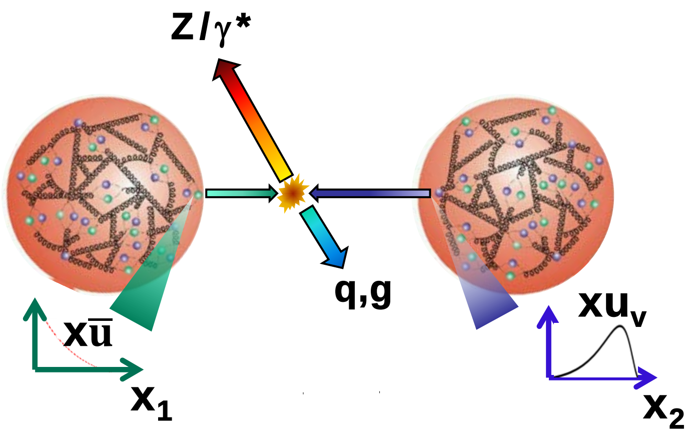
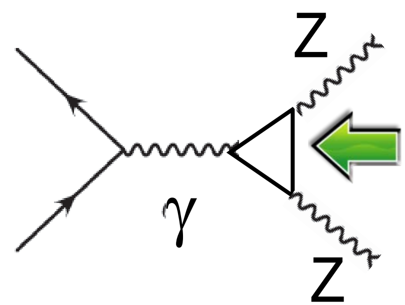
Recent Highlights of Electroweak Physics in ATLAS

Manuella G. Vincter (Carleton University)
on behalf of the ATLAS Collaboration

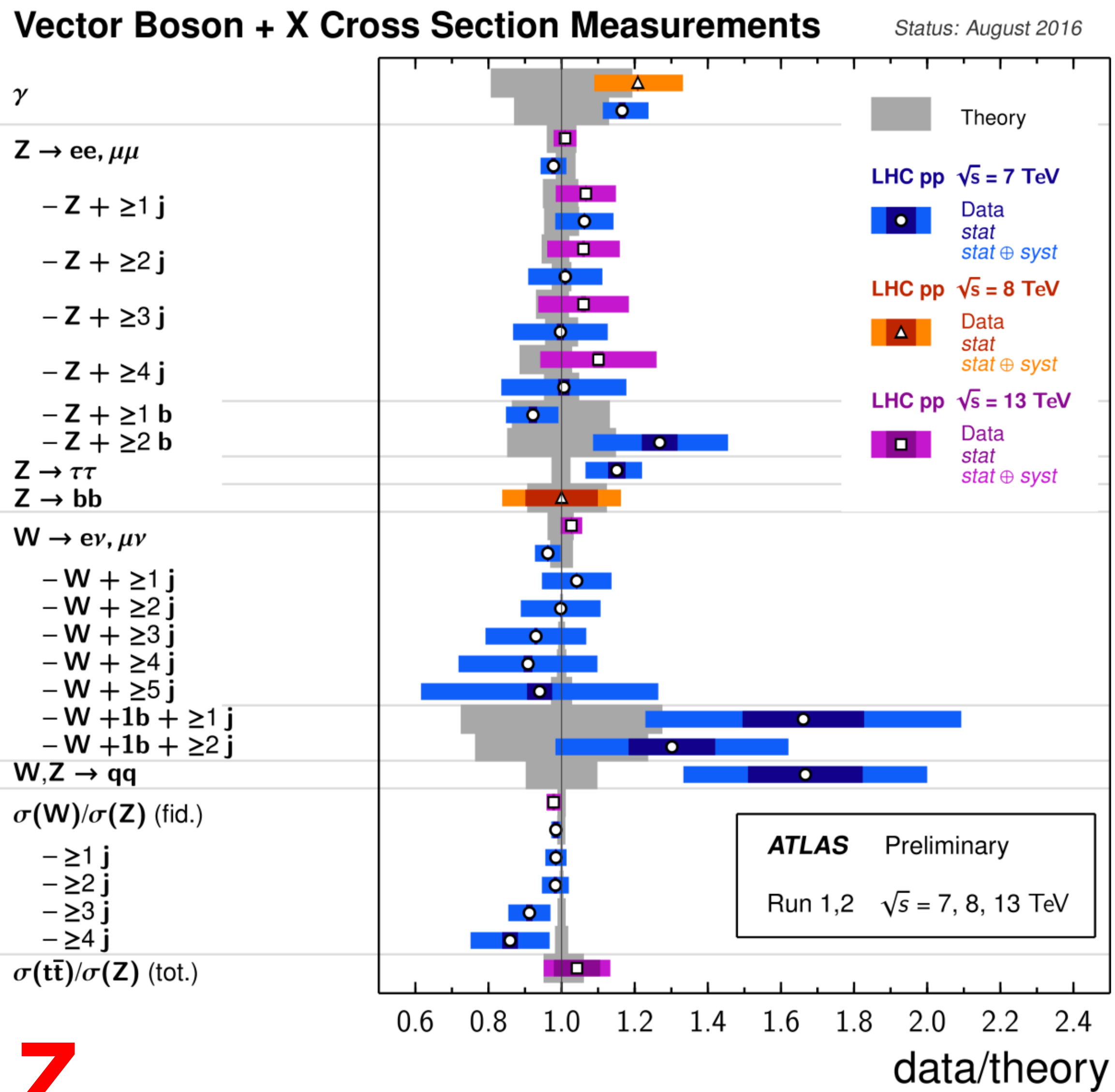
- Focus on the most recent ATLAS single and multi-boson W,Z results
  - LHC pp collisions at $\sqrt{s}$ = 8, 13TeV:
    - cross sections and their ratios,
    - Z angular coefficients,
    - anomalous Triple/Quartic Gauge Couplings (aT/QGC)

Standard Model and *Beyond...*





Processes involving jets → Bogdan Malaescu this afternoon

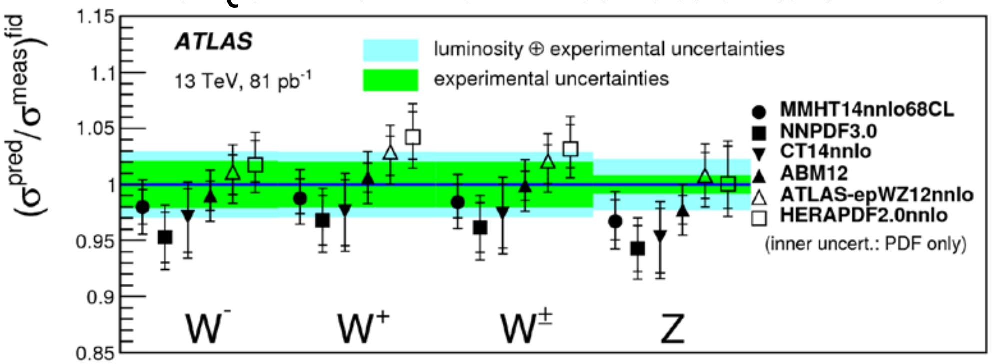


SINGLE W,Z PRODUCTION

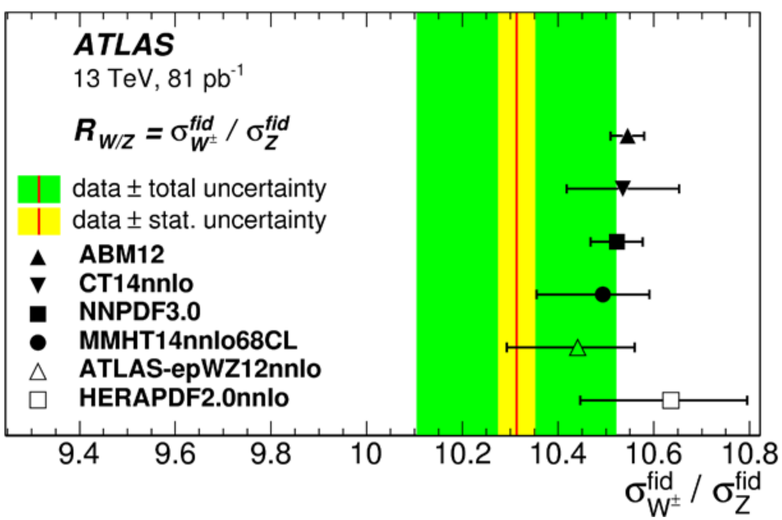
W,Z production: cross sections, ratios at 13TeV

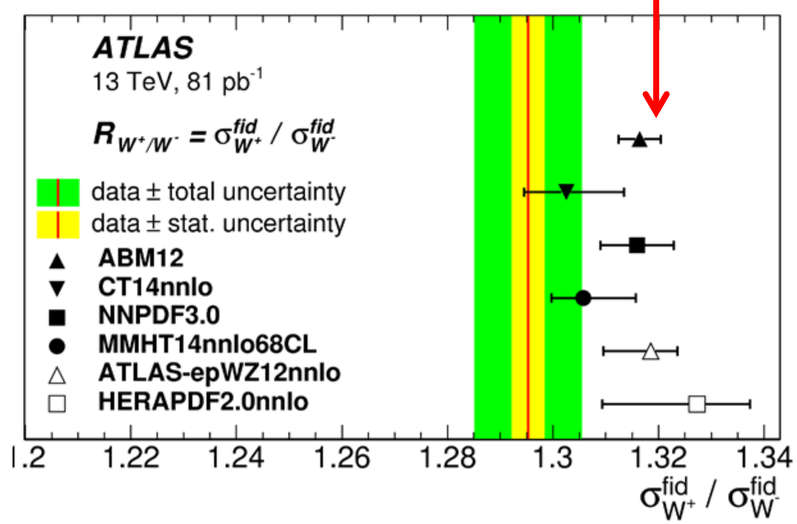
W,Z@13TeV Phys. Lett. B 759 (2016) 601

- Inclusive $W \to \ell\nu$, $Z \to \ell\ell$ (ℓ=e,μ) cross sections with very first 13TeV LHC data
- Uncert: lumi=2.1%, expt<3% (<1%) W (Z)
- agree with theoretical calculations based on NNLO QCD with NLO EW correction and PDFs



