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Saturation and geometrical scaling in high energy collisions

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Kitzbuehel 1.7.2016.

Is multiparticle production still interesting in the LHC era?

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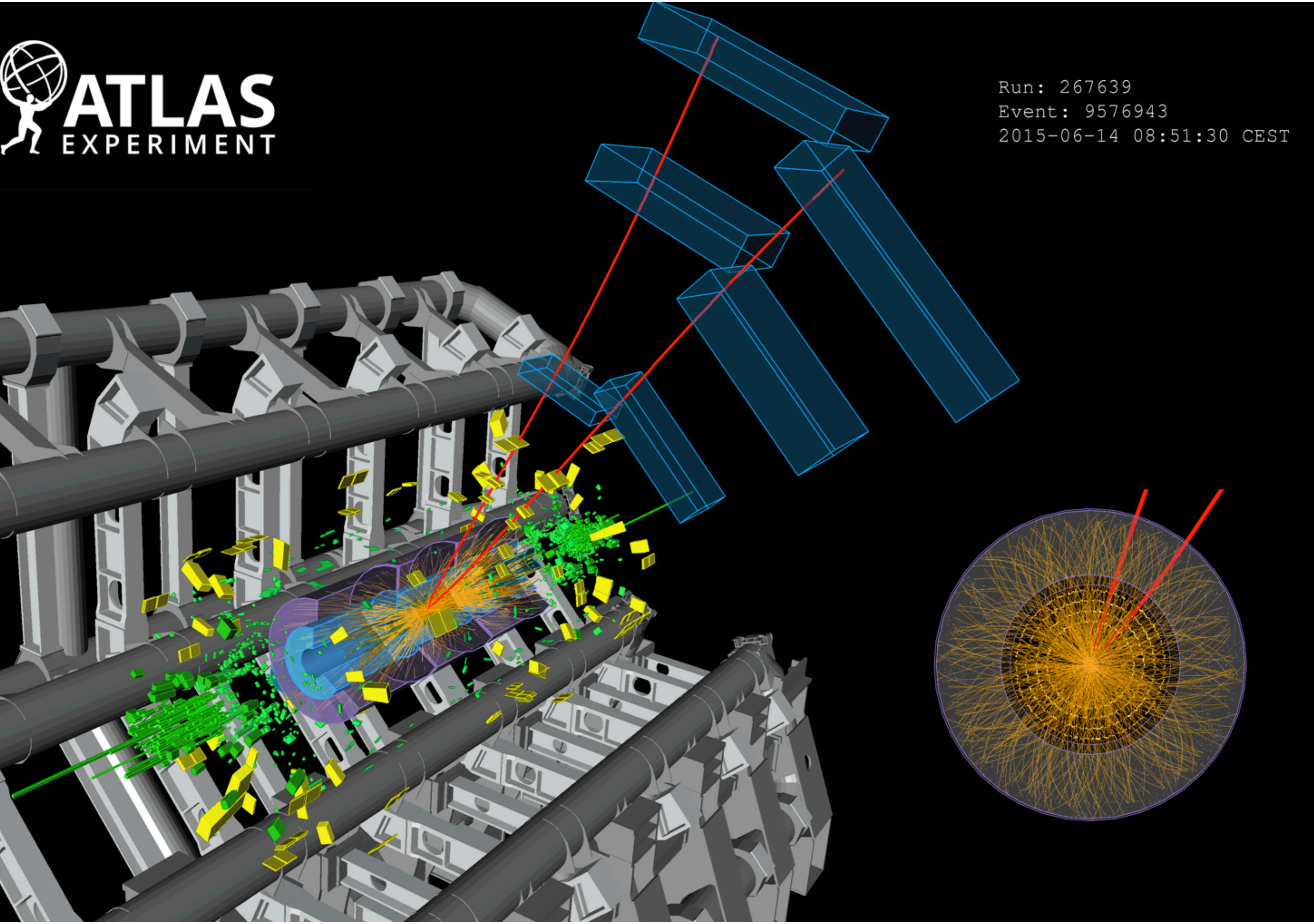

ATLAS
EXPERIMENT
Run: 267639
Event: 9576943
2015-06-14 08:51:30 CEST

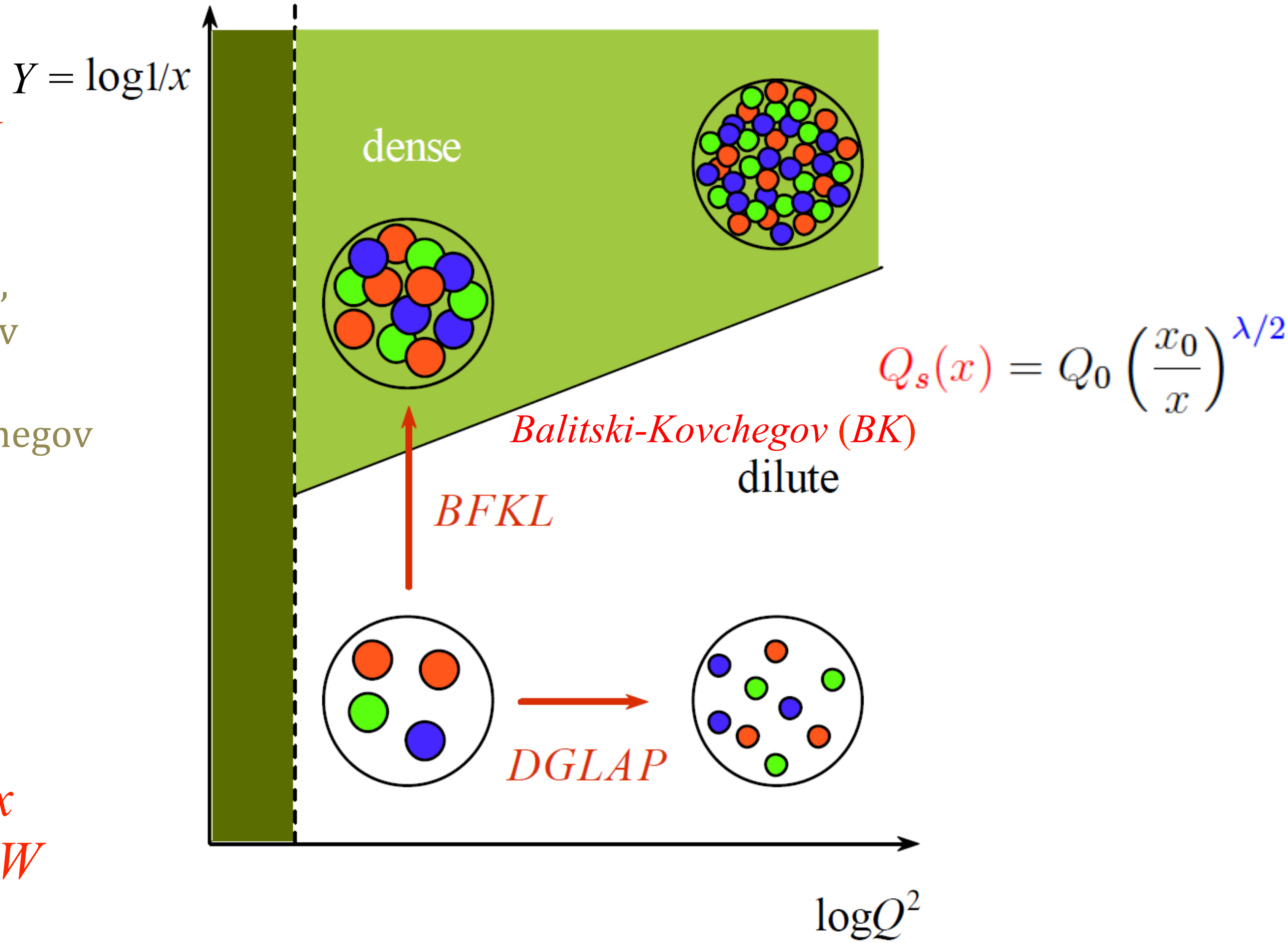
DGLAP vs BFKL Evolution

small x
large W

Balitsky, Fadin,
Kuraev, Lipatov

Balitsky, Kovchegov

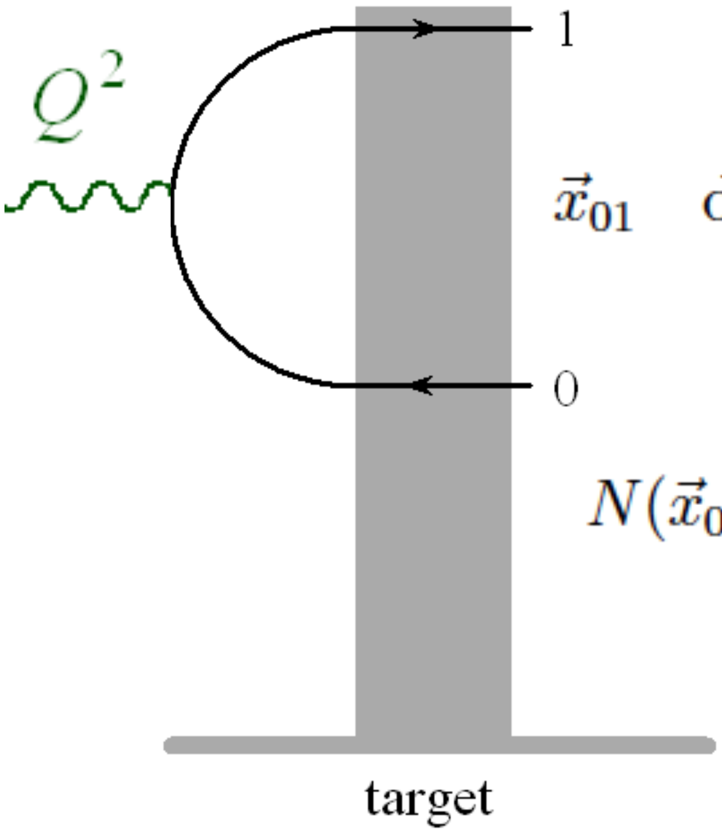
large x
small W



Dipole Picture

BFKL equation has very simple form and interpretation in the dipole picture of A. Mueller

A.H. Mueller and J.-w. Qiu,
Nucl. Phys. B 268 (1986) 427



$\vec{x}_{01}$ dipole transverse size $Y = \log 1/x$

$N(\vec{x}_{01}, Y)$ dipole-target forward amplitude

BK Equation

in terms of a Fourier transform: $$N(x,Y) \;=\; x^2 \int \frac{d^2\vec{k}}{2\pi} e^{i\vec{k}\cdot\vec{x}}\, \tilde{N}(k,Y)$$

$$\frac{\partial}{\partial Y}\tilde{N}(k,Y) \;=\; \overline{\alpha}_s\, \chi(-\partial/\partial \ln k^2)\, \tilde{N}(k,Y) - \overline{\alpha}_s\, \tilde{N}^2(k,Y)$$

here χ is a BFKL characteristic function

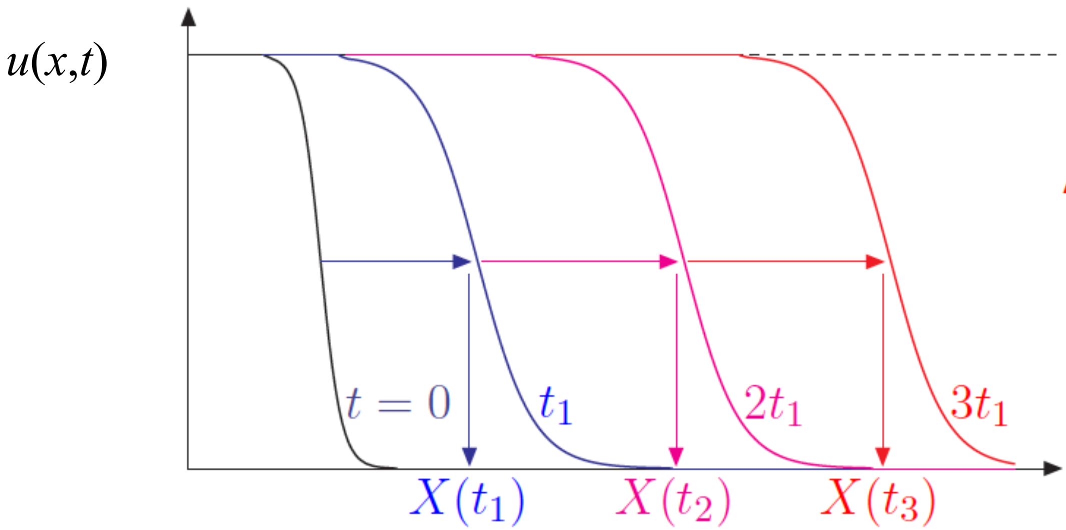
$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

there exists a theorem from the ‘30 (Fisher, Kolomogorov, Petrovsky, Piscounov) that non-linear equations of this sort have asymptotically travelling wave solutions

