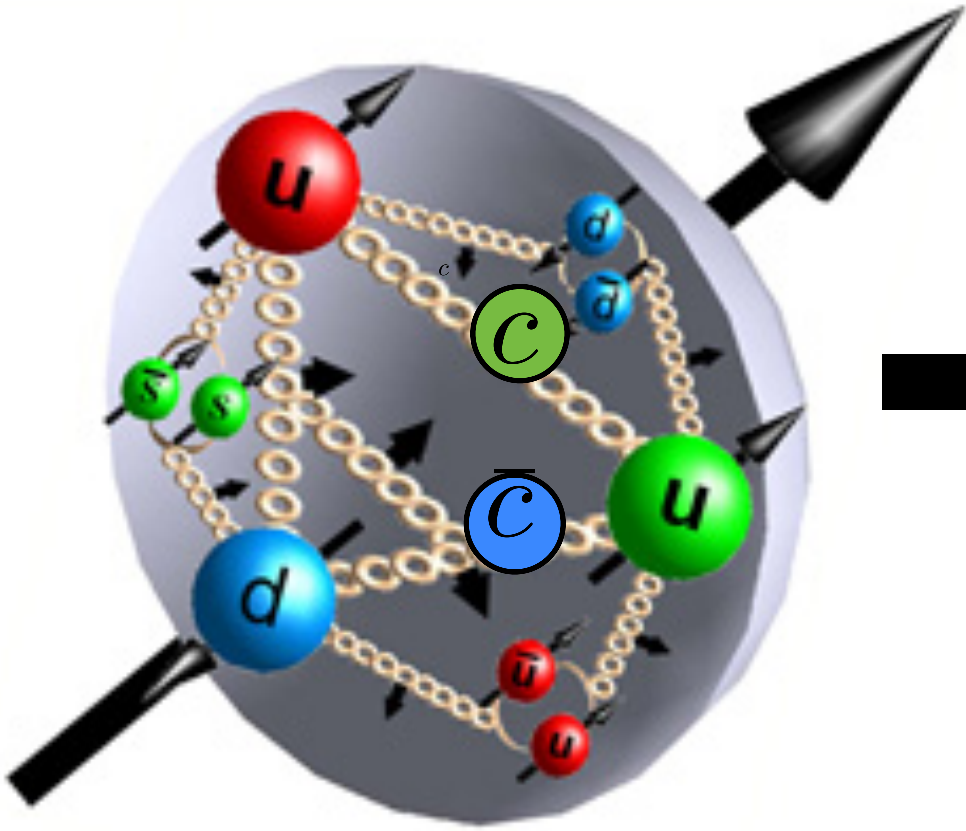
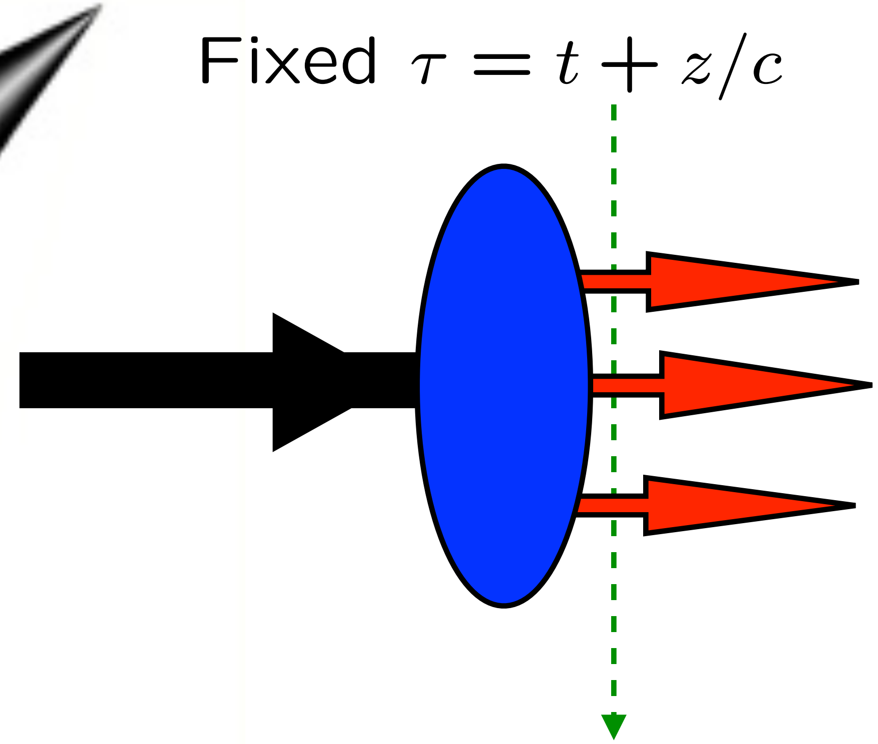
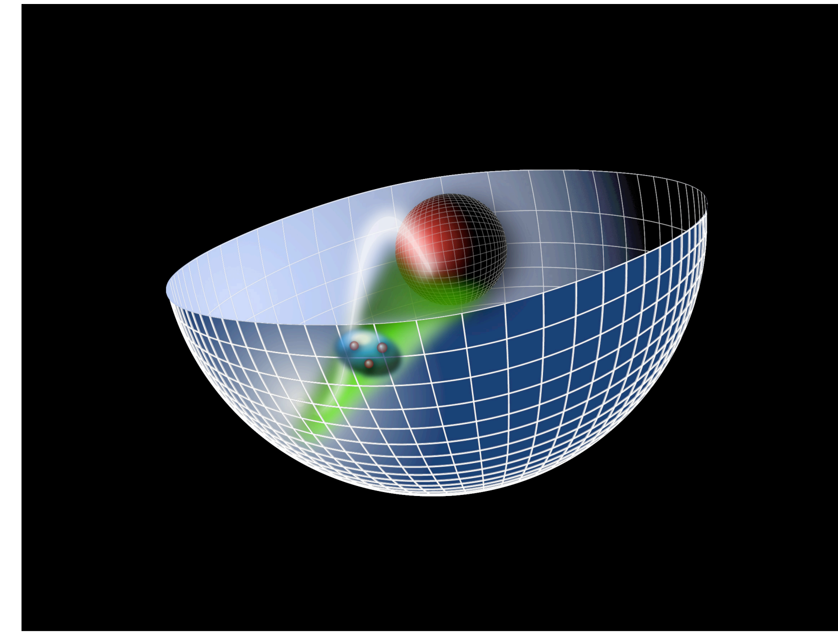
The QCD Vacuum, Color Confinement, and Superconformal Properties of Hadron Physics







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Stanford University

Humboldt Kolleg on Particle Physics
From the Vacuum to the Universe
Kitzbühel Austria
June 29, 2016

with Guy de Tèramond, Hans Günter Dosch, Cedric Lorcè, Alexandre Deur

Craig Roberts, Robert Shrock, Peter Tandy, Prem Srivistava

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f} \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale if $m_q=0$

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

$$H_{LF} = P^+P^- - \vec{P}^2_\perp$$

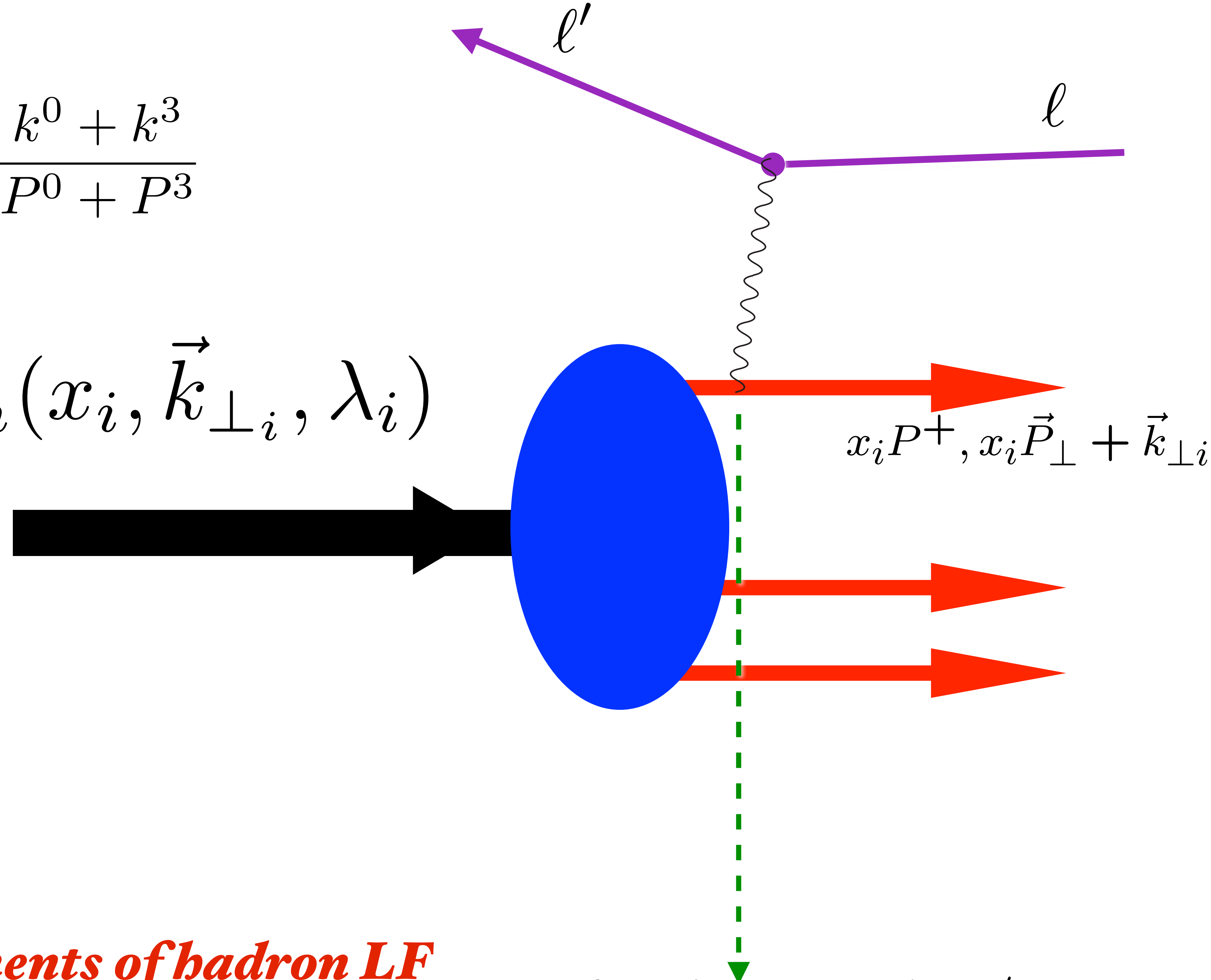
$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$P^+, \vec{P}_\perp$



Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

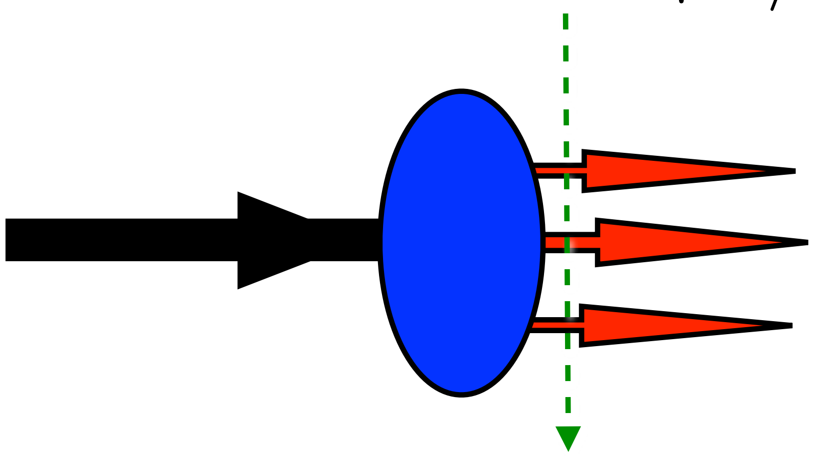
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

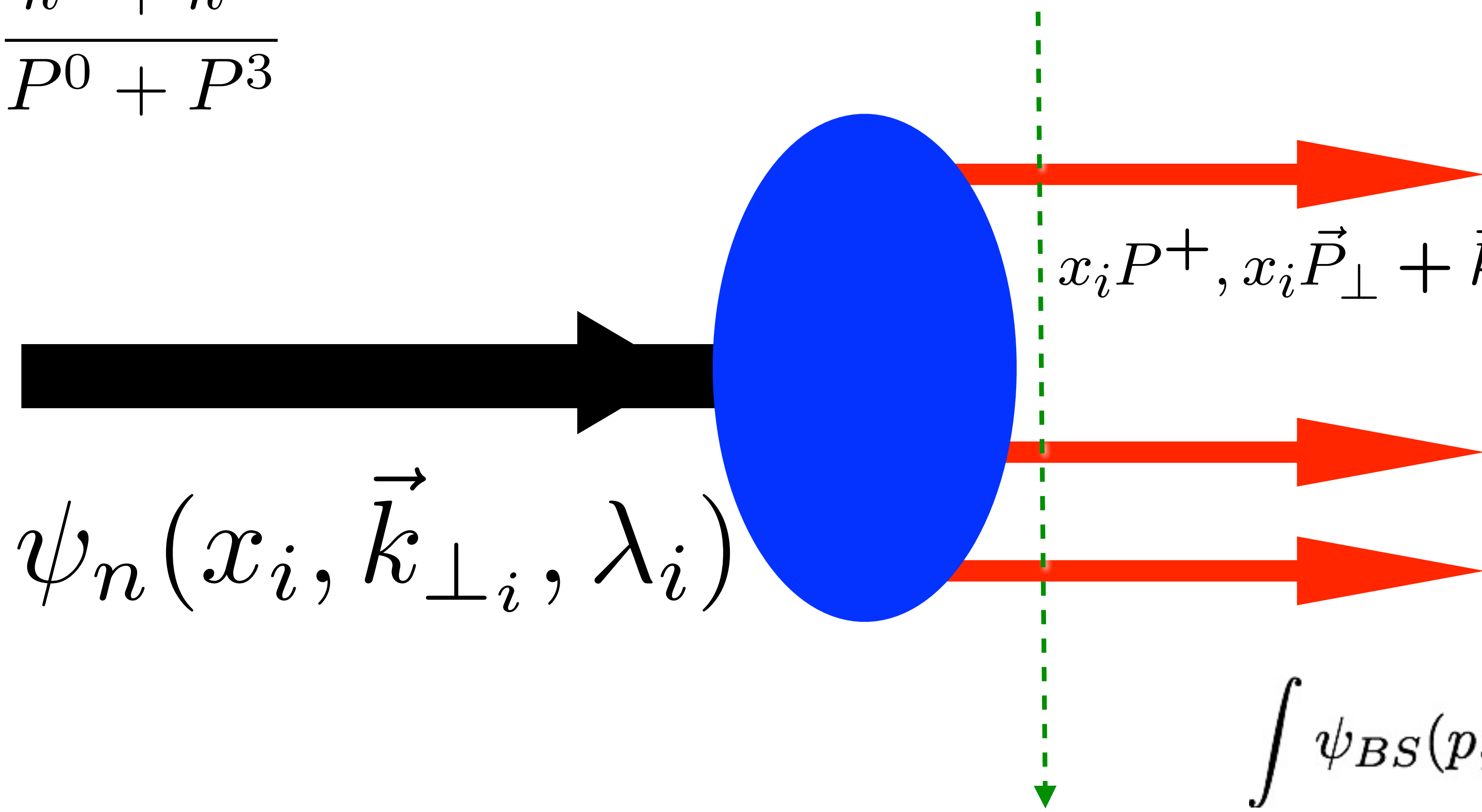
Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$\int \psi_{BS}(p,k) dk^- \to \psi_{LF}$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

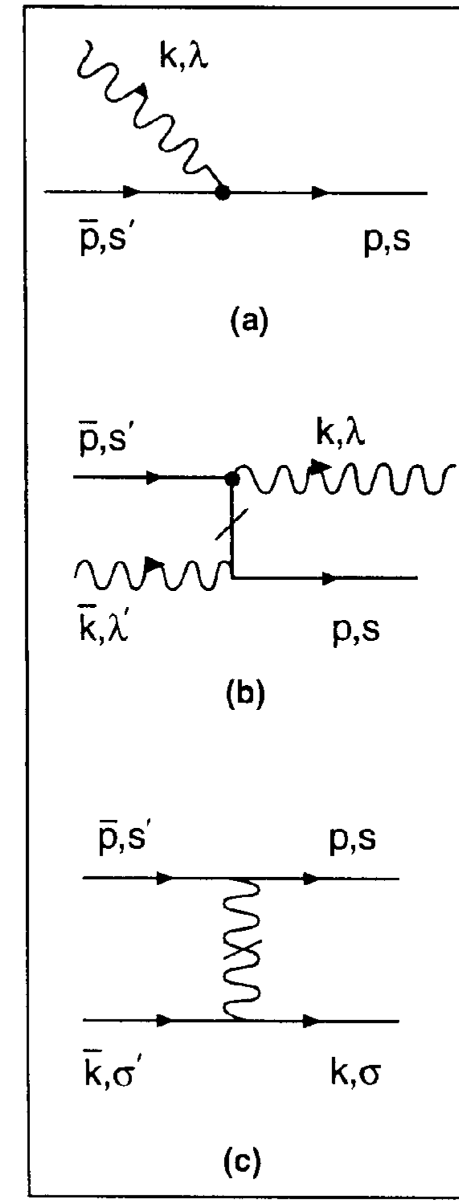
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i \left[\frac{m^2 + k^2_\perp}{x}\right]_i + H^{int}_{LF}$$

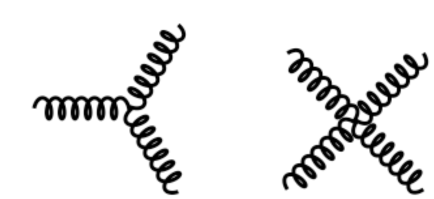
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h > = \mathcal{M}^2_h |\Psi_h >$$

$$|p, J_z > = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i >$$



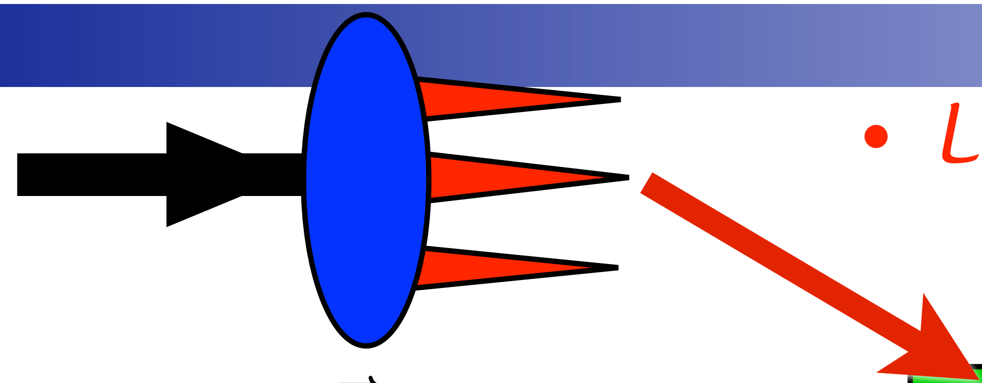
Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



