FERMIONIC WARM DARK MATTER: REVISITING LIMITS FROM THE SMALLEST DWARF GALAXIES

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Limits on Fermionic Warm Dark Matter mass from the smallest dwarf spheroidal galaxies

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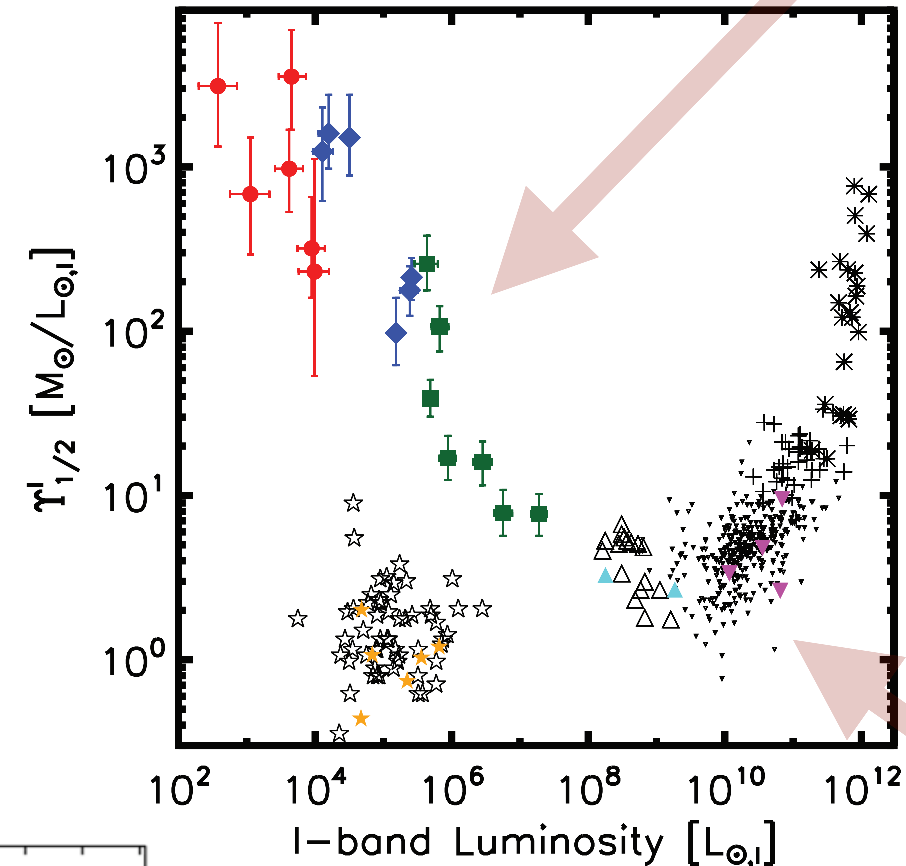
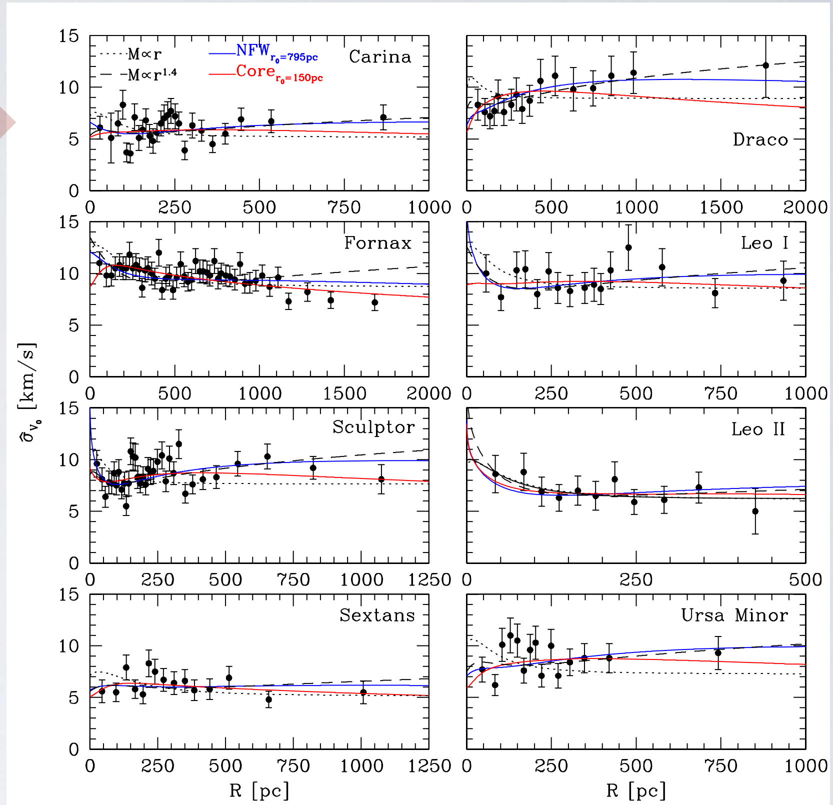
ONE-SLIDE RECAP

- Mass-to-Light ratio
  striking evidence of DM
- (Globular Clusters likely rule out modifications of gravity for DM)
- Cored profile preferred for small galaxies. . . ?

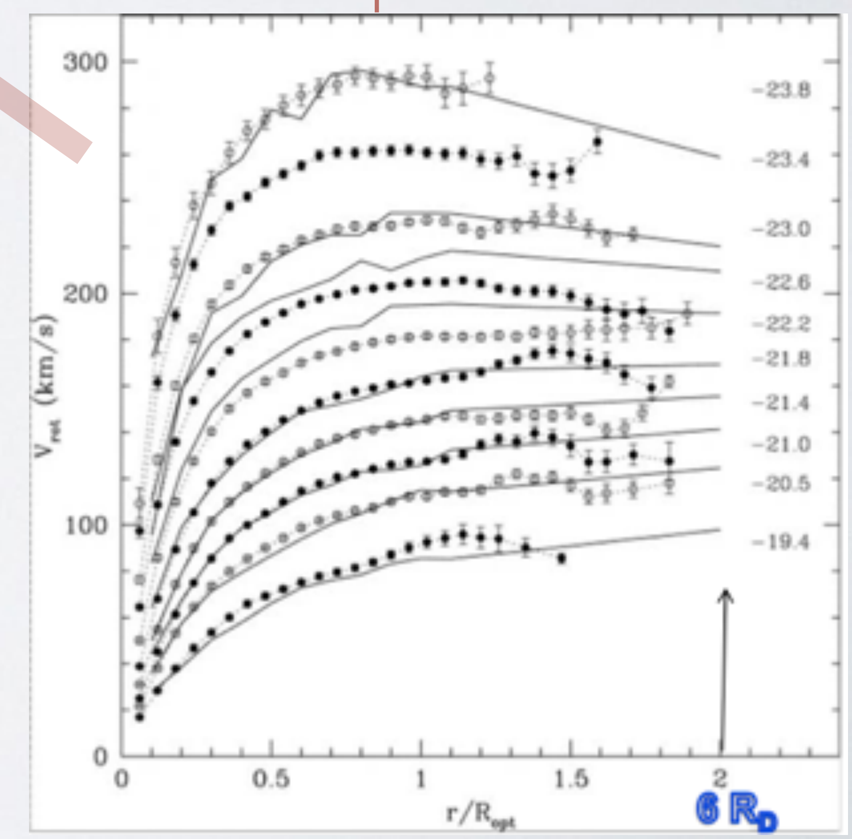


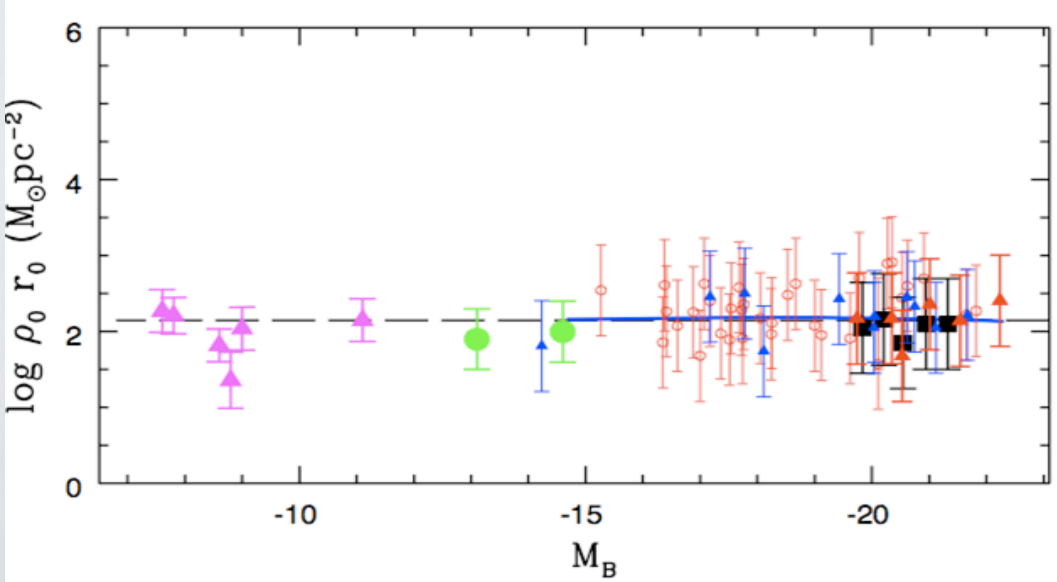
Dwarf Spheroidals



[Walker+ '09]

Spirals





Fits with Burkert:

Constant surface density

← $\Sigma_0 = \rho_0 r_0 \simeq 120 M_\odot/\mathrm{pc}^2$

(over many magnitudes !?)

HYPOTHESIS OF DEGENERATE FERMIONIC DARK MATTER HALOS

- Pauli exclusion can forbid a central density cusp

fermionic DM can explain cored profiles

The largest density is observed in smallest systems, so

we need to focus on the smallest dwarf galaxies

Dwarf galaxies, dark matter dominated, could be

quantum degenerate spheres of fermi particles (10^{70} of them)

The particle mass is bounded from below, *à la* Tremaine-Gunn

[…,Chavanis+ ‘97, Bilic+ ’99,…,Destri DeVega Sanchez ’13; Merafina Alberti ’14; Domcke Urbano ’14]

AUTO GRAVITATING FERMION GAS

Spherical symmetry, isothermal. $\phi(r)$: average gravitational potential

Fermi Dirac Statistics:

$$\rho(r) = m n(r) = \frac{gm}{2\pi^2\hbar^3}\int_0^\infty p^2\, dp\, f\left[\frac{p^2}{2m} - \mu(r)\right]$$

$$f_{FD}(E) = \frac{1}{1 + exp(E/T_0)}$$

Poisson equation

$$\begin{cases} \dfrac{d\phi(r)}{dr} = G\dfrac{M(r)}{r^2} \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \rho(r) \end{cases}$$

r-dependent chemical potential:

$$\mu(r) = \mu_0 - m\phi(r)$$

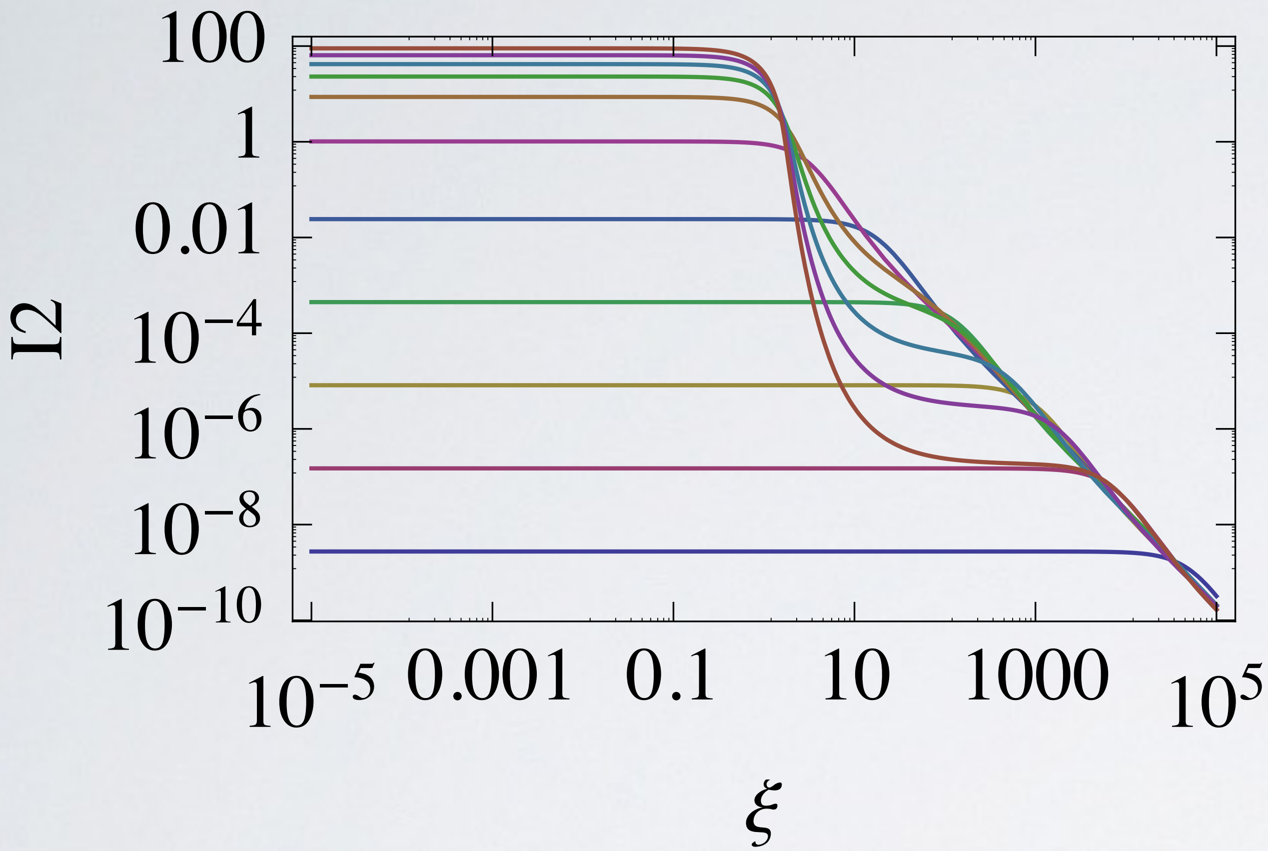
Thomas-Fermi Equation

$$\frac{d^2\mu}{dr^2} + \frac{2}{r}\frac{d\mu}{dr} = -4\pi G m \rho(r) = -\frac{2gGm^2}{\pi\hbar^3}\int_0^\infty dp\, p^2 f\left[\frac{p^2}{2m} - \mu(r)\right]$$

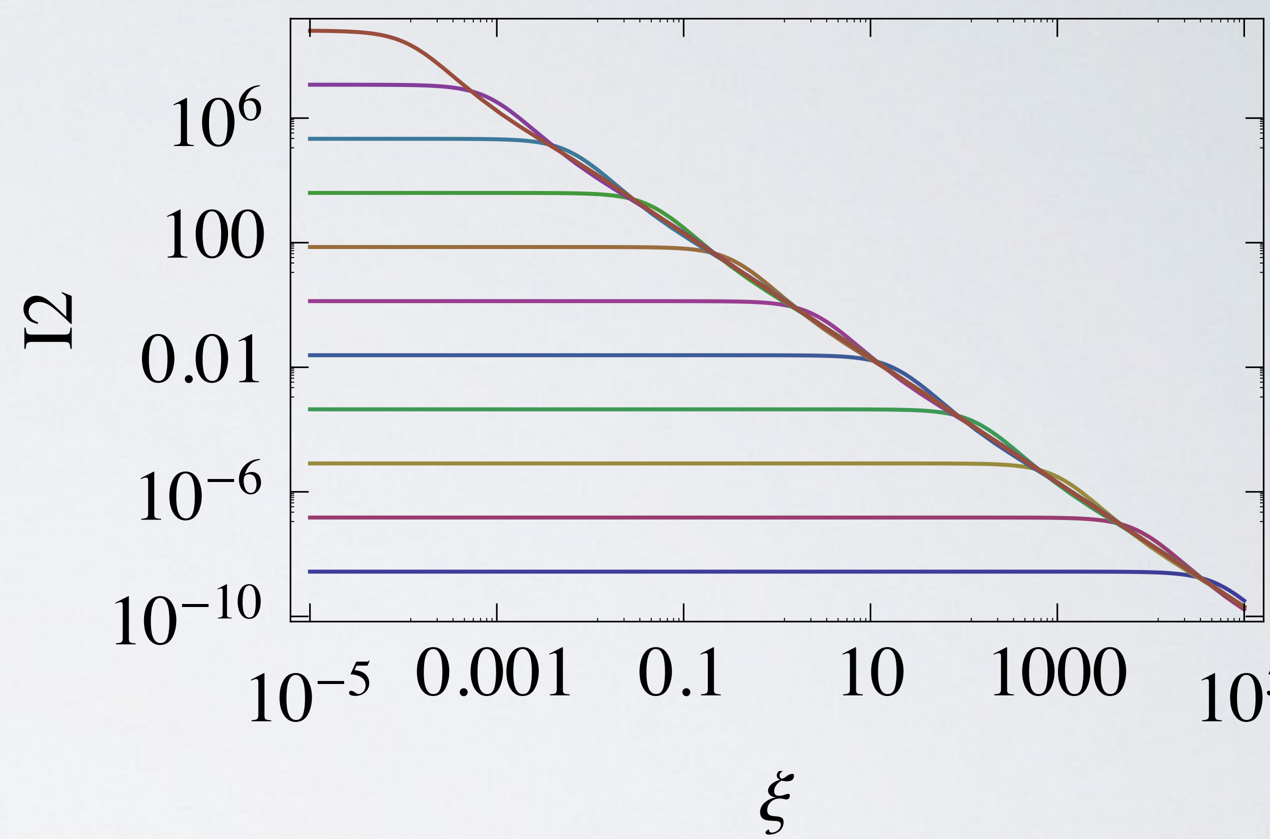
b.c. $\begin{cases} \dfrac{d\mu}{dr}(0) = 0 \\ \mu(0) \end{cases}$ free parameter: degeneracy at origin

$I_2(\xi)$ ~ mass density profile $\rho(r)$

Fermi-Dirac



Maxwell-Boltzmann



core for high ν_0

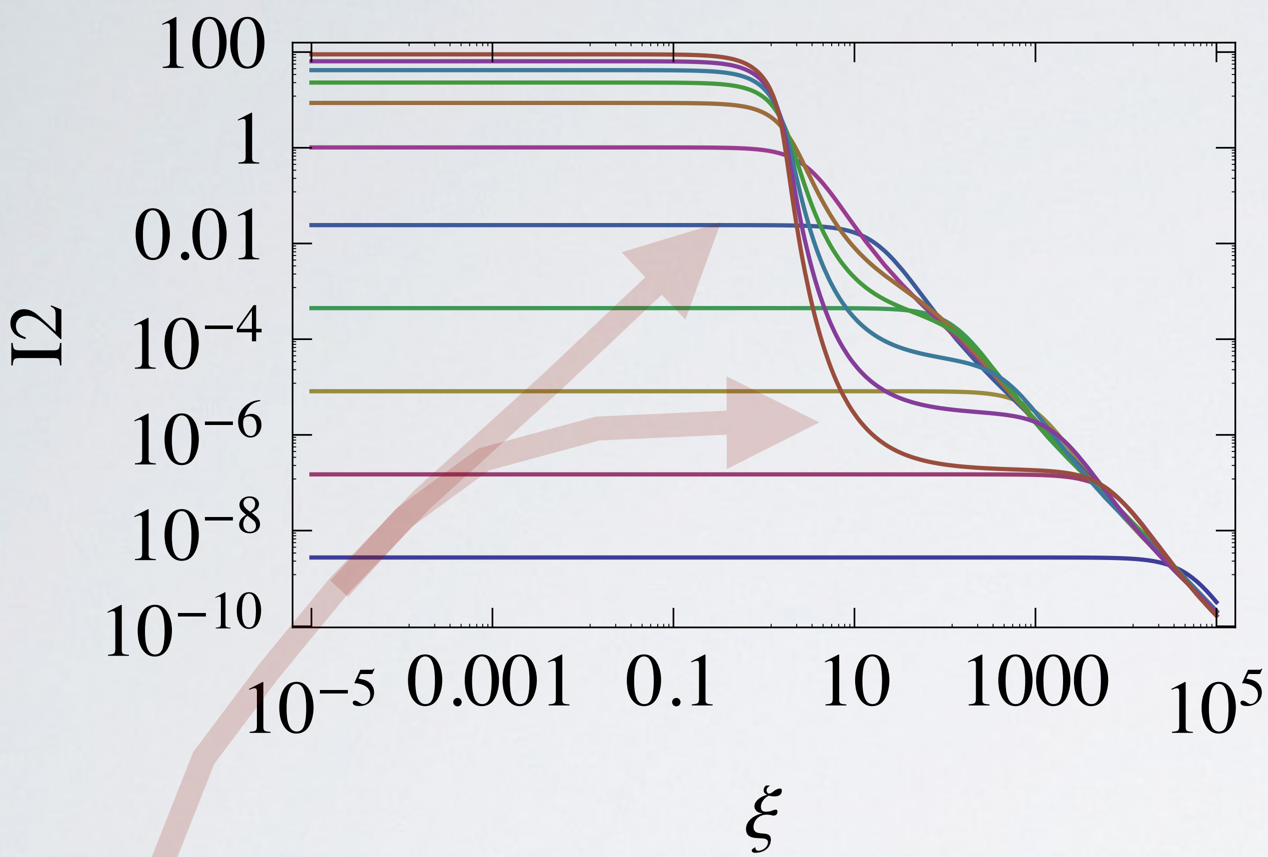
cusp for high ν_0

$\nu_0 \lesssim -4$ ► Classical isothermal halo

$\nu_0 \gtrsim -4$ ► Double feature:
degenerate core plus isothermal halo

$I_2(\xi)$ ~ mass density profile $\rho(r)$

Fermi-Dirac



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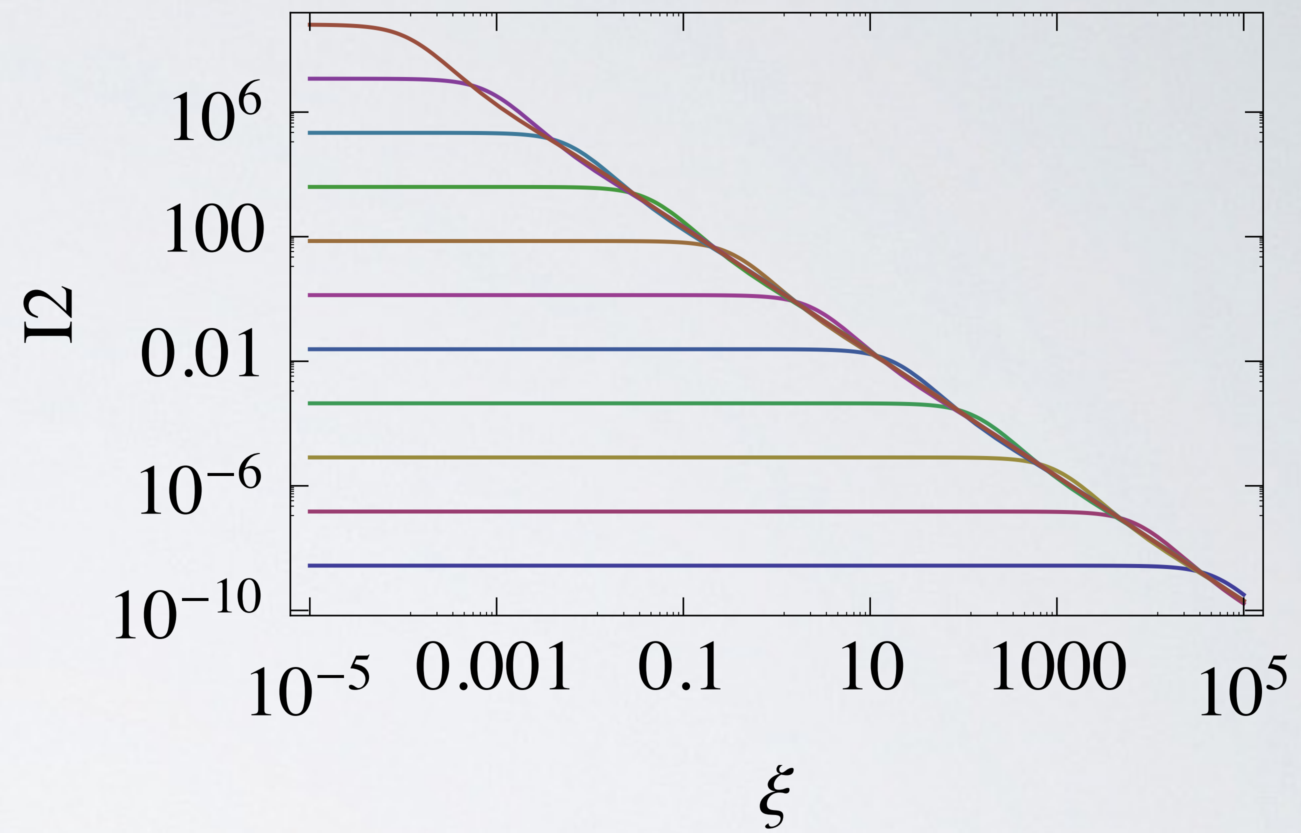

