Gravitational wave data analysis




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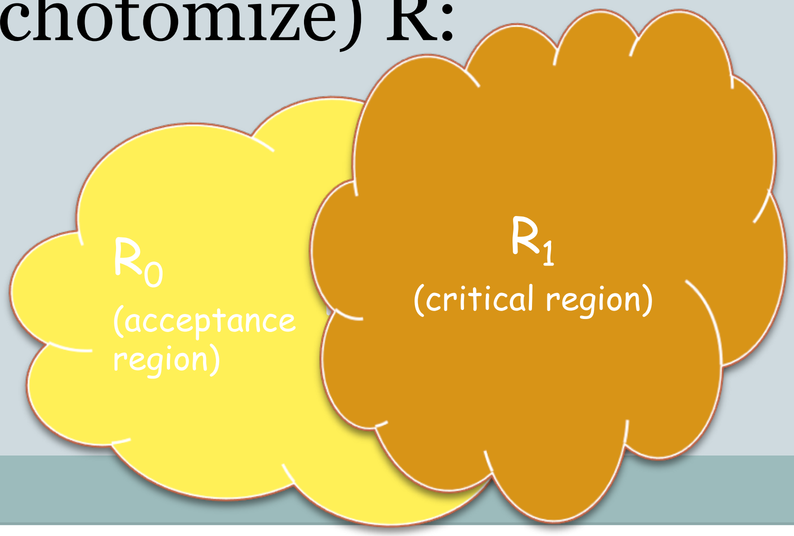
This presentation

- Aim: make it possible for you to read and understand the observational papers of LIGO and Virgo.

  - Very brief basics of signal detection
  - Searches for compact binary inspiral signals

Hypothesis testing

- Our data $\{y_i\} \in R$
- Consider a detection problem: H_0 signal absent, H_1 signal present
- Question: has hypothesis H_0 or H_1 produced our data ?
- Deciding means finding a way to partition (dichotomize) R:



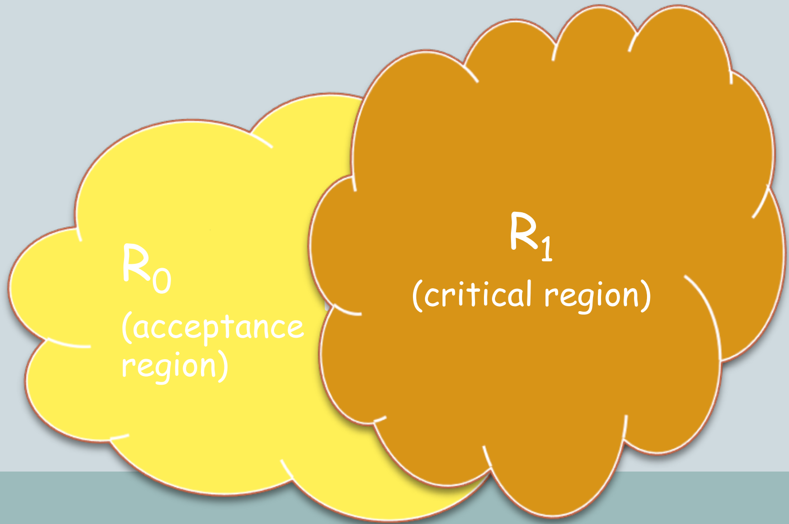
if $\{y_i\} \in R_0 \rightarrow D_0$

if $\{y_i\} \in R_1 \rightarrow D_1$

Hypothesis testing- types of errors

- Type I error: decide D_1, when H_0 holds
  - False alarm probability, $P_{fa}=P(D_1|H_0)$, size of the test

- Type II errors:decide D_0, when H_1 holds
  - False dismissal probability, $P_{fd}=P(D_0|H_1)$, $1-P_{fd}$ is the power of the test



if $\{y_i\} \in R_0 \rightarrow D_0$

if $\{y_i\} \in R_1 \rightarrow D_1$

Neymann-Pearson criterium

- The decision should be such that at fixed P_{fa} the P_{fd} is the smallest.