H^{int}_{LF}

LFWFs: Off-shell in P- and invariant mass

- *Light Front Wavefunctions:*



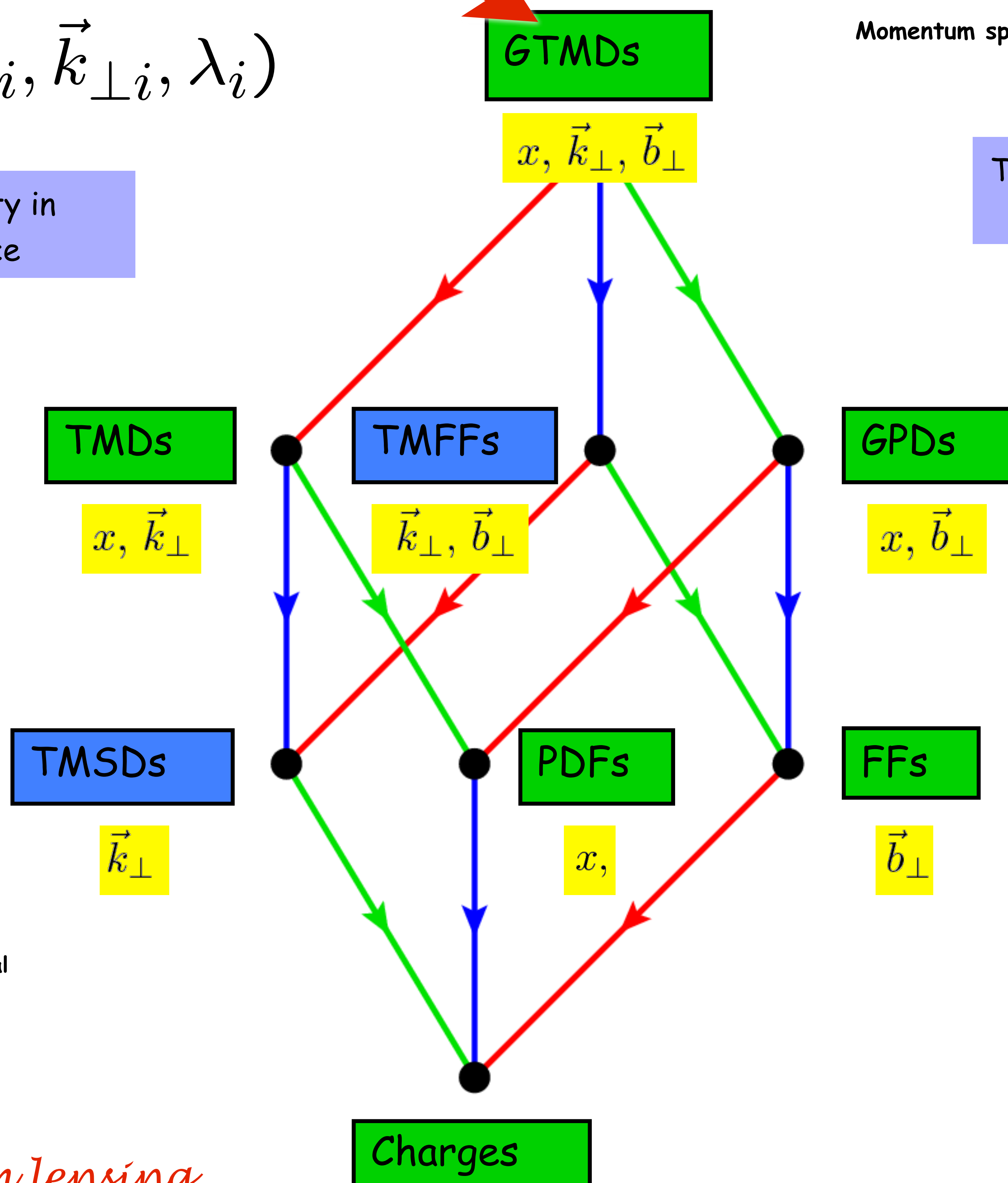
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

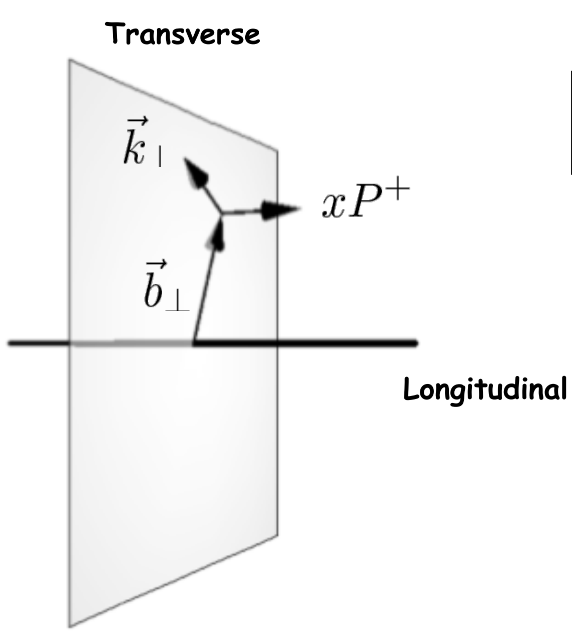
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

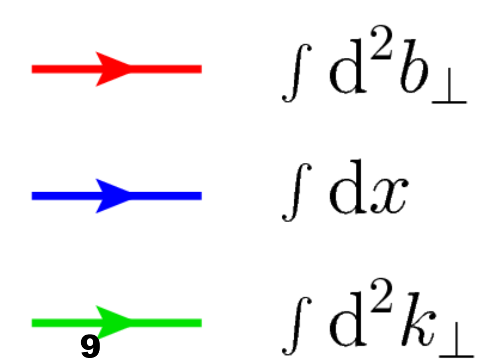
Transverse density in position space



Lorce, Pasquini

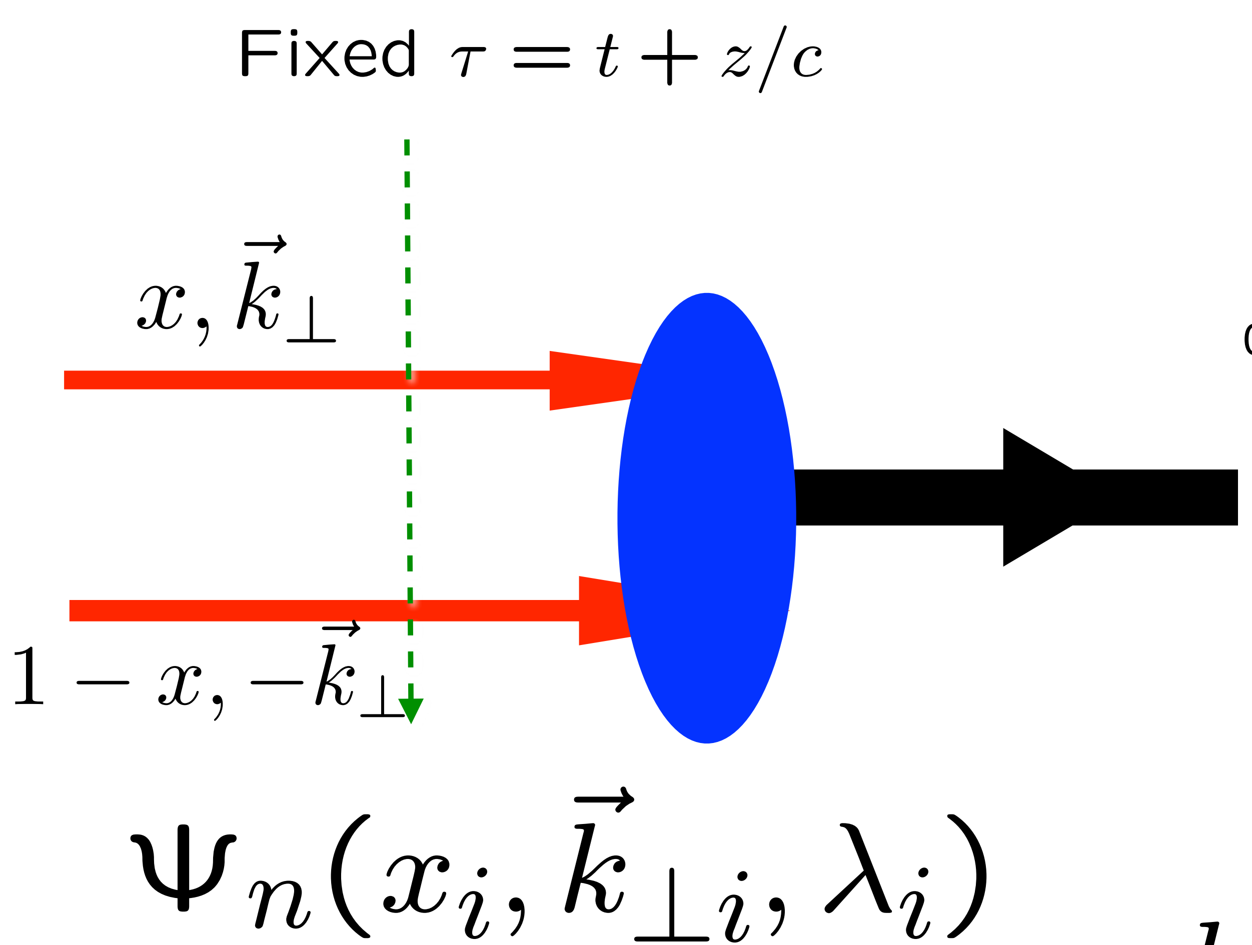


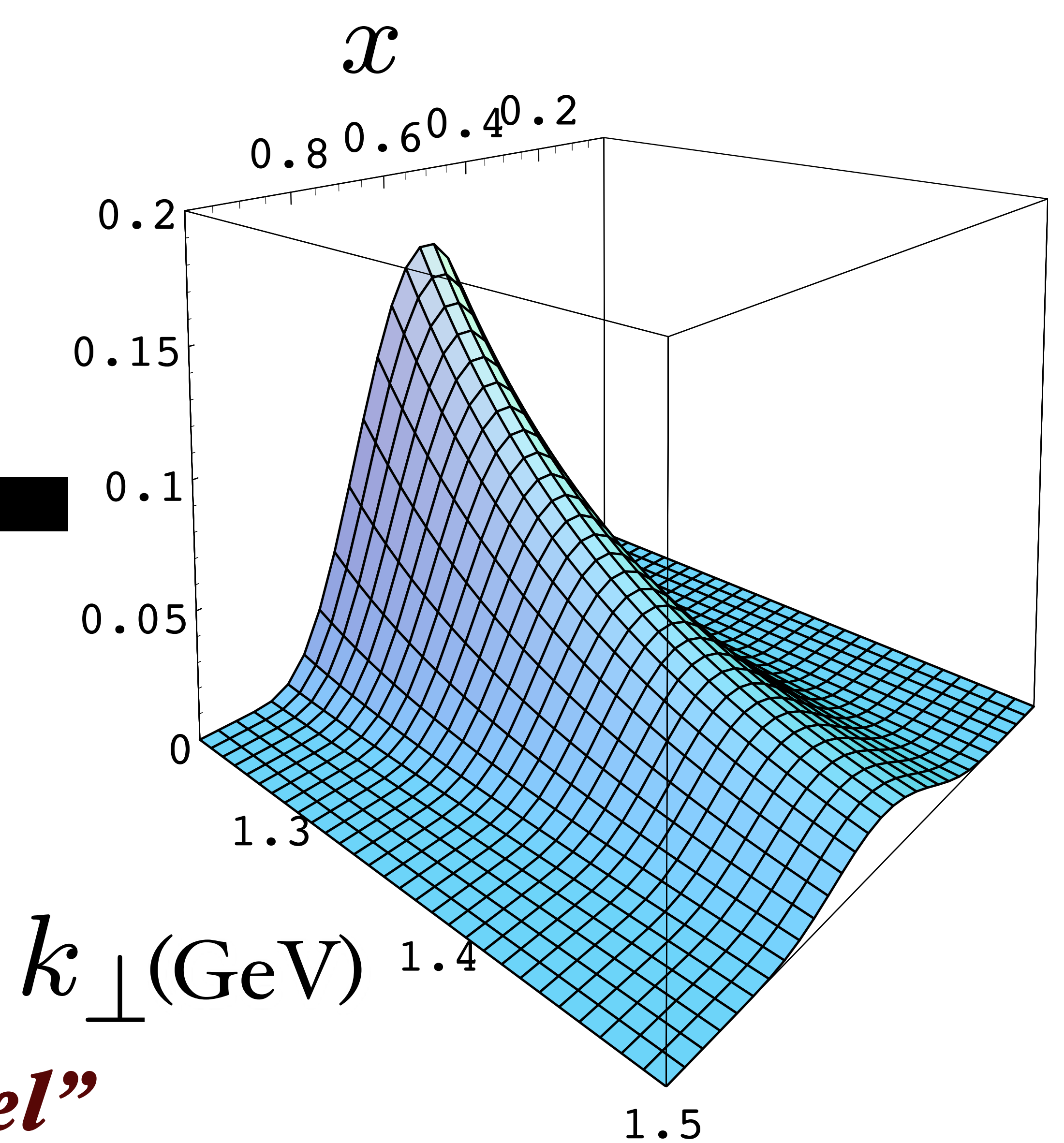
Sivers, T-odd from lensing



- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$





"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

Advantages of the Dirac's Front Form for Hadron Physics

Independent of Observer's Motion

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent: no boosts, no pancakes!**
- **Same structure function measured at an e p collider and Deep Inelastic Lepton Scattering in the proton rest frame**
- **No dependence on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no vacuum condensates!**
- **Profound implications for Cosmological Constant**

Roberts, Shrock, Tandy, sjb

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

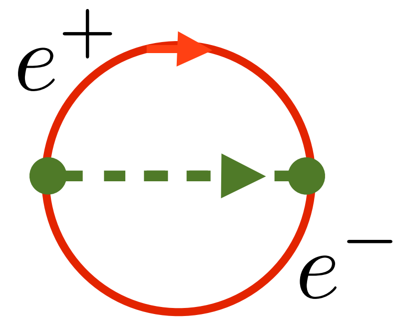
Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD "Condensates"

(C) Higgs Light-Front Zero Mode

Instant-Form Vacuum in QED



- Loop diagrams of all orders contribute
- Huge vacuum energy: $\rho_\Lambda^{QED} \simeq 10^{120} \rho_\Lambda^{Observed}$
- $\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3} \sqrt{\vec{k}^2 + m^2}$ Cutoff the quadratic divergence at M_{Planck}
- Why not impose :Normal Ordering: ? Causality issues.
- Divide S-matrix by disconnected vacuum diagrams?
- In Contrast: Light-Front Vacuum trivial since plus momenta are positive and conserved: $k^+ = k^0 + k^3 > 0$

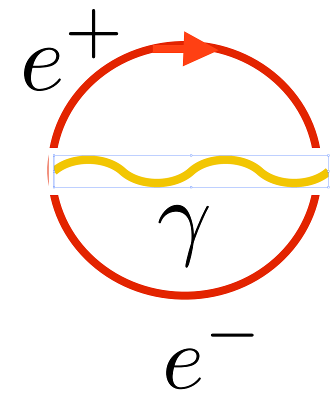
We view the universe as light reaches us along the light-front at fixed

$$\tau = t + z/c$$

Front Form Vacuum Describes the Empty, Causal Universe

Front-Form Vacuum in QED

$P^+ = 0$



$\sum_i k_i^+ \neq P^+ = 0$

$k_i^+ > 0$

- **All Light-Front Vacuum Graphs Vanish!**
- **Light-Front Vacuum is trivial since all plus momenta are positive and conserved.**
- **Zero modes (k^+=0) in vacuum allowed in some theories with massless fermions.**
- **Zero contribution to Λ from QED LF Vacuum**
- **Instant Form gives zero result if one normal orders.**

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0 >= E_0|\psi_0 >, E_0 = min\{E_i\}$$

Eigenstate defined at one time t over all space; Acausal! Frame-Dependent

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0 >_{LF}= M_0^2|\psi_0 >_{LF}, M_0^2 = 0.$$

Frame-independent eigenstate at fixed LF time $\tau = t+z/c$ within causal horizon

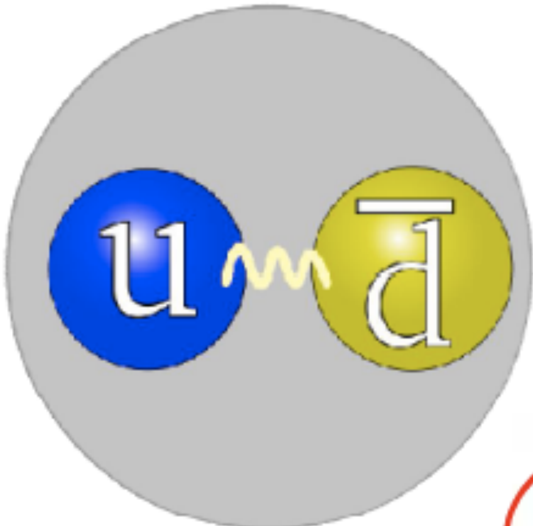
Front Form Vacuum Describes the Causal Universe

Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state M= 0.
- Trivial up to k^+=0 zero modes-- already normal-ordering
- Higgs theory consistent with LF theory (Srivastava, sjb)
- QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD, EW

What is the evidence for a nonzero vacuum quark condensate?



Gell-Mann - Oakes - Renner Relation (1968)

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

- Pion's leptonic decay constant, mass-dimensioned <u>observable</u> which describes rate of process $\pi^+ \to \mu^+ \nu$
- *Vacuum quark condensate*

ζ: renormalization scale

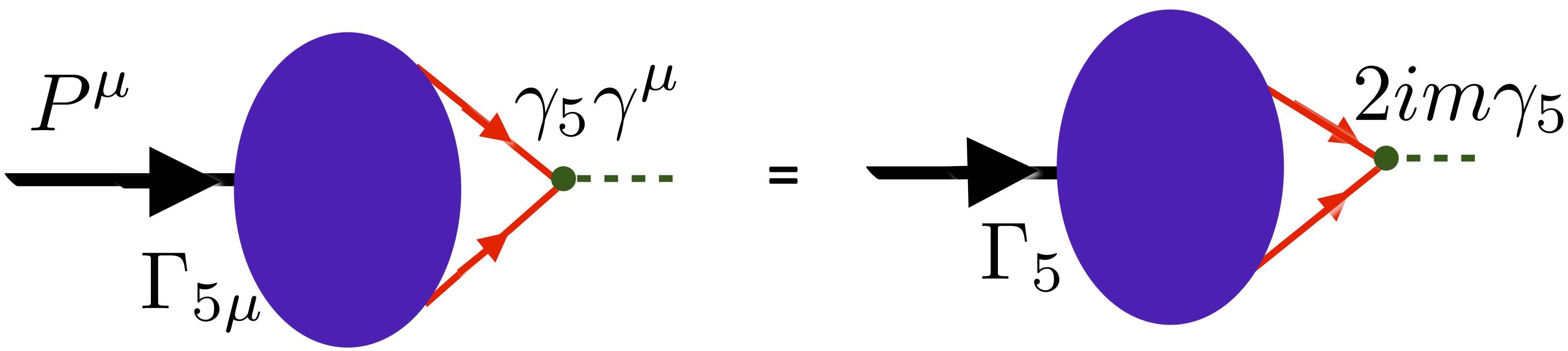
Derived in current algebra using an effective pion field

How is this modified in QCD for a composite pion?