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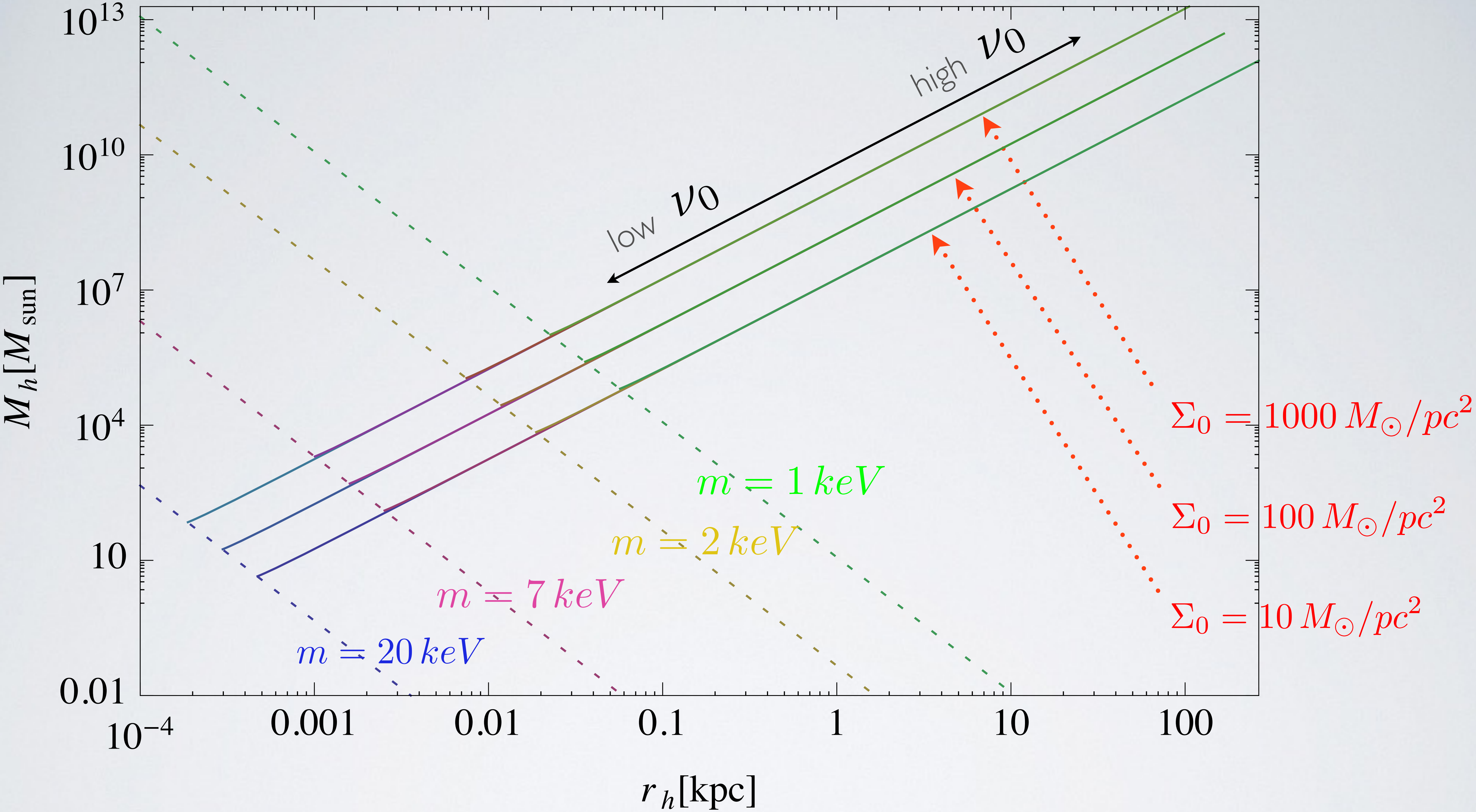
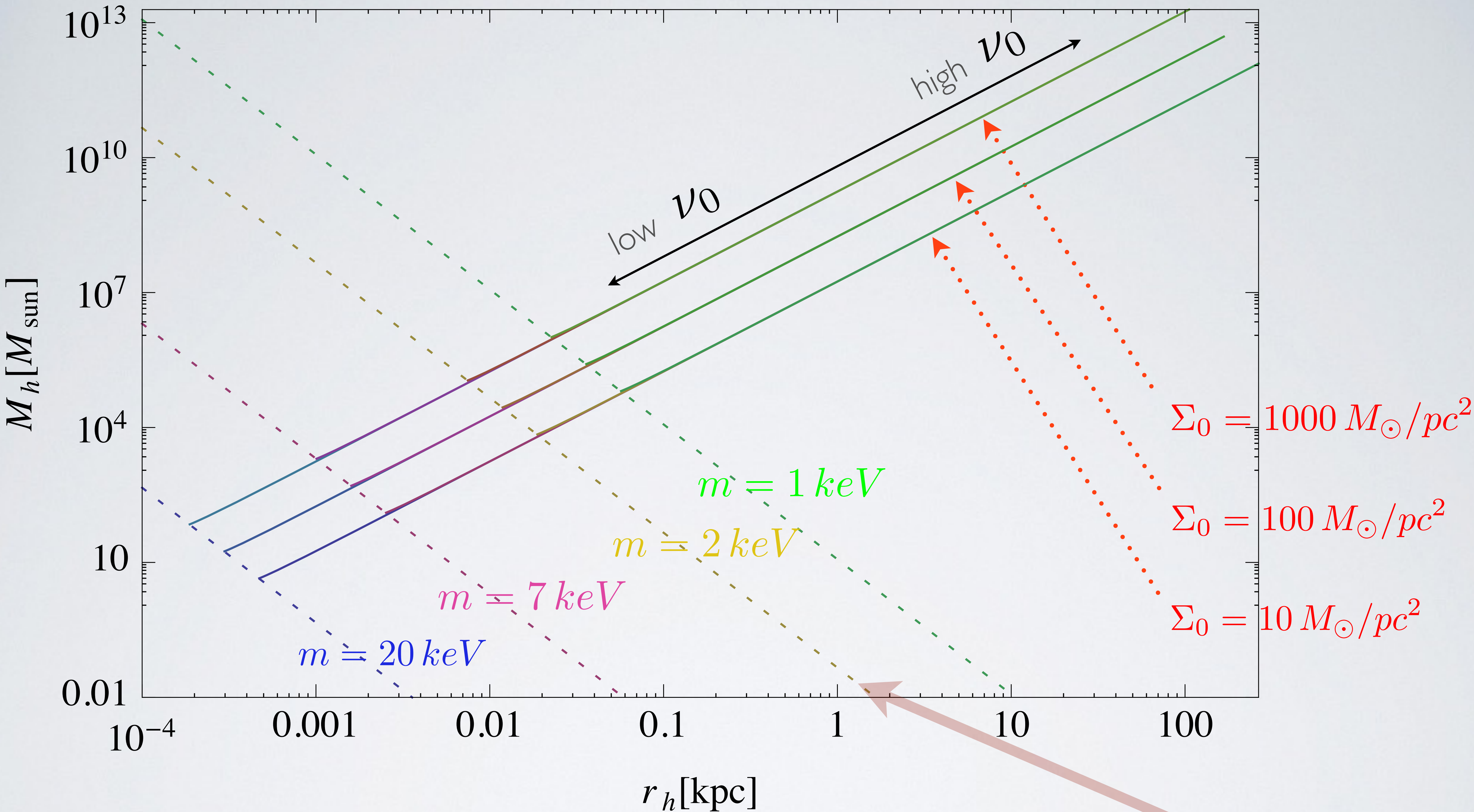


There are limiting values for high ν_0 : $r_{h,\,min}$ $M_{h,\,min}$

...a minimal galaxy, for given mass *m*



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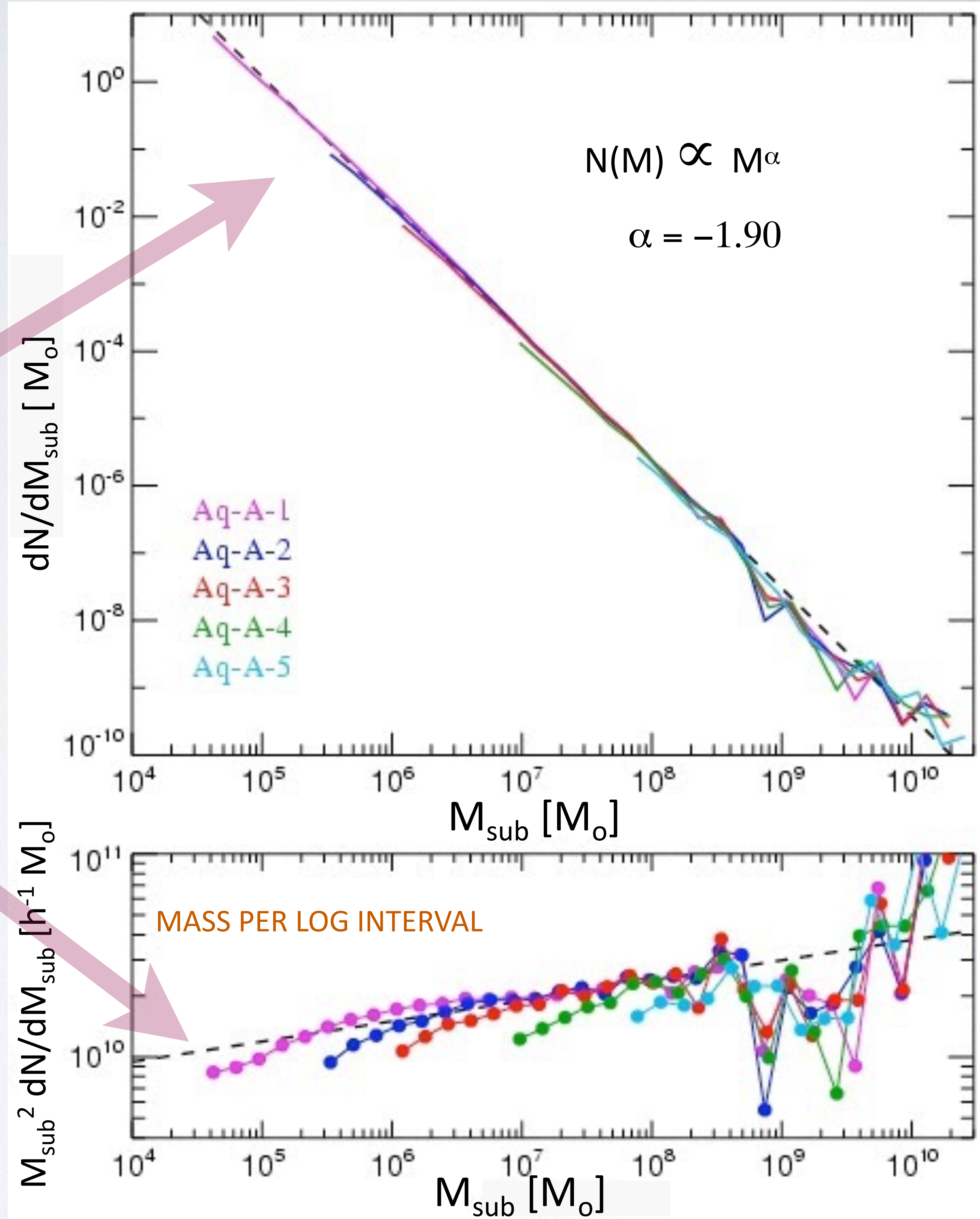
...a minimal galaxy, for given mass *m*

CDM MASS DISTRIBUTION OF SUBSTRUCTURES

unseen low masses

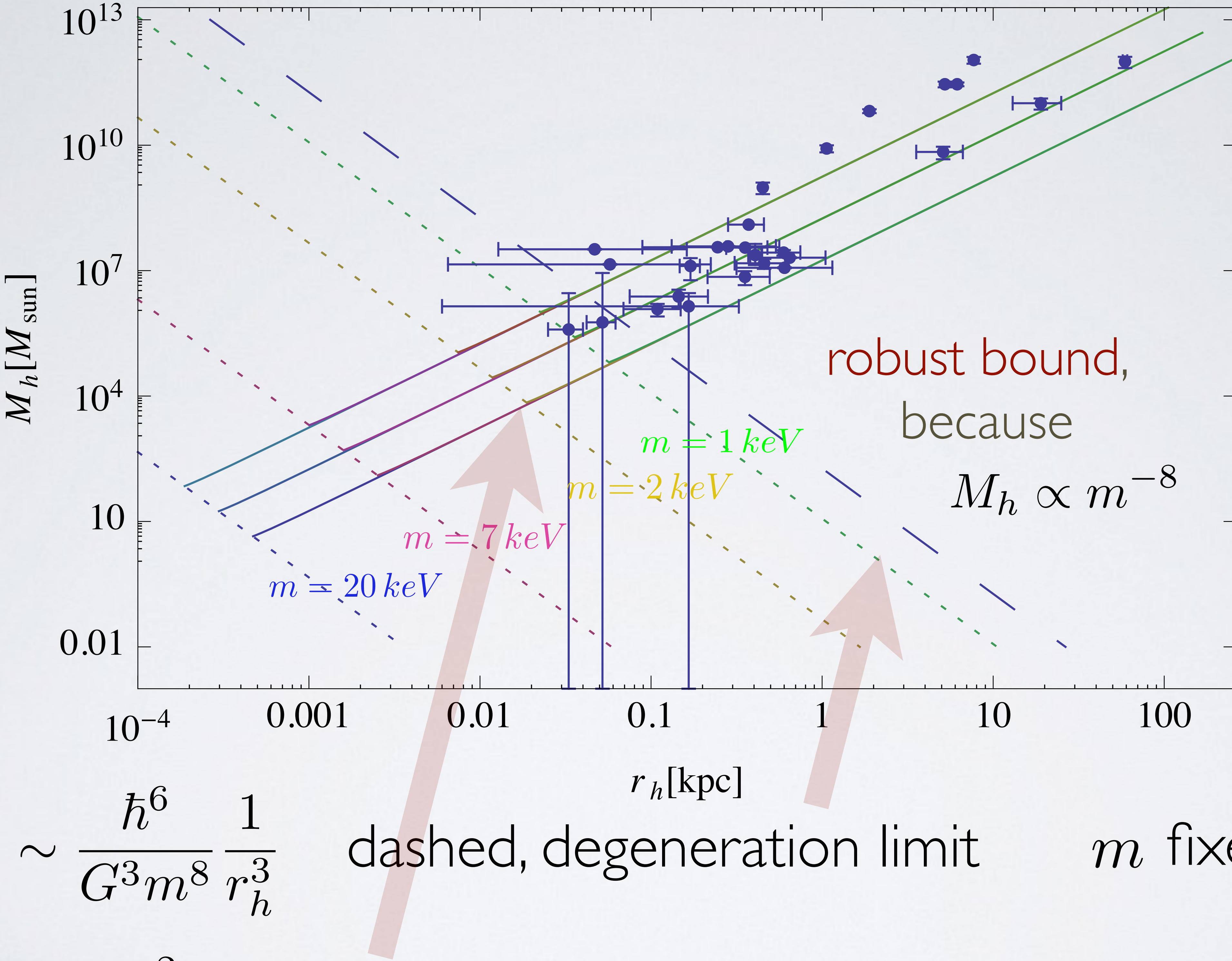
Dramatic slope,
if extrapolated
to low masses

(btw - impossible to predict annihilation, due to unknown amount of subhalos)



[Springel+ '08]

SO : LOWER LIMIT ON m



$$M_h \sim \frac{\hbar^6}{G^3 m^8} \frac{1}{r_h^3}$$ dashed, degeneration limit m fixed

$$M_h \sim \Sigma_0 r_h^2$$ continuous lines Σ_0 fixed

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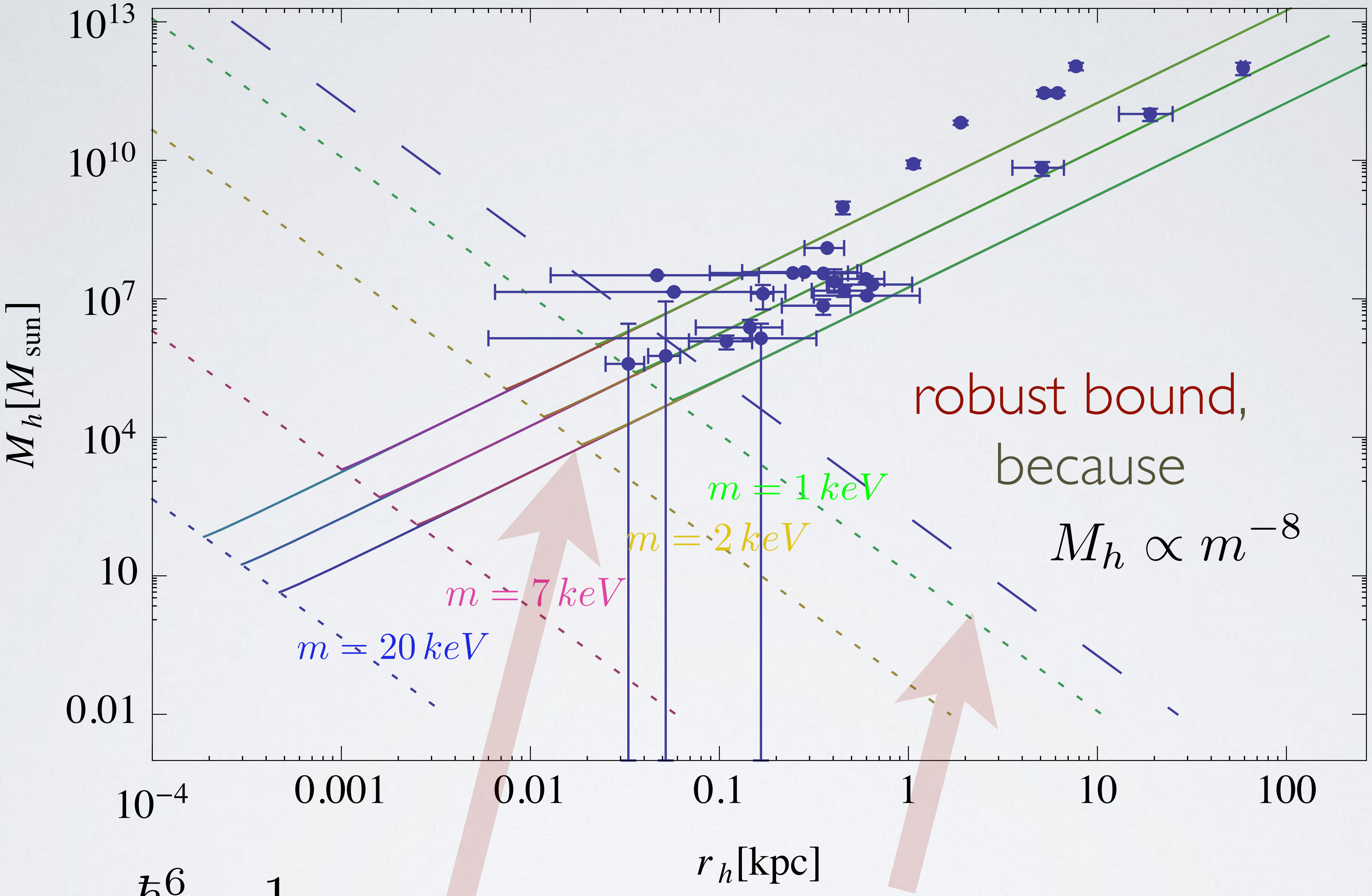

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$M_h \sim \Sigma_0 r_h^2$ continuous lines Σ_0 fixed

(btw a mistery, compare with neutron star, $M_h \sim R_h^3$ ...)

HOW DOES THE GALAXY PROFILE CHANGES

fix two parameters

$$\Sigma_0 = 120\, M_\odot/pc^2$$

$$M_h \simeq 2.6 \times 10^6 M_\odot$$

Only parameter left: m

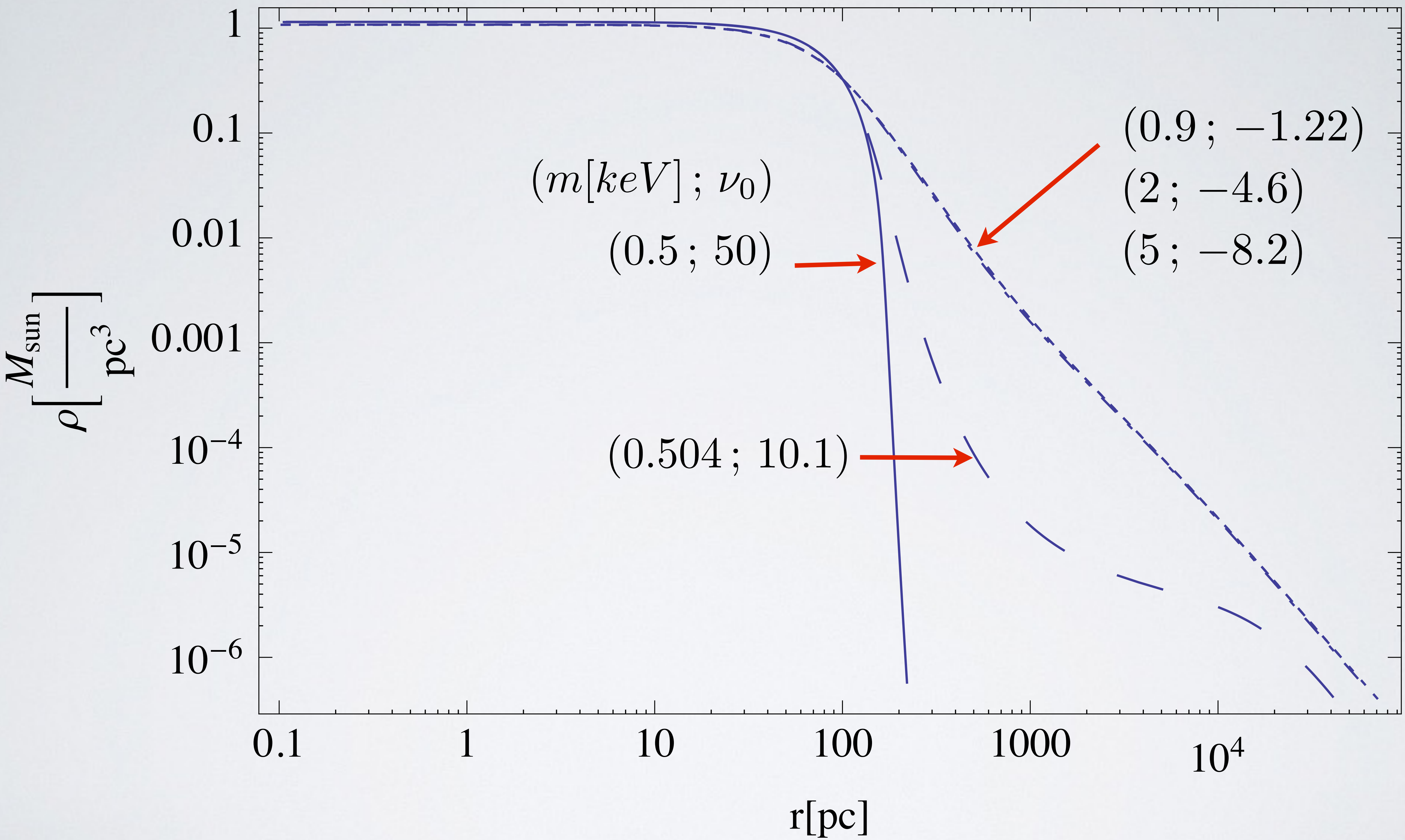
Or: ν_0

And let's see if we can discriminate *m* with the profile.

Recall

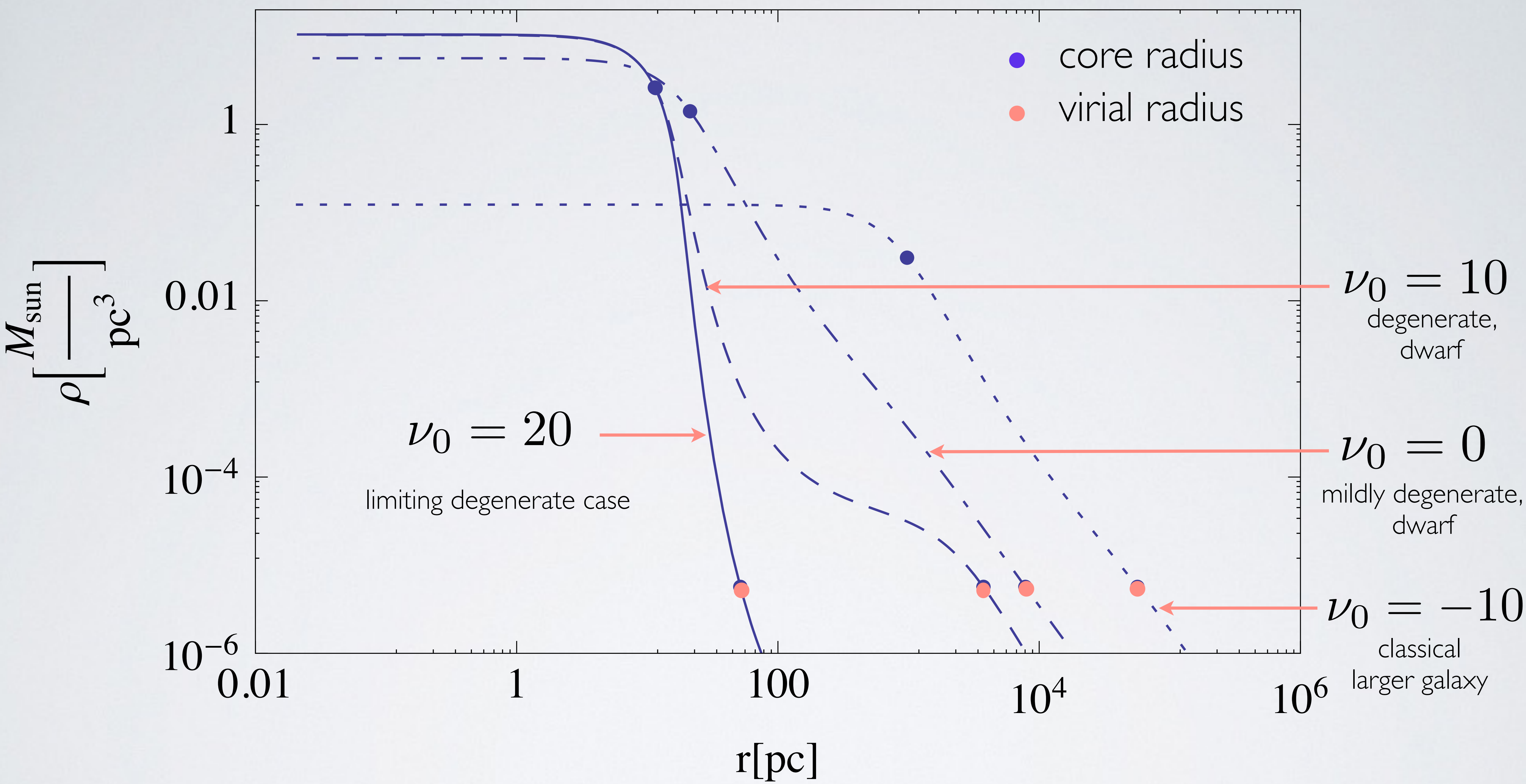
- $\nu_0 \lesssim -4$ ► classical
- $\nu_0 \gtrsim -4$ ► starts to be degenerate
- $\nu_0 \gtrsim 10$ ► highly degenerate, T_0 independent

FIX A GALAXY, VARY DM MASS m



transition to degeneration fast: m varies by 1% only

FIX m, GALAXIES OF DIFFERENT MASS



smallest are near the degeneration

Can we probe these profiles?

