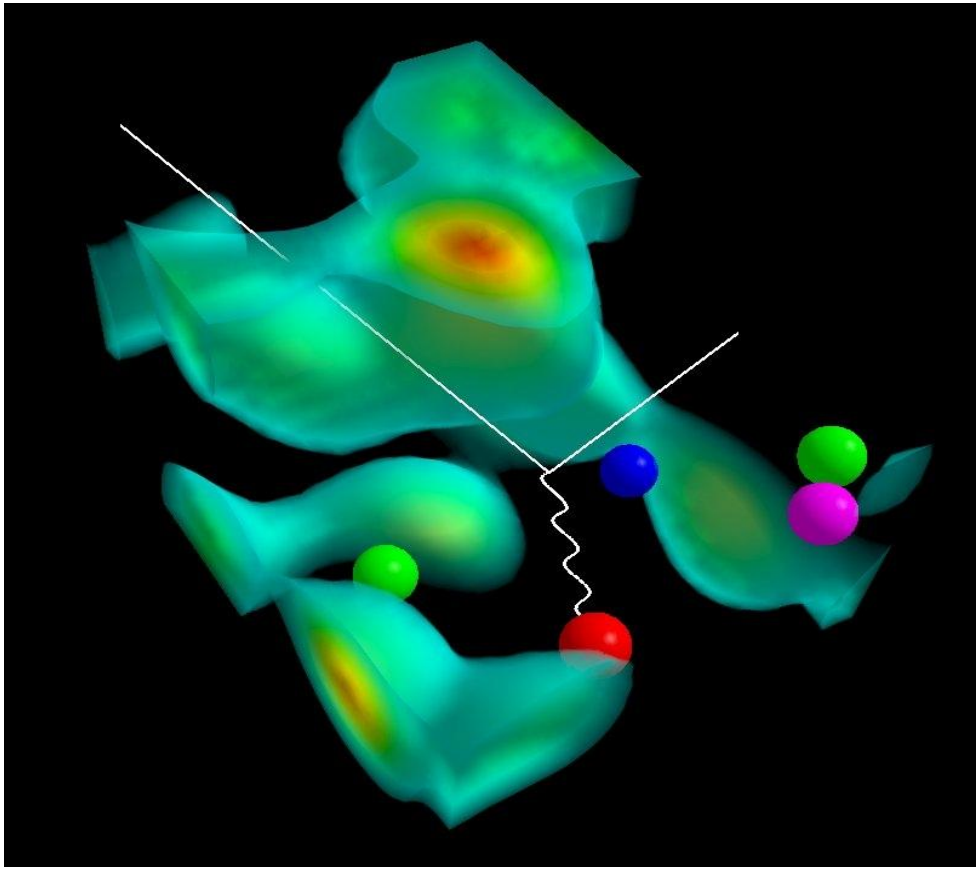
QCD, Atomic Nuclei and Neutron Stars



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Australian Government
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Anthony W. Thomas

Humboldt Kolleg : “From the Vacuum to the Universe”
Kitzbühel : 28th June 2016

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SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Outline

- **Start from a QCD-inspired model of *hadron* structure**

- **Ask how that internal structure is modified in-medium**

- **This naturally leads to saturation**
  + predictions for all hadrons (e.g. hypernuclei...)

- **Derive effective forces (Skyrme type): apply to finite nuclei**

- **Test predictions for quantities sensitive to internal structure: DIS structure functions, form factors in-medium....**

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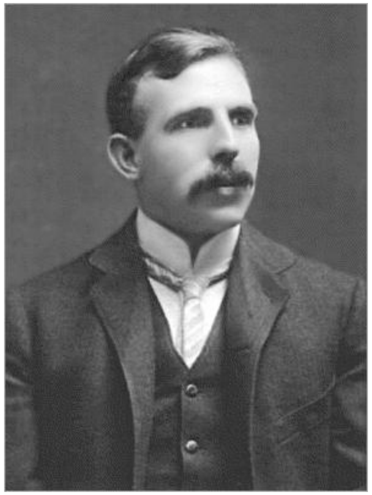
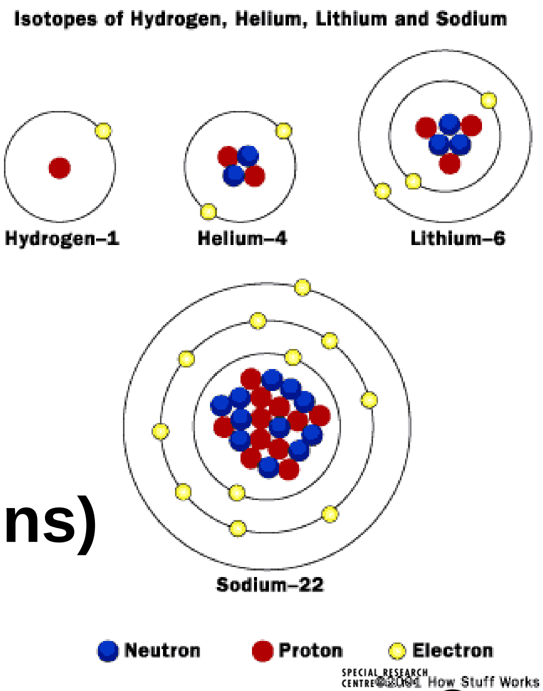
Rutherford



Discovered that alpha particles went straight through matter – most of the time

Occasionally scatter very hard – back the way they came!



Concluded matter is mainly empty space!

There is a heavy nucleus surrounded at a (comparatively great distance by electrons)

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- **Since the neutron was discovered by Chadwick, nuclei have been built from neutrons and protons*, with exactly the same properties in-medium as outside*, interacting through the exchange of pions and other mesons**

- **BUT is that the whole story?**

- **After all, along came QCD in the 1970s!**

BUT regarded as irrelevant to nuclear structure............

D. Alan Bromley (Yale) to Stan Brodsky in 1982

"Stan, you have to understand -- in nuclear physics we are only interested in how protons and neutrons make up a nucleus.

We are <u>not</u> interested in what is inside of a proton."

D. Alan Bromley (Yale) to Stan Brodsky in 1982

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Fundamental Question for Nuclear Physics

- **Is the nucleon immutable ?**

  FlamingText.com

- **i.e. When immersed in a nuclear medium with *applied scalar field strength of order half its mass* is it really unchanged??**

- **When looked at in the context of QCD as the theory of the strong force clearly NO**

- **Is this irrelevant to nuclear structure? NO**

- **Indeed, we argue it is of fundamental importance.....**

A different approach : QMC Model

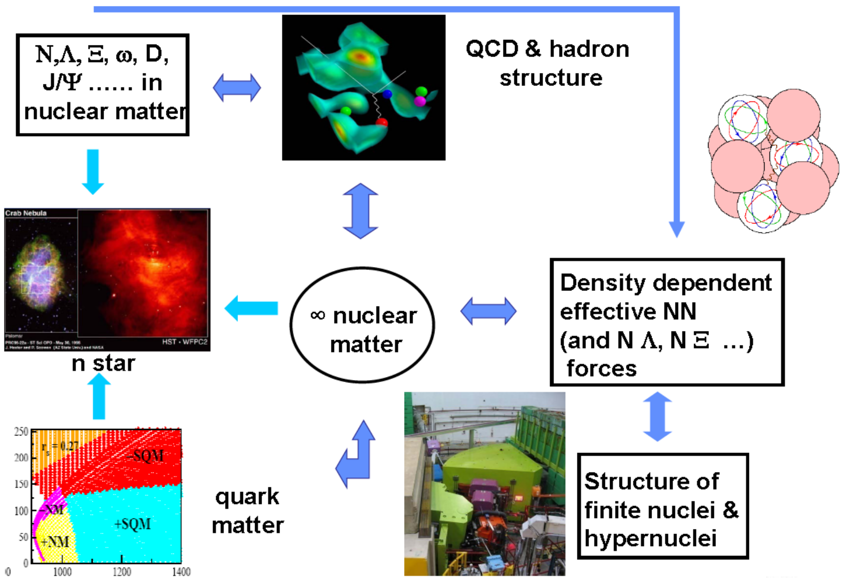
(Guichon, Saito, Tsushima et al., Rodionov et al.
- see Saito et al., Progress Part. Nucl. Phys. 58 (2007) 1 for a review)

- **Start with quark model (MIT bag/NJL…) for all hadrons**

- **Introduce a relativistic Lagrangian with σ, ω and ρ mesons coupling to non-strange quarks**

- **Hence <u>only 3 parameters : </u> $g^{q}_{\sigma,\omega,\rho}$**

  − **determine by fitting to saturation properties of nuclear matter (ρ_0 , E/A and symmetry energy)**



- **Must solve self-consistently for the internal structure of baryons in-medium**

Effect of scalar field on quark spinor

- **MIT bag model: quark spinor modified in bound nucleon**

$$\Psi = \frac{\mathcal{N}}{4\pi} \begin{pmatrix} j_0(xu'/R_B) \\ i\beta_q \vec{\sigma} \cdot \hat{u}' j_1(xu'/R_B) \end{pmatrix} \chi_m$$

- **Lower component enhanced by attractive scalar field**

$$\beta_q = \sqrt{\frac{\Omega_0 - m_q^* R_B}{\Omega_0 + m_q^* R_B}}$$

- **This leads to a *very small* (~1% at ρ_0) *increase in bag radius***

- **It also *suppresses the scalar coupling to the nucleon as the scalar field increases***

$$\frac{\Omega_0/2 + m_q^* R_B(\Omega_0 - 1)}{\Omega_0(\Omega_0 - 1) + m_q^* R_B/2} = \int \bar{\psi}\, \psi \; dV$$

- **This is the "scalar polarizability": a new saturation mechanism for nuclear matter**

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Quark-Meson Coupling Model (QMC):
Role of the Scalar Polarizability of the Nucleon

The response of the nucleon internal structure to the scalar field is of great interest… and importance

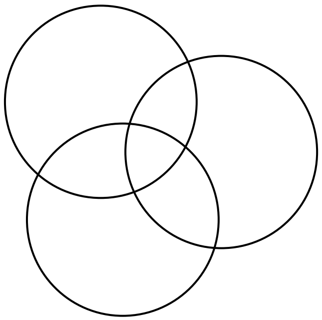
$$M^*(\vec{R}) = M - g_\sigma \sigma(\vec{R}) + \frac{d}{2}\left(g_\sigma \sigma(\vec{R})\right)^2$$

Non-linear dependence through the scalar polarizability
d ~ 0.22 R in original QMC (MIT bag)

Indeed, in nuclear matter at mean-field level (e.g. QMC), this is the ONLY place the response of the internal structure of the nucleon enters.