S. Munier, R.B. Peschanski
PRL 91 (2003) 232001
PRD 69 (2004) 034008

Travelling waves

identify time : $t = Y$, position : $x = \ln k^2$



Asymptotic solution:
travelling wave

$u(x,t) = u(x - v_c t)$

Position: $X(t) = X_0 + v_c t$ © G. Soyez

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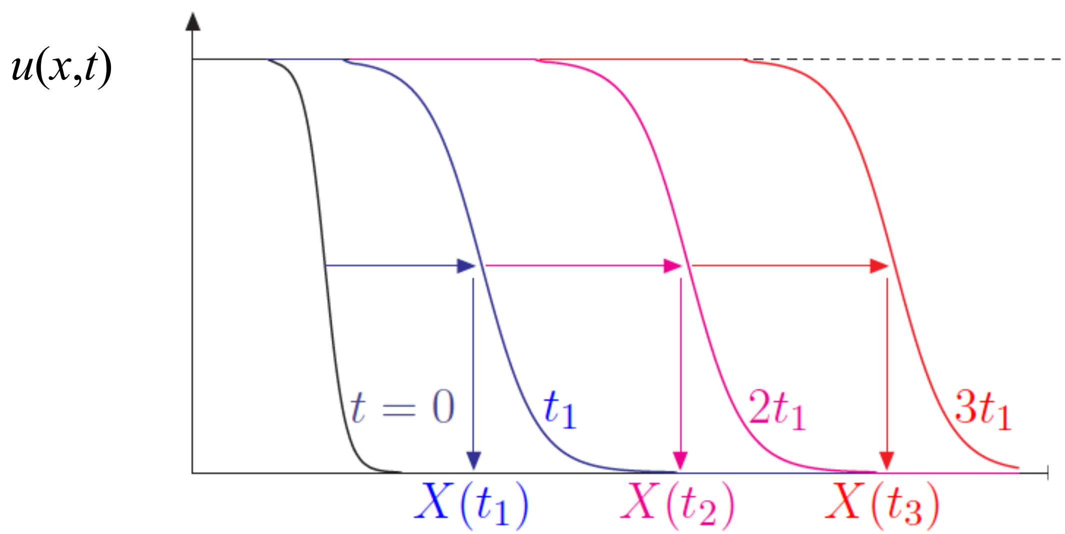

Position: $X(t) = X_0 + v_c t$ © G. Soyez

Asymptotic solution: travelling wave

$$u(x,t) = u(x - v_c t)$$

$$\begin{aligned} x - v_c t &= \log\left(\frac{k^2}{k_0^2}\right) - v_c \log\left(\frac{1}{x}\right) \\ &= \log\left[k^2 \times \frac{1}{k_0^2}\left(\frac{1}{x}\right)^{-v_c}\right] \\ &= \log\left(\frac{k^2}{Q_{\rm sat}^2(x)}\right) \end{aligned}$$

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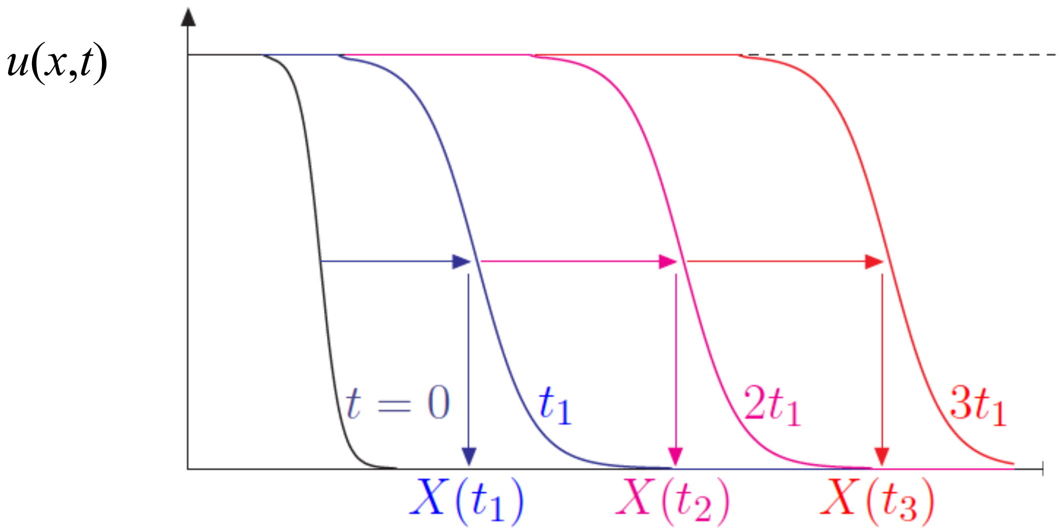
Travelling waves

identify $\quad \text{time}: \quad t = Y, \quad \text{position}: \quad x = \ln k^2$



Position: $X(t) = X_0 + v_c t$ © G. Soyez

$$Q^2_{\text{sat}}(x) = Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda}$$

Asymptotic solution:

travelling wave

$$u(x,t) = u(x - v_c t)$$

$$\begin{aligned} x - v_c t &= \log\left(\frac{k^2}{k_0^2}\right) - v_c \log\left(\frac{1}{x}\right) \\ &= \log\left[k^2 \times \frac{1}{k_0^2}\left(\frac{1}{x}\right)^{-v_c}\right] \\ &= \log\left(\frac{k^2}{Q^2_{\text{sat}}(x)}\right) \end{aligned}$$

Conlusions

- Nonlinear BK equation generates sauration scale $Q_s(x)$

Travelling waves in QCD imply Geometrical Scaling

$$f(x, k^2) = \mathcal{F}\left(\frac{k^2}{Q_s^2(x)}\right)$$

$$Q_s(x) = Q_0 \left(\frac{x_0}{x}\right)^{\lambda/2}$$

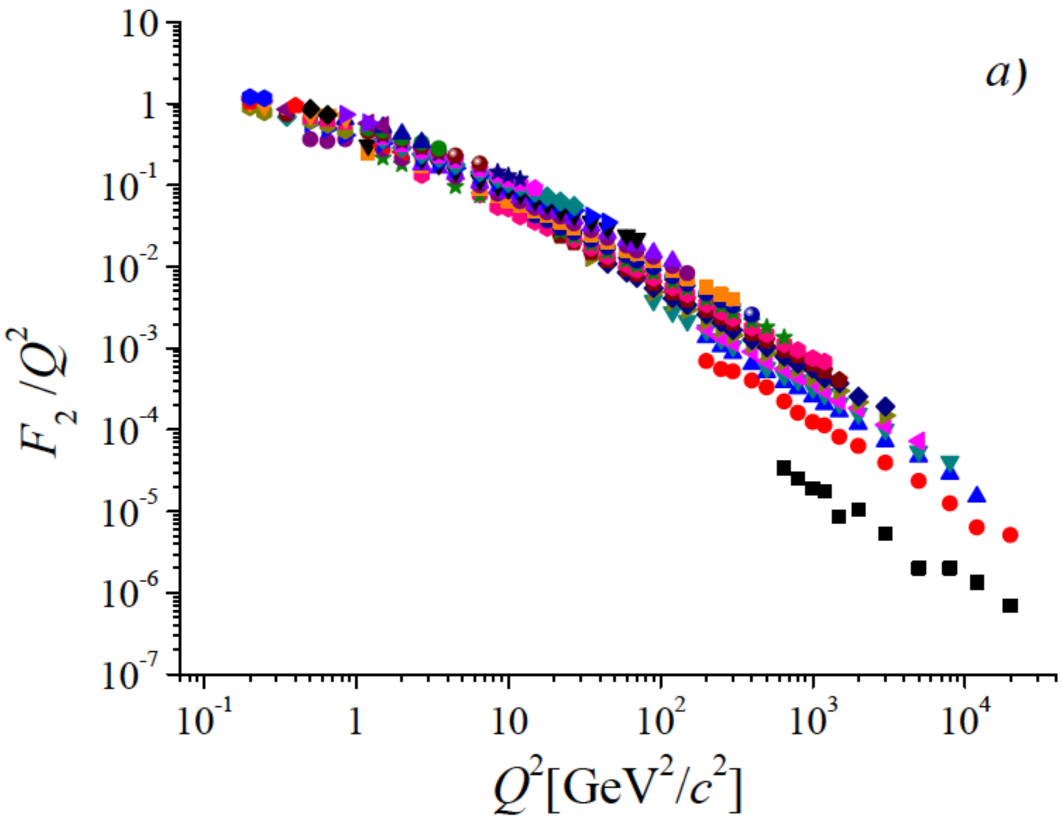
Conlusions

- Nonlinear BK equation generates sauration scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges

Deep Inealstic Scattering

Saturation scale: energy and x dependence

$$Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$

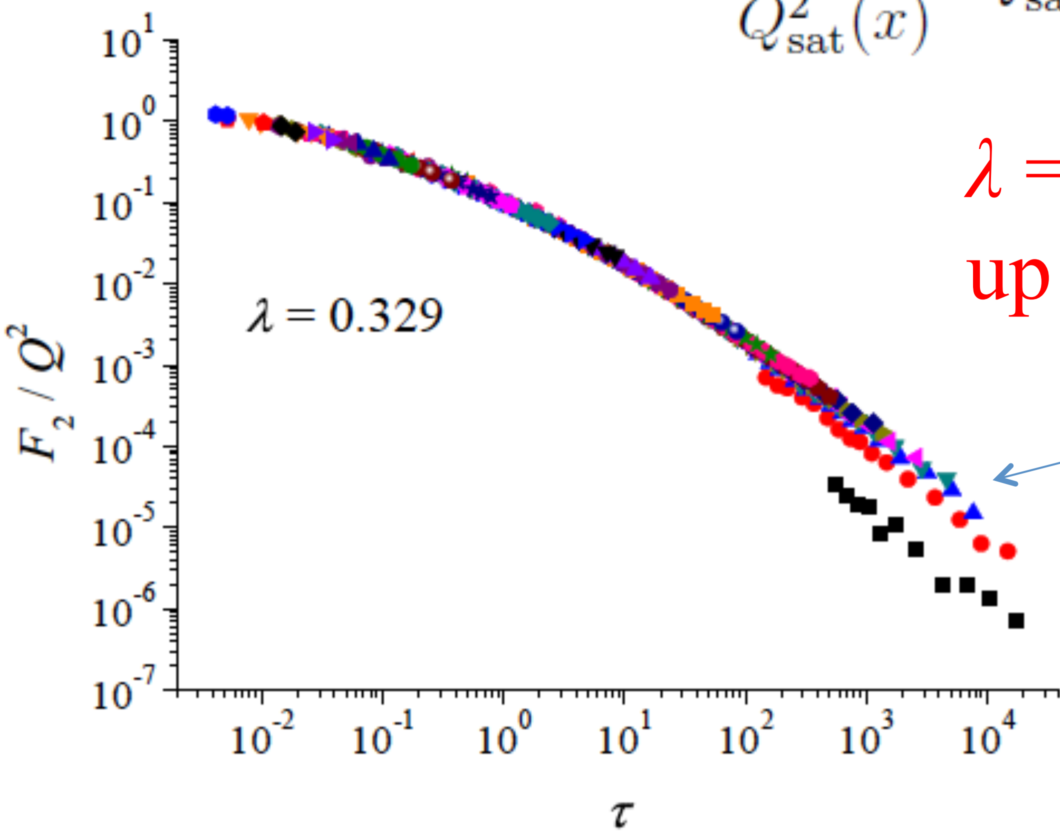


A.M. Stasto, K. J. Golec-Biernat, J. Kwiecinski
PRL 86 (2001) 596-599

M.Praszalowicz and T.Stebel
JHEP 1303, 090 (2013)
arXiv:1211.5305 [hep-ph]
and
JHEP 1304, 169 (2013)
arXiv:1302.4227 [hep-ph]

