Quantum Simulation of High Energy Physics Models using Cold Atoms

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MPQ
Max-Planck-Institut für Quantenoptik

From the vacuum to the universe
Humboldt Kolleg for Particle Physics
Kitzbühl, Austria, July 1st, 2016

QUANTUM SIMULATORS

Simulating Physics with Computers

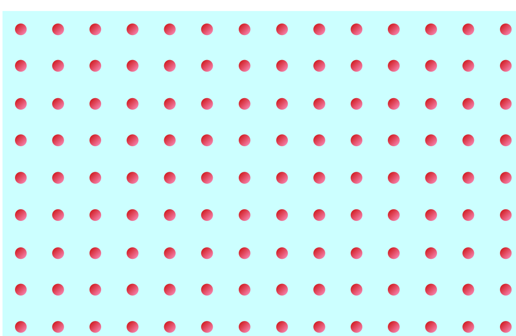
Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should

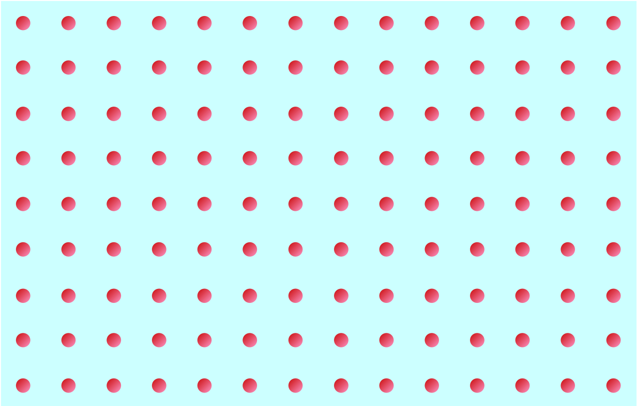


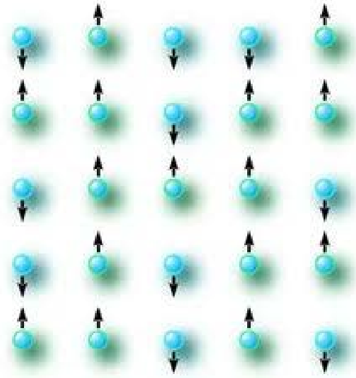
$$c_1\,|000\ldots0\rangle + c_2\,|000\ldots1\rangle + \ldots + c_{2^N}\,|111\ldots1\rangle$$

QUANTUM SIMULATORS
ANALOG

MODEL

QUANTUM SIMULATOR





Model Hamiltonian

$$H = \ldots$$

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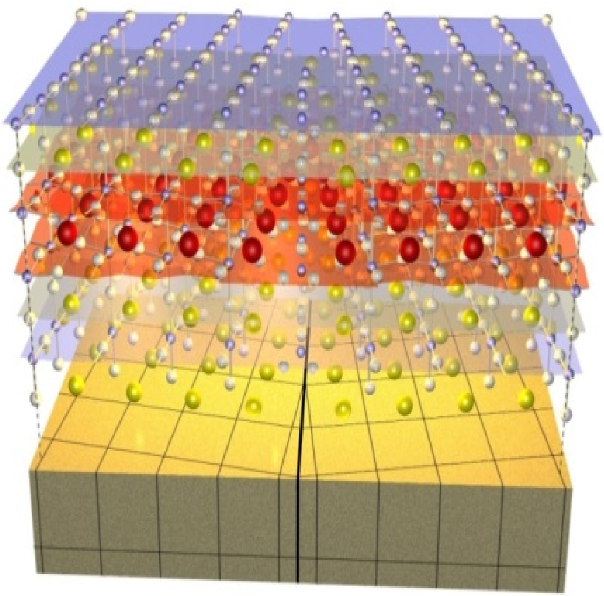
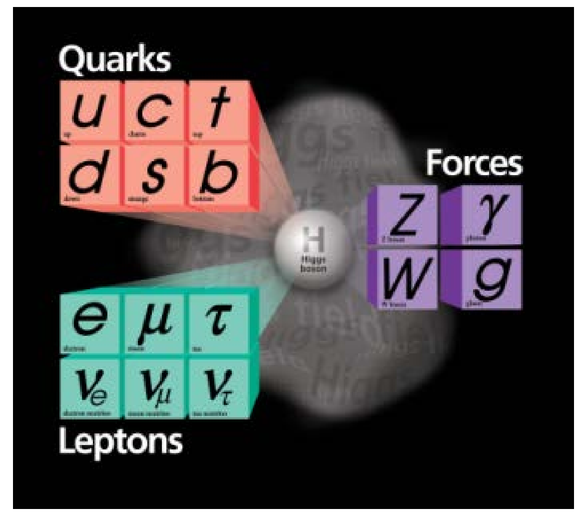
$$H = \ldots$$