Ward-Takahashi Identity for axial current

$$P^\mu \Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

Identify pion pole at $P^2 = m_\pi^2$

$$P^\mu < 0|\bar{q}\gamma_5\gamma^\mu q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u+m_d)\rho_\pi$$

Revised Gell Mann-Oakes-Renner Formula in QCD

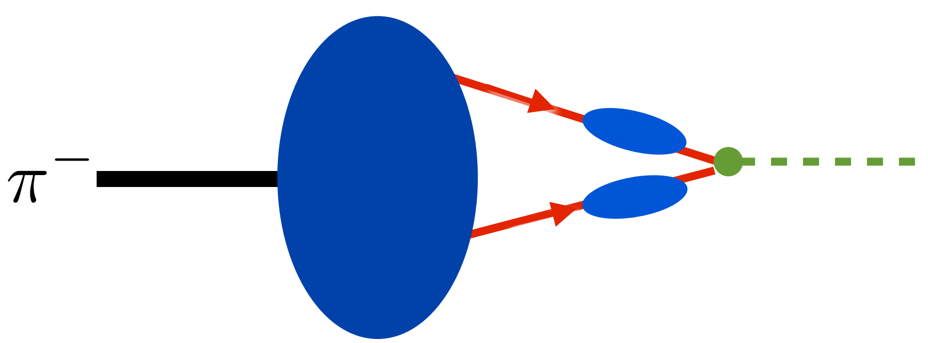
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"

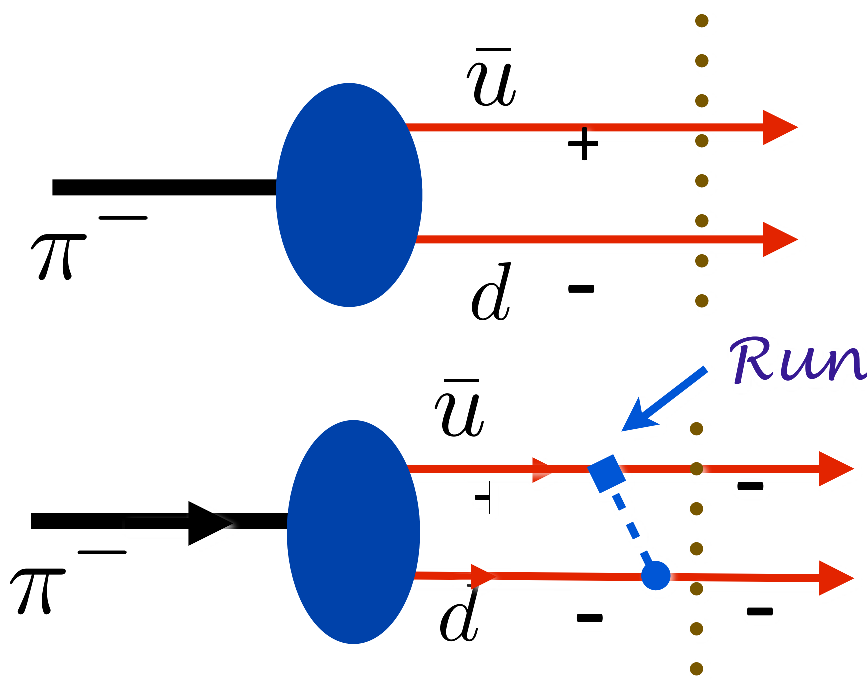


$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$



Running constituent mass at vertex

$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

Couples to

$$L^z = 0, S^z = 0 \quad < \pi|\bar\gamma^\mu q \gamma_5 q|0 > \sim f_\pi$$

Couples to

$$L^z = +1, S^z = -1 \quad < \pi|\bar q \gamma_5 q|0 > \sim \rho_\pi$$

Angular Momentum Conservation

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

Summary on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena -- consistent with LF Theory
- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: "In-Hadron Condensates"
- Find:

$$\frac{< 0|\bar{q}q|0 >}{f_\pi} \to - < 0|i\bar{q}\gamma_5 q|\pi >= \rho_\pi$$

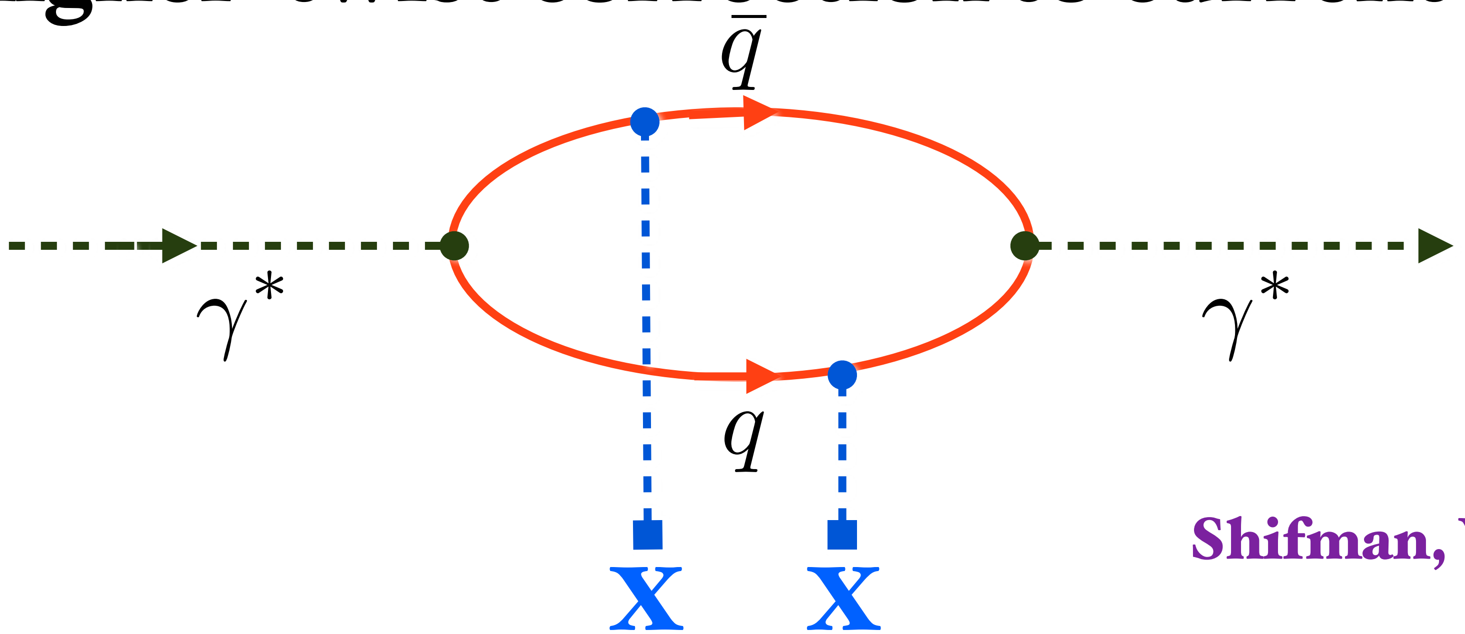
$$< 0|\bar{q}i\gamma_5 q|\pi > \text{ similar to } < 0|\bar{q}\gamma^\mu\gamma_5 q|\pi >$$

- Zero contribution to cosmological constant! Included in hadron mass
- ρ_π survives for small m_q -- enhanced running mass from gluon loops / multiparton Fock states

Is there empirical evidence for a gluon vacuum condensate?

$$< 0|\frac{\alpha_s}{\pi}G^{\mu\nu}(0)G_{\mu\nu}(0)|0 >$$

Look for higher-twist correction to current propagator



Shifman, Vainshtein, Zakharov

$e^+e^- \to X$, τ decay, $Q\bar{Q}$ phenomenology

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi}\frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

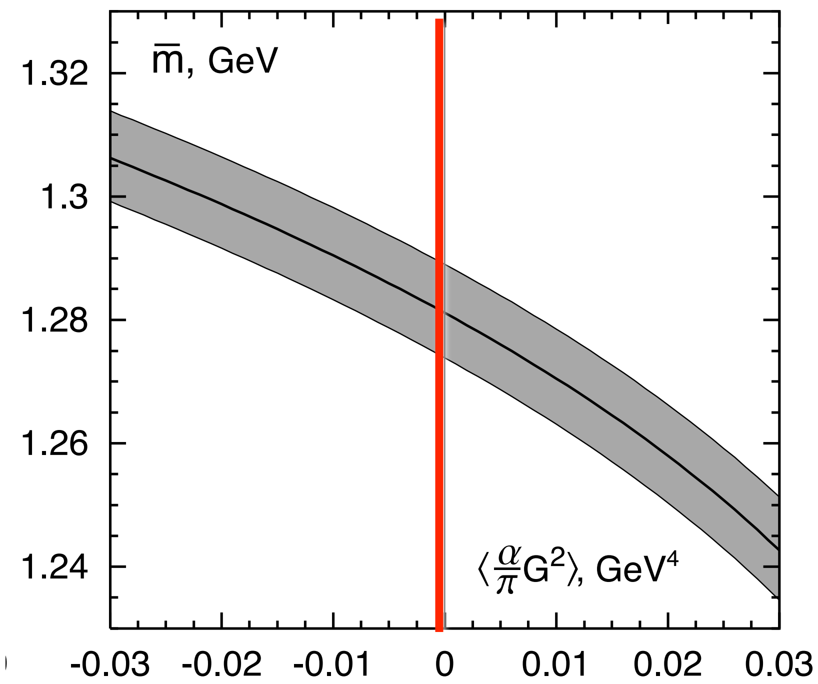
Determinations of the vacuum Gluon Condensate

$$< 0|\frac{\alpha_s}{\pi}G^2|0 > [\mathrm{GeV}^4]$$

-0.005 ± 0.003 from τ decay. Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules Ioffe, Zyablyuk



Consistent with zero vacuum condensate

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind **Maris, Roberts, Tandy** **Shrock and sjb**

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

Light-Front Formalism

PHYSICAL REVIEW C 85, 065202 (2012)

Confinement contains condensates

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Dynamical chiral symmetry breaking and its connection to the generation of hadron masses has historically been viewed as a vacuum phenomenon. We argue that confinement makes such a position untenable. If quark-hadron duality is a reality in QCD, then condensates, those quantities that have commonly been viewed as constant empirical mass scales that fill all space-time, are instead wholly contained within hadrons; i.e., they are a property of hadrons themselves and expressed, e.g., in their Bethe-Salpeter or light-front wave functions. We explain that this paradigm is consistent with empirical evidence and incidentally expose misconceptions in a recent Comment.

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Confinement contains condensates

Light-Front Quantization of the Standard Model

- SU(2) X U(1) GWS Model of Weak Interactions
- Non-Abelian Higgs Model in LG Gauge
- Unitary, renormalizable, no Gupta-Bleuler, Fadeev-Popov ghosts
- SSB: Perturbative vacuum plus zero mode field
- t'Hooft conditions satisfied
- Higgs field: Real field creates Higgs particle; imaginary components identified with longitudinal components of W, Z
- Higgs VEV replaced by zero mode

Abelian U(1) LF Model with Spontaneous Symmetry Breaking

$$\mathcal{L} = \partial_+ \phi^\dagger \partial_- \phi + \partial_- \phi^\dagger \partial_+ \phi - \partial_\perp \phi^\dagger \partial_\perp \phi - \mathcal{V}(\phi^\dagger \phi)$$

$$\text{where } V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \qquad \text{with } \lambda > 0,\ \mu^2 < 0$$

$$\text{Constraint equation: } \int d^2x_\perp dx^- \left[\partial_\perp \partial_\perp \phi - \frac{\delta V}{\delta \phi^\dagger}\right] = 0$$

$$\phi(\tau, x^-, x_\perp) = \omega(\tau, x_\perp) + \varphi(\tau, x^-, x_\perp)$$

$$\omega(\tau, x_\perp) \text{ is a } k^+ = 0 \text{ zero mode}$$

$$\omega = v/\sqrt{2} \text{ where } v = \sqrt{-\mu^2/\lambda}$$

Thus a c-number in LF replaces conventional Higgs VEV

Higgs coupling to gravity?

$$\text{Possibility: } \partial_\perp \omega \neq 0$$

P. Srivastava, sjb

Standard Model on the Light-Front

- **Same phenomenological predictions**
- **Higgs field has three components**
- **Real part creates Higgs particle**
- **Imaginary part (Goldstone) become longitudinal components of W, Z**
- ***Higgs VEV of instant form becomes $k^+=0$ LF zero mode!***
- **Analogous to a background static classical Zeeman or Stark Fields**
- **Zero contribution to T^μ_μ ; zero coupling to gravity**

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Semiclassical first approximation to QED

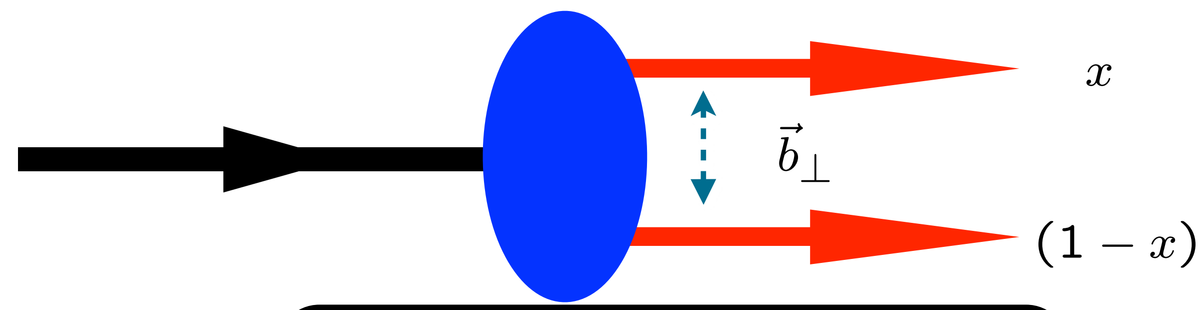
Coulomb potential

Bohr Spectrum

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b^2_\perp$$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

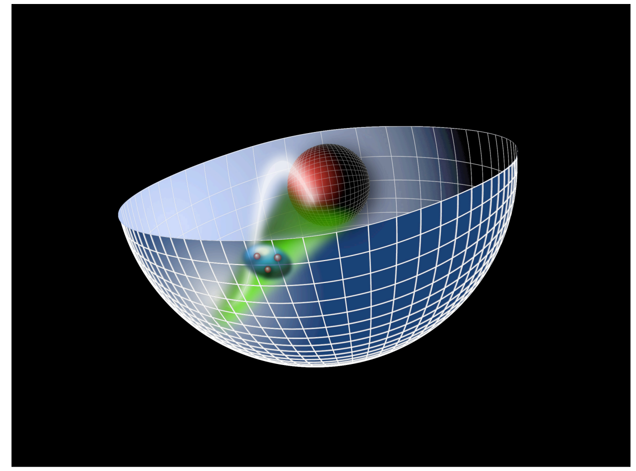
Sums an infinite # diagrams

Semiclassical first approximation to QCD

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

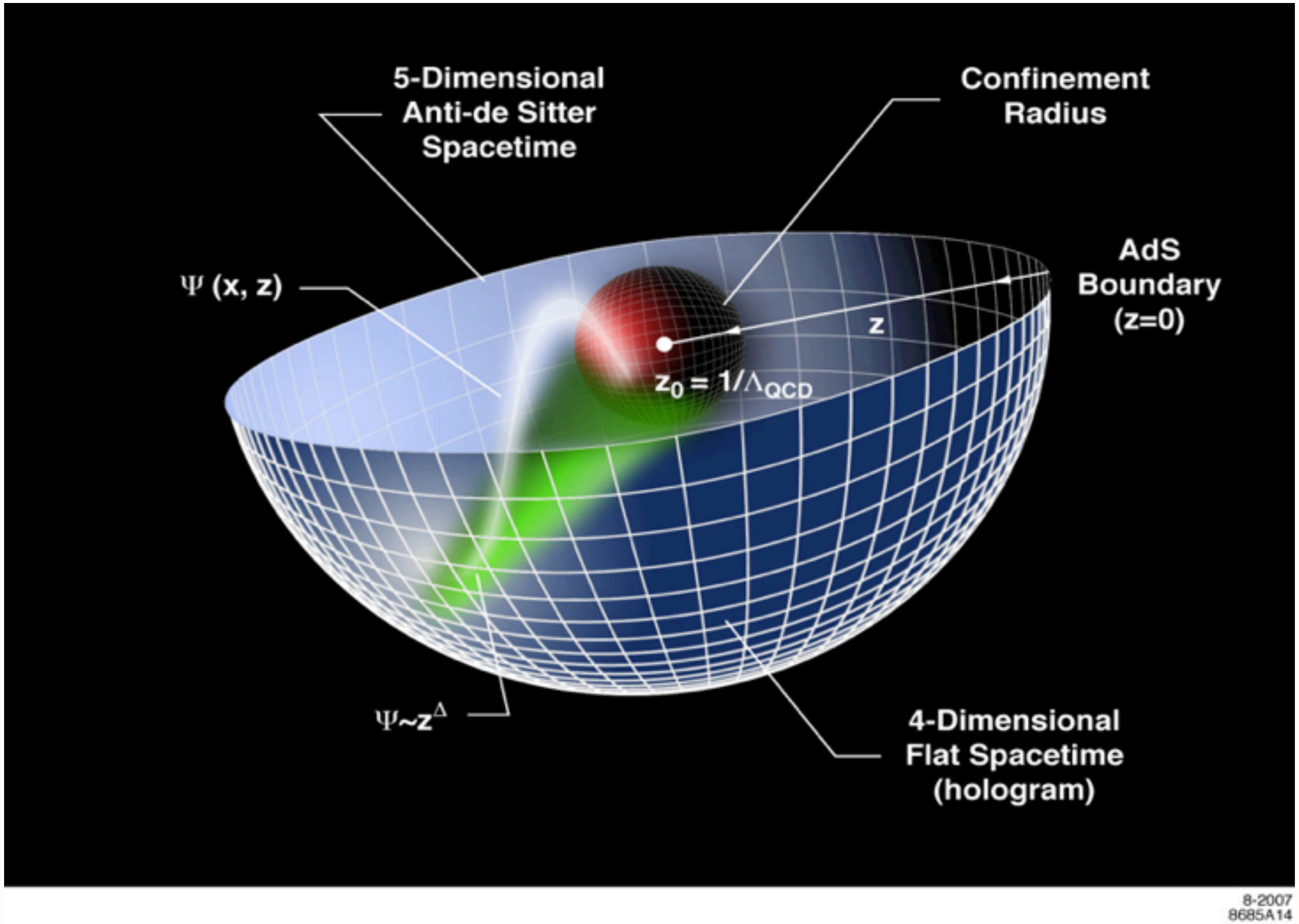
$$\kappa \simeq 0.5\ GeV$$

Confinement scale:

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

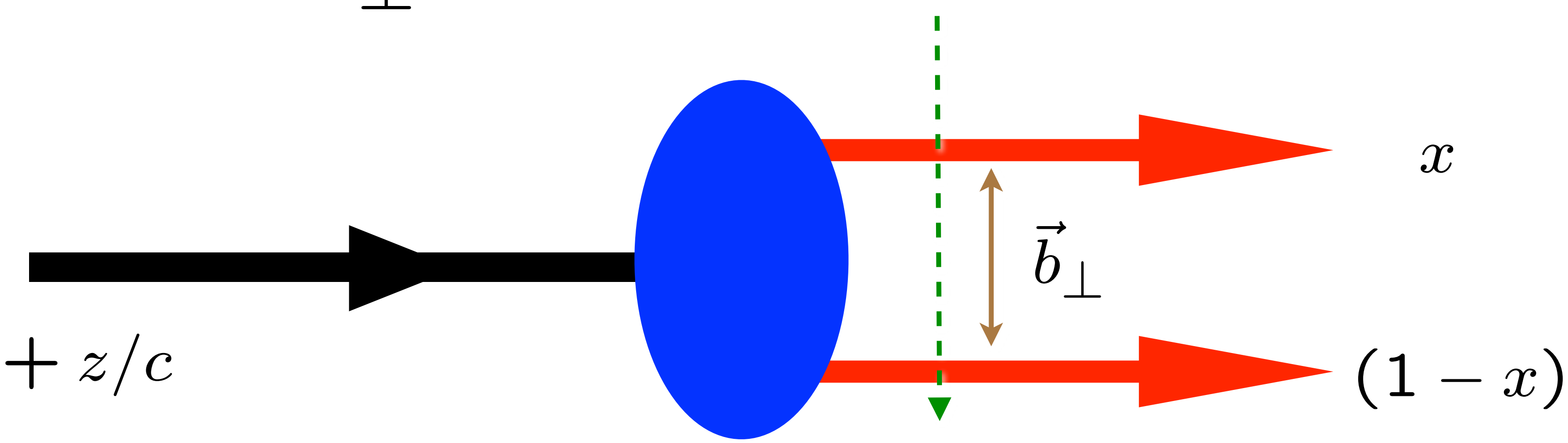
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x,\vec b_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec b_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J-1)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

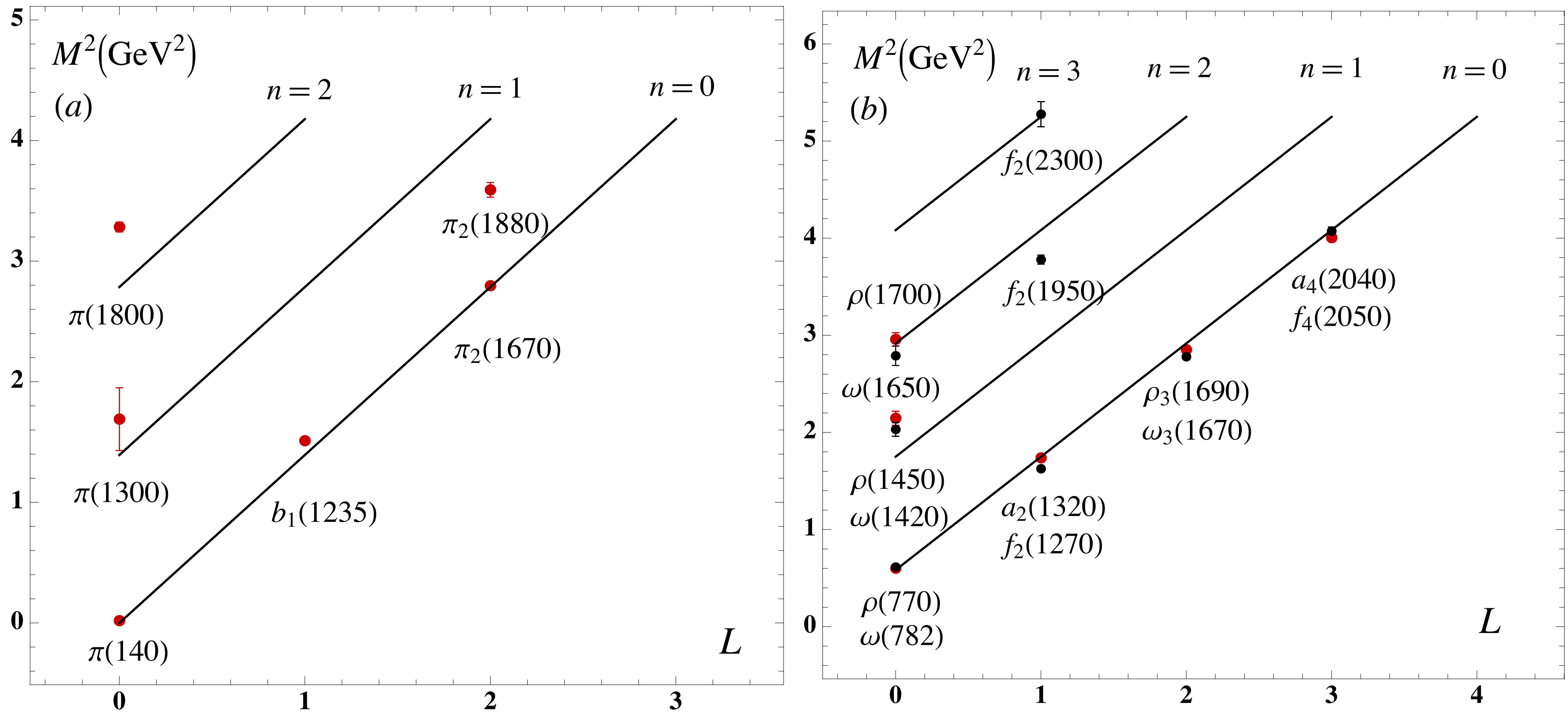
- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

$m_u = m_d = 0$

de Tèramond, Dosch, sjb

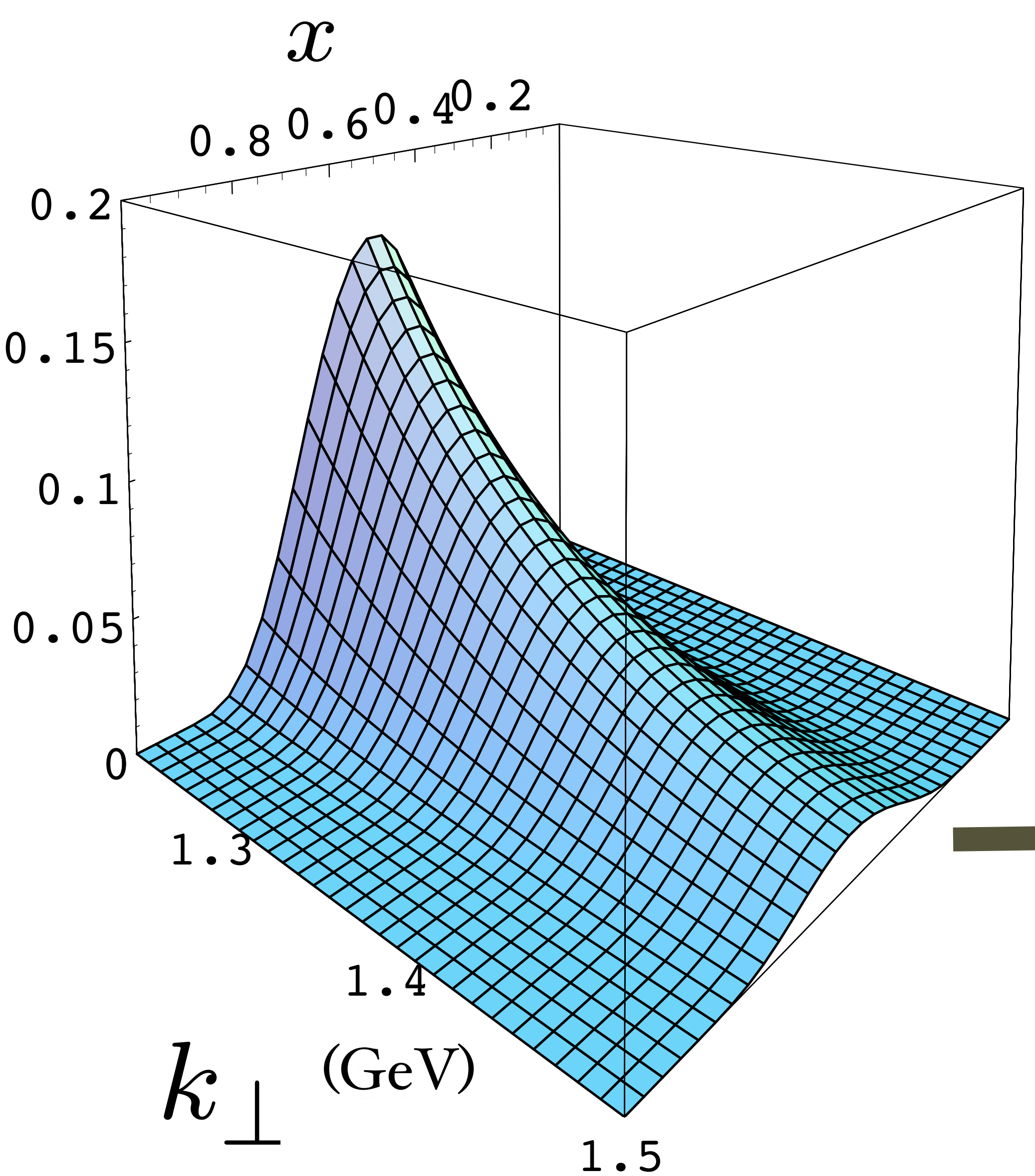


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Prediction from AdS/QCD: Meson LFWF

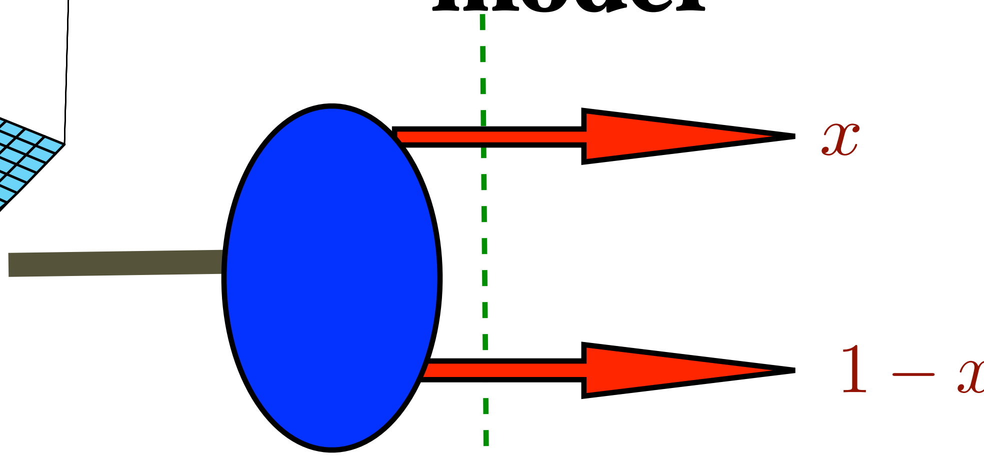
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

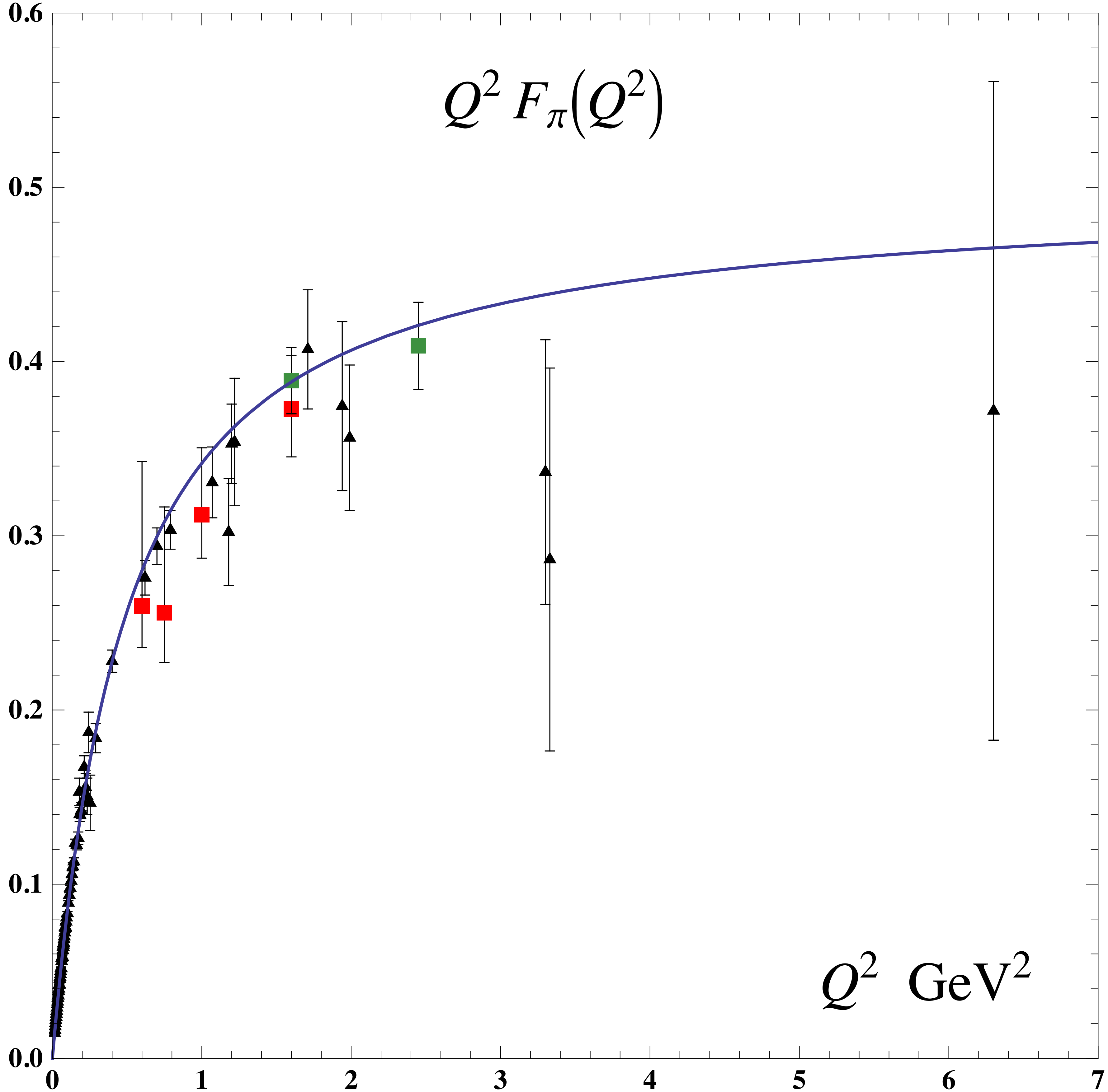
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE!

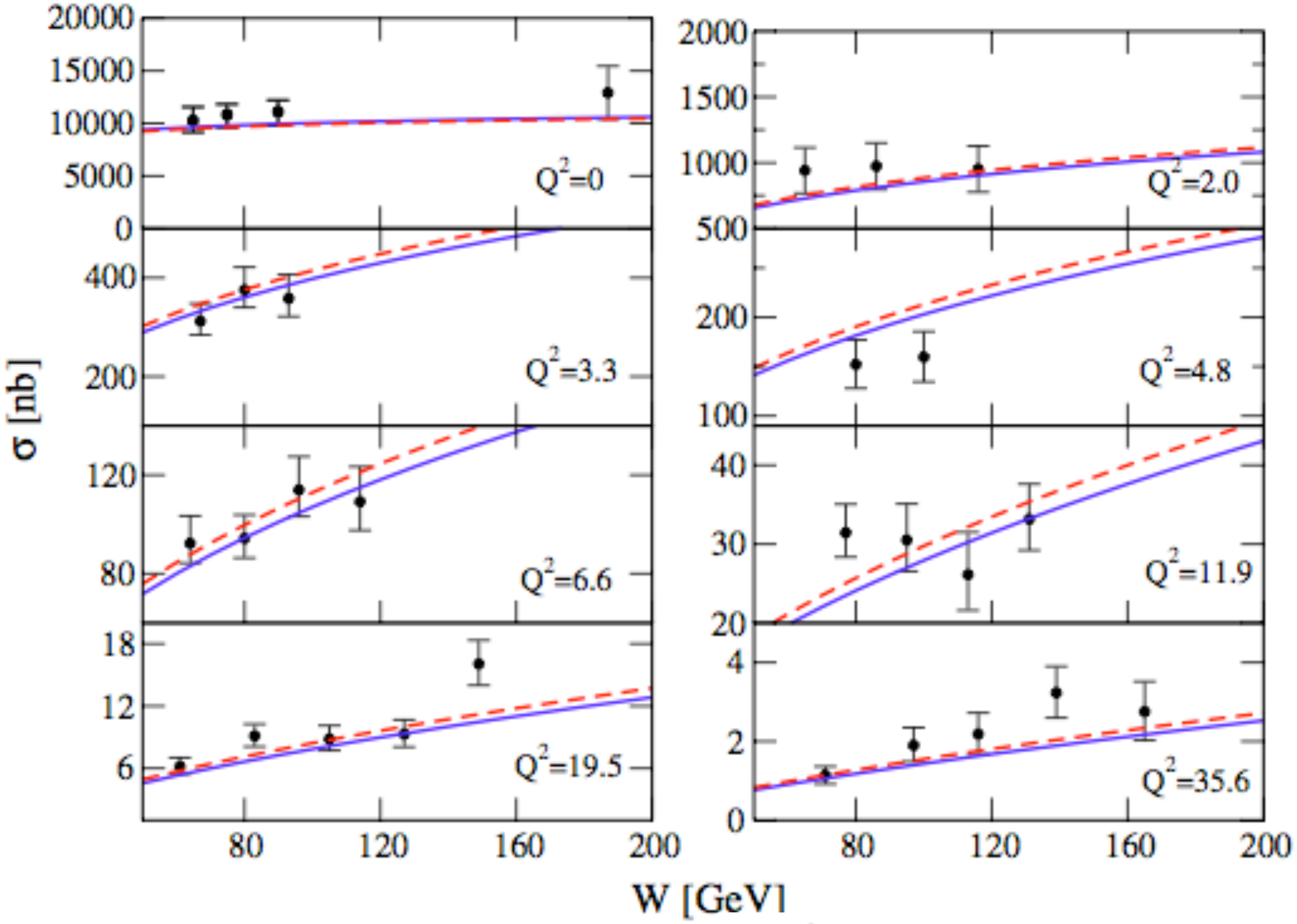
Provides Connection of Confinement to Hadron Structure