WE OBSERVE THE STAR VELOCITY DISPERSION

(LINE OF SIGHT ONLY, $\boldsymbol{\sigma}_r$)

- **No: nontrivial problem already to estimate the DM mass**

- use Jeans equation for matter, from given mass model $M(r)$

$$\left(\frac{\partial}{\partial r} + \frac{2\beta}{r}\right)(n_* \sigma_r^2) = -n_* \frac{GM(r)}{r^2}$$

- *Dispersion anisotropy* $\boldsymbol{\beta}$ *unknown* $\beta \equiv 1 - \sigma_\perp^2 / \sigma_r^2$

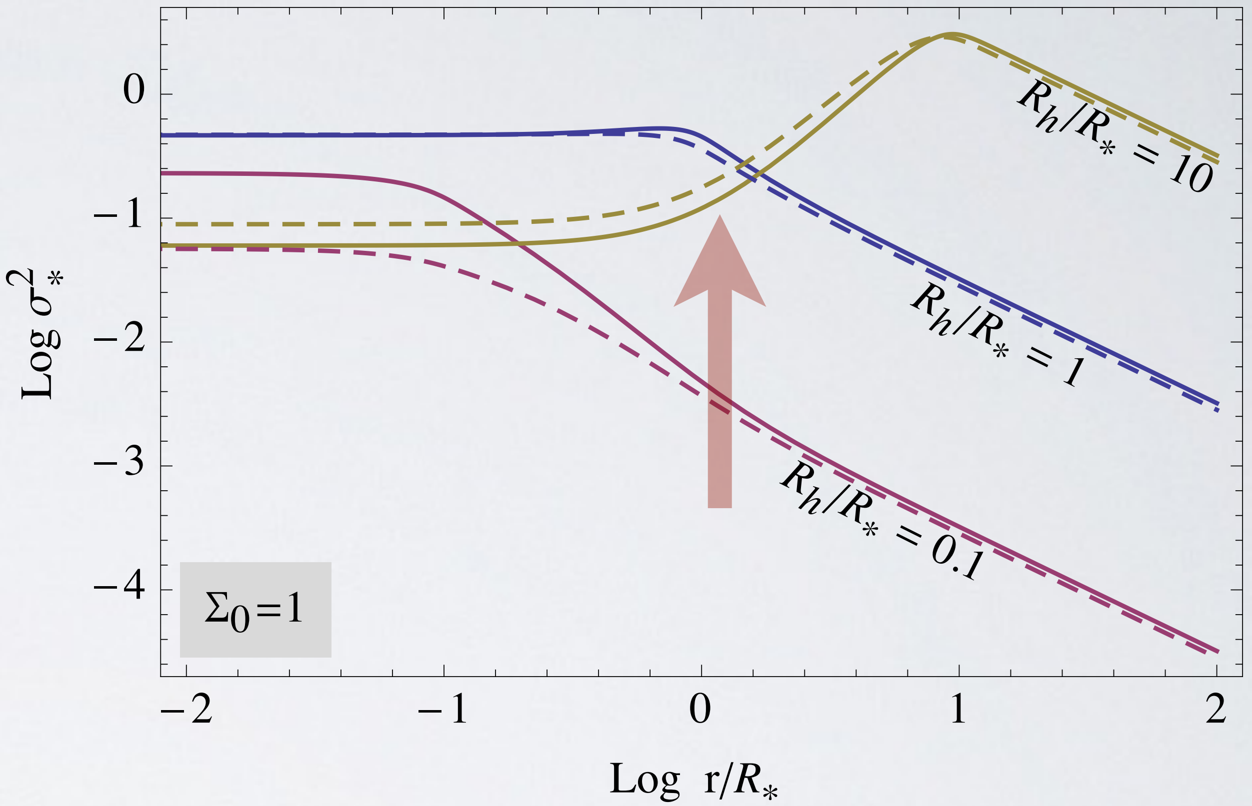
- *hard to measure stars for small galaxies*

PREDICTED STAR VELOCITY DISPERSION

WITH OR WITHOUT ANISOTROPY

- Small or large DM core R_h

too large cores ruled out by constant observed $\sigma_{*,r}$



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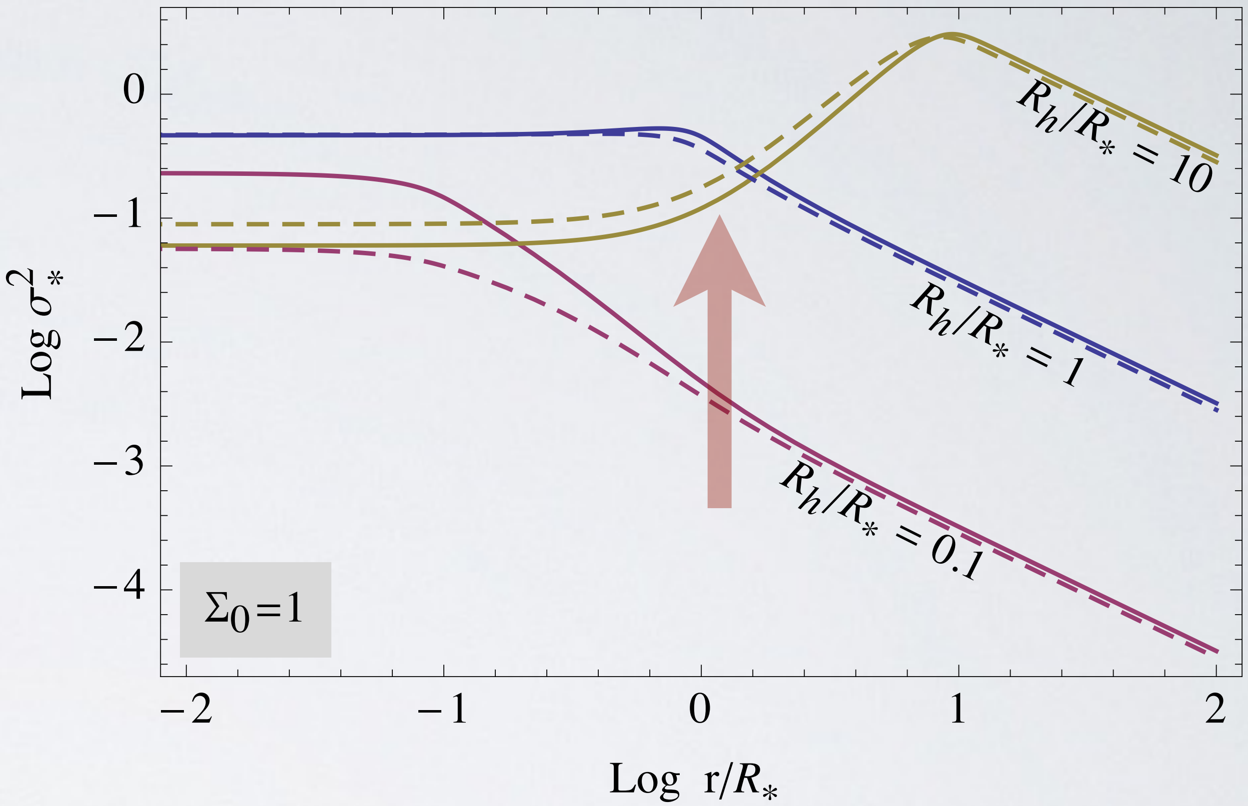
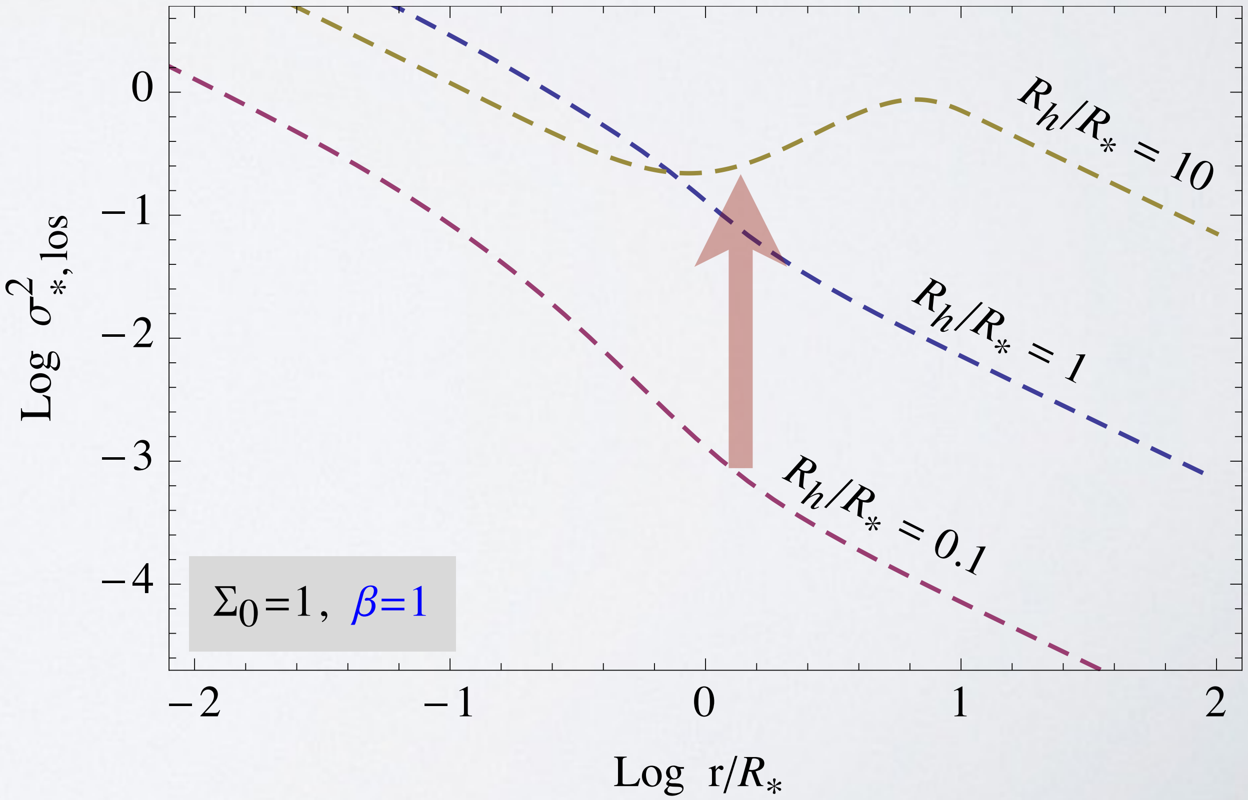


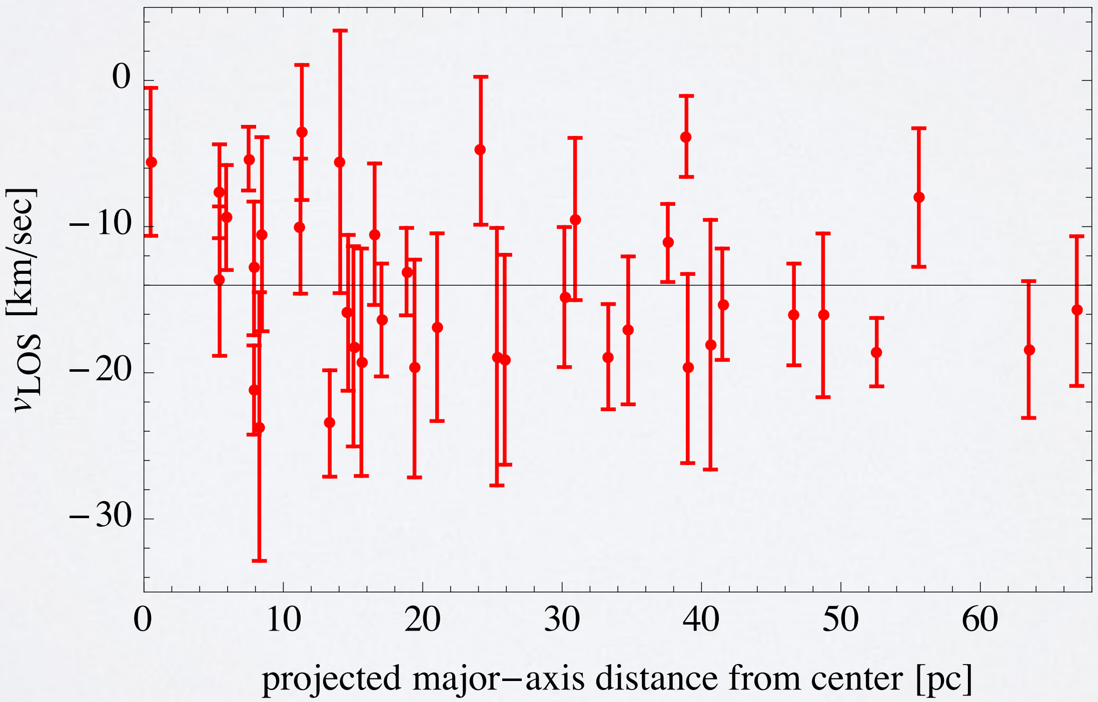
- Effect of anisotropy, e.g. $\beta=1$

  large core gives again a flattish $\sigma_{*,r}$ … !

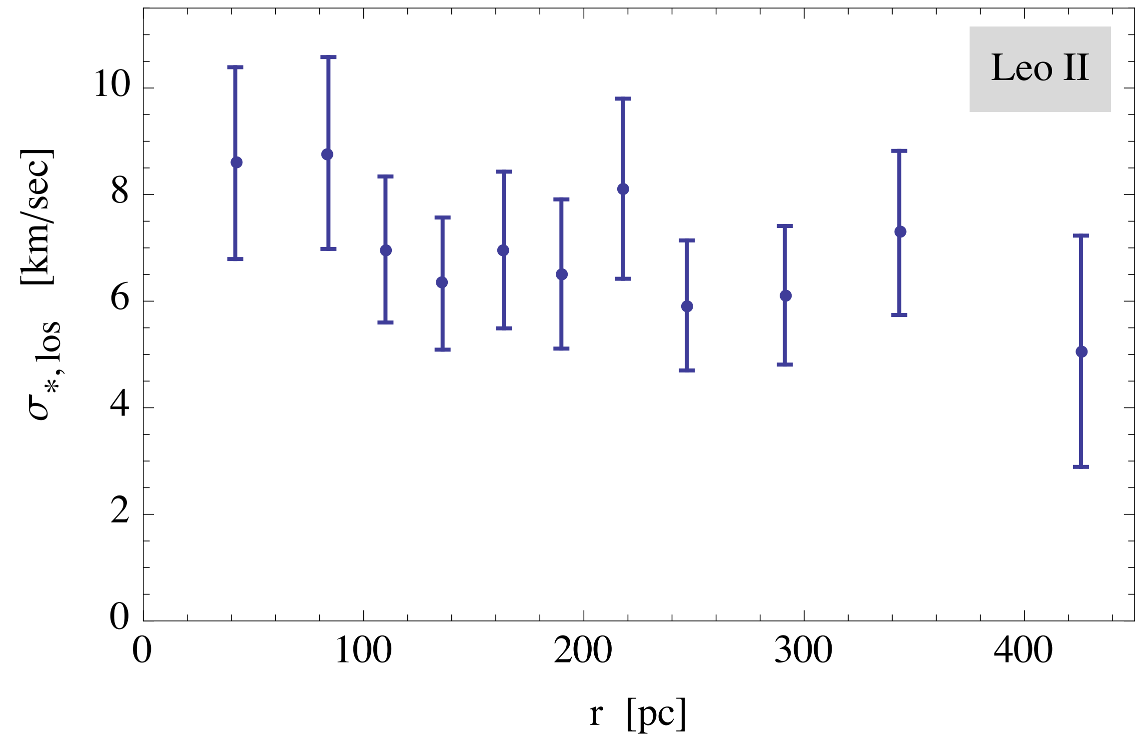


Are rising velocity dispersion profiles allowed?

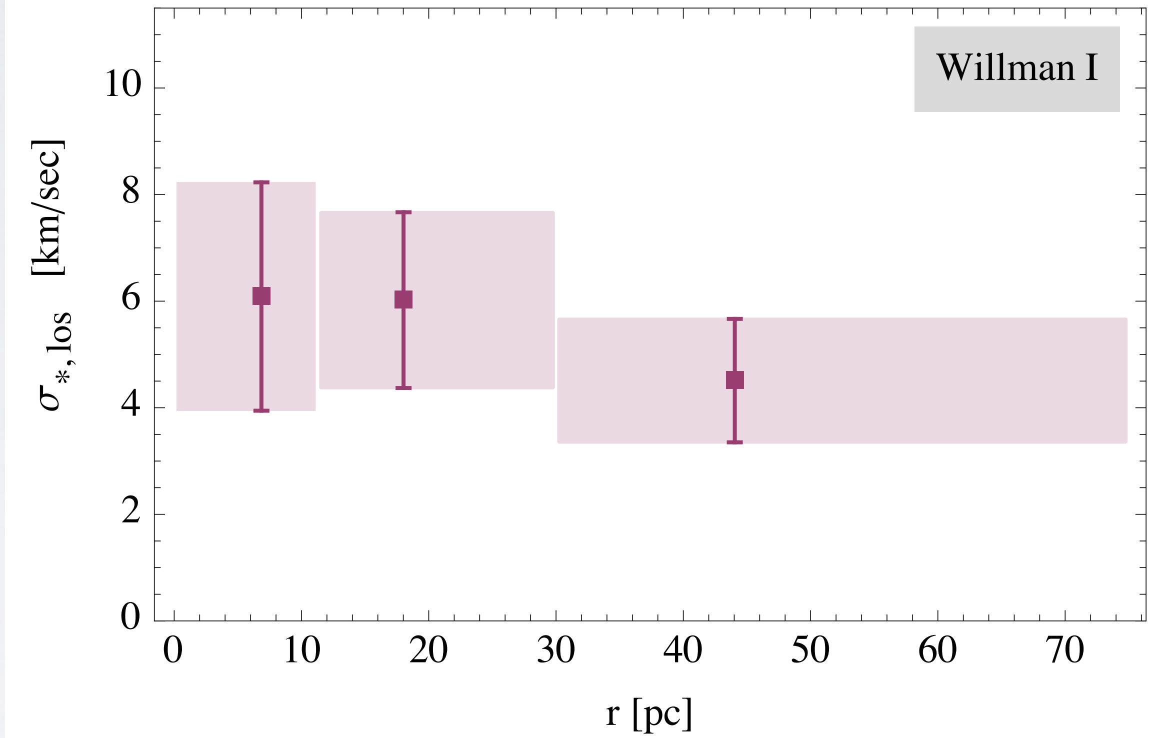
Compare with data



LEO II , WILLMAN I

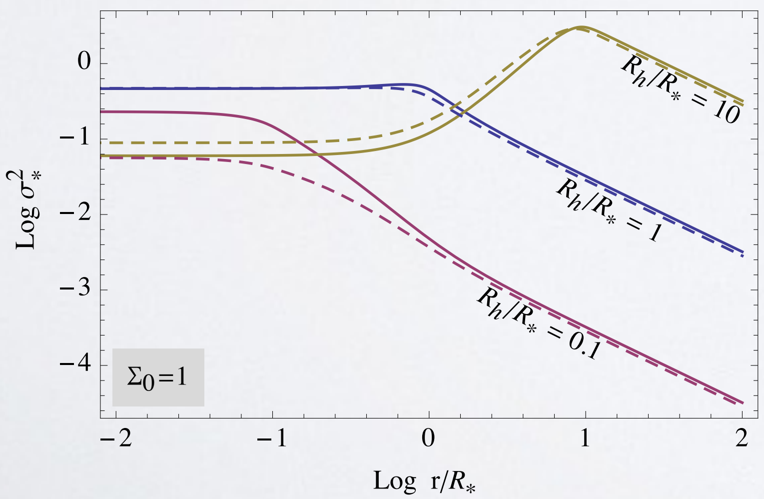


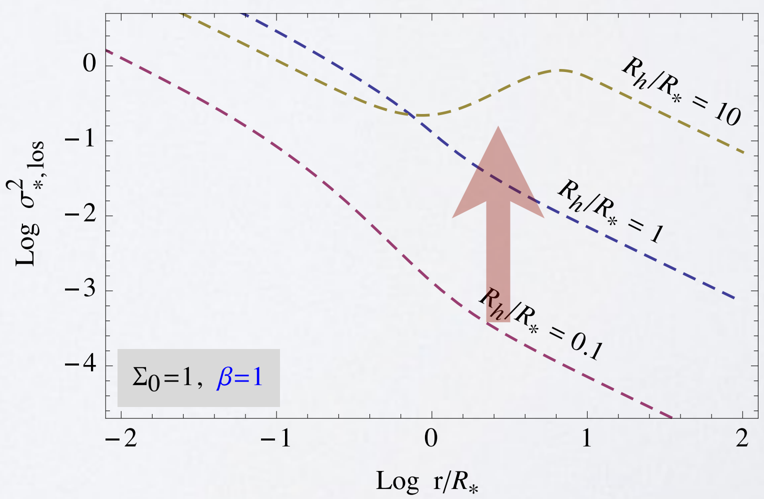
r_*=177pc, distance=205 kpc



r_*=20pc, distance=38 kpc

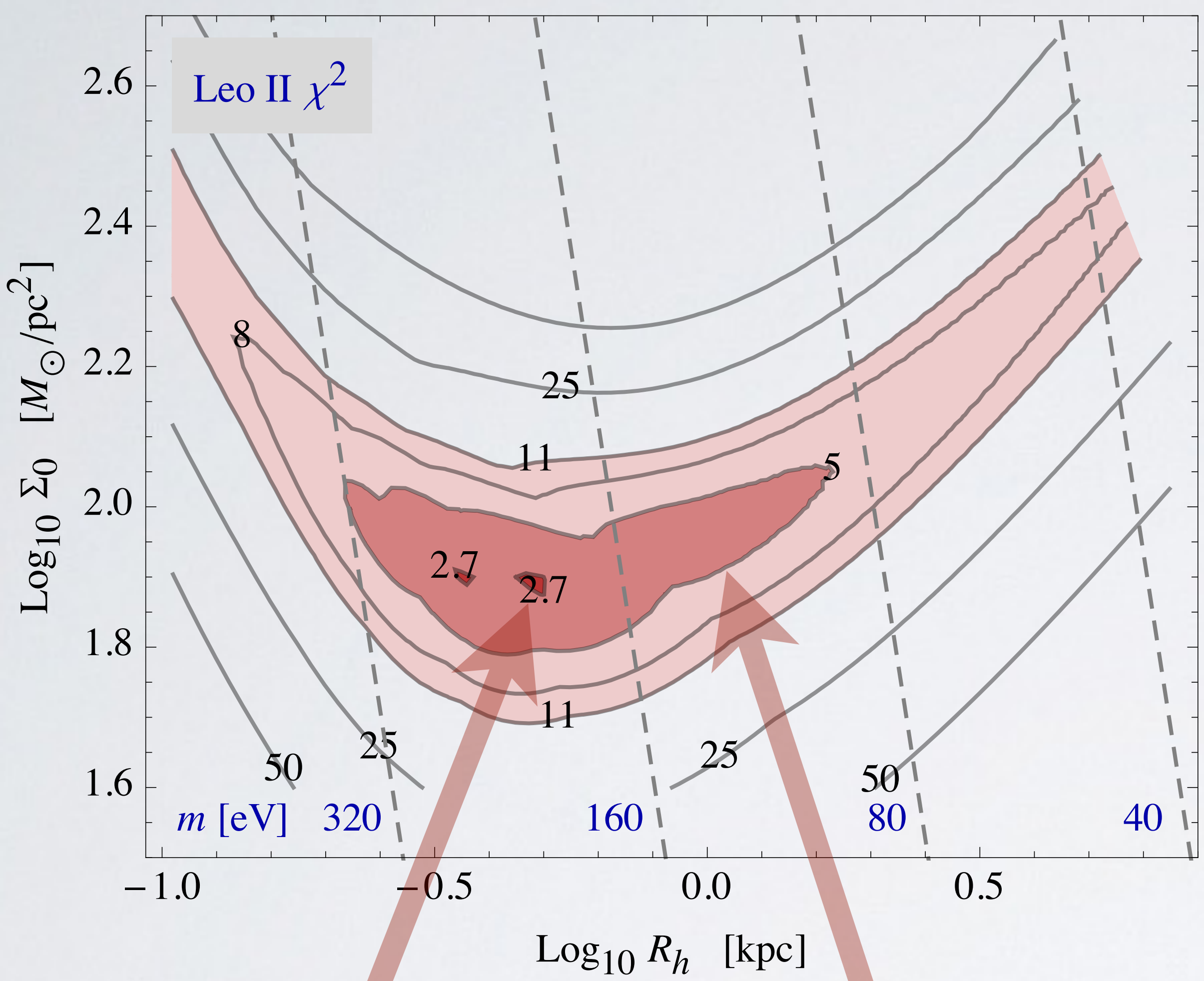
constrain R_h, M_h, m by fitting with predicted profiles





MARGINALIZING BETA β - LEO II

Parameter estimation



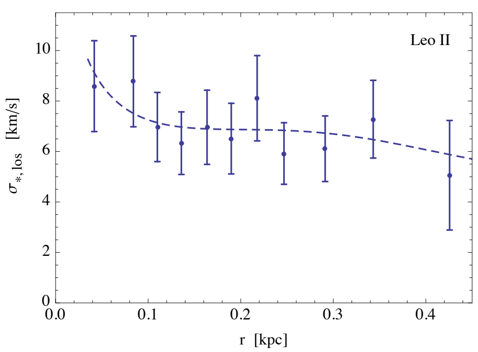


FIG. 4. Stellar velocity dispersion for Leo II. The dashed line represents the best fit, achieved for $\beta = 0.7$.