Material Science



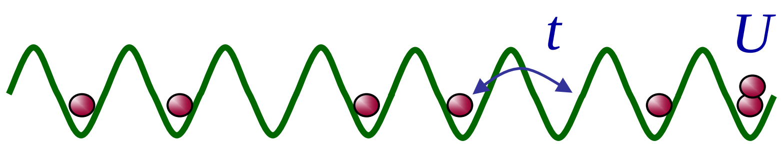
HEP?

COLD ATOMS IN OPTICAL LATTICES

COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u_{\sigma_i} \int \Psi_{\sigma_1}^\dagger \Psi_{\sigma_2}^\dagger \Psi_{\sigma_3} \Psi_{\sigma_4}$$



Lattice theory: Bose/Fermi-Hubbard model

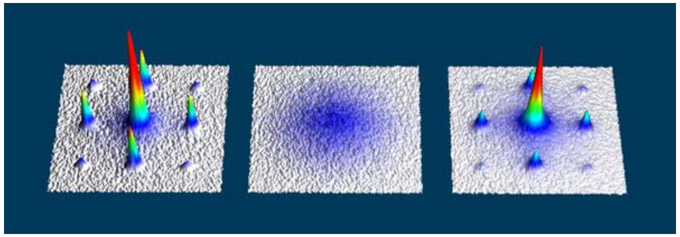
$$H = -t\sum_n \left(a_n^\dagger a_{n+1} + h.c\right) + U\sum_n a_n^{\dagger 2} a_n^2$$

articles

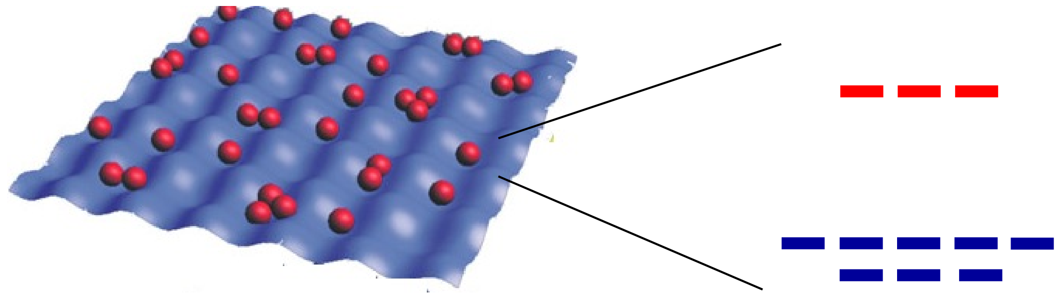
Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

* *Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*
† *Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland*