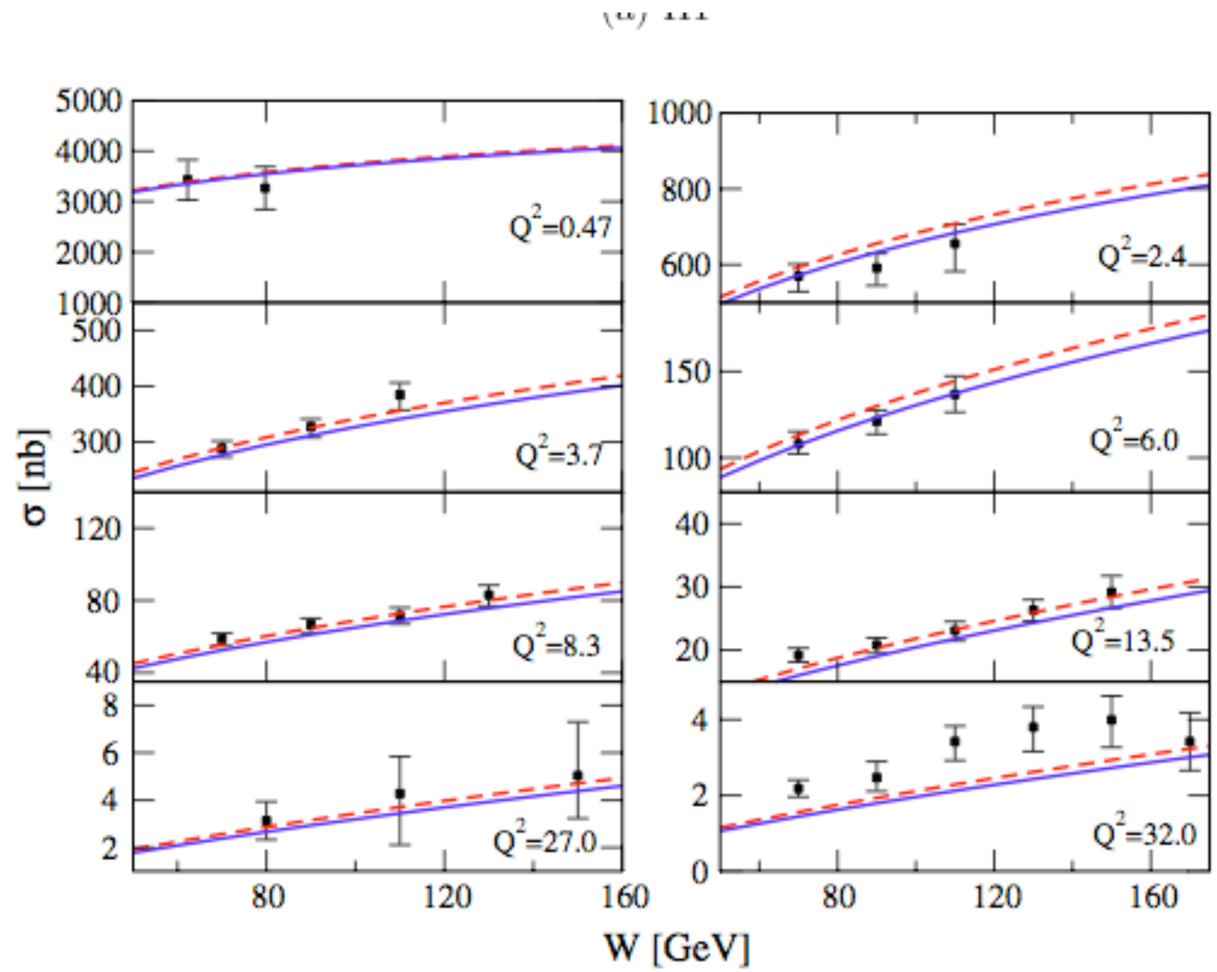
$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a)

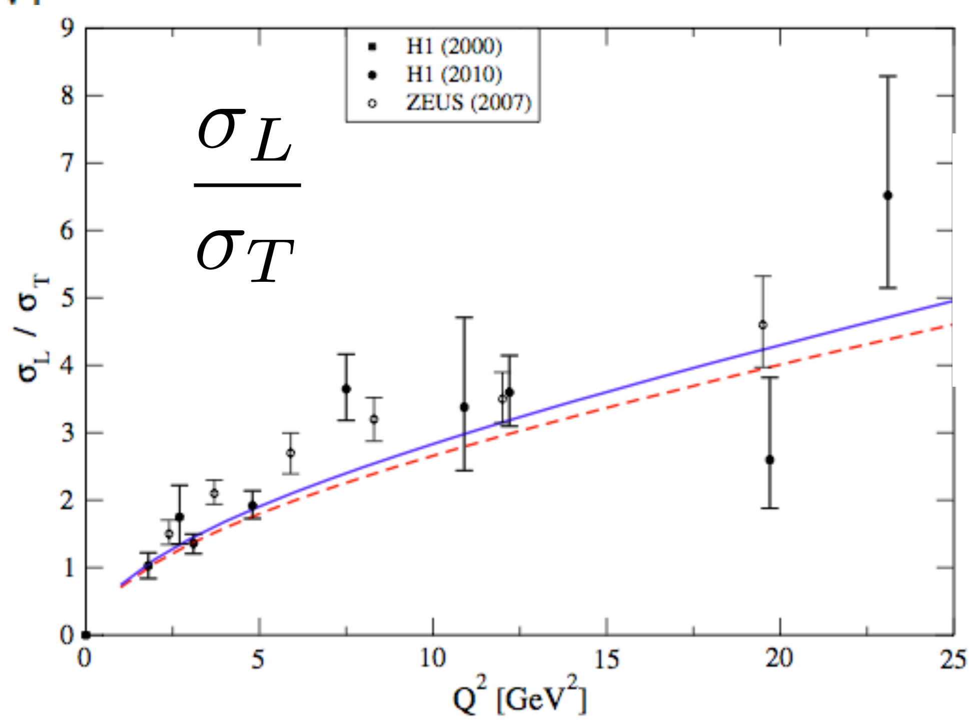


(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \rightarrow \rho^0 p'$$

$$\frac{\sigma_L}{\sigma_T}$$



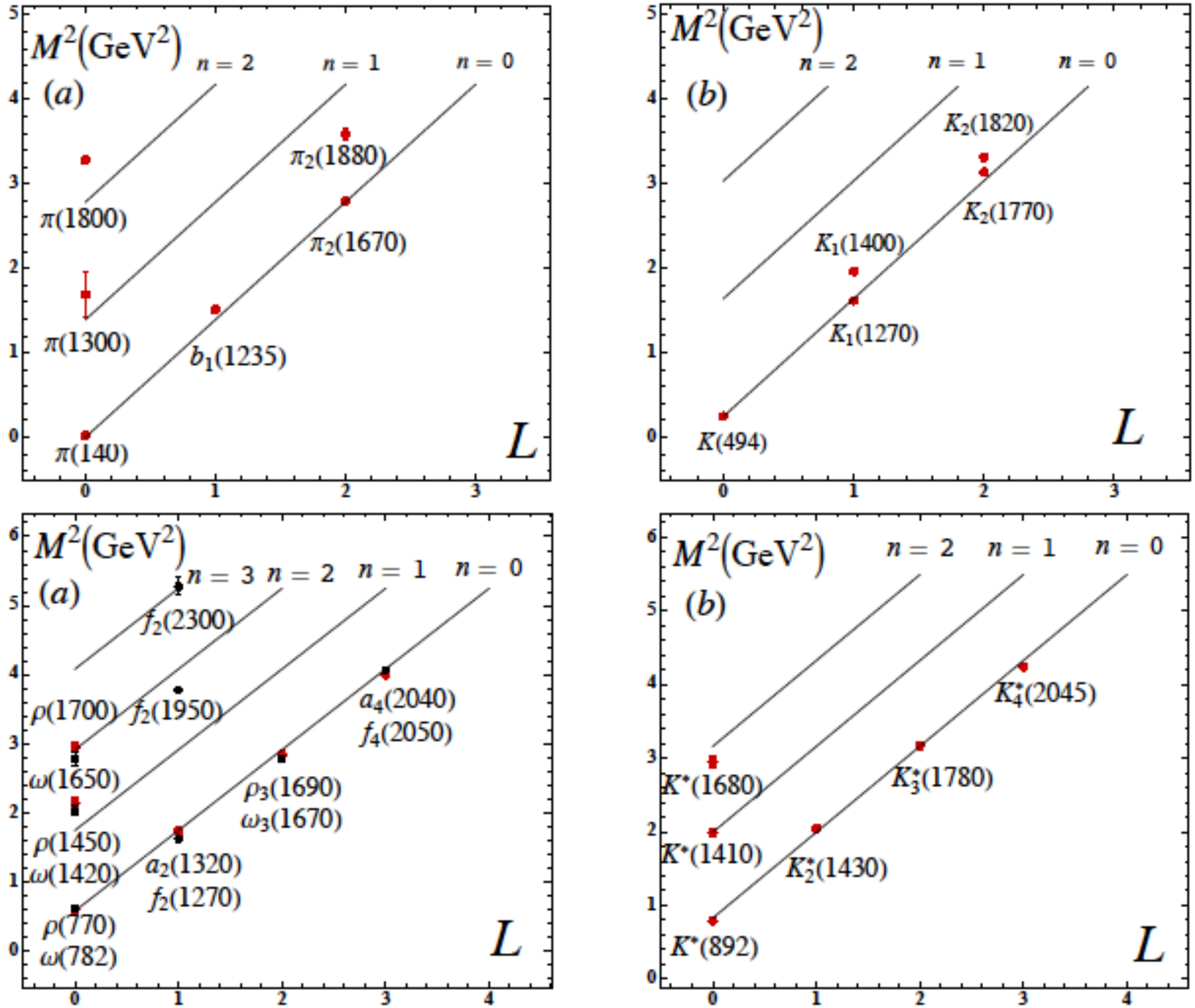
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

De Tèramond, Dosch, sjb

$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$



- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- Identify with difference of LF time $\Delta x^+/P^+$ between constituents
- Finite range
- Measure in Double-Parton Processes

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates the conformal algebra

[H,D]= i H,   [H, K] =2 i D,   [K, D] = - i K

Haag, Lopuszanski, Sohnius (1974)

Fubini and Rabinovici

Superconformal Quantum Mechanics

de Teramond Dosch, Lorce, sjb

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Superconformal Quantum Mechanics

Baryon Equation

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action ***Fubini and Rabinovici***

New Extended Hamiltonian G is diagonal:

$$G_{11} = \Big(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\Big)$$

$$G_{22} = \Big(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\Big)$$

Identify $f - \frac{1}{2} = L_B$, $w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

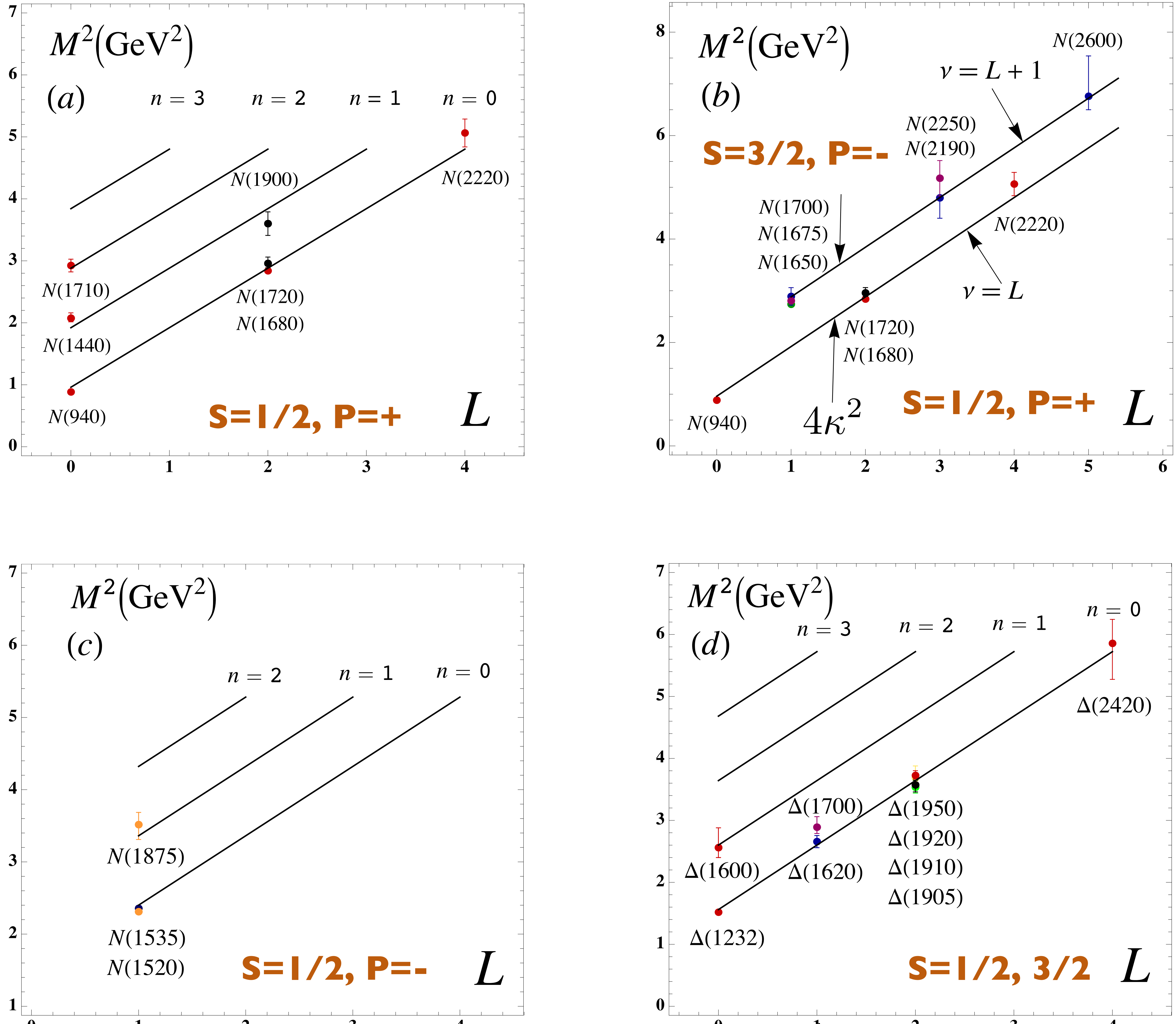
$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ!

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

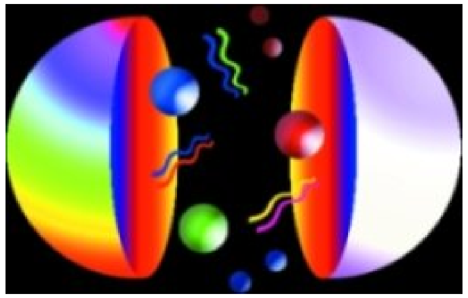


M²(GeV²)
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
M²(GeV²)
(b)
N(2600)
ν = L + 1
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
ν = L
N(1720)
N(1680)
N(940)
4κ²
S=1/2, P=+
L
M²(GeV²)
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
M²(GeV²)
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Nucleon: Equal Probability for L=0,1

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

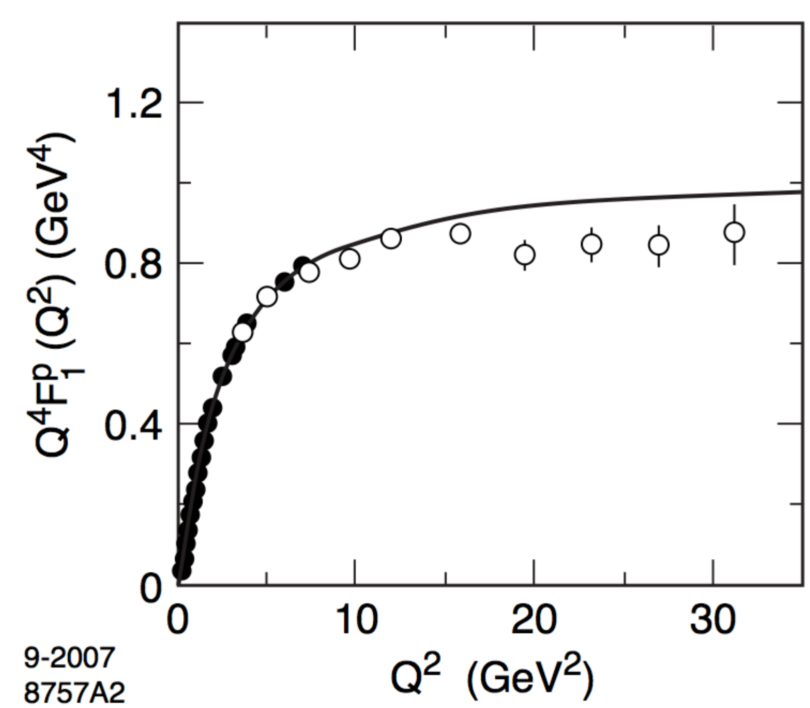
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$