1σ range

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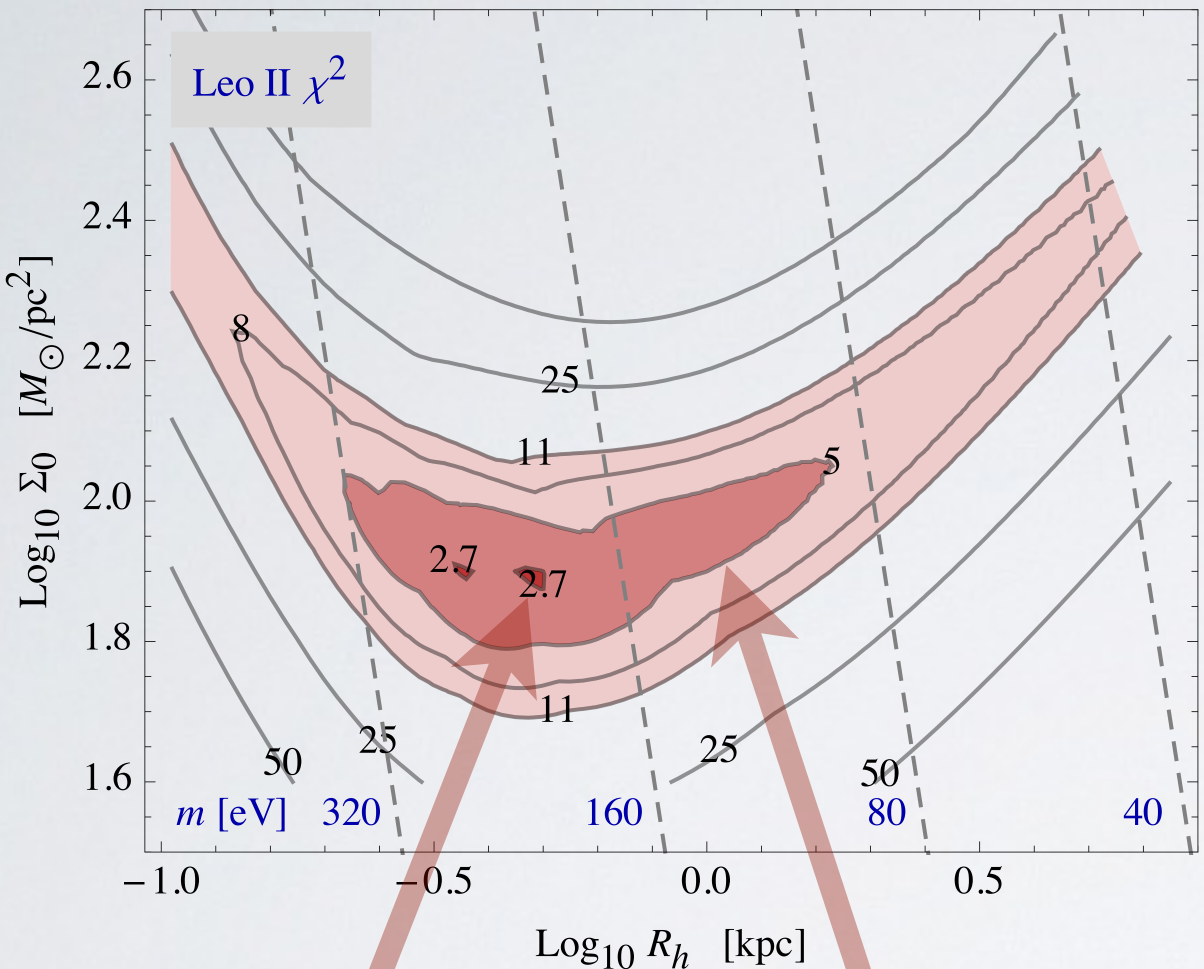

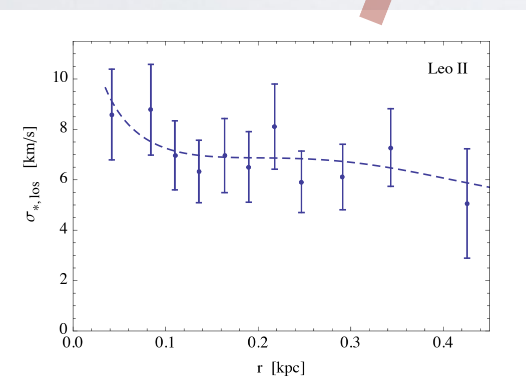

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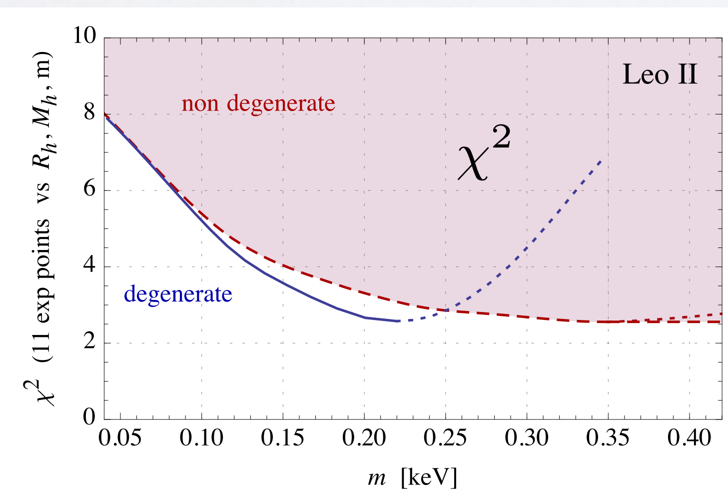

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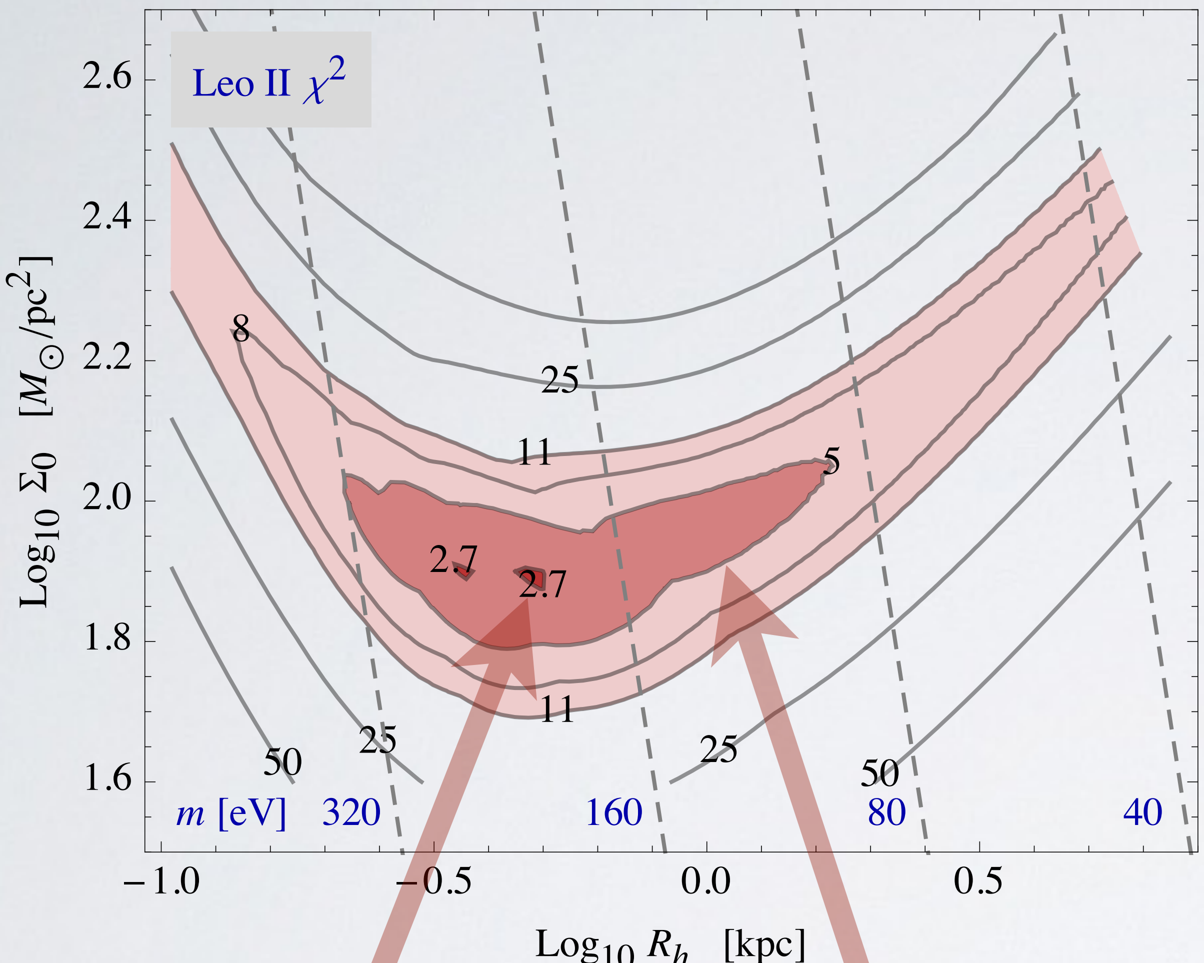
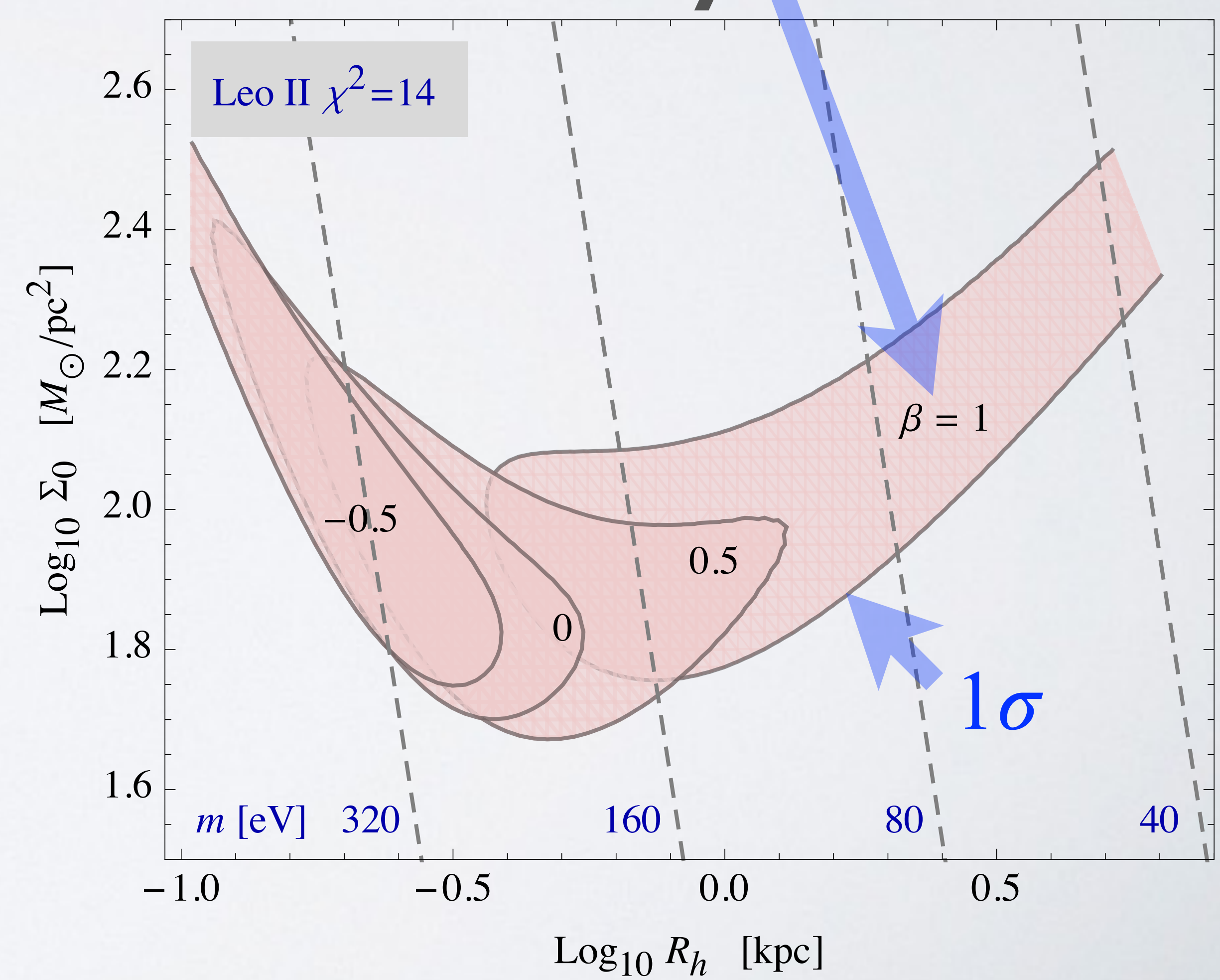


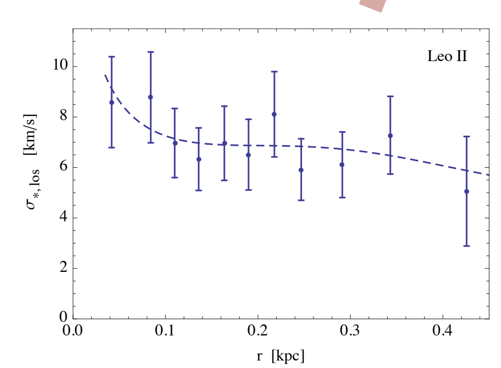
due to $\boldsymbol{\beta} \to 1$

No upper limit on R_h

No lower limit on m

Consistency with data





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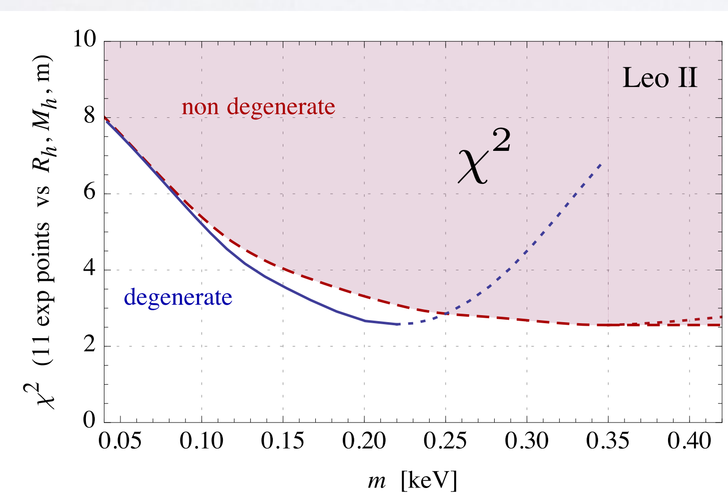

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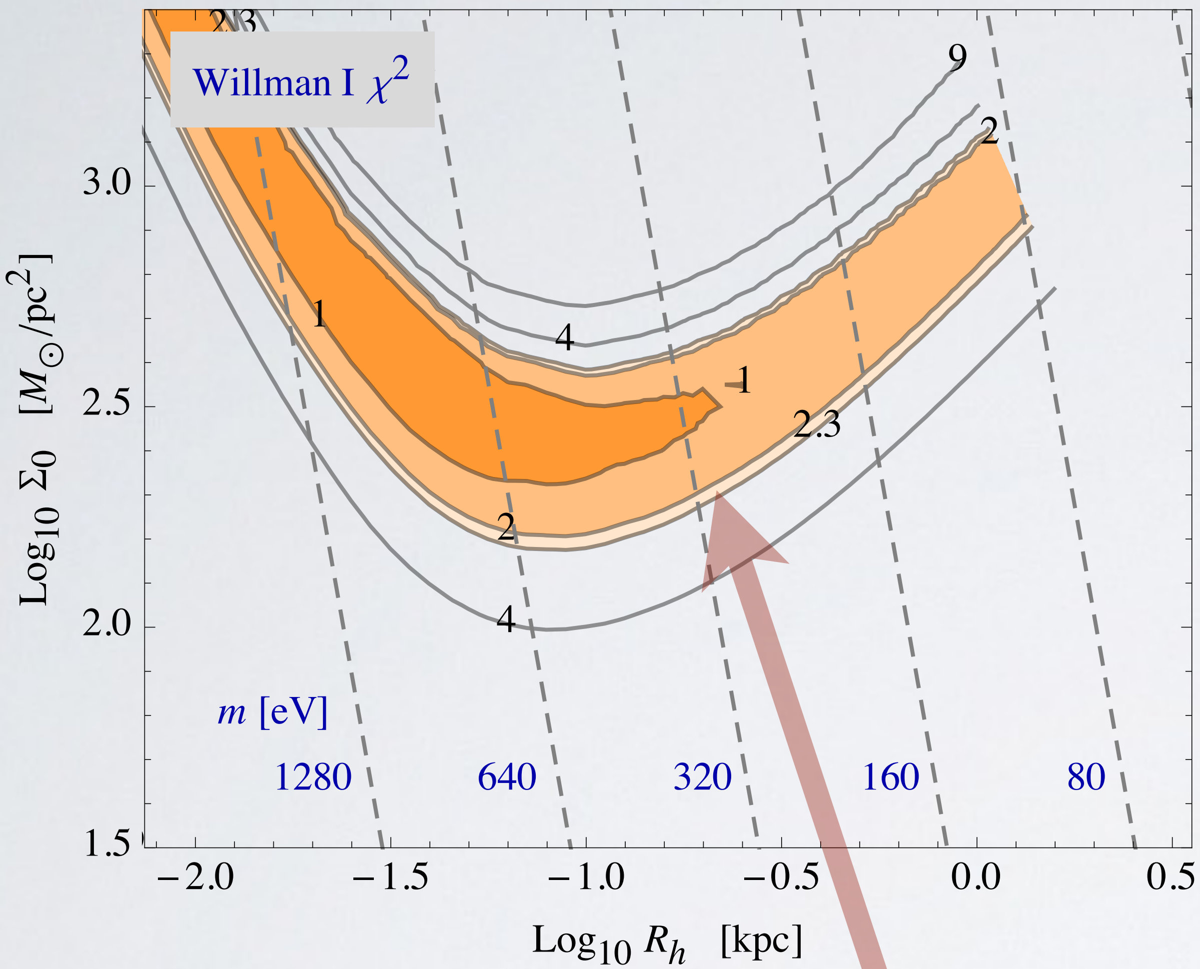

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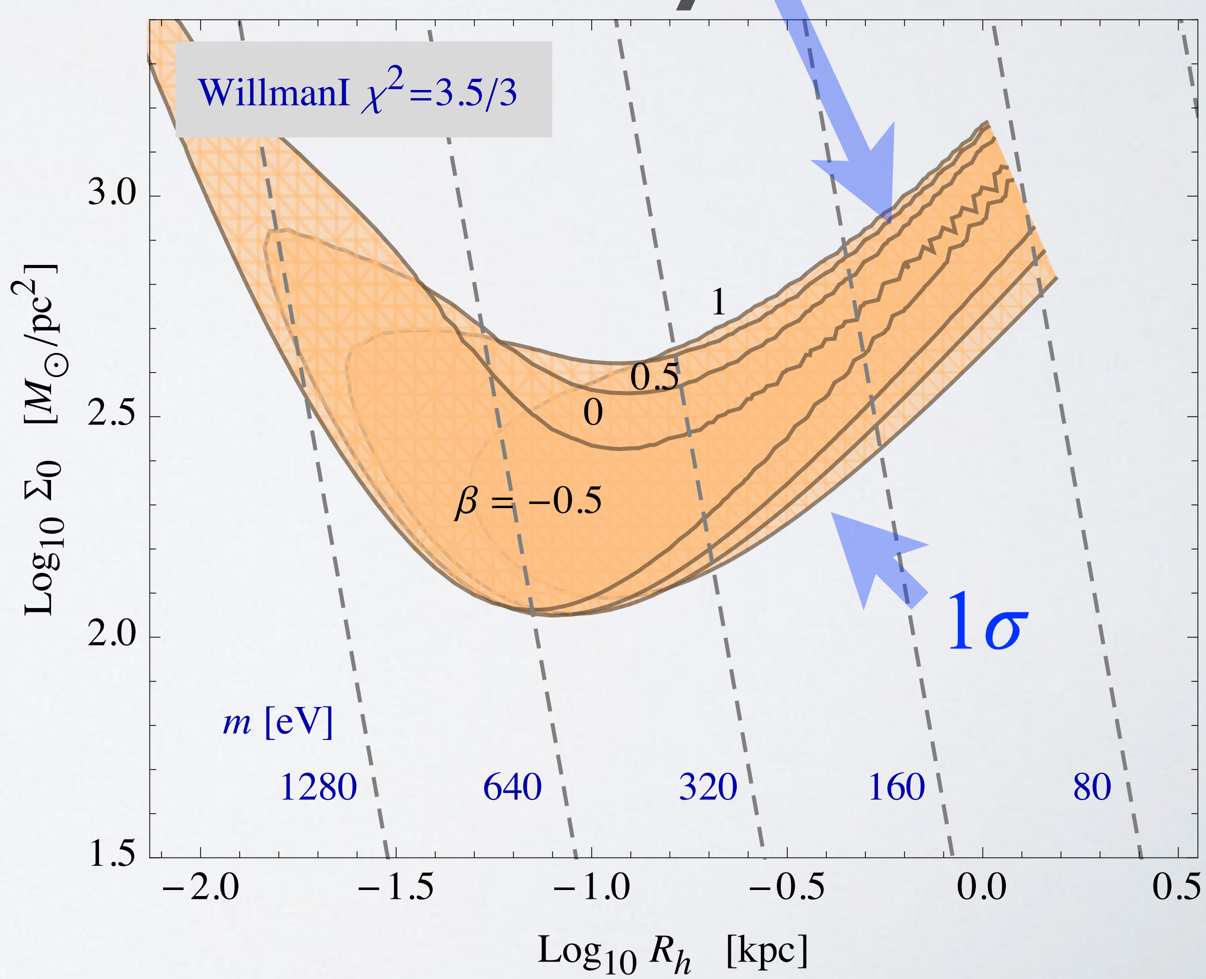


1σ

TOTAL MASS LIMITED BY DYNAMICAL FRICTION

Satellites would have fallen in the MW halo…
…due to gravitational friction

[Chandrasekar formula, e.g. Binney Tremaine 2008 Read+ '06; Just '11, etc]

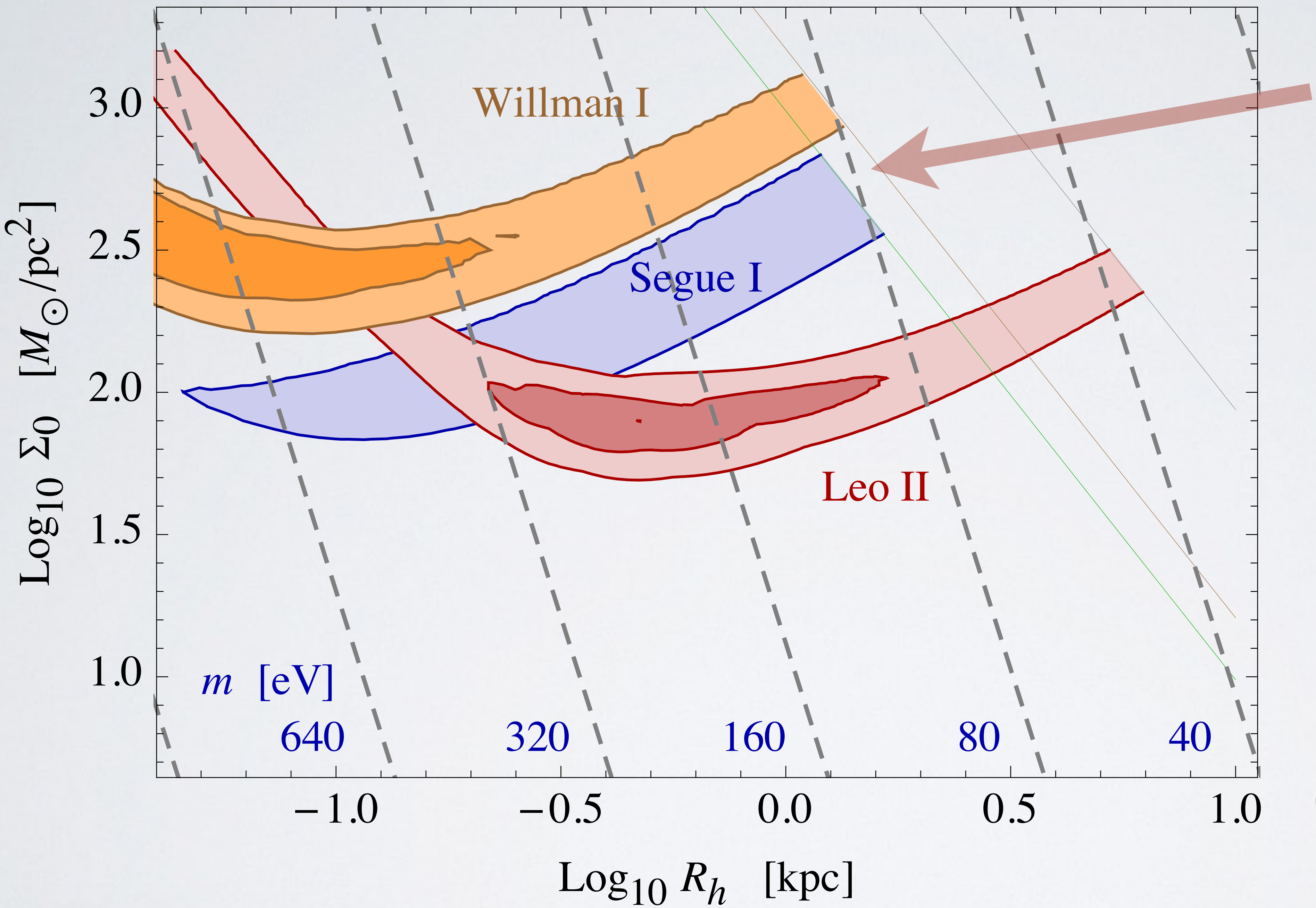
- Time:

$$t_{\rm fric} = \frac{10^{10}{\rm y}}{\ln\Lambda}\left(\frac{D}{60{\rm kpc}}\right)\left(\frac{v}{220{\rm km/s}}\right)\left(\frac{2\cdot 10^{10}\,M_\odot}{M_{\rm h}}\right)$$

should be larger than the age of Galaxy $\sim 10^{10}$ y.