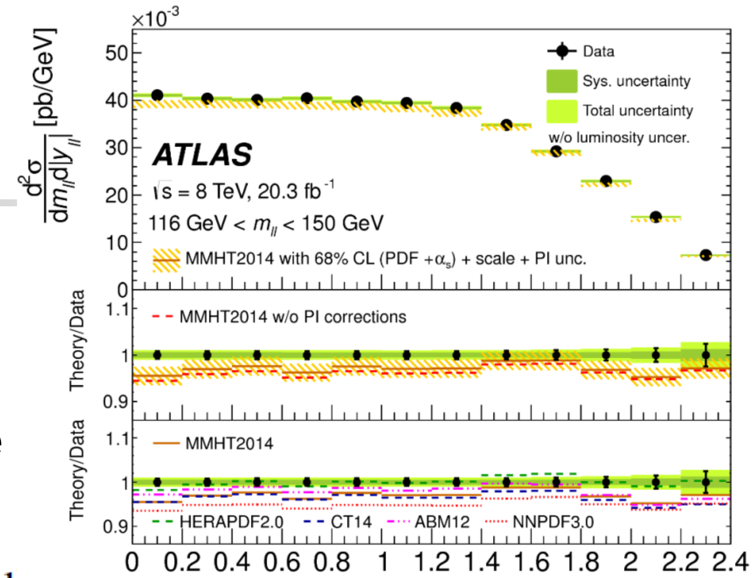
- Cross-section ratios benefit from cancellation of some expt. uncertainties, including LHC luminosity
- Powerful tool to constrain PDF fits
  - partonic content of the proton
- W/Z: constrains strange-quark sea
- W^+/W^-: sensitive to difference of u_v, d_v valence-quark at low x
  - Measured to 0.8% precision
  - Discriminates amongst PDFs





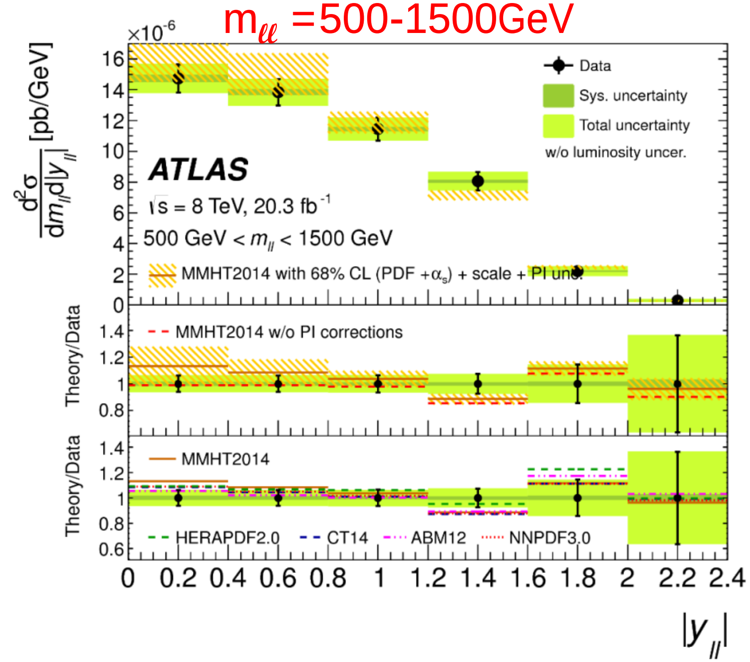
Drell-Yan and photon-induced dilepton production at 8TeV

- Off-shell neutral DY measurements:
  - dominated by coupling to γ, offer different sensitivity to up, down quarks than on-shell (Z)
  - Photon-induced (PI) process $\gamma\gamma \rightarrow \ell\ell$ expected to be significant at large $m_{\ell\ell}$, small $|y_{\ell\ell}|$
  - Sensitivity to new physics e.g Z'
- Double diff dilepton cross section (ee,$\mu\mu$)@8TeV: $\frac{d}{dm_{\ell\ell}\, d|y_{\ell\ell}|}$
  - Show here 2 bins of $m_{\ell\ell}$
  - Sensitivity to PI corrections
  - Impact of data on γ PDF
    - Reduction of uncertainty is significant!

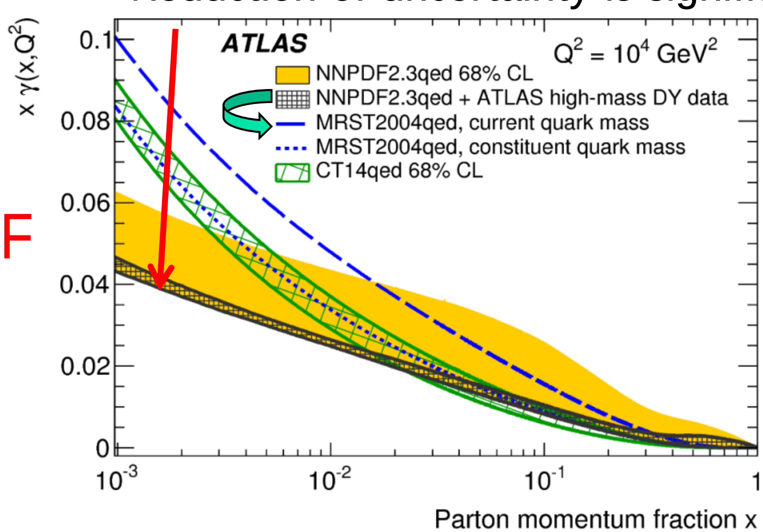


$m_{\ell\ell}$ =116-150GeV

$m_{\ell\ell}$ =500-1500GeV



γ PDF



High-mass DY@8TeV JHEP 08 (2016) 009

Z production / decay: pp $\to Z(/\gamma^*) \to \ell\ell$ at 8TeV

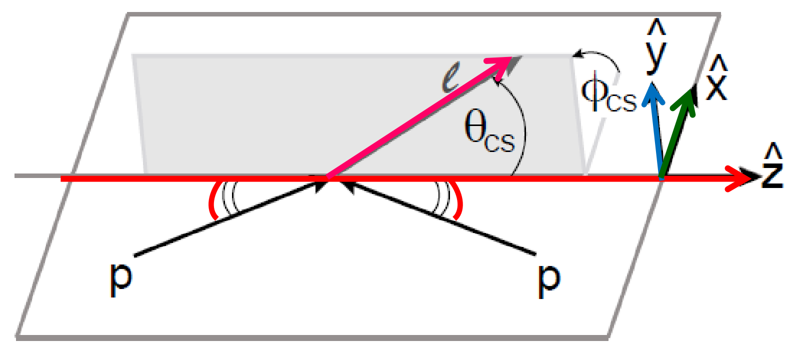
Z@8TeV JHEP 08 (2016) 159

A_i	Polynomials P_i	Couplings	Non-zero
A_0	$P_0 = \frac{1}{2}\,[1-3\cos^2\theta]$	$(v_l^2+a_l^2)\cdot(v_q^2+a_q^2)$	$\mathcal{O}(\alpha_S^1)$
A_1	$P_1 = \sin 2\theta \cos\varphi$		$\mathcal{O}(\alpha_S^1)$
A_2	$P_2 = \frac{1}{2}\sin^2\theta \cos 2\varphi$		$\mathcal{O}(\alpha_S^1)$
A_3	$P_3 = \sin\theta\cos\varphi$	$(v_l a_l)\cdot(v_q a_q)$ $\sim \sin^2\theta_W$	$\mathcal{O}(\alpha_S^1)$
A_4	$P_4 = \cos\theta$		$\mathcal{O}(\alpha_S^0)$
A_5	$P_5 = \sin^2\theta \sin 2\varphi$	$(v_l^2+a_l^2)\cdot(v_q a_q)$	$\mathcal{O}(\alpha_S^2)$
A_6	$P_6 = \sin 2\theta \sin\varphi$		$\mathcal{O}(\alpha_S^2)$
A_7	$P_7 = \sin\theta\sin\varphi$	$(v_l a_l)\cdot(v_q^2+a_q^2)$	$\mathcal{O}(\alpha_S^2)$

- Initial-state parton, final-state lepton spin correlations carry info about Z polarisation
- 5-dim cross section decomposed into 1+ 8 harmonic polynomials $P_i(\cos\theta, \varphi)$ dependent on the lepton angles multiplied by dimensionless coeffs. $A_i(p_T^Z, y^Z, m^Z)$ (ratios of helicity cross-sections)

$$\frac{d\sigma}{dp_T^Z\,dy^Z\,dm^Z\,d\cos\theta\,d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\,dy^Z\,dm^Z}\left\{(1+\cos^2\theta) + \sum_{i=0}^{7} A_i(p_T^Z, y^Z, m^Z)\cdot P_i(\cos\theta, \varphi)\right\}.$$

  - dynamics from production described within structure of A_i, and factorised from Z decay kinematics.
  - On Z peak: $m_{\ell\ell}$ =80-100GeV