COLD ATOMS
QUANTUM SIMULATION

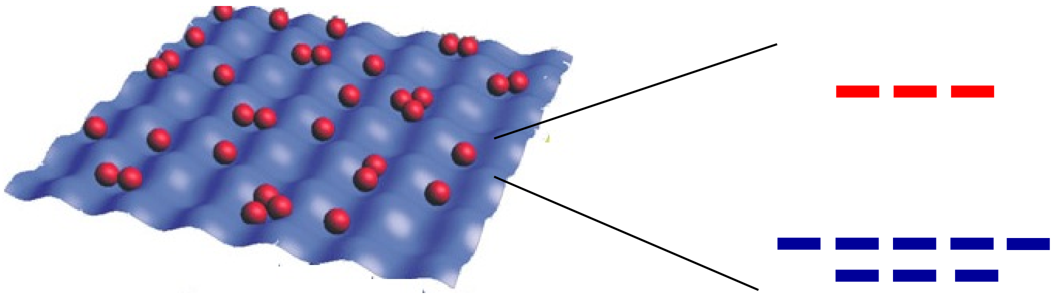


- Bosons/Fermions: $H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$

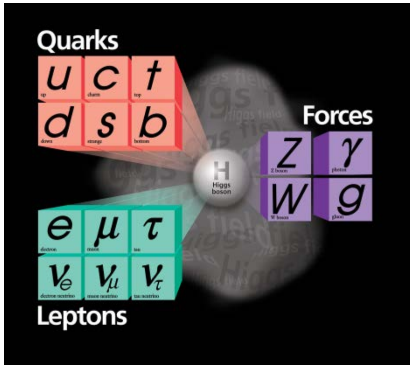
- Spins: $H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$

→ CONDENSED MATTER PHYSICS

COLD ATOMS
QUANTUM SIMULATION



HIGH ENERGY PHYSICS?



QUANTUM SIMULATIONS OF HEP MODELS

QUANTUM SIMULATION HEP MODELS
INGREDIENTS

$$S = \int \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi - Q\int A_{\mu}\overline{\Psi}\gamma^{\mu}\Psi - \frac{1}{4}\int F_{\mu\nu}F^{\mu\nu} + ...$$

- Matter + Gauge Fields
- Relativistic theory
- Gauge invariant
- Hamitonian formulation: $i\partial_t |\Psi\rangle = H |\Psi\rangle$
  - Gauss law $G(x)|\Psi\rangle = 0$

$$[H, G(x)] = 0$$

INGREDIENTS

Lattice → J. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).
J. B. Kogut, Rev. Mod. Phys. **51**, 659 (1979).
J. B. Kogut, Rev. Mod. Phys. **55**, 775 (1983).

$$H = \int \Psi^\dagger_\sigma \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi^\dagger_\mu \Phi_{\sigma'} \Psi^\dagger_\sigma \Psi_{\sigma'} + v\int \Phi^\dagger_\sigma \Phi^\dagger_{\sigma'} \Phi_{\sigma'} \Phi_\sigma + \ldots$$