Saturation scale: energy and x dependence

$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$



$\lambda = 0.329 \pm 0.005$

up to $x = 0.08$ (!)

large x

more "sophisticated" scaling variables do not work well

Conlusions

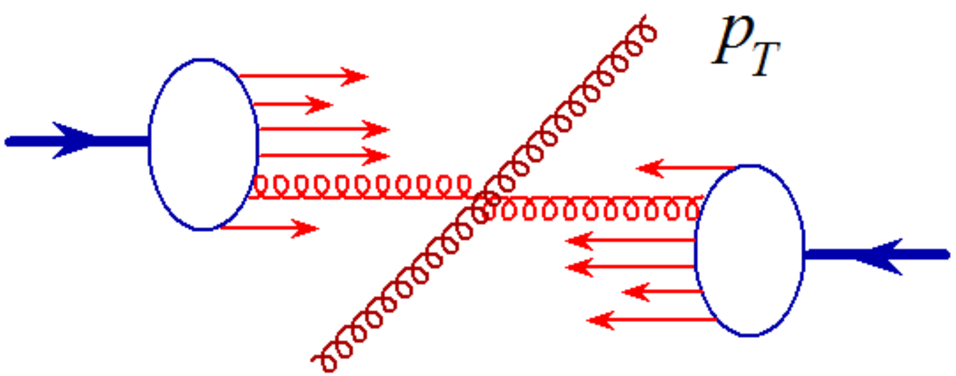
- Nonlinear BK equation generates sauration scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$

proton-proton @ LHC

Basics of geometrical scaling

Gribov, Levin Ryskin, *High p_T Hadrons In The Pionization Region In QCD*. Phys.Lett.B100:173-176,1981.

$$\frac{d\sigma}{dy d^2 p_{\rm T}} = \frac{3\pi \alpha_s}{2 p_{\rm T}^2} \int d^2 \vec{k}_{\rm T} \, \varphi_1(x_1, \vec{k}_{\rm T}^2) \varphi_2(x_2, (\vec{k} - \vec{p})_{\rm T}^2)$$



$$x_{1,2} = \frac{p_{\rm T}}{\sqrt{s}} e^{\pm y}$$

gluon distribution

unintegrated glue

$$xG(x, Q^2) = \int^{Q^2} dk_{\rm T}^2 \, \varphi(x, k_{\rm T}^2)$$

Kharzeev, Levin
Phys.Lett.B523:79-87,2001.

Basics of geometrical scaling

gluon distribution — unintegrated glue

$$xG(x,Q^2) = \int^{Q^2} dk_T^2\, \varphi(x,k_T^2)$$

Golec-Biernat – Wuesthoff (DIS)

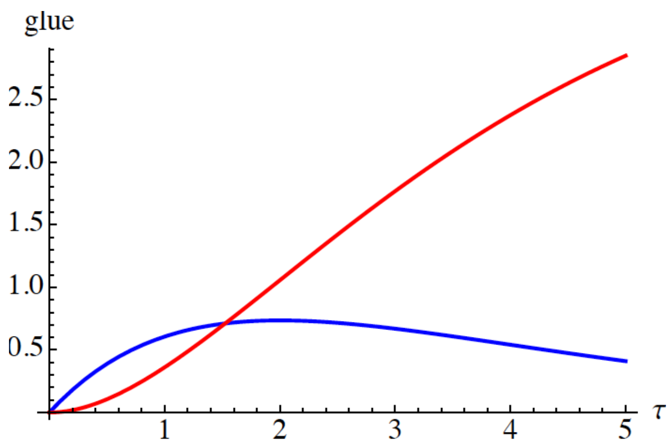
$$\varphi(x,k_T^2) = S_\perp \frac{3}{4\pi^2}\frac{k_T^2}{Q_s(x)^2}\exp\left(-k_T^2/Q_s(x)^2\right)$$

$$S_\perp = \sigma_0$$

Kharzeev – Levin (AA)

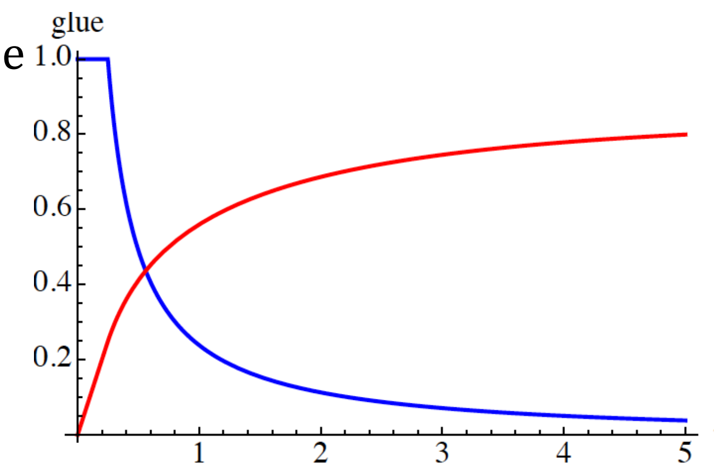
$$\varphi(x,k_T^2) = S_\perp \begin{cases} 1 & \text{for} \quad k_T^2 < Q_s(x)^2 \\ Q_s(x)^2/k_T^2 & \text{for} \quad Q_s(x)^2 < k_T^2 \end{cases}$$

$S_\perp$ is the transverse size given by geometry



scaling variable

$$\tau = \frac{p_T^2}{Q_s^2(x)}$$



Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dy d^2 p_{\mathrm{T}}} = \frac{3\pi\alpha_{\mathrm{s}}}{2} \frac{Q_s^2(x)}{p_{\mathrm{T}}^2} \int \frac{d^2\vec{k}_{\mathrm{T}}}{Q_s^2(x)} \varphi_1 \left(\vec{k}_{\mathrm{T}}^2 / Q_s^2(x) \right) \varphi_2 \left((\vec{k} - \vec{p})_{\mathrm{T}}^2 / Q_s^2(x) \right)$$

