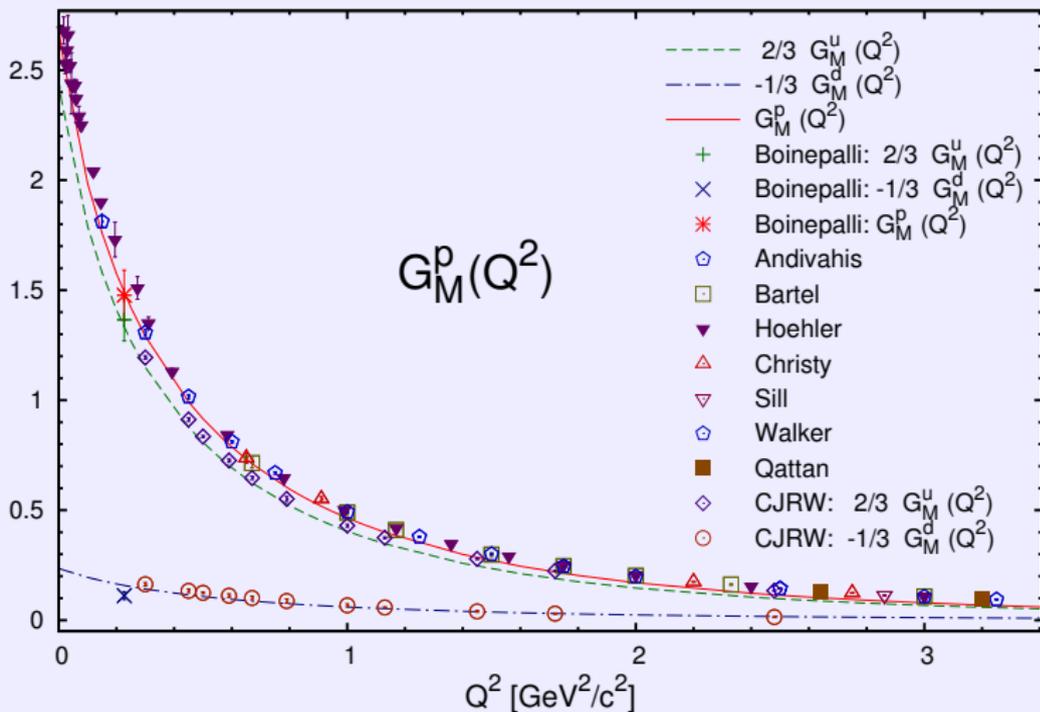
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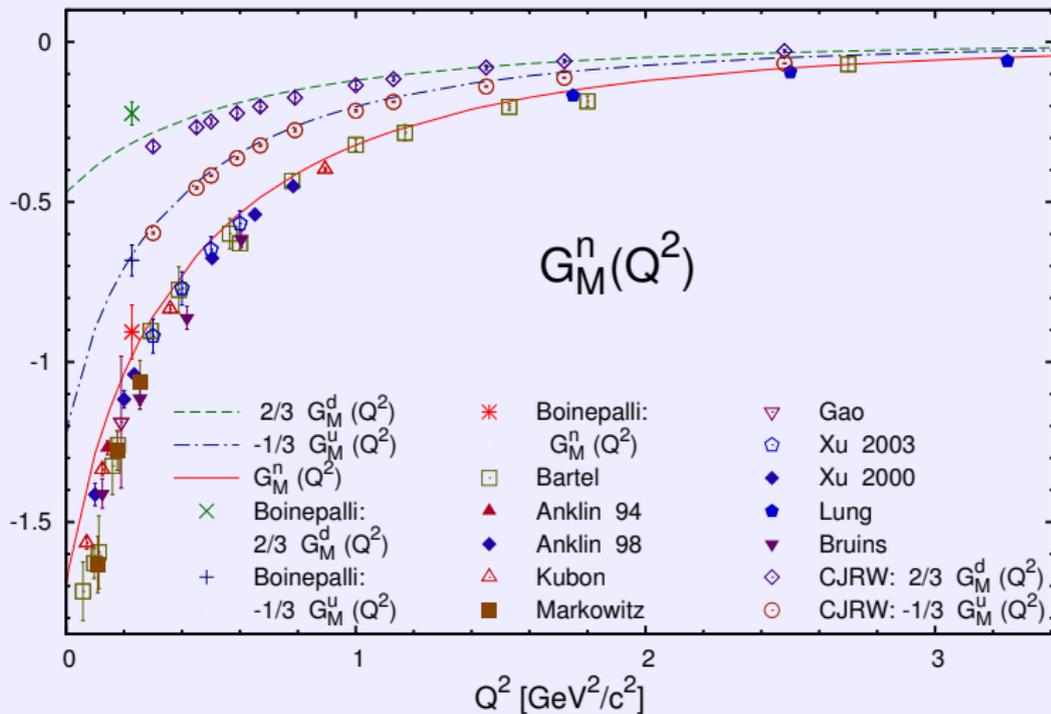
M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