- Puts a bound on halo mass $M_{\rm h}$ [Gerhard Spergel '92]

BOUND ON DM MASS *m* ?



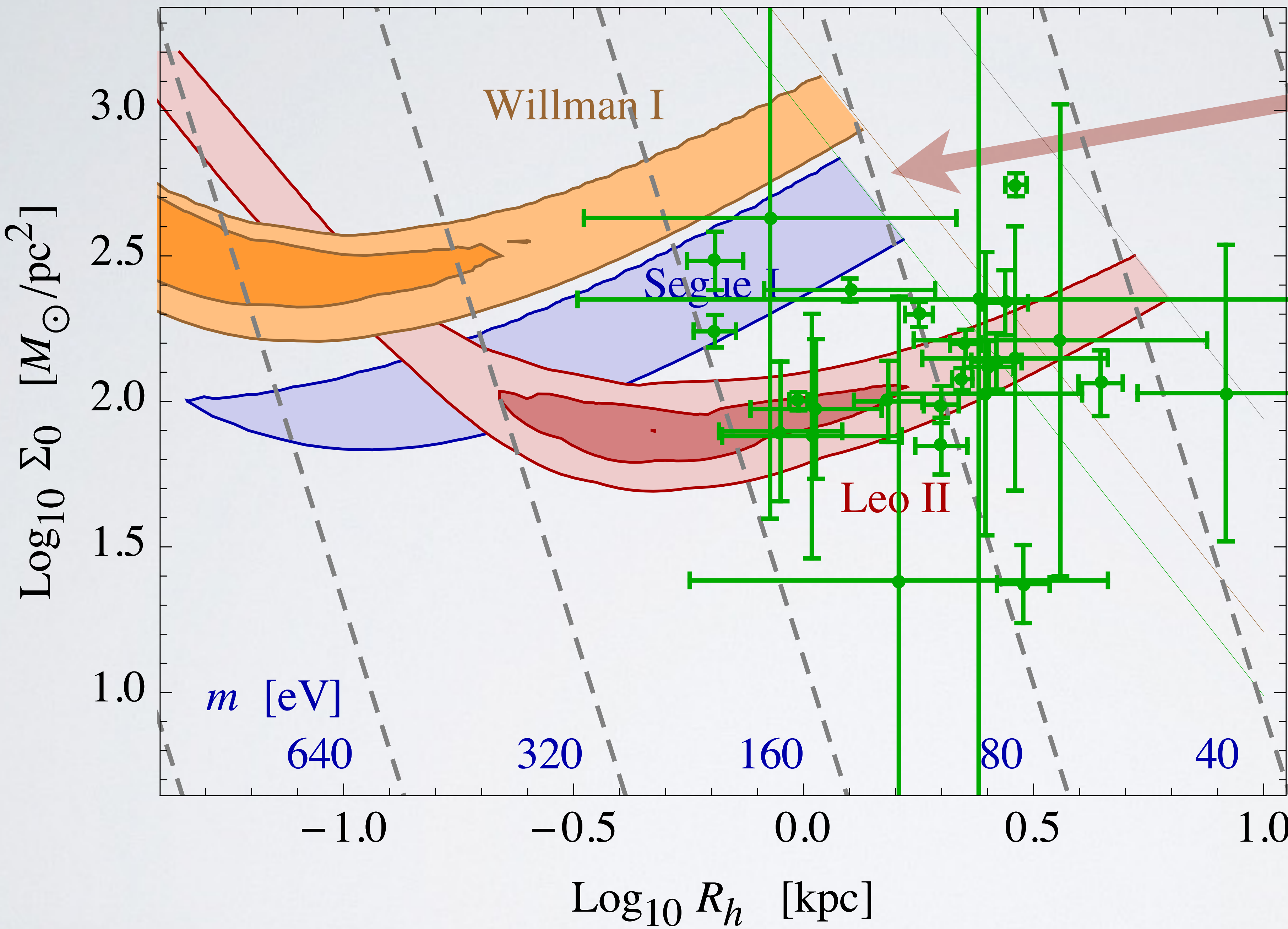
dSph
may be larger…

Tremaine Gunn
saved by
Dynamical Friction

…substantially
weakened to
$m \gtrsim 100$ eV ?

(even if one does not accept ≳ kpc size)

BOUND ON DM MASS *m* ?



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Nothing stronger from Dwarf Disk galaxies [Little Things '15 HI survey]

E.G. SEARCHES FOR X-RAY LINES

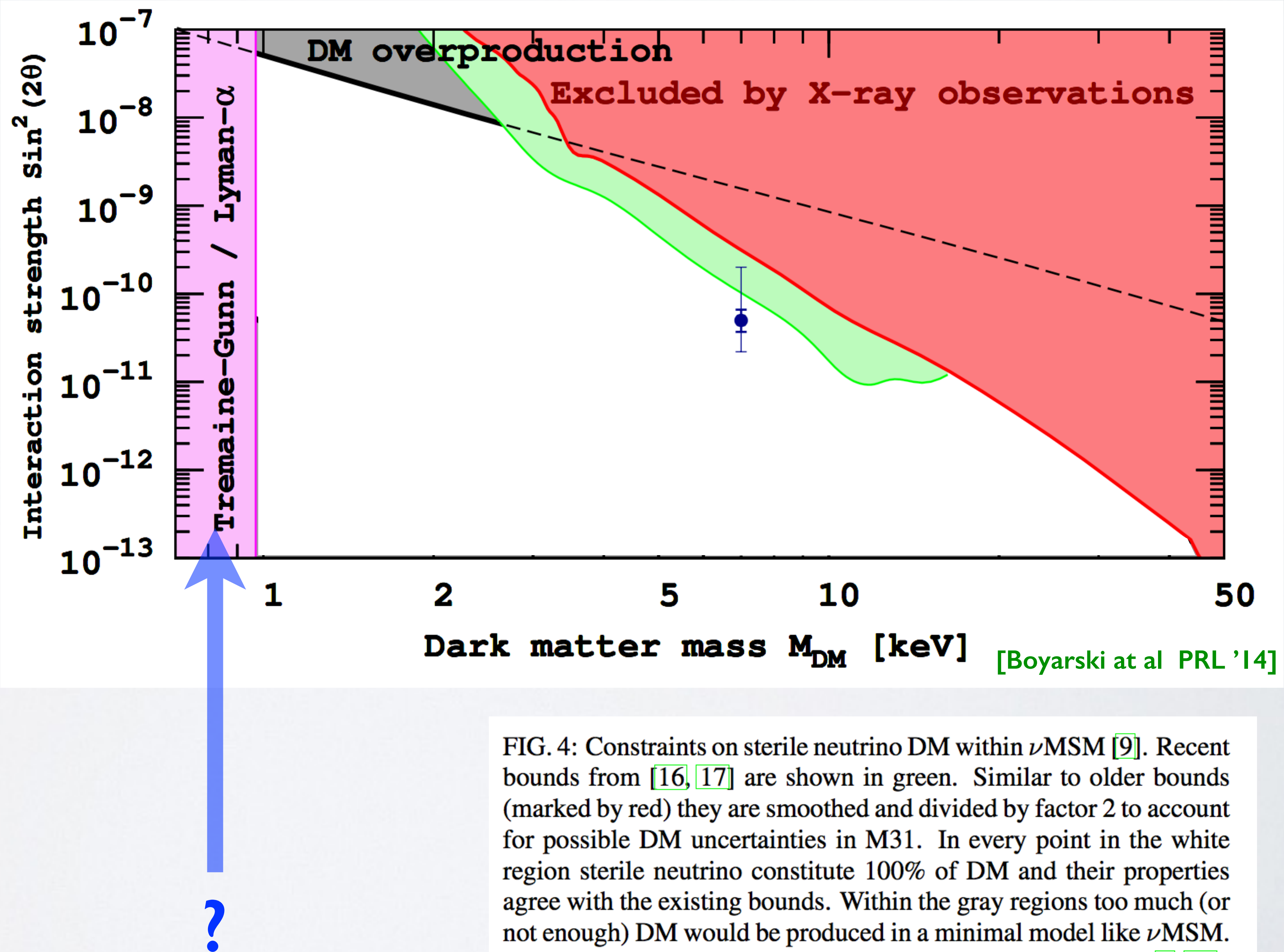

FIG. 4: Constraints on sterile neutrino DM within νMSM [9]. Recent bounds from [16, 17] are shown in green. Similar to older bounds (marked by red) they are smoothed and divided by factor 2 to account for possible DM uncertainties in M31. In every point in the white region sterile neutrino constitute 100% of DM and their properties agree with the existing bounds. Within the gray regions too much (or not enough) DM would be produced in a minimal model like νMSM. At masses below $\sim$ 1 keV dwarf galaxies would not form [4, 48]. The blue point would corresponds to the best-fit value from M31 if the line comes from DM decay. Thick errorbars are $\pm 1\sigma$ limits on the flux. Thin errorbars correspond to the uncertainty in the DM distribution in the center of M31.

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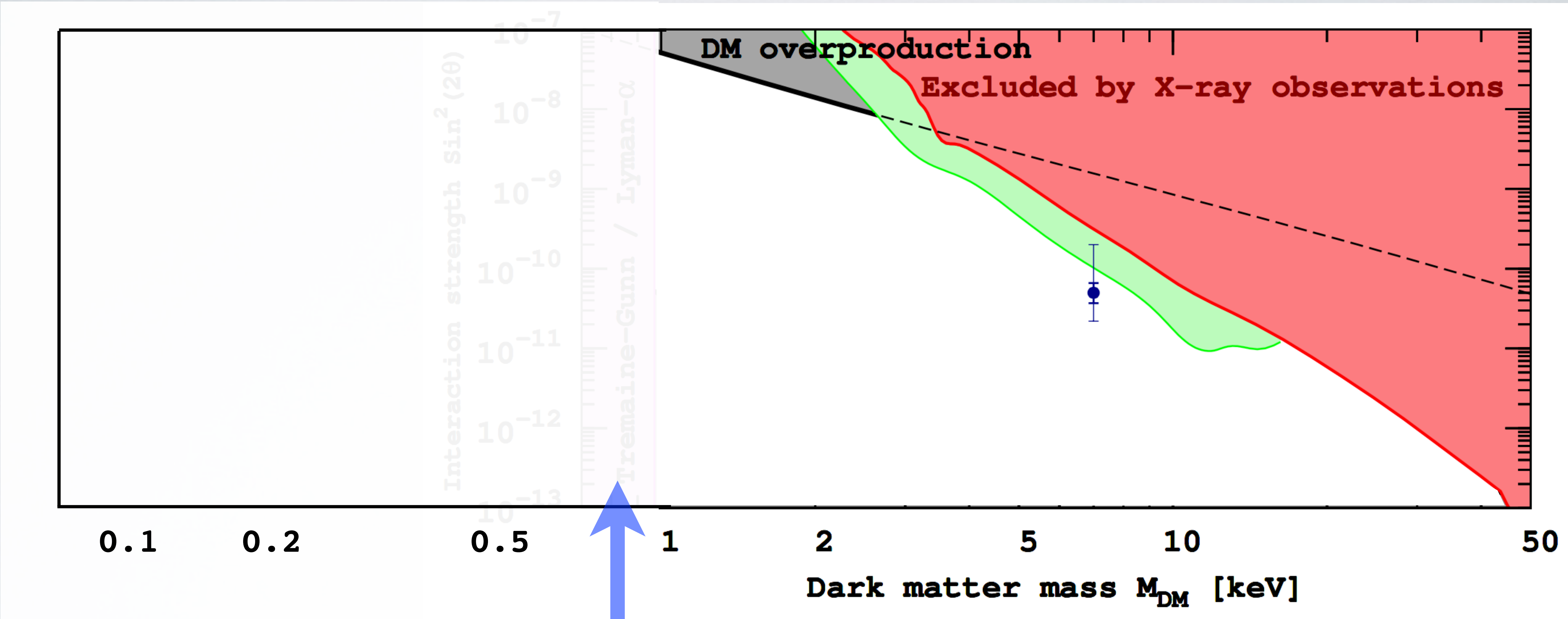

is there
more space
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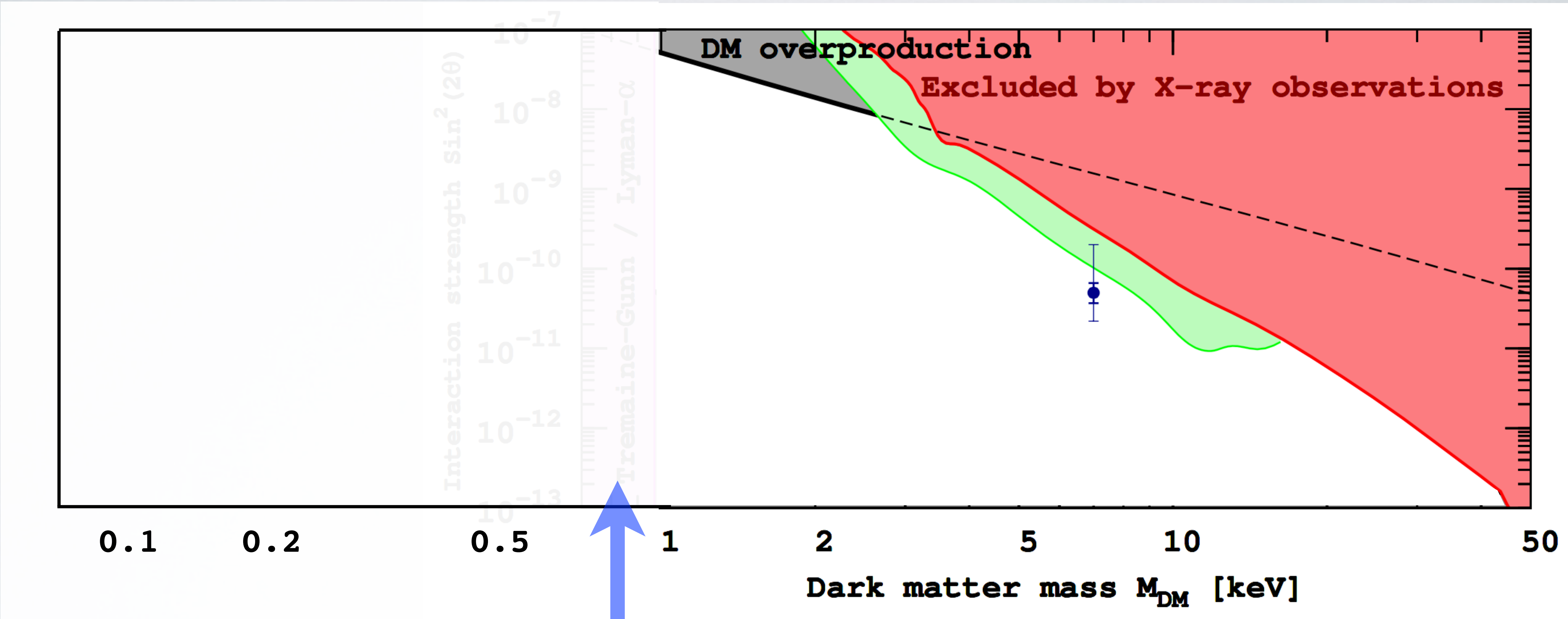

is there
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…now go tell to
Lyman-alpha collegues…

[$m > O(1)$ keV]

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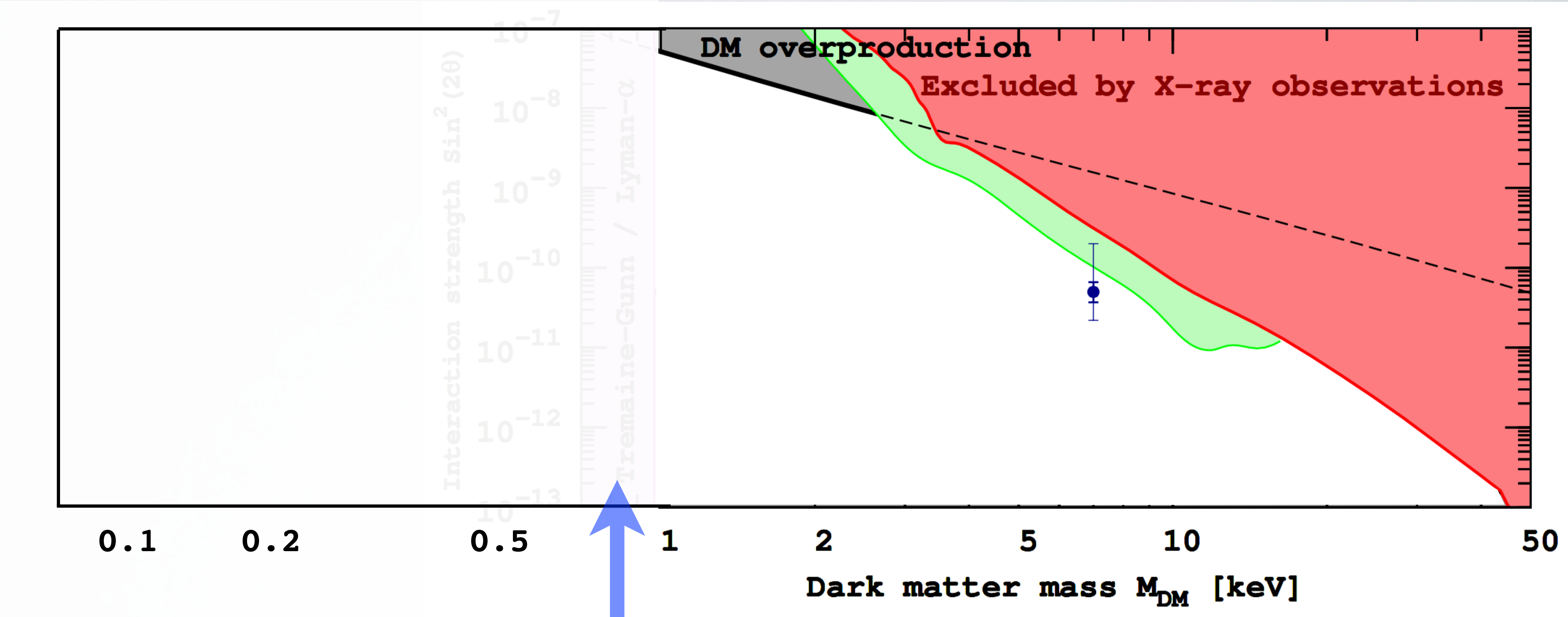


is there more space for sterile neutrinos ?

(and btw how to search for them?)

…now go tell to Lyman-alpha collegues…

[$m > O(1)$ keV]

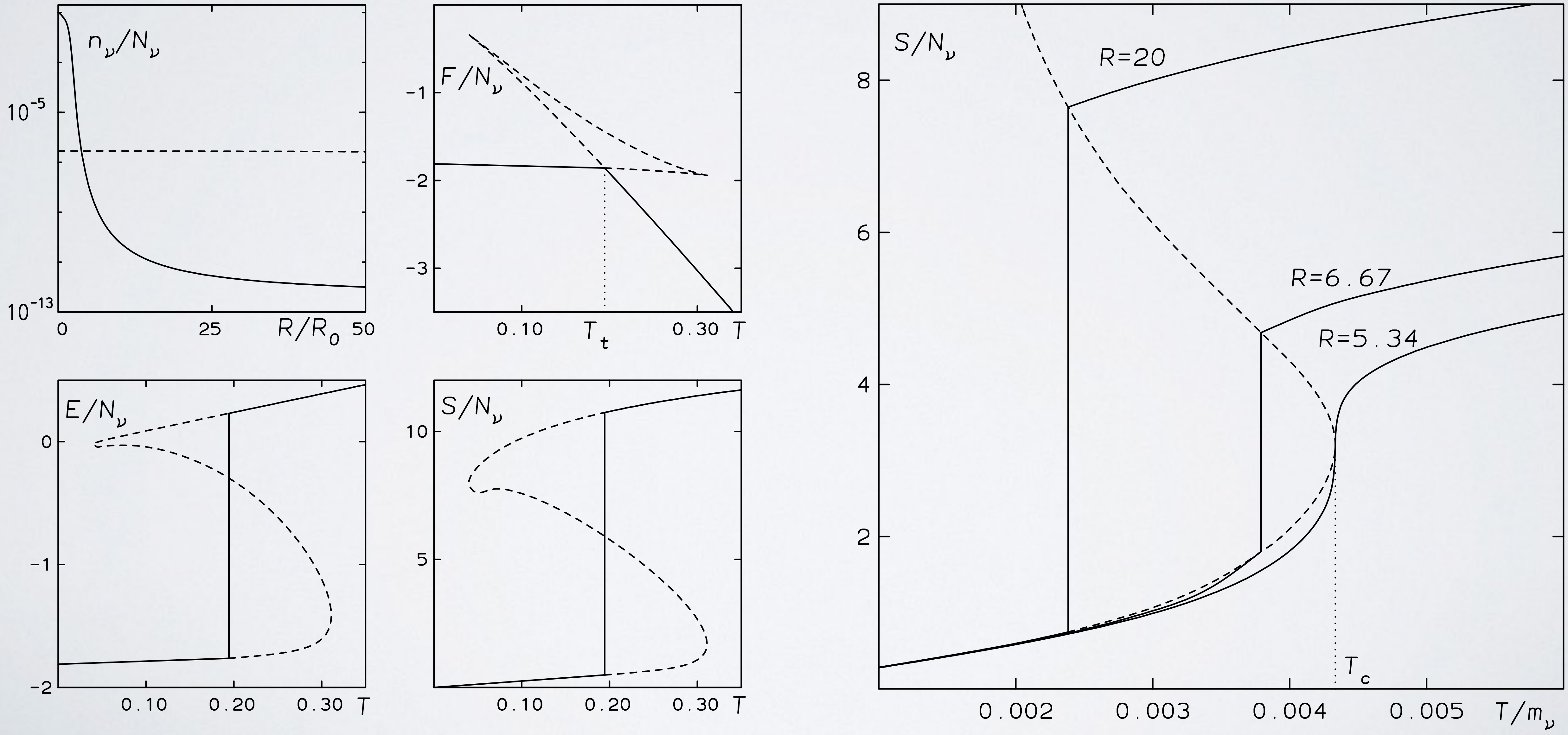
That’s all from data.

Then, a serious question is

Are degenerate fermionic galaxies physical?

PHASE TRANSITION TO DEGENERATE?

possible, because gravity is attractive [Hertel Thirring '71]



[Bilić Viollier '98]

PHASE SPACE DISTRIBUTIONS

[Navarro Eke Frenk ’96]

- For classical models (~maxwellian, intermediate momenta dominate)

- To be compared with degenerate FD

  Lower momenta, Denser.

Will the distribution collapse? How?