Summary : Scalar Polarizability

Consequence of polarizability in atomic physics is many-body forces:



$$V = V_{12} + V_{23} + V_{13} + V_{123}$$

- **same is true in nuclear physics:**
- **scalar polarizability is natural source of 3-body force**

Binding of hadrons immersed in matter

Mesons in Nuclei

- At Hartree level mesons like ω, η and η' contain light quark-anti-quark pairs

- Repulsive vector potential cancels for q and $\bar{q}$
  (s and $\bar{s}$ do not couple to σ, ω and ρ)

- Thus mesons must feel attraction associated with the mean scalar field (Saito et al., Phys.Rev. C55 (1997) 2637-2648)

- Initial estimates significantly underestimated absorption of the ω, which adds repulsion

  – but V. Metag finds hint of mild attraction in C : -20 ± 25 ± 10 MeV

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.... more later in this session from Metag

Hyperons

- Derive Λ N, Σ N, Λ Λ … effective forces in-medium with **no** additional free parameters

- Attractive and repulsive forces (σ and ω mean fields) both decrease as # light quarks decreases

- NO Σ hypernuclei are bound!

- Λ bound by about 30 MeV in nuclear matter (~Pb)

- Miniscule spin-orbit force for Λ is natural

- Nothing known about Ξ hypernuclei – JPARC!

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Λ- and Ξ-Hypernuclei in QMC

	$^{89}_{\Lambda}$Yb (Expt.)	$^{91}_{\Lambda}$Zr	$^{91}_{\Xi^0}$Zr	$^{208}_{\Lambda}$Pb (Expt.)	$^{209}_{\Lambda}$Pb	$^{209}_{\Xi^0}$Pb
$1s_{1/2}$	-22.5	-24.0	-9.9	-27.0	-26.9	-15.0
$1p_{3/2}$		-19.4	-7.0		-24.0	-12.6
$1p_{1/2}$	-16.0 ($1p$)	-19.4	-7.2	-22.0 ($1p$)	-24.0	-12.7
$1d_{5/2}$		-13.4	-3.1	—	-20.1	-9.6
$2s_{1/2}$		-9.1	—	—	-17.1	-8.2
$1d_{3/2}$	-9.0 ($1d$)	-13.4	-3.4	-17.0 ($1d$)	-20.1	-9.8
$1f_{7/2}$		-6.5	—	—	-15.4	-6.2
$2p_{3/2}$		-1.7	—	—	-11.4	-4.2
$1f_{5/2}$	-2.0 ($1f$)	-6.4	—	-12.0 ($1f$)	-15.4	-6.5
$2p_{1/2}$		-1.6	—	—	-11.4	-4.3

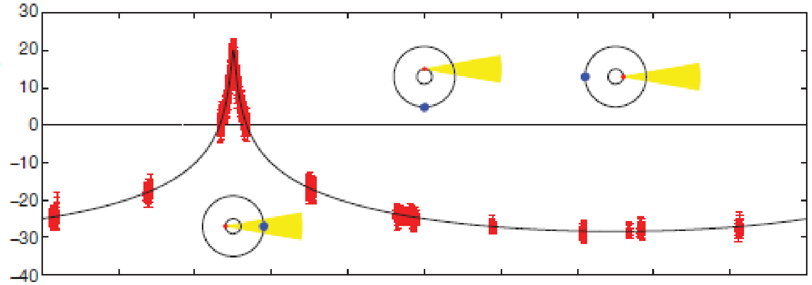
Predicts Ξ – hypernuclei bound by 10-15 MeV to be tested at J-PARC

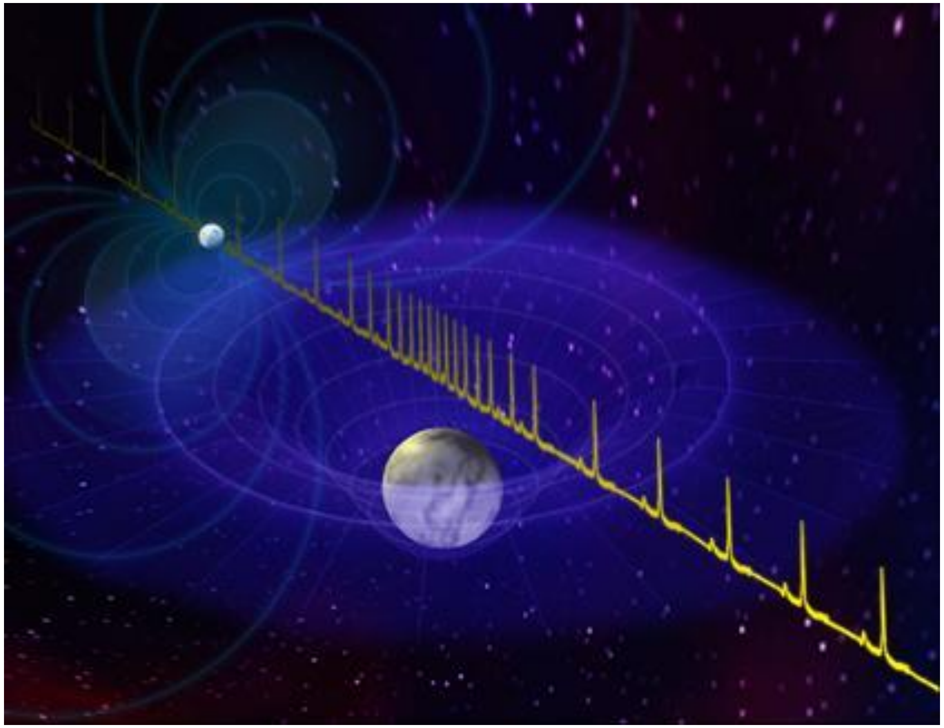
LETTER

doi:10.1038/nature09466

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]





Report a very accurate pulsar mass much larger than seen before : 1.97 ± 0.04 solar mass

Claim it rules out hyperons (particles with strange quarks - ignored our *published* work!)

J1614-2230

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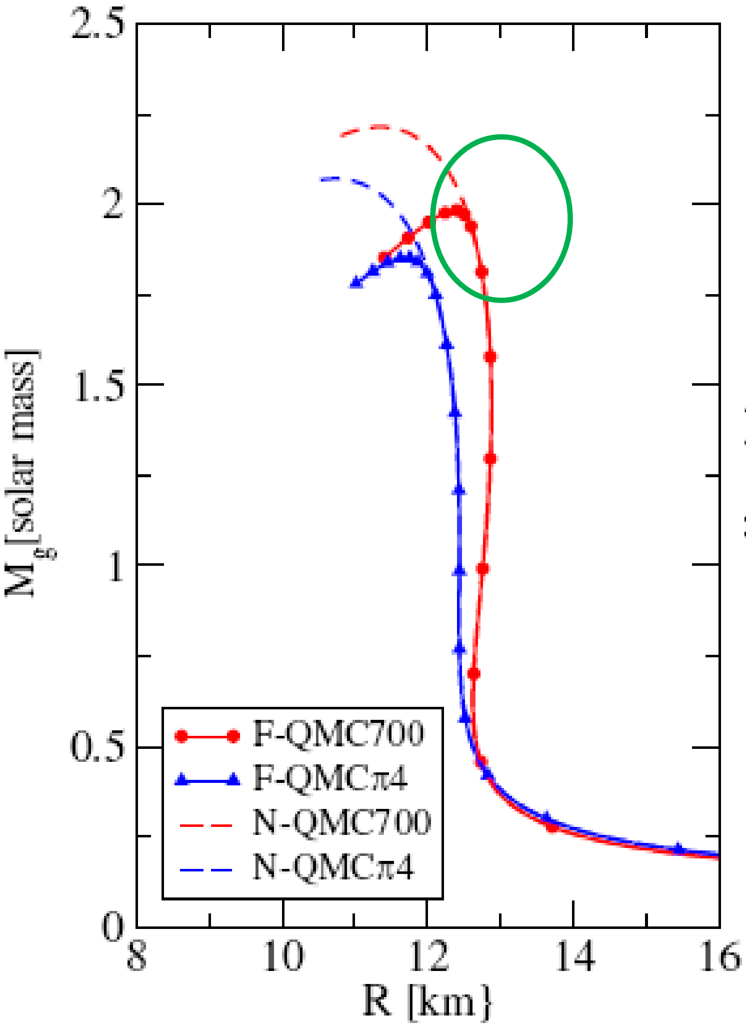
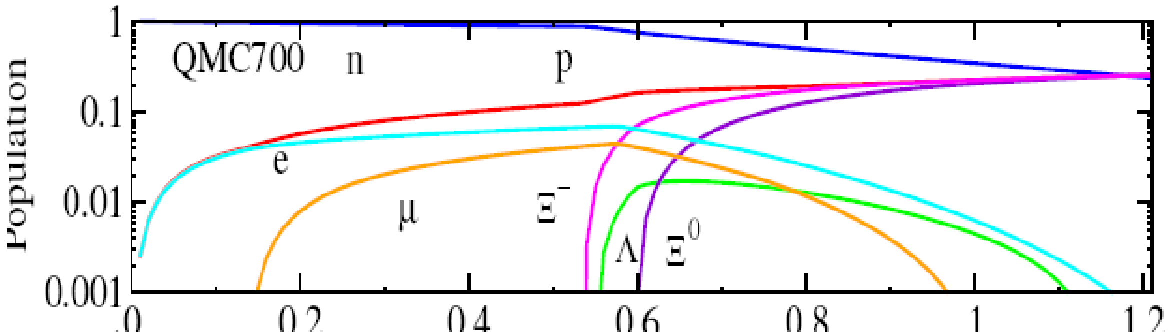
Consequences of QMC for Neutron Star