- It can be demonstrated that the corresponding partition is any level surface of a function of the data called the likelihood:

$$\Lambda(y) = \frac{p_1(y)}{p_0(y)}$$

prob given H_1

prob given H_0

Neymann-Pearson criterium

- The specific level surface that one takes depends on convenience and defines the detection statistic.
- The partition is a threshold on the detection statistic that determines the P_{fa} and the P_{fd} .

$$R_1 \supset y \mid \Lambda(y) > \Lambda^*$$

$$P\left(\Lambda(y) > \Lambda^* \mid H_0\right) = P_{fa}$$

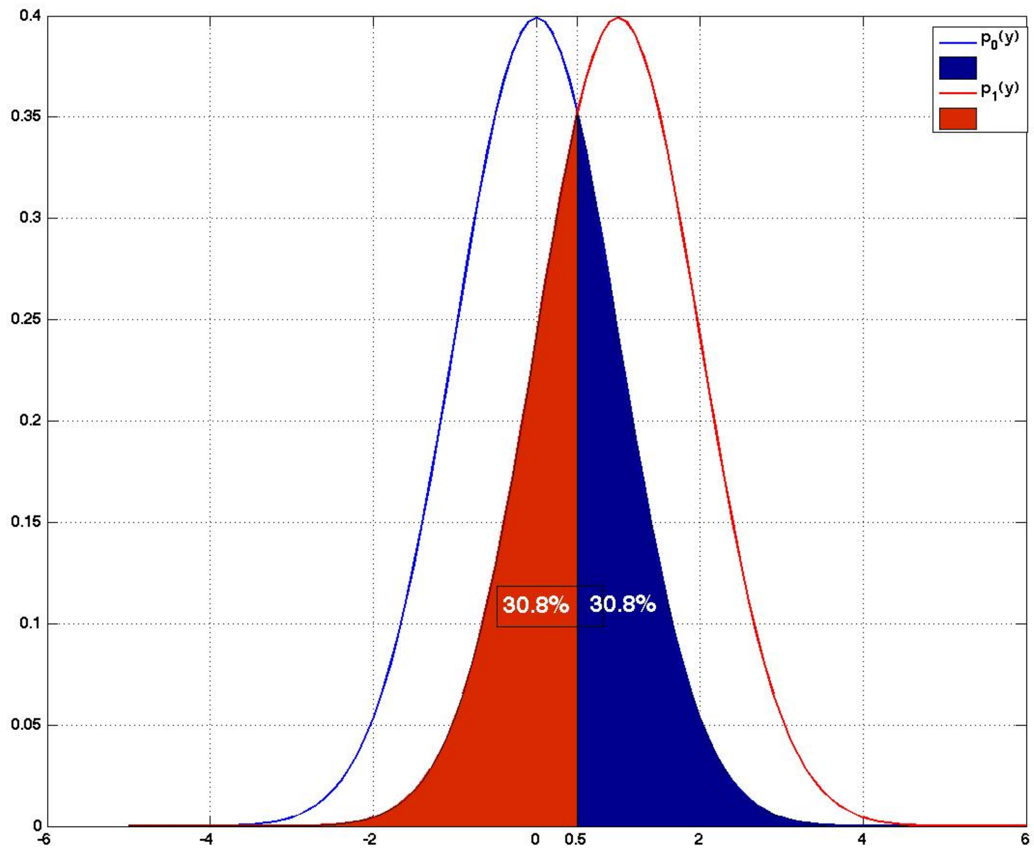
A very simple example

- Consider a single measurement
- y=n+s
  - n is Gaussian noise, zero mean and unit variance
  - s=1 is a constant, our signal

