QCD with functional methods

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June 30, 2016

FWF Der Wissenschaftsfonds. UNI GRAZ NAWI Graz Natural Sciences INSTITUT FÜR PHYSIK

QCD: A simple theory?

$$\frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu} + \bar{q}(i\not{D} - M)q$$

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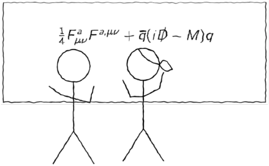

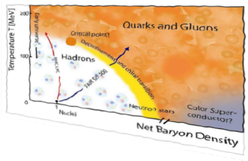

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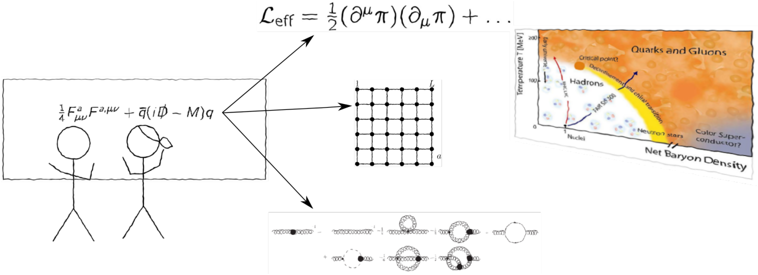

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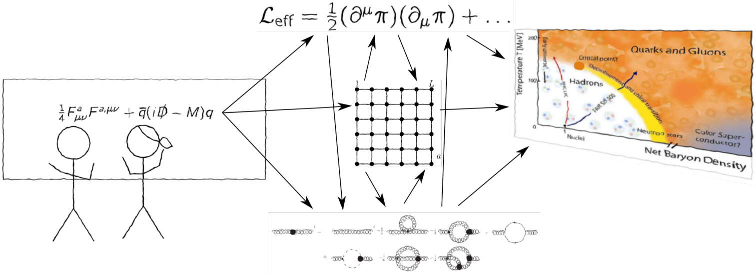

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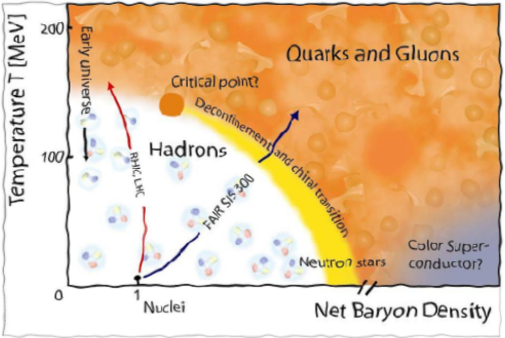
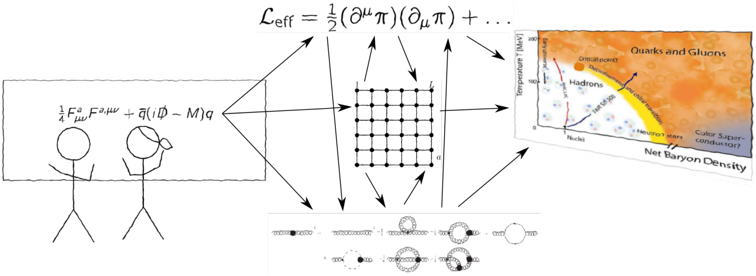


- Phases: hadronic phase, quark-gluon plasma, color superconductor, quarkyonic?
- Transitions: first order line, crossover at $\mu = 0$
- Critical point: existence? position?

QCD: A simple theory?



- Challenges for all methods at $\mu > T$, e.g.
  - Lattice QCD: complex action problem
  - Models: parameters
  - Functional methods: reliability of truncations

Functional methods

Functional equations: Exact equations derived from QCD action.

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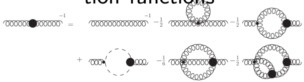
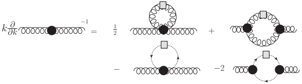
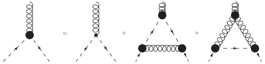
Dyson-Schwinger eqs.: eqs. of motion of correlation functions





funct. renorm. group eqs.

eqs. of motion from 3PI eff. action



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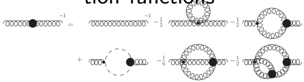
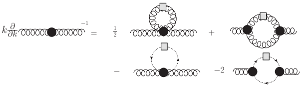


- Chiral limit accessible.
- No sign problem.
- Large scale separations easy.



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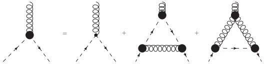

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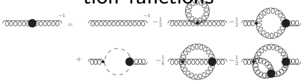

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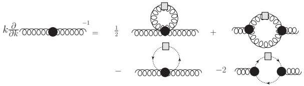

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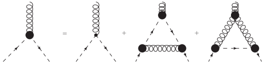


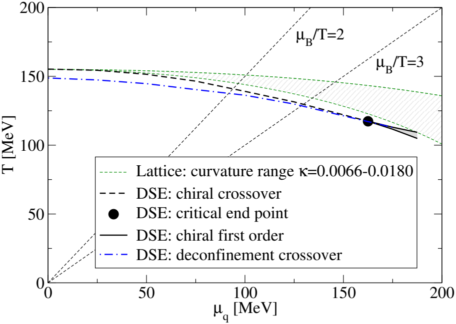
Difficulty

Infinitely large systems of equations without obvious ordering scheme.