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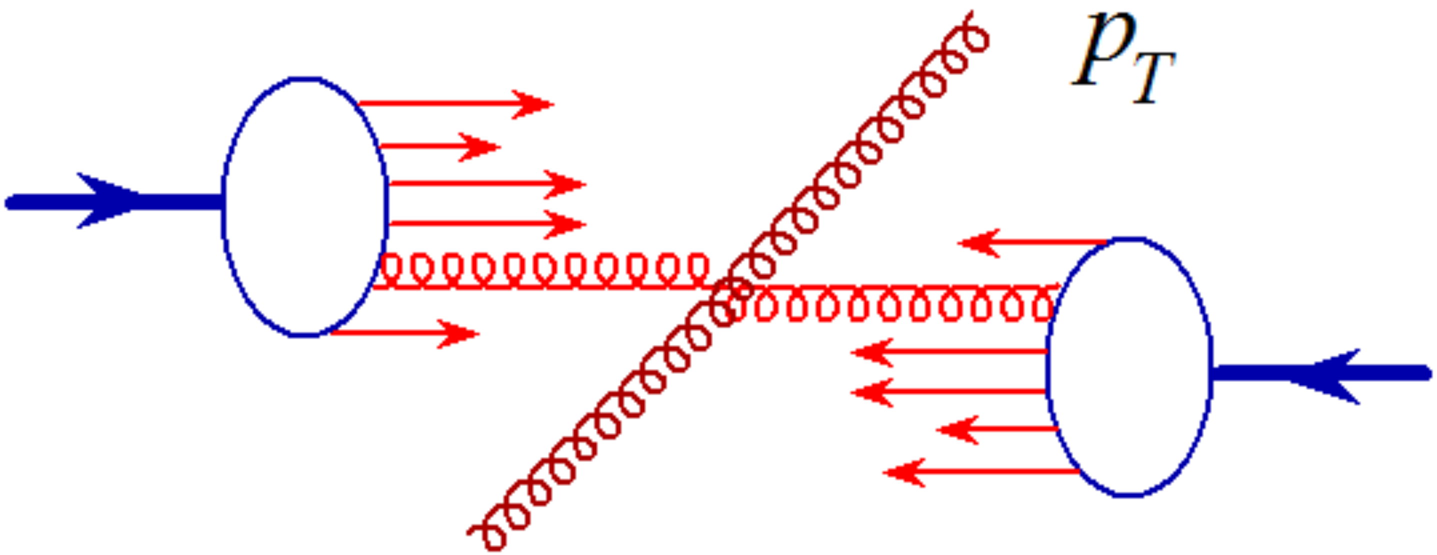
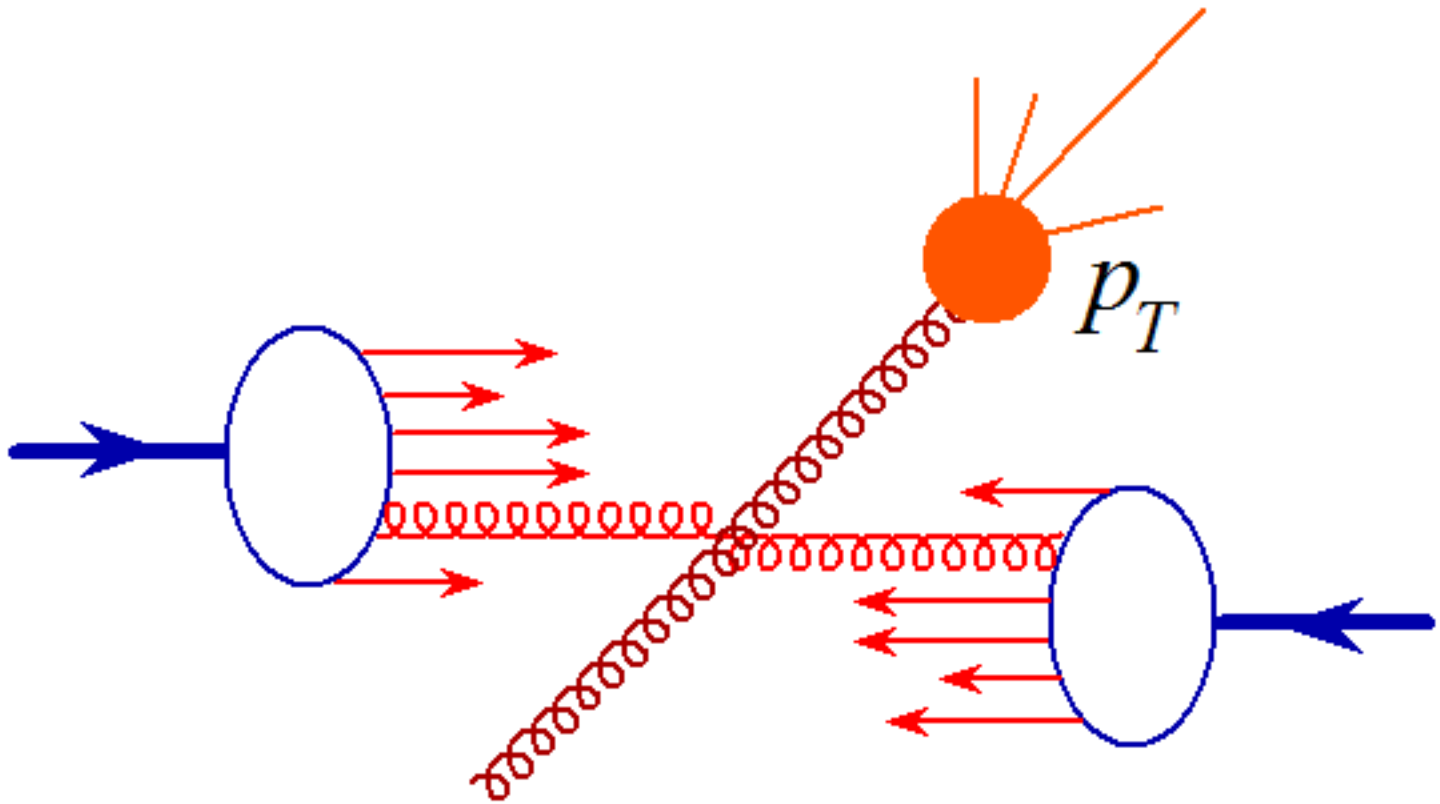
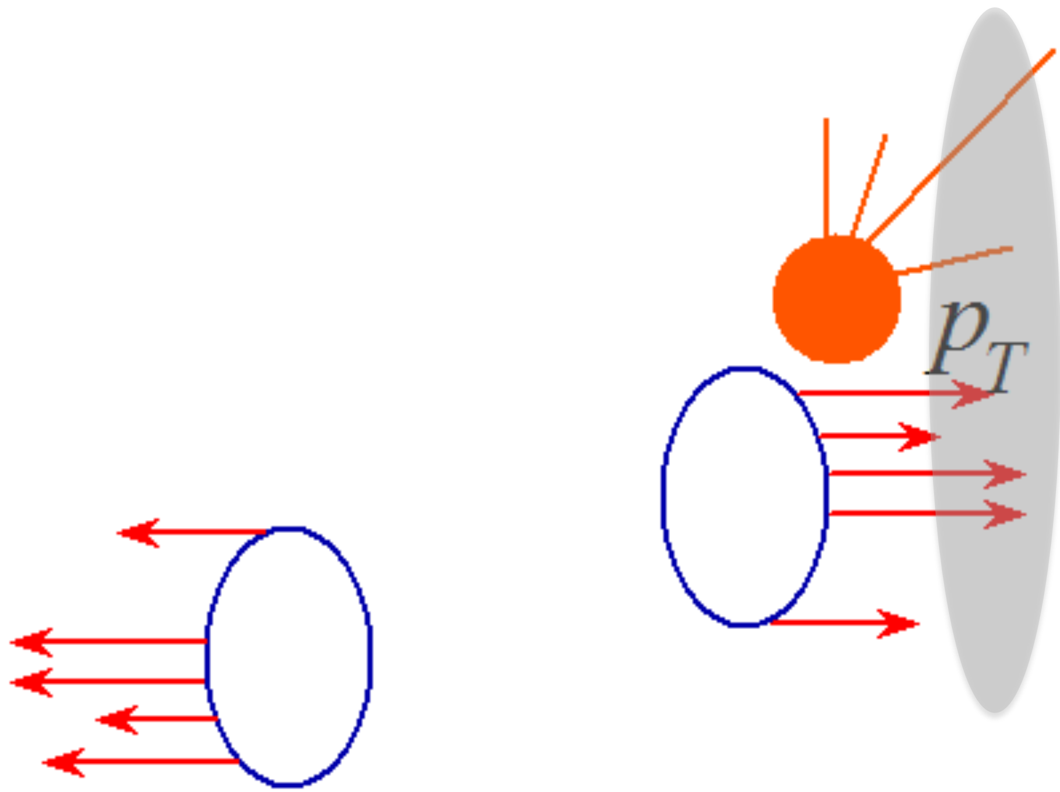
parton – hadron duality: power-like growth of particle multiplicity


p_T


p_T


p_T

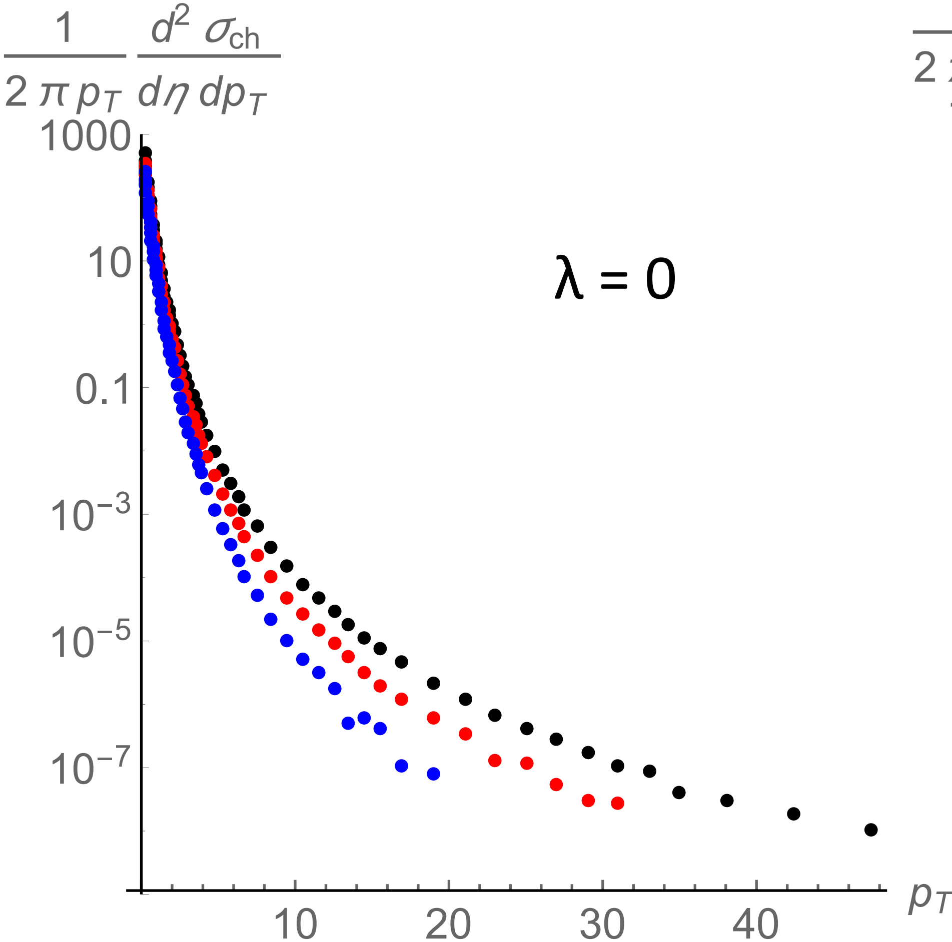
Geometrical scaling of p_T distribution

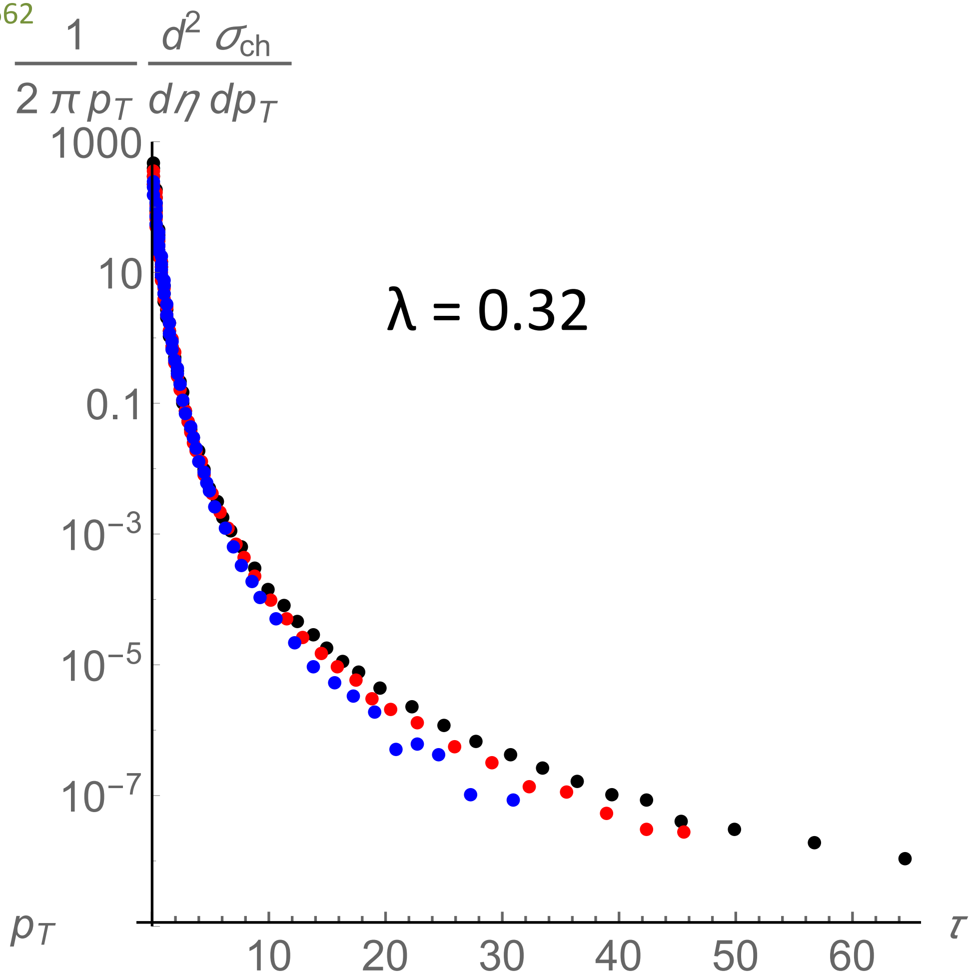
L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011
M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566
Phys.Rev. D87 (2013) 071502(R)

$$\tau = \frac{p_{\mathrm{T}}^2}{Q_{\mathrm{sat}}^2(p_{\mathrm{T}}/\sqrt{s})} = \frac{p_{\mathrm{T}}^2}{1\,\mathrm{GeV}^2}\left(\frac{p_{\mathrm{T}}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

