Hi!

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Impact of LHC dark matter searches on new physics scenarios

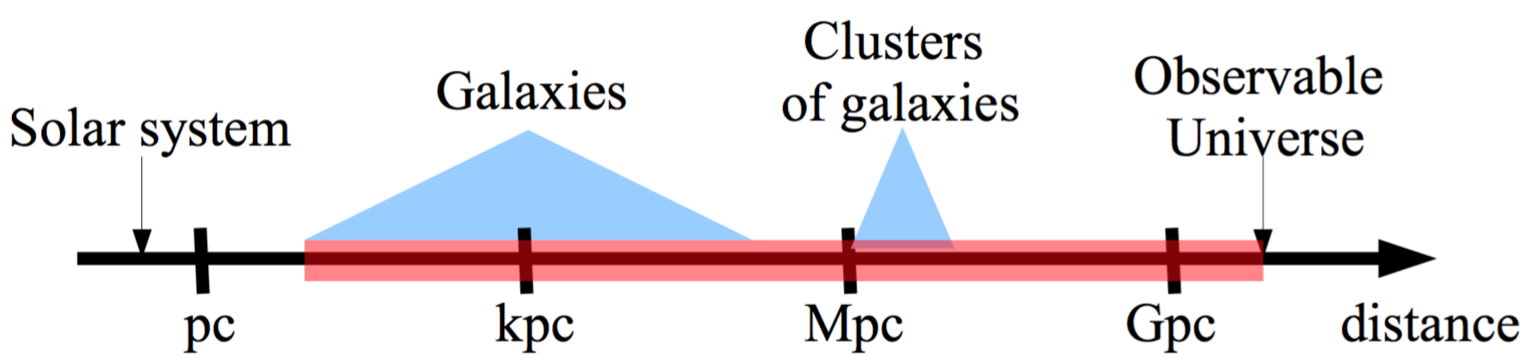
Suchita Kulkarni
HEPHY, Vienna

Based on:
arXiv:1512.06842 (D. Barducci, A. Goudelis, D. Sengupta) [Published JHEP]
arXiv:1605.02684 (D. Barducci, A. Bharucha, N. Desai, M. Frigerio, B. Fuks, A. Goudelis, S. Lacroix, G. Polesello, D. Sengupta) [work in progress]

28 June 2016, From Vacuum to the Universe, Kitzbühel, Austria

Dark matter

- Strong evidence on all scales



- But apart from that…

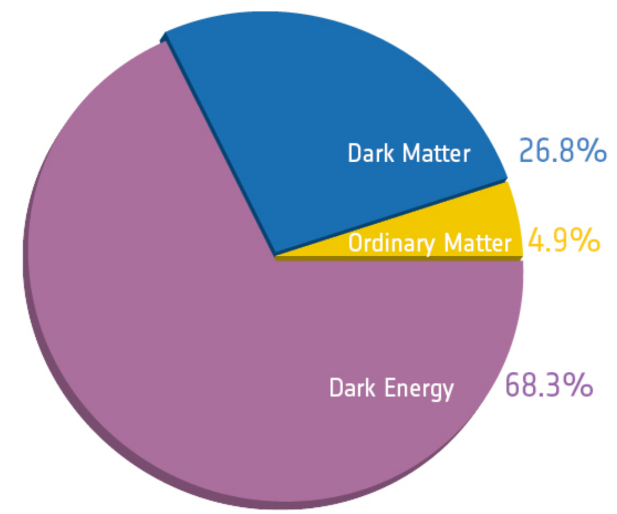
DARK MATTER

$J =$?

Mass $m =$?
Mean life $\tau =$?

DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
?	?	?	?

- Precise measurements of relic density



After Planck

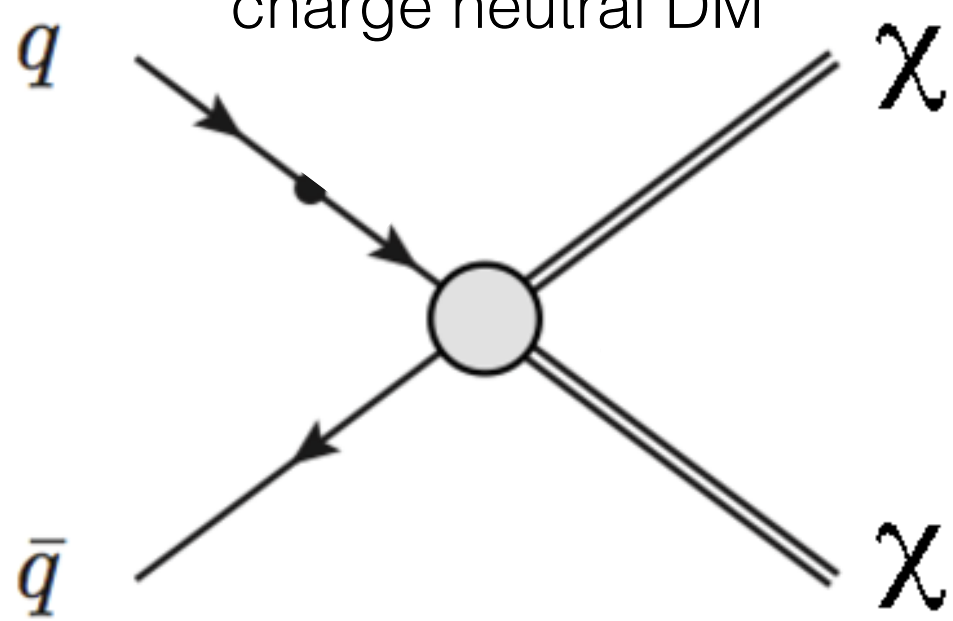
Goal for 21st century: identify the properties of dark matter

- Talk by J. Schieck

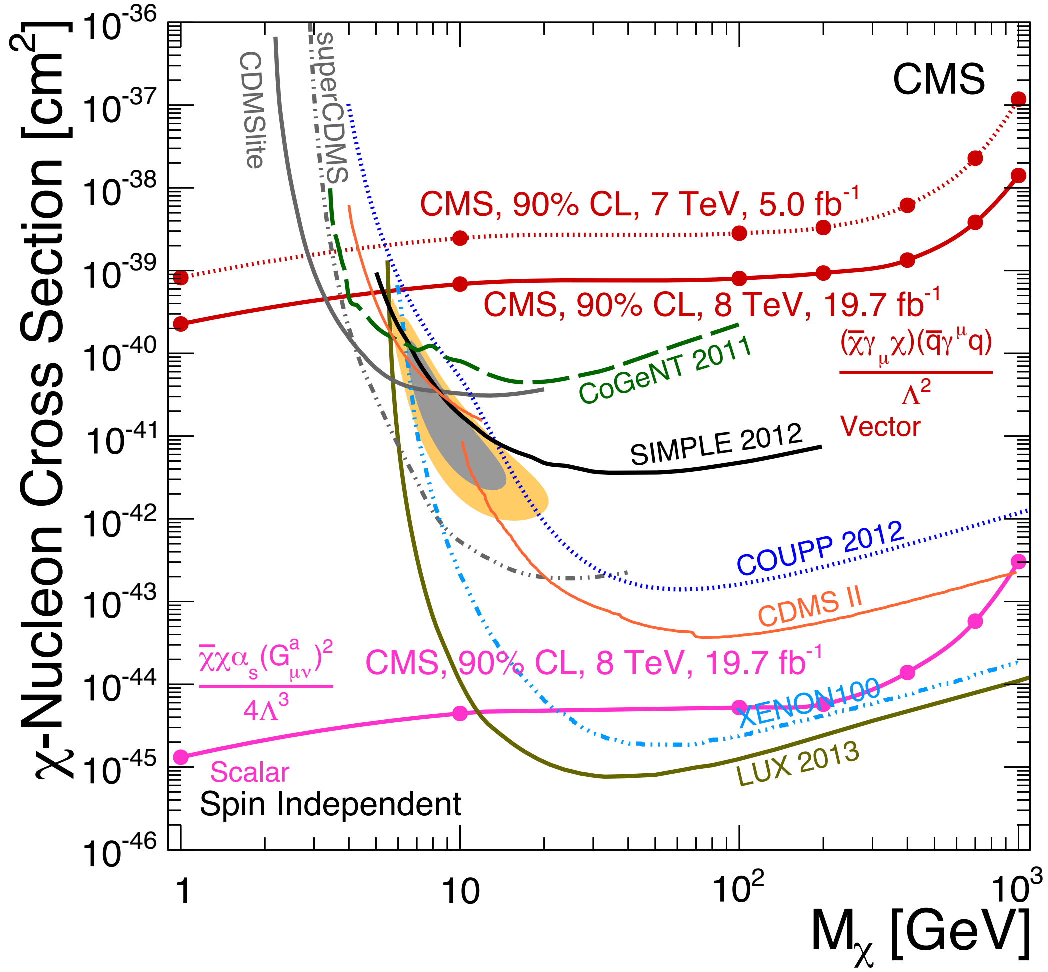
- This talk: If dark matter is related to TeV scale physics, what can LHC tell us?

Hunt at the LHC

Stable, thermal, charge neutral DM



arXiv:1408.3583



- Derived limits often depend on exact theoretical scenario, e.g. couplings, masses
- Necessary to reinterpret mono jet limits within a given theoretical scenario (extensive use of additional information given by LHC collaborations)
- This talk: usage of monojet searches to constrain two new physics scenarios
  - NB: Effective operator description at the LHC is a dangerous way to set limits, interpret plots carefully

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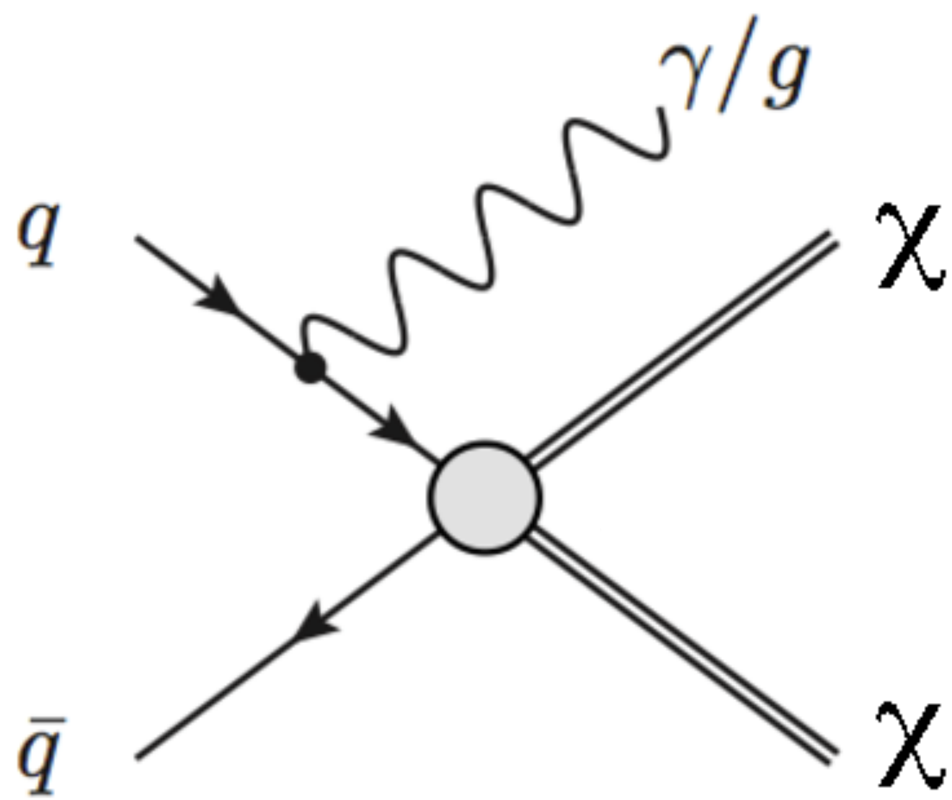
Hunt at the LHC



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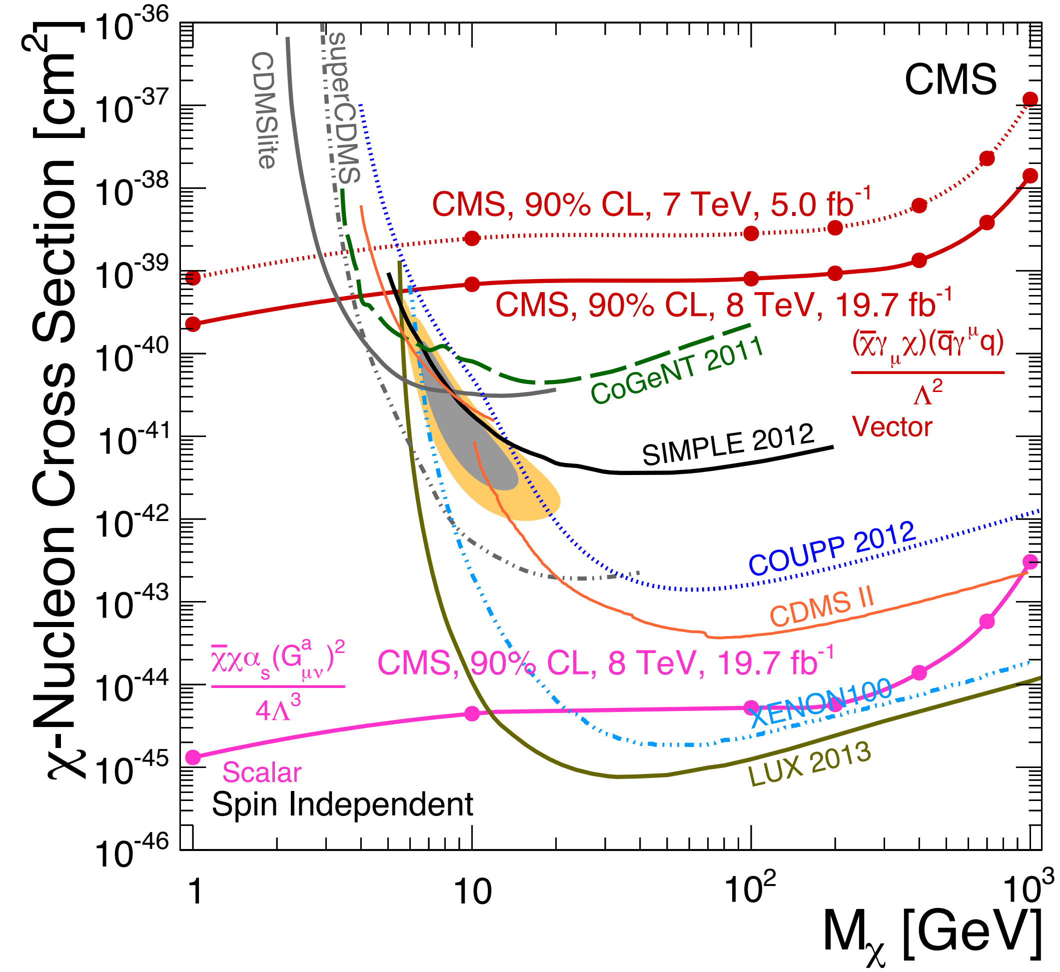