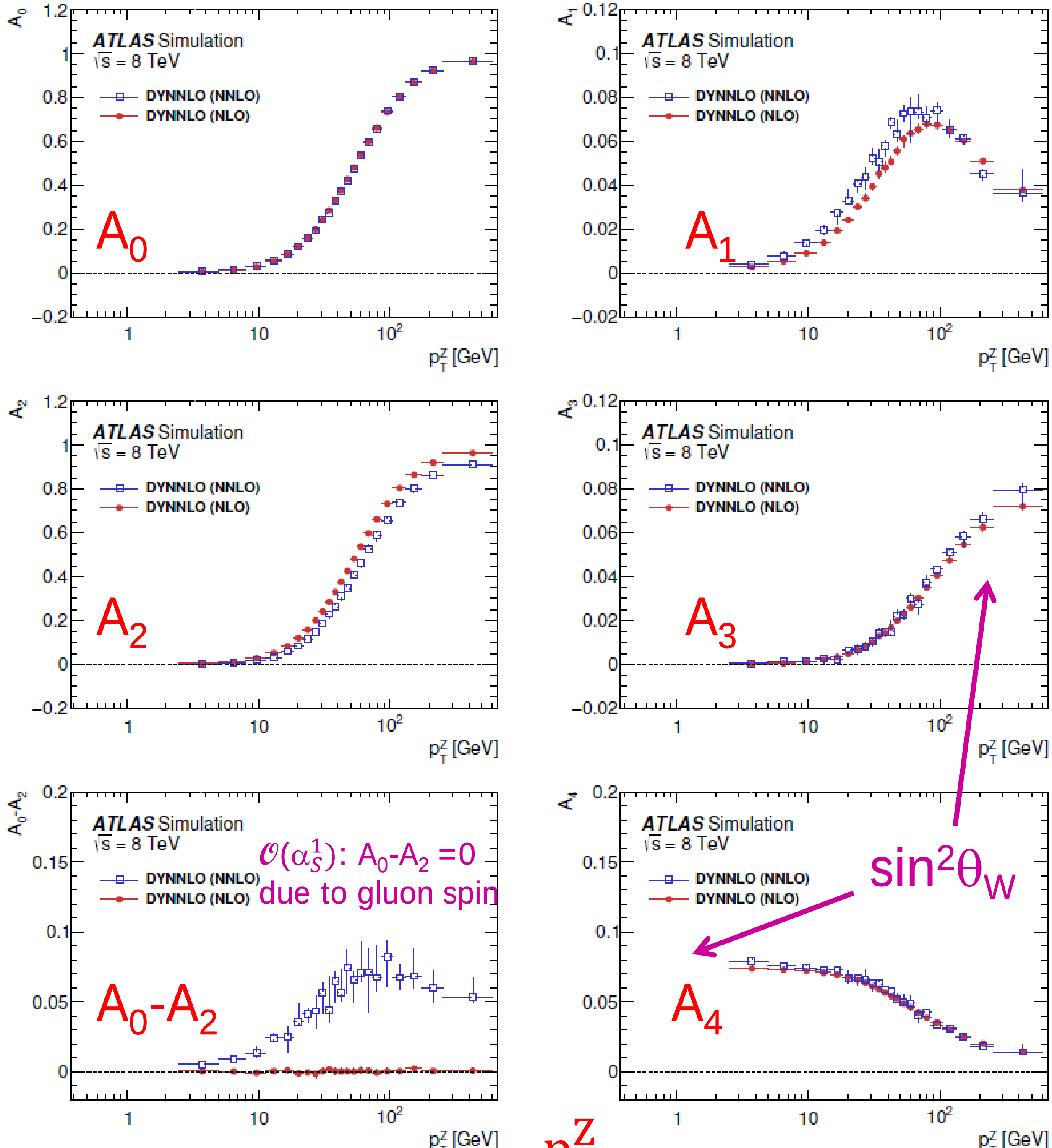
θ, φ measured in Collins-Soper (CS) frame

- z-axis: in Z rest frame, external bisector of angle between the two protons
  - +z: direction of positive longitudinally-polarised Z in lab frame
- y-axis: normal to plane spanned by the two incoming protons
- x-axis: right-handed cartesian system
- Polar θ_{CS} and azimuthal φ_{CS} angles: with respect to negatively charged lepton

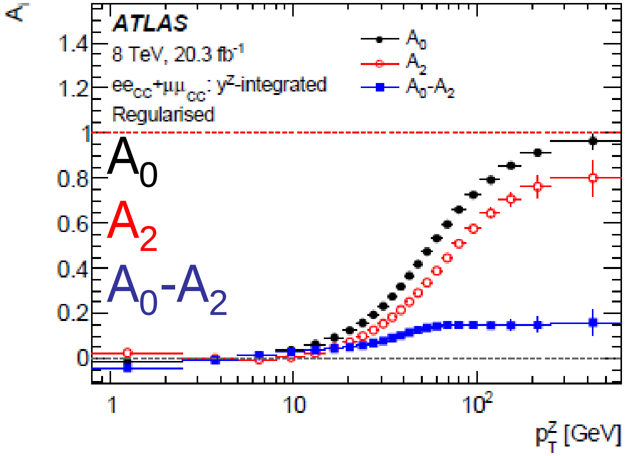
Some A_i values: DYNNLO at $\mathcal{O}(\alpha_S^1)$ (NLO), $\mathcal{O}(\alpha_S^2)$ (NNLO)

Order	Process	Info
$\mathcal{O}(\alpha_S^0)$	$q\bar{q}\rightarrow Z$	
$\mathcal{O}(\alpha_S^1)$	$q\bar{q}\rightarrow Zg$	Annihilation
	$qg\rightarrow Zq$	Compton
$\mathcal{O}(\alpha_S^2)$	$q\bar{q}\rightarrow Zgg$	
	$q\bar{q}\rightarrow Zq\bar{q}$	
	$qg\rightarrow Zqg$	
	$qq\rightarrow Zqq$	
	$gg\rightarrow Zq\bar{q}$	
	$q\bar{q}\rightarrow Zg$	Loop
	$qg\rightarrow Zq$	Loop

$A_{5,6,7}$ ~0.005 at higher values of p_T^Z



p_T^Z

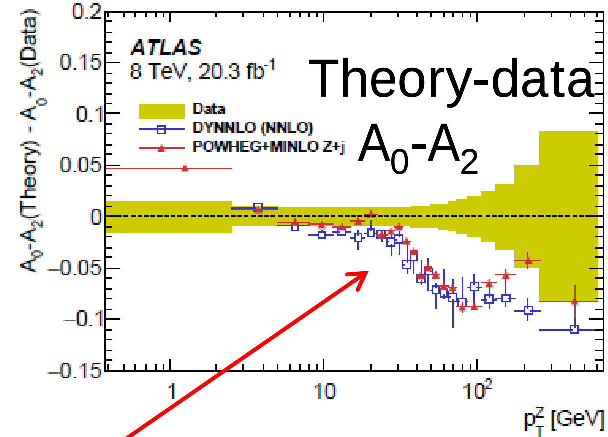


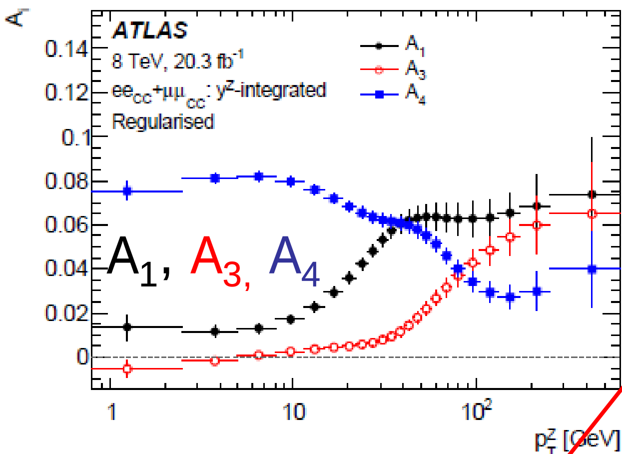
Some results...

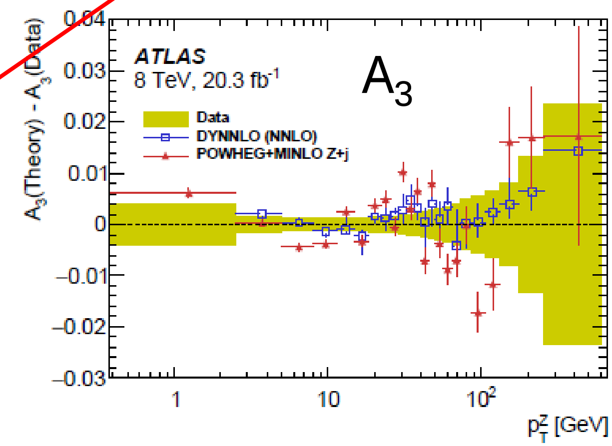
(also in 3 bins of $|y^Z|$)

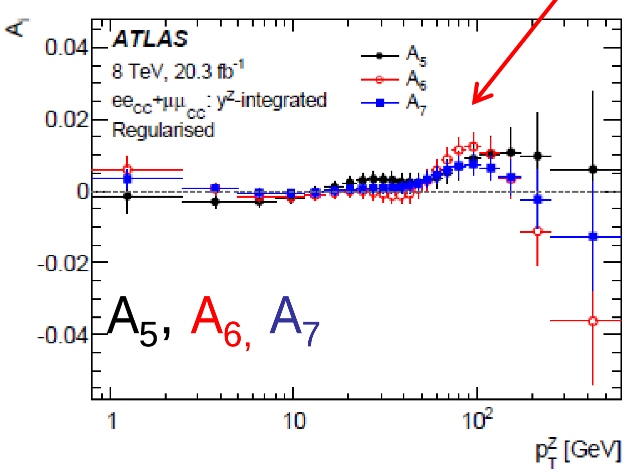
Observations

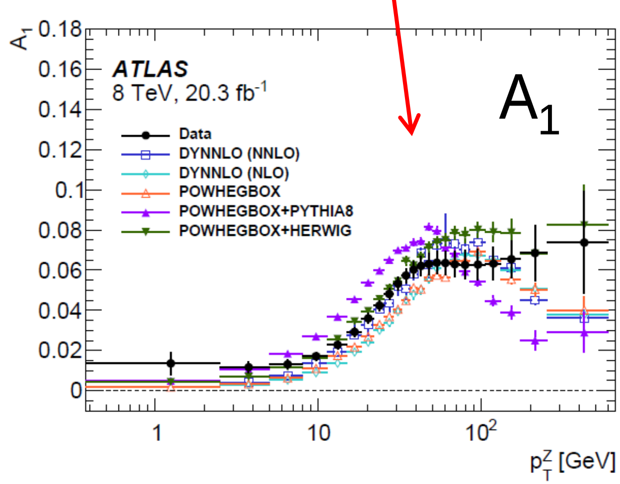
- Most cases: stats dominated even in most populated bins which contain 100,000s of events
- A_0-A_2 factor 2 larger than NNLO expectations, likely due to higher-order effects
- A_5,A_6,A_7 non-zero: 3σ level
- Some coefficients sensitive to parton-shower models