QCD phase diagram from functional equations

2+1 flavor QCD from DSEs

[Fischer, Lücker, Welzbacher '14]:



Positions of critical endpoint:

$\sim (168\,\text{MeV}, 115\,\text{MeV})$

lattice gluon from $T = 0$, vertex model

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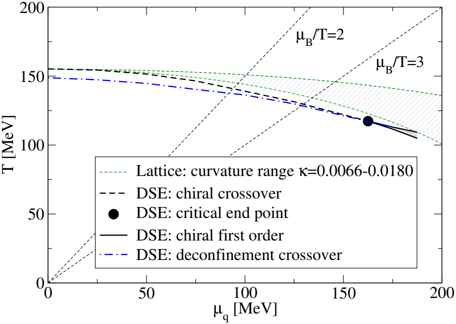
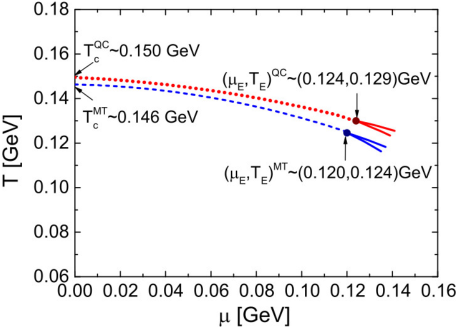


2 flavor QCD from DSEs

[Xin, Qin, Liu '15]:



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lattice gluon from $T = 0$, vertex model

$\sim (122\,\text{MeV}, 126\,\text{MeV})$

rainbow approximation

Landau gauge QCD

$$\mathcal{L} = \bar{q}(-\not{D} + m)q + \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\,g\,[A_\mu, A_\nu]$$

Landau gauge QCD

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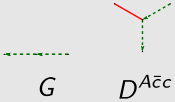
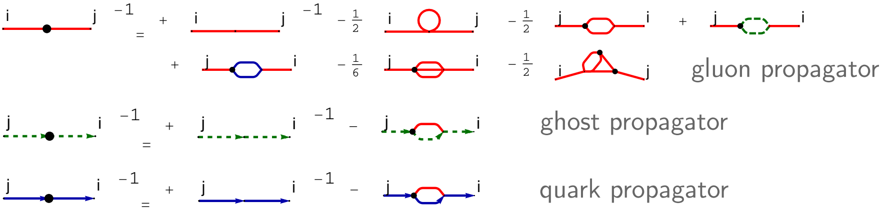
Landau gauge

- simplest one for functional equations
- $\partial_\mu A_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu A_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{c}\,(-\Box + g\,A\,\times)\,c$



The tower of DSEs



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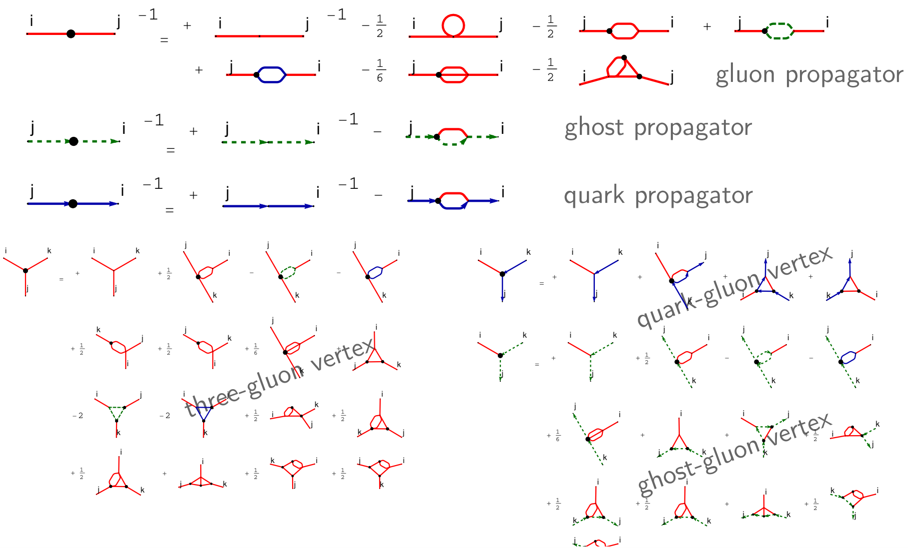