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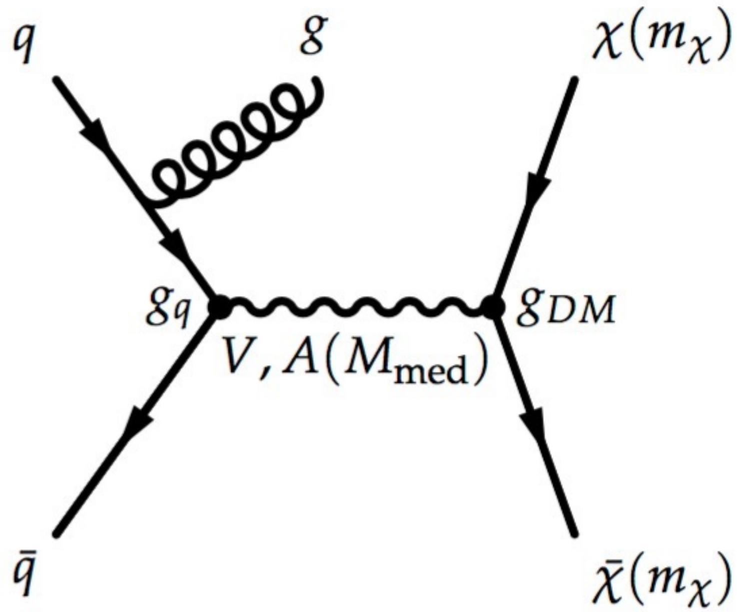
Hunt at the LHC

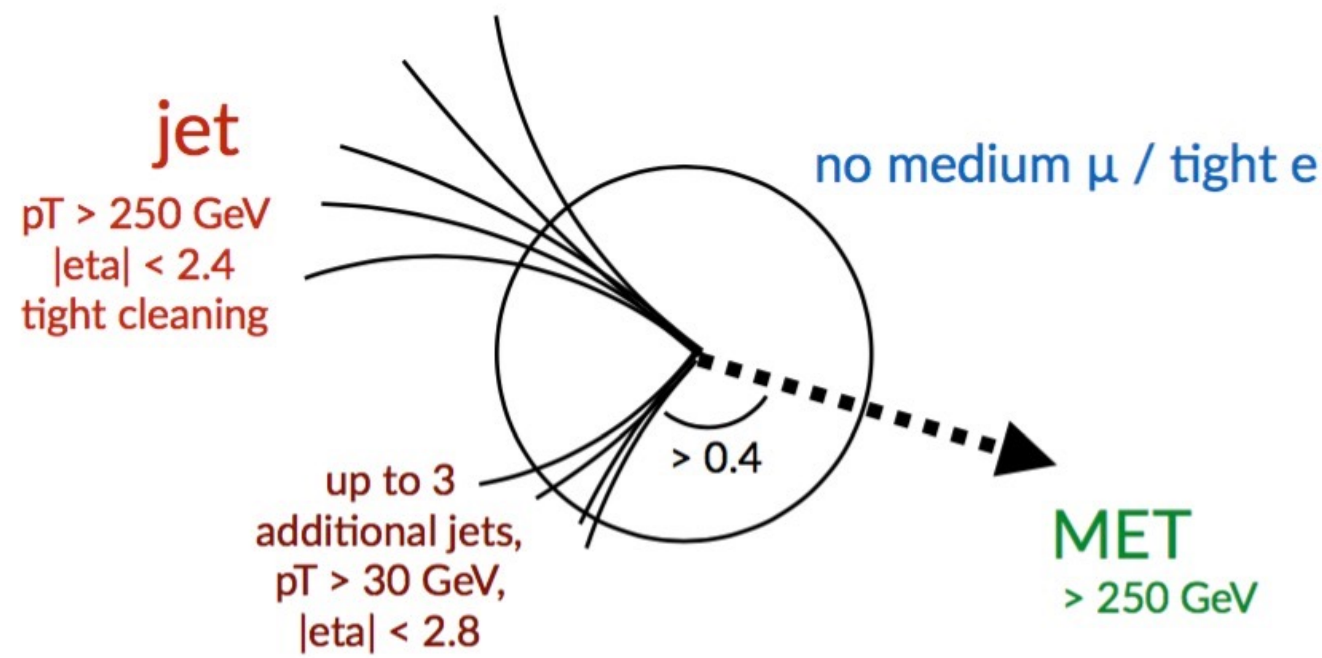
Figures curtesy A. Boveia's talk

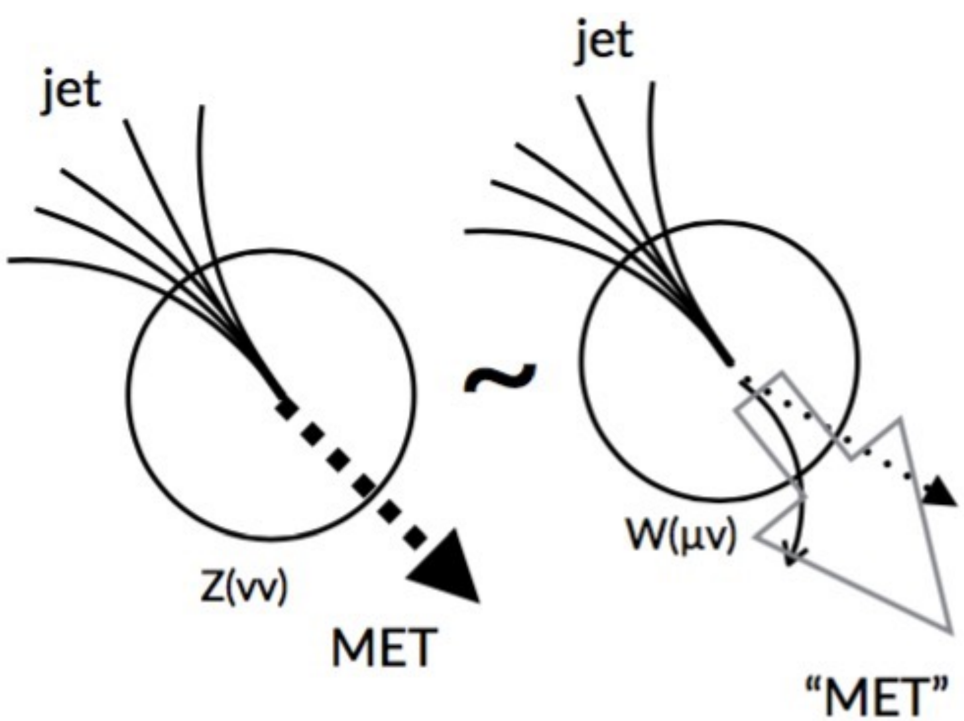


- Signal regions with successively larger MET requirement

CMS-EXO-12-048

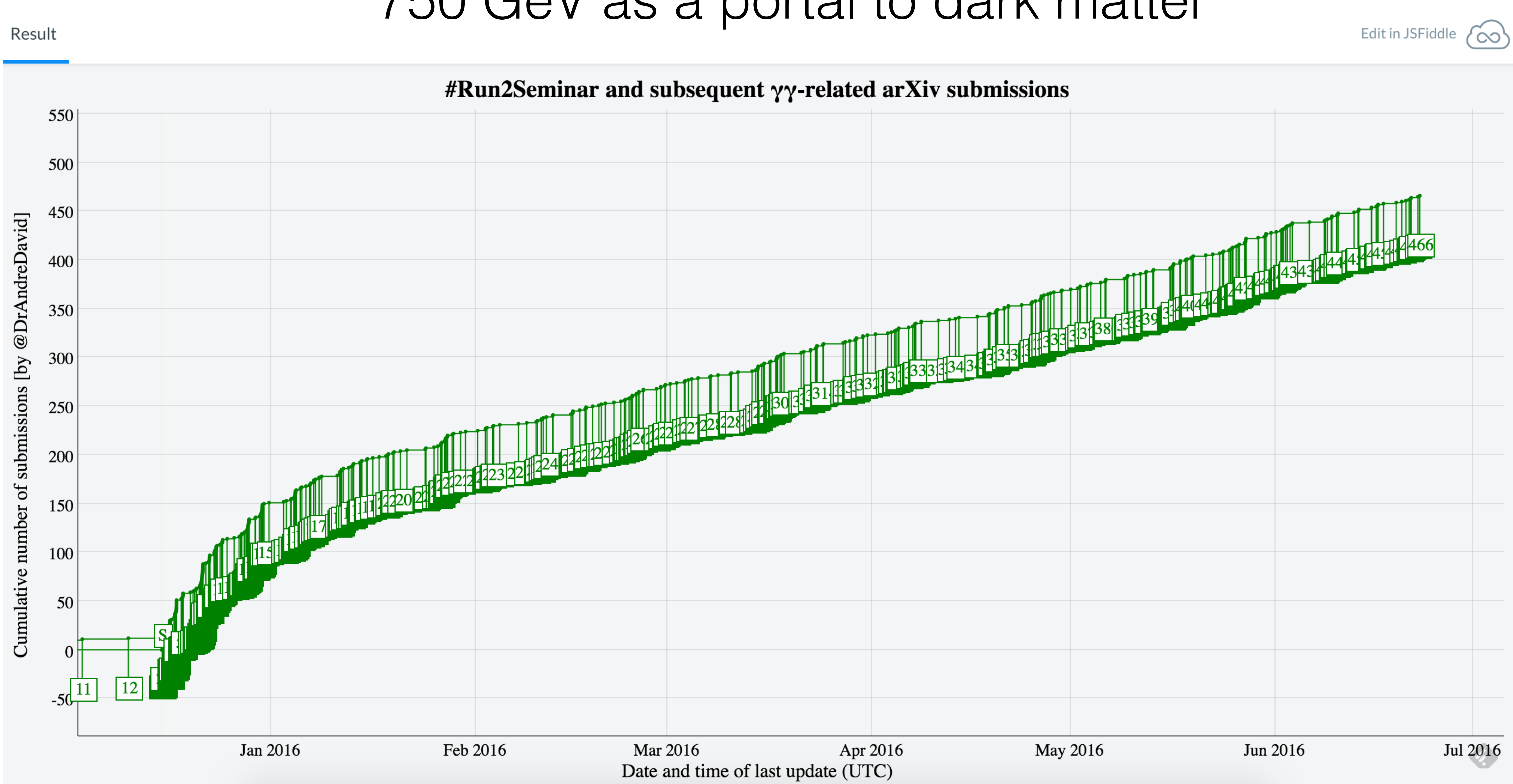
$\text{MET} > 250$ GeV
$\text{MET} > 300$ GeV
$\text{MET} > 350$ GeV
$\text{MET} > 400$ GeV
$\text{MET} > 450$ GeV
$\text{MET} > 500$ GeV
$\text{MET} > 550$ GeV





Test case: I

750 GeV as a portal to dark matter



André David

http://jsfiddle.net/adavid/bk2tmc2m/show/

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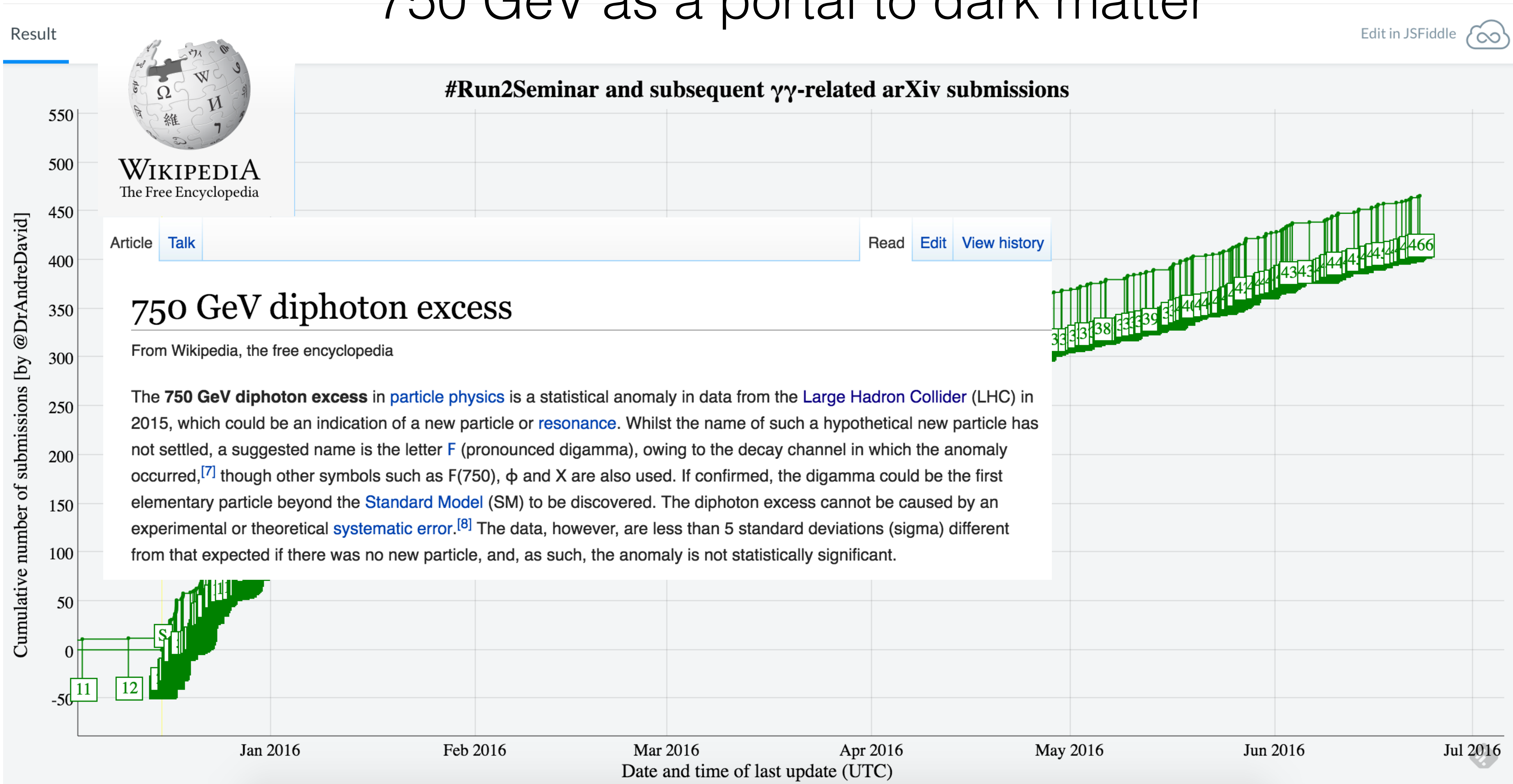

750 GeV diphoton excess

From Wikipedia, the free encyclopedia

The **750 GeV diphoton excess** in particle physics is a statistical anomaly in data from the Large Hadron Collider (LHC) in 2015, which could be an indication of a new particle or resonance. Whilst the name of such a hypothetical new particle has not settled, a suggested name is the letter Ϝ (pronounced digamma), owing to the decay channel in which the anomaly occurred,[7] though other symbols such as F(750), ϕ and X are also used. If confirmed, the digamma could be the first elementary particle beyond the Standard Model (SM) to be discovered. The diphoton excess cannot be caused by an experimental or theoretical systematic error.[8] The data, however, are less than 5 standard deviations (sigma) different from that expected if there was no new particle, and, as such, the anomaly is not statistically significant.