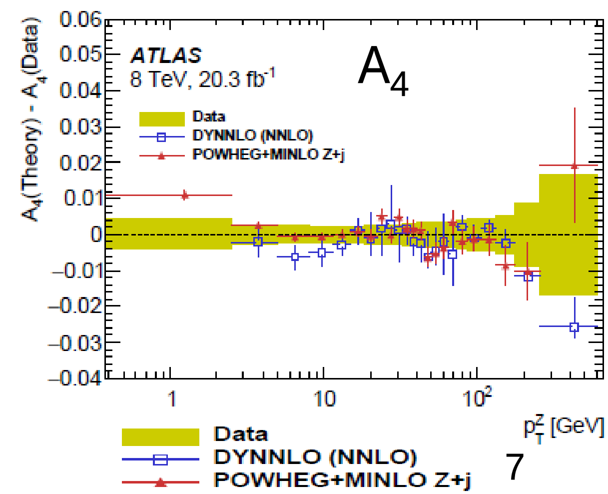


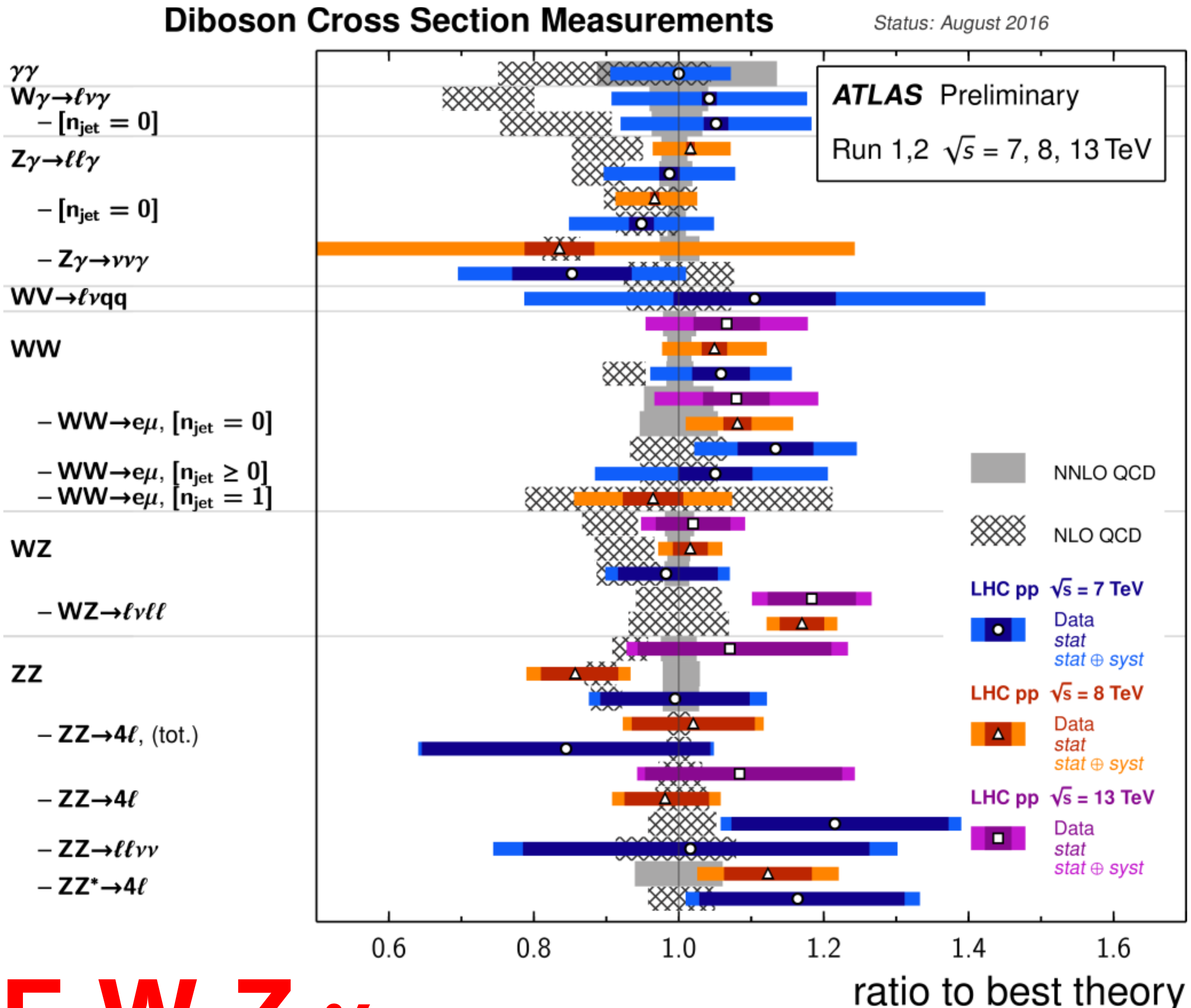








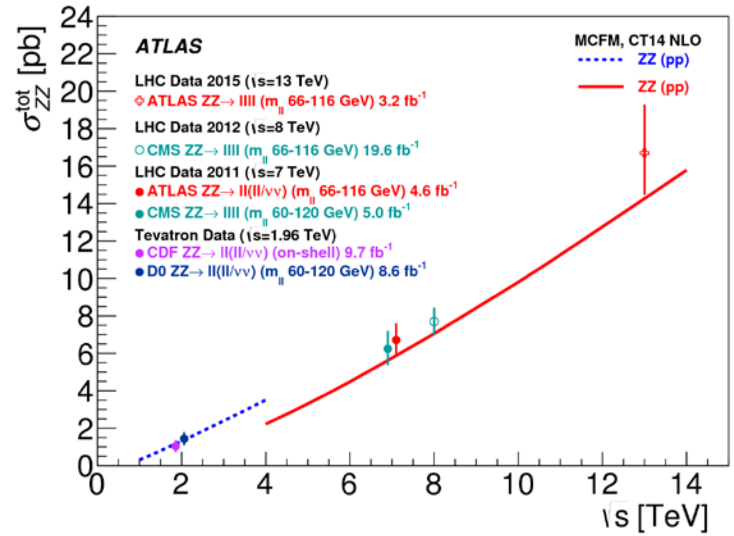




MULTIPLE W,Z,γ PRODUCTION:

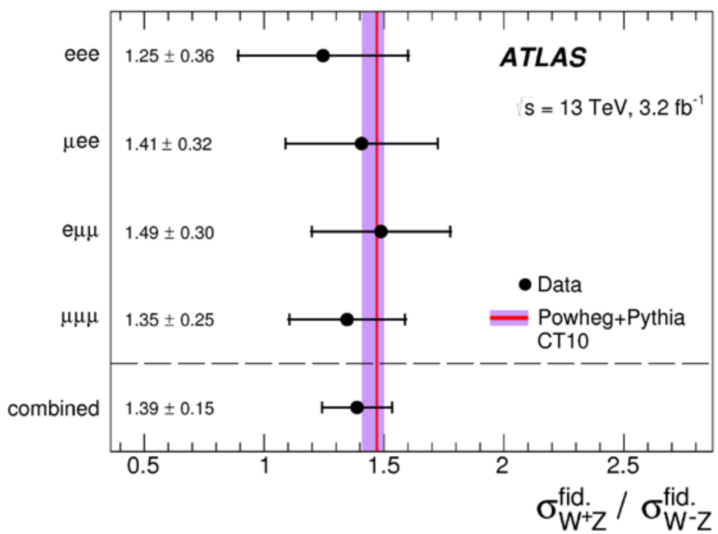
cross sections and aT/QGC

Recent diboson cross section measurements



WZ@13TeV Phys. Lett. B 762 (2016) 1

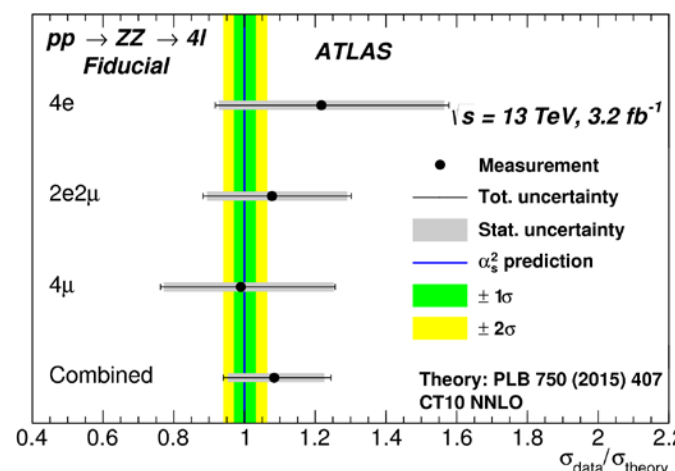
- Fiducial cross-section ratios W^+Z/W^-Z:
  - sensitive to PDFs
- Benefit from cancellation of uncertainties
- Syst+lumi: 5% → 2% (but stats dominated)



- Results also used to test pQCD in low bkg (unlike WW), high cross-section (unlike ZZ) environment

ZZ@13TeV Phys. Rev. Lett. 116 (2016) 101801

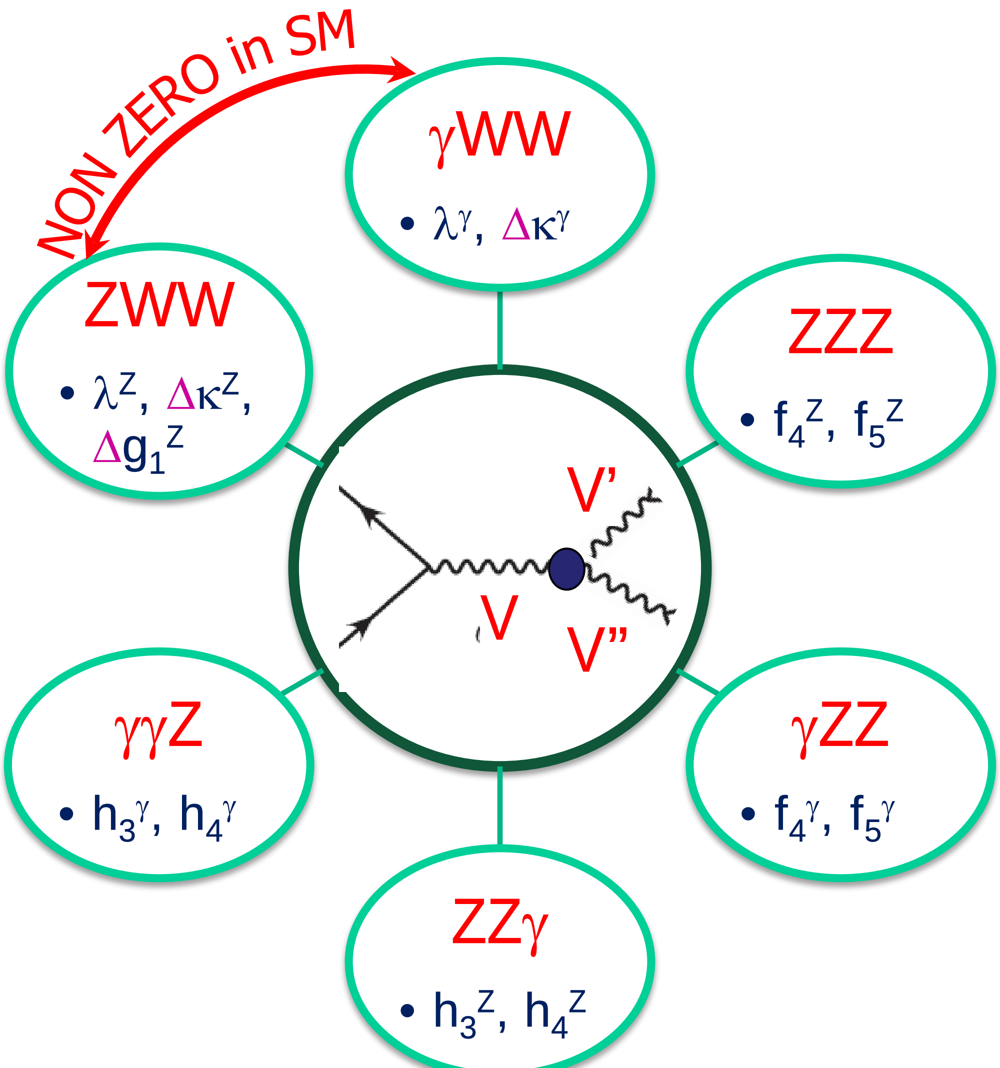
- Cross sections: fiducial and total
- Very much stats dominated
- ~ equal contributions from theory, experiment, luminosity