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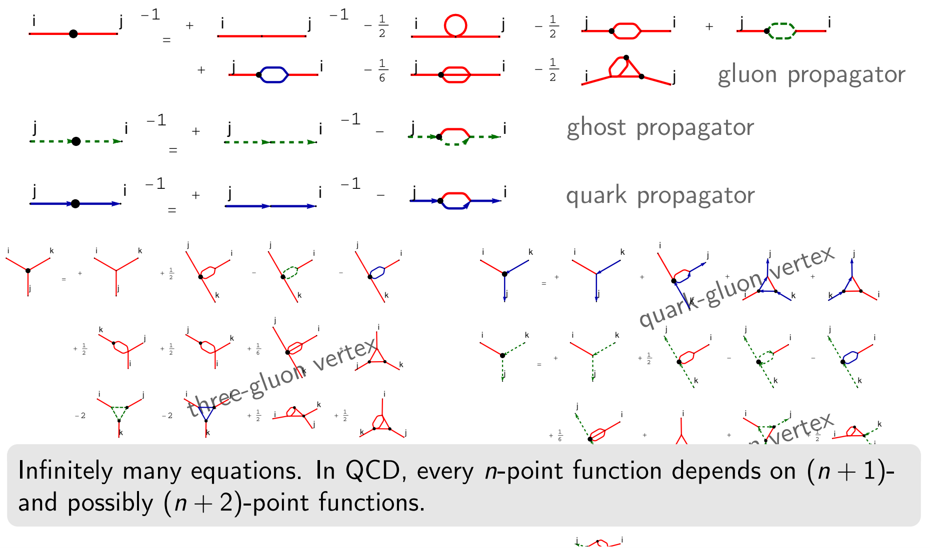

Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Truncating the equations

Guides

- Perturbation theory
- Symmetries
- Lattice
- Analytic results

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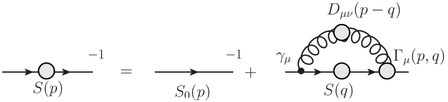
Truncation

- Drop quantities (unimportant?)
- Use fits
- Model quantities (good models available? 'true' or 'effective'?)

Ideally: Find a truncation that has (I) no parameters and yields (II) quantitative results.

Example: Bottom-up approach

Quark gap equation:



Required input:

- Gluon propagator $D(p^2)$
- Quark-gluon interaction $\Gamma(p,q)$

Effective interaction $D(p^2)\Gamma(p,q) \to \mathcal{G}(p^2)$

Example: Maris-Tandy interaction with parameters ω, D

$$\frac{\mathcal{G}(s)}{s} = \frac{4\pi^2 D}{\omega^6} s\, \mathrm{e}^{-s/\omega^2} + \frac{4\pi\, \gamma_m \pi\, \mathcal{F}(s)}{1/2 \ln[\tau + (1 + s/\Lambda^2_{\mathrm{QCD}})^2]}.$$

$\to$ Use for bound state studies.

Example: Top-down for Yang-Mills theory

Neglect all non-primitively divergent Green functions. $\rightarrow$ Self-contained.

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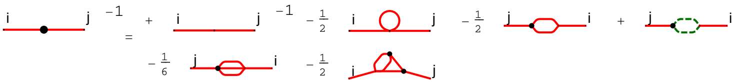
Full propagator equations (two-loop diagrams!):



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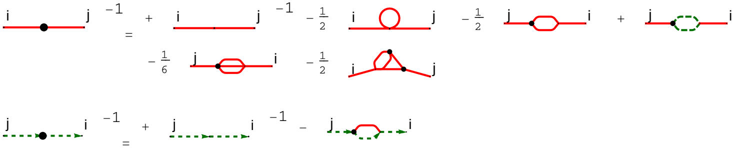
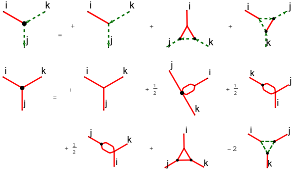
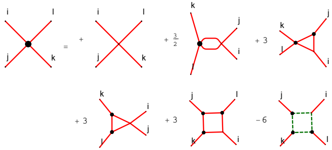


Truncated three-point functions:



Truncated four-gluon vertex:



Automated derivation

Derivation by hand becomes tedious:

- Large Lagrangians.
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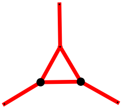