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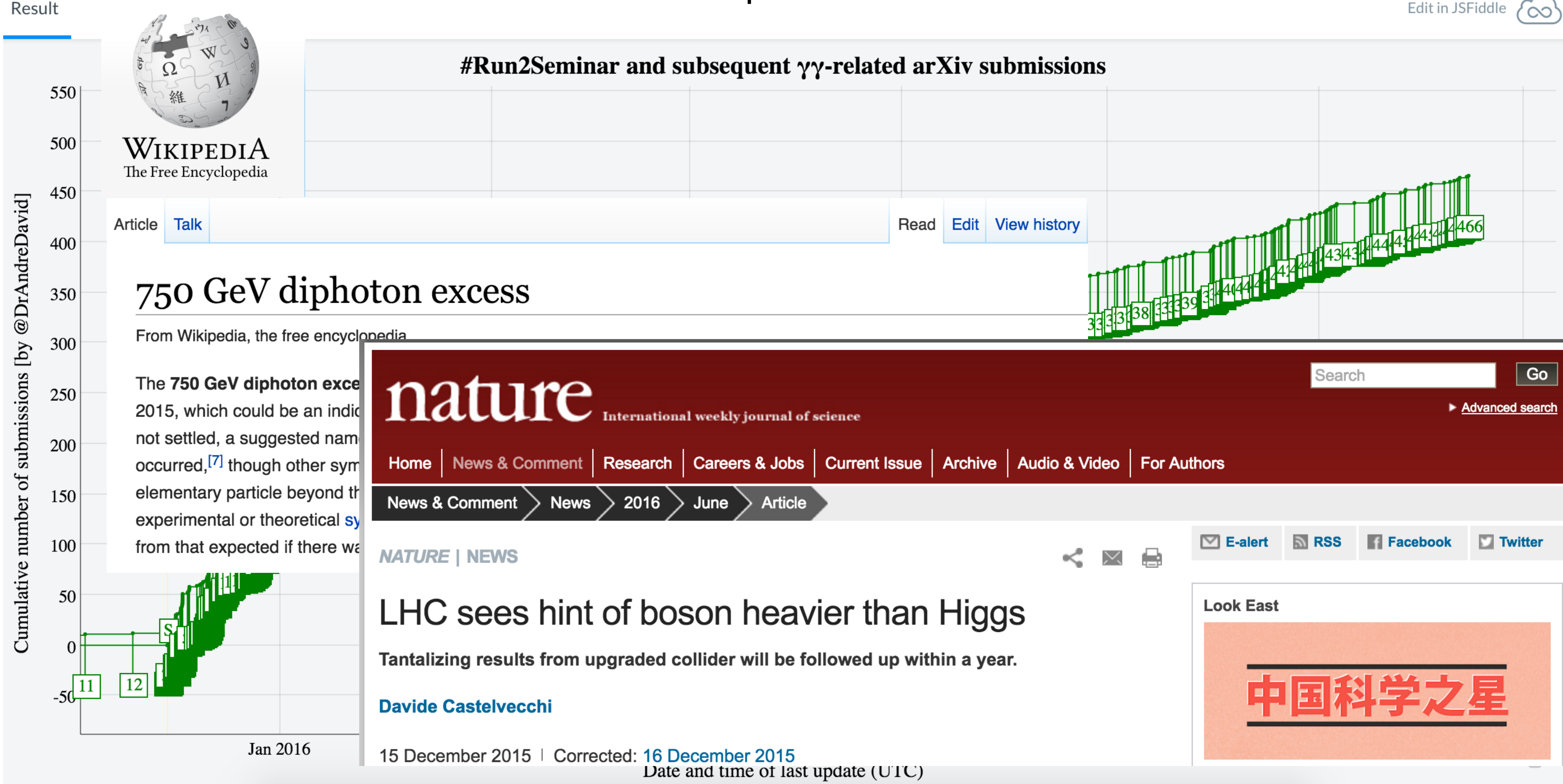
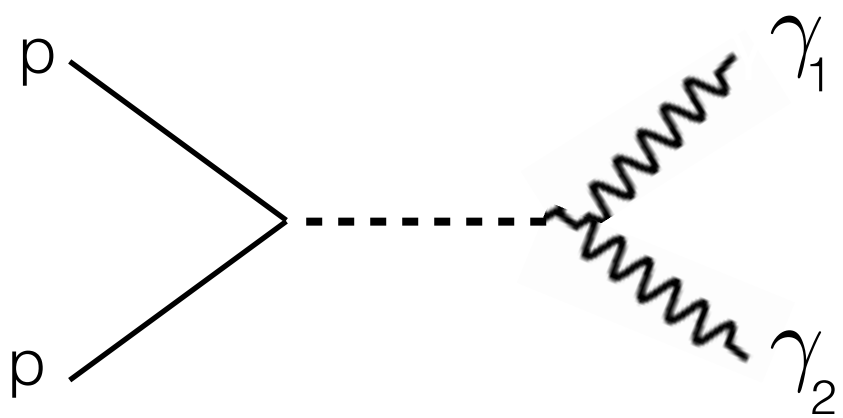
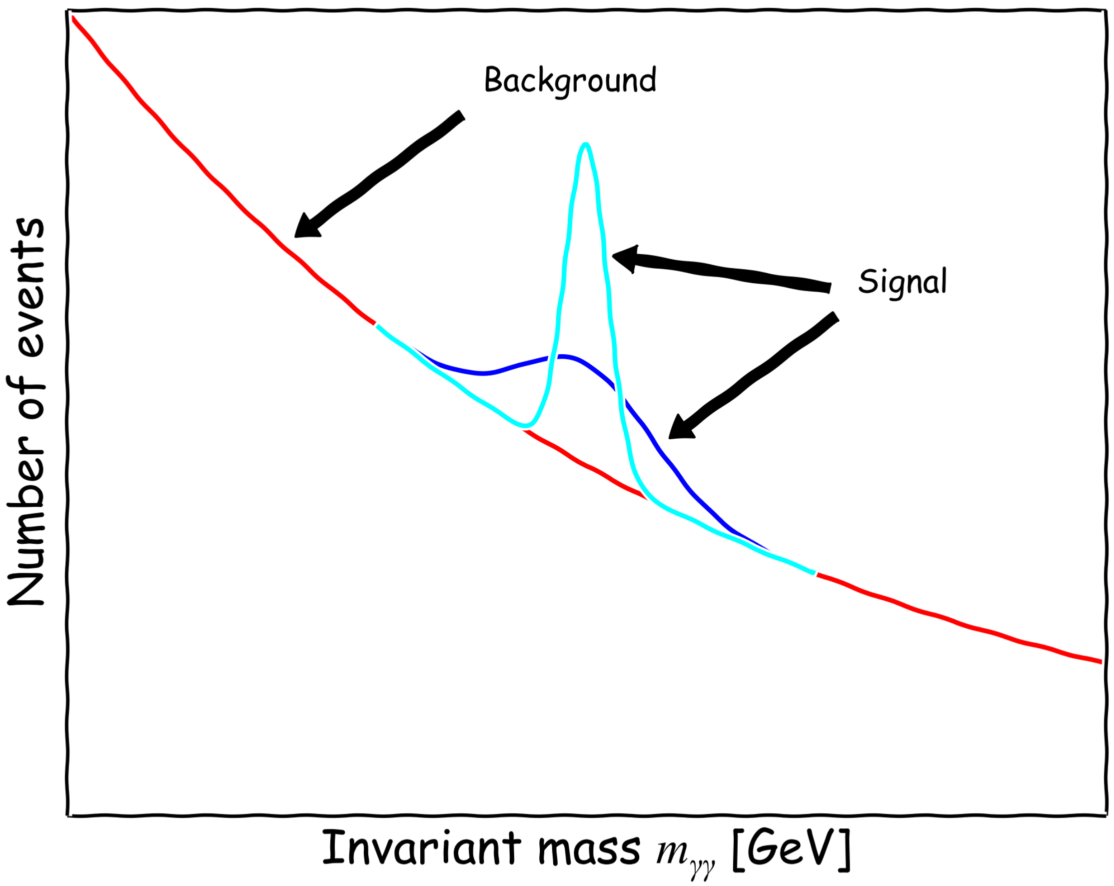


The 750 GeV buzz (fuss)



$$m_{\gamma\gamma}^2 = 2E_{\gamma_1}E_{\gamma_2}(1-\cos\theta_{\gamma_1\gamma_2})$$



- ATLAS: 3.9**σ** local and 2.1**σ** global significance (13 TeV)
- CMS: 3.4**σ** local and 1.4**σ** global significance (8 + 13 TeV)
- Excess with large cross sections ~ few fb
- Not possible to obtain large width, cross section with couplings only to SM particles → additional decay modes necessary

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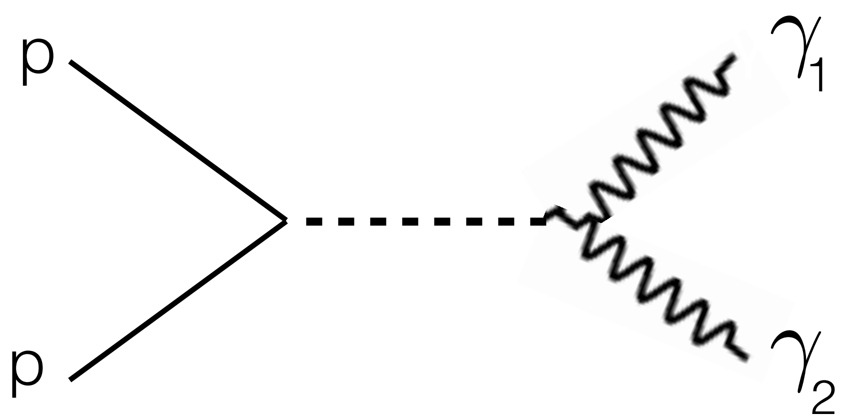

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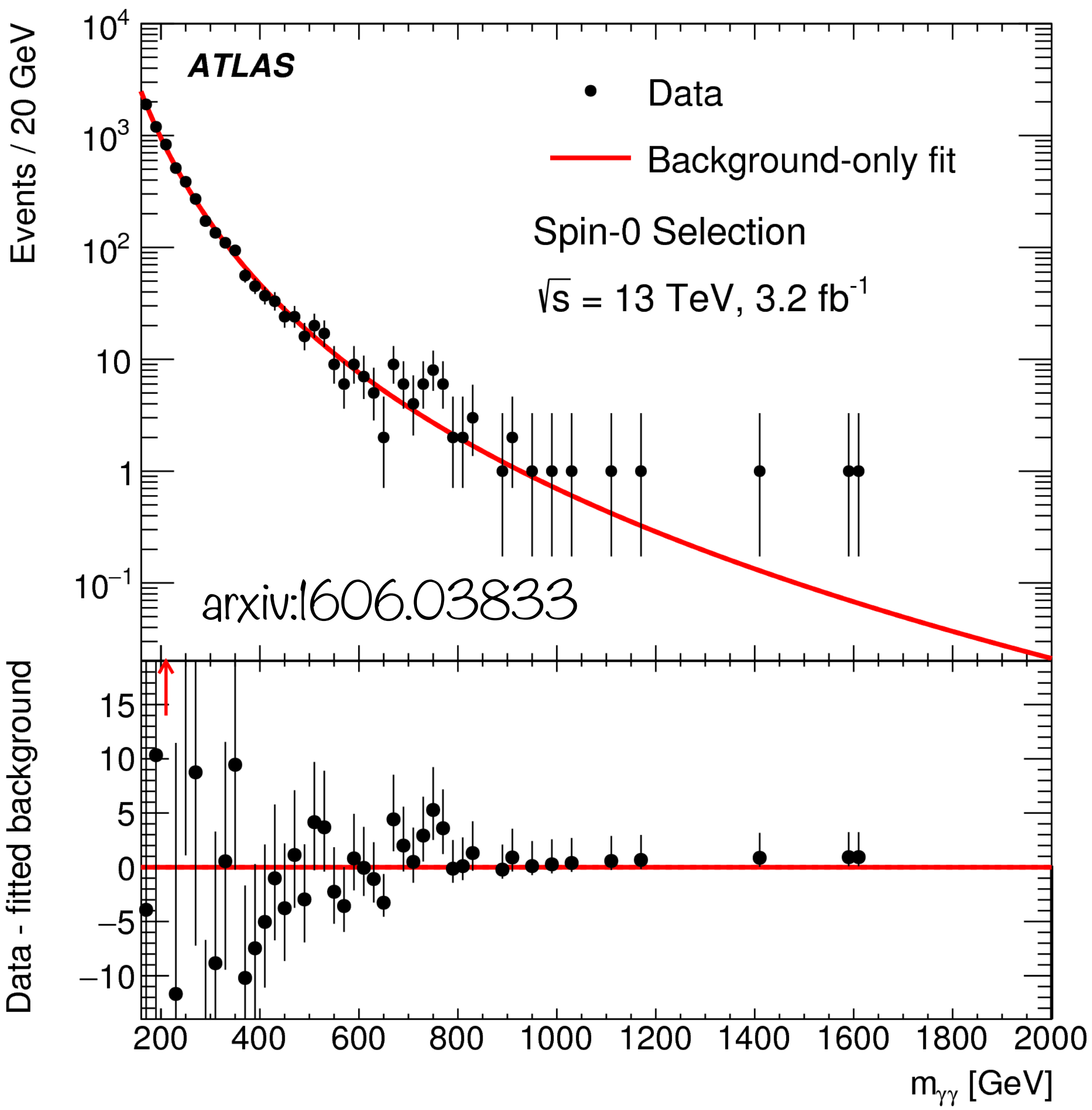