Rikovska-Stone et al., NP A792 (2007) 341

Finite nuclei

Derivation of Density Dependent Effective Force

Physical origin of density dependent forces of Skyrme type within the quark meson coupling model

P.A.M. Guichon [a,*], H.H. Matevosyan [b,c], N. Sandulescu [a,d,e], A.W. Thomas [b]

Nuclear Physics A 772 (2006) 1–19

- **Start with classical theory of MIT-bag nucleons with structure modified in medium to give M_{eff} (σ).**

- **Quantise nucleon motion (non-relativistic), expand in powers of derivatives**

- **Derive equivalent, local energy functional:**

$$\langle H(\vec{r}\,)\rangle = \rho M + \frac{\tau}{2M} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm so}$$

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Derivation of effective Force (cont.)

$$\mathcal{H}_0 + \mathcal{H}_3 = \rho^2\left[\frac{-3G_\rho}{32} + \boxed{\frac{G_\sigma}{8(1+d\rho G_\sigma)^3}} - \frac{G_\sigma}{2(1+d\rho G_\sigma)} + \frac{3G_\omega}{8}\right]$$
$$+ (\rho_n - \rho_p)^2\left[\frac{5G_\rho}{32} + \boxed{\frac{G_\sigma}{8(1+d\rho G_\sigma)^3}} - \frac{G_\omega}{8}\right],$$

$$\mathcal{H}_{\rm eff} = \left[\left(\frac{G_\rho}{8m_\rho{}^2} - \frac{G_\sigma}{2m_\sigma{}^2} + \frac{G_\omega}{2m_\omega{}^2} + \frac{G_\sigma}{4M_N{}^2}\right)\rho_n + \left(\frac{G_\rho}{4m_\rho^2} + \frac{G_\sigma}{2M_N^2}\right)\rho_p\right]\tau_n$$
$$+ p \leftrightarrow n,$$

$$\mathcal{H}_{\rm fin} = \left[\left(\frac{3G_\rho}{32m_\rho{}^2} - \frac{3G_\sigma}{8m_\sigma{}^2} + \frac{3G_\omega}{8m_\omega{}^2} - \frac{G_\sigma}{8M_N{}^2}\right)\rho_n\right.$$
$$\left.+ \left(\frac{-3G_\rho}{16m_\rho{}^2} - \frac{G_\sigma}{2m_\sigma{}^2} + \frac{G_\omega}{2m_\omega{}^2} - \frac{G_\sigma}{4M_N{}^2}\right)\rho_p\right]\nabla^2(\rho_n) + p \leftrightarrow n,$$

$$\mathcal{H}_{\rm so} = \nabla\cdot J_n\left[\left(\frac{-3G_\sigma}{8M_N{}^2} - \frac{3G_\omega(-1+2\mu_s)}{8M_N{}^2} - \frac{3G_\rho(-1+2\mu_v)}{32M_N{}^2}\right)\rho_n\right.$$
$$\left.+ \left(\frac{-G_\sigma}{4M_N{}^2} + \frac{G_\omega(1-2\mu_s)}{4M_N{}^2}\right)\rho_p\right] + p \leftrightarrow n.$$

Spin-orbit force predicted!

Note the totally new, subtle density dependence

Systematic Study of Finite Nuclei

Systematic approach to finite nuclei

J.R. Stone, P.A.M. Guichon, P. G. Reinhard & A.W. Thomas:

(Phys Rev Lett, 116 (2016) 092501)

- **Constrain 3 basic quark-meson couplings ($g_\sigma{}^q$, $g_\omega{}^q$, $g_\rho{}^q$) so that nuclear matter properties are reproduced within errors**

$$-17 < E/A < -15 \text{ MeV}$$
$$0.14 < \rho_0 < 0.18 \text{ fm}^{-3}$$
$$28 < S_0 < 34 \text{ MeV}$$
$$L > 20 \text{ MeV}$$
$$250 < K_0 < 350 \text{ MeV}$$

- **Fix at overall best description of finite nuclei** (+2 pairing pars)

- **Benchmark comparison: SV-min 16 parameters (11+5)**

Overview of 106 Nuclei Studied – Across Periodic Table

Element	Z	N	Element	Z	N
C	6	6 -16	Pb	82	116 - 132
O	8	4 -20	Pu	94	134 - 154
Ca	20	16 – 32	Fm	100	148 - 156
Ni	28	24 – 50	No	102	152 - 154
Sr	38	36 – 64	Rf	104	152 - 154
Zr	40	44 -64	Sg	106	154 - 156
Sn	50	50 – 86	Hs	108	156 - 158
Sm	62	74 – 98	Ds	110	160
Gd	64	74 -100			

N	Z	N	Z
20	10 – 24	64	36 - 58
28	12 – 32	82	46 - 72
40	22 – 40	126	76 - 92
50	28 – 50		

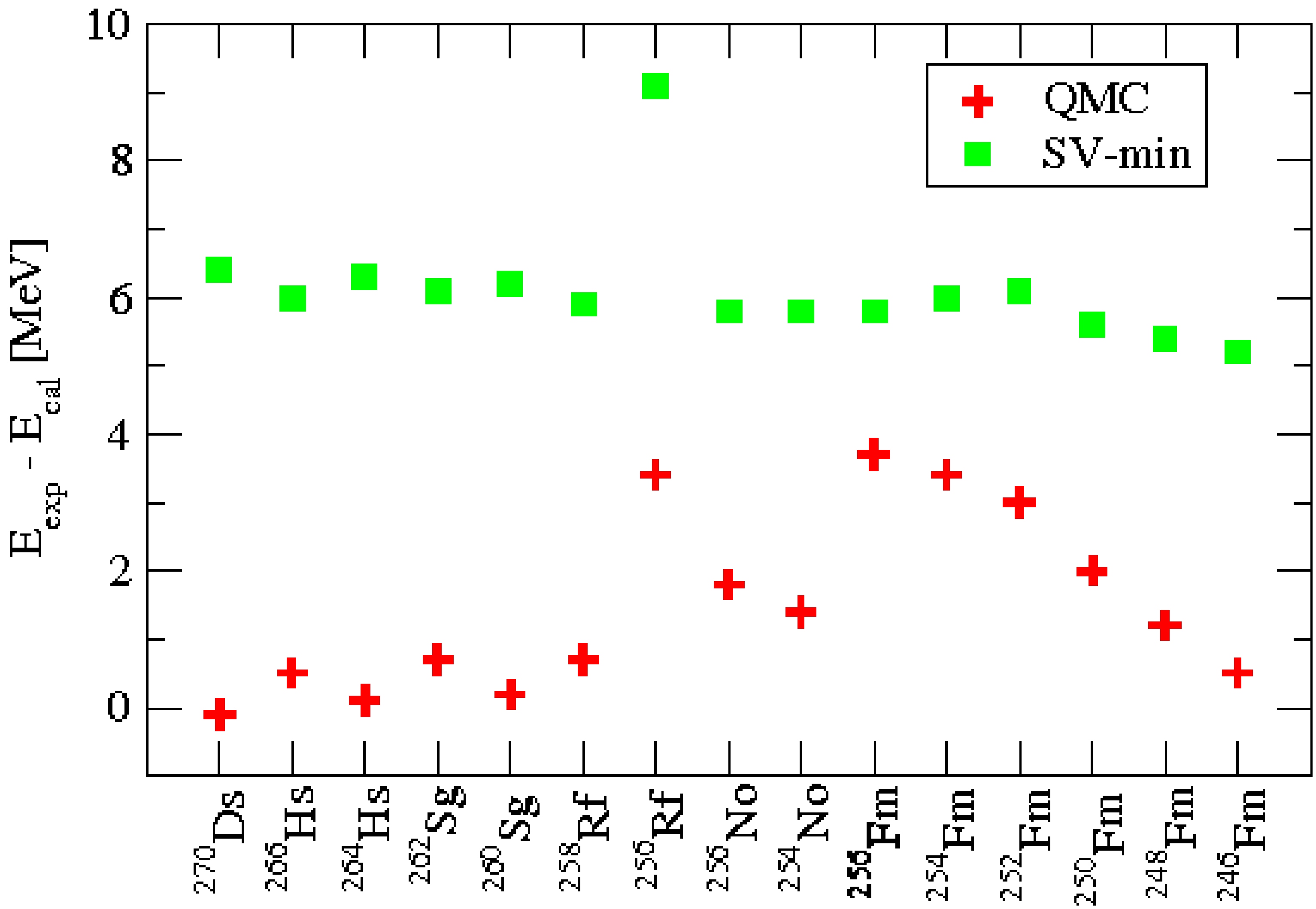
i.e. We look at most challenging cases of p- or n-rich nuclei

Overview

data	rms error %	
	QMC	SV-min
fit nuclei:		
binding energies	0.36	0.24
diffraction radii	1.62	0.91
surface thickness	10.9	2.9
rms radii	0.71	0.52
pairing gap (n)	57.6	17.6
pairing gap (p)	25.3	15.5
ls splitting: proton	15.8	18.5
ls splitting: neutron	20.3	16.3
superheavy nuclei:	0.1	0.3
N=Z nuclei	1.17	0.75
mirror nuclei	1.50	1.00
other	0.35	0.26