Lattice

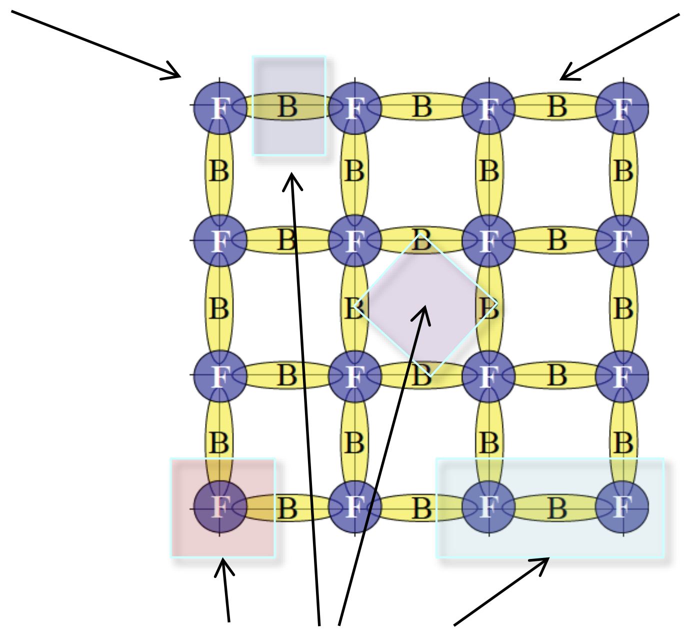
Fermion-gauge field coupling

Gauge field dynamics

HAMILTONIAN FORMULATION

Matter (Fermions): can move

Gauge fields (Bosons): Static

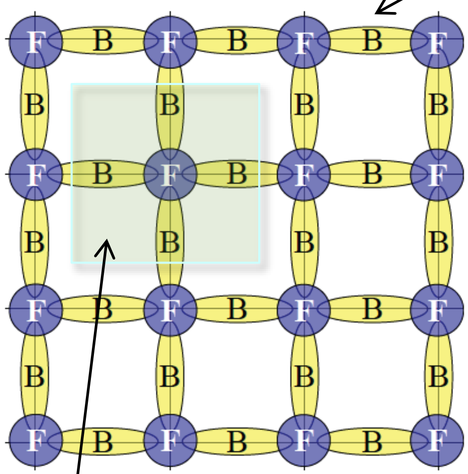


- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$

HAMILTONIAN FORMULATION

Matter (Fermions): can move

Gauge fields (Bosons): Static

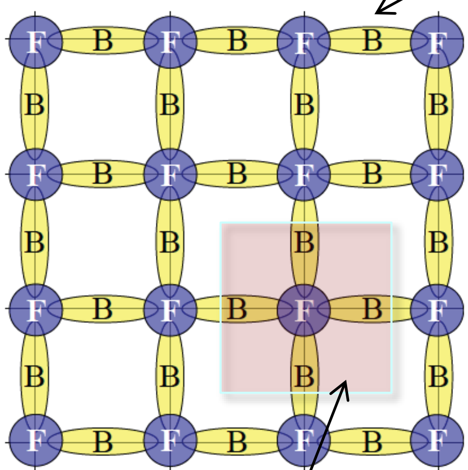


- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$
- Gauge invariance: Gauge group: U(1), Z_N,SU(N), etc

HEP LATTICE MODELS
HAMILTONIAN FORMULATION

Matter (Fermions): can move

Gauge fields (Bosons): Static



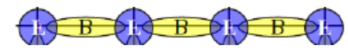
- Hamiltonian: $H = H_M + H_{KS} + H_{\text{int}}$
- Gauge invariance: Gauge group: U(1), Z_N,SU(N), etc
- Gauss law: $G_{plaquette} \mid phys\rangle = 0$

HEP LATTICE MODELS

HAMILTONIAN FORMULATION

- Example: compact-QED in 1D

  - Hamiltonians:



$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi^\dagger_{\mathbf{n}} \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}} \right)$$

$$H_{KS} = H_E = \frac{g^2}{2} \sum_{n,k} E_n^2$$

$$[E_{\mathbf{n},k}, \phi_{\mathbf{m},l}] = -i\delta_{\mathbf{nm}}\delta_{kl}$$ (ie, compact)

  - Gauss law: $G_n \,|\, phys\rangle = 0$

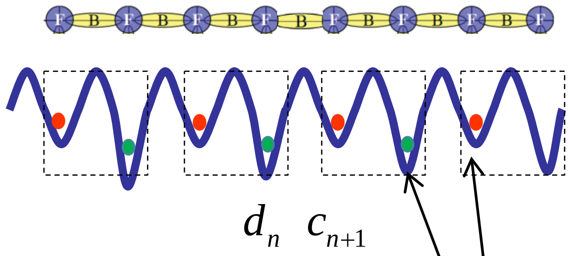
$$G_n = E_{n+1} - E_n - \psi^\dagger_n \psi_n$$

  - Gauge invariance: $e^{-i\theta G_n} H e^{i\theta G_n} = H$

SCHWINGER MODEL

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Fermions:



$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

$$H_M = M \sum_n (-1)^n \psi_n^\dagger \psi_n$$

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v \int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

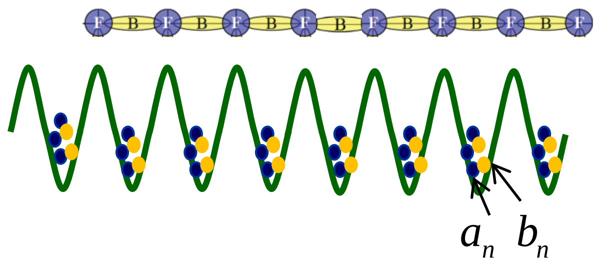
- Even sites: hole = particle
- Odd sites: fermion = antiparticle

Staggered Fermions:

L. Susskind, Phys. Rev. D **16**, 3031 (1977).
G. 't Hooft, Nucl. Phys. B **75**, 461 (1974)

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- **Bosons**:



internal states

$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

- Schwinger rep:

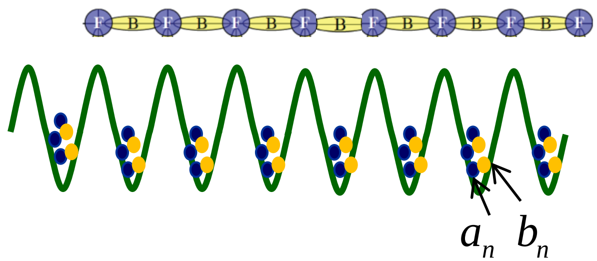
$$L_+ = a^\dagger b$$
$$L_z = \frac{1}{2}(N_a - N_b)$$
$$\ell = \frac{1}{2}(N_a + N_b)$$

$$\xrightarrow{\ell >> 1}$$

$$L_+ \approx \ell e^{i\phi}$$
$$L_z \approx i\partial_\phi$$

- If ℓ is small (eg 2 atoms), we obtain a truncated version
- One can also use a single atom with few internal levels (Z_M is the gauge group)

- Bosons:



internal states

$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

Schwinger rep:

$$L_+ = a^\dagger b$$

$$L_z = \frac{1}{2}(N_a - N_b)$$

$$\ell = \frac{1}{2}(N_a + N_b)$$

$$H_E = \frac{g^2}{2}\sum_n L_{z,n}^2$$

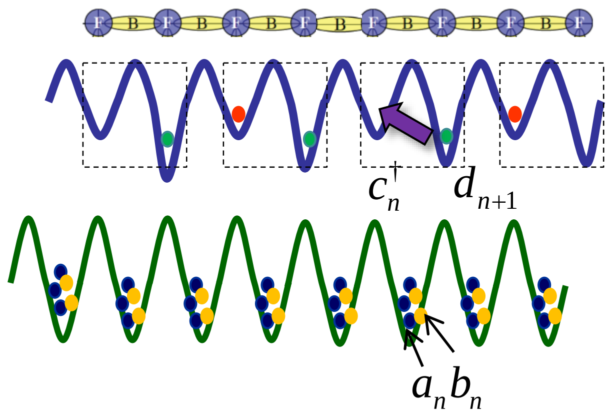
$$= \frac{g^2}{8}\sum_n \left(N_{a,n}^2 + N_{b,n}^2 - 2N_{a,n}N_{b,n}\right)$$

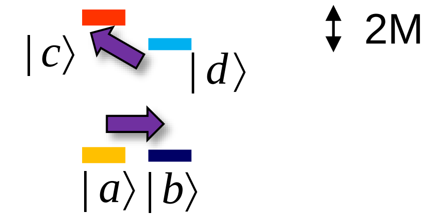
$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

SCHWINGER MODEL 1+1

- Interactions:

$c_n^\dagger a_n^\dagger b_n d_{n+1}$





$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

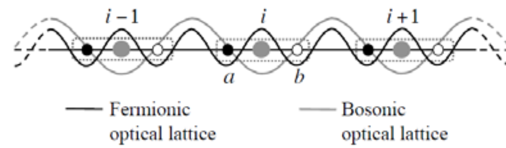
conserves angular momentum locally

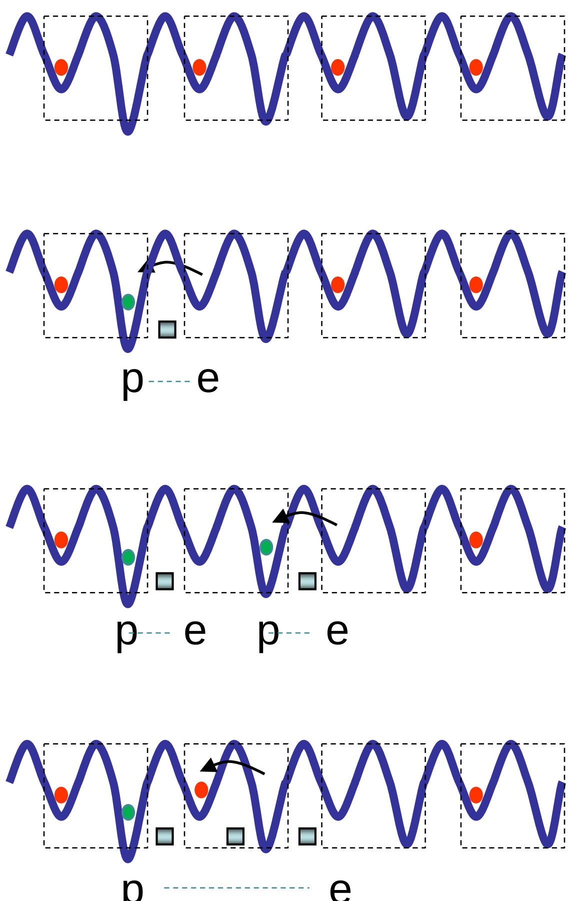
→ Gauge invariance

$$H_{int} \quad \frac{1}{\sqrt{\ell(\ell+1)}} \psi_n^\dagger a_n^\dagger b_n \psi_{n+1} \approx \psi_n^\dagger e^{i\phi_n} \psi_{n+1}$$

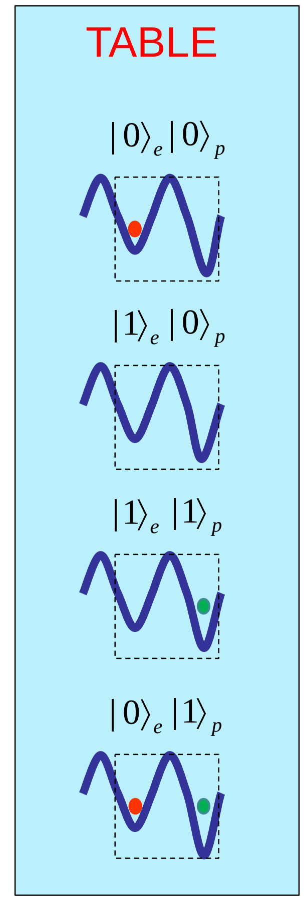
QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Physical processes:



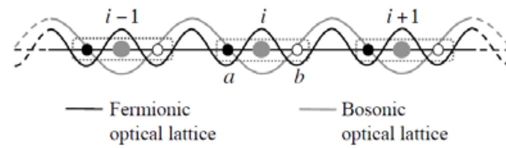


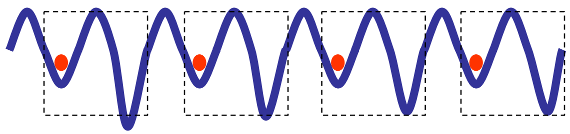
non-interacting vacuum



QUANTUM SIMULATION
SCHWINGER MODEL 1+1

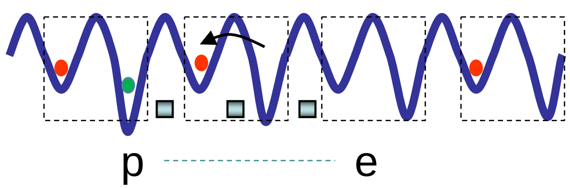
- Preparation:





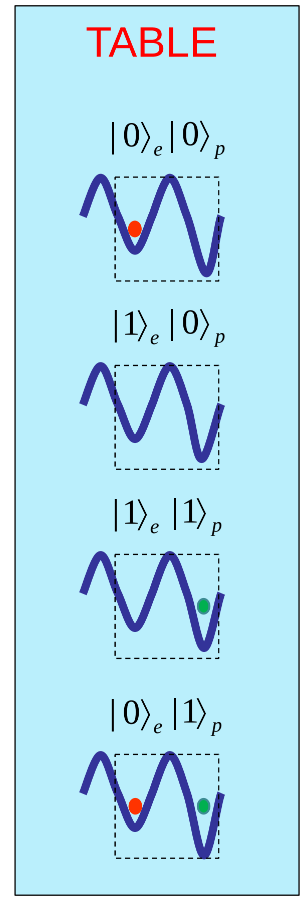
non-interacting vacuum

switch on interactions



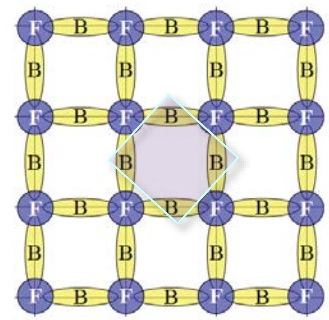
interacting vacuum

- Confinement
- Excitations: vector + scalar
- Time-dependent phenomena
- First experiments: few bosonic atoms



QUANTUM SIMULATION

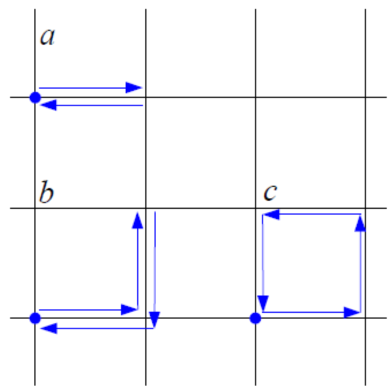
HIGHER DIMENSIONS, NON-ABELIAN



- Plaquette interactions:

$$H_B = -\frac{2\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\left(U_{\mathbf{n},1}U_{\mathbf{n}+\hat{1},2}U^\dagger_{\mathbf{n}+\hat{2},1}U^\dagger_{\mathbf{n},2} + h.c.\right) =$$

$$-\frac{4\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\cos\left(\phi_{\mathbf{n},1}+\phi_{\mathbf{n}+\hat{1},2}-\phi_{\mathbf{n}+\hat{2},1}-\phi_{\mathbf{n},2}\right)$$

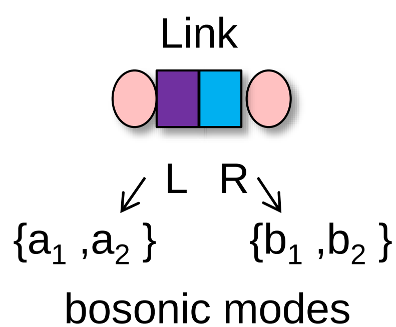


- Non-abelian gauge theories:

$$H_E = \frac{g^2}{2}\sum_{\mathbf{n},k,a}\left(E_{\mathbf{n},k}\right)_a\left(E_{\mathbf{n},k}\right)_a$$

$$H_B = -\frac{1}{g^2}\sum_{\text{plaquettes}}\left(\text{Tr}\left(U_1U_2U_3^\dagger U_4^\dagger\right)+h.c.\right)$$