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750 GeV portal to DM

- Coupling of a 750 scalar resonance to gauge bosons and dark matter particle
- d = 5 effective Lagrangian, Majorana fermion dark matter

$$\mathcal{L}_{\mathrm{NP,CPE}} = \frac{1}{2}(\partial_\mu s)^2 - \frac{\mu_s^2}{2} s^2 + \frac{1}{2}\bar{\psi}(i\not{\partial}_\mu - m_\psi)\psi - \frac{y_\psi}{2} s\bar{\psi}\psi \\ - \frac{g_1^2}{4\pi}\frac{1}{4\Lambda_1} s\ B_{\mu\nu}B^{\mu\nu} - \frac{g_2^2}{4\pi}\frac{1}{4\Lambda_2} s\ W_{\mu\nu}W^{\mu\nu} - \frac{g_3^2}{4\pi}\frac{1}{4\Lambda_3} s\ G_{\mu\nu}G^{\mu\nu}$$

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Effective couplings to gauge bosons

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$$- \frac{g_1^2}{4\pi}\frac{1}{4\Lambda_1}s\, B_{\mu\nu}B^{\mu\nu} - \frac{g_2^2}{4\pi}\frac{1}{4\Lambda_2}s\, W_{\mu\nu}W^{\mu\nu} - \frac{g_3^2}{4\pi}\frac{1}{4\Lambda_3}s\, G_{\mu\nu}G^{\mu\nu}$$

Mass scales for effective couplings

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SM gauge couplings

750 GeV portal to DM

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$$\mathcal{L}_{\mathrm{NP,CPE}} = \frac{1}{2}(\partial_\mu s)^2 - \frac{\mu_s^2}{2}s^2 + \frac{1}{2}\bar{\psi}(i\partial\!\!\!/_\mu - m_\psi)\psi - \frac{y_\psi}{2}s\bar{\psi}\psi$$
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Coupling to DM
(controls width of s)

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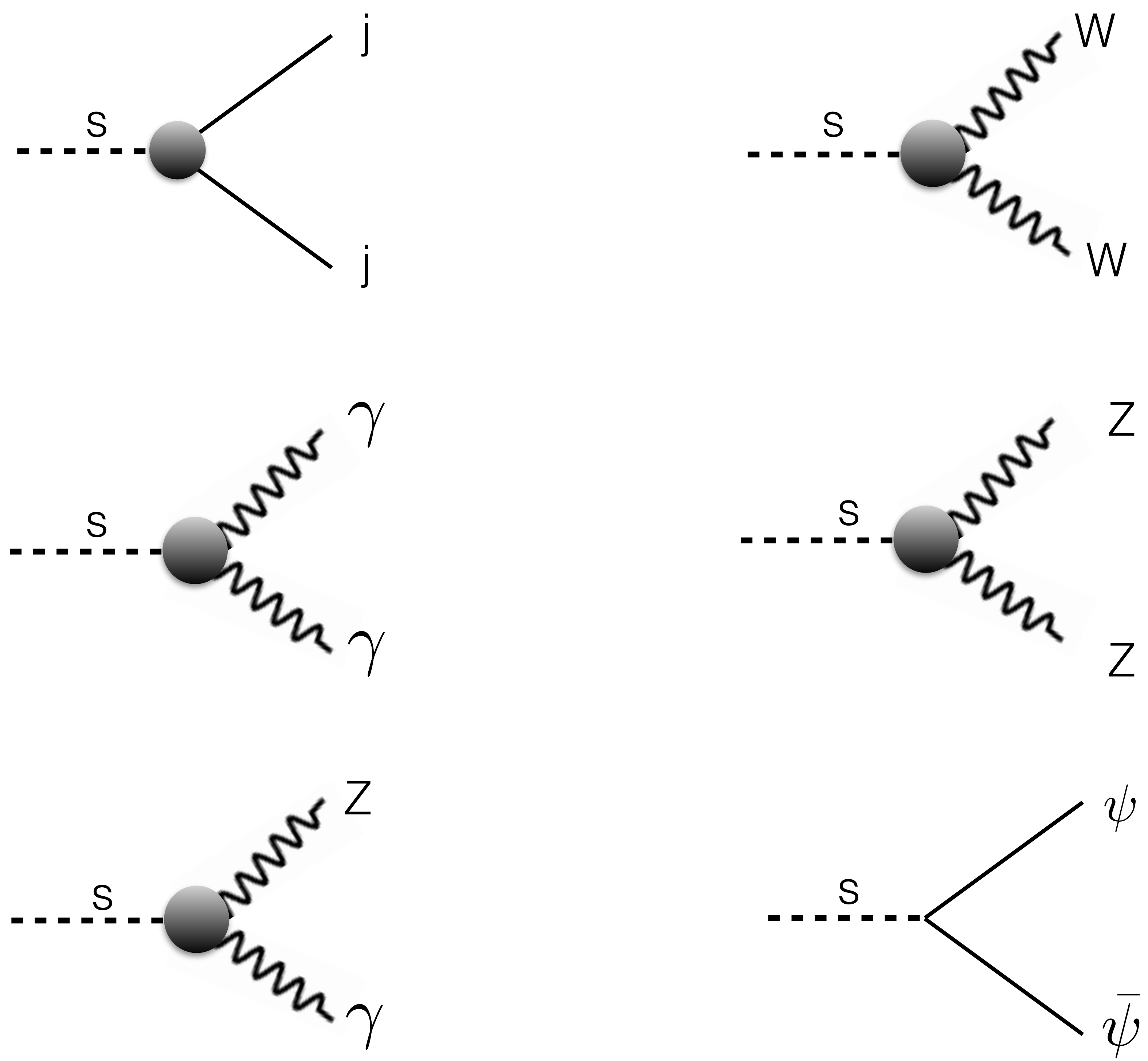
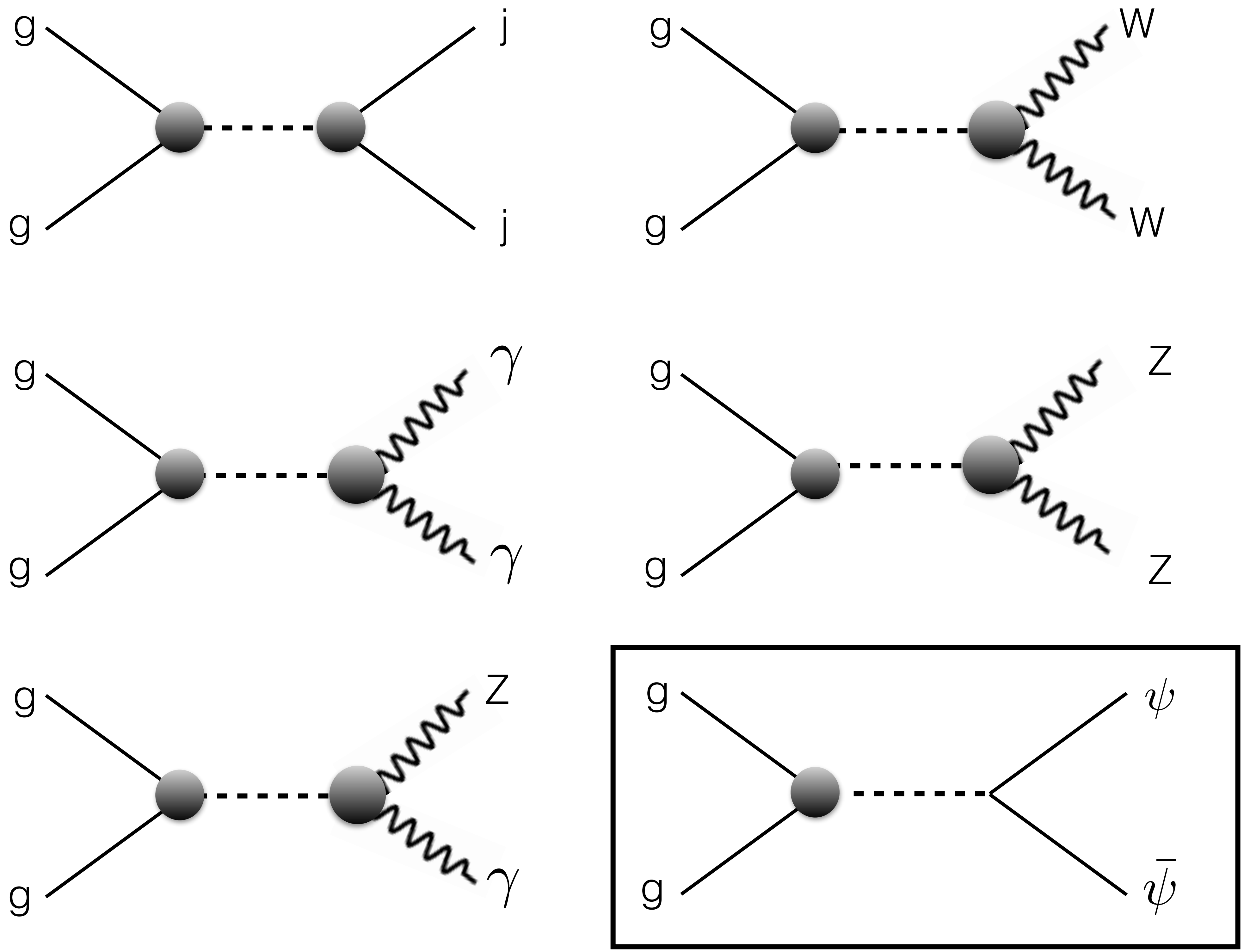
Mass term 750 GeV