Data/Pred.

TGC

- Test EW sector of SM: gauge boson self-interactions
  - anomalous Triple Gauge Couplings (aTGC)
- SM multiboson production a source of bkg for:
  - Higgs production (e.g. H→WW, ZZ)
  - New physics (e.g. new resonances→VV)



Effective Lagrangian formalism

- General V V' V" vertex
- Tree level some TGCs non-zero:
  - γWW, ZWW
- Other TGCs zero at tree level but non-zero contributions at higher order
- Couplings such that SM values = 0 or 1
  - For SM=1 → deviation Δ from 1
- Leptonic decays of V ($W\to\ell\nu$, $Z\to\ell\ell,\nu\nu$)
- Note: terms in Lagrangian would lead to unitarity violation vs. $\sqrt{s}$. New physics interactions at scale Λ needed
  - form factor parameterisation

$$f_i^V = f_{i0}^V/(1+\hat{s}/\Lambda^2)^n$$

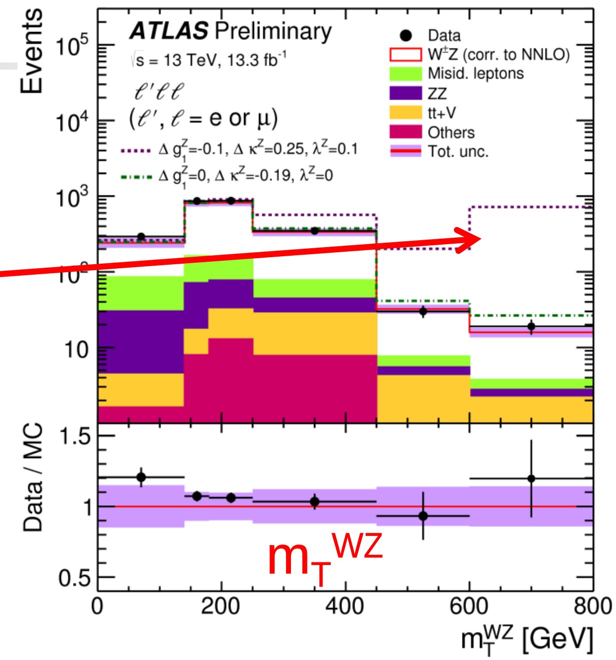
Effect Field Theory approach

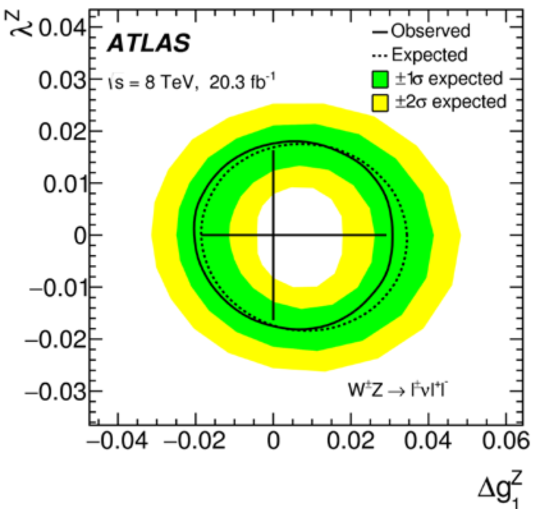
- Particle content of SM unchanged, add to Lagrangian linear combination of dimension–six operators: c_i/Λ^2
- Not considered here

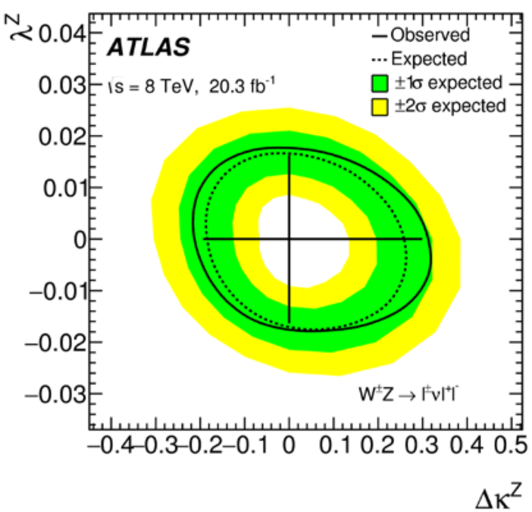
aTGC methodology.
An example: WZ@8 and 13TeV

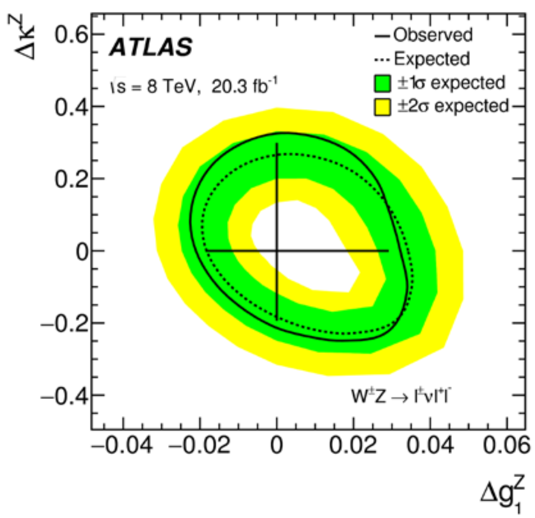
WZ@8TeV Phys. Rev. D 93 (2016) 092004
WZ@13TeV ATLAS-CONF-2016-043

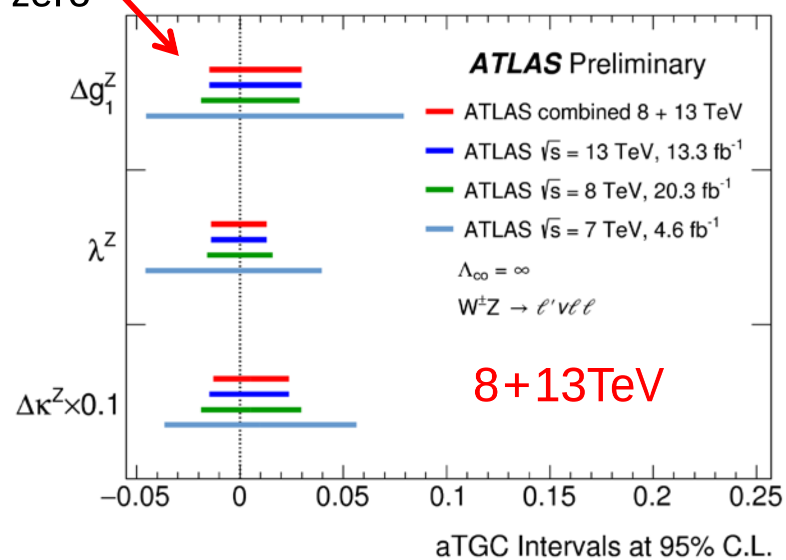
- Measure diboson kinematic distributions/cross section vs. variables sensitive to aTGCs
  - WZ $\rightarrow \ell\nu+\ell\ell$: m_T^{WZ}
- Presence of aTGC distorts shape
- Use MC@NLO MC to reweight to distributions with aTGCs
- Set limits on each coupling:
  - assuming others are zero or
  - pairs assuming others zero











Recent results: aTGCs

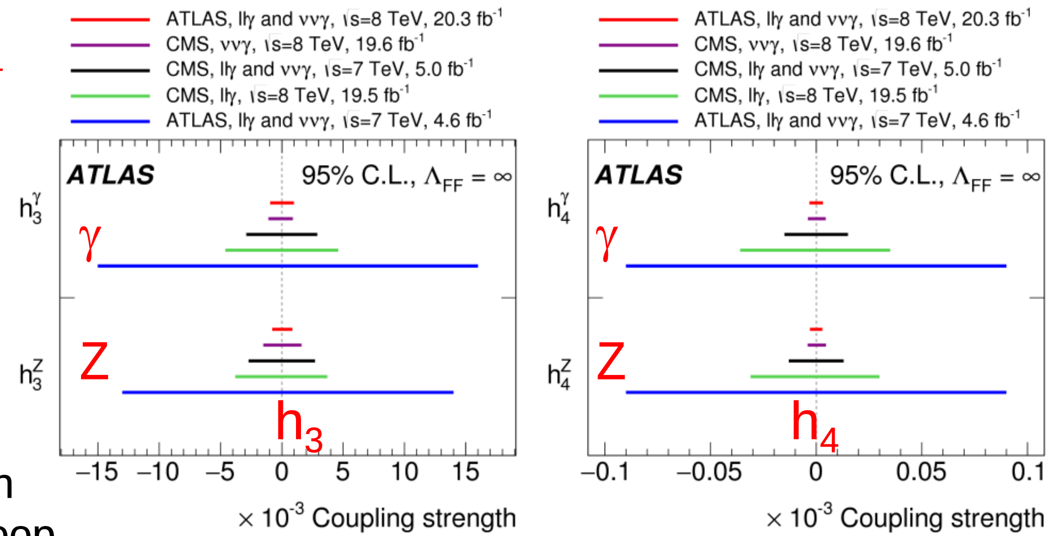
WW@8TeV JHEP 09 (2016) 029

- aTGCs extracted from leading lepton p_T
- NLO EW corrections (signif. at high p_T)