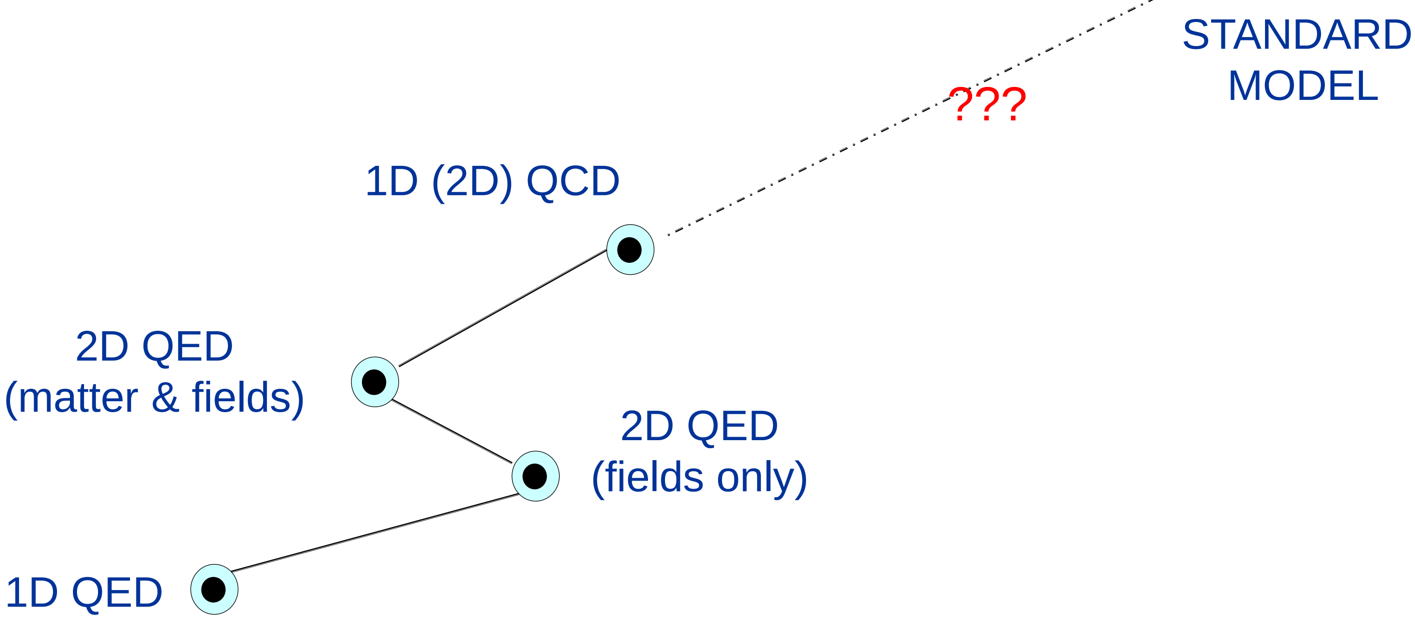
$$H_{int} = \epsilon\sum_{\mathbf{n},k}\left(\psi^\dagger_{\mathbf{n}}U^r_{\mathbf{n},k}\psi_{\mathbf{n}+\hat{\mathbf{k}}}+h.c.\right)$$




WARNING

QUANTUM SIMULATION
HIGH ENERGY MODELS

ROAD MAP



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Zohar, IC, Reznik, PRA **88**, 023617 (2013)
Zohar, Farace, Reznik, JIC, in preparation

See also:

Kapit,Mueller, PRA**83**, 033625 (2011)
Banerjee,…, Wiese, Zoller, PRL**109**, 175302 (2013)
Banerjee,…, Wiese, Zoller, PRL**110**, 125303 (2013)
Gauge fields: Lewenstein et al