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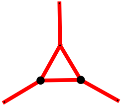

```
(2 g^2 Nc Z1 DAAA[y, qs + y + 2 sp[q, q1], (-y - sp[q, q1])/Sqrt[y (qs + y + 2 sp[q, q1])]]
  DAAA[x2 + y + 2 sp[p, q], qs + x2 - 2 sp[p, q1], (-x2 - sp[p, q] + sp[p, q1] + sp[q, q1])/Sqrt[(x2 + y + 2 sp[p, q]) (qs + x2 - 2 sp[p, q1])]] Dgl[qs] Dgl[qs + x2 - 2 sp[p, q1]] Dgl[
   sp[p, q]^4 (sp[p, q1]^2 sp[q, q1] (y + sp[q, q1]) + qs x2 (y (9 qs + 6 (x2 + y)) + (5 qs + 6 x2 + 10 y) sp[q, q1]) - sp[p, q1] (qs y (5 qs
   sp[p, q]^3 (2 sp[p, q1]^3 (qs y - sp[q, q1]^2) + sp[p, q1] (qs y (10 qs^2 + (-5 x2 - 3 y) y + qs (19 x2 + 3 y)) + (3 qs^3 + 8 qs x2 y + 21 qs^2 (
     qs x2 (y (-9 qs^2 + 3 x2^2 + 7 x2 y + 3 y^2 + 2 qs (x2 + y)) + (-10 qs^2 + qs (-3 x2 - 19 y) + x2 (3 x2 + 5 y)) sp[q, q1] + (-16 qs - 7 x2 - 11 y
     sp[p, q1]^2 (qs (-16 qs - 11 x2 - 7 y) y + (-5 qs^2 + qs (-9 x2 - 19 y) + 2 y (5 x2 + 3 y)) sp[q, q1] + (-5 qs + 12 (x2 + y)) sp[q, q1]^2 +
   sp[p, q]^2 (sp[p, q1]^4 sp[q, q1] (qs + sp[q, q1]) + sp[p, q1]^3 (qs y (7 qs + 11 x2 + 16 y) + (-6 qs^2 + y (9 x2 + 5 y) + qs (-10 x2 + 19 y)
     qs x2 (y (-3 qs^3 - 10 qs^2 (x2 + y) - 6 x2 y (x2 + y) + qs (-3 x2^2 - 19 x2 y - 3 y^2)) + (-6 qs^3 + qs^2 (-21 x2 - 32 y) + qs (-9 x2^2 - 60 x2 y
       (-15 qs^2 - 15 x2^2 + qs (-46 x2 - 41 y) - 41 x2 y - 12 y^2) sp[q, q1]^2 + (-7 qs - 16 x2 - 11 y) sp[q, q1]^3) + sp[p, q1]^2 (qs y (-15 q
       (3 qs^3 + qs^2 (5 x2 - 39 y) + qs (-81 x2 - 39 y) y + y^2 (5 x2 + 3 y)) sp[q, q1] + (12 qs^2 + 12 x2^2 + 3 x2 y + 12 y^2 + 3 qs (x2 + y)) sp[q
     sp[p, q1] (qs y (6 qs^3 + qs^2 (32 x2 + 21 y) + qs (25 x2^2 + 60 x2 y + 9 y^2) + x2 (3 x2^2 + 25 x2 y + 15 y^2)) + (15 qs^3 (x2 + y) + x2 y (-3 x
       (-3 qs^3 + x2^2 (-3 x2 - 5 y) + qs^2 (39 x2 - 5 y) + qs x2 (39 x2 + 81 y)) sp[q, q1]^2 + (-6 qs^2 + qs (19 x2 - 10 y) + x2 (5 x2 + 9 y)) sp[
   x2 y (-sp[p, q1]^5 (qs + sp[q, q1]) + sp[p, q1]^4 (qs (6 qs + 6 x2 + 9 y) + (10 qs + 6 x2 + 5 y) sp[q, q1]) - qs (qs y - sp[q, q1]^2) (x2 (-
       (6 qs + 9 x2 + 6 y) sp[q, q1]^2 + sp[q, q1]^3) + sp[p, q1]^3 (qs (-3 qs^2 - 3 x2^2 + qs (-7 x2 - 2 y) - 2 x2 y + 9 y^2) + (-3 x2^2 + 3 x2 y +
     sp[p, q1]^2 (qs (-3 qs^2 (2 x2 + y) + qs (-6 x2^2 - 19 x2 y - 10 y^2) + y (-3 x2^2 - 10 x2 y - 3 y^2)) + (-3 qs^3 - 25 qs^2 (x2 + y) + qs (-15 x2
       (-12 qs^2 - 15 x2^2 - 46 x2 y - 15 y^2 - 41 qs (x2 + y)) sp[q, q1]^2 + (-11 qs - 16 x2 - 7 y) sp[q, q1]^3) +
```

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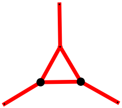

- Automated derivation of DSEs and flow equations:
  Mathematica package *DoFun* [Alkofer, MQH, Schwenzer '08; MQH, Braun '11]
  `http://tinyurl.com/dofun2`
- Framework for numeric handling:
  C++ program *CrasyDSE* [MQH, Mitter '11]
  `http://tinyurl.com/crasydse`

Go ahead and calculate . . .