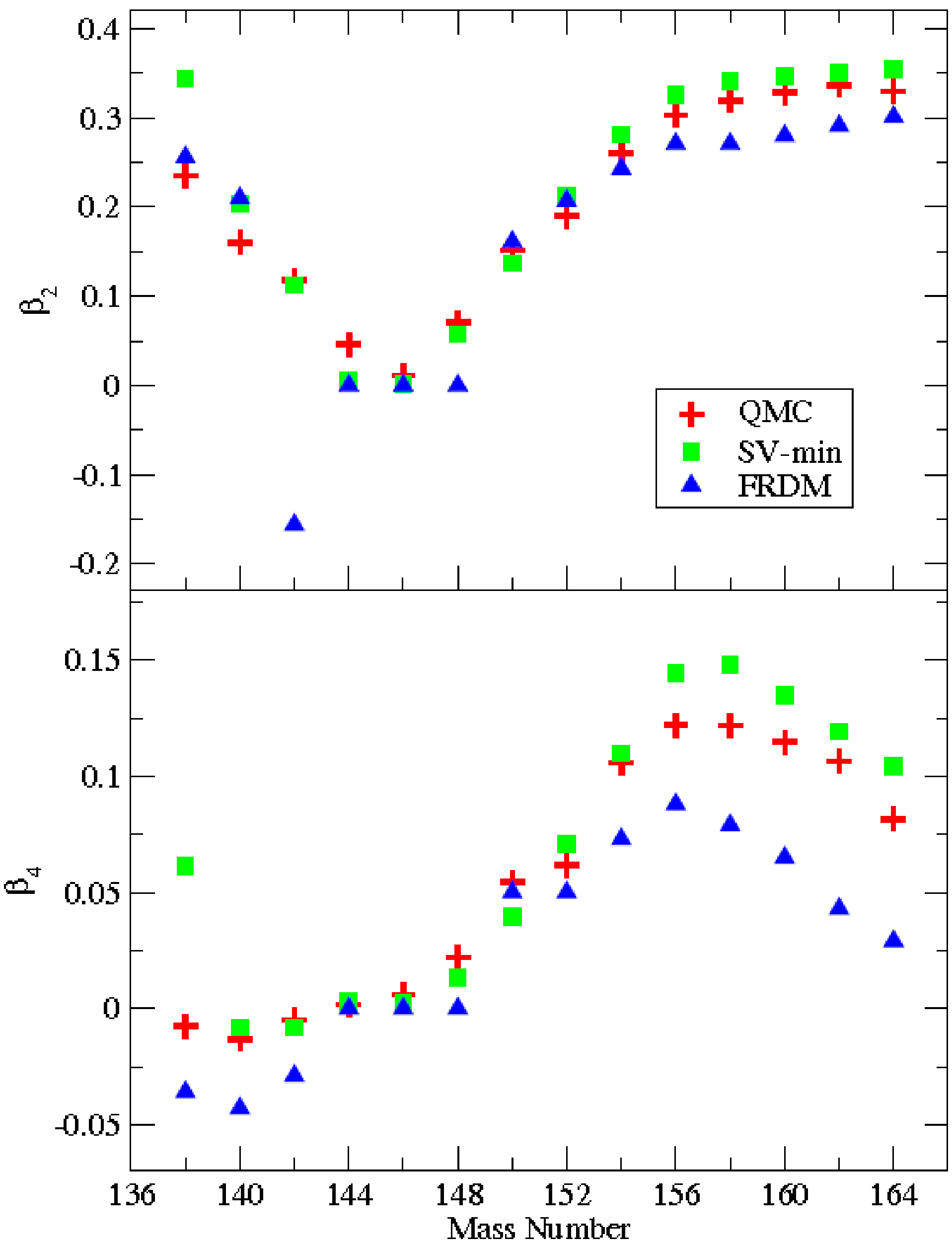
Stone et al., PRL (2016)

Superheavies : 0.1% accuracy



Stone et al., PRL (2016)

Deformation in Gd (Z=64) Isotopes



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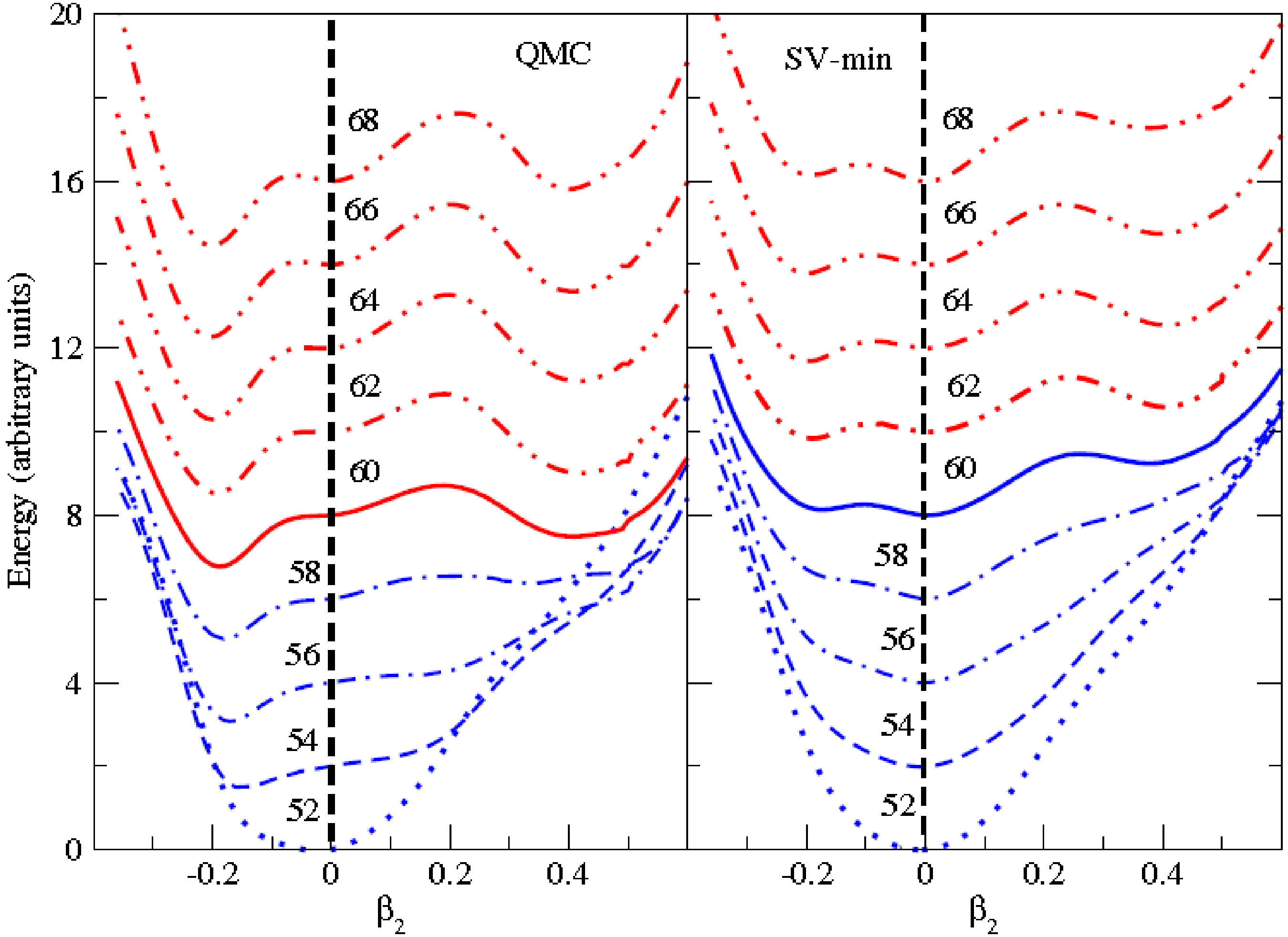
Spin-orbit splitting

Element		States	Exp [keV]	QMC [keV]	SV-bas [keV]
O16	proton	$1p_{1/2}$ - $1p_{3/2}$	6.3 (1.3)a)	5.8	5.0
	neutron	$1p_{1/2}$ - $1p_{3/2}$	6.1 (1.2)a)	5.7	5.1
Ca40	proton	$1d_{3/2}$ - $1d_{5/2}$	7.2 [b)]	6.3	5.7
	neutron	$1d_{3/2}$ - $1d_{5/2}$	6.3 [b)]	6.3	5.8
Ca48	proton	$1d_{3/2}$ - $1d_{5/2}$	4.3 [b)]	6.3	5.2
	neutron	$1d_{3/2}$ - $1d_{5/2}$		5.3	5.2
Sn132	proton	$2p_{1/2}$ - $2p_{3/2}$	1.35(27)[a)]	1.32	1.22
	neutron	$2p_{1/2}$ - $2p_{3/2}$	1.65(13)[a)]	1.47	1.63
	neutron	$2d_{3/2}$ - $2d_{5/2}$		2.71	2.11
Pb208	proton	$2p_{1/2}$ - $2p_{3/2}$		0.91	0.93
	neutron	$3p_{1/2}$ - $3p_{3/2}$	0.90(18)[a)]	1.11	0.89

Shape evolution of Zr (Z=40) Isotopes



- **Shape co-existence sets in at N=60 – Sotty *et al*.,PRL115 (2015)172501**
- **Usually difficult to describe**
  - **– e.g. Mei et al., PRC85, 034321 (2012)**

Stone et al., PRL (2016)

Summary: Finite Nuclei

- **The effective force was *derived* at the quark level *based upon changing structure of bound nucleon***

- **Has many less parameters but reproduces nuclear properties at a level comparable with the best phenomenological Skyrme forces**

- **Looks like standard nuclear force**

- **BUT underlying theory also predicts modified internal structure and hence modified**
  - **DIS structure functions**
  - **elastic form factors......**