- **Flavor analysis of nucleon e.m. form factors** in a relativistically invariant framework (point form).

- The **GBE RCQM** predicts flavor contributions in reasonable agreement with **experimental data**.

- The GBE RCQM relies on $\{QQQ\}$ degrees of freedom only; no explicit $\{QQQQ\bar{Q}\}$ etc.

- No explicit **meson-cloud effects** are included.

- No **strangeness content** in the nucleon for the low momentum transfers considered here.

- With respect to F_2^d/F_2^u three different phenomenological analyses give **distinct answers**.

- Details:

  M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

  W. Plessas: Mod. Phys. Lett. A **28**, 136022 (2013)

Δ

Λ, Σ, Ξ

Σ^*, Ξ^*, Ω

Electric Δ Form Factors

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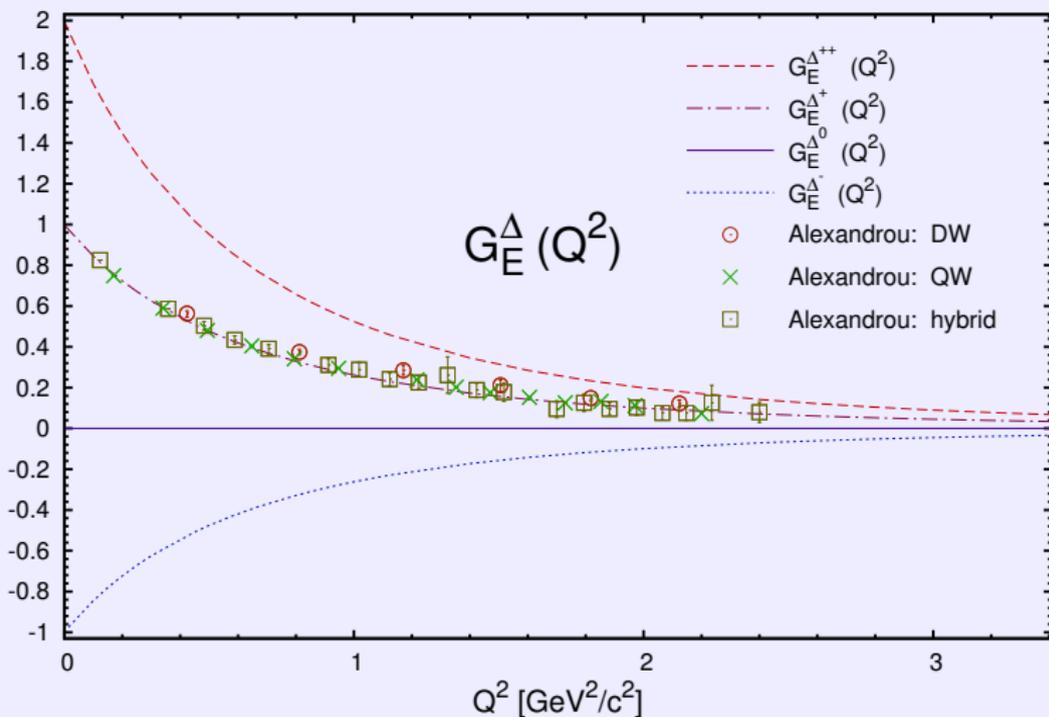
GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507
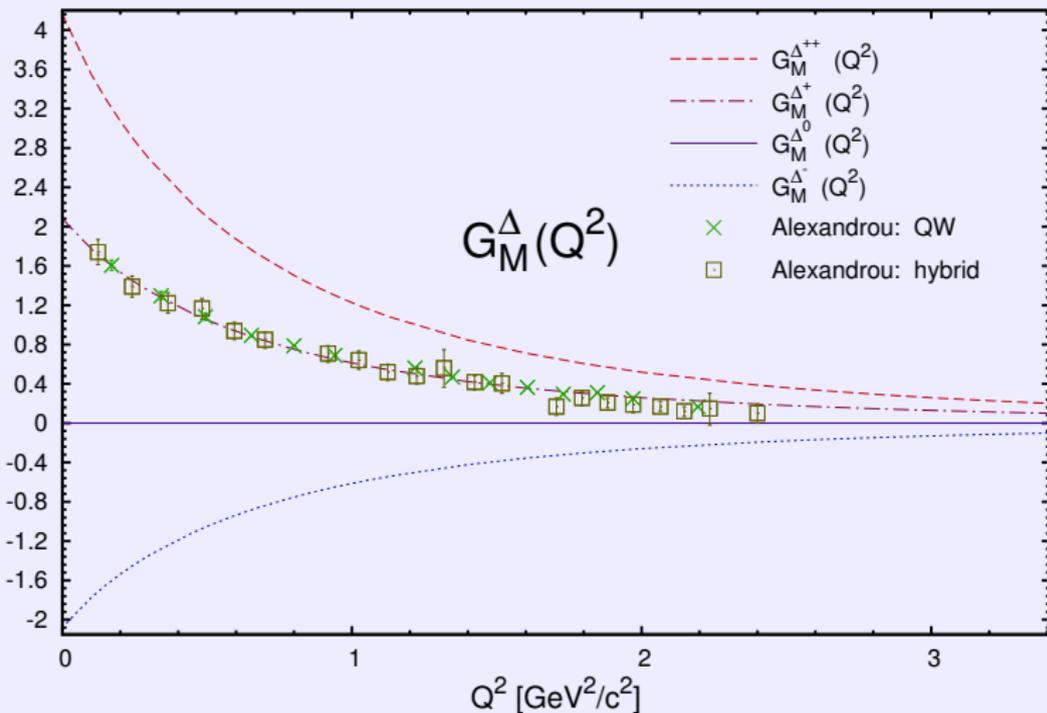
GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

Axial **Charges** and Axial **Form Factors**

of

N Ground State and *N*[*] Resonances

as well as

$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Axial Nucleon Form Factors

Covariant predictions of the GBE RCQM:
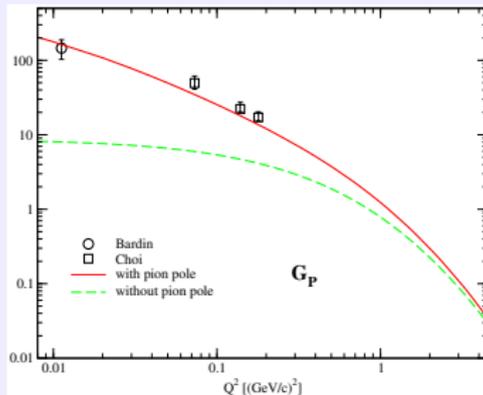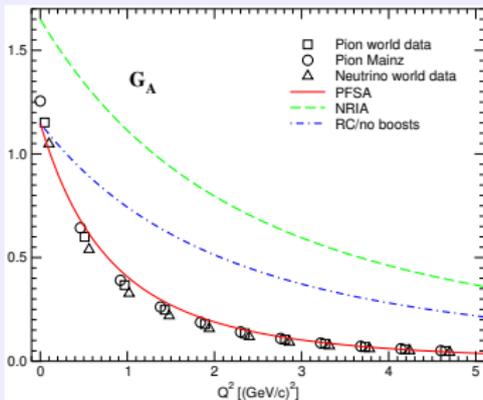
$$g_A^{GBE} = 1.15 \quad \text{vs.} \quad g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Microscopic Description