Cross-section scaling in pp

ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662

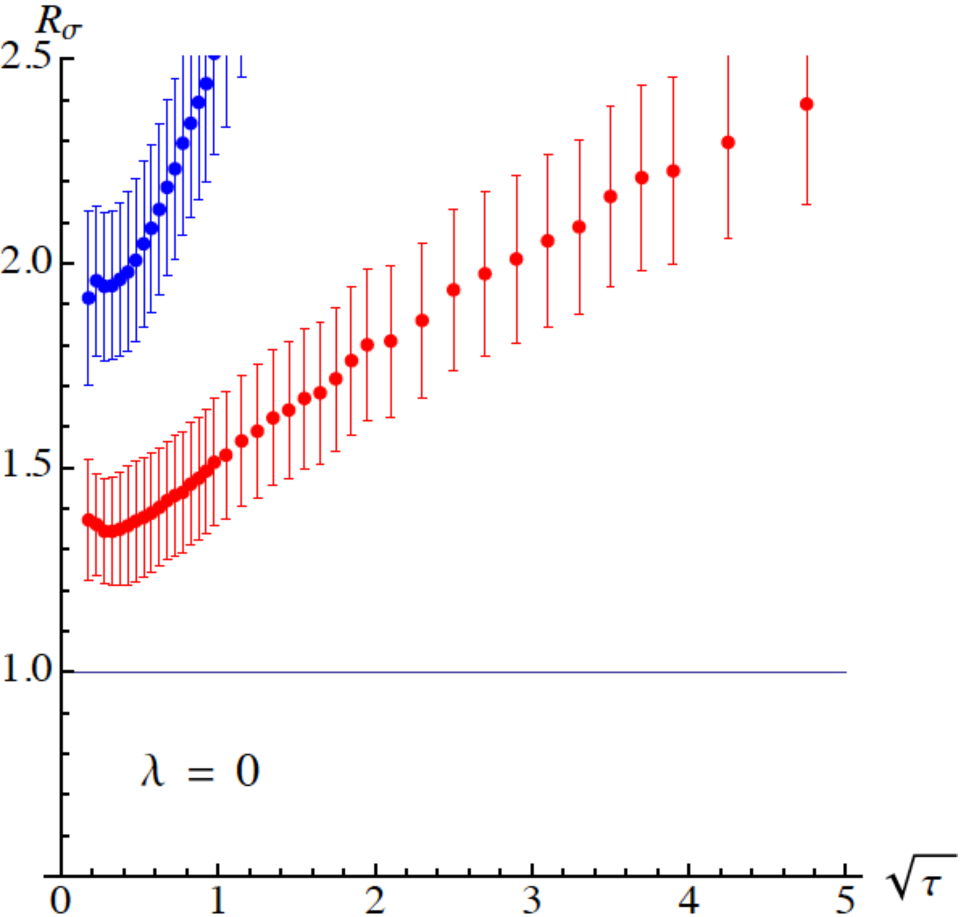




ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662

Determination of lambda

$$\tau = \frac{p_{\mathrm{T}}^2}{Q_{\mathrm{sat}}^2(p_{\mathrm{T}}/\sqrt{s})} = \frac{p_{\mathrm{T}}^2}{1\,\mathrm{GeV}^2}\left(\frac{p_{\mathrm{T}}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$



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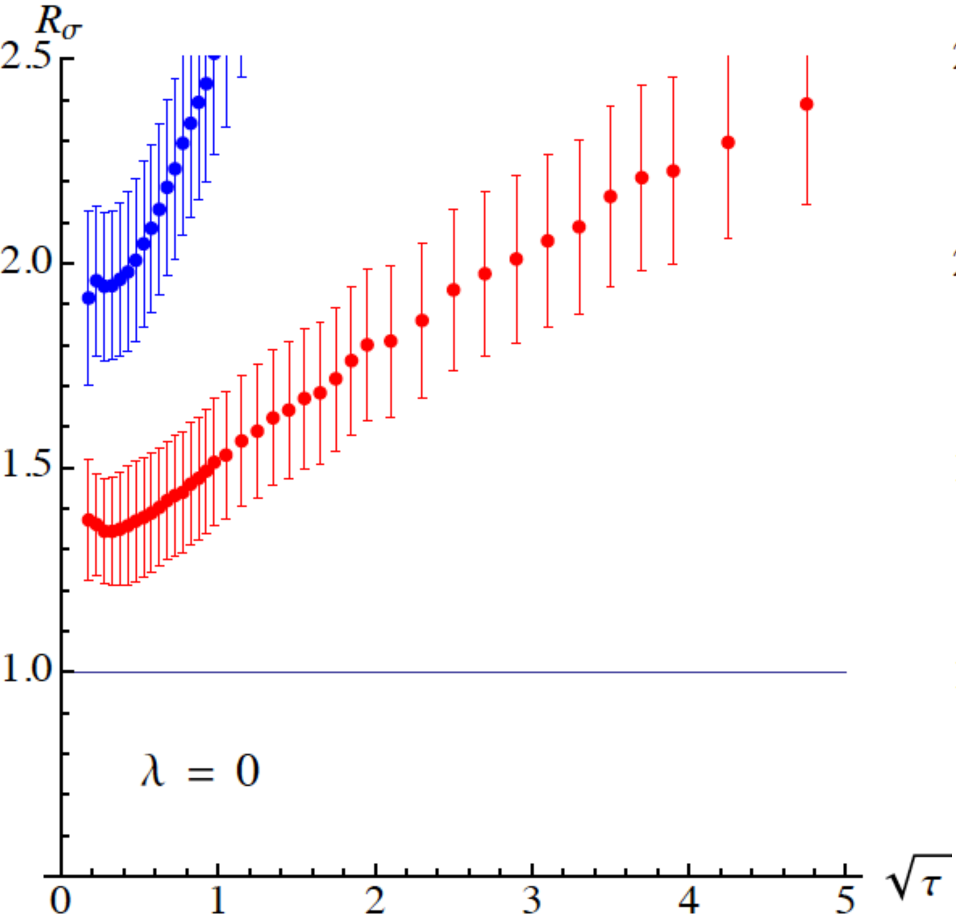

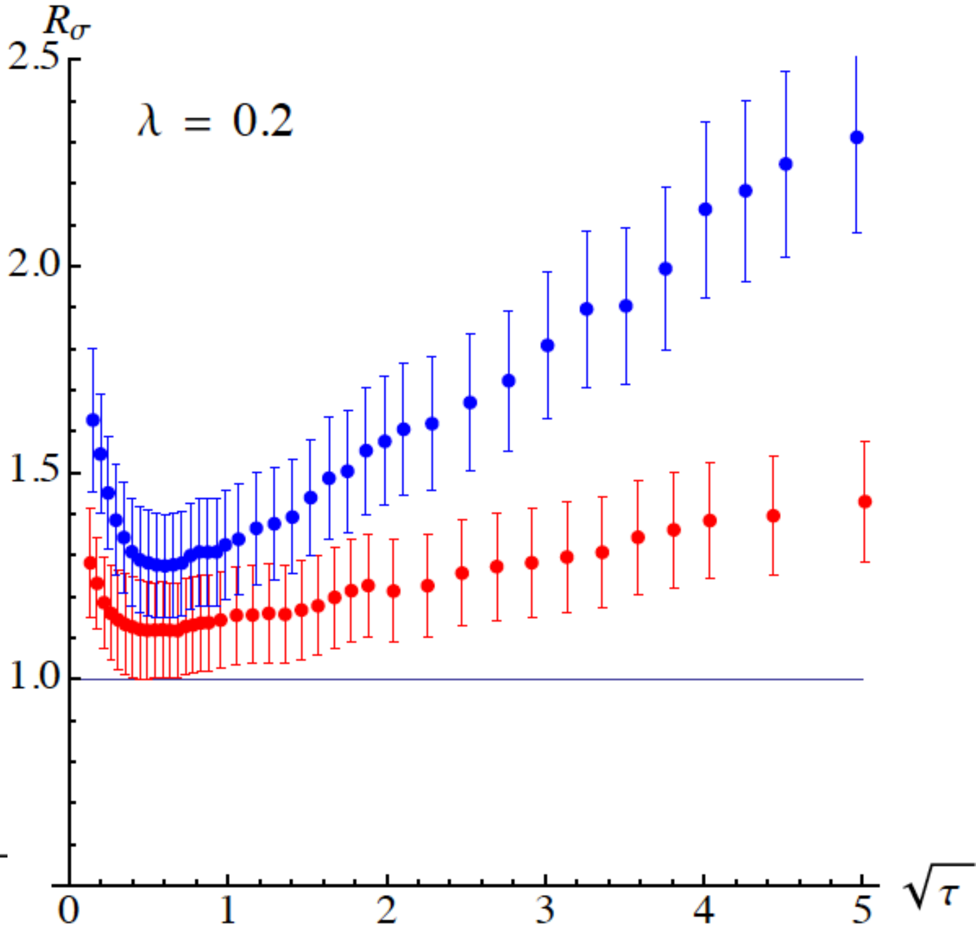

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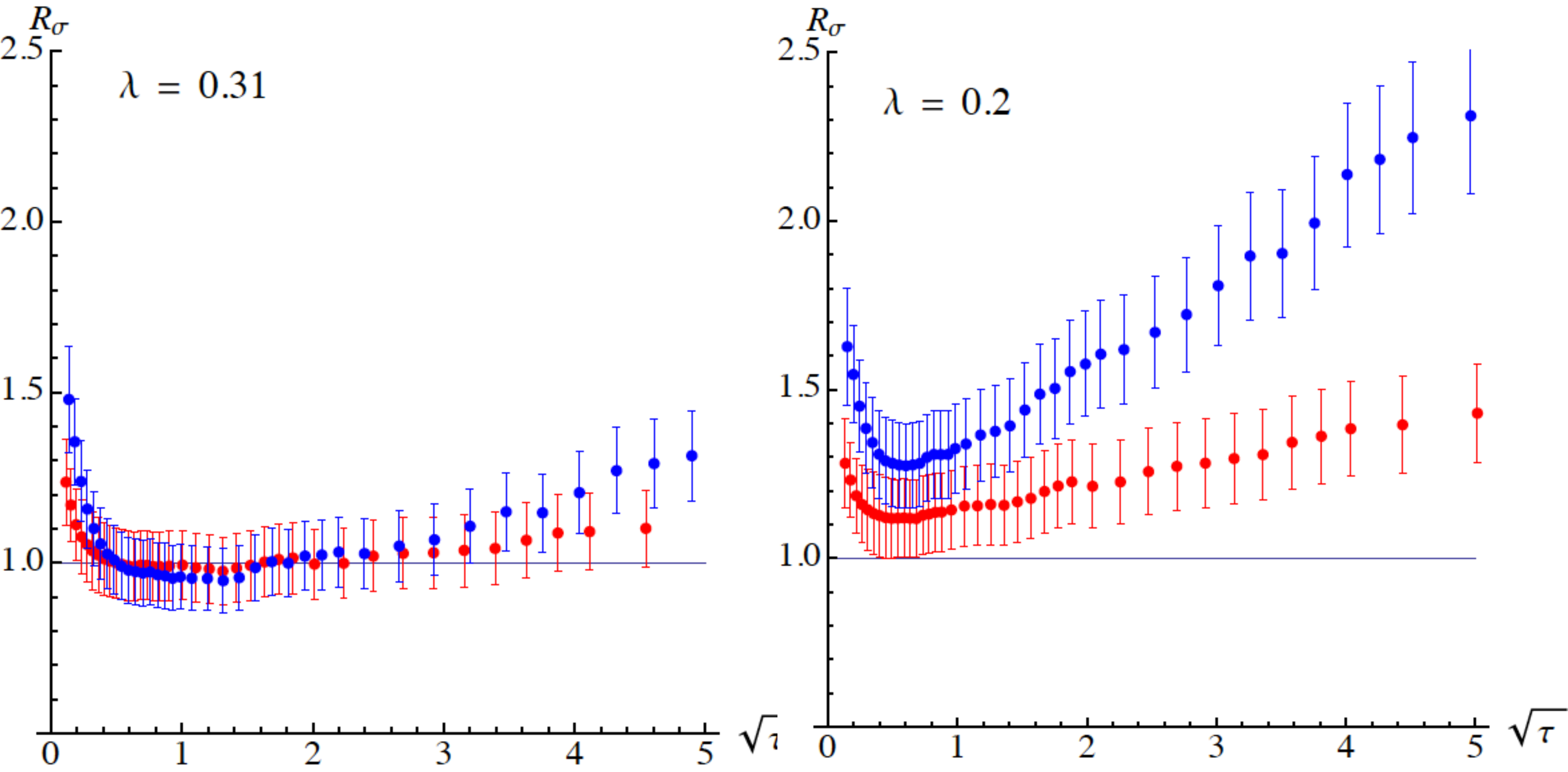