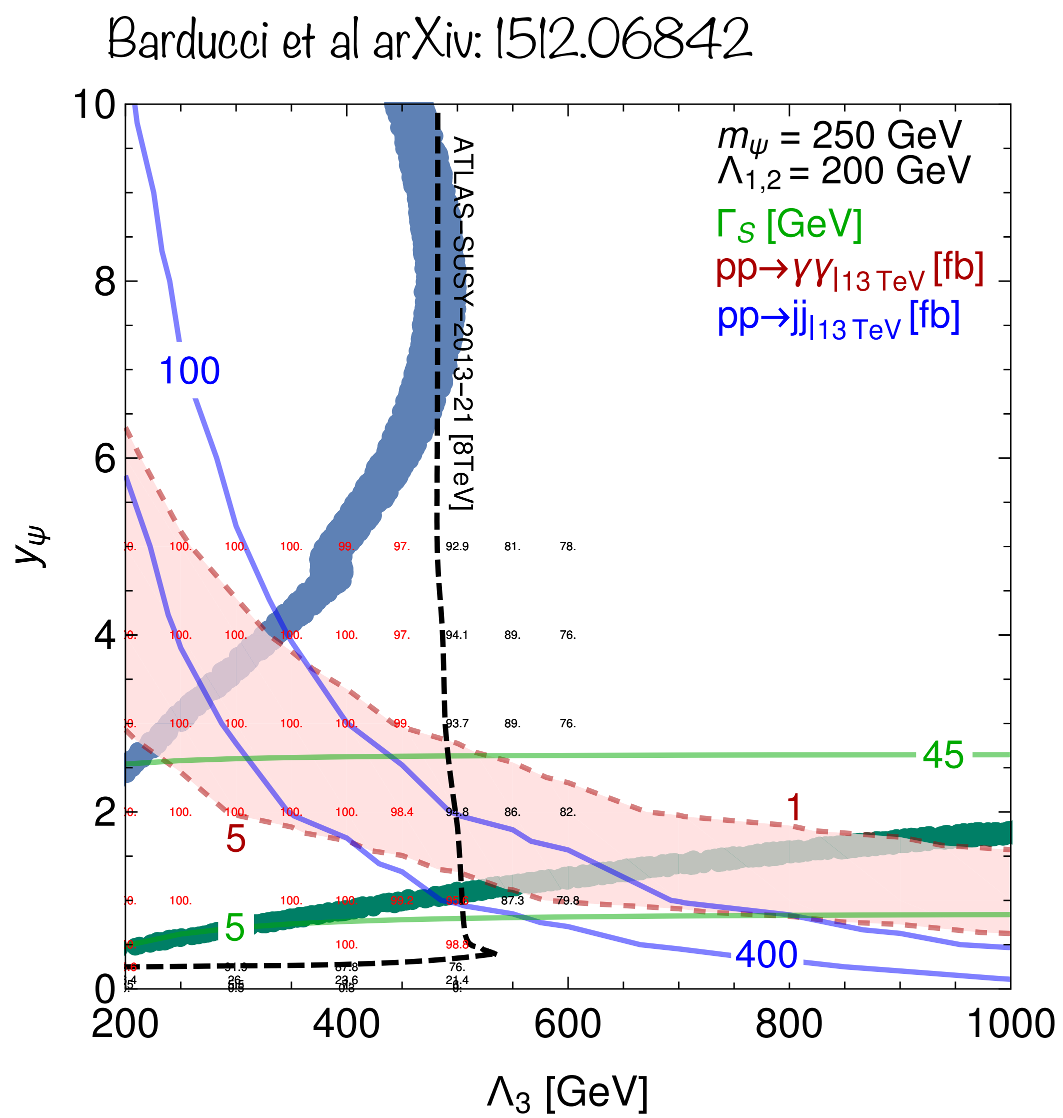
Decays



LHC phenomenology



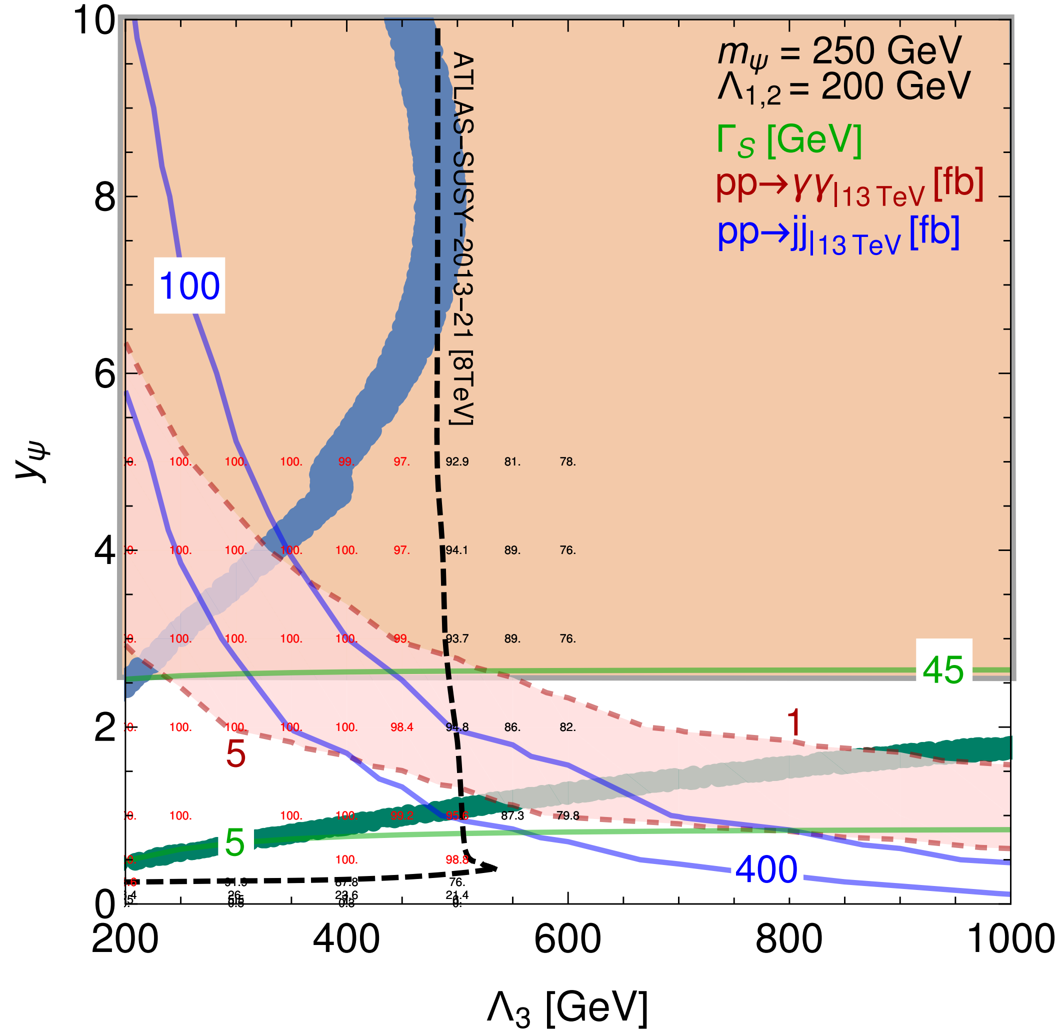
Melange des constraints



- Using ATLAS SUSY analysis it is possible to constrain the parameter space for diphoton excess models connected to dark matter

Melange des constraints

Barducci et al arXiv: 1512.06842



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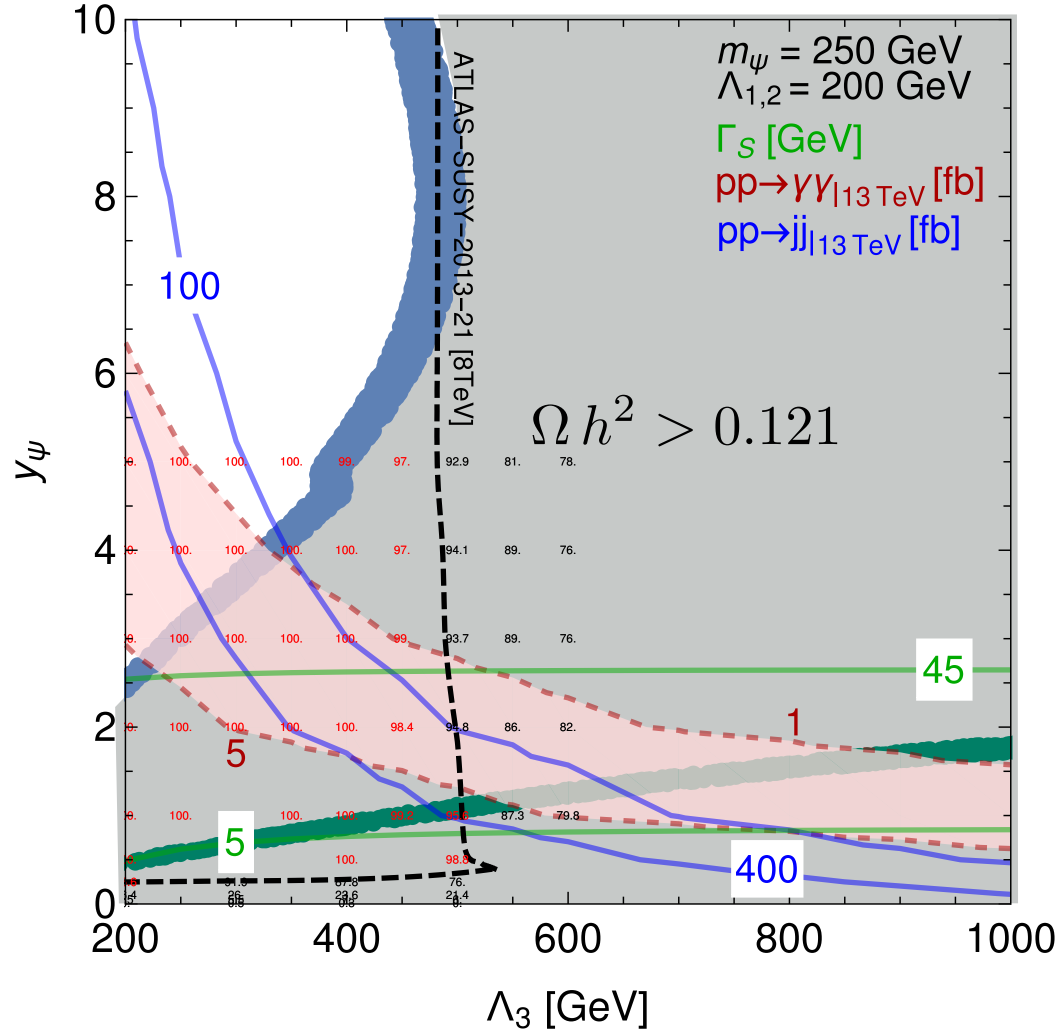

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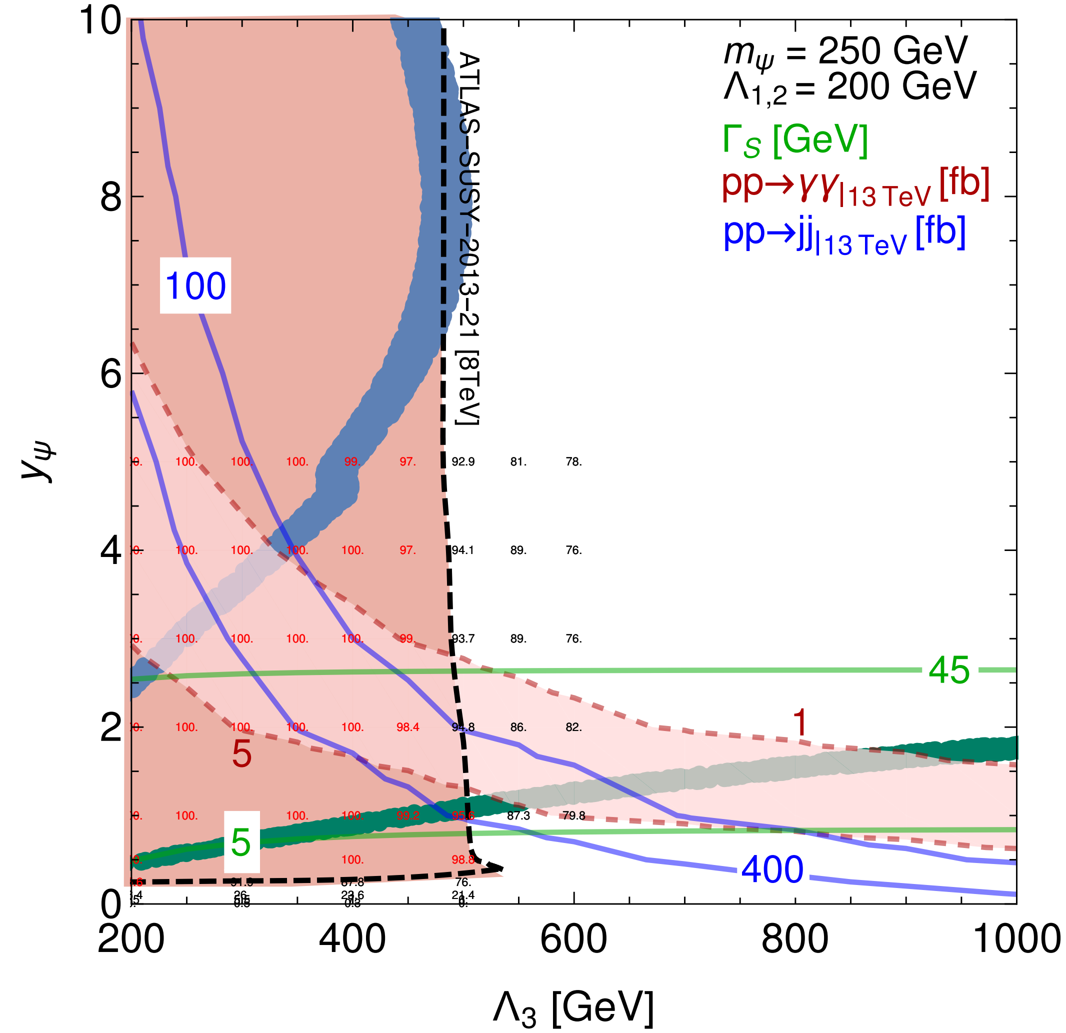

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Test case: II

Exploring momentum dependent dark matter couplings

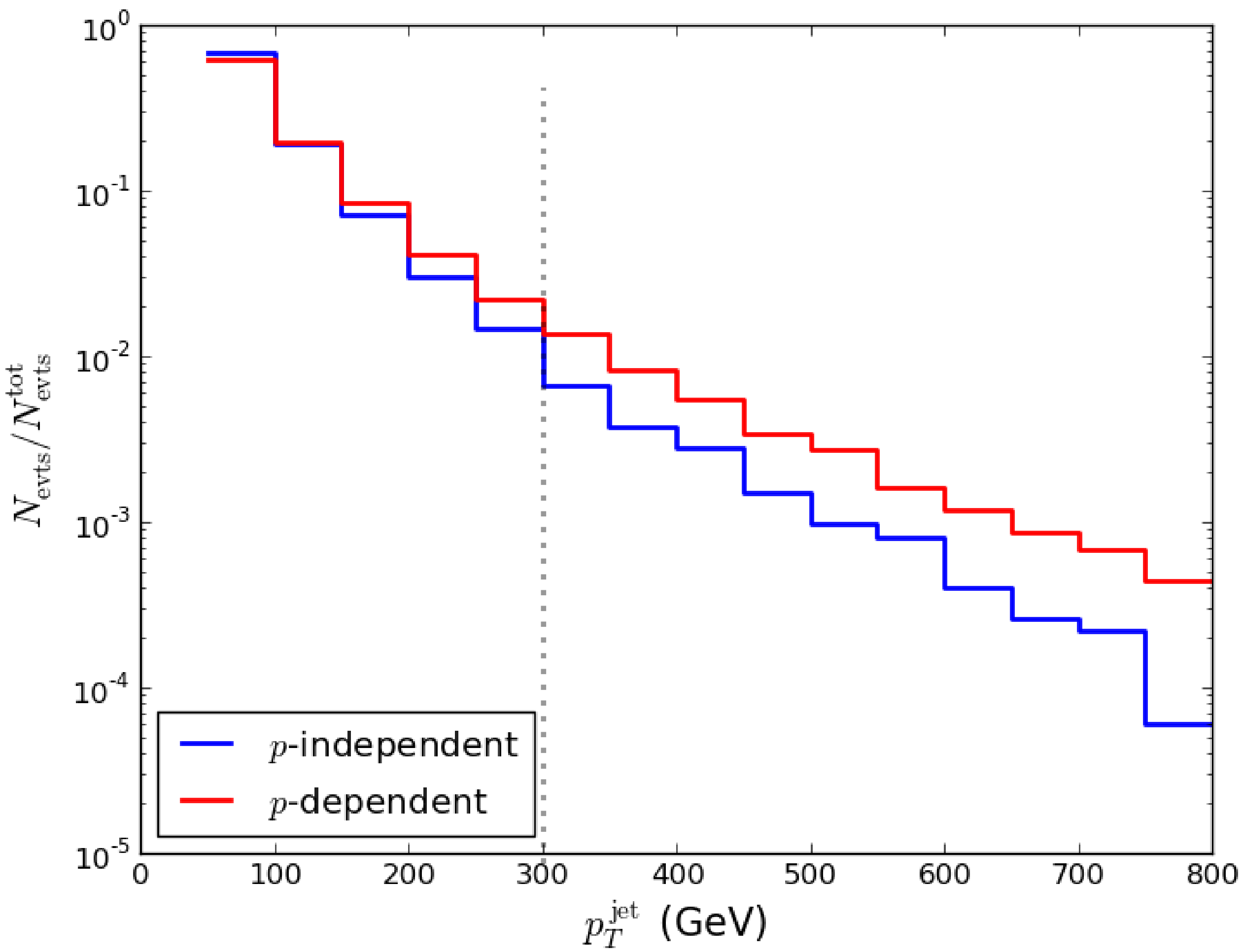
Impact on monojets

- Operators generally considered at the LHC

$$\mathcal{L}_{\text{vector}} = g_{\text{q}} \sum_{q=u,d,s,c,b,t} Z'_\mu \bar{q}\gamma^\mu q + g_\chi Z'_\mu \bar{\chi}\gamma^\mu \chi$$

$$\mathcal{L}_{\text{axial-vector}} = g_{\text{q}} \sum_{q=u,d,s,c,b,t} Z'_\mu \bar{q}\gamma^\mu \gamma^5 q + g_\chi Z'_\mu \bar{\chi}\gamma^\mu \gamma^5 \chi.$$

- What about more complicated interactions, e.g. momentum dependent ones?