of

Meson-Baryon Interaction Vertices

Interaction vertices
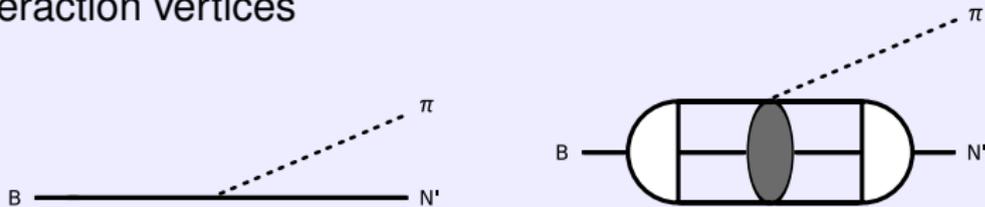
$$F_{i \to f} = (2\pi)^4 \langle f | \mathcal{L}_I (0) | i \rangle \qquad \equiv \qquad \langle V', M', J', \Sigma' | \hat{D}_{\mathrm{rd}}^{\pi} | V, M, J, \Sigma \rangle$$

where

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \left| \hat{D}_{\mathrm{rd}}^{\pi} \right| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle =$$

$$3 \mathcal{N}_{\mathrm{S}} \frac{i g_{qqm}}{2 m_1 (2\pi)^{\frac{3}{2}}} \bar{u}(p_1', \sigma_1') \gamma_5 \gamma_\mu \lambda_m u(p_1, \sigma_1) \tilde{q}^\mu 2 p_{20} \delta(\vec{p}_2 - \vec{p}_2') 2 p_{30} \delta(\vec{p}_3 - \vec{p}_3') \delta_{\sigma_2 \sigma_2'} \delta_{\sigma_3 \sigma_3'}$$

and

$$G_{\pi NN} \left(Q^2 \right) = \frac{1}{f_{\pi NN}} \frac{m_\pi \sqrt{2\pi}}{\sqrt{2 M_N}} \frac{\sqrt{E_N' + M_N'}}{E_N' + M_N' + \omega} \frac{F_{i \to f}}{Q_z}$$

$$G_{\pi N\Delta} \left(Q^2 \right) = -\frac{1}{f_{\pi N\Delta}} \frac{3\sqrt{2\pi}}{2} \frac{m_\pi}{\sqrt{E_N' + M_N'} \sqrt{2 M_\Delta}} \frac{F_{i \to f}}{Q_z}$$

πNN and $\pi N\Delta$ Interaction Vertices
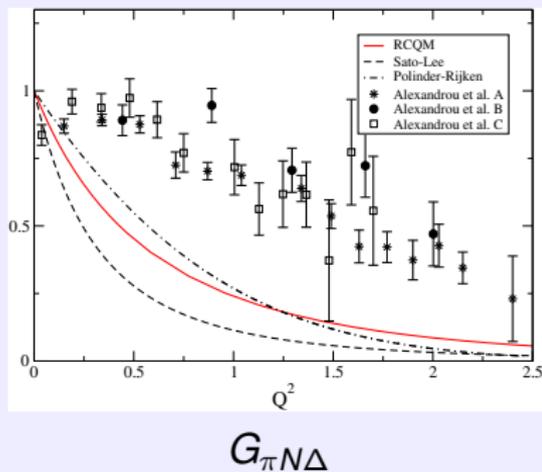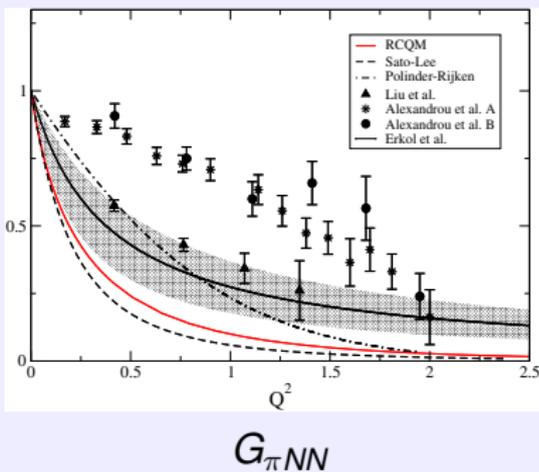
$G_{\pi NN}$