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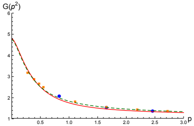
Landau gauge, vacuum:



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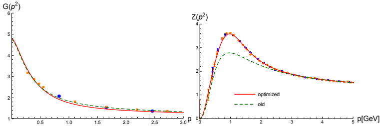

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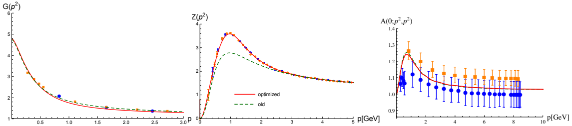

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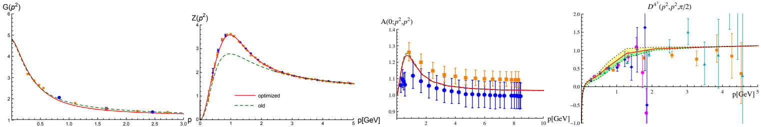

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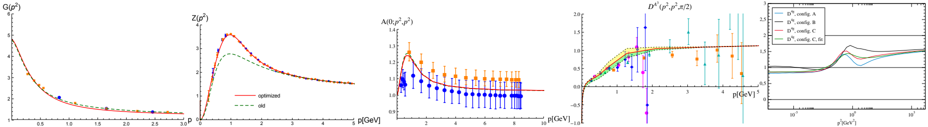

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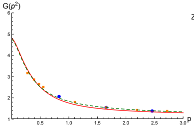
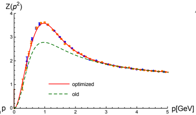
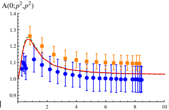
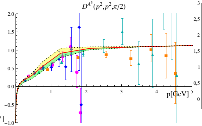
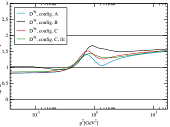


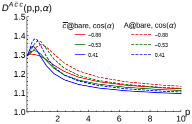








Coulomb and linear covariant gauges, vacuum:



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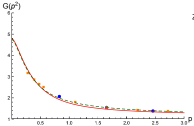

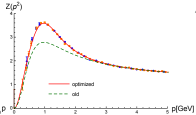

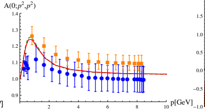

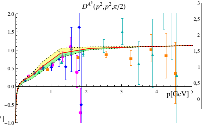

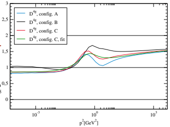

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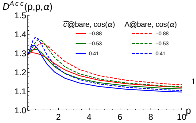

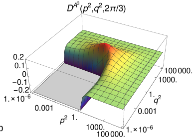

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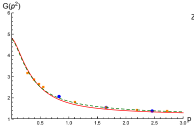

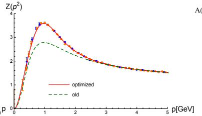

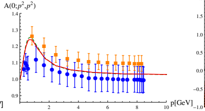

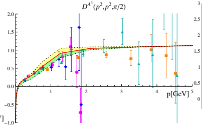

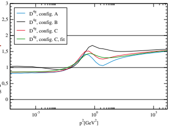

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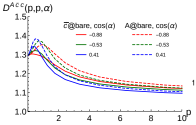

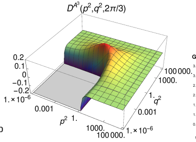

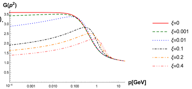

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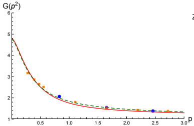

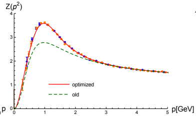

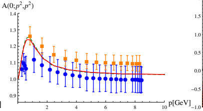

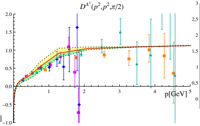

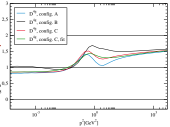

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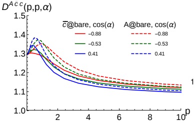

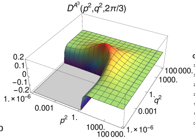

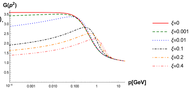

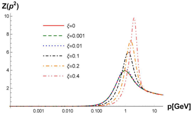

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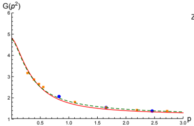

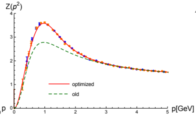

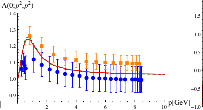

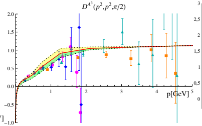

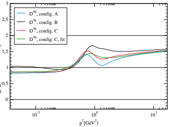

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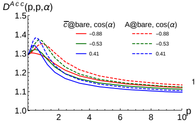

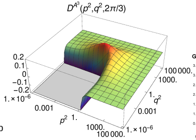

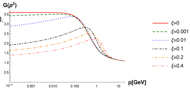

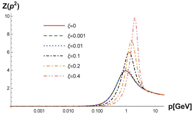

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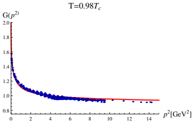