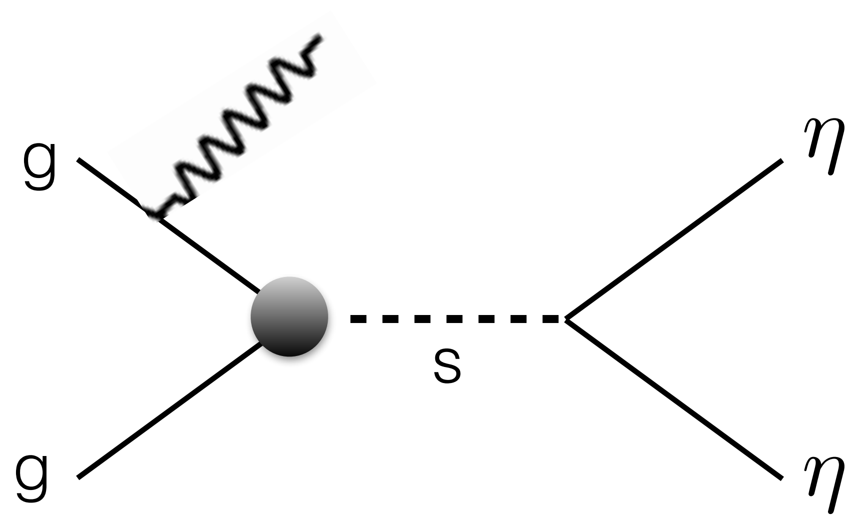
$$i g_{h\eta\eta} = 2iv\left[\lambda_{\text{mi}} + \frac{p_h^2}{f^2}\right]$$

Model

Barducci et al, arXiv:1605.02684

- Z_2 odd real singlet scalar dark matter particle couplings to the Standard Model with Z_2 even scalar singlet

$$\mathcal{L}_{\eta,s} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}m_\eta^2\eta\eta + \frac{1}{2}\partial_\mu s\partial^\mu s - \frac{1}{2}m_s^2 ss$$
$$+ \frac{c_{s\eta}f}{2}s\eta\eta + \frac{c_{\partial s\eta}}{f}(\partial_\mu s)(\partial^\mu\eta)\eta + \frac{\alpha_s}{16\pi}\frac{c_{sg}}{f}sG^a_{\mu\nu}G^{a\mu\nu}$$



For consistent model constructions and detailed dark matter phenomenology see Fonseca et al. arXiv:1501.05957

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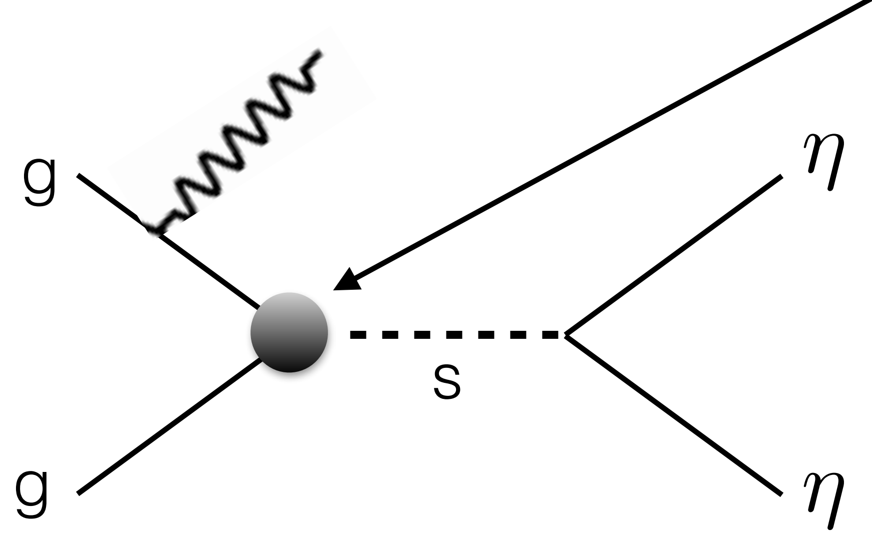

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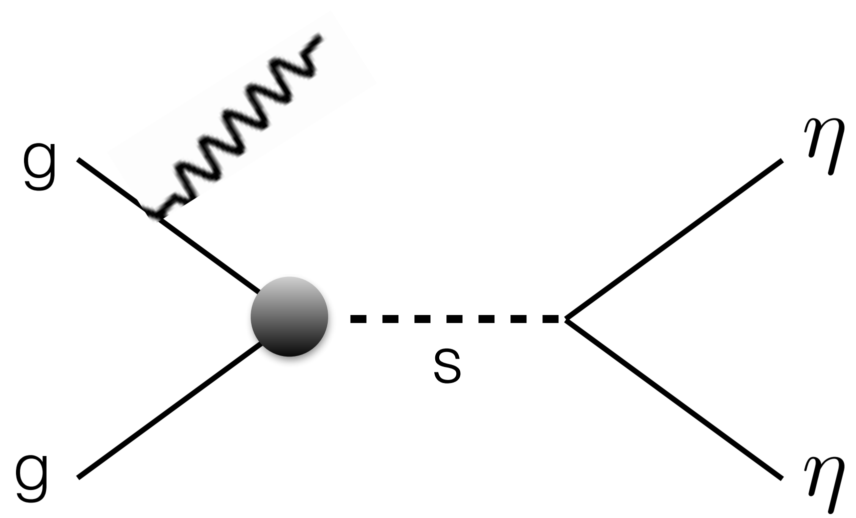

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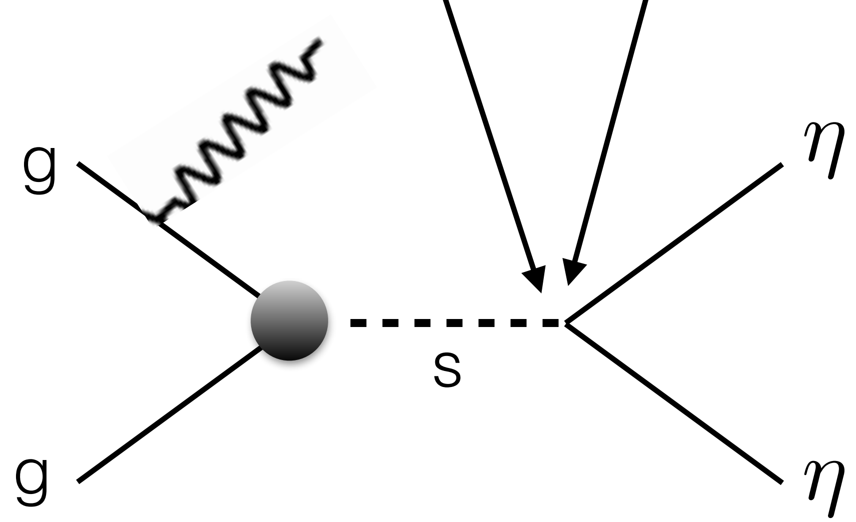

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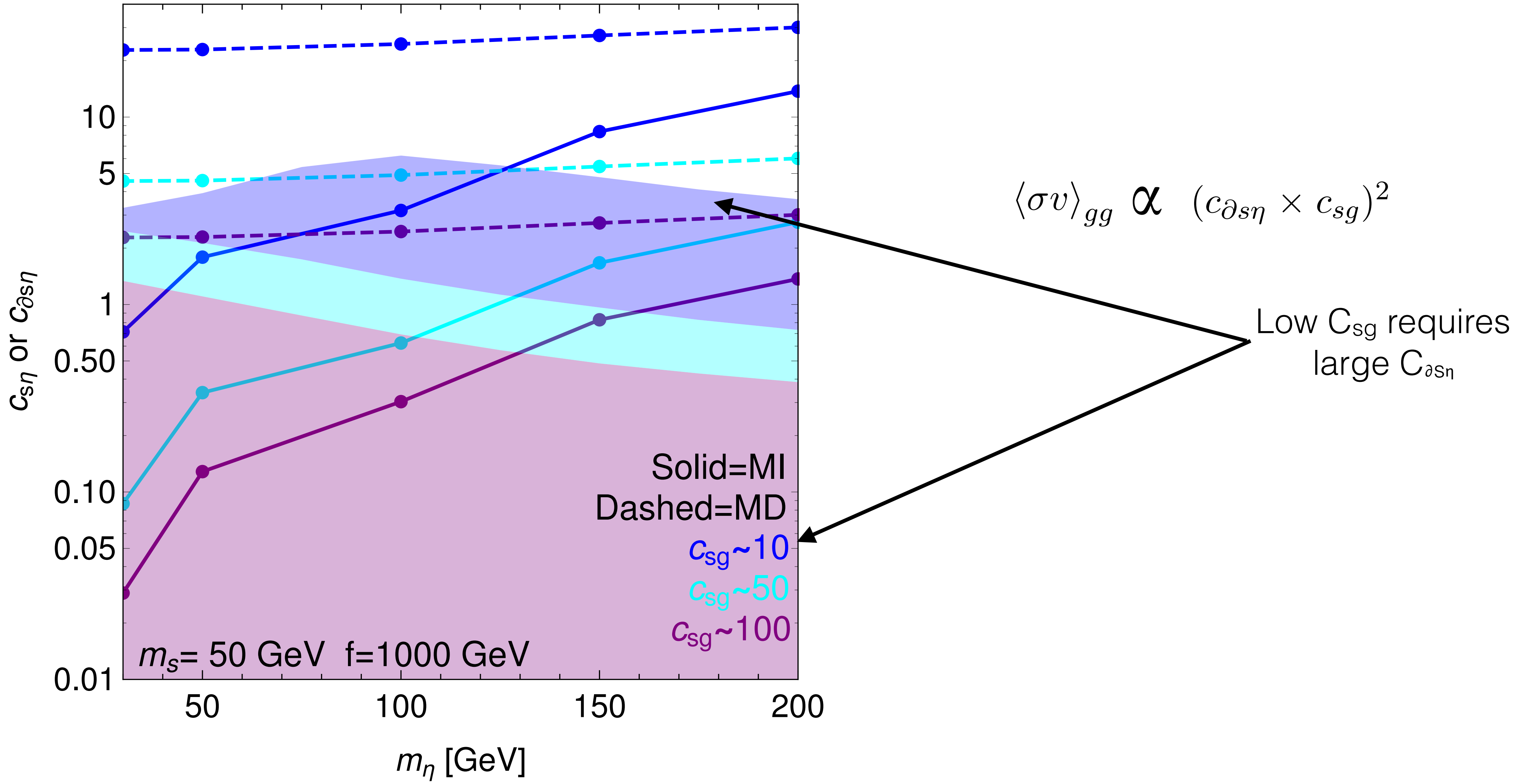
Limits



$$\langle \sigma v \rangle_{gg} \propto (c_{\partial s\eta} \times c_{sg})^2$$

- Very different limits for momentum dependent and independent couplings
- LHC can not yet probe regions compatible relic density, situation might be more optimistic at 13 TeV searches (work in progress)

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Conclusions

- Identifying the properties of dark matter is one the challenges of 21st century
- Searches at the LHC can help constrain the properties of thermal dark matter
- The dark matter motivated explanations of 750 GeV diphoton excess are well constrained by the monojet searches
- Dark matter can also have momentum dependent couplings e..g. if dark matter is pNGB boson
  - The momentum dependent and independent couplings yield genuine differences in the pT distributions of the jets and hence in the limits derived from monojet searches
  - Current limits from 8TeV monojet searches do not probe relic compatible region

Backup

Test case: I

750 GeV as a portal to dark matter

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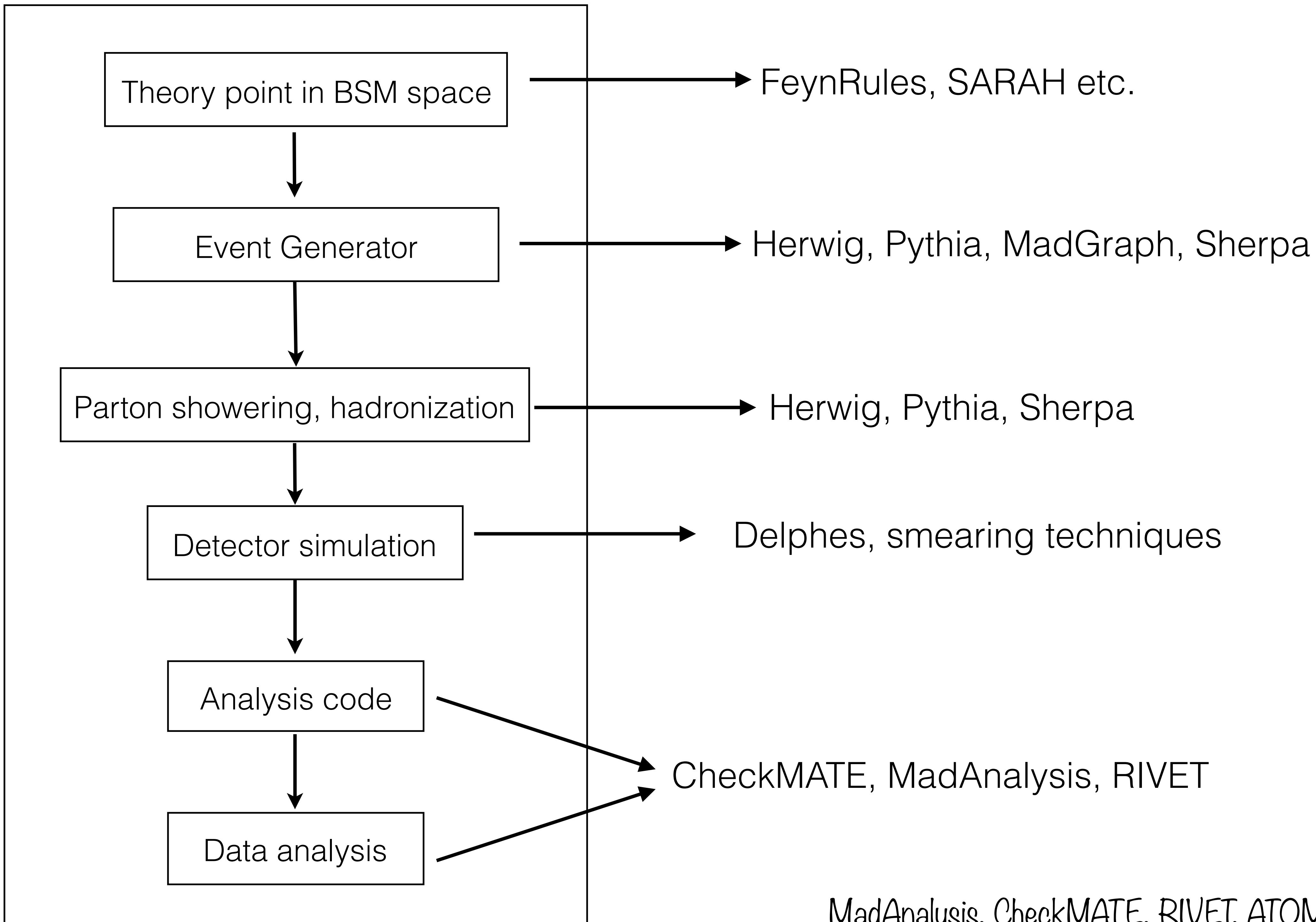
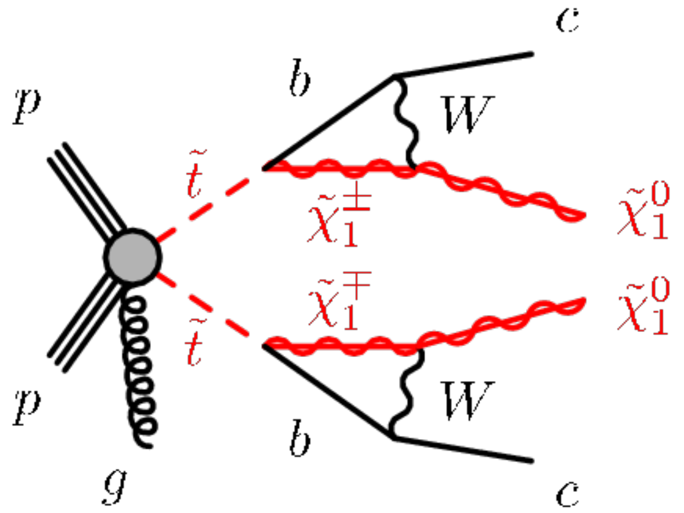
Analysis reinterpretation