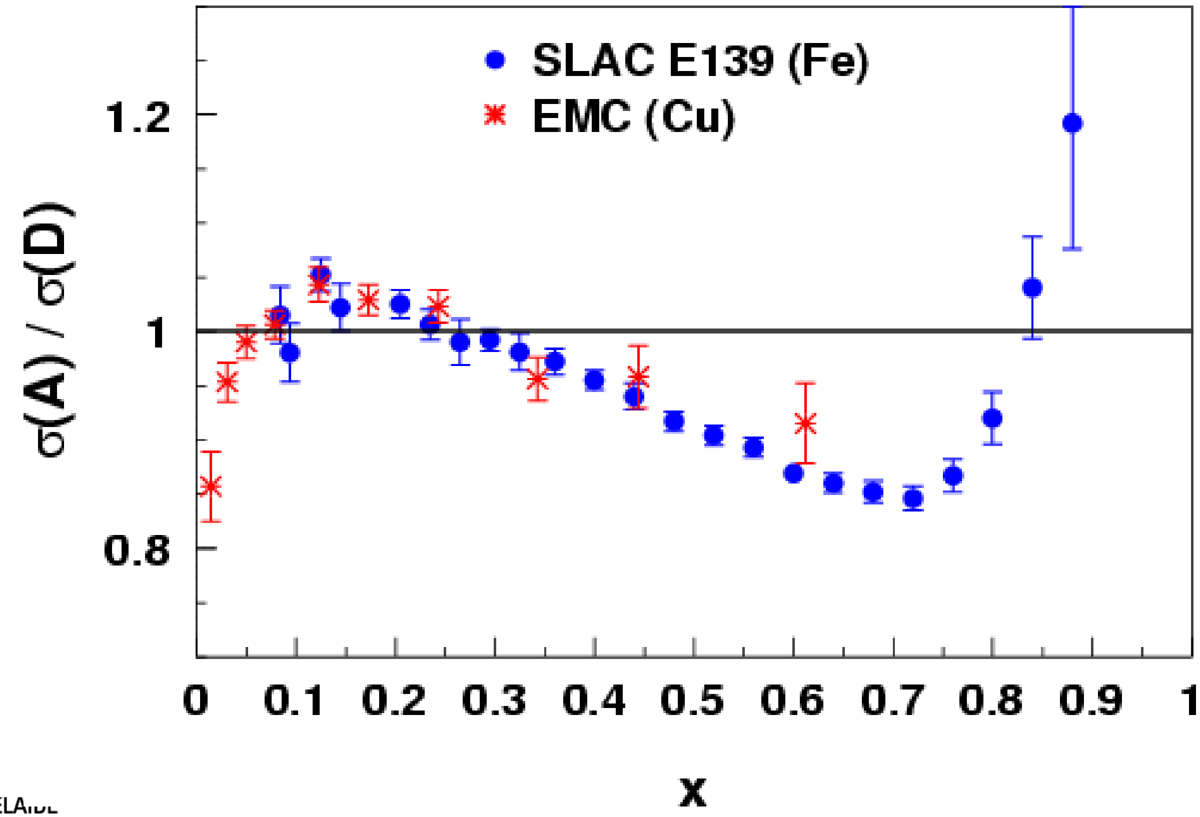
Nuclear DIS Structure Functions

To address questions like this one MUST start with a theory that quantitatively describes nuclear structure – very, very few examples.....

The EMC Effect: Nuclear PDFs

- Observation **stunned and electrified** the
  HEP and Nuclear communities 30 years ago

- What is it that alters the quark momentum in the nucleus?



J. Ashman *et al*., Z. Phys. C57, 211 (1993)

J. Gomez *et al*., Phys. Rev. D49, 4348 (1994)

Theoretical Understanding

- **Still numerous proposals but few consistent theories**
- **Initial studies used MIT bag[1] to estimate effect of self-consistent change of structure in-medium − but better to use a covariant theory**
- **For that Bentz and Thomas[2] re-derived change of nucleon structure in-medium in the NJL model**
- **This set the framework for sophisticated studies by Cloët and collaborators over the last decade**

[1] Thomas, Michels, Schreiber and Guichon, Phys. Lett. B233 (1989) 43
[2] Bentz and Thomas, Nucl. Phys. A696 (2001) 138

Calculations for Finite Nuclei

(Spin dependent EMC effect TWICE as large as unpolarized)

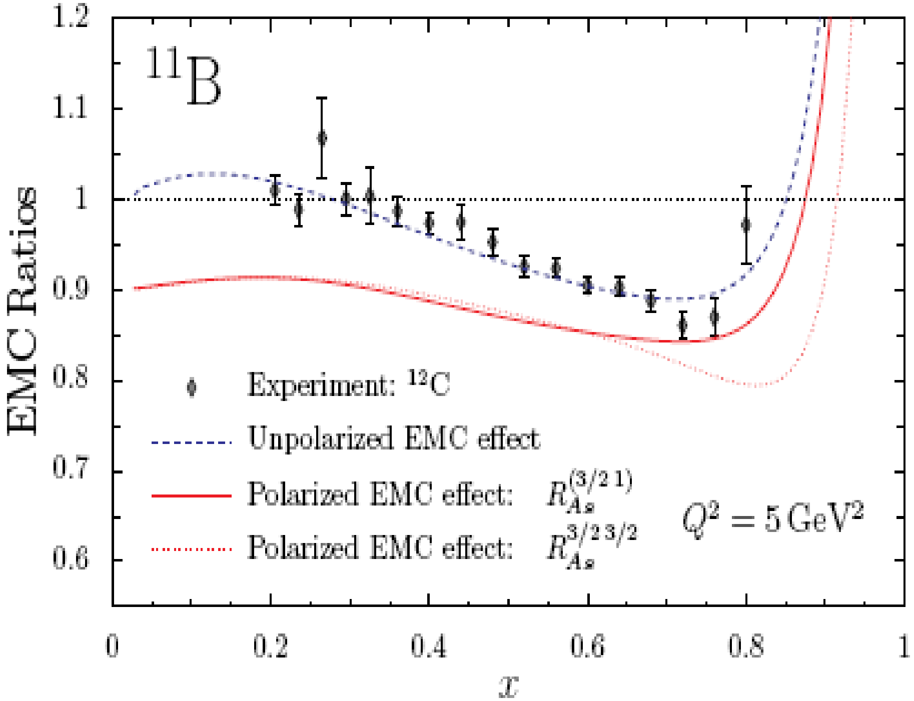


FIG. 7: The EMC and polarized EMC effect in ^{11}B. The empirical data is from Ref. [31].

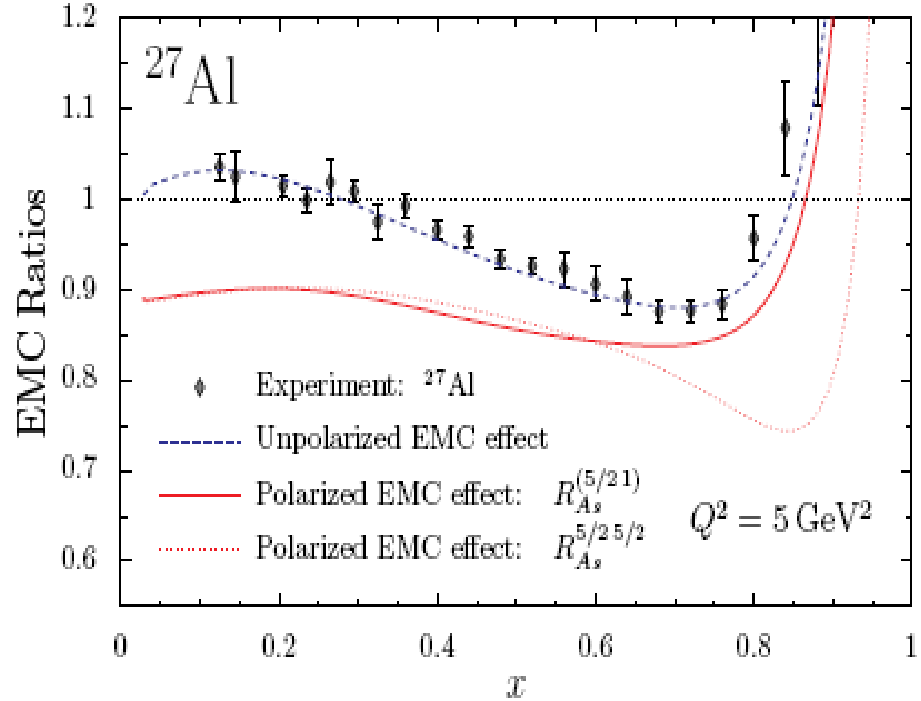


FIG. 9: The EMC and polarized EMC effect in ^{27}Al. The empirical data is from Ref. [31].

Cloët, Bentz &Thomas, Phys. Lett. B642 (2006) 210 (nucl-th/0605061)