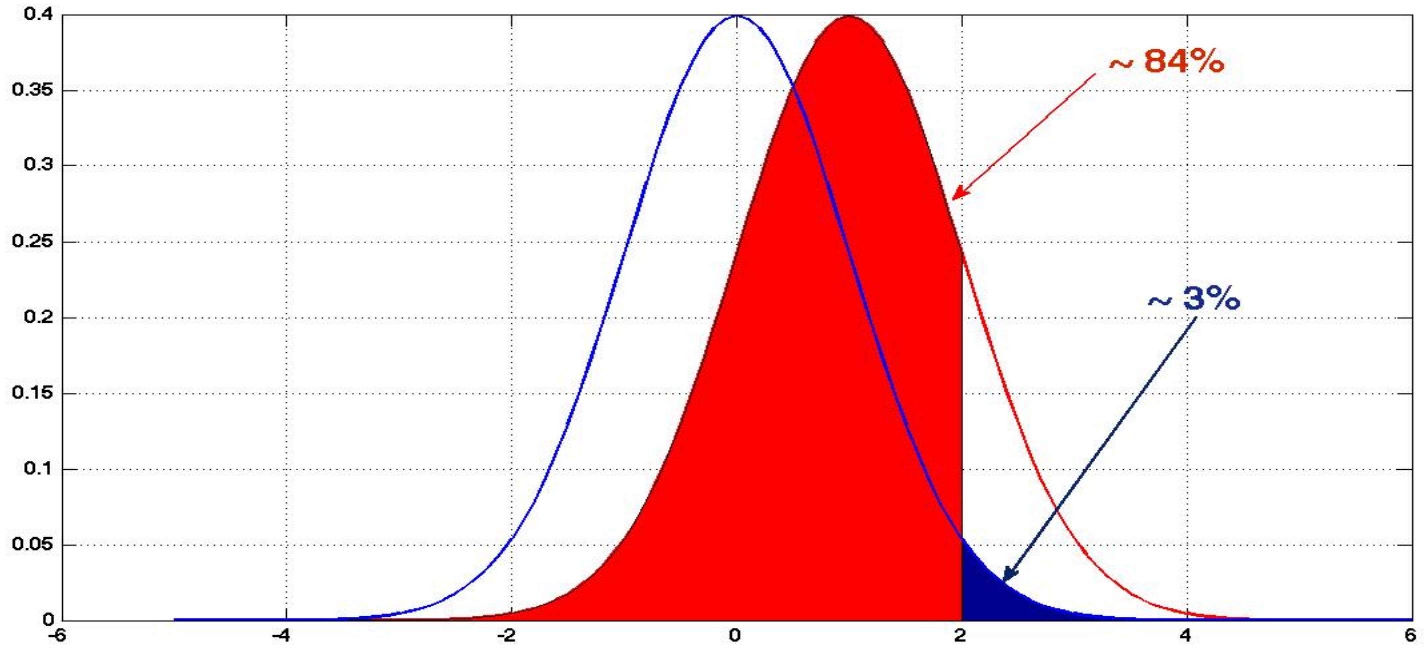
$$p_0 = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad p_1 = \frac{1}{\sqrt{2\pi}} e^{-(y-1)^2/2} \text{ and } \Lambda(y) = \frac{p_1(y)}{p_0(y)}$$

- Neymann-Pearson Criterium:
  - $\Lambda(y) = e^{(y-1/2)}$ ➔ Rule is: threshold on y