$G_{\pi N\Delta}$

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Form-Factor Parametrizations

Low-energy
QCD

RCQM
GBE RCQM

Spectroscopy
Light, strange,
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Structure
Nucleon E.m.
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Axial FFs
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Summary

$$G\left(\vec{q}^{\,2}\right) = \frac{1}{1+\left(\frac{\vec{q}}{\Lambda_1}\right)^2+\left(\frac{\vec{q}}{\Lambda_2}\right)^4} \qquad\qquad G\left(Q^2\right) = \frac{1}{1+\left(\frac{Q}{\Lambda}\right)^2}$$

		RCQM	SL	PR		LIU	ERK	ALX
	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075		0.0649	0.0481	0.0412
N	Λ_1	0.451	0.453	0.940	Λ	0.747	0.614	1.65
	Λ_2	0.931	0.641	1.102		-	-	-
	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478				
Δ	Λ_1	0.594	0.458	0.853				
	Λ_2	0.998	0.648	1.014				

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Nucleon **Scalar** Form Factor

and

π-*N* **Sigma** Term

$\underline{S(Q^2)}$

Invariant ME of $\hat{S} = \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d)$:

$$\langle P'J\Sigma'|\hat{S}|PJ\Sigma\rangle = \bar{U}(P')U(P)S(Q^2)$$

$\underline{\sigma_N}$

Scalar form factor at zero momentum transfer:

$$\sigma_N = S(Q^2 = 0)$$
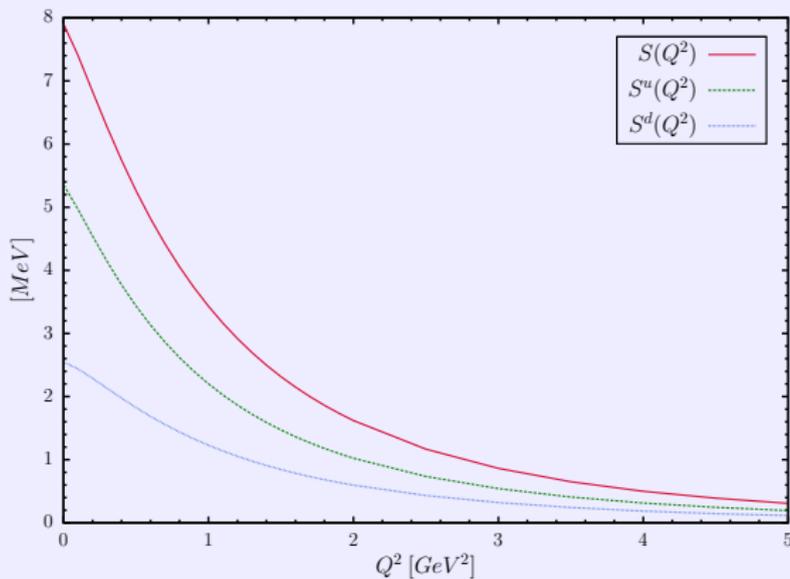
red line   scalar form factor
green line   u-contribution to scalar form factor
blue line   d-contribution to scalar form factor

π-N Sigma Term

Prediction of the Goldstone-boson-exchange RCQM:

$$\sigma_N \approx 8.0 \text{ MeV}$$

To be compared, e.g., with:

$$
\begin{aligned}
\sigma_N \quad &\approx \quad 35 \pm 5 \quad \text{MeV} \quad [1] \\
&\quad\quad 18 \pm 5 \quad \text{MeV} \quad [2] \\
&\quad\quad 31 \pm 7 \quad \text{MeV} \quad [3] \\
&\quad\quad \text{etc.}
\end{aligned}
$$

[1] J. Gasser and H. Leutwyler: Phys. Rep. 87 (1982) 77

[2] S. Güsken et al. (SESAM Coll.): Phys. Rev. D 59 (1999) 054504

[3] R. Horsley et al. (QCDSF-UKQCD Coll.): Phys. Rev. D 85 (2012) 034506

etc.