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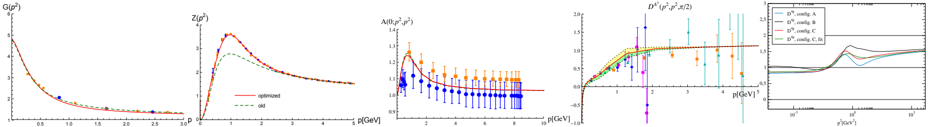

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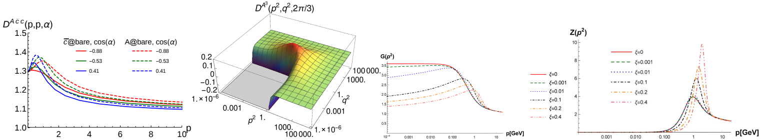

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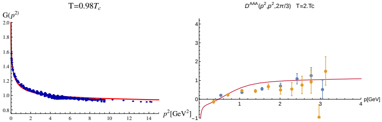

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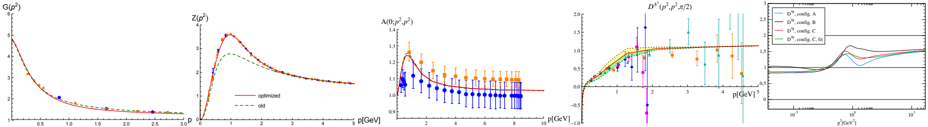

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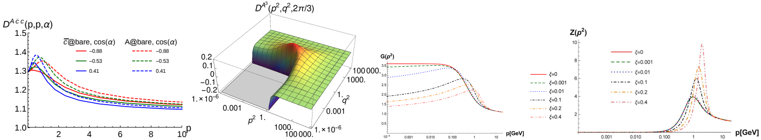

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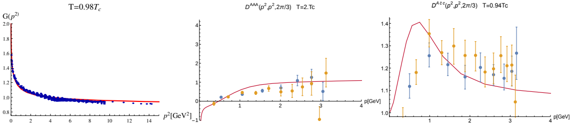

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- Propagators
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- $\mu > 0$?
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→ Comparison with lattice is helpful, but finally self-consistent checks are required.

Two words of caution:

- One cannot assume naturally that the hierarchy is the same for all T and μ.
- Even the effect of a single correlation function is difficult to estimate.

Example: What do we need to go beyond modern QCD phase diagram calculations?

Beyond effective interaction approximation: ✓ [Fischer, Lücker, Welzbacher '14]

Input for DSEs:

- model for quark-gluon vertex (parameters fixed at $\mu = 0$)
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Ultimately, full control over Yang-Mills part required!

Yang-Mills theory

Up to now
(separately and partly combined):

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- Three-point functions
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Challenges:

- Find truncation.
- Solve large system of equations.
- Spurious (quadratic) divergences in gluon propagator: Consistent subtraction [MQH, von Smekal, '14]?

Testing truncations

Switch to three dimensions:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle

$\Rightarrow$ Many complications from $d = 4$ absent!

In particular: Spurious divergences of 'simple' enough form ($\propto a\Lambda + b \ln \Lambda$).

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Vary equations and truncations.

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Truncation

Complete system of propagators, three-point function and four-gluon vertex.

Varying the four-gluon vertex

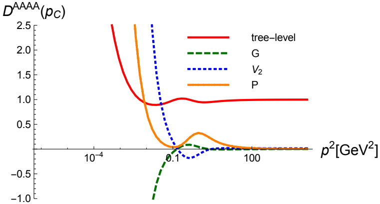
How stable is the truncation?

Varying the four-gluon vertex

How stable is the truncation?

Compare:

- full four-gluon vertex
- bare four-gluon vertex



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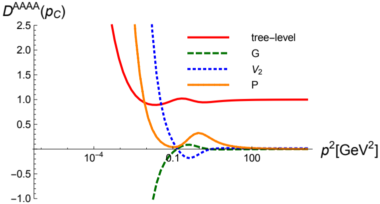

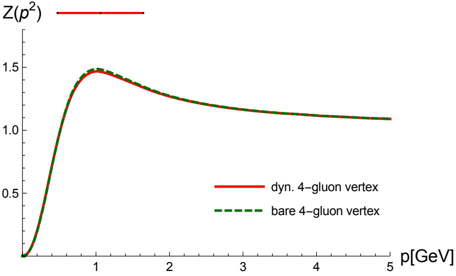

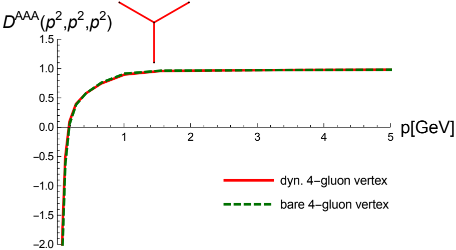

$\rightarrow$ Very similar results.