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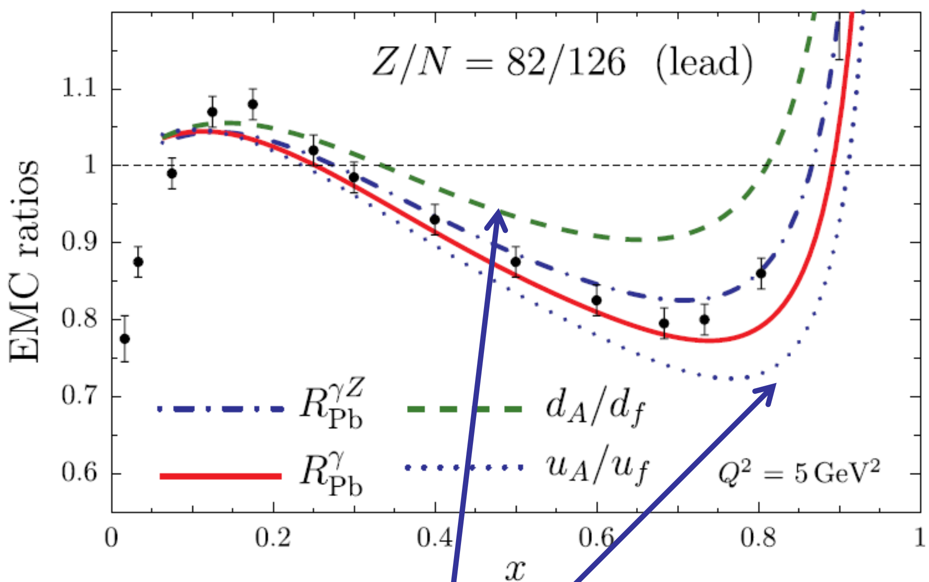
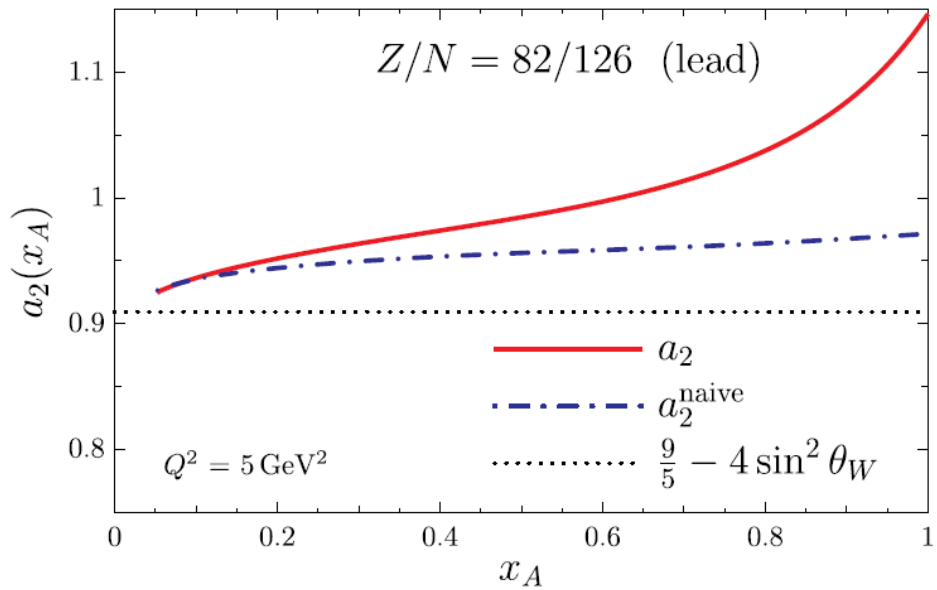
PRL **109,** 182301 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

Parity-Violating Deep Inelastic Scattering and the Flavor Dependence of the EMC Effect

I. C. Cloët,[1] W. Bentz,[2] and A. W. Thomas[1]



$$A_{PV} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha_{em}}\left[a_2(x_A) + \frac{1-(1-y)^2}{1+(1-y)^2}a_3(x_A)\right]$$



Ideally tested at EIC with CC reactions

Parity violating EMC will test this at JLab 12 GeV

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Modified Electromagnetic Form Factors In-Medium

Recent Calculations Motivated by:

E01-015, PR-04-015 – Chen, Choi & Meziani

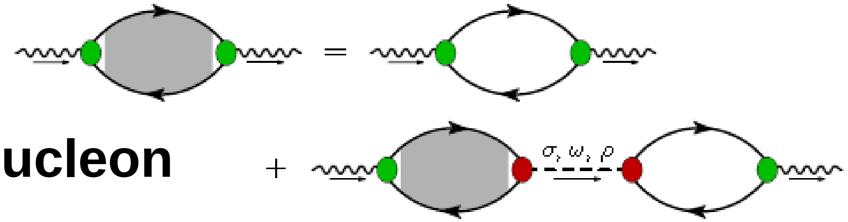
- **Using NJL model with nucleon structure self-consistently solved in-medium**

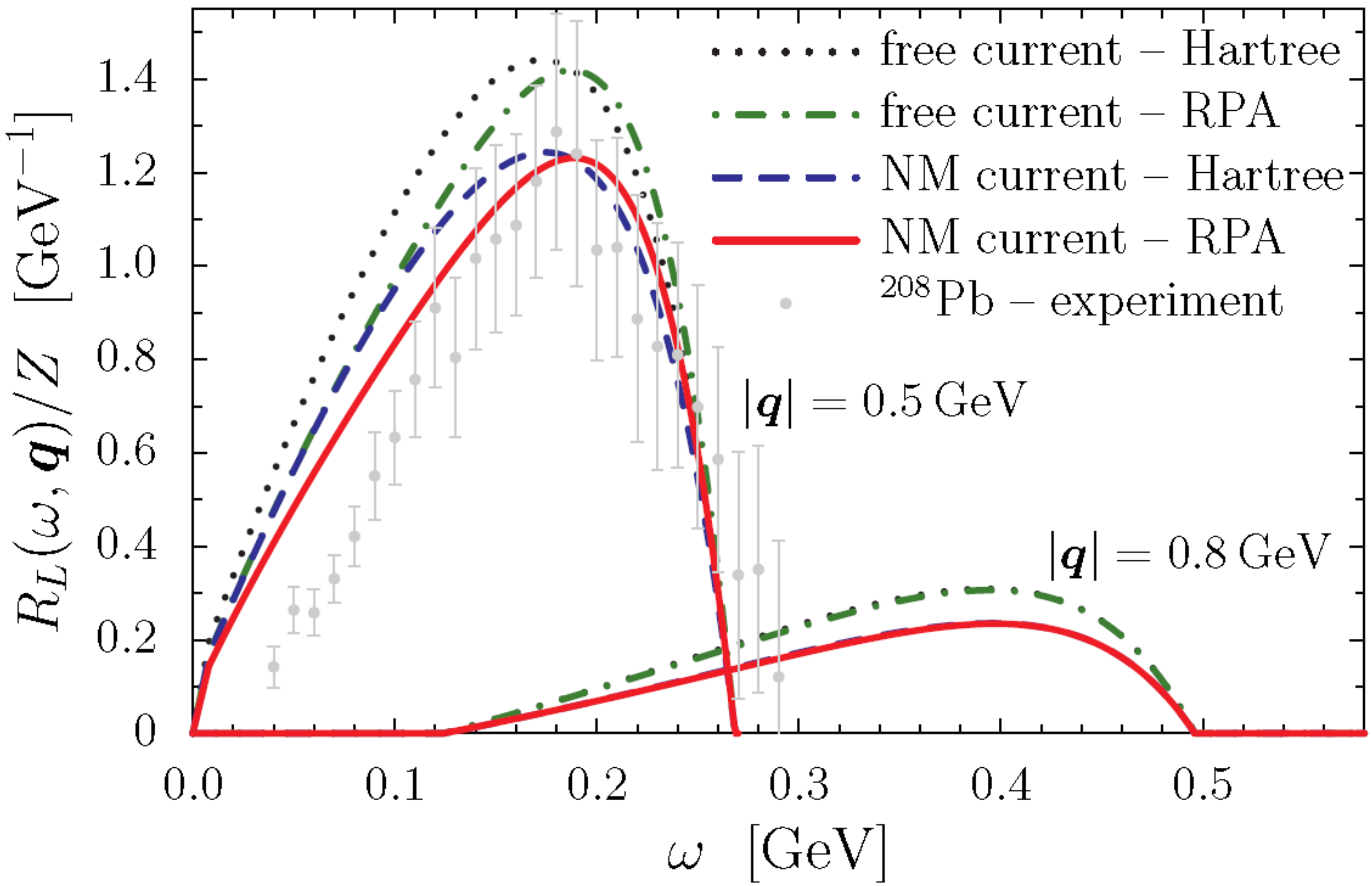
- **Same model describing free nucleon form factors, structure functions and EMC effect**

Response Function

$$\frac{d^2\sigma}{d\Omega\, d\omega} = \sigma_{\text{Mott}}\left[\frac{q^4}{|\boldsymbol{q}|^4} R_L(\omega,|\boldsymbol{q}|) + \left(\frac{q^2}{2|\boldsymbol{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(\omega,|\boldsymbol{q}|)\right]$$

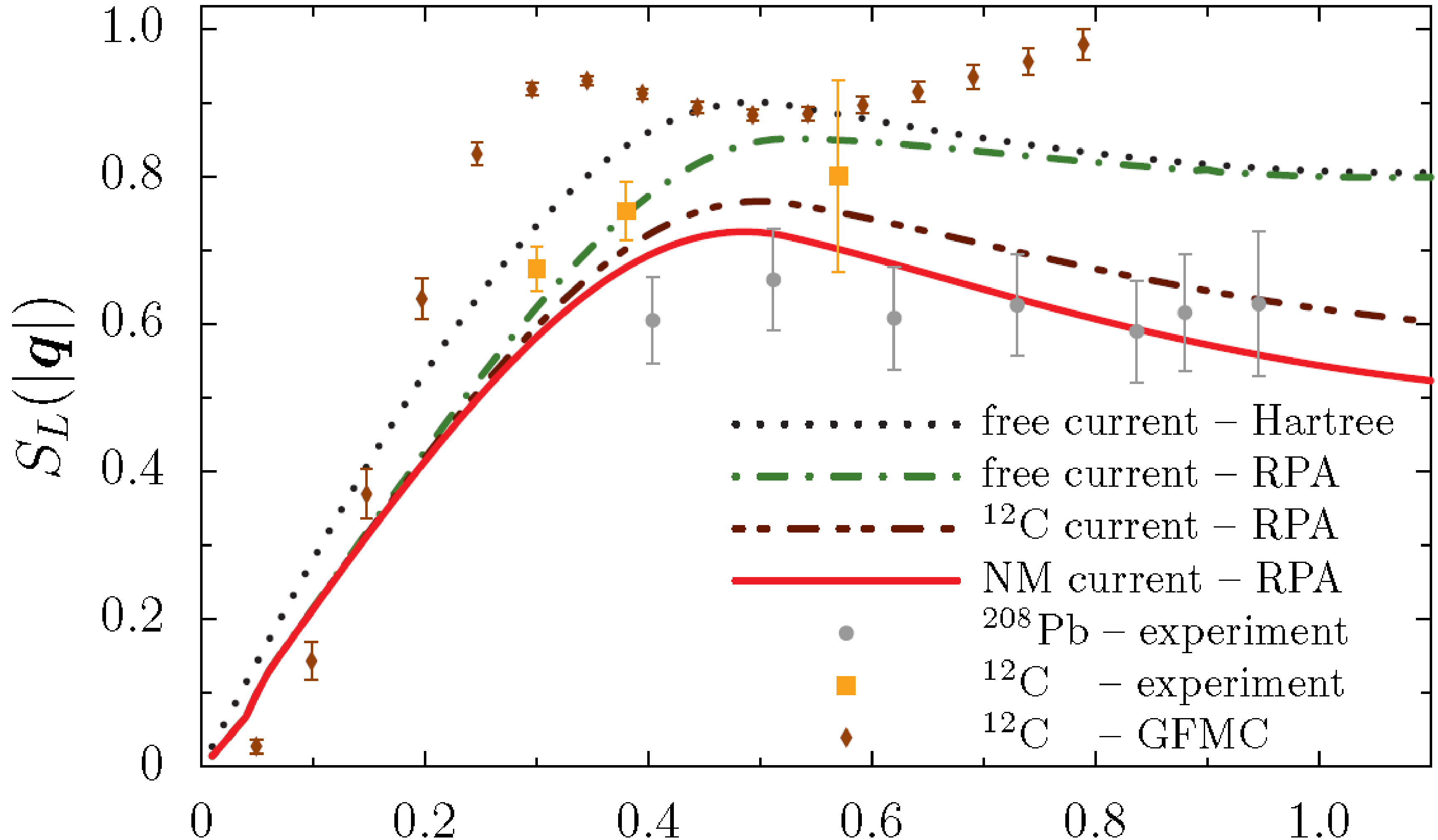
RPA correlations repulsive
Significant reduction in Response Function from modification of bound-nucleon