Scenario	Parameter
No constraints scenario	Δg_1^Z, Δk^Z, λ^Z, Δk^γ, λ^γ
LEP	Δg_1^Z, Δk^Z, λ^Z
HISZ	Δk^Z, λ^Z
Equal Couplings	Δk^Z, λ^Z

Scenarios:

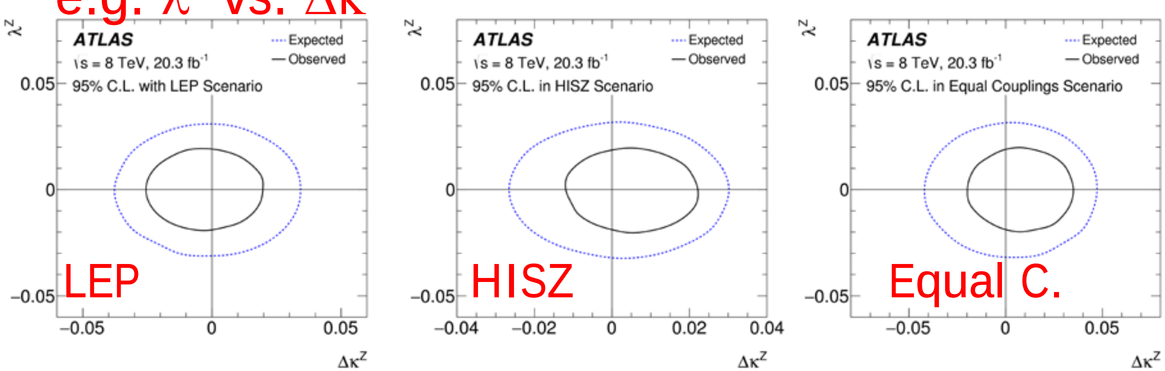
- No constraints
- LEP: SU(2)xU(1) gauge invariance
- HISZ: absence of cancellation between tree-level and one-loop
- Equal couplings: params same for WWZ, WWγ



Zγ@8TeV Phys. Rev. D 93, 112002 (2016)

- Includes Z→νν!
  - Missing E_T requirements
- aTGCs extracted from Zγ fiducial cross section with high E_T^γ (>250GeV for $\ell\ell\gamma$, >400GeV $\nu\bar{\nu}\gamma$) with exclusive zero-jet
  - Stats dominated
- aTGC predictions from MCFM

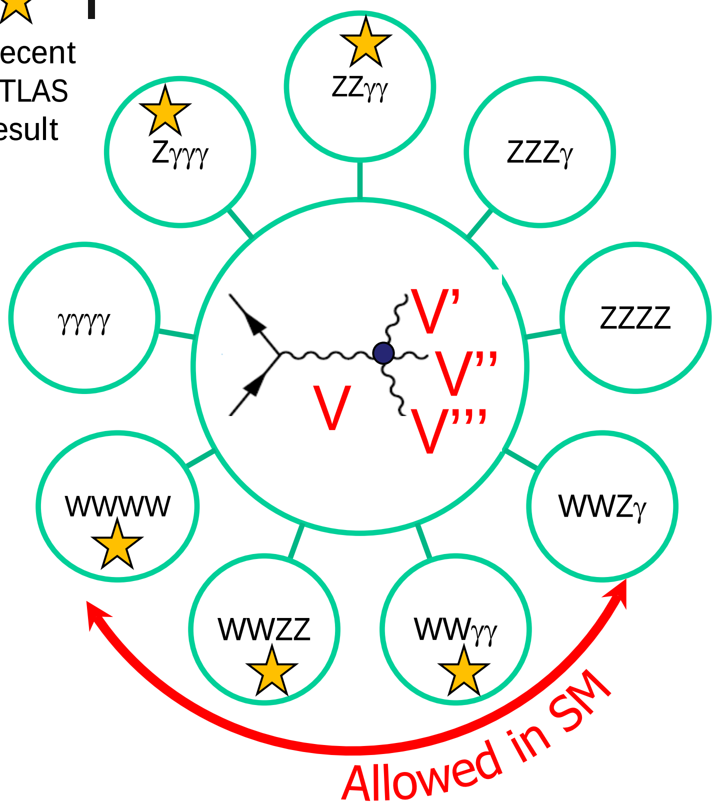
e.g. λ^Z vs. $\Delta\kappa^Z$



anomalous Quartic Gauge Couplings

QGC

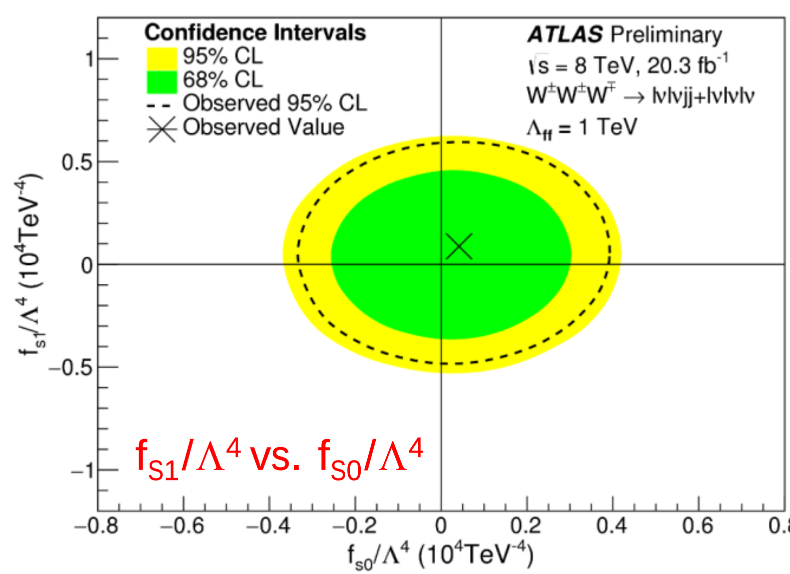
★ Recent ATLAS result



- 18 dim-8 effective operators involving gauge bosons built from
  - Higgs field: f_{Si}/Λ^4, i=0,1
  - Field strengths $SU(2)_L$, $U(1)_Y$: f_{Ti}/Λ^4, i=0-2,5-9
  - Both: f_{Mi}/Λ^4, i=0-7

WWW@8TeV (preliminary plots from STDM-2015-07)

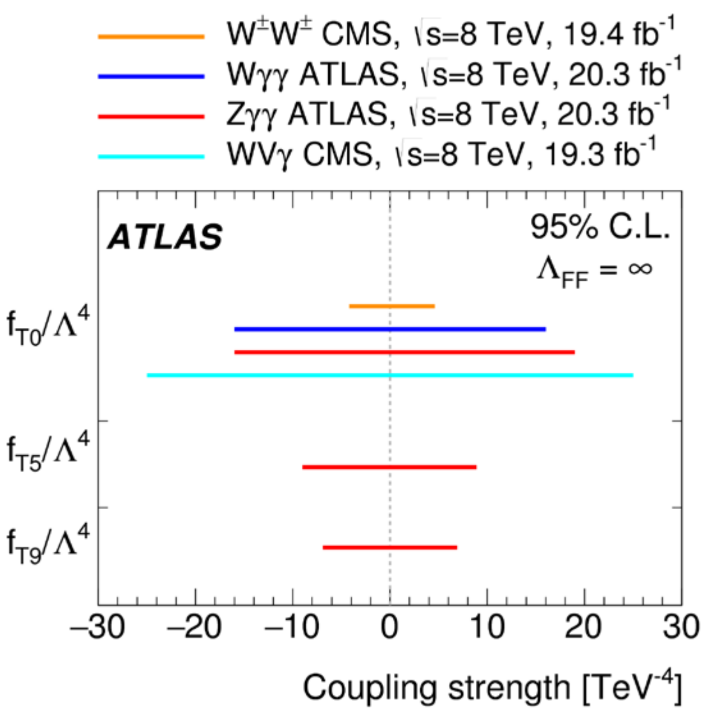
- $\ell\nu\ell\nu\ell\nu$: categorised by number of same-flavour opposite-sign leptons SFOS=0, 1, 2
- $\ell\nu\ell\nu jj$: $e^\pm e^\pm$, $e^\pm\mu^\pm$, $\mu^\pm\mu^\pm$ +2 jets consistent with m_W
  - Specific requirements on missing E_T and $m_{\ell\ell}$ to enhance signal, and veto Z background
- VBFNLO to produce events with aQGCs with coefficients f_{S0}/Λ^4, f_{S1}/Λ^4
- profile likelihood incorporates observed and expected numbers of events for different aQGCs

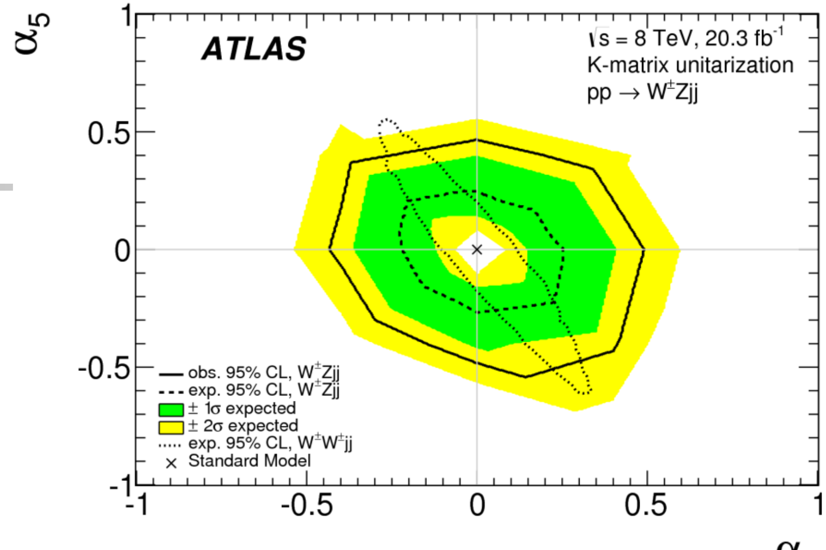


Recent results: aQGCs

$Z\gamma\gamma$@8TeV Phys. Rev. D 93 (2016) 112002

- VBFNLO to produce events with aQGCs with coefficients:
  - f_{T0}/Λ^4, f_{T5}/Λ^4, f_{T9}/Λ^4, f_{M2}/Λ^4, f_{M3}/Λ^4
- aQGCs extracted from exclusive zero-jet fiducial cross section for $m_{\gamma\gamma}$>300 (200)GeV for $\nu\nu\gamma\gamma$ ($\ell\ell\gamma\gamma$)