If we set y* = 0.5

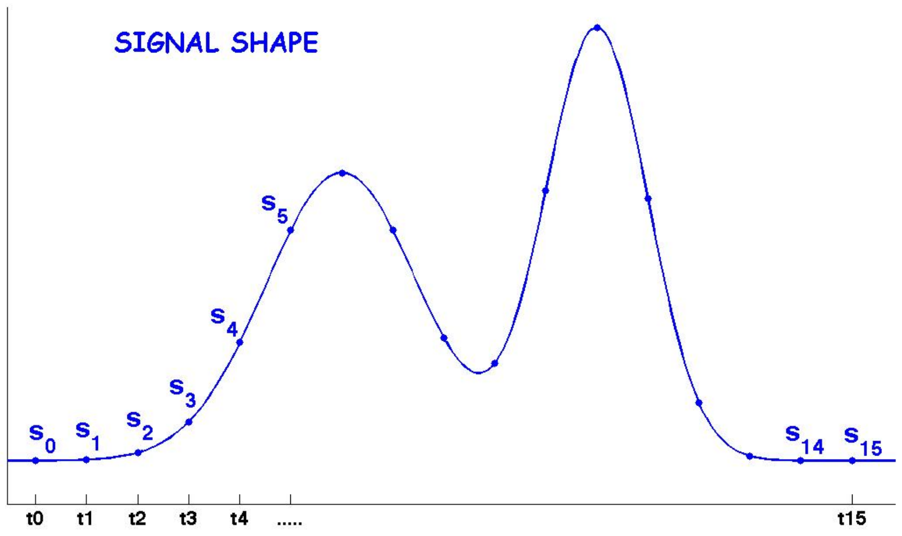


If we set y*=2



Another example, a set of measurements

- Consider some measurements
- $y_i(i_o) = n_i + \varepsilon s_{i-io}$
  - n is Gaussian noise, zero mean and unit variance
  - s_{i-io} is a signal of known shape, arriving at time i_o



the pdfs are:

$$p_0 = \prod_k \frac{1}{\sqrt{2\pi}} e^{-|y_k|^2/2}$$

$$p_1 = \prod_k \frac{1}{\sqrt{2\pi}} e^{-|y_k - \varepsilon s_{k-i0}|^2/2}$$

a convenient level surface of the likelihood is $\rho(\{y\}, i_0) = \sum_k \varepsilon s_{k-i_0} y_k$