Cloët, Bentz & Thomas (PRL 116 (2016) 032701)

Comparison with Unmodified Nucleon & Data

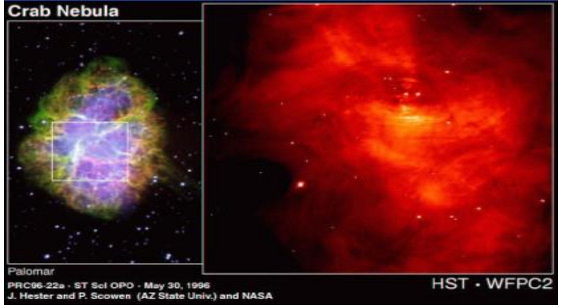


$$S_L(|\boldsymbol{q}|) = \int_{\omega+}^{|\boldsymbol{q}|} d\omega \, \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\, G_{Ep}^2(Q^2) + N\, G_{En}^2(Q^2)}$$

Data: Morgenstern & Meziani

Calculations: Cloët, Bentz & Thomas (PRL 116 (2016) 032701)

Summary



- Intermediate range NN attraction is STRONG Lorentz scalar

- This modifies the intrinsic structure of the bound nucleon
  − profound change in shell model :
  what occupies shell model states are NOT free nucleons

- Scalar polarizability is a natural source of three-body force/ density dependence of effective forces
  − clear physical interpretation

- Derived, density-dependent effective force gives results better than most phenomenological Skyrme forces

Summary

- Initial systematic study of finite nuclei very promising
  - Binding energies typically within 0.3% across periodic table

- Super-heavies (Z > 100) especially good
  (average difference 0.1%)

- Deformation, spin-orbit splitting and charge distributions all look good)

- BUT need empirical confirmation:
  - Response Functions & Coulomb sum rule (soon)
  - Isovector EMC effect; spin EMC
  - Your idea here......................

Special Mentions……

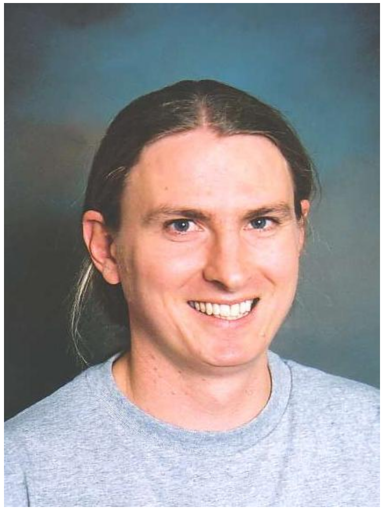
Guichon

Tsushima

Saito

Stone

Bentz



Cloët

Whittenbury

Key papers on QMC

- **Two major, recent papers:**
  1. Guichon, Matevosyan, Sandulescu, Thomas, Nucl. Phys. A772 (2006) 1.
  2. Guichon and Thomas, Phys. Rev. Lett. 93 (2004) 132502

- **Built on earlier work on QMC: e.g.**
  3. Guichon, Phys. Lett. B200 (1988) 235
  4. Guichon, Saito, Rodionov, Thomas, Nucl. Phys. A601 (1996) 349

- **Major review of applications of QMC to many nuclear systems:**
  5. Saito, Tsushima, Thomas, Prog. Part. Nucl. Phys. 58 (2007) 1-167 (hep-ph/0506314)

References to: Covariant Version of QMC

- Basic Model: (Covariant, chiral, confining version of NJL)

•Bentz & Thomas, Nucl. Phys. A696 (2001) 138

- Bentz, Horikawa, Ishii, Thomas, Nucl. Phys. A720 (2003) 95

- Applications to DIS:

- Cloet, Bentz, Thomas, Phys. Rev. Lett. 95 (2005) 052302

- Cloet, Bentz, Thomas, Phys. Lett. B642 (2006) 210

- Applications to neutron stars – including SQM:

- Lawley, Bentz, Thomas, Phys. Lett. B632 (2006) 495

- Lawley, Bentz, Thomas, J. Phys. G32 (2006) 667

Most recent studies

- Whittenbury, Carrillo-Serrano & Thomas, arXiv: 1606.03158
- Whittenbury, Matevosyan & Thomas, Phys. Rev. C93 (2016) 035807
- Whittenbury, Carroll, Thomas, Tsushima and Stone, Phys. Rev. C89 (2014) 065801

Can we Measure Scalar Polarizability in Lattice QCD ?

- IF we can, then in a real sense we would be linking nuclear structure to QCD itself, because scalar polarizability is sufficient in simplest, relativistic mean field theory to produce saturation

- Initial ideas on this published :
  the trick is to apply a <u>chiral invariant</u> scalar field
  − do indeed find polarizability opposing applied σ field

18th Nishinomiya Symposium: nucl-th/0411014
− published in Prog. Theor. Phys.

CoEPP ARC Centre of Excellence for Particle Physics at the Terascale
ADELAIDE UNIVERSITY AUSTRALIA
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Explicit Demonstration of Origin of 3-Body Force

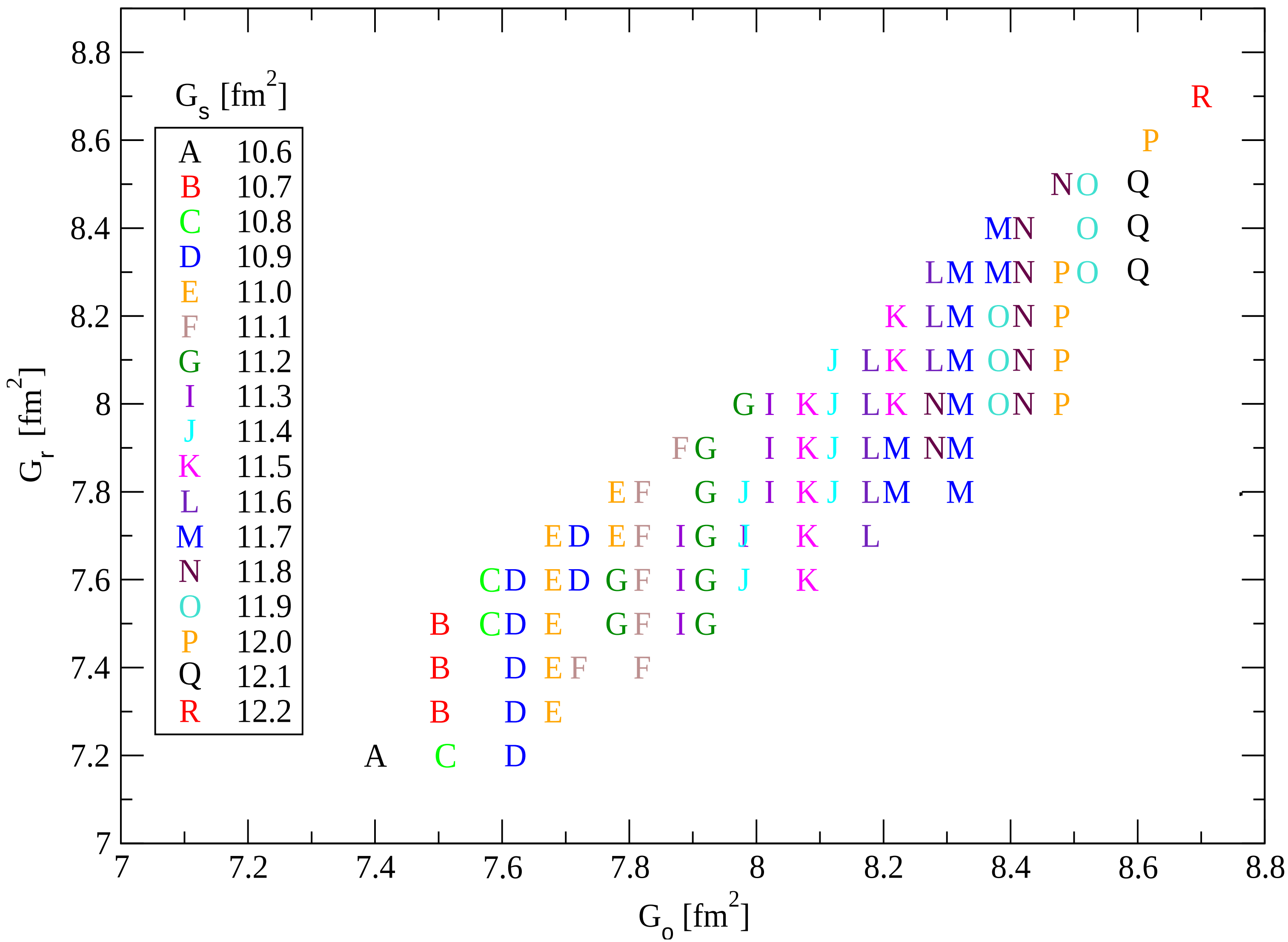
Since early 70's tremendous amount of work in nuclear theory is based upon effective forces