Gravitational Form Factors

of

the Nucleon

Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P'J\Sigma'|\hat{\Theta}^{\mu\nu}|PJ\Sigma\rangle = \bar{U}(P')\left[\gamma^{(\mu}\bar{P}^{\nu)}A(Q^2) + \right.$$

$$\left.\frac{i}{2M}\bar{P}^{(\mu}\sigma^{\nu)}B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M}C(Q^2)\right]U(P)$$

$$A(Q^2) \sim \langle P'J\Sigma'|\Theta^{00}|PJ\Sigma\rangle$$

Nucleon Gravitational Form Factor $A(Q^2)$

J.P. Day: PhD Thesis, Univ. Graz, August 2013
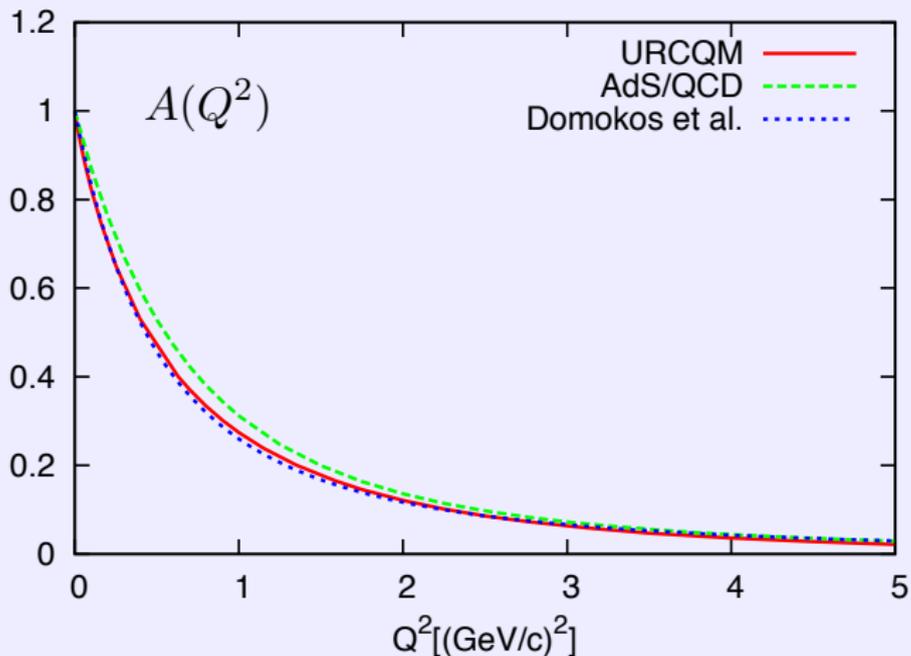
Summary and Conclusions

- Surprisingly **good agreement** of predictions by the $\{QQQ\}$ GBE RCQM with experimental data (wherever such data are available)

- **Small deviations** left in some observables, such as electric radii and magnetic moments

- Surprisingly **good agreement** of predictions by the GBE RCQM with lattice-QCD results

- Most important symmetries of the GBE RCQM:
  - **SB$_\chi$S**
  - **Lorentz invariance**
  - **time-reversal invariance**
  - **current conservation**

- The **non-relativistic** quark model **does not work** in any instance

Graz

K. Berger, J.P. Day, K.-S. Choi, L. Glozman, T. Melde,
M. Rohrmoser, R.C. Schardmüller, B. Sengl,
R.F. Wagenbrunn

(Theoretical Physics, University of Graz)

Pavia

S. Boffi and M. Radici

(INFN, Sezione di Pavia)

Padova

L. Canton

(INFN, Sezione di Padova)

Iowa City

W. Klink

(Department of Physics, University of Iowa, USA

Thank you very much

for

your attention!