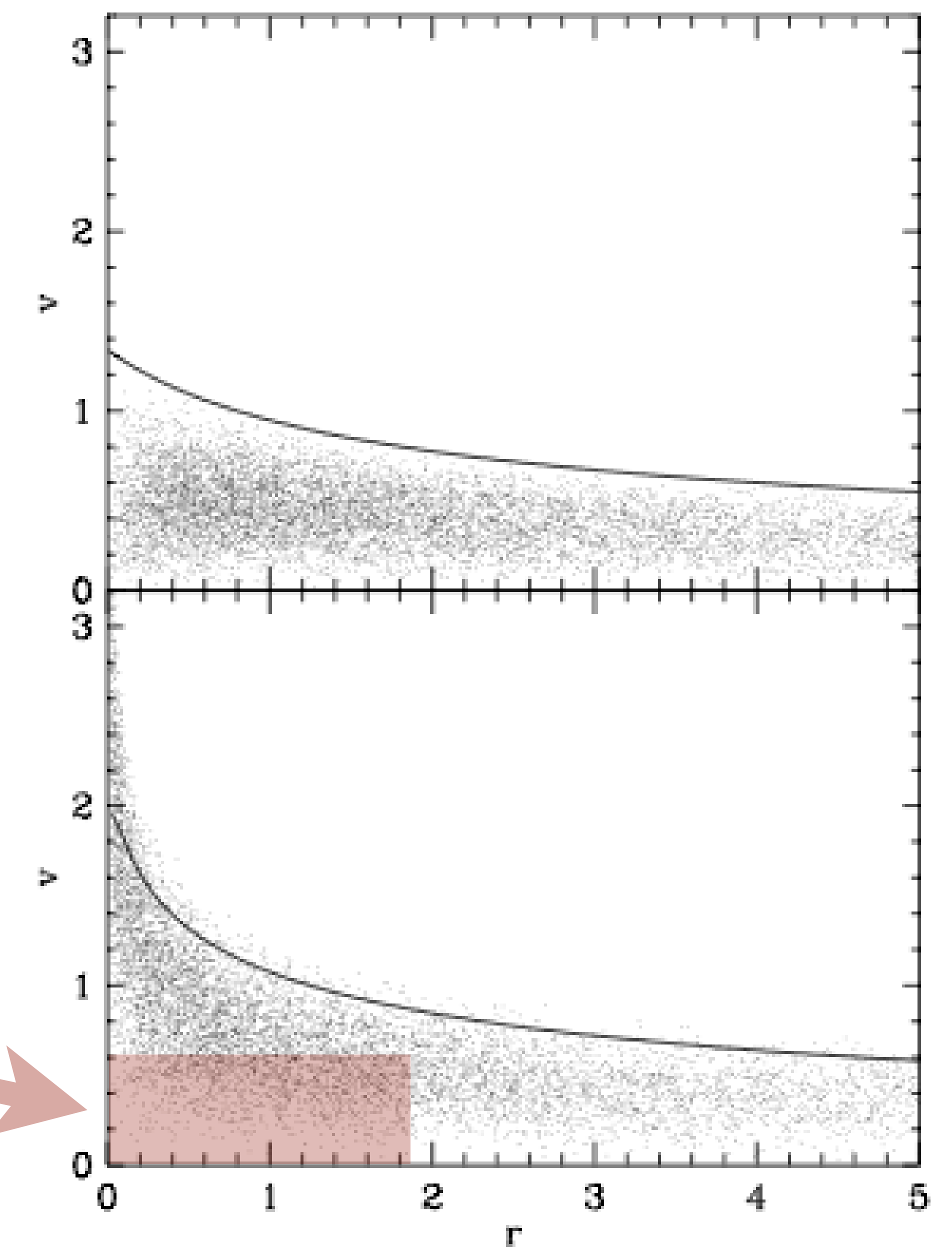
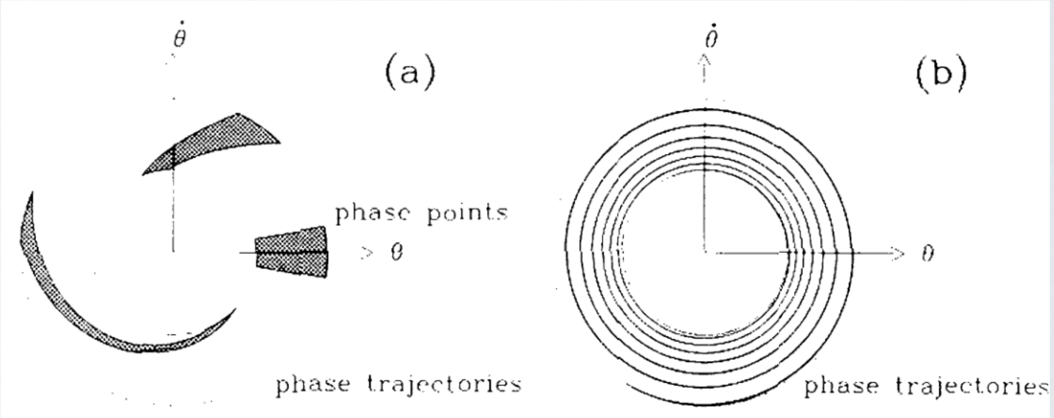


Figure 2. “Phase-space” distribution of halo particles before and after imposing the disk potential (upper and lower panel, repectively). The solid line in each panel shows the escape velocity

RELAXATION IN GALACTIC DYNAMICS

- **Encounters?** No - play a role only for few objects
(T = $T_{crossing}$ 0.1N/logN, here N~10^70, very large)
Thus, we are *collisionless*

- **Phase mixing?** *Relaxation for ignorance.*
Probably not relevant to get degeneration
(phase space has to be fully filled)



- **Violent relaxation?**
changes energy per unit mass (i.e. independent of mass)
(collision independent - assumes motion in a changing potential)

RELAXATION IN GALACTIC DYNAMICS

What about for fermions?

- Fermions interact only with near-fermi-surface states, so even reduced encounters? (and slow ones bounce off)

- Violent relaxation only possibility? (collisionless interaction) But are timescales of Potential variation sufficiently long? Still an open problem it seems [Chavanis '01-'03]

- BTW: violent relaxation leads to Fermi Dirac - like distribution, even for bosons...... [Lynden-Bell '67]
(thus, we may say it's compatible)

SO IS IT ACTUALLY REALIZED?

Favourable (free)energy budget necessary for phase transition, not sufficient.

Self-gravitating systems like DM halos are **intrinsically non equilibrium…**

So what matter are the timescales... **Relaxation, thermalization, evaporation. ?**

- **Fermionic jeans instability has lower k bound, degeneracy historically relevant** [Chavanis+ 1409xxxx]
- Ideal violent relaxation leads to core plus 1/r^2 [Lynden Bell '67]
  but incomplete violent relaxation can lead to large distance cutoff
  as also evaporation
- Simulations of classical violent relaxation lead to core plus 1/r^4
  [Henon '64;van Albada+ '82; Roy+ '04; Joyce+'09]
  due to thermalization + evaporation after core formation (but it appears to be slow?).

SO IS IT ACTUALLY REALIZED?

Favourable (free)energy budget necessary for phase transition, not sufficient.

Self-gravitating systems like DM halos are **intrinsically non equilibrium…**

So what matter are the timescales... **Relaxation, thermalization, evaporation. ?**

- **Fermionic jeans instability has lower k bound, degeneracy historically relevant** [Chavanis+ 1409xxxx]
- Ideal violent relaxation leads to core plus 1/r^2 [Lynden Bell '67]
  but incomplete violent relaxation can lead to large distance cutoff
  as also evaporation
- Simulations of classical violent relaxation lead to core plus 1/r^4 [Henon '64;van Albada+ '82; Roy+ '04; Joyce+'09]
  due to thermalization + evaporation after core formation (but it appears to be slow?).

Looking forward for quantum simulations?

STOP

- Quantum degenerate fermionic DM may avoid cusps in dwarfs
- Basic Thomas-Fermi approach

- Revisiting Tremaine-Gunn-like bound
  from existence of small galaxies:
  $m > 100\mathrm{eV}$
  even more challenging Direct Search? Lyman alpha?

  - Missing satellite problem:
    hint to upper bound $m < \text{few keV}$?

- Smallest galaxies are the frontier - confrontation with data hard dispersion anisotropy the main nuisance.

- Physics of fermionic galaxy formation the outstanding question

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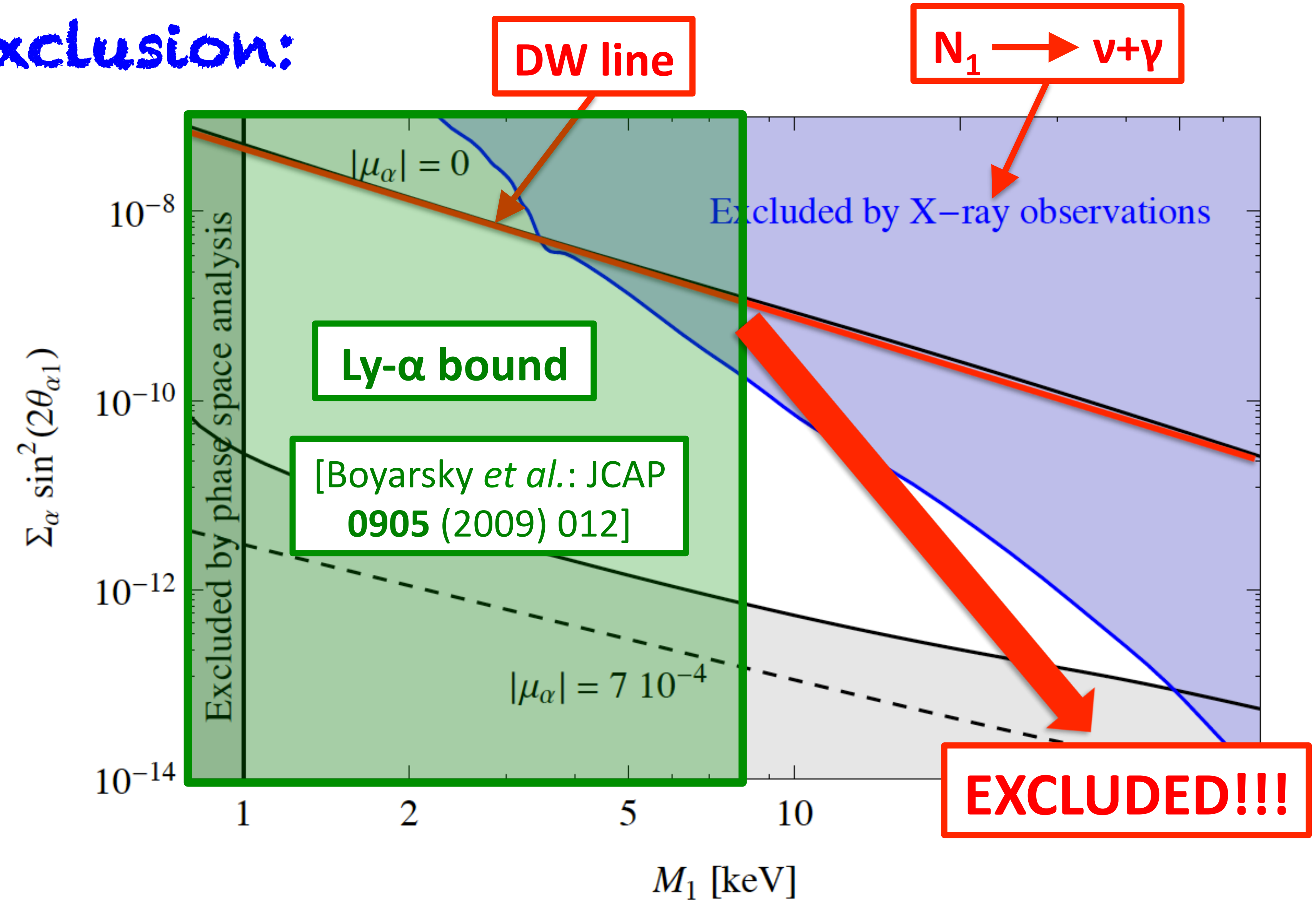
Thanks!

E.g.
Alexander Merle
ICTP, 2015

2. Production Mechanisms

Exclusion:



[Canetti *et al.*: Phys. Rev. **D87** (2013) 093006]

E.G. BULBUL+, 3.5 KEV ANALYSIS

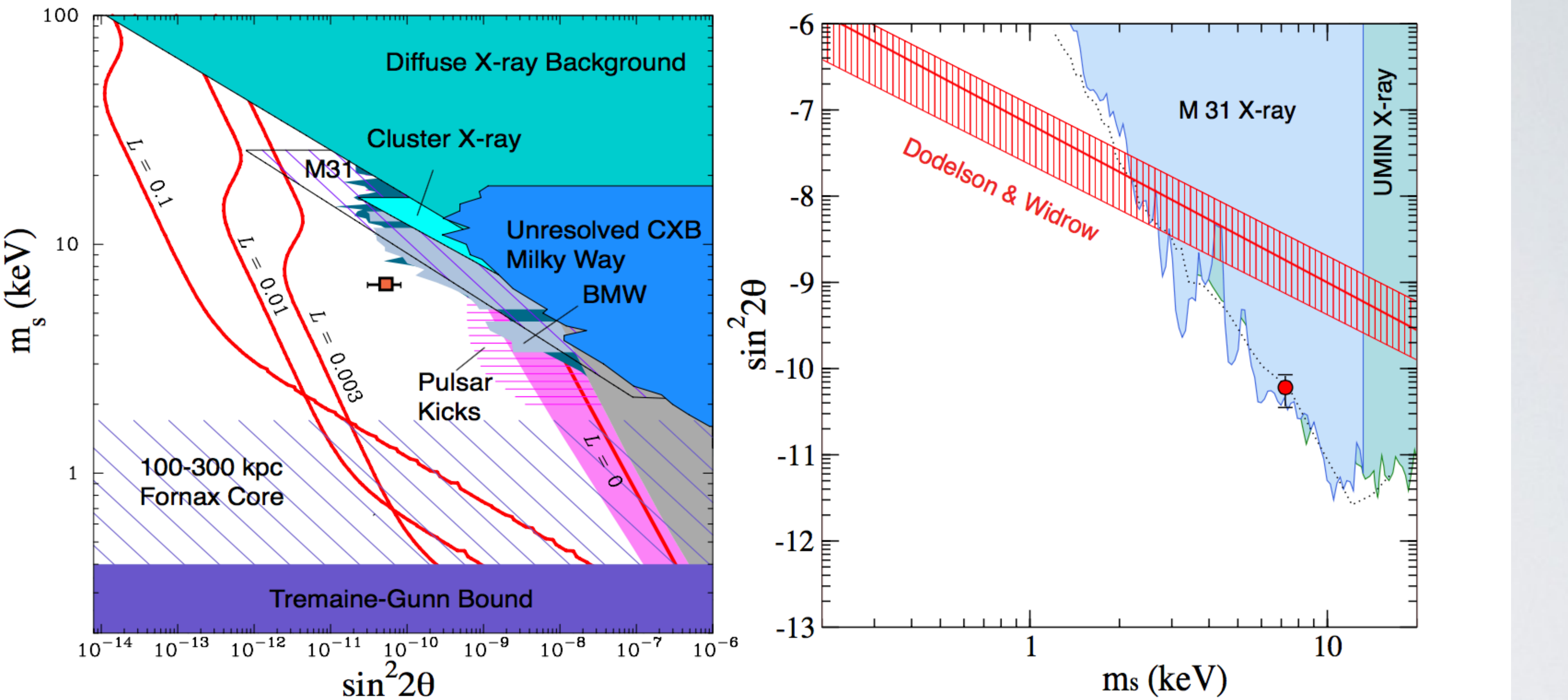


Figure 13. Constraints on the sterile neutrino model from the literature. Our full-sample MOS line detection (assuming that the line is from sterile neutrino and that all dark matter is in sterile neutrino) is shown by red symbols in both panels; error bar is statistical 90%. *Left:* historic constraints from Abazajian (2009). Red curves show theoretical predictions for the Dodelson-Widrow mechanism assuming sterile neutrinos constitute the dark matter with lepton numbers L=0, 0.003, 0.01, 0.1. See Abazajian (2009) for explanation of the various observational constraints that come from Tremaine & Gunn (1979); Bode et al. (2001); Boyarsky et al. (2006); Strigari et al. (2006); Abazajian et al. (2007). *Right:* most recent X-ray constraints (reproduced from Horiuchi et al. (2014)), based on deep *Chandra* (Horiuchi et al. 2014) and *XMM-Newton* (Watson et al. 2012) observations of M31 and *Suzaku* observations of Ursa Minor (Loewenstein et al. 2009). The red band marked "Dodenson & Widrow" is same as the $L = 0$ curve in left panel. Our measurement lays at the boundary of the constraints from M31.

DETECTION OF AN UNIDENTIFIED EMISSION LINE IN THE STACKED X-RAY SPECTRUM OF GALAXY CLUSTERS

Esra Bulbul[1,2], Maxim Markevitch[3], Adam Foster[1], Randall K. Smith[1] Michael Loewenstein[2,4], and Scott W. Randall[1]

SOME MEANING OF Σ_0 ?

Basic Virial estimate:

$$E_{grav} \sim -2\,E_{kin}$$

$$\frac{GM_h^2}{r_h} \sim Nmv^2 \sim M_h v^2 \sim M_h \sigma^2$$

Gives velocity dispersion

$$\sigma^2 \sim \frac{GM_h}{r_h}$$

But, for cored profile

$$\begin{cases} \rho_0 \sim M_h / r_h^3 \\ \sigma^2 \simeq P/\rho_0 \end{cases}$$

thus the observed constant surface density $\Sigma_0 \sim \frac{M_h}{r_h^2}$

implies a
Constant Pressure (?)

$$P \sim G\Sigma_0^2$$

THOMAS FERMI - DIMENSIONLESS

$$r = l_0\,\xi \quad , \quad \mu(r) = T_0\,\nu(\xi) \qquad y = p/\sqrt{2mT_0}$$

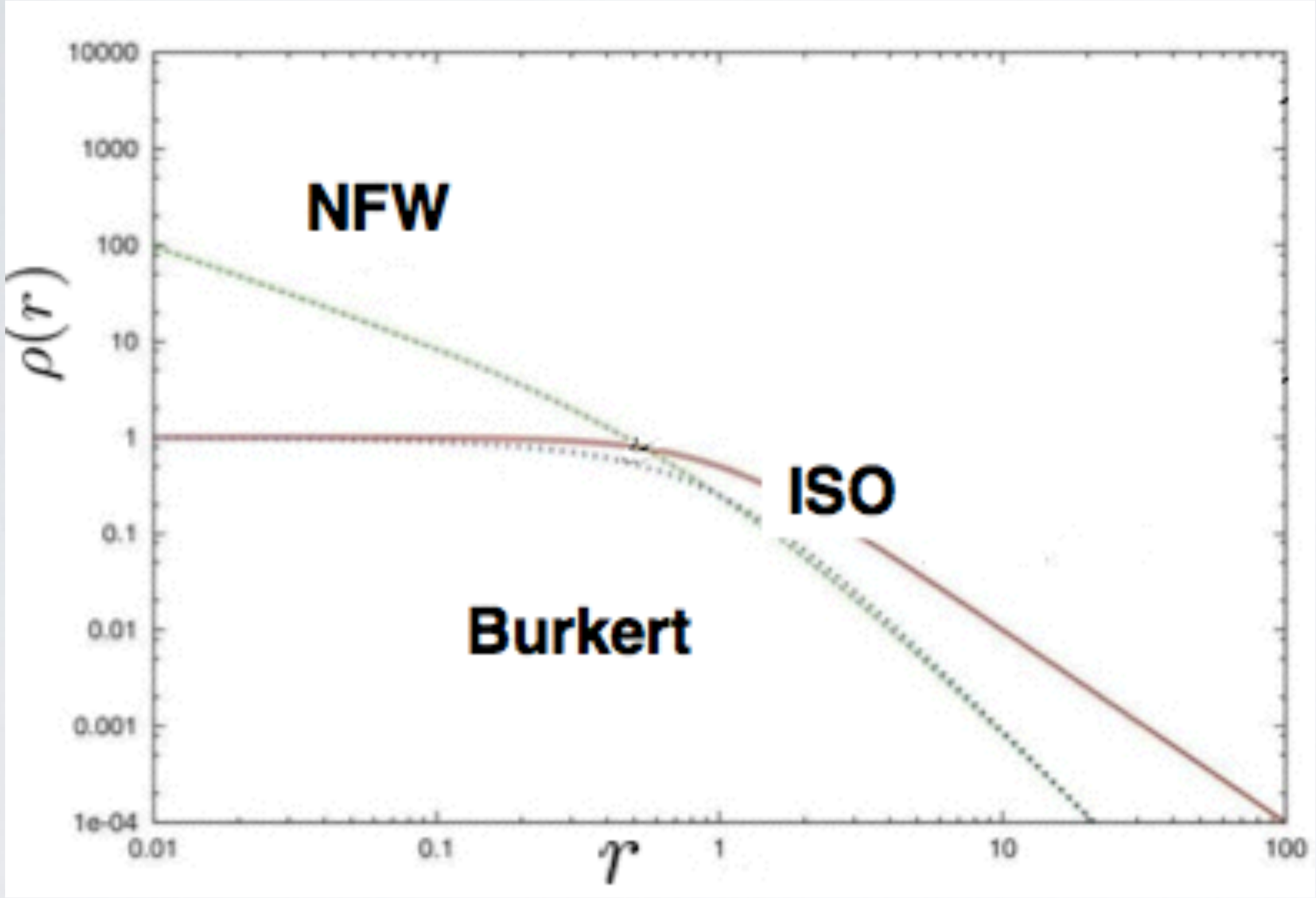
$$\frac{d^2\nu}{d\xi^2} + \frac{2}{\xi}\frac{d\nu}{d\xi} = -I_2(\nu) \qquad\qquad I_2(\nu) = 3\int_0^\infty y^2\,dy\,\Psi(y^2-\nu)$$

b.c. $\begin{cases} \nu'(0) = 0 \\ \nu_0 \end{cases}$ only one free parameter

determines the dimensionless potential $\nu(\xi)$

…and all solutions will be just rescalings.

REPRESENTATIVE DM PROFILES



NFW

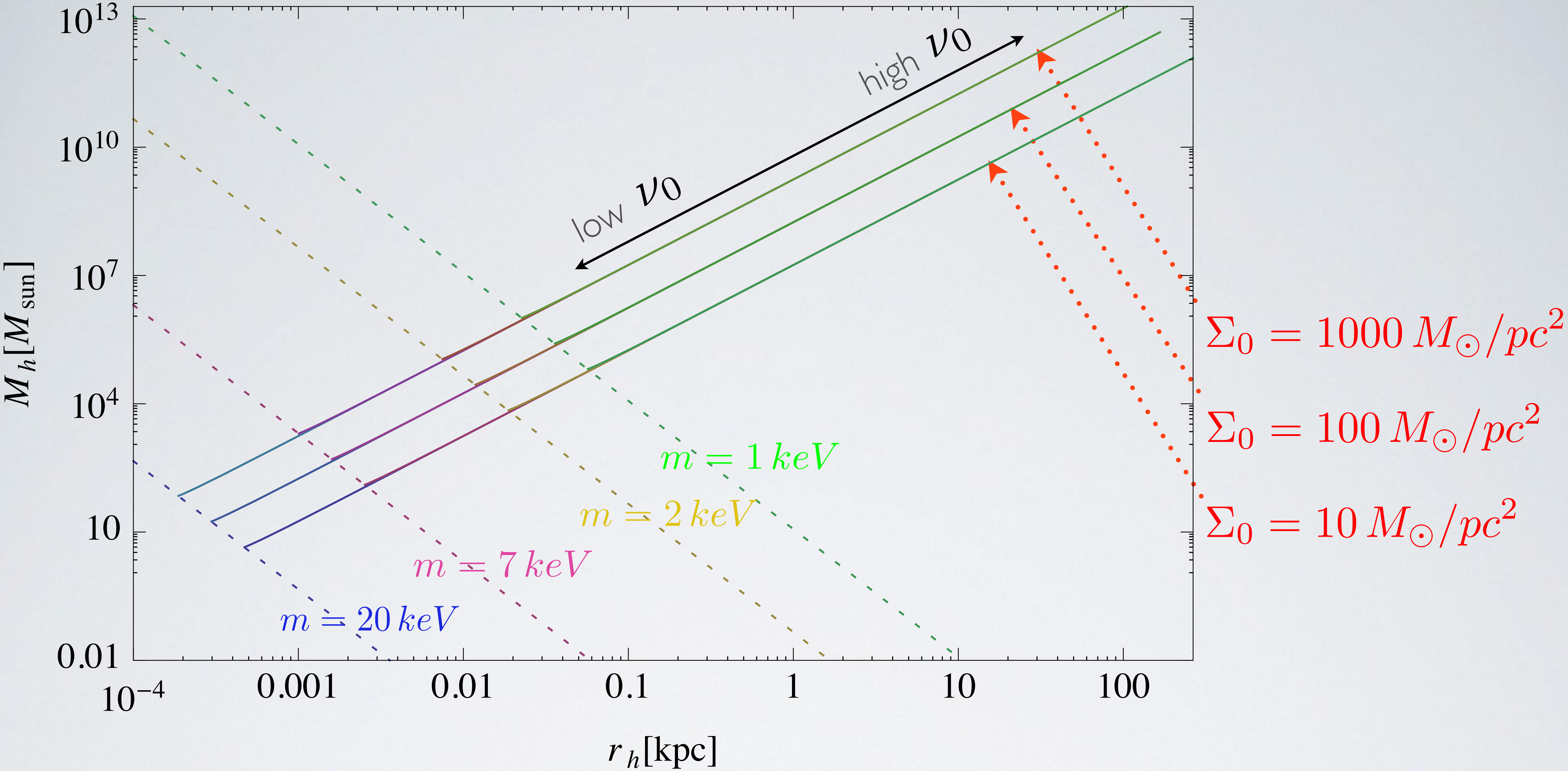
$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}(1+\frac{r}{R_s})^2}$$