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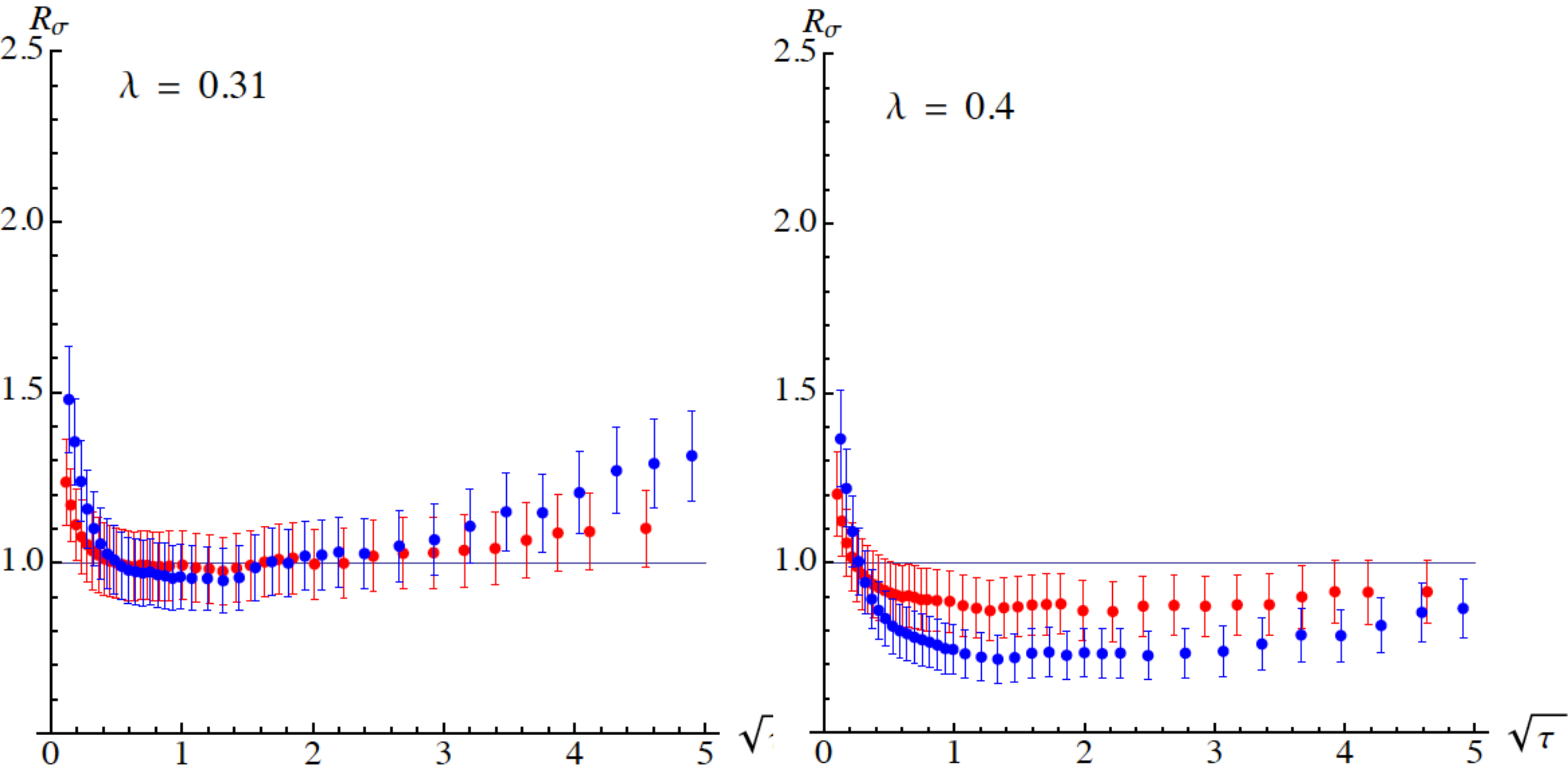

Conlusions

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- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08 \text{ with } \lambda \sim 0.33$
- GS for the cross-section compatible with DIS

Basics of geometrical scaling

for $y \sim 0$ (central rapidity) *i.e.* for $x_1 \sim x_2 = x$ and for symmetric systems

$$\frac{d\sigma}{dyd^2p_{\mathrm{T}}} = \frac{3\pi\alpha_{\mathrm{s}}}{2}\frac{Q_s^2(x)}{p_{\mathrm{T}}^2}\int\frac{d^2\vec{k}_{\mathrm{T}}}{Q_s^2(x)}\varphi_1\left(\vec{k}_{\mathrm{T}}^2/Q_s^2(x)\right)\varphi_2\left((\vec{k}-\vec{p})_{\mathrm{T}}^2/Q_s^2(x)\right)$$

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$$\bar{Q}_s(W) = Q_0\left(\frac{W}{Q_0}\right)^{\lambda/(2+\lambda)}$$

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$$\frac{d\sigma}{dy} = S_\perp^2 \int \mathcal{F}(\tau) d^2 p_{\rm T} = S_\perp^2 \bar{Q}_s^2(W) \int \mathcal{F}(\tau)\ldots d\tau = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W)$$

Basics of geometrical scaling

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$$\frac{d\sigma}{dy} = S_\perp \frac{dN}{dy} = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W) \rightarrow \bar{Q}_s^2(W) = \frac{\kappa}{S_\perp}\frac{dN}{dy}$$

Basics of geometrical scaling

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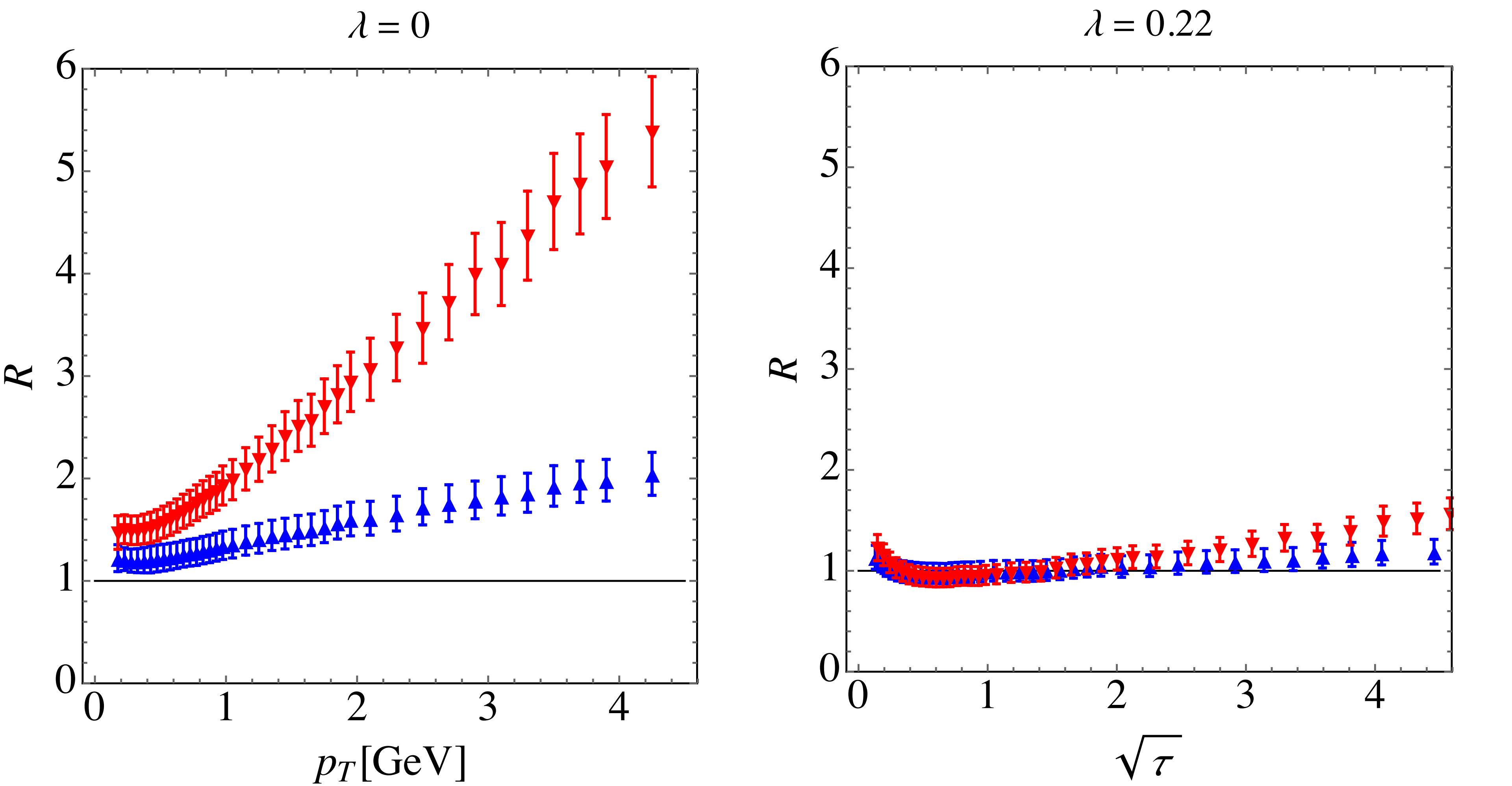
$$\frac{d\sigma}{dy} = \sigma^{\rm MB}(W) \frac{dN}{dy} = \frac{1}{\kappa} S_\perp^2 \bar{Q}_s^2(W) \rightarrow S_\perp \frac{\bar{Q}_s^2(W)}{\sigma^{\rm MB}(W)} = \frac{\kappa}{S_\perp} \frac{dN}{dy}$$

Determination of lambda

$$\frac{dN_{\rm ch}}{dy d^2 p_{\rm T}} = S_\perp \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$