[MQH '16]

Solution from the 3PI effective action

Different set of functional equations:
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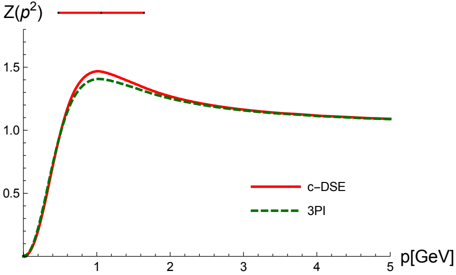
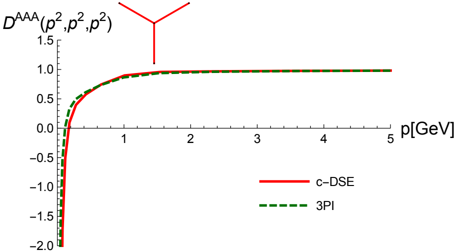




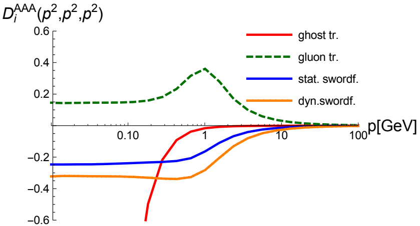
→ Very similar results. [MQH '16]

Summary results $d = 3$ Yang-Mills system

- Stability: Deforming the system has a minor effect.
- Vertices close to tree-level.
- Two-loop diagrams: Important in gluon propagator, much less in vertices.

Cancellations in gluonic vertices

Three-gluon vertex:



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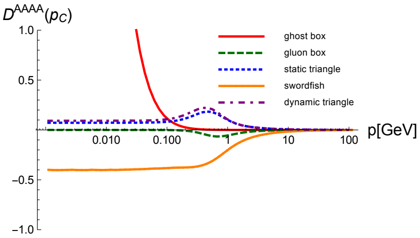

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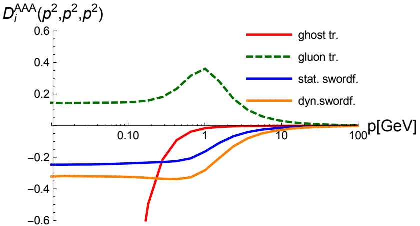

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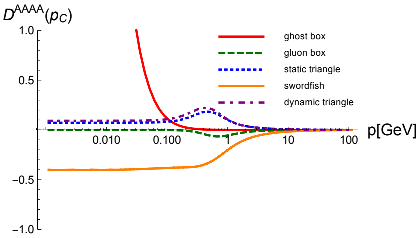

Higher contributions:

- Higher vertices close to 'tree-level'? → Small.
- If pattern changes (higher vertices large): cancellations required.

[MQH '16]

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Common lore: Quarks are easy in functional equations.

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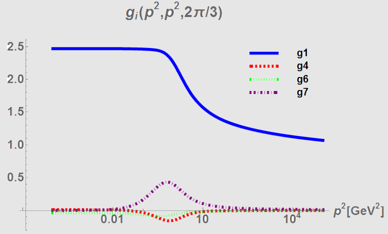
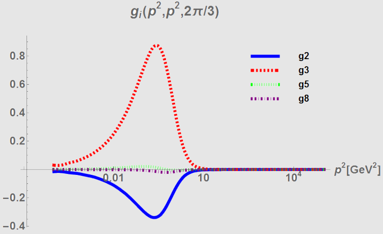
Explicit quark-gluon vertex solution

[Hopfer '14; Windisch '14; Mitter, Pawlowski, Strodthoff '14; Williams, Fischer, Heupel '15]



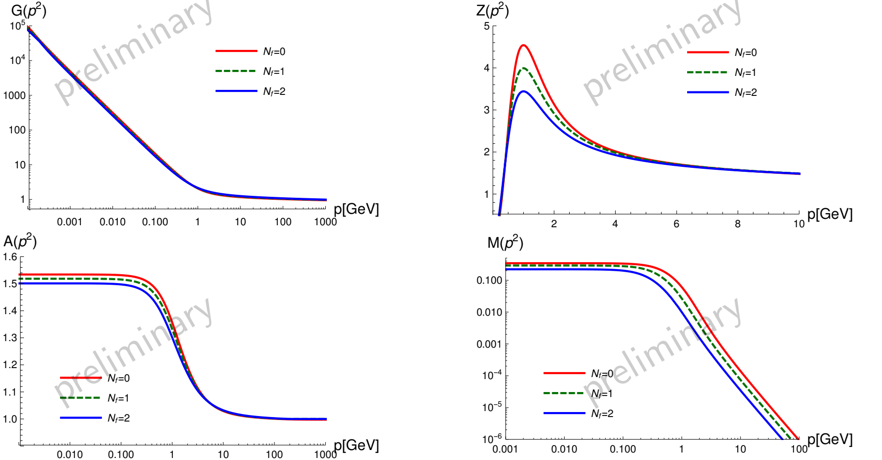


[Blum, Alkofer, MQH, Windisch '15]

→ Also non-tree-level dressings become important.

First step: Unquenching of propagators

Subtraction of spurious divergences as in Yang-Mills part [MQH, von Smekal '14]!
Models for vertices.