Burkert

$$\rho(r) = \frac{\rho_0 r_h^3}{(r+r_h)(r^2+r_h^2)}$$

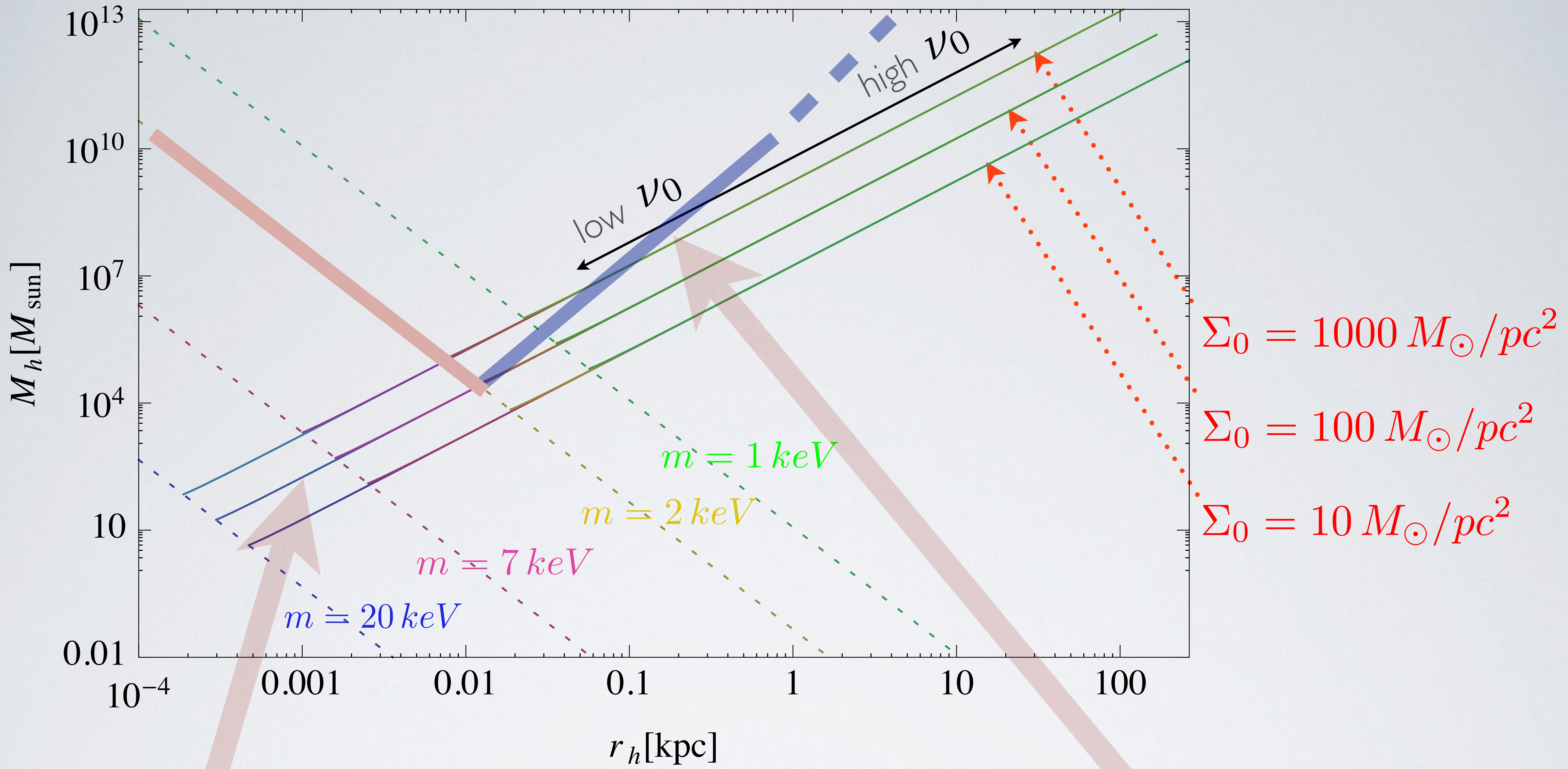
Isothermal

$$\rho(r) = \rho_0 \frac{r_h^2}{(r^2+r_h^2)}$$



$M_h \sim \Sigma_0 r_h^2$ continuous lines Σ_0 fixed

$M_h \sim \dfrac{\hbar^6}{G^3 m^8}\dfrac{1}{r_h^3}$ dashed, degeneration limit m fixed



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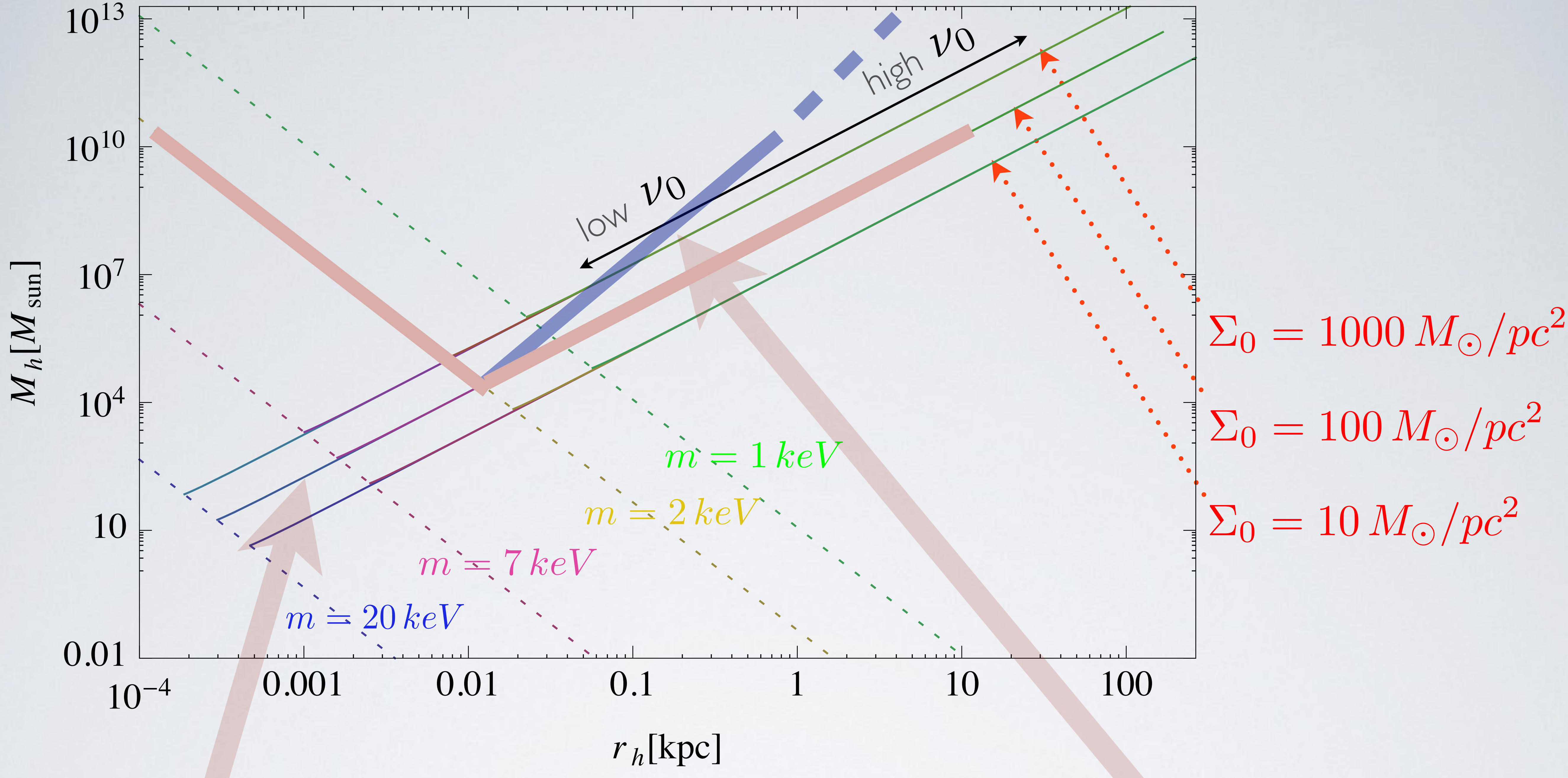
(Still, mistery of Σ_0 compare with neutron star, $M_h \sim R_h^3$)



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(Still, mistery of Σ_0 compare with neutron star, $M_h \sim R_h^3$)

THERMODYNAMIC QUANTITIES

- $f(E)$
- non relativistic DM

$$P(r) = \frac{g}{6\pi^2 m \hbar^3} \int_0^\infty dp\, p^4 f\left[\frac{p^2}{2m} - \mu(r)\right]$$

$$\sigma^2(r) = \frac{1}{3}\langle v^2 \rangle(r) = \frac{1}{3m^2} \frac{\int_0^\infty dp\, p^4 f\left[\frac{p^2}{2m} - \mu(r)\right]}{\int_0^\infty dp\, p^2 f\left[\frac{p^2}{2m} - \mu(r)\right]}$$

Thomas-Fermi automatically includes

1) $P(r) = \frac{1}{3}\langle v^2 \rangle(r)\, \rho(r) = \sigma^2(r)\rho(r)$ Local eq of state

2) $\frac{dP}{dr} + \rho(r)\frac{d\phi}{dr} = 0$ Idrostatic equilibrium (newtonian)

THOMAS FERMI - DIMENSIONLESS

$$r = l_0\,\xi \quad , \quad \mu(r) = T_0\,\nu(\xi) \qquad\qquad y = p/\sqrt{2mT_0}$$

$T_0 =$ temperature

$l_0 =$ characteristic length scale

$$l_0 \;=\; \frac{\hbar}{\sqrt{8G}}\left(\frac{2}{g}\right)^{\frac{1}{3}}\left[\frac{9\pi I_2(\nu_0)}{m^8\rho_0}\right]^{\frac{1}{6}} \;=$$

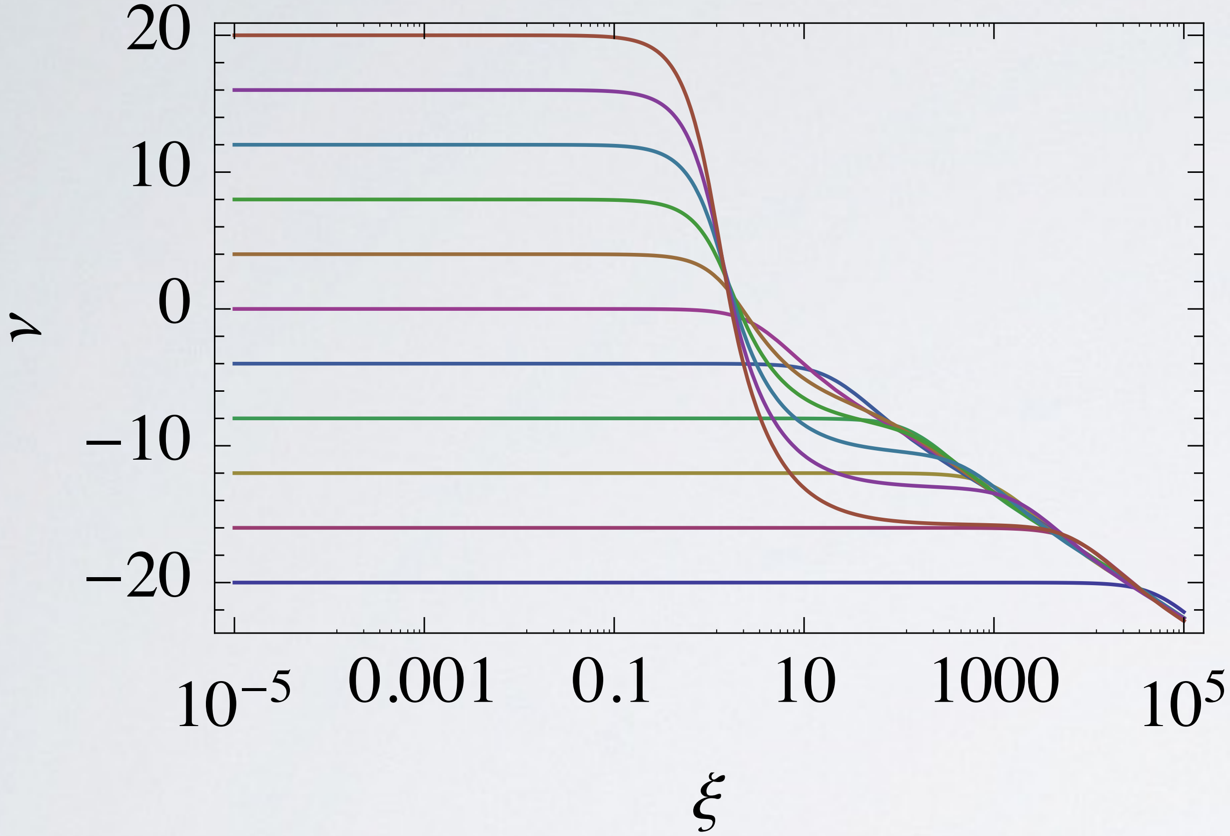
$$=\; R_0\left(\frac{2\,keV}{m}\right)^{\frac{4}{3}}\left(\frac{2}{g}\right)^{\frac{1}{3}}\left[\frac{I_2(\nu_0)}{\rho_0}\,\frac{M_\odot}{pc^3}\right]^{\frac{1}{6}} \qquad\qquad R_0 = 7.425\,pc$$

$$I_n(\nu) = (n+1)\int_0^\infty y^n\,dy\,\Psi(y^2-\nu) \;\;,\;\; n = 1, 2, ...$$

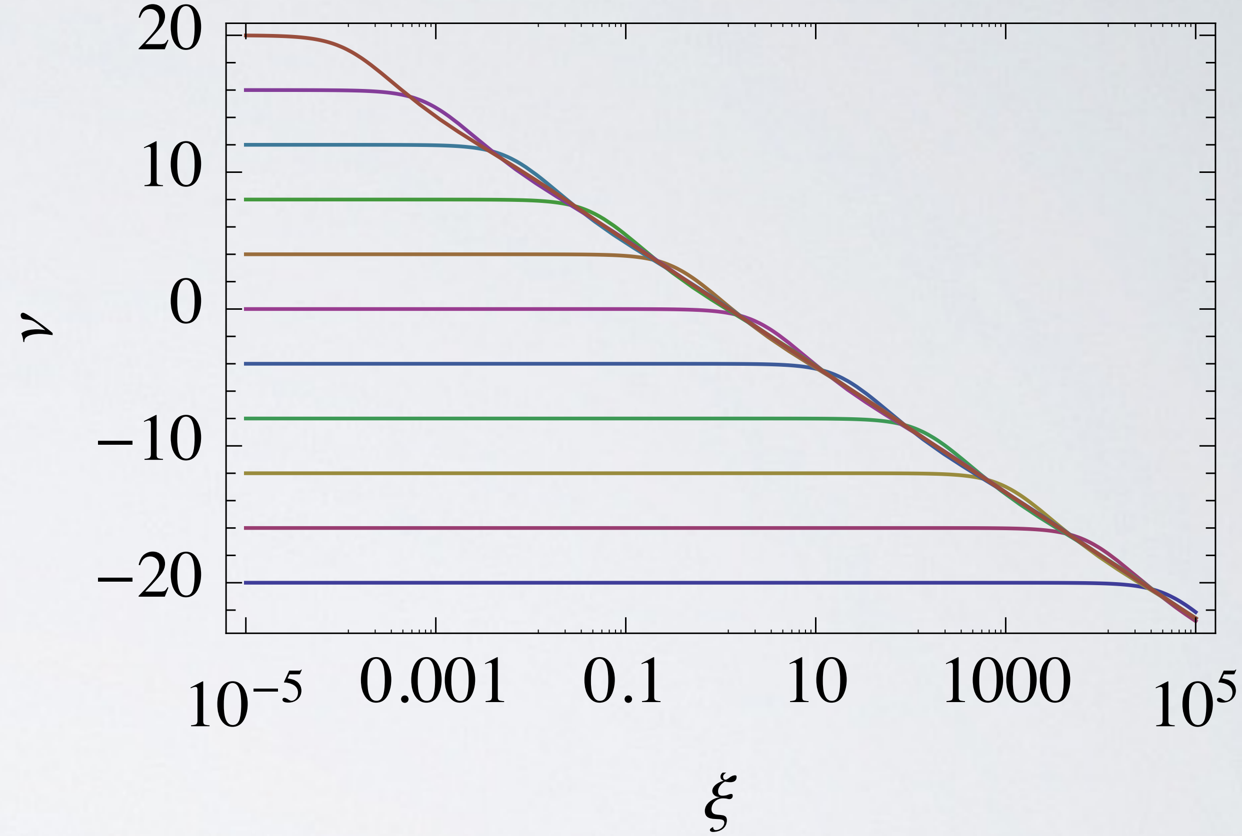
$g = 2$ for spin 1/2 particles

$\nu(\xi)$ associated to (chemical) potential

Fermi-Dirac



Maxwell-Boltzmann



$\nu_0 \lesssim -4$ ► quantum and classical regimes coincide

l_0

$\nu_0 \gtrsim -4$ ► degeneration starts to appear

Back to physical units

$$\begin{cases} \nu(\xi) & \nu'(\xi) \\ I_2(\nu(\xi)) & I_4(\nu(\xi)) \end{cases} \quad \blacktriangleright \quad \rho(r),\ M(r)\ ,\ \sigma(r),\ P(r)$$

$$\rho(r) = \frac{m^{\frac{5}{2}}}{3\pi^2\hbar^3}(2T_0)^{\frac{3}{2}} I_2(\nu(\xi)) = \rho_0 \frac{I_2(\nu(\xi))}{I_2(\nu_0)}$$

$$\rho_0 = \frac{m^{\frac{5}{2}}}{3\pi^2\hbar^3}(2T_0)^{\frac{3}{2}} I_2(\nu_0)$$ connection between ρ_0 and T_0

three free parameters $\nu_0 \quad m \quad T_0$

to be traded for

$\nu_0 \quad m \quad \rho_0$

Pressure

$$P(r) = \frac{m^{\frac{3}{2}}}{15\pi^2\hbar^3}(2T_0)^{\frac{5}{2}} I_4(\nu(\xi)) = \frac{1}{5}\,(9\pi^4)^{\frac{1}{3}}\left(\frac{\hbar^6}{m^8}\right)^{\frac{1}{3}}\left[\frac{\rho_0}{I_2(\nu_0)}\right]^{\frac{5}{3}} I_4(\nu(\xi))$$

Velocity dispersion

$$\sigma^2(r) = \frac{P(r)}{\rho(r)} = \frac{2T_0}{5m}\frac{I_4(\nu(\xi))}{I_2(\nu(\xi))}$$

nontrivial dependence only on ν_0

Enclosed mass

$$M(r) = 4\pi\int_0^r r^2 dr\rho(r) = 4\pi\frac{\rho_0 l_0^3}{I_2(\nu_0)}\int_0^\xi dx\, x^2 I_2(\nu(x))$$

$$= 4\pi\frac{\rho_0 l_0^3}{I_2(\nu_0)}\xi^2\,|\nu'(\xi)|$$

$$= M_0\xi^2|\nu'(\xi)|\left(\frac{2\,keV}{m}\right)^4\sqrt{\frac{\rho_0}{I_2(\nu_0)}\frac{pc^3}{M_\odot}}$$

with $M_0 = 4\pi M_\odot\left(\frac{R_0}{pc}\right)^3 = 5.144\ 10^3\ M_\odot$

Finally we wish to use surface density

$$\begin{cases} \rho(r_h) = \dfrac{\rho_0}{4} & \text{Burkert-like definition of } r_h \\ \Sigma_0 = r_h \rho_0 & \text{surface density} \end{cases}$$

$\nu_0 \quad m \quad \rho_0$ ► $\nu_0 \quad m \quad \Sigma_0$

$$l_0 = \left(\frac{9\pi}{2^9}\right)^{\frac{1}{5}} \left(\frac{\hbar^6}{G^3 m^8}\right)^{\frac{1}{5}} \left[\frac{\xi_h I_2(\nu_0)}{\Sigma_0}\right]^{\frac{1}{5}} = 4.2557\,[\xi_h I_2(\nu_0)]^{\frac{1}{5}} \left(\frac{2\,keV}{m}\right)^{\frac{8}{5}} \left(\frac{120\,M_\odot}{\Sigma_0\,pc^2}\right)^{\frac{1}{5}} pc$$

$$T_0 = \left(18\pi^6 \frac{\hbar^6 G^2}{m^3}\right)^{\frac{1}{5}} \left[\frac{\Sigma_0}{\xi_h I_2(\nu_0)}\right]^{\frac{4}{5}} = \frac{7.12757\,10^{-3}}{[\xi_h I_2(\nu_0)]^{\frac{4}{5}}} \left(\frac{2\,keV}{m}\right)^{\frac{3}{5}} \left(\frac{\Sigma_0\,pc^2}{120\,M_\odot}\right)^{\frac{4}{5}} K$$

$$r = l_0 \xi = 4.2557\,\xi\,[\xi_h I_2(\nu_0)]^{\frac{1}{5}} \left(\frac{2\,keV}{m}\right)^{\frac{8}{5}} \left(\frac{120\,M_\odot}{\Sigma_0\,pc^2}\right)^{\frac{1}{5}} pc$$

$$r_h = l_0 \xi_h$$

$$\rho(r) = \left(\frac{2^9 G^3 m^8}{9\pi\hbar^6}\right)^{\frac{1}{5}} \left[\frac{\Sigma_0}{\xi_h I_2(\nu_0)}\right]^{\frac{6}{5}} I_2(\nu(\xi))$$

$$= 28.1967 \frac{I_2(\nu(\xi))}{[\xi_h I_2(\nu_0)]^{\frac{6}{5}}} \left(\frac{m}{2\,keV}\right)^{\frac{8}{5}} \left(\frac{\Sigma_0\, pc^2}{120\, M_\odot}\right)^{\frac{6}{5}} \frac{M_\odot}{pc^3}$$

$$M(r) = 4\pi \left(\frac{9\pi\hbar^6}{2^9 G^3 m^8}\right)^{\frac{2}{5}} \left[\frac{\Sigma_0}{\xi_h I_2(\nu_0)}\right]^{\frac{3}{5}} \xi^2 |\nu'(\xi)|$$

$$= \frac{27312\, \xi^2}{[\xi_h I_2(\nu_0)]^{\frac{3}{5}}} |\nu'(\xi)| \left(\frac{2\,keV}{m}\right)^{\frac{16}{5}} \left(\frac{\Sigma_0\, pc^2}{120\, M_\odot}\right)^{\frac{3}{5}} M_\odot$$

$$\sigma^2(r) = \frac{11.0402}{[\xi_h I_2(\nu_0)]^{\frac{4}{5}}} \frac{I_4(\nu(\xi))}{I_2(\nu(\xi))} \left(\frac{2\,keV}{m}\right)^{\frac{8}{5}} \left(\frac{\Sigma_0\, pc^2}{120\, M_\odot}\right)^{\frac{4}{5}} \left(\frac{km}{s}\right)^2$$

$$P(r) = \frac{8\pi}{5} G \left[\frac{\Sigma_0}{\xi_h I_2(\nu_0)}\right]^2 I_4(\nu(\xi)) \quad = \frac{311.310}{[\xi_h I_2(\nu_0)]^2} I_4(\nu(\xi)) \left(\frac{\Sigma_0\, pc^2}{120\, M_\odot}\right)^2 \frac{M_\odot}{pc^3} \left(\frac{km}{s}\right)^2$$