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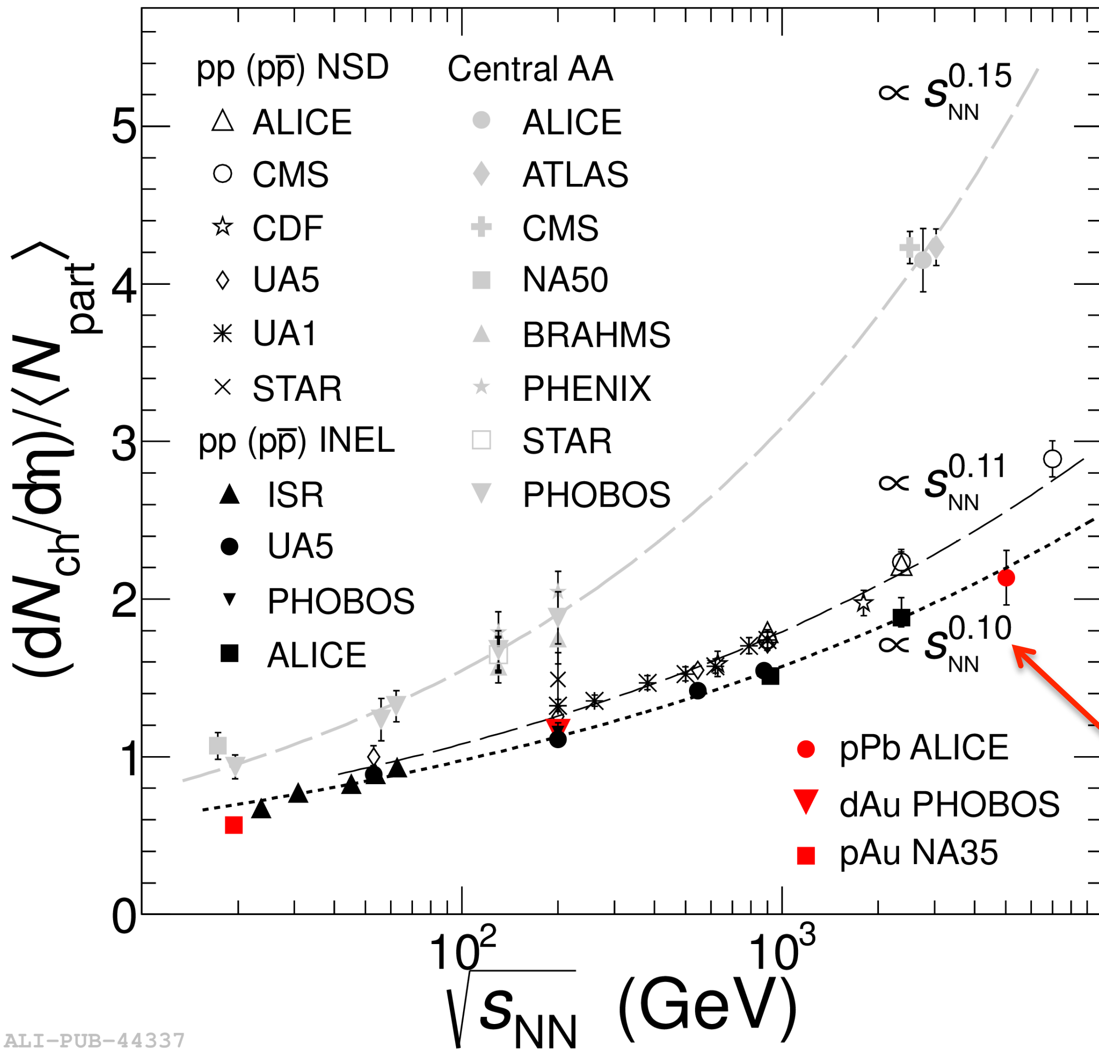
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with with $\lambda \sim 0.22$ (!)

continue

with multiplicity scaling…

Power-like growth of multiplicity

http://th-www.if.uj.edu.pl/school/2014/talks/braun-munzinger_1.pdf



plot: P. Braun-Munzinger,
54 Cracow School of
Theoretical Physics
(from ALICE-PUB-44337)

$$\frac{dN_{ch}}{dy} \sim S_\perp \bar{Q}_s^2(W)$$

$$\sim S_\perp Q_0^2 \left(\frac{W^2}{Q_0^2}\right)^{\lambda/(2+\lambda)}$$

transverse area is energy indpendent

$$\lambda/(2+\lambda) \simeq 0.099$$

Conlusions

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- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with with $\lambda \sim 0.22$ (!)
- As a consequence total multiplicity grows with energy as $s^{0.1}$

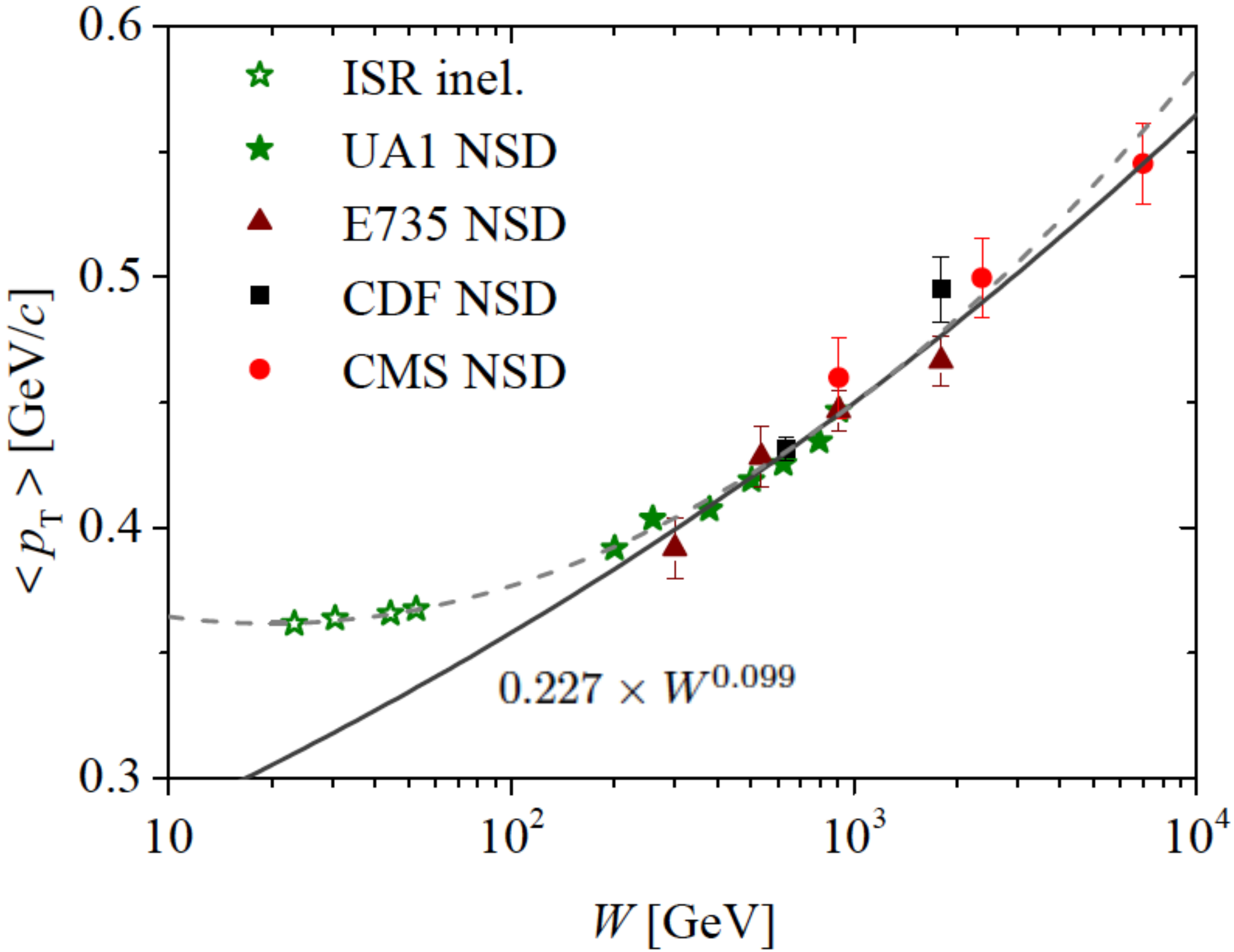
Average transverse momentum

Average transverse momentum

$$\frac{dN_{\rm ch}}{dyd^2p_{\rm T}} = S_\perp \mathcal{F}(\tau)$$

$$\langle p_{\rm T}\rangle = \frac{\int p_T \frac{dN_{\rm g}}{dyd^2p_{\rm T}} d^2p_T}{\int \frac{dN_{\rm g}}{dyd^2p_{\rm T}} d^2p_T} \sim \bar{Q}_{\rm s}(W) \sim Q_0 \left(\frac{W}{Q_0}\right)^{\lambda/(2+\lambda)}$$

Average transverse momentum



Conlusions

- Nonlinear BK equation generates sauration scale $Q_s(x)$
- In the region with no other scales Geometrical Scaling emerges
- GS works well in DIS up to relatively large $x \sim 0.08$ with $\lambda \sim 0.33$
- GS for the cross-section compatible with DIS
- In pp GS works for multiplicity distributions with with $\lambda \sim 0.22$ (!)
- As a consequence total multiplicity grows with energy as $s^{0.1}$
- geometrical scaling predicts energy dependence of $\langle p_T \rangle$

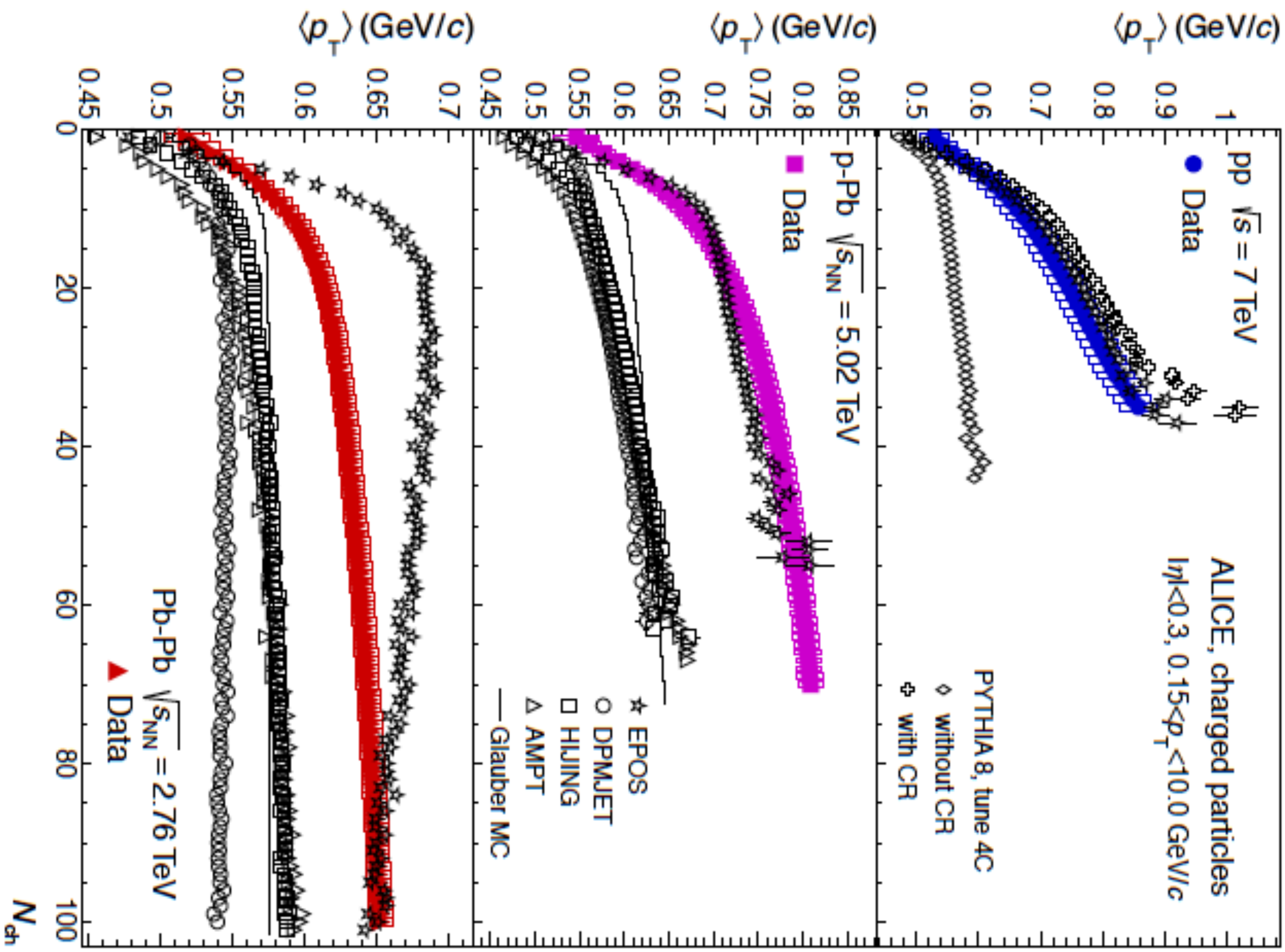
Mean p_T as a function of N_{ch}

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-PH-EP-2013-111
July 2, 2013