- **Used for everything from nuclear astrophysics to collective excitations of nuclei**
- **Skyrme Force: Vautherin and Brink**

$$
\begin{aligned}
H_{QMC} &= \sum_i \frac{\overleftarrow{\nabla}_i \cdot \overrightarrow{\nabla}_i}{2M} + \frac{G_\sigma}{2M^2} \sum_{i\neq j} \overleftarrow{\nabla}_i \delta(\vec{R}_{ij}) \cdot \overrightarrow{\nabla}_i \\
&+ \frac{1}{2} \sum_{i\neq j} \left[\nabla_i^2 \delta(\vec{R}_{ij})\right] \left[\frac{G_\omega}{m_\omega^2} - \frac{G_\sigma}{m_\sigma^2} + \frac{G_\rho}{m_\rho^2} \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4}\right] \\
&+ \frac{1}{2} \sum_{i\neq j} \delta(\vec{R}_{ij}) \left[G_\omega - G_\sigma + G_\rho \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4}\right] \\
&+ \frac{dG_\sigma^2}{2} \sum_{i\neq j\neq k} \delta^2(ijk) - \frac{d^2 G_\sigma^3}{2} \sum_{i\neq j\neq k\neq l} \delta^3(ijkl) \\
&+ \frac{i}{4M^2} \sum_{i\neq j} A_{ij} \overleftarrow{\nabla}_i \delta(\vec{R}_{ij}) \times \overrightarrow{\nabla}_i \cdot \vec{\sigma}_i \, ,
\end{aligned}
$$

Guichon and Thomas, Phys. Rev. Lett. 93, 132502 (2004)

Constraints from nuclear matter



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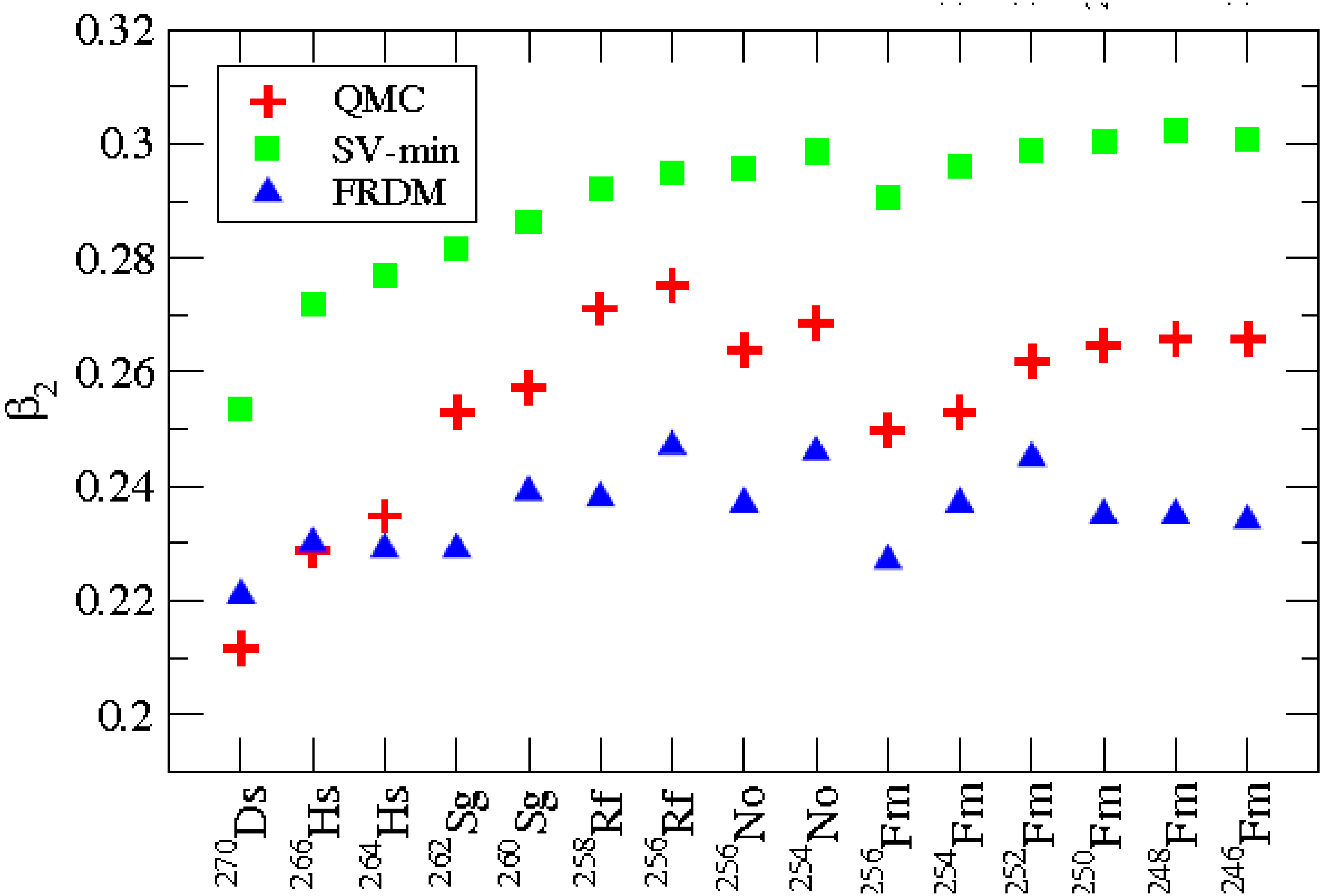
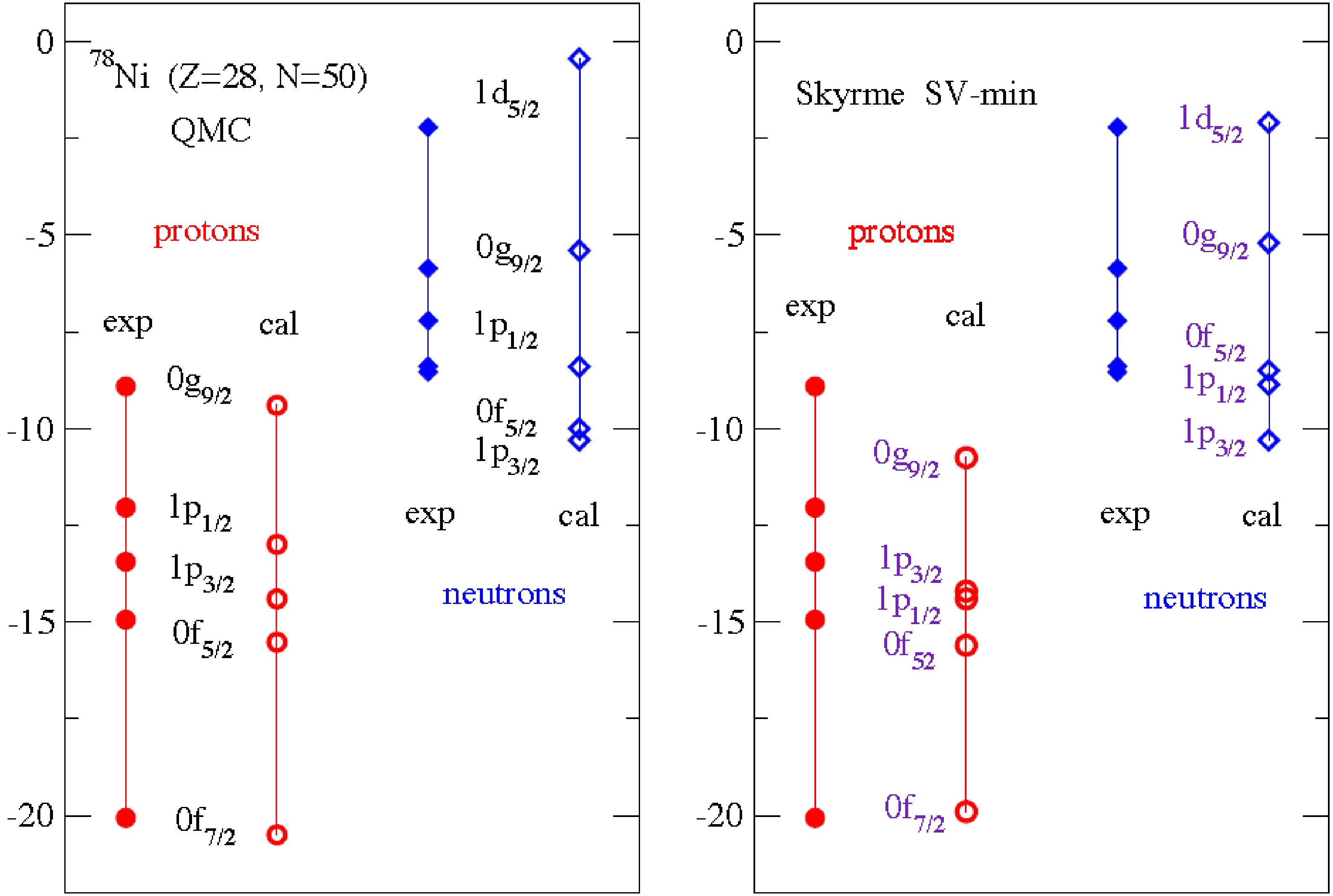
Quadrupole Deformation of Superheavies



Stone et al., PRL (2016)

"Hot off the press"



Traditionally very hard to describe