in general, or in classical regime

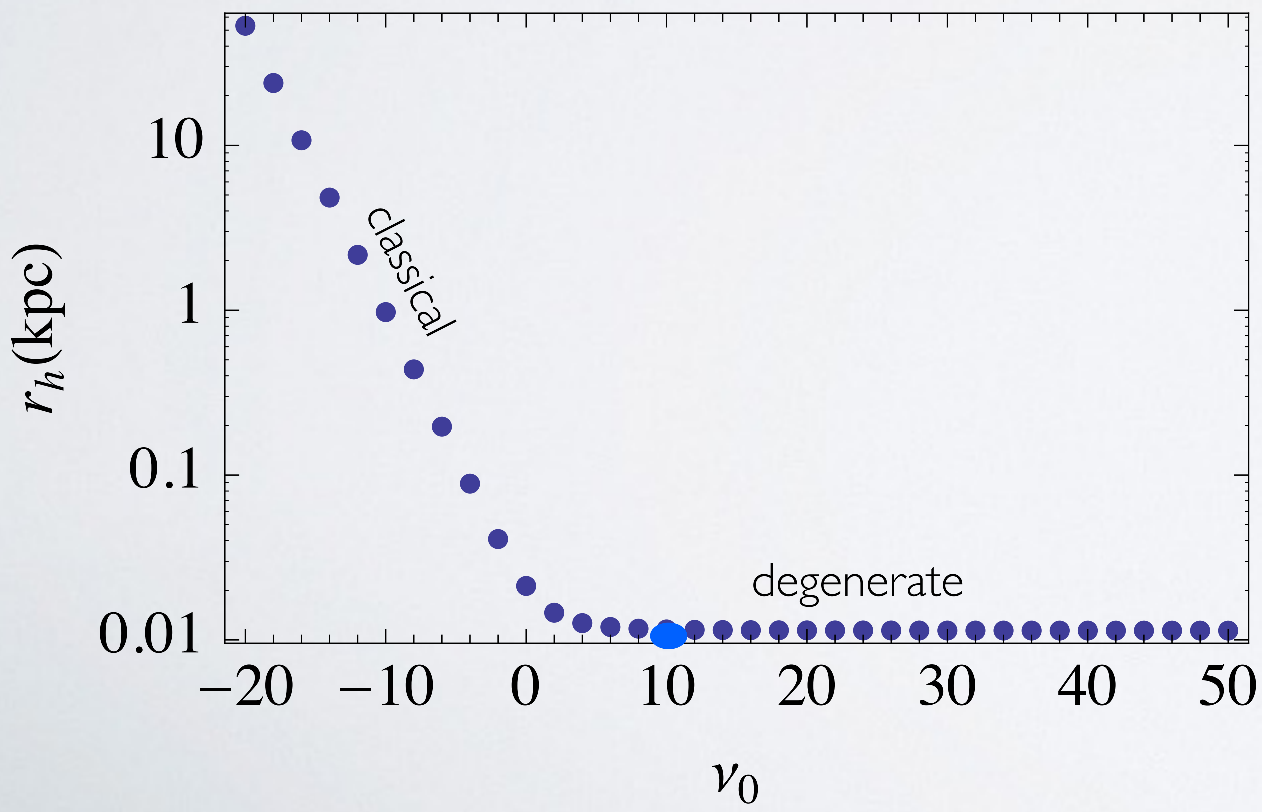
3 parameters

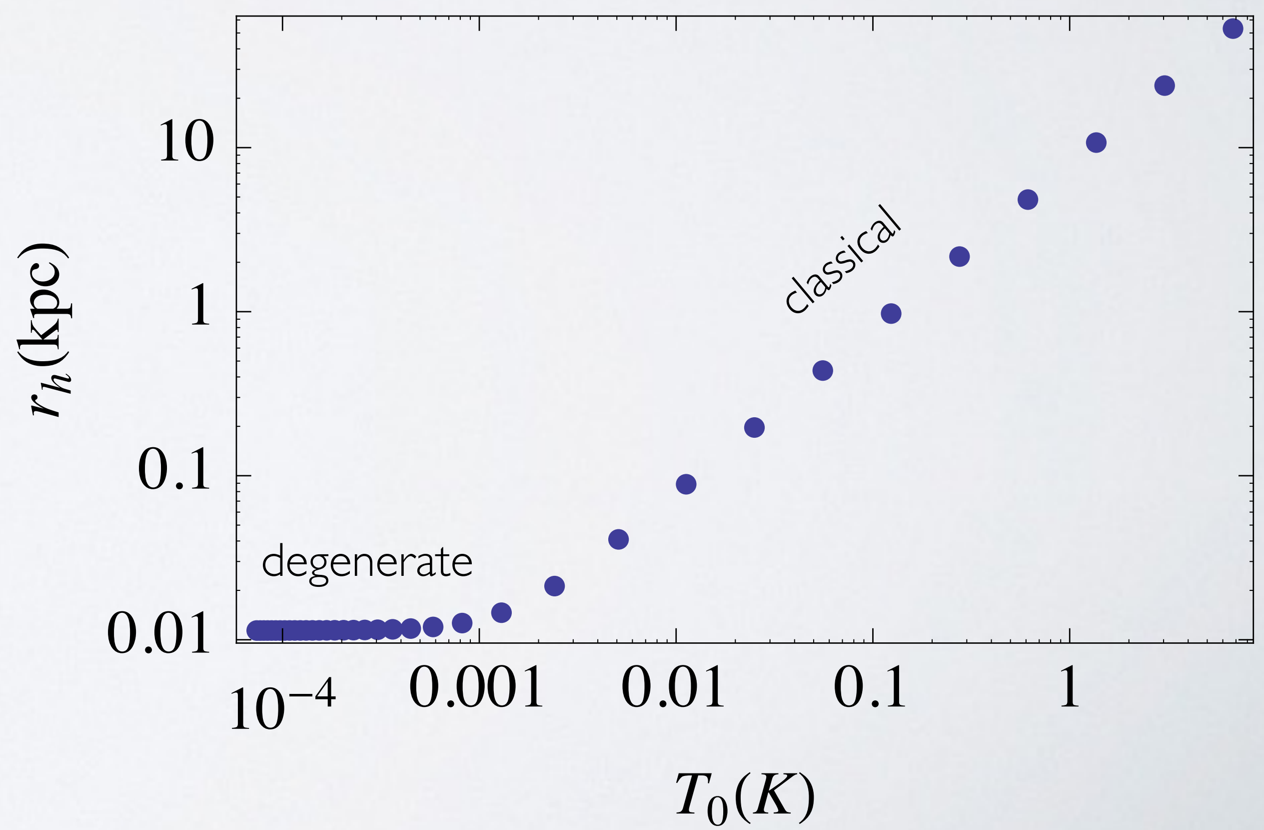
quantum degenerate limit

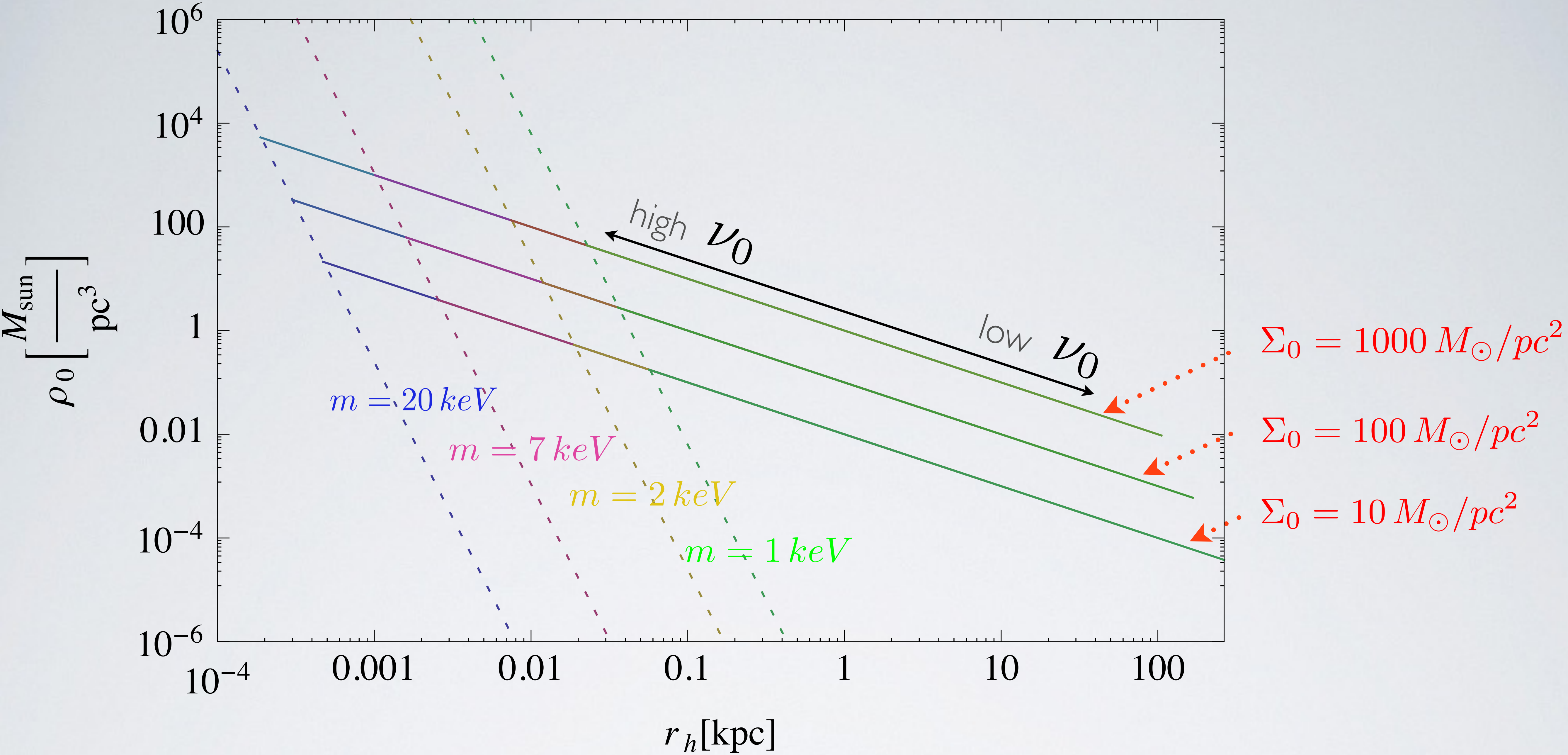
2 parameters

$\nu_0 \quad m \quad \Sigma_0 \quad \rightarrow \quad m \quad \Sigma_0$

$T_0 \quad m \quad M_h \quad \rightarrow \quad m \quad M_h$





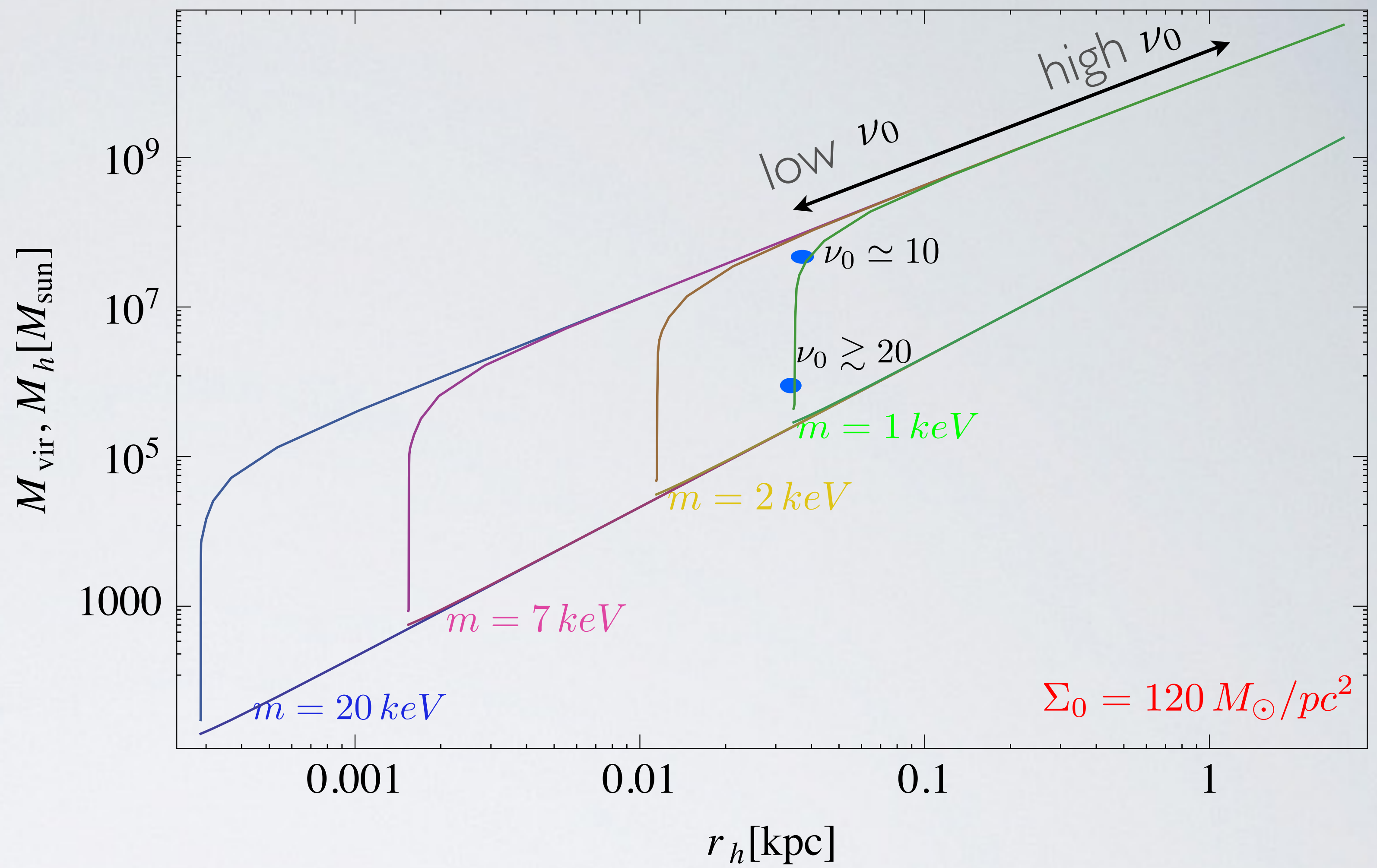


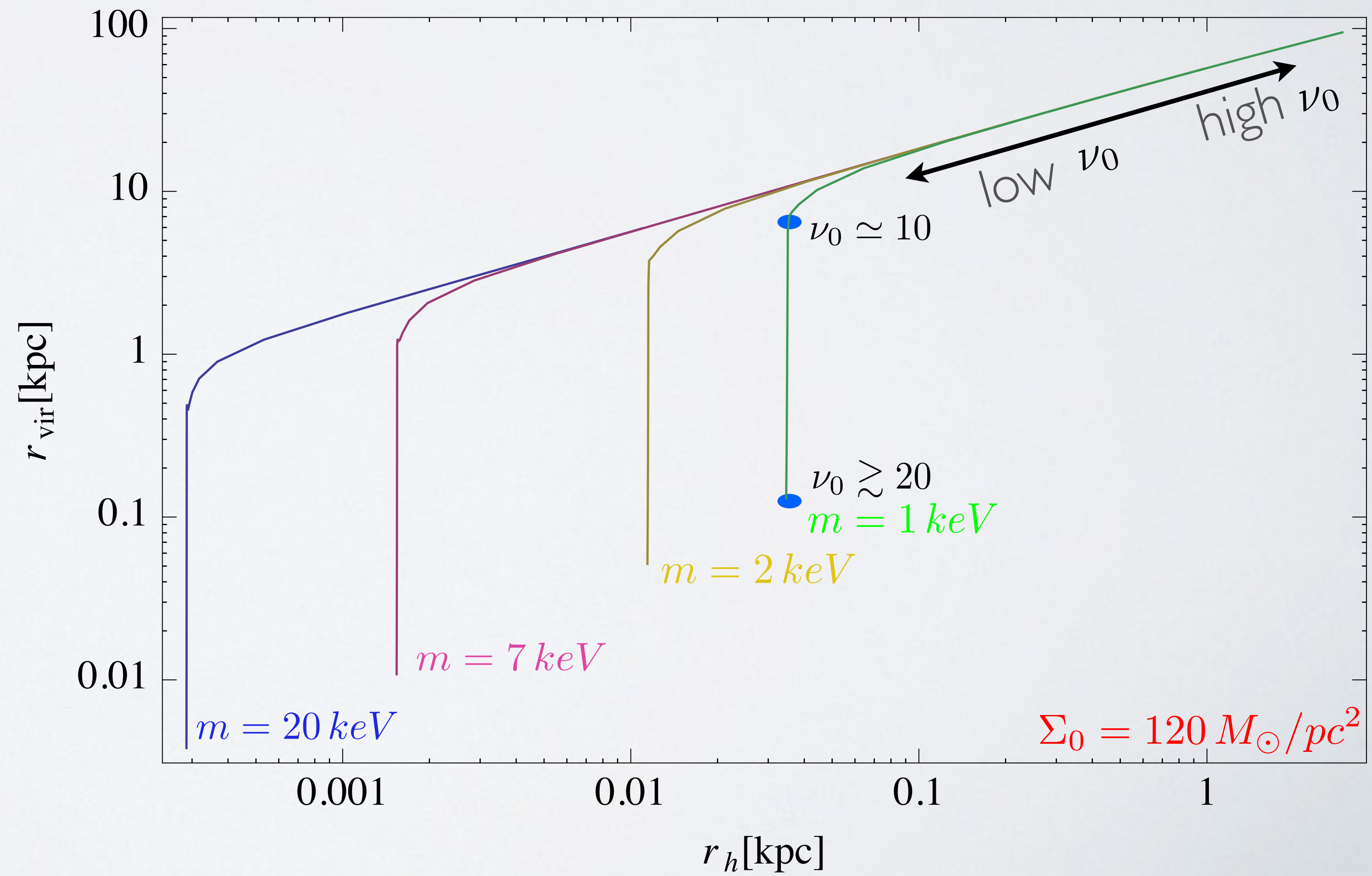
$\rho_0 = \frac{\Sigma_0}{r_h}$	continuous lines	Σ_0 fixed
$\rho_0 \sim \frac{\hbar^6}{G^3 m^8} \frac{1}{r_h^6}$	dashed, degeneration limit	m fixed

VIRIAL MASS & RADIUS

M_{vir}

r_{vir}





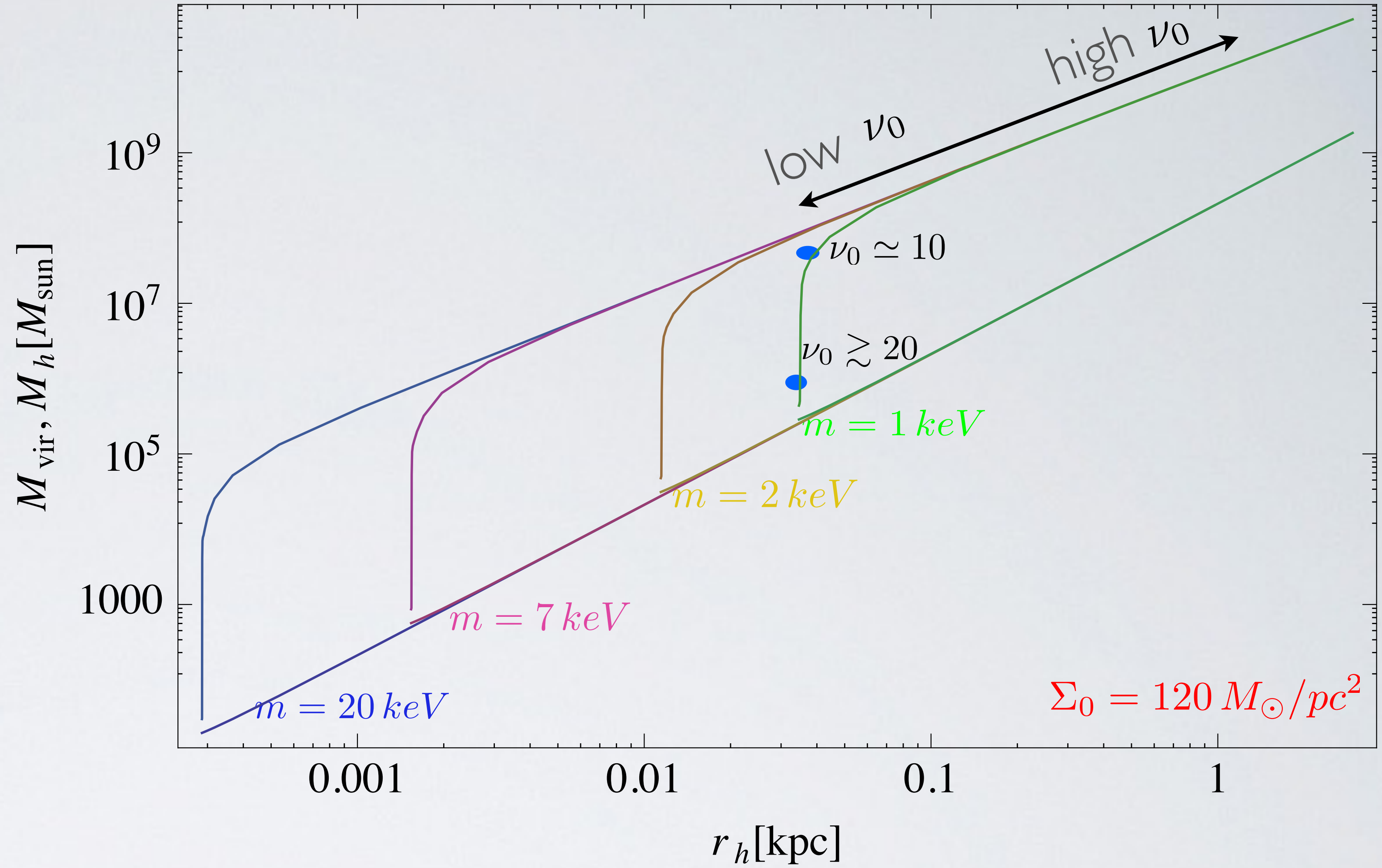
VIRIAL MASS & RADIUS

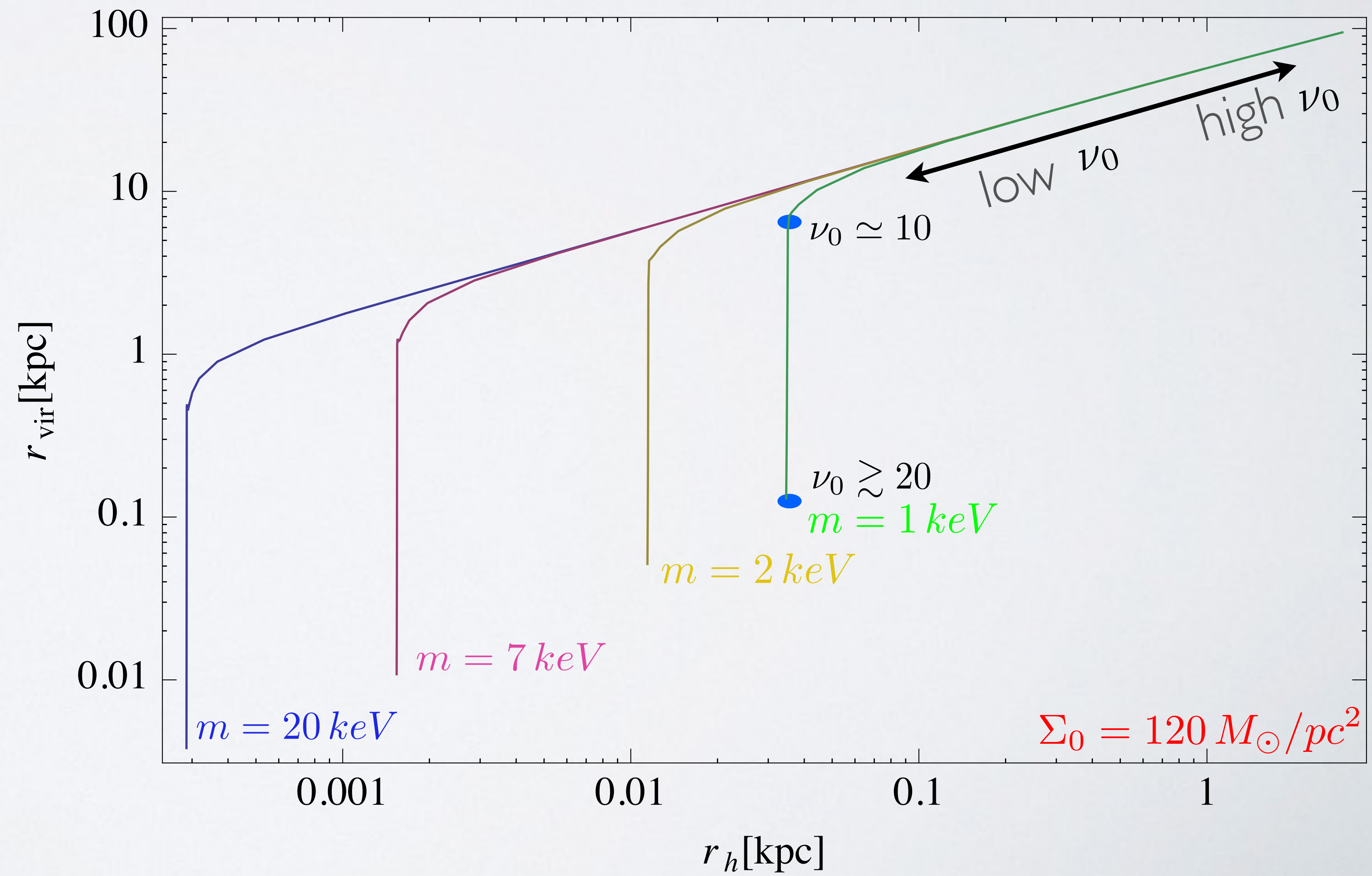
$M_{\rm vir}$

$r_{\rm vir}$

both could identify m

...unfortunately, not directly measurable.





VIRIAL MASS & RADIUS

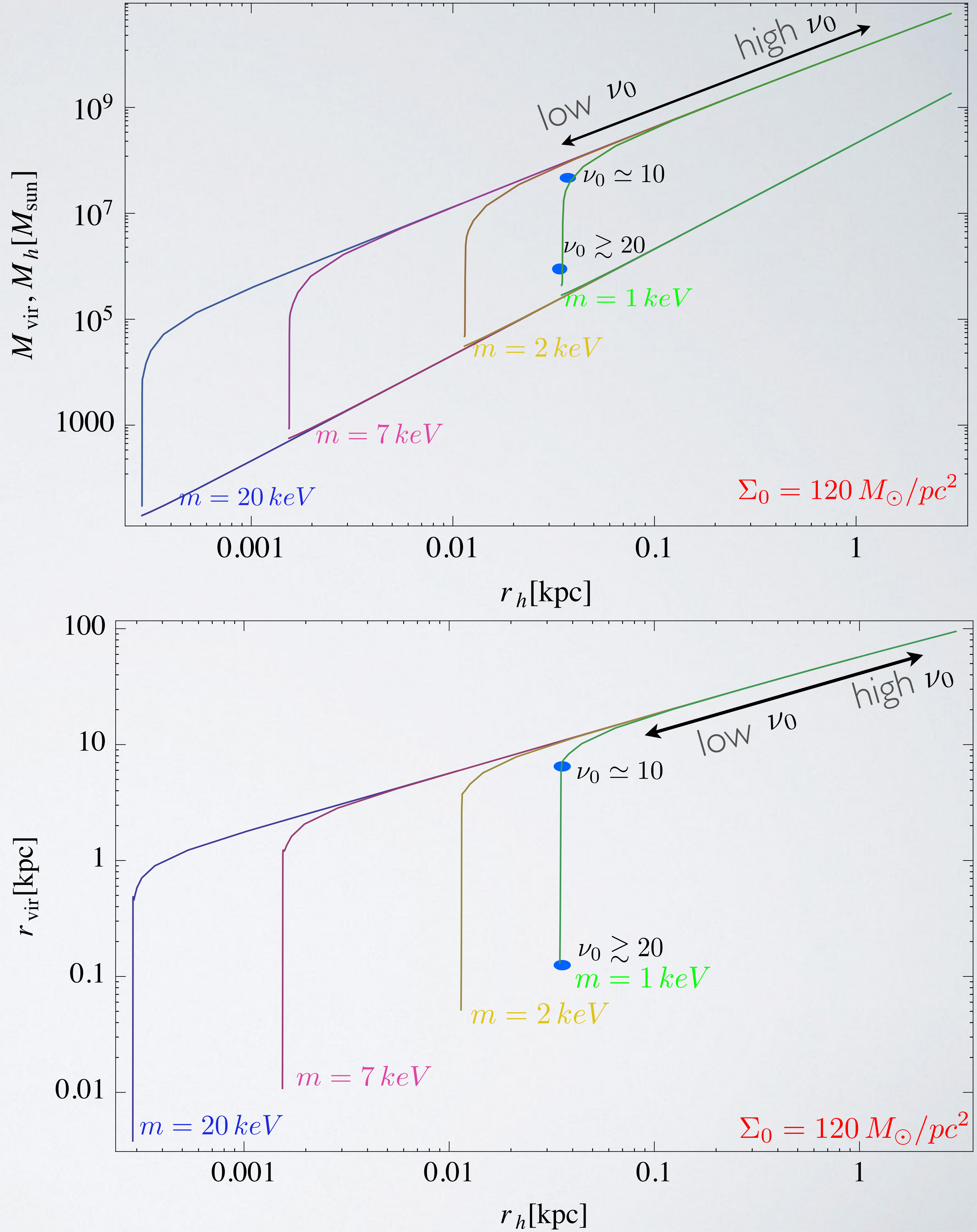
$M_{\rm vir}$

$r_{\rm vir}$

both could identify m

...unfortunately, not directly measurable.

Still, prediction:
a lower limit on total mass



so Bimodal profile of dispersion velocity
inside and outside the core,
for ***degenerate galaxies***

a) Quantum drives the dispersion in the core $\sigma = \frac{\hbar}{m^{\frac{4}{3}}}\rho_0^{\frac{1}{3}}$

b) Classical dispersion in the halo: $\sigma = \sqrt{T_0/m}$

Recall, small m galaxies are more degenerate

by looking at profiles - could we find m ?