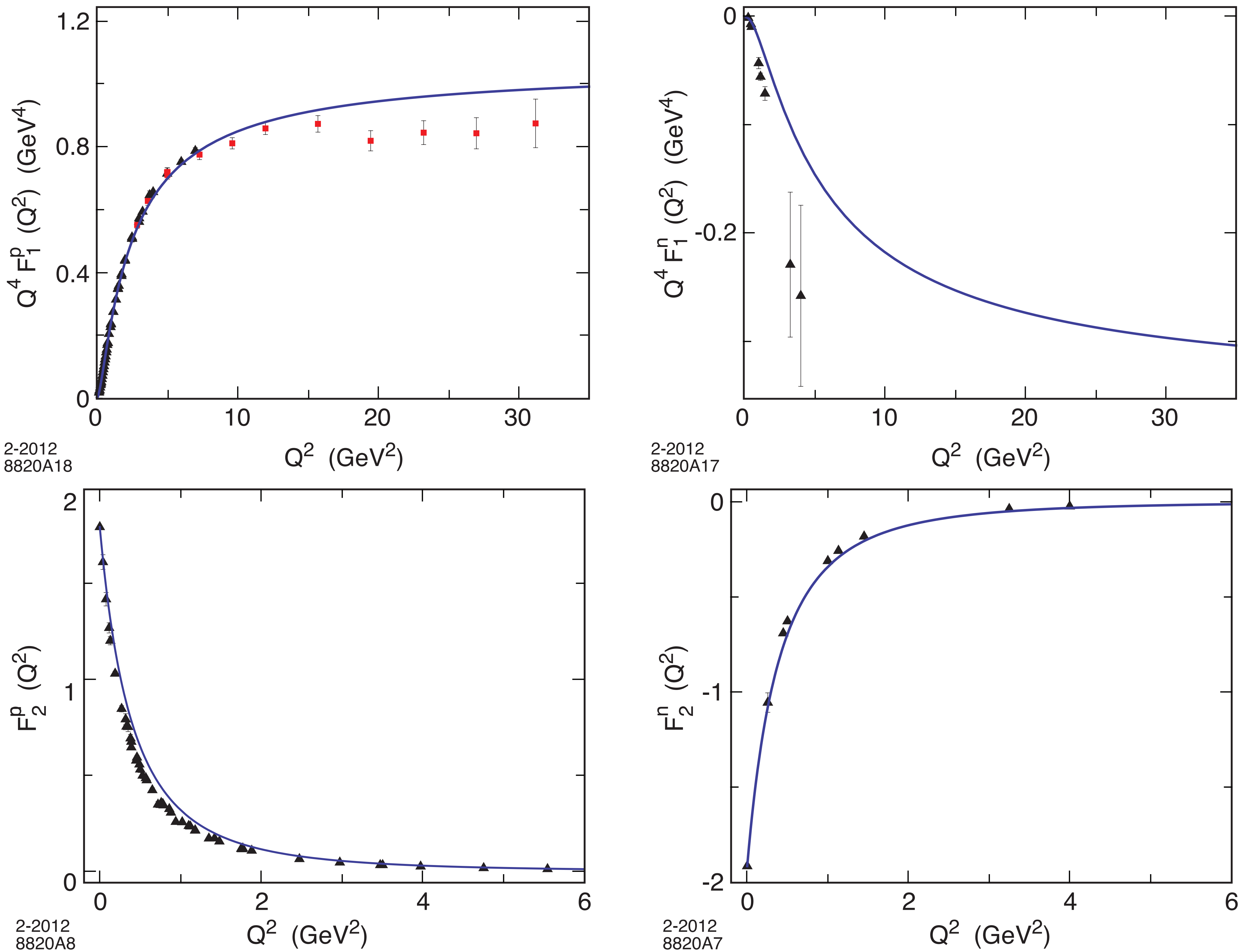
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$

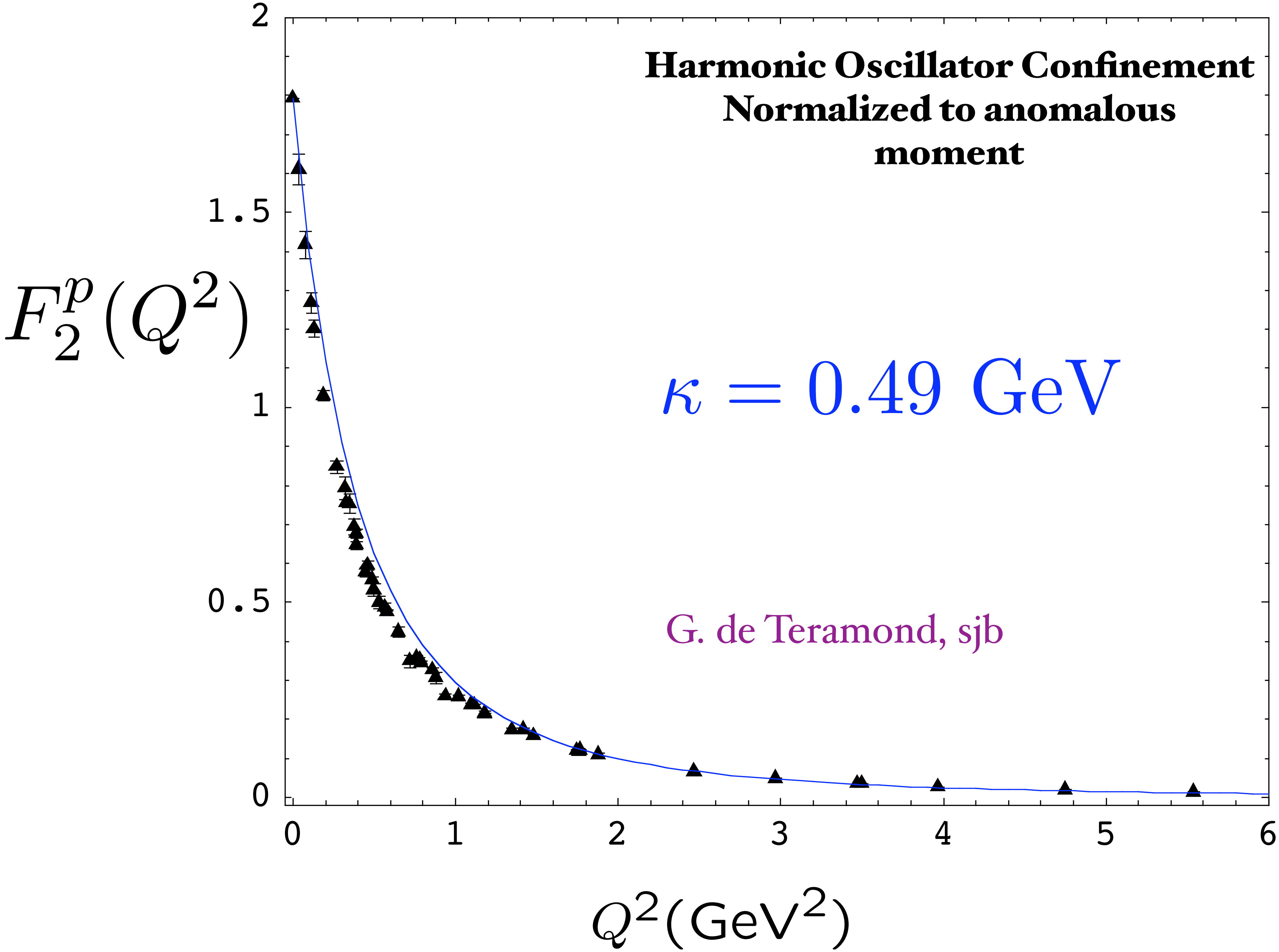


Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

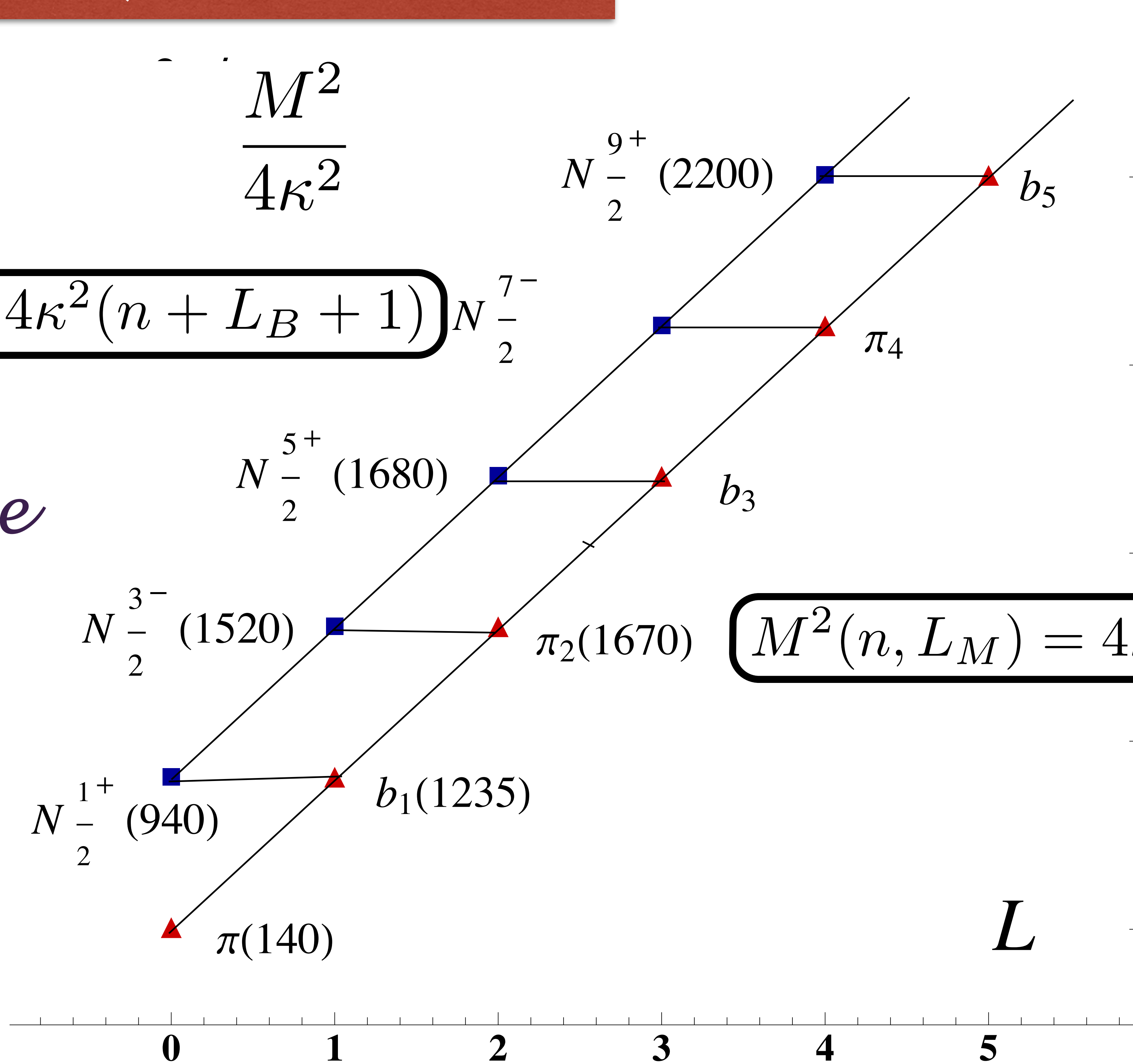
No mass -degenerate parity partners!

Superconformal Quantum Mechanics

de Tèramond, Dosch, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



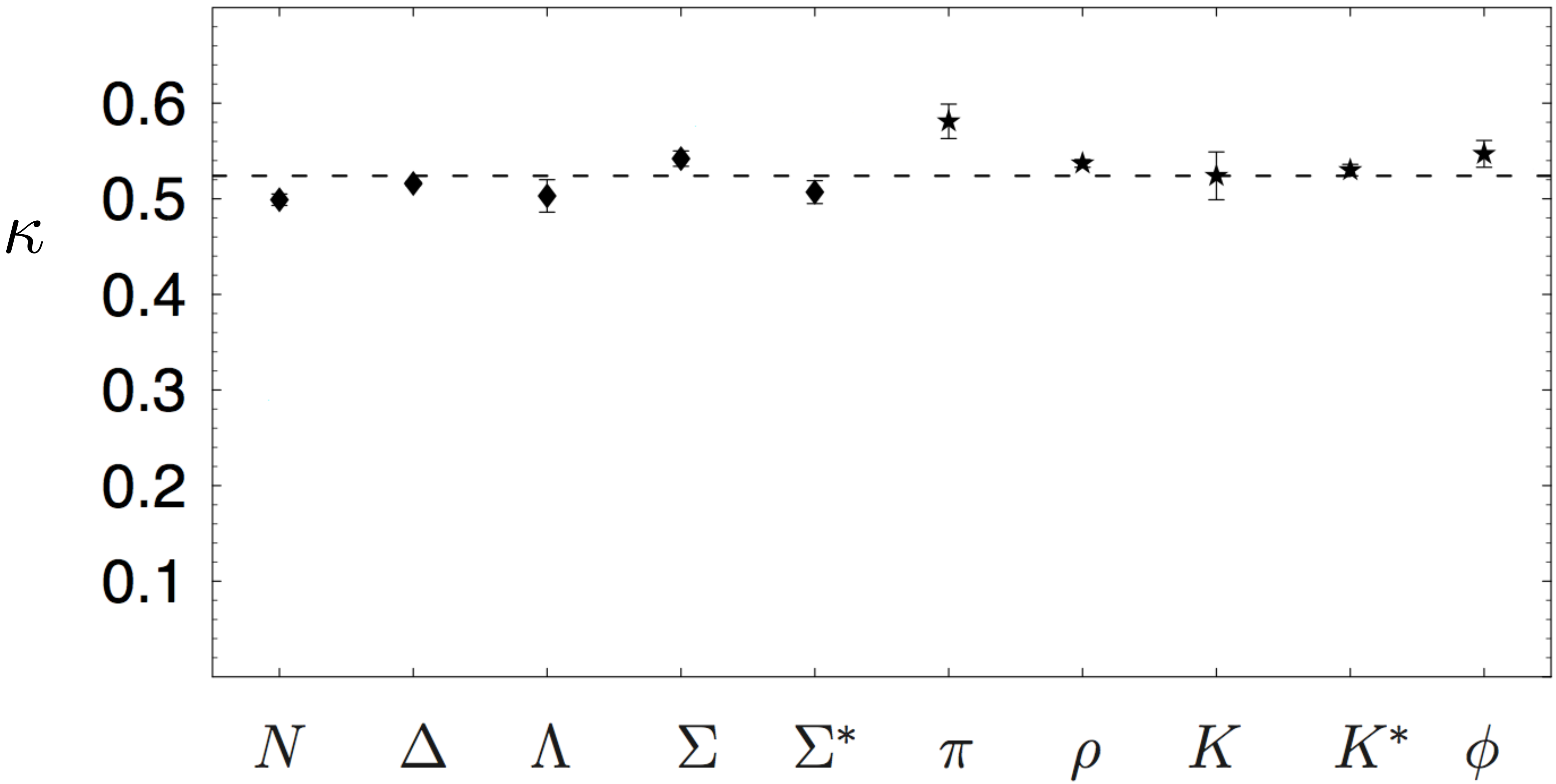
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Dosch, de Teramond, Lorce, sjb

$$m_u = m_d = 46 \text{ MeV}, \, m_s = 357 \text{ MeV}$$



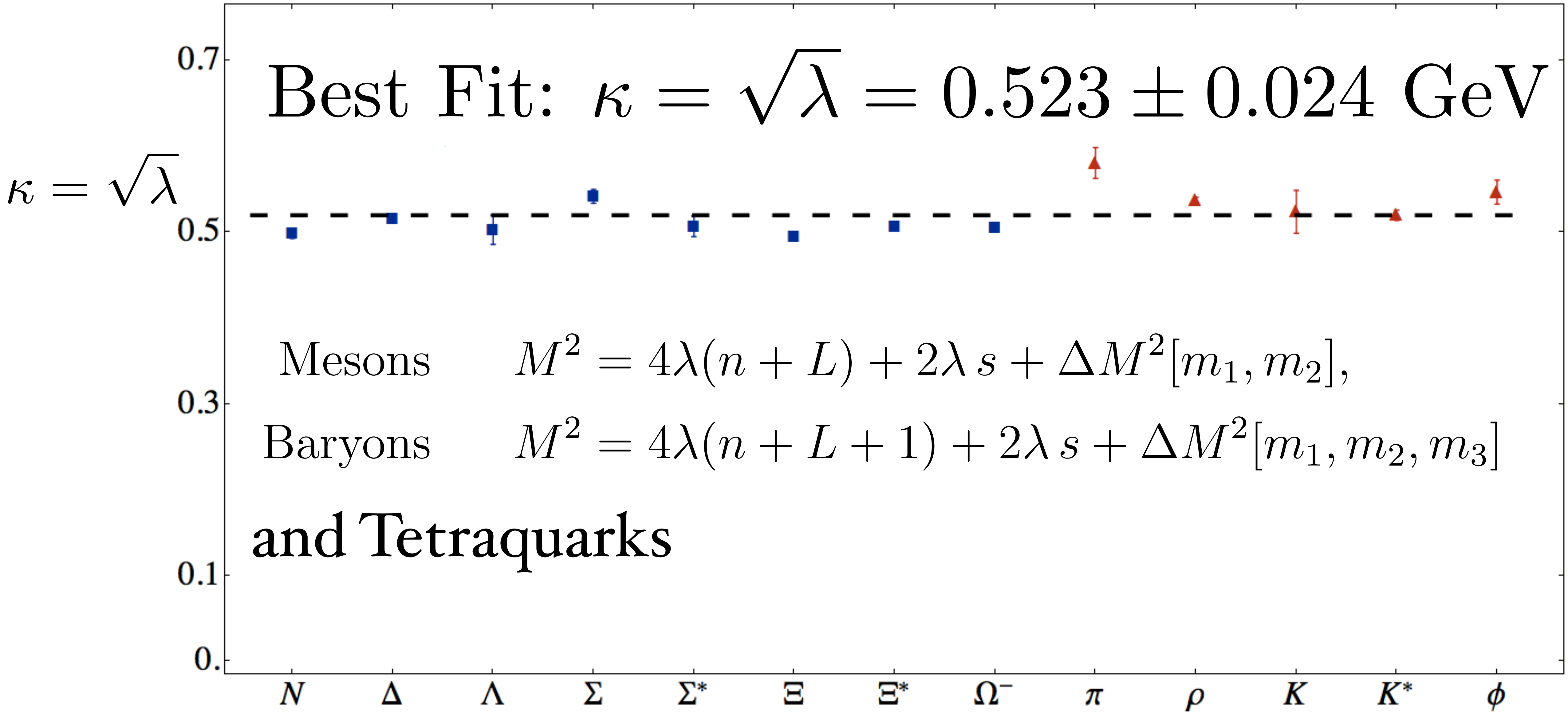
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

- Universal Regge slopes

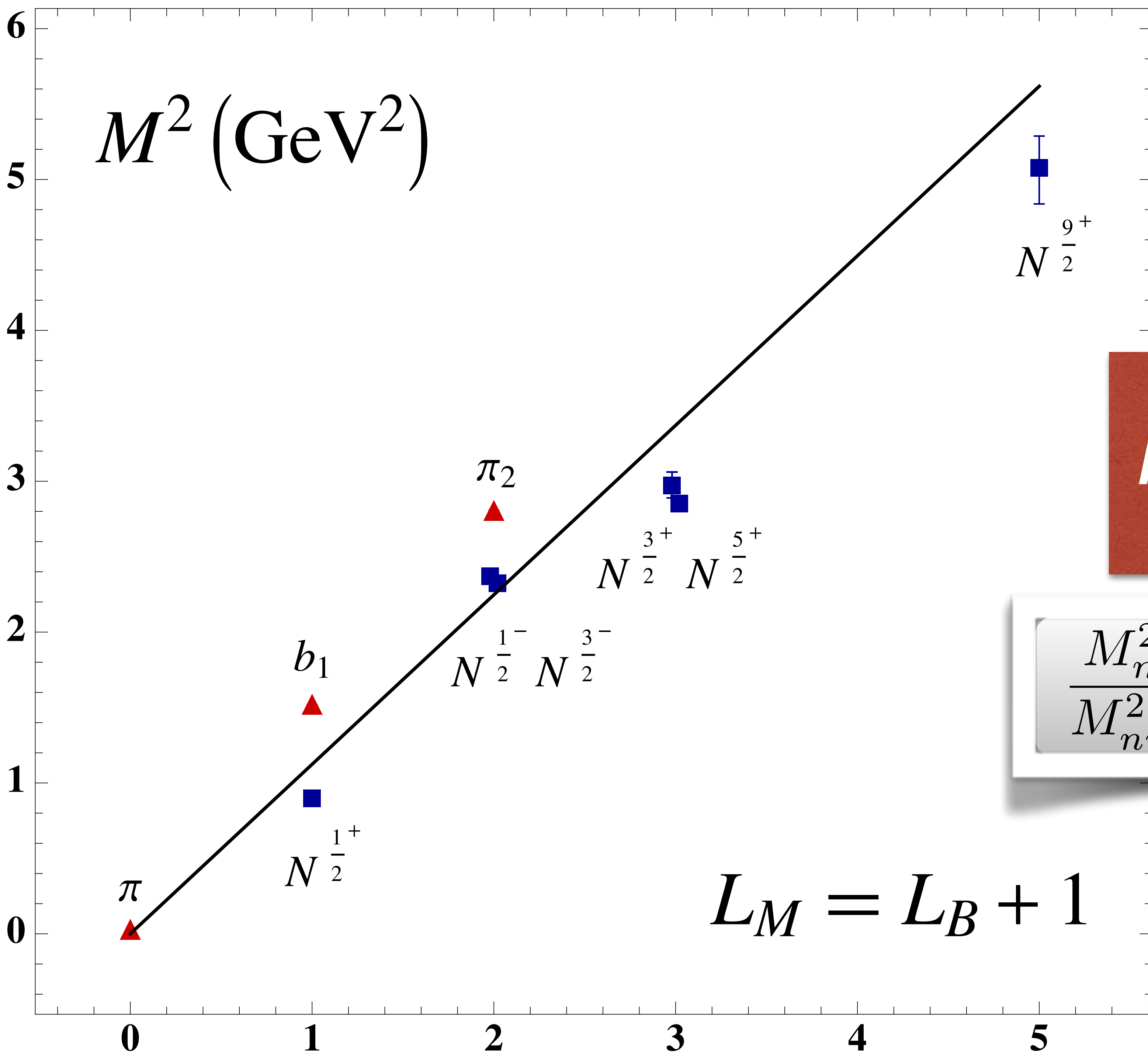
Dosch, de Teramond
Lorce, sjb

$$M_H^2 = 4\lambda(n+L) + \cdots$$



Best fit for the value of the hadronic scale $\sqrt{\lambda}$ for the different Regge trajectories for baryons and mesons including all radial and orbital excitations using Eqs. (23) and (24). The dotted line is the average value $\sqrt{\lambda} = 0.523$ GeV; it has the standard deviation $\sigma = 0.024$ GeV. For the baryon sample alone the values are 0.509 ± 0.015 GeV and for the mesons 0.524 ± 0.023 GeV.