WZ@8TeV Phys. Rev. D 93, 092004 (2016)

- WHIZARD to produce events with aQGCs with coefficients:
  - $\alpha_4 \to f_{S0}/\Lambda^4$, $\alpha_5 \to f_{S1}/\Lambda^4$
- aQGCs extracted from fiducial cross section of WZ production with at least 2 jets, in phase space:
  - $|\Delta\varphi(WZ)|>2$, $\Sigma|p_T^{\ell}|>250$GeV
  - Without these requirements, set limit on vector-boson scattering production WZjj-EW

95% CL upper limit on $\sigma^{\text{fid.}}_{W^\pm Zjj\text{-EW}\to\ell'\nu\ell\ell}$ [fb]	
Observed	0.63
Expected	0.45

Summary

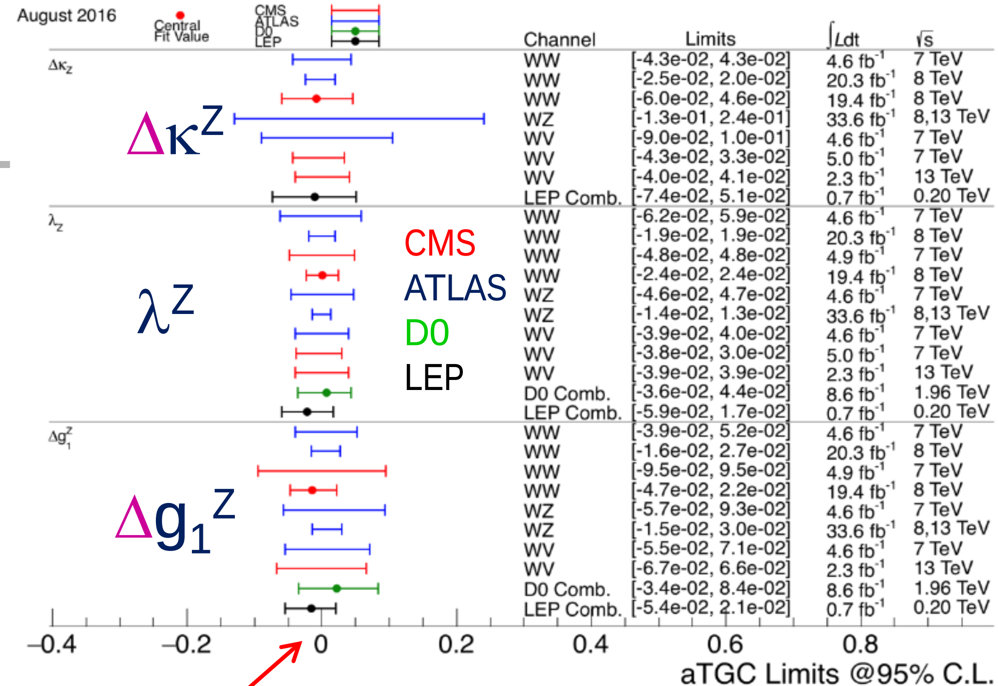
Cross sections and their ratios

- Data have differing sensitivity to different aspects of PDFs
  - EW boson production sensitive to valence, sea quarks
  - Compare to state-of-the-art PDFs
- Test pQCD at NNLO (at perhaps beyond?)

Test predictions of EW sector at TeV scale

- Self interactions between gauge bosons
  - Neutral aTGCs: LHC leading here
  - Charged aTGC: surpassing LEP limits
  - aQGC: benefit significantly from LHC run 2 datasets

Thanks



Citations

- W,Z@13TeV PLB 759 (2016) 601
  - Cross sections
- HM DY@8TeV JHEP 08 (2016) 009
  - Cross sections
- Z@8TeV JHEP 08 (2016) 159
  - Z angular coefficients
- WZ@8, 13TeV PRD93 (2016) 092004, ATLAS-CONF-2016-043, Phys. Lett. B 762 (2016) 1,
  - Cross section, aTGC, aQGC
- ZZ@13TeV PRL 116 (2016) 101801
  - Cross section
- WW@8TeV JHEP 09 (2016) 029
  - aTGCs
- $Z\gamma$,$Z\gamma\gamma$@8TeV PRD 93 (2016) 112002
  - aTGCs, aQGCs
- WWW@8TeV (prelim from STDM-2015-07)
  - aQGCs

EXTRA MATERIAL

Angular decomposition of the differential cross section

Initial-state parton and final-state lepton spin correlations in pp $\to$Z$\to$ $\ell\ell$ described by contraction of lepton tensor $L_{\mu\nu}$ with parton-level hadron tensor $H^{\mu\nu}$.
$\to$ $L_{\mu\nu}$ acts as analyser of the structure of $H^{\mu\nu}$, which carries info about polarisation of Z.

Five-dimensional differential cross section describing kinematics of the two Born-level leptons from Z decay can be decomposed as 9 harmonic polynomials (think $Y_l^{\ m}$!), dependent on the lepton polar θ, azimuthal φ multiplied by helicity cross sections that depend on Z transverse momentum (p_T^Z), rapidity (y^Z), invariant mass (m^Z).
$\to$ The decomposition is true to all orders.

Standard convention: factorise out unpolarised cross-section, σ^{U+L}. Differential cross section: expansion into 1+ 8 harmonic polynomials $P_i\,(\cos\theta, \varphi)$ and dimensionless angular coefficients $A_i(p_T^Z, y^Z, m^Z)$ (ratios of helicity cross-sections with respect to σ^{U+L})

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi} \frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z} \left\{(1+\cos^2\theta) + \sum_{i=0}^{7} A_i(p_T^Z, y^Z, m^Z)\cdot P_i(\cos\theta, \varphi)\right\}.$$

$\to$ Hadronic dynamics from production mechanism described within the structure of A_i coefficients, and are factorised from Z decay kinematics.

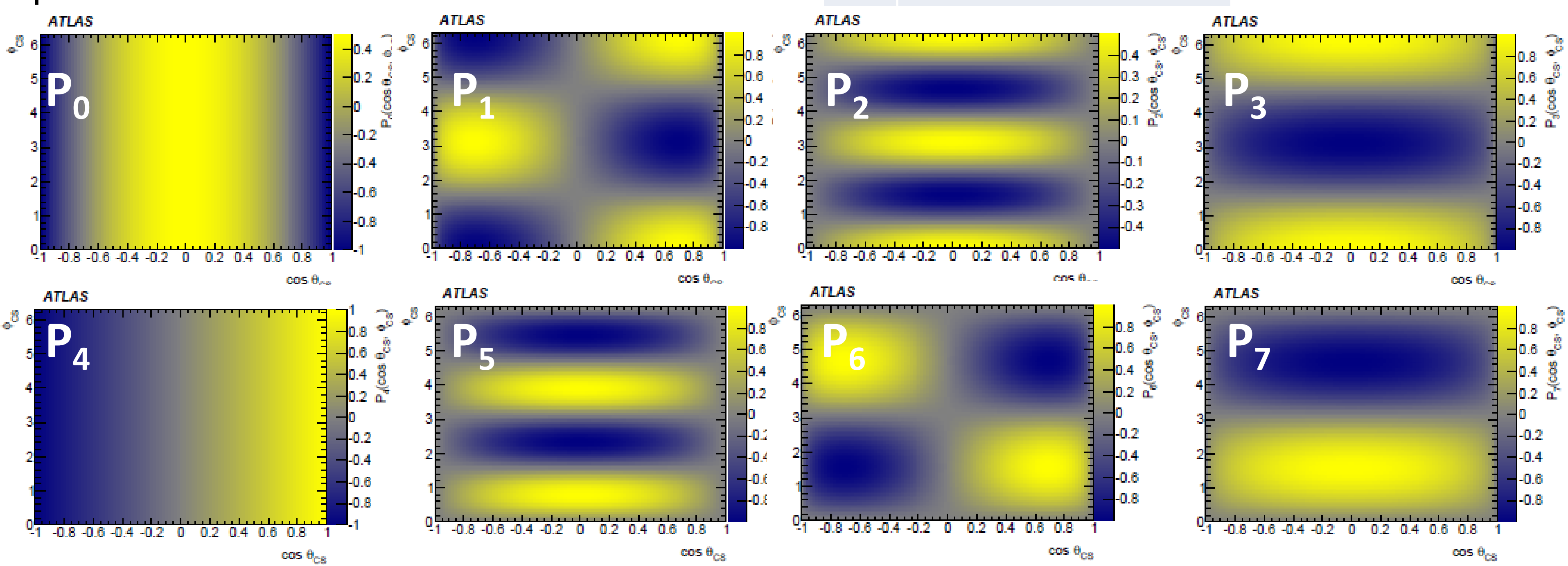
Differential cross section

$$\frac{d\sigma}{dp_T^Z\, dy^Z\, dm^Z\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T^Z\, dy^Z\, dm^Z}$$

$$\text{x} \quad \left\{(1+\cos^2\theta) + \sum_{i=0}^{7} A_i \cdot P_i \right\}.$$

A_i	Polynomials P_i
A_0	$P_0 = \frac{1}{2}\,[1-3\cos^2\theta]$
A_1	$P_1 = \sin 2\theta \cos\varphi$
A_2	$P_2 = \frac{1}{2}\sin^2\theta \cos 2\varphi$
A_3	$P_3 = \sin\theta \cos\varphi$
A_4	$P_4 = \cos\theta$
A_5	$P_5 = \sin^2\theta \sin 2\varphi$
A_6	$P_6 = \sin 2\theta \sin\varphi$
A_7	$P_7 = \sin\theta \sin\varphi$