ATLAS-SUSY-2013-21

- Search for compressed stops
- Considers monojet (ISR) and c-tagging
- Only monojet analysis implemented in MadAnalysis5



ATLAS-SUSY-2013-21

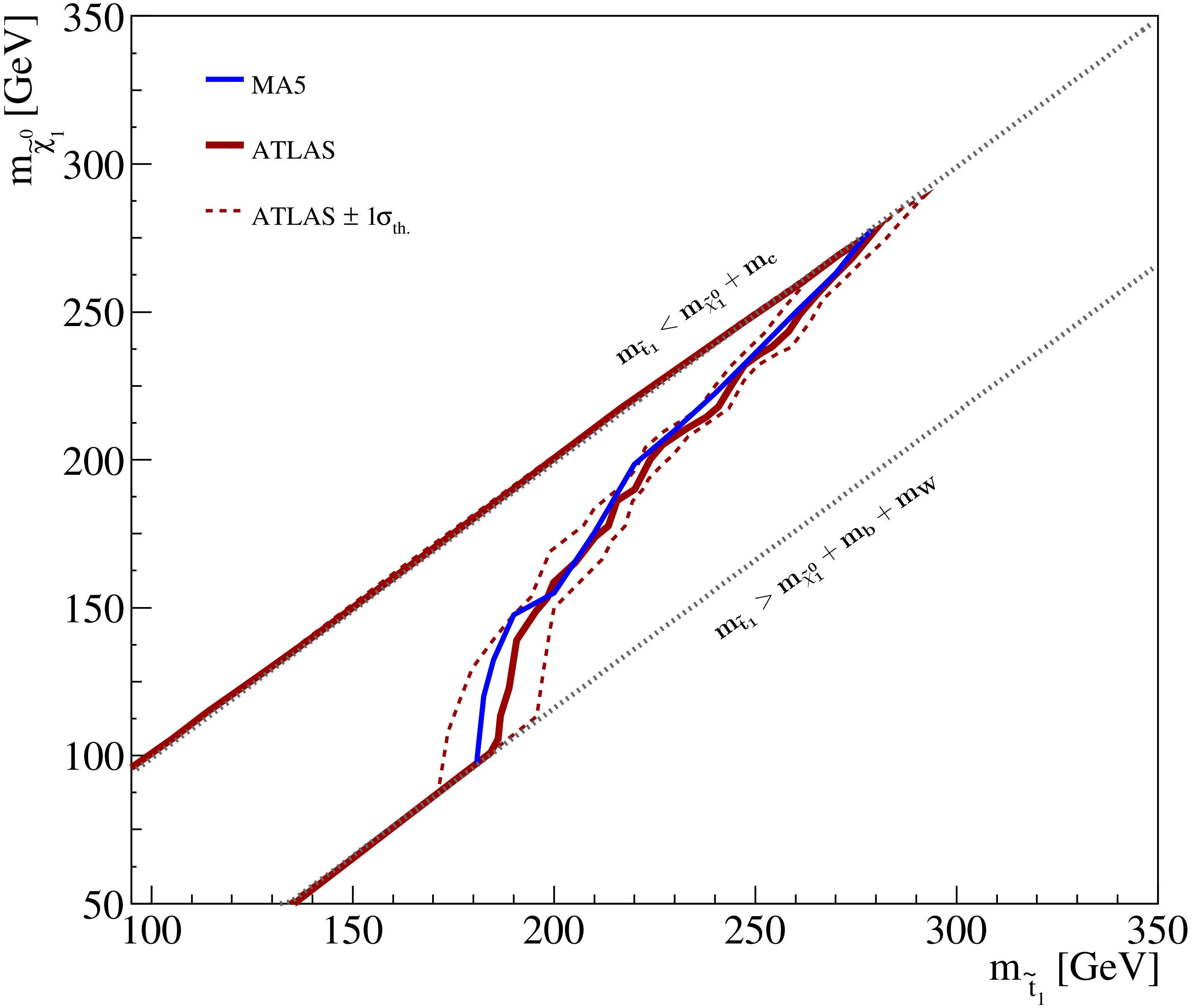
Sengupta et. al. https://inspirehep.net/record/1388797

$\tilde{t} \to c\tilde{\chi}_1^0$ (200/125) cutflow

cut	# events (scaled to σ and $\mathcal{L}$)	relative change	# events (official)	relative change (official)
Initial number of events	376047.3	376047.3		
$E_T^{\text{miss}} > 80$ GeV Filter	192812.8	-48.7%	181902.0	181902.0
$E_T^{\text{miss}} > 100$ GeV	136257.1	-29.3%	97217.0	-46.6%
Trigger, Event cleaning...	-	-	82131.0	
Lepton veto	134894.2	-1.0%	81855.0	-15.8%
$N_{\text{jets}} \leq 3$	101653.7	-24.6%	59315.0	-27.5%
$\Delta\phi(E_T^{\text{miss}}, \text{jets}) > 0.4$	95568.8	-2.1%	54295.0	-8.5%
Leading jet $p_T > 150$ GeV	17282.8	-81.9%	14220.0	-73.8%
$E_T^{\text{miss}} > 150$ GeV	10987.8	-36.4%	9468.0	-33.4%
M1 Signal Region				
Leading jet $p_T > 280$ GeV	2031.2	-81.5%	1627.0	-82.8%
$E_T^{\text{miss}} > 220$ GeV	1517.6	-25.3%	1276.0	-21.6%
M2 Signal Region				
Leading jet $p_T > 340$ GeV	858.0	-92.2%	721.0	-92.4%
$E_T^{\text{miss}} > 340$ GeV	344.4	-59.9%	282.0	-60.9%
M3 Signal Region				
Leading jet $p_T > 450$ GeV	204.3	-98.1%	169.0	-98.2%
$E_T^{\text{miss}} > 450$ GeV	61.3	-70.0%	64.0	-62.1%

ATLAS-SUSY-2013-21

Sengupta et. al. https://inspirehep.net/record/1388797

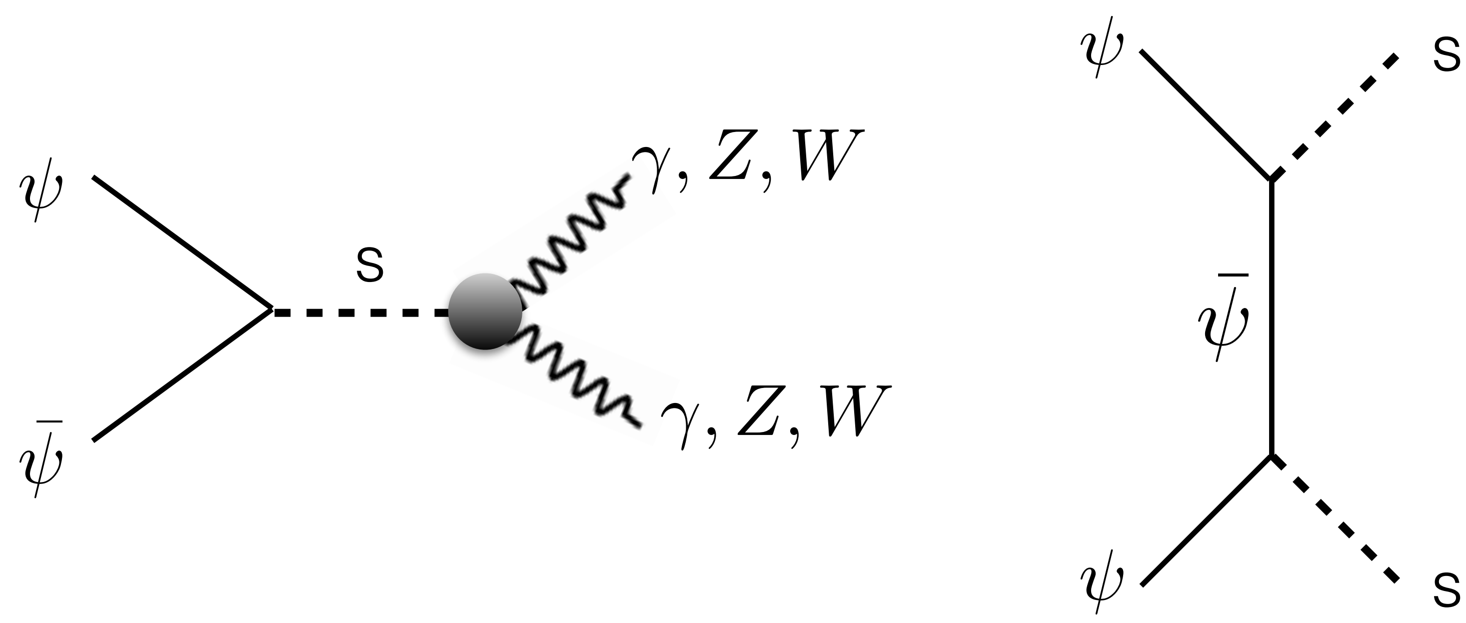


Dark matter relic

- CP even and CP odd couplings make a big difference for relic density
- CP even case: p-wave suppression for annihilation cross section
  - Needs large values of Yukawa couplings to achieve relic
- CP odd case: No velocity suppression much smaller values of Yukawa couplings work

- No couplings to fermions present
- For dark matter mass less than 375 GeV, only gauge bosons, photons, and gg final states in s-channel present
- For dark matter mass greater than 750 GeV t-channel annihilation into scalar resonance possible
  - Contribution up to 20% of the first scenario