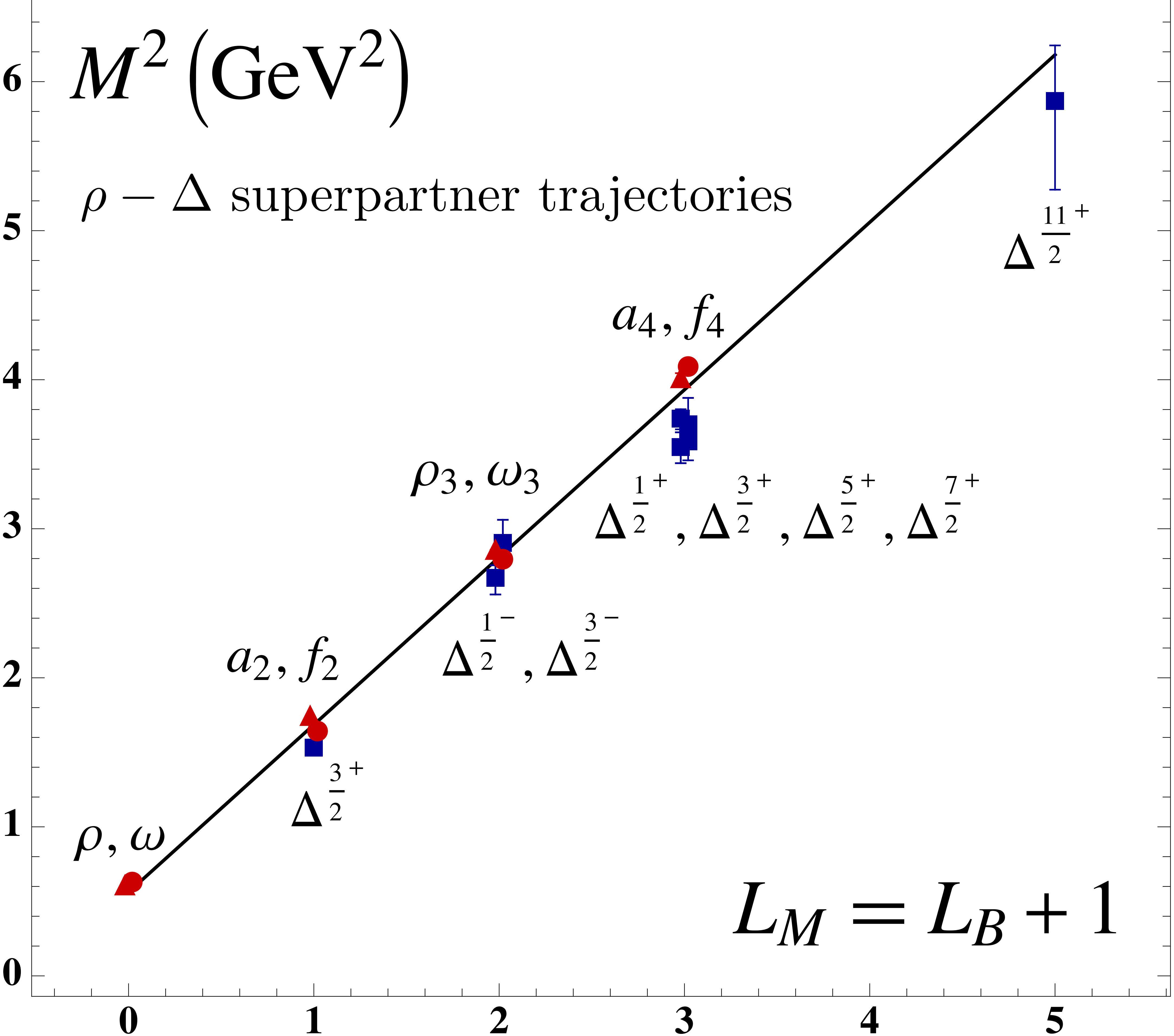
Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

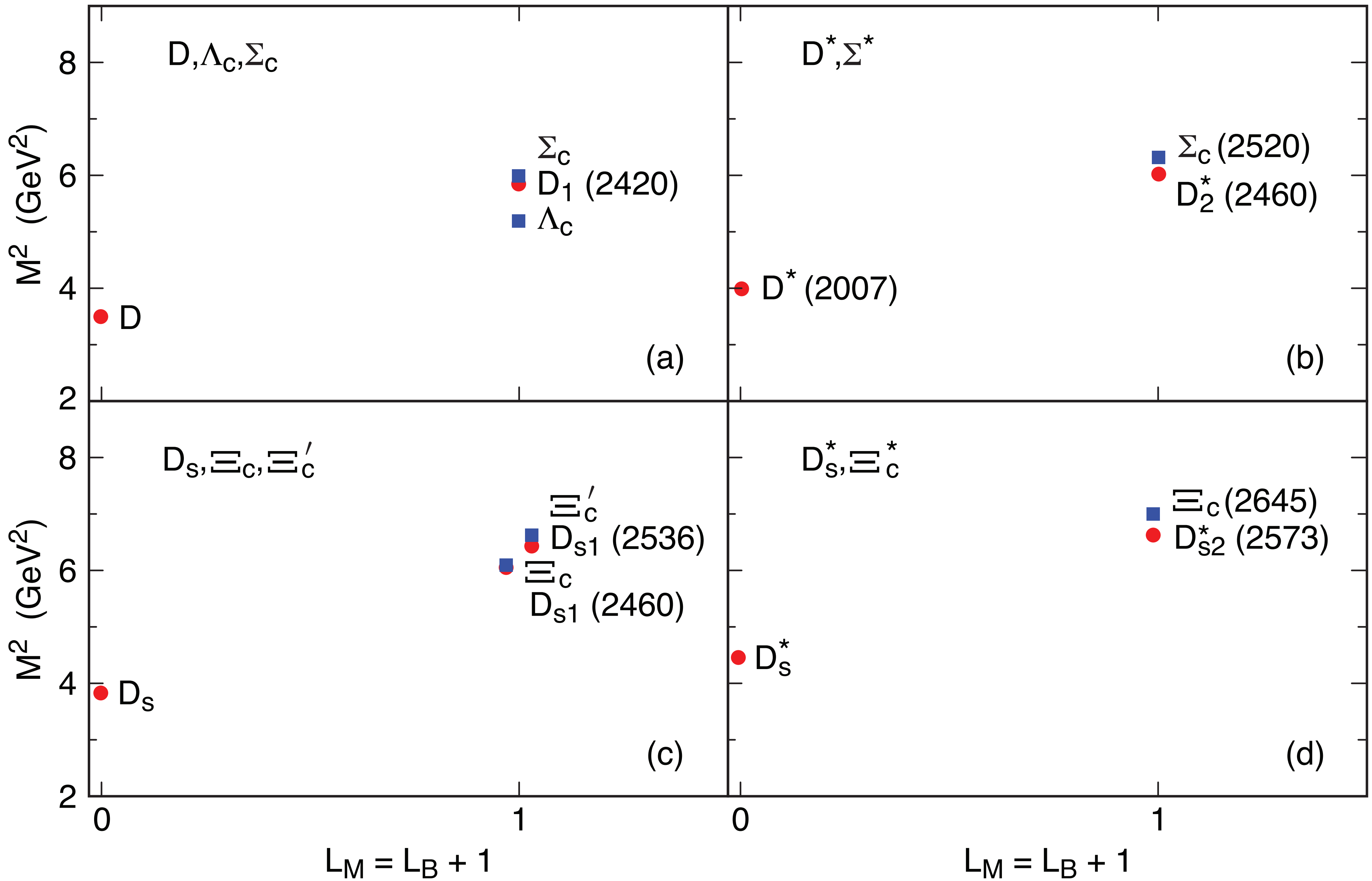


Dosch, de Teramond, sjb

Dosch, de Teramond, sjb

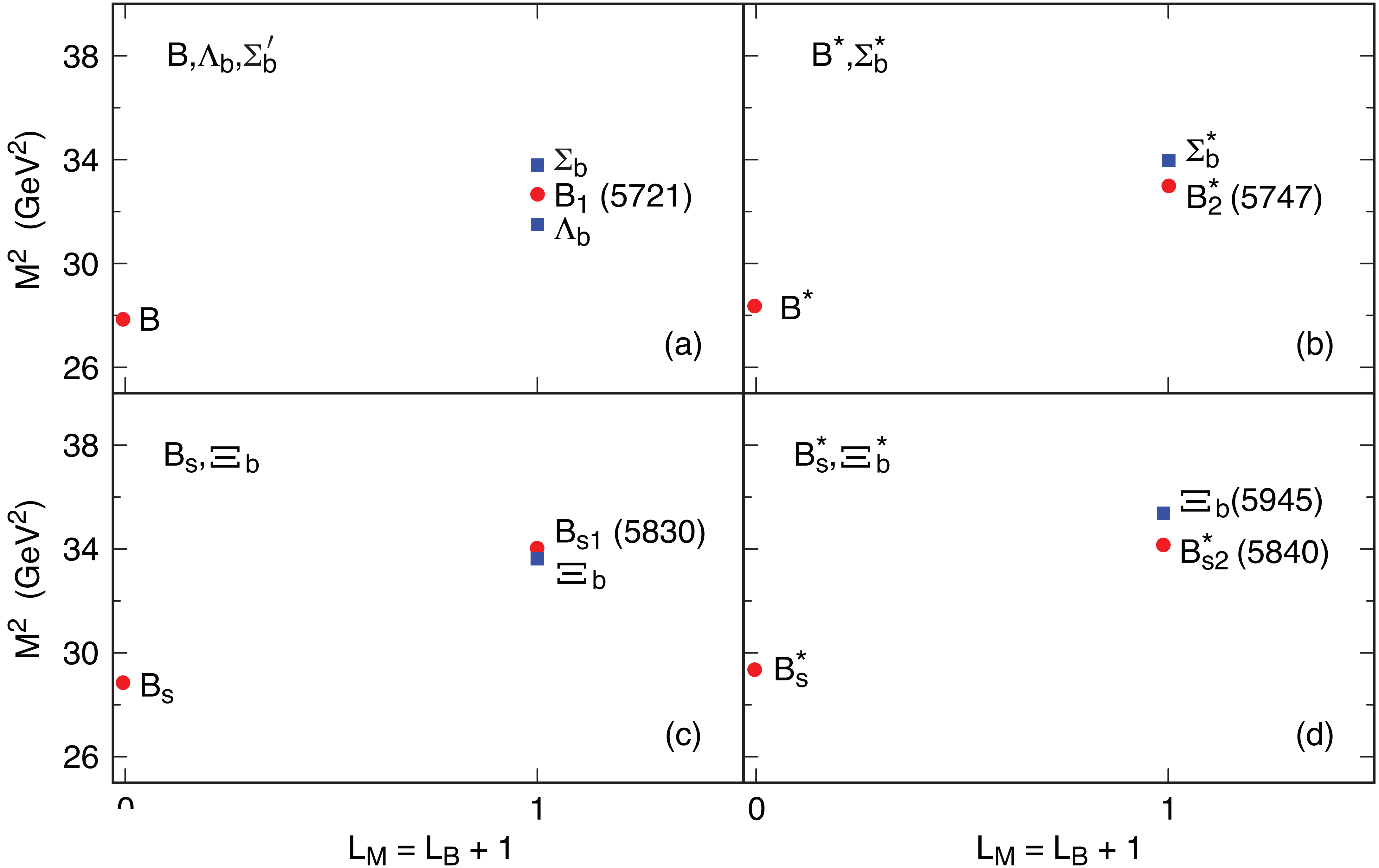
Supersymmetry across the light and heavy-light spectrum

- Introduction of quark masses breaks conformal symmetry without violating supersymmetry



Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Supersymmetric relations for mesons and baryons with b quarks

Features of Supersymmetric Equations

- $J = L + S$ baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\qquad S^z = \pm 1/2$

- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$

- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

Meson and baryon have same κ !

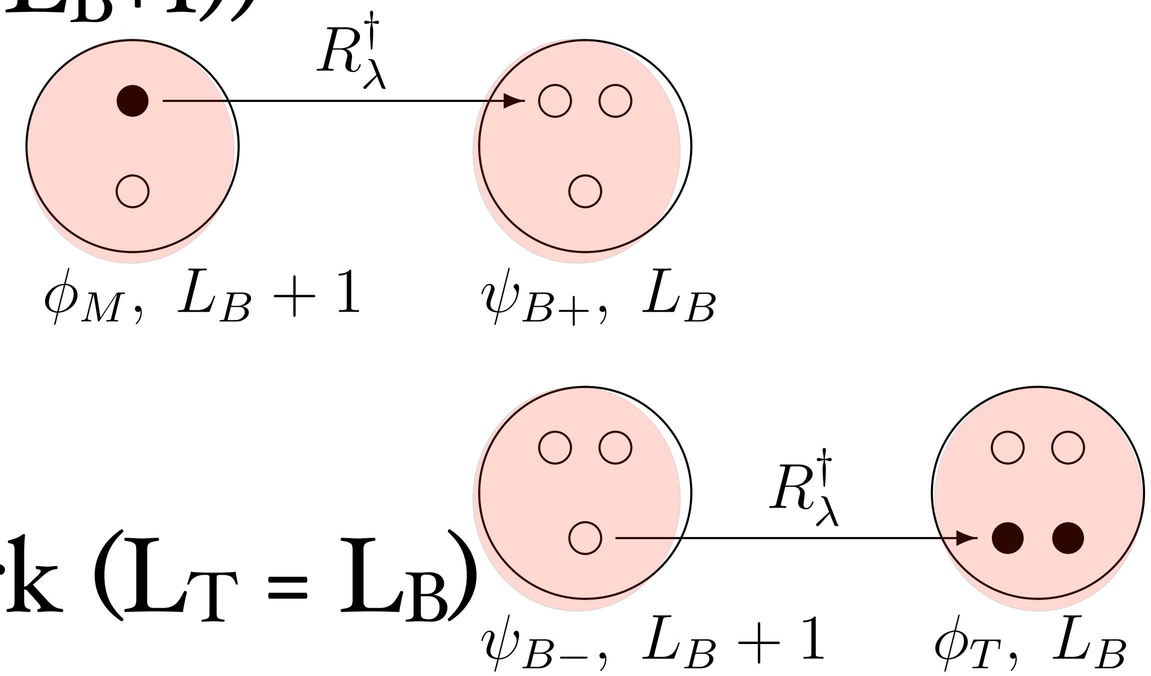
de Tèramond, Dosch, Lorce, sjb

Superconformal Algebra

2X2 Hadronic Multiplets

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

- quark-antiquark meson ($L_M = L_B$+1))
- quark-diquark baryon (L_B)
- quark-diquark baryon (L_B+1)
- diquark-antidiquark tetraquark ($L_T = L_B$)
- Universal Regge slopes $\lambda = \kappa^2$



$$M_H^2/\lambda = \overbrace{\underbrace{(2n + L_H + 1)}_{kinetic} + \underbrace{(2n + L_H + 1)}_{potential}}^{contribution\ from\ 2\text{-}dim\ light\text{-}front\ harmonic\ oscillator} + \overbrace{2(L_H + s) + 2\chi}^{contribution\ from\ AdS\ and\ superconformal\ algebra} + < \sum_i \frac{m_i^2}{x_i} >$$

$\chi(mesons) = -1$ $\chi(baryons, tetraquarks) = +1$

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

- Diquark: Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states $\bar{3}_C \times 3_C = 1_C$

$$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

$$2[\sigma([\{qq\}N) + \sigma(qN)] - [\sigma(qN) + \sigma(\bar{q}N)] = [\sigma(\{qq\}N) + \sigma(\{qq\}N)]$$

Candidates $f_0(980) I = 0, J^P = 0^+$, partner of proton

$a_1(1260) I = 0, J^P = 1^+$, partner of $\Delta(1233)$

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

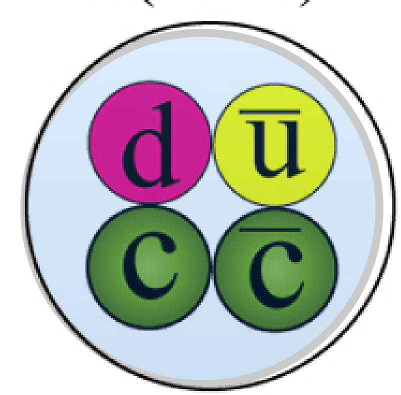
- Diquark Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states
- Confinement Force Similar to quark-antiquark mesons $\bar{3}_C \times 3_C = 1_C$
- Isospin $I = 0, \pm 1, \pm 2$ Charge $Q = 0, \pm 1, \pm 2$

Complete Regge spectrum in n, L

$uu\bar{d}\bar{d}$ $\quad$ $uu\bar{s}\bar{d}$ $\quad$ $uu\bar{s}\bar{s}$

Q= +2

Z(4430)



Q= - 1

Universal Hadronic Features

- **Universal quark light-front kinetic energy**

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

Equal: Virial Theorem!