φ vs. cos θ



What these A_i mean…

A_i	σ^k/σ^{U+L}	$H_{mm'}$	Parity	Couplings	Non-zero
A_0	L	H_{00}	C	$(v_l^2+a_l^2)\cdot(v_q^2+a_q^2)$	$\mathcal{O}(\alpha_S^1)$
A_1	I	$\frac{1}{4}(H_{+0}+H_{0+}-H_{-0}-H_{0-})$	C		$\mathcal{O}(\alpha_S^1)$
A_2	T	$\frac{1}{2}(H_{+-}+H_{-+})$	C		$\mathcal{O}(\alpha_S^1)$
A_3	A	$\frac{1}{4}(H_{+0}+H_{0+}+H_{-0}+H_{0-})$	C	$(v_l a_l)\cdot(v_q a_q)$ ~ $\sin^2\theta_W$	$\mathcal{O}(\alpha_S^1)$
A_4	P	$(H_{++}-H_{--})$	C		$\mathcal{O}(\alpha_S^0)$
A_5	7	$-\frac{i}{2}(H_{+-}-H_{-+})$	V	$(v_l^2+a_l^2)\cdot(v_q a_q)$	$\mathcal{O}(\alpha_S^2)$
A_6	8	$-\frac{i}{4}(H_{+0}-H_{0+}+H_{-0}-H_{0-})$	V		$\mathcal{O}(\alpha_S^2)$
A_7	9	$-\frac{i}{4}(H_{+0}-H_{0+}-H_{-0}+H_{0-})$	V	$(v_l a_l)\cdot(v_q^2+a_q^2)$	$\mathcal{O}(\alpha_S^2)$

Helicity density matrix elements:

$H_{mm'} = \varepsilon_\mu^*(m)H^{\mu\nu}\varepsilon_\nu(m')$, $\quad$ m,m'=+,0,-

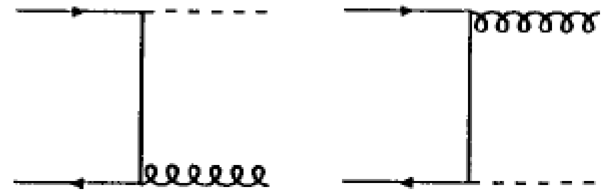
$H^{\mu\nu}$ = hadron tensor, ε_μ = polarisation vectors of Z

$\varepsilon_\mu(\pm)= \sqrt{\frac{1}{2}}\,(0; \pm1,-i,0)$

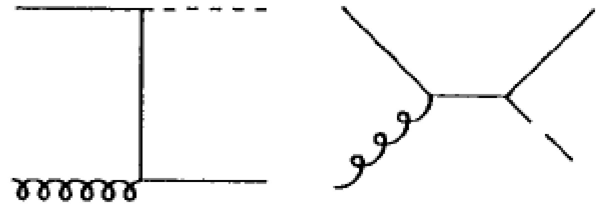
$\varepsilon_\mu(0)= (0; 0,0,1)$

Order	A_i non-zero	Process	Comment
$\mathcal{O}(\alpha_S^0)$	A_4	$q\bar{q}\to Z$	
$\mathcal{O}(\alpha_S^1)$	A_0, A_1, A_2, A_3, A_4	$q\bar{q}\to Zg$	Annihilation
		$qg\to Zq$	Compton
$\mathcal{O}(\alpha_S^2)$	$A_0, A_1, A_2, A_3, A_4,$ A_5, A_6, A_7	$q\bar{q}\to Zgg$	
		$q\bar{q}\to Zq\bar{q}$	
		$qg\to Zqg$	
		$qq\to Zqq$	
		$gg\to Zq\bar{q}$	
		$q\bar{q}\to Zg$	Loop
		$qg\to Zq$	

$\mathcal{O}(\alpha_S^1)$: examples

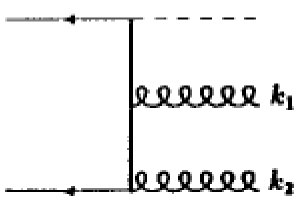


Annihilation

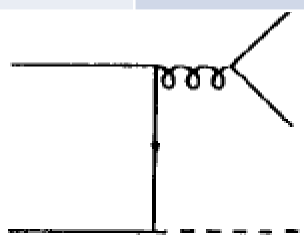


Compton

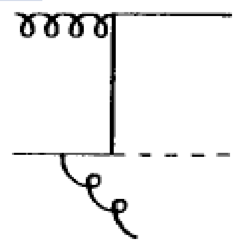
$\mathcal{O}(\alpha_S^2)$: examples



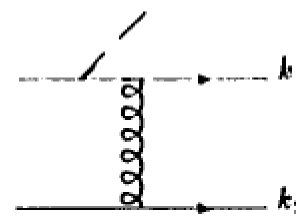
$q\bar{q}\to Zgg$



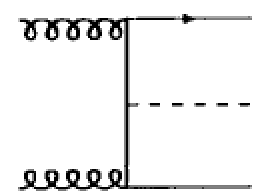
$q\bar{q}\to Zq\bar{q}$



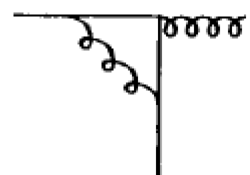
$qg\to Zqg$



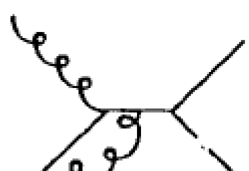
$qq\to Zqq$



$gg\to Zq\bar{q}$



$q\bar{q}\to Zg$



$qg\to Zq$

Mirkes, NPB 387 (1992) 3-85

Relation between Y_l^m and harmonic polynomials P_i

		$Y_l^m(\theta,\varphi)$	Spherical Harmonics Polynomials	$\rightarrow P_i$
l=0	m=0	Y_0^0	$\frac{1}{2}\sqrt{\frac{1}{\pi}}$	
l=1	m=-1	Y_1^{-1}	$\frac{1}{2}\sqrt{\frac{3}{2\pi}}$ $\sin\theta\,[\cos\varphi - i\sin\varphi]$	$(P_3,-P_7)$
	m=0	Y_1^0	$\frac{1}{2}\sqrt{\frac{3}{\pi}}$ $\cos\theta$	(P_4)
	m=+1	Y_1^{+1}	$-\frac{1}{2}\sqrt{\frac{3}{2\pi}}$ $\sin\theta\,[\cos\varphi + i\sin\varphi]$	(P_3,P_7)
l=2	m=-2	Y_2^{-2}	$\frac{1}{4}\sqrt{\frac{15}{2\pi}}$ $\sin^2\theta\,[\cos 2\varphi - i\sin 2\varphi]$	$(P_2,-P_5)$
	m=-1	Y_2^{-1}	$\frac{1}{2}\sqrt{\frac{15}{2\pi}}$ $\sin\theta\cos\theta[\cos\varphi - i\sin\varphi]$	$(P_1,-P_6)$
	m=0	Y_2^0	$\frac{1}{4}\sqrt{\frac{5}{\pi}}$ $[3\cos^2\theta - 1]$	(P_0)
	m=+1	Y_2^{+1}	$-\frac{1}{2}\sqrt{\frac{15}{2\pi}}$ $\sin\theta\cos\theta\,[\cos\varphi + i\sin\varphi]$	(P_1,P_6)
	m=+2	Y_2^{+2}	$\frac{1}{4}\sqrt{\frac{15}{2\pi}}$ $\sin^2\theta\,[\cos 2\varphi + i\sin 2\varphi]$	(P_2,P_5)

Dimension-8 operators and quartic vertices

M. Baak et al. arXiv:1310.6708

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0}$, $\mathcal{O}_{S,1}$	X	X	X						
$\mathcal{O}_{M,0}$, $\mathcal{O}_{M,1}$,$\mathcal{O}_{M,6}$,$\mathcal{O}_{M,7}$	X	X	X	X	X	X	X		
$\mathcal{O}_{M,2}$,$\mathcal{O}_{M,3}$, $\mathcal{O}_{M,4}$,$\mathcal{O}_{M,5}$		X	X	X	X	X	X		
$\mathcal{O}_{T,0}$,$\mathcal{O}_{T,1}$,$\mathcal{O}_{T,2}$	X	X	X	X	X	X	X	X	X
$\mathcal{O}_{T,5}$,$\mathcal{O}_{T,6}$,$\mathcal{O}_{T,7}$		X	X	X	X	X	X	X	X
$\mathcal{O}_{T,8}$,$\mathcal{O}_{T,9}$			X			X	X	X	X

Table 1-16. *Quartic vertices modified by each dimension-8 operator are marked with X.*

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] ,$$

$$\mathcal{O}_{M,0} = \mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,1} = \mathrm{Tr}\left[W_{\mu\nu}W^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,2} = \left[B_{\mu\nu}B^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,3} = \left[B_{\mu\nu}B^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu} D^\mu\Phi\right] \times B^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu} D^\nu\Phi\right] \times B^{\beta\mu} ,$$
$$\mathcal{O}_{M,6} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu} W^{\beta\nu} D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu} W^{\beta\mu} D^\nu\Phi\right] ,$$

$$\mathcal{O}_{T,0} = \mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \times \mathrm{Tr}\left[W_{\alpha\beta}W^{\alpha\beta}\right] ,$$
$$\mathcal{O}_{T,1} = \mathrm{Tr}\left[W_{\alpha\nu}W^{\mu\beta}\right] \times \mathrm{Tr}\left[W_{\mu\beta}W^{\alpha\nu}\right] ,$$
$$\mathcal{O}_{T,2} = \mathrm{Tr}\left[W_{\alpha\mu}W^{\mu\beta}\right] \times \mathrm{Tr}\left[W_{\beta\nu}W^{\nu\alpha}\right] ,$$
$$\mathcal{O}_{T,5} = \mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \times B_{\alpha\beta}B^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \mathrm{Tr}\left[W_{\alpha\nu}W^{\mu\beta}\right] \times B_{\mu\beta}B^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \mathrm{Tr}\left[W_{\alpha\mu}W^{\mu\beta}\right] \times B_{\beta\nu}B^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}$$
$$\mathcal{O}_{T,9} = B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha} .$$