Dark matter relic



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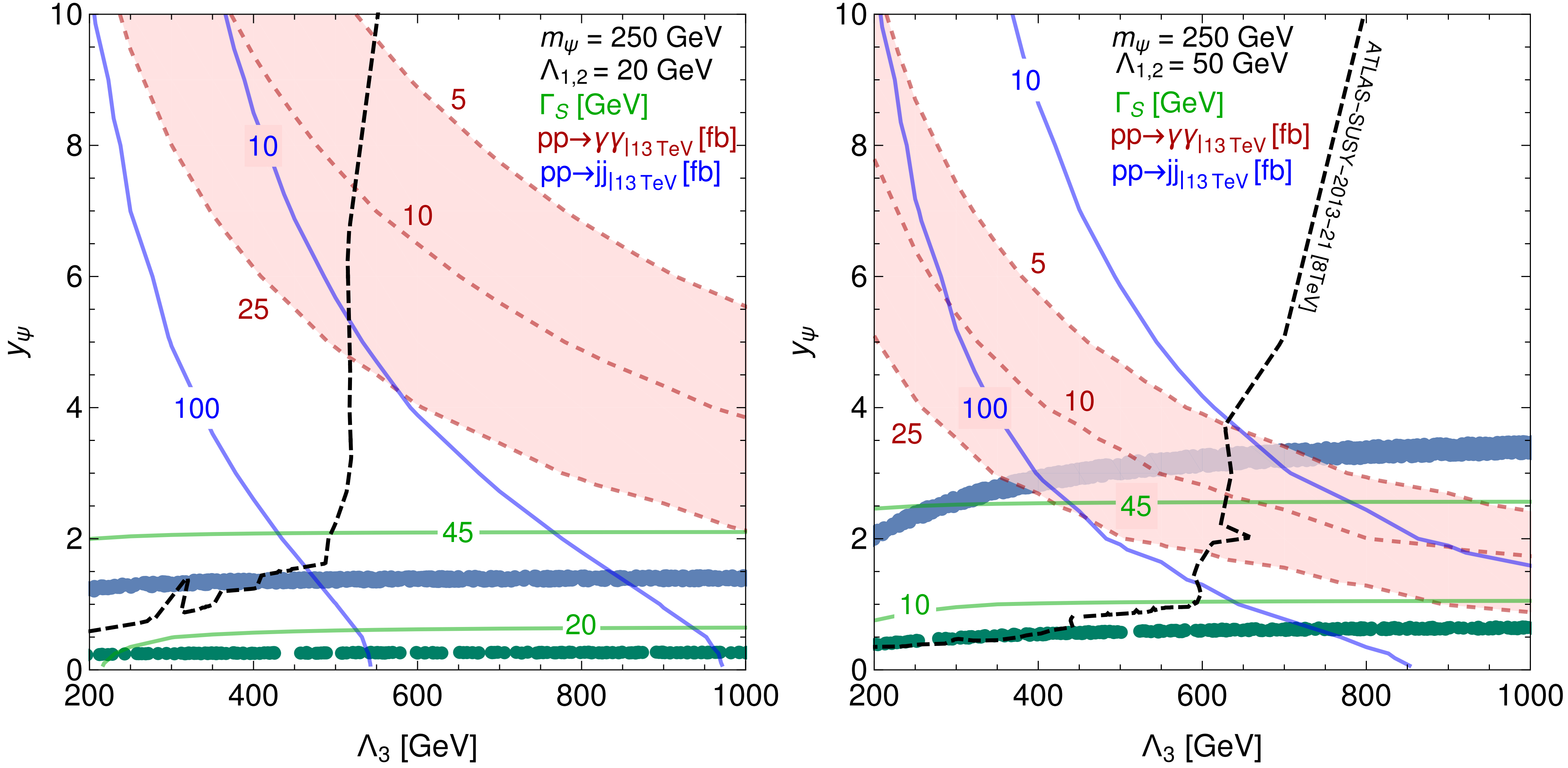
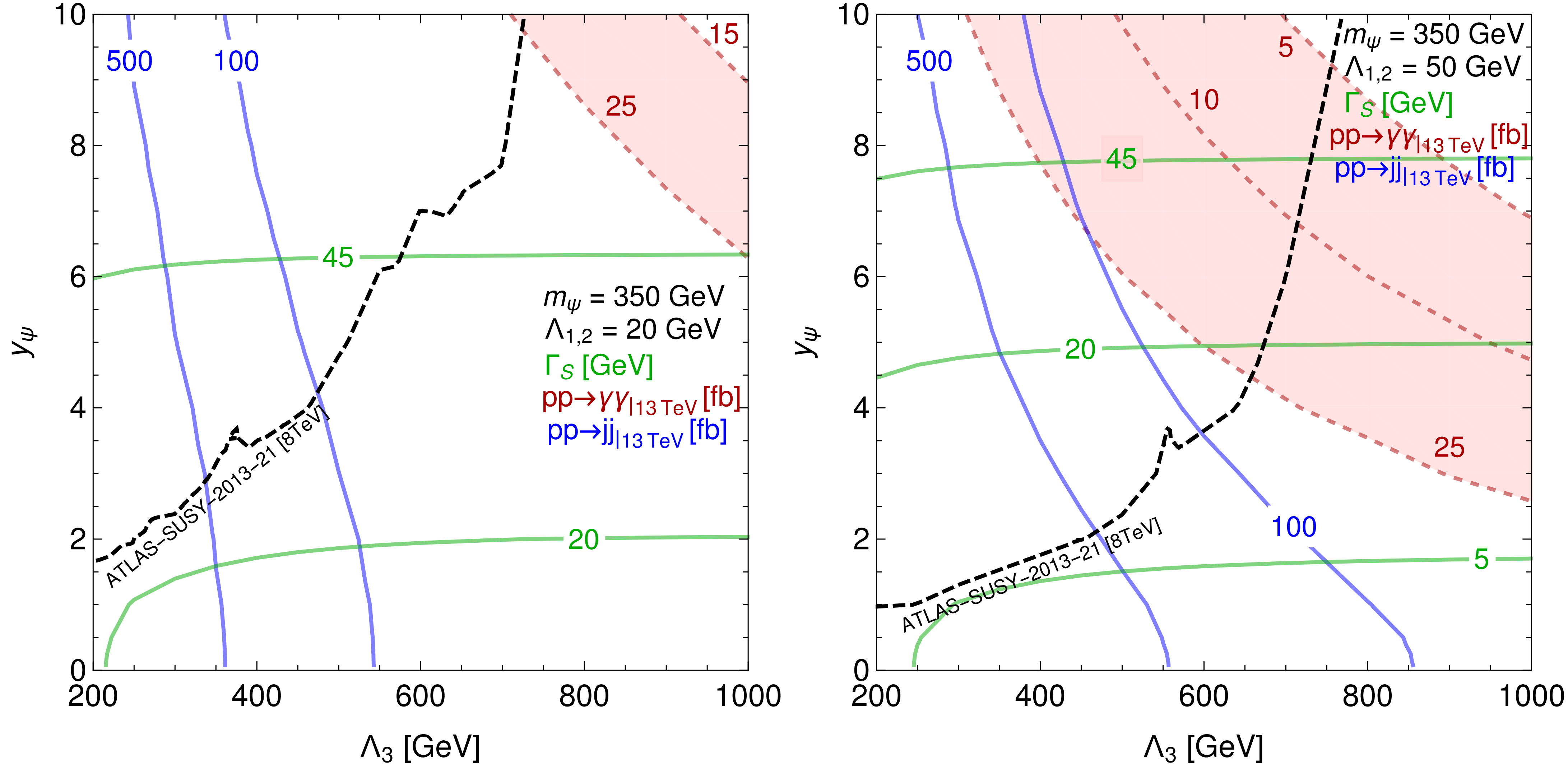
Additional results



Additional results



Test case: II

Exploring momentum dependent dark matter couplings

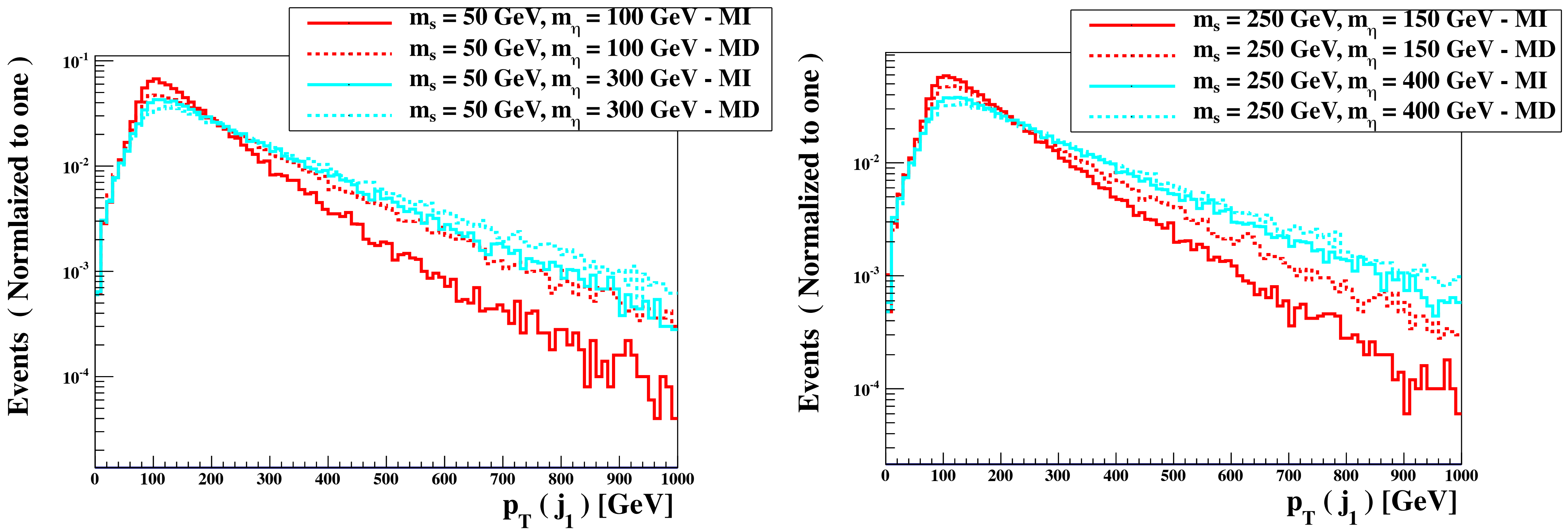
Propagator dependence

- The upper limits on the cross section are independent of the propagator mass

m_η	m_s	$\mathcal{A} \times \epsilon$ (SR1)		$\mathcal{A} \times \epsilon$ (SR2)		$\mathcal{A} \times \epsilon$ (SR3)		$\sigma_{UL}^{95\%\mathrm{CL}}$[pb]	
		MD	MI	MD	MI	MD	MI	MD	MI
200	50	0.123	0.101	0.073	0.056	0.033	0.023	0.317	0.465
200	250	0.124	0.104	0.069	0.054	0.031	0.022	0.349	0.487

Effect of momentum dependence

Barducci et al, to appear



$$c_{gs} = 100 \quad f = 1\ \mathrm{TeV} \quad c_{\partial s\eta} = 2.5 \quad c_{s\eta} = 0.5$$

Production cross section of 2.9 pb after generator cut of jet pT > 80 GeV

For $p_T > 300$ GeV,
Luminosity $300\ \mathrm{fb}^{-1}$

#events MI	#events MD
131300	196533

Momentum dependent expected to yield 50% better sensitivity

Simple case

Barducci et al, arXiv:1605.02684

- Extension of the Standard Model by gauge singlet real scalar field

$$\mathcal{L}_\eta = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}\mu_\eta^2\eta^2 - \frac{1}{4}\lambda_\eta\eta^4 - \frac{1}{2}\lambda\eta^2 H^\dagger H + \frac{1}{2f^2}(\partial_\mu\eta^2)\partial^\mu(H^\dagger H)$$

- After electroweak symmetry breaking

$$\mathcal{L}_\eta \supset -\frac{1}{4}(v+h)^2\left(\lambda\eta^2 + \frac{1}{f^2}\partial_\mu\partial^\mu\eta^2\right)$$

$$m_\eta^2 = \mu_\eta^2 + \lambda v^2/2$$

scale of spontaneous symmetry breaking

Momentum dependent coupling

For mono-Higgs signature study of similar model see e.g. arXiv:1312.2592

Simple case

- Extension of the Standard Model by gauge singlet real scalar field

$$\mathcal{L}_\eta = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu \eta \partial^\mu \eta - \frac{1}{2}\mu_\eta^2 \eta^2 - \frac{1}{4}\lambda_\eta \eta^4 - \frac{1}{2}\lambda \eta^2 H^\dagger H + \frac{1}{2f^2}(\partial_\mu \eta^2)\partial^\mu (H^\dagger H)$$

- Monojet production cross section

$$\hat{\sigma}(gg \to gh^* \to g\eta\eta) \propto \frac{\theta(p_h^2 - 4m_\eta^2)}{(p_h^2 - m_h^2)^2 + \Gamma_h^2 m_h^2} \left(\frac{p_h^2}{f^2} - \lambda\right)^2 \sqrt{1 - \frac{4m_\eta^2}{p_h^2}}$$

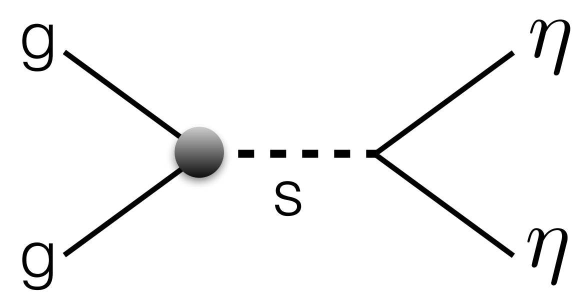
cross section < 1 fb	momentum dependent	On-shell production $p_h^2 = m_h^2$
cross section < 0.5 fb	momentum independent	No momentum dependence

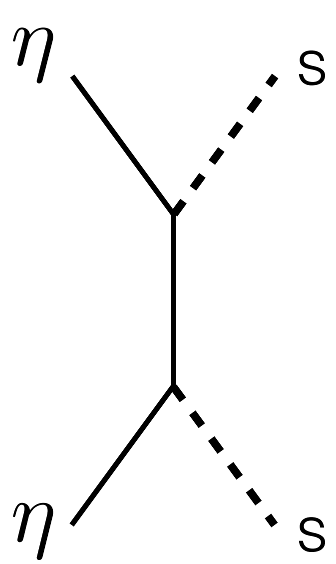
- Good measurements of Higgs production cross sections limit ggh couplings, decreasing the total cross section for monojet production

Relic density

- Unlike LHC constraints, relic density depends on the propagator mass
- Two annihilation channels

$$\langle \sigma v \rangle_{gg} \simeq \frac{\alpha_s^2 c_{sg}^2 \left(c_{s\eta} f^2 + 4 c_{\partial s\eta} m_s^2\right)^2}{256 \pi^3 f^4 \left(m_s^2 - 4 m_\eta^2\right)^2}$$





$$\langle \sigma v \rangle_{ss} \simeq \frac{\sqrt{1 - \frac{m_s^2}{m_\eta^2}} \left(c_{\partial s\eta} m_s^2 + c_{s\eta} f^2\right)^4}{16 \pi f^4 m_\eta^2 \left(m_s^2 - 2 m_\eta^2\right)^2}$$

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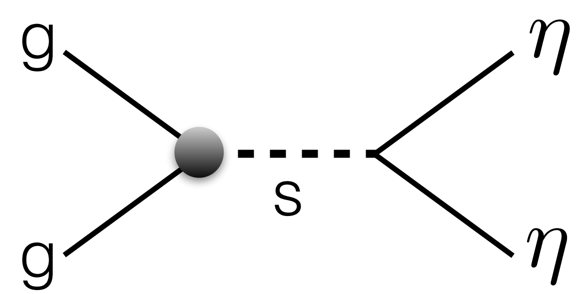
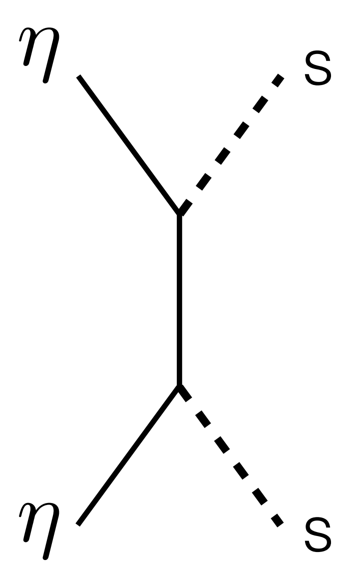
Mostly drives relic

$$\langle \sigma v \rangle_{gg} \simeq \frac{\alpha_s^2 c_{sg}^2 \left(c_{s\eta} f^2 + 4 c_{\partial s \eta} m_s^2\right)^2}{256 \pi^3 f^4 \left(m_s^2 - 4 m_\eta^2\right)^2}$$





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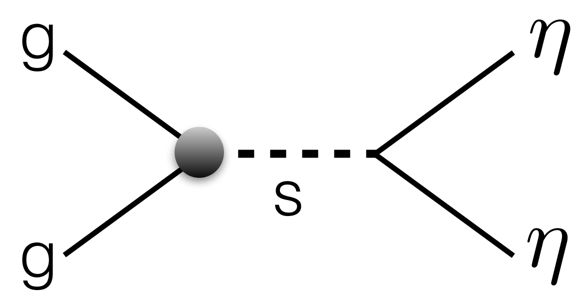
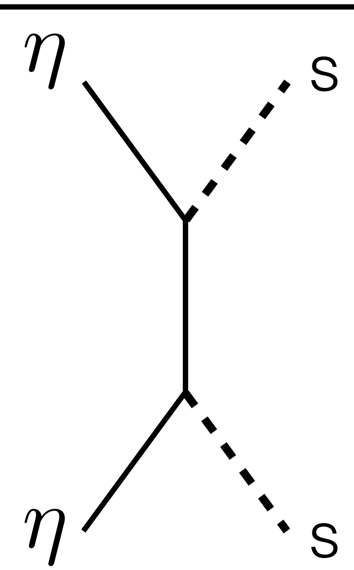


Contributes up to 15%



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