- **Universal quark light-front potential energy**

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- **Universal Constant Term**

$$\mathcal{M}^2_{spin} = 2\kappa^2(S + L - 1 + 2n_{diquark})$$

$$M^2 = \Delta\mathcal{M}^2_{LFKE} + \Delta\mathcal{M}^2_{LFPE} + \Delta\mathcal{M}^2_{spin}$$

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons, baryons***
- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

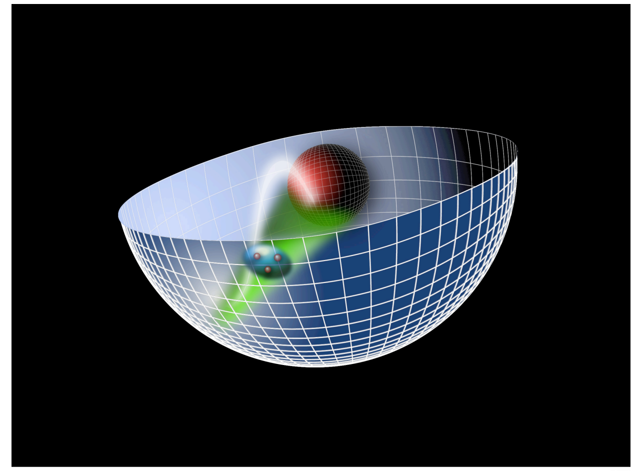
Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

$$\kappa \simeq 0.5 \ GeV$$

Confinement scale:

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

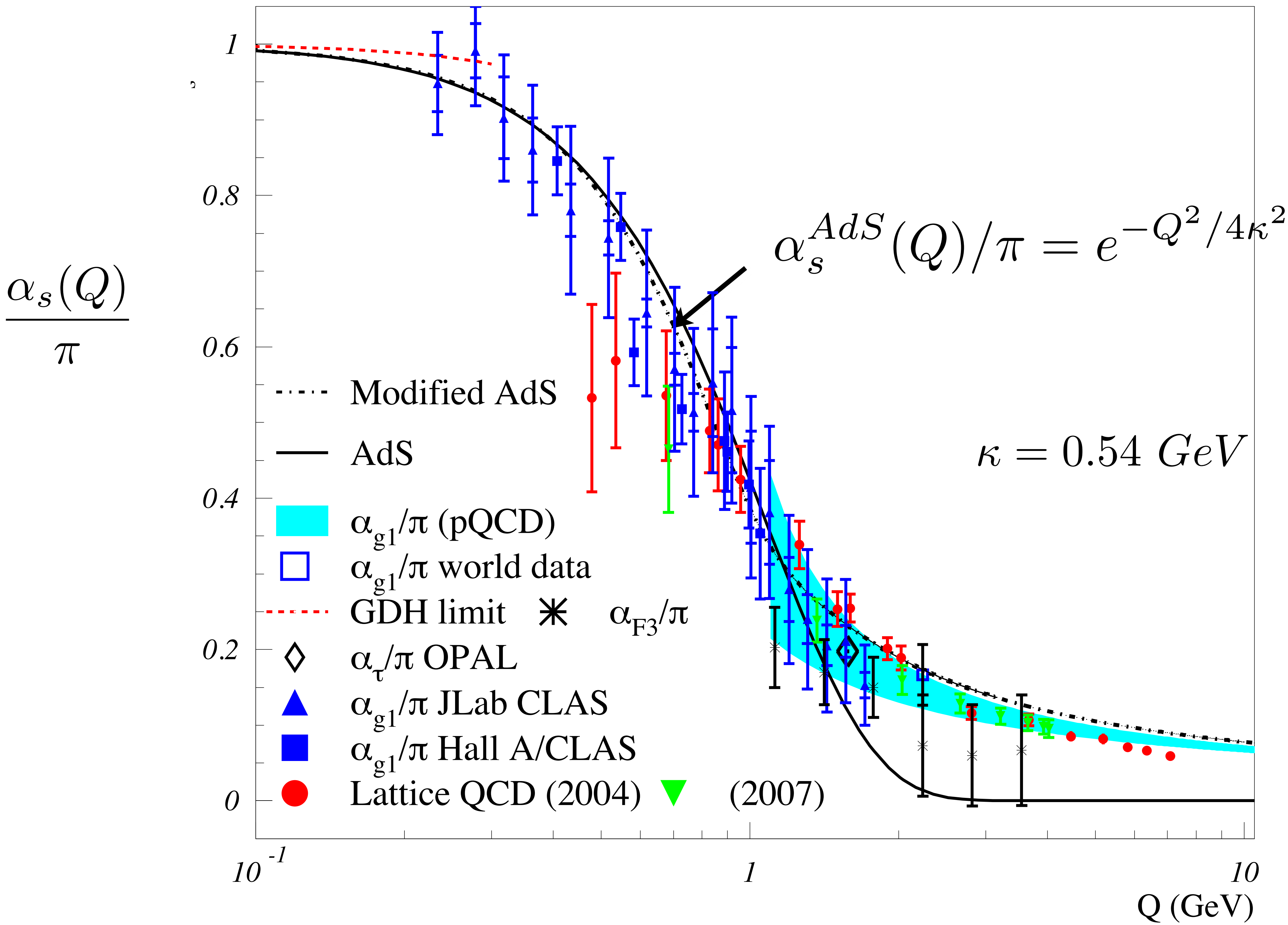
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Deur, de Tèramond, sjb

All-Scale QCD Coupling

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

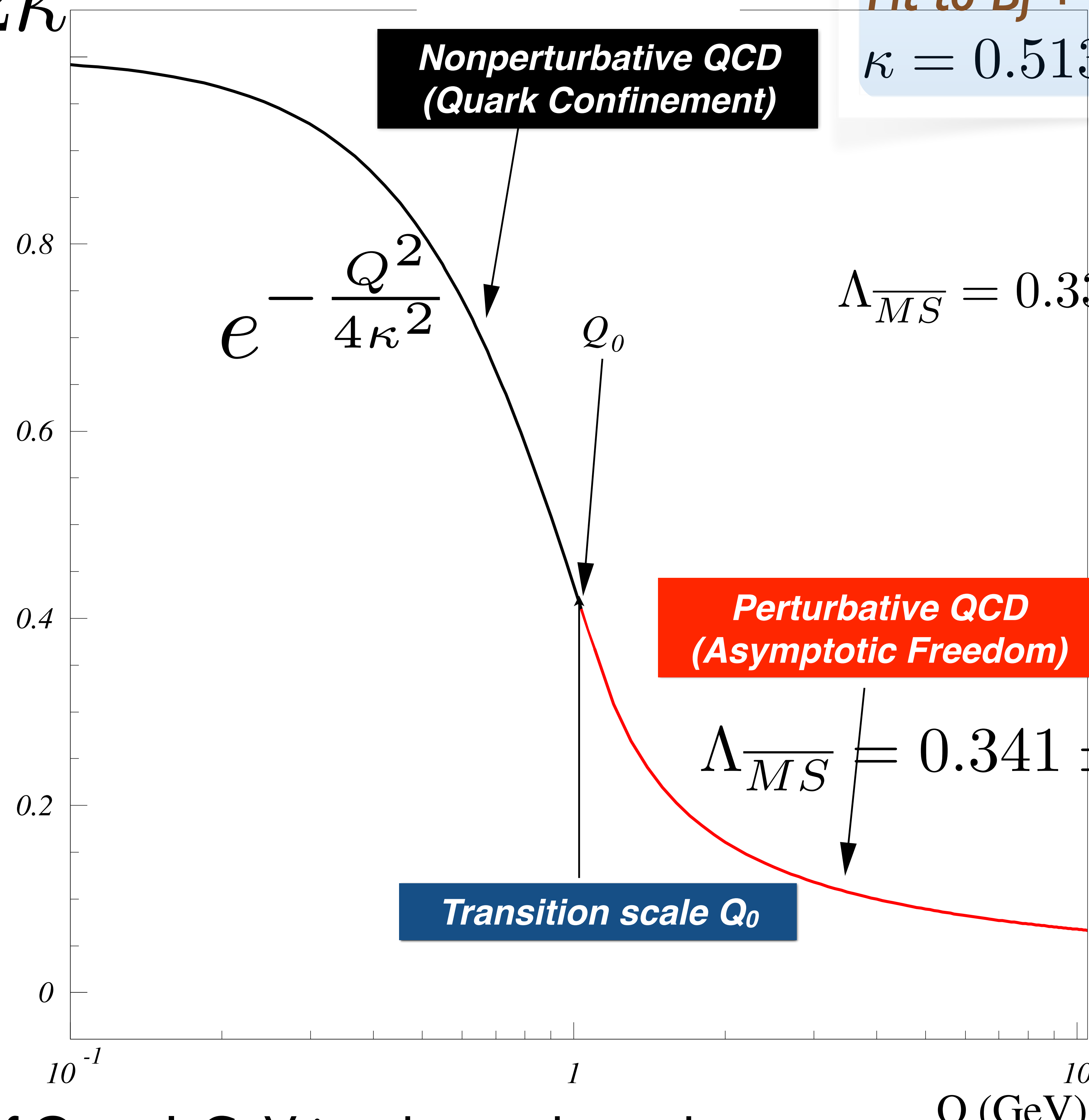
Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$

Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



$\overline{MS}$ scheme

$$\lambda \equiv \kappa^2$$

The value of Q_0 ~ 1 GeV is scheme dependent

Features of AdS/QCD

de Tèramond, Dosch, Deur, sjb

- **Color confining potential $\kappa^4\zeta^2$ and universal mass scale from dilaton** $e^{\phi(z)} = e^{\kappa^2 z^2}$ $\alpha_s(Q^2) \propto \exp -Q^2/4\kappa^2$
- **Dimensional transmutation** $\Lambda_{\overline{MS}} \leftrightarrow \kappa \leftrightarrow m_H$
- **Chiral Action remains conformally invariant despite mass scale** *DAFF*
- **Light-Front Holography: Duality of AdS and frame-independent LF QCD**
- **Reproduces observed Regge spectroscopy — same slope in n, L, and J for mesons and baryons**
- **Massless pion for massless quark**
- **Supersymmetric meson-baryon dynamics and spectroscopy: $L_M = L_B + 1$**
- **Dynamics: LFWFs, Form Factors, GPDs**

Superconformal Quantum Mechanics
Fubini and Rabinovici

de Tèramond, Dosch, sjb

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21\ \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2\ \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

$$(m_q = 0)$$
$$m_\pi = 0$$

$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Fundamental Features of Hadrons

Virial Theorem

- Partition of the Proton's Mass: Potential vs. Kinetic Contributions

$$\Delta \mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$
$$\Delta \mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- Color Confinement $U(\zeta^2) = \kappa^4 \zeta^2$
- Role of Quark Orbital Angular Momentum in the Proton

Equal L=0,1

- Quark-Diquark Structure
- Quark Mass Contribution $\Delta M^2 = < \frac{m_q^2}{x} >$

from the Yukawa coupling to the Higgs zero mode

- Baryonic Regge Trajectory $M^2_p(n, L_B) = 4\kappa^2(n + L_B + 1)$
- Mesonic Supersymmetric Partners $L_M = L_B + 1$
- Proton Light-Front Wavefunctions and Dynamical Observables

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

- Form Factors, Distribution Amplitudes, Structure Functions
- Non-Perturbative - Perturbative QCD Transition $Q_0 = 0.87 \pm 0.08\ GeV\ \overline{MS}\ scheme$
- Dimensional Transmutation:

$$m_p \simeq 3.21\ \Lambda_{\overline{MS}}$$
$$m_\rho \simeq 2.2\ \Lambda_{\overline{MS}}$$

de Tèramond, Dosch, Lorce, sjb

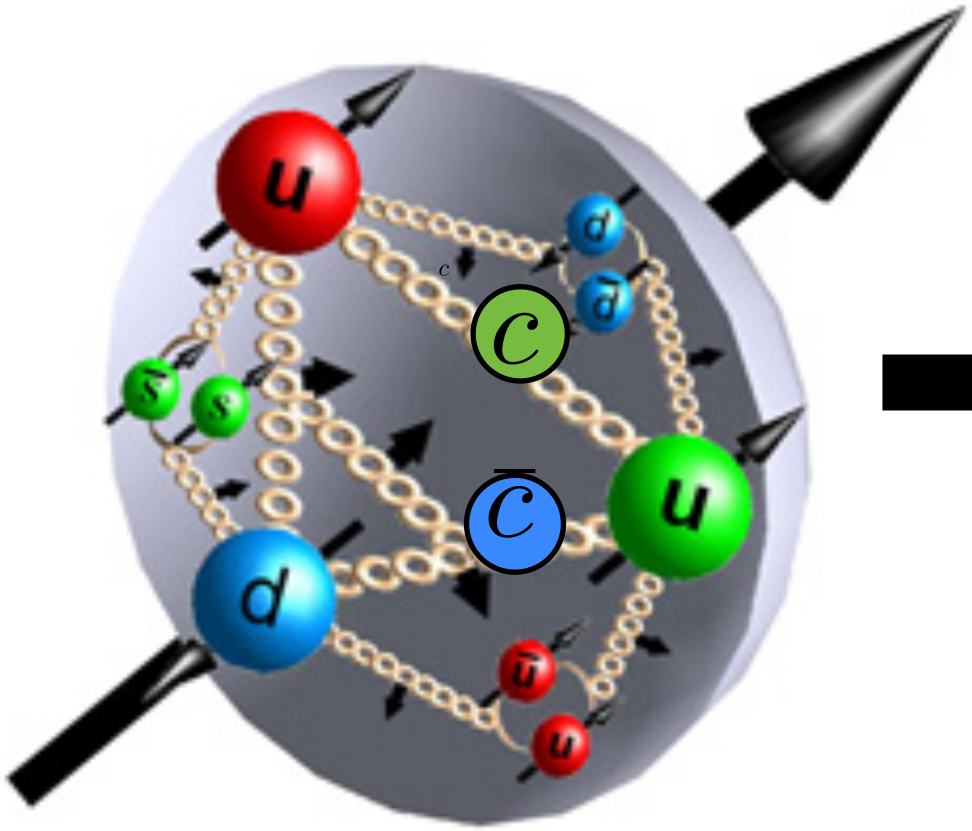
Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffractive dissociation of pion and proton to jets**
- **Factorization Scale for ERBL, DGLAP evolution: Q_0**
- **Calculate Sivers Effect including FSI and ISI**
- **Compute Tetraquark Spectroscopy: Sequential Clusters**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization** *Vary, sjb*

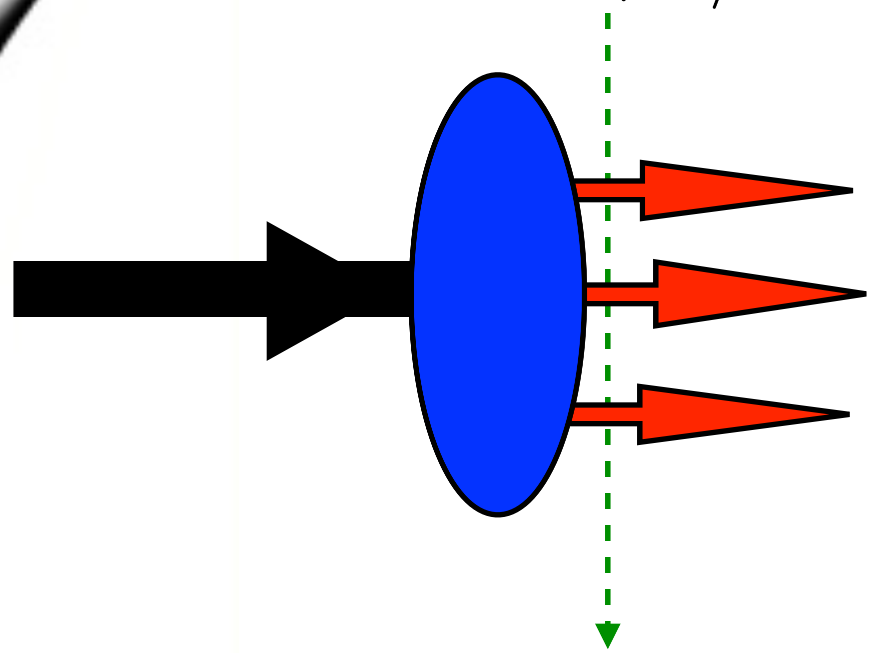
QCD Myths

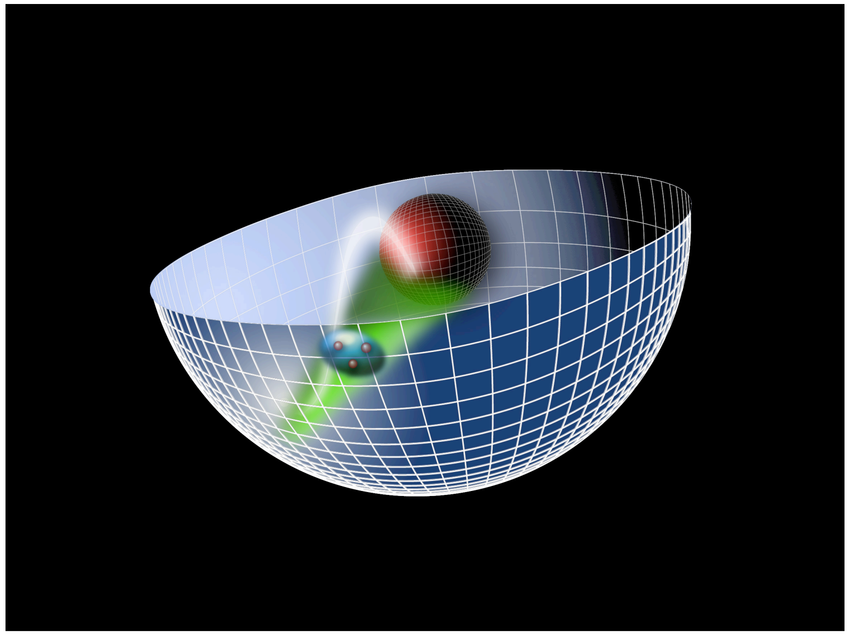
- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- QCD gives 10^{42} to the cosmological constant
- Colliding Pancakes at RHIC

The QCD Vacuum, Color Confinement, and Superconformal Properties of Hadron Physics



Fixed $\tau = t + z/c$





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