→ Poster by Contant.

[Contant, MQH, unpublished]

Summary and conclusions

- Functional equations: Non-perturbative approach to QCD.
- Calculations of propagators, vertices and partially mixed systems show a coherent picture.
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- Vertices 'simple'.
- Proper control of gluonic sector important for full QCD.

QCD

- Quark-gluon vertex 'complicated'.

- Gluon propagator: basic quantity, still challenging: spurious divergences, RG resummation.
- Truncation tests in $d = 3$: Stability reached (in vacuum).
  - Varying *system* of equations.
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  $\rightarrow$ Similar behavior in $d = 4$ expected.

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QCD

- Quark-gluon vertex 'complicated'.

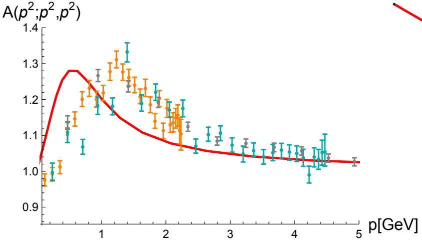
- Gluon propagator: basic quantity, still challenging: spurious divergences, RG resummation.
- Truncation tests in $d = 3$: Stability reached (in vacuum).
  - Varying *system* of equations.
  - Varying *equations* of system.

  $\rightarrow$ Similar behavior in $d = 4$ expected.

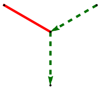
Thank you for your attention.

Results: Three-point functions

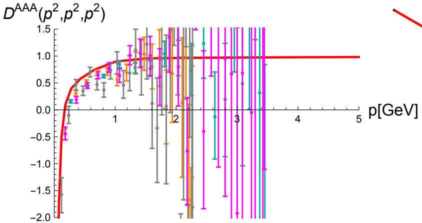
Dressings:



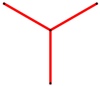
[MQH '16; lattice: Maas, unpublished]



- Maximum position shifted.
- Bump height ok.

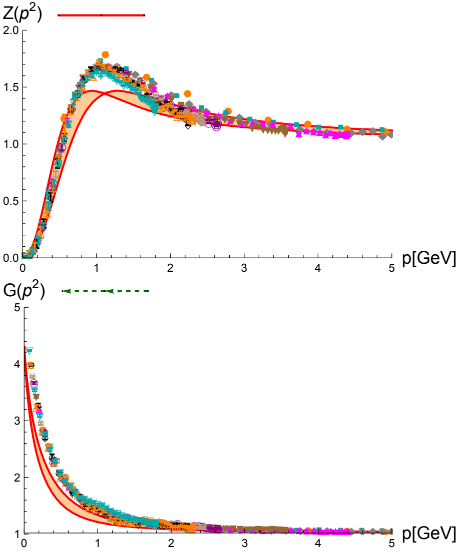


[MQH '16; lattice: Cucchieri, Maas, Mendes



- Good agreement with lattice data.
- Linear IR divergence.

Results: Propagators



Bands from uncertainty in setting the physical scale.

[MQH '16; lattice: Maas '14]

Summary results $d = 3$ Yang-Mills system

- Stability: Deforming the system has a minor effect.
- Vertices close to tree-level.
- Two-loop diagrams: Important in gluon propagator, much less in vertices.

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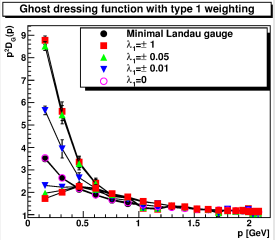
Reasons unclear.

Possible explanation: Gribov problem.

Example: Various solutions for ghost dressing found.



[Maas '13]

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Why is it so stable?

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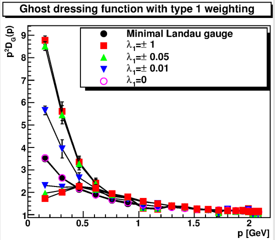

[Maas '13]

Couplings

Perturbatively the couplings must agree due to Slavnov-Taylor identities.

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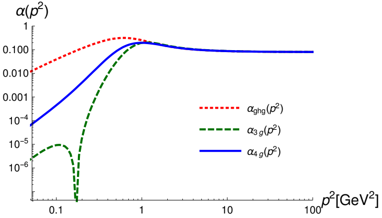
$$\alpha_{\text{ghg}}(p^2) = \frac{g^2}{4\pi} D^{A\bar{c}c}(p^2, p^2, p^2)^2 G(p^2)^2 Z(p^2)$$

$$\alpha_{\text{3g}}(p^2) = \frac{g^2}{4\pi} D^{AAA}(p^2, p^2, p^2)^2 Z(p^2)^3$$

$$\alpha_{\text{4g}}(p^2) = \frac{g^2}{4\pi} D^{AAAA}(p^2, p^2, p^2) Z(p^2)^2$$



$\Rightarrow$ Good agreement down to a few GeV!

[MQH '16]