Multiplicity dependence of the average transverse momentum in pp, p–Pb, and Pb–Pb collisions at the LHC

The ALICE Collaboration*



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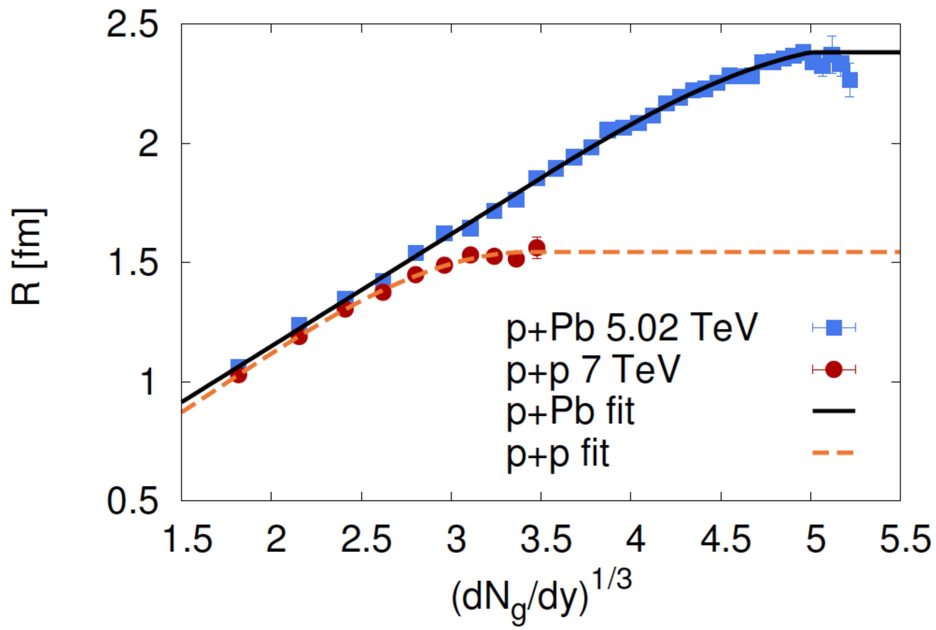
Mean p_T as a function of N_{ch}

$$\langle p_T \rangle \sim \bar{Q}_s(W) \sim \sqrt{\frac{dN/dy}{S_\perp}} \sim \frac{1}{R}\sqrt{\frac{dN}{dy}}$$

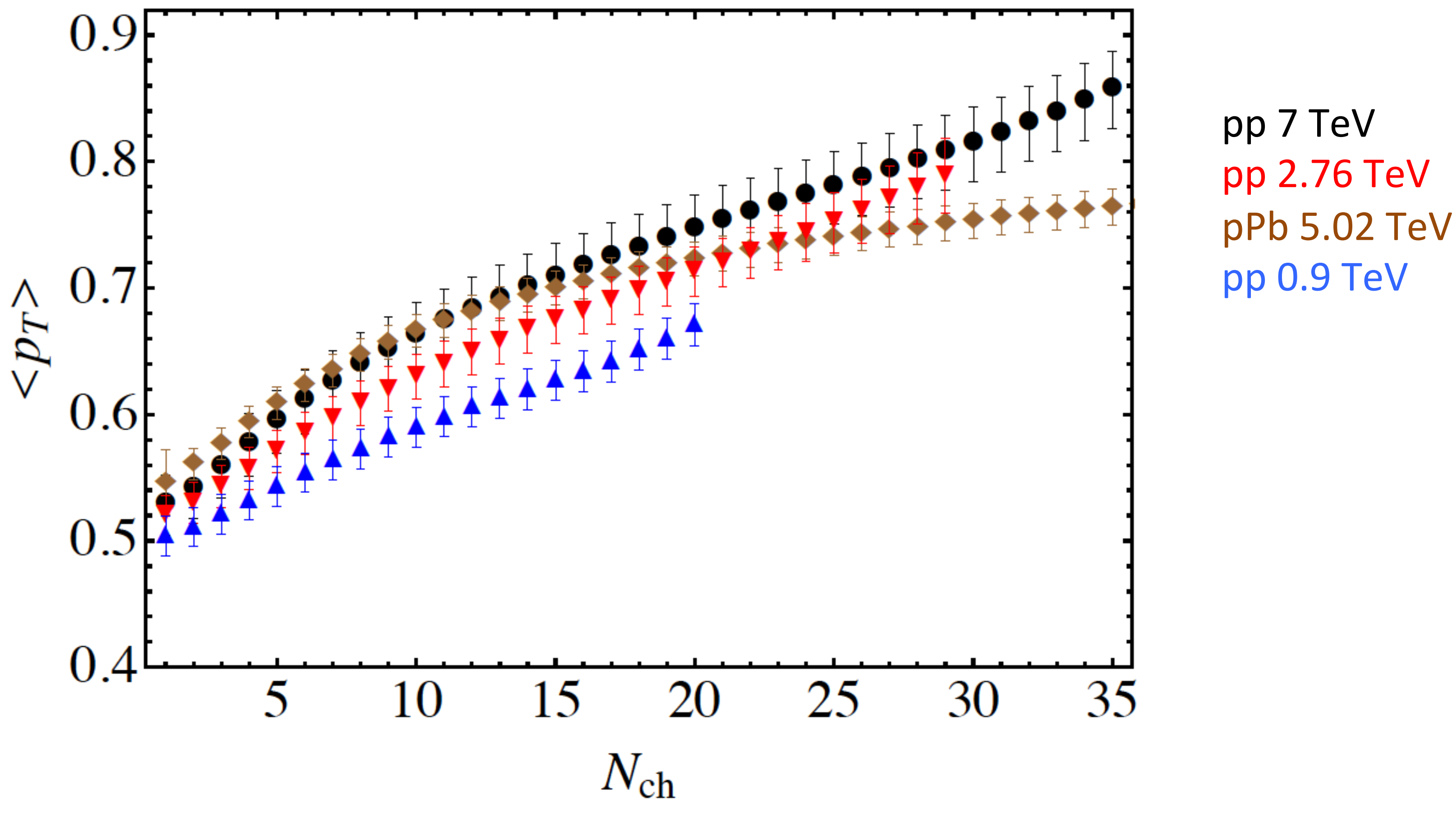
interaction radius



A. Bzdak, B. Schenke, P. Tribedy and R. Venugopalan, *Initial state geometry and the role of hydrodynamics in proton-proton, proton-nucleus and deuteron-nucleus collisions*, Phys. Rev. C 87 (2013) 064906, [arXiv:1304.3403 [nucl-th]].

Mean p_T scaling

ALICE Collaboration, Phys. Lett. B727 (2013) 371 [arXiv:1307.1094 [nucl-ex]]



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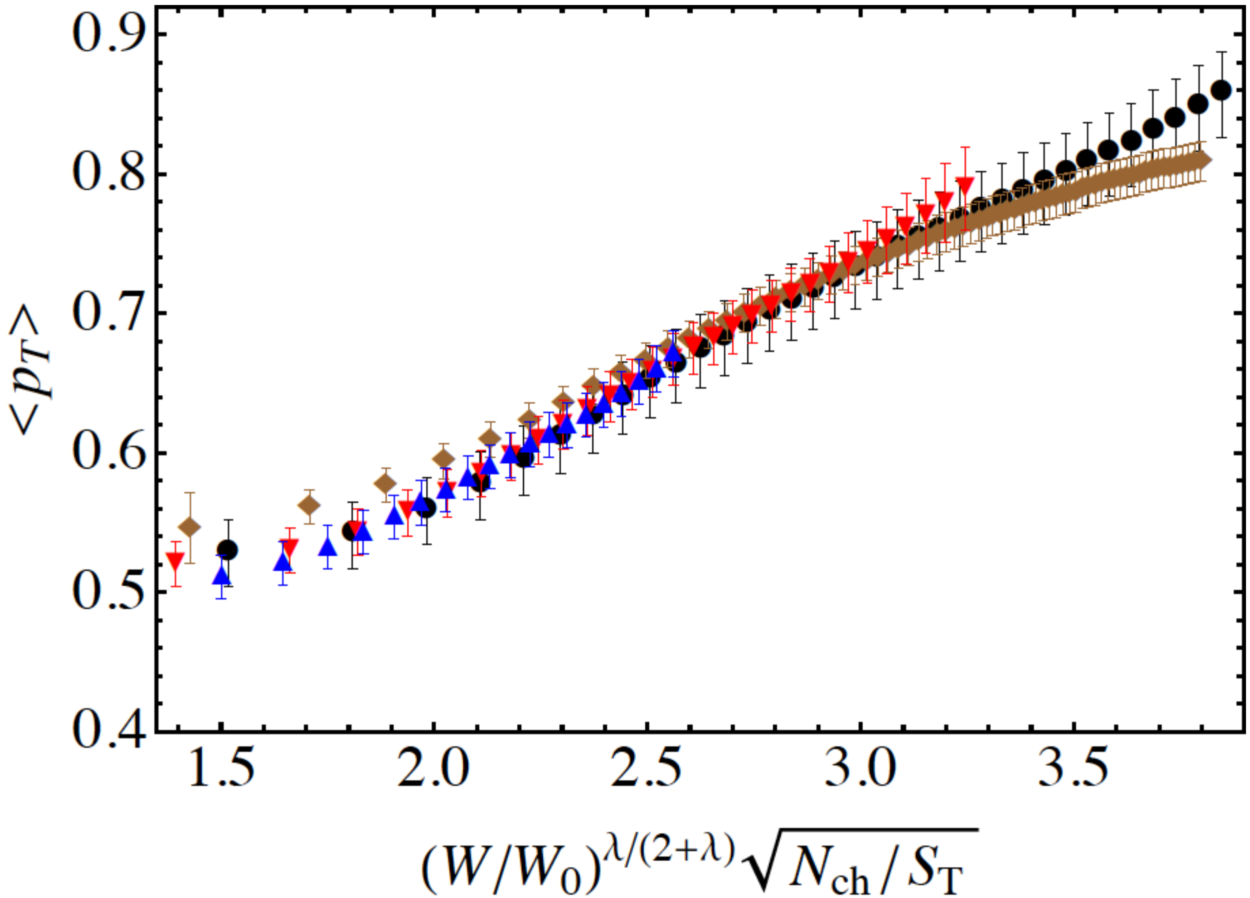

pp 7 TeV
pp 2.76 TeV
pPb 5.02 TeV
pp 0.9 TeV

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- scaling of $\langle p_{\rm T} \rangle(N_{\rm ch})$ induced by energy dependence of $Q_{\rm sat}$ + CGC

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- $<p_T>(N_{ch})$ difficult to describe by untuned MonteCarlos
- scaling of $<p_T>(N_{ch})$ induced by energy dependence of Q_{sat} + CGC

Not discussed

- Consequences of GS for F_L
- Scaling violations in pp due to $y \neq 0$
- Scaling violations in pp due to $\lambda(Q^2)$
- Scaling in pp for identified particles
- Connection with Tsallis distribution
- $<p_T>(N)$ for identified particles
- Fluctuations of the saturation scale – diffusive scaling in DIS?
- Fluctuations of Q_{sat} in small systems (pA)
- GS in heavy ion collisions – scaling with energy and N_{part}