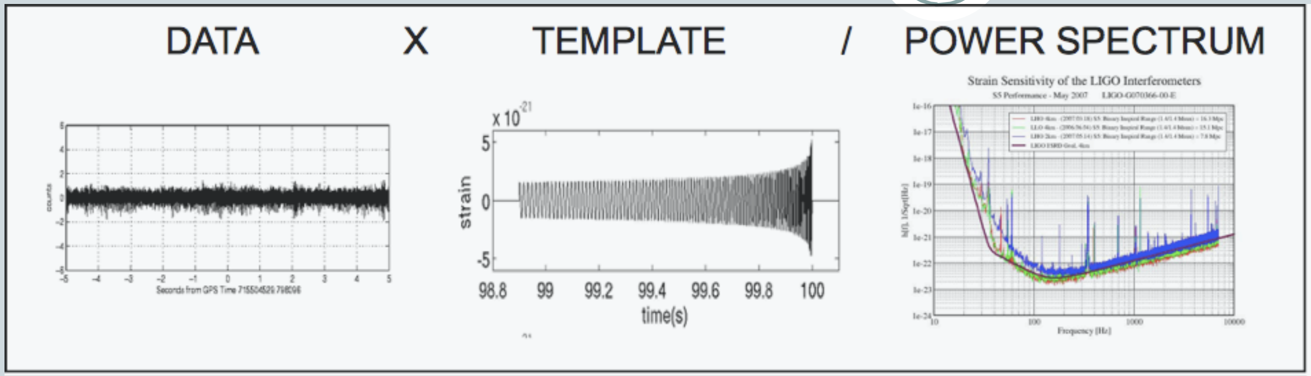
Likelihood and matched filtering

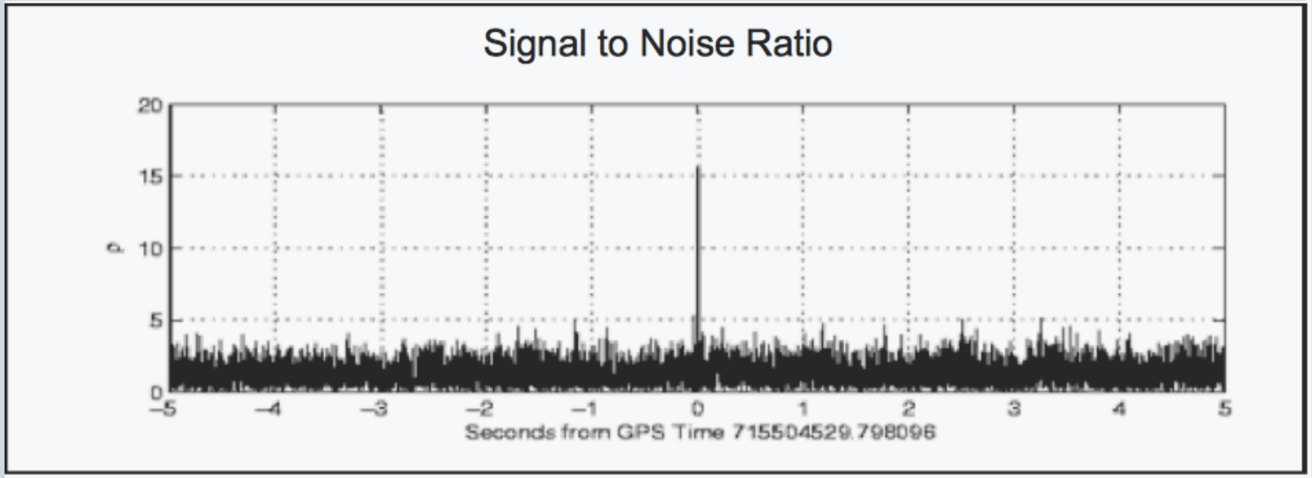
- so our detection statistic is $D(\{y\},i_0) = \sum_k \varepsilon s_{k-i_o} y_k$
- in the continuum $\sum_k \varepsilon s_{k-i_o} y_k \quad \Rightarrow \quad \int_{-\infty}^{+\infty} dt\, s(t-\tau_0) y(t)$
- and in the Fourier domain:

$$\rho(\tau_0) = \int d\omega \frac{S^*(\omega,\tau_0) Y(\omega)}{N(\omega)}$$

the standard expression for matched filtering, N being the noise spectrum

Matched filter





FT of waveform with parameters a, at t_o

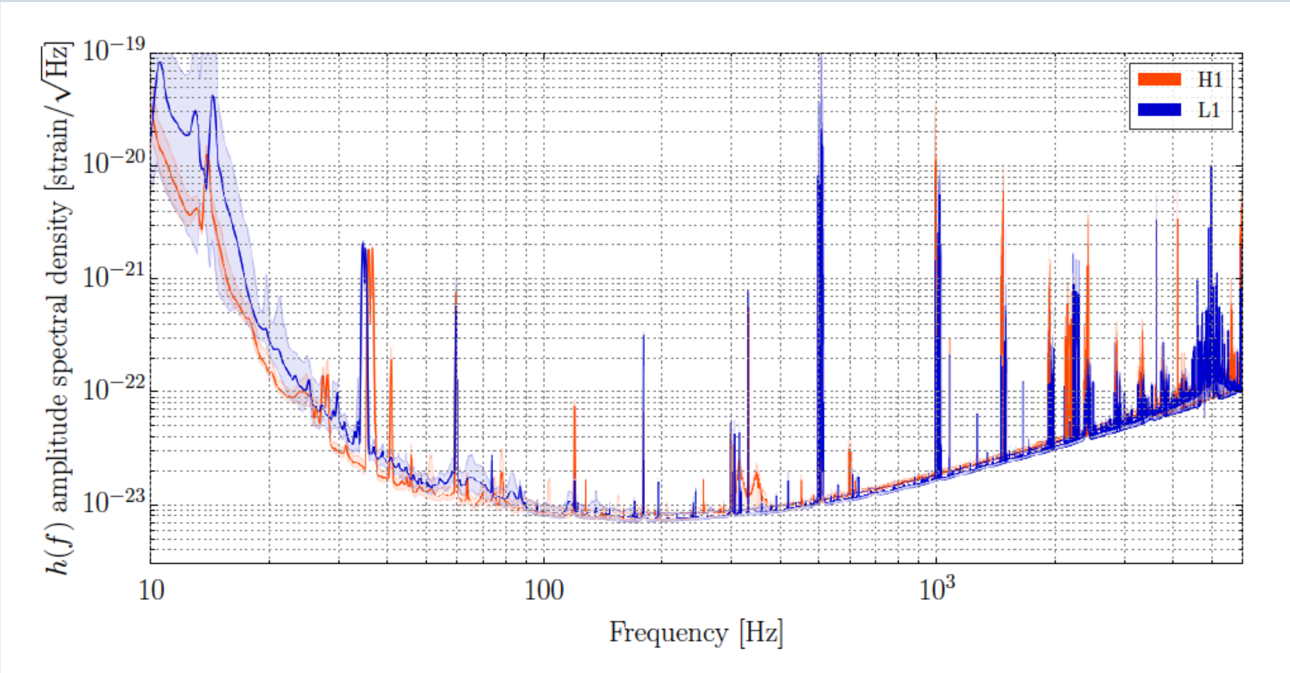
FT of data

$$\rho(t_0,a) = \int d\omega \frac{H^*(\omega,a,t_0)\,X(\omega)}{S_h(\omega)}$$

noise

What signals may de detectable ?

time scales of ms to s, compact objects, high accelerations:

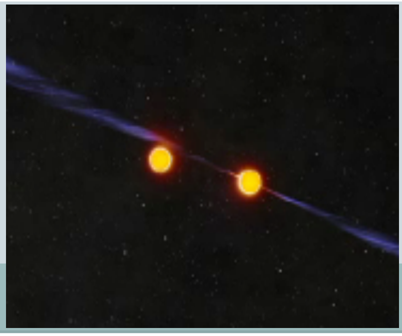


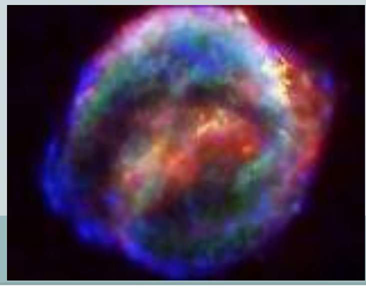
from inspiraling compact objects

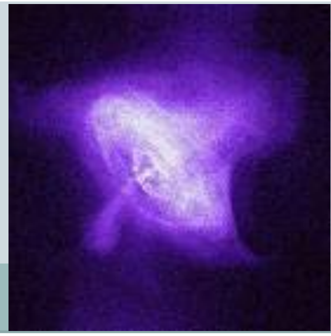
bursts , typically arising from catastrophic events

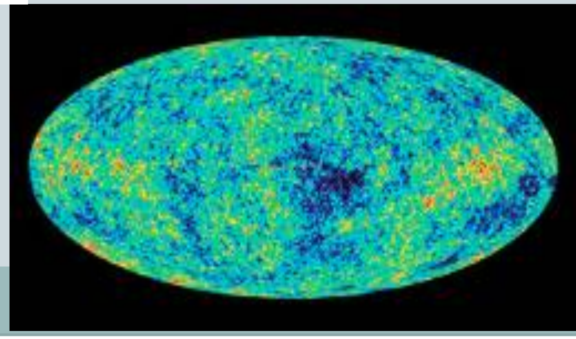
continuous quasi-periodic waves

stochastic background of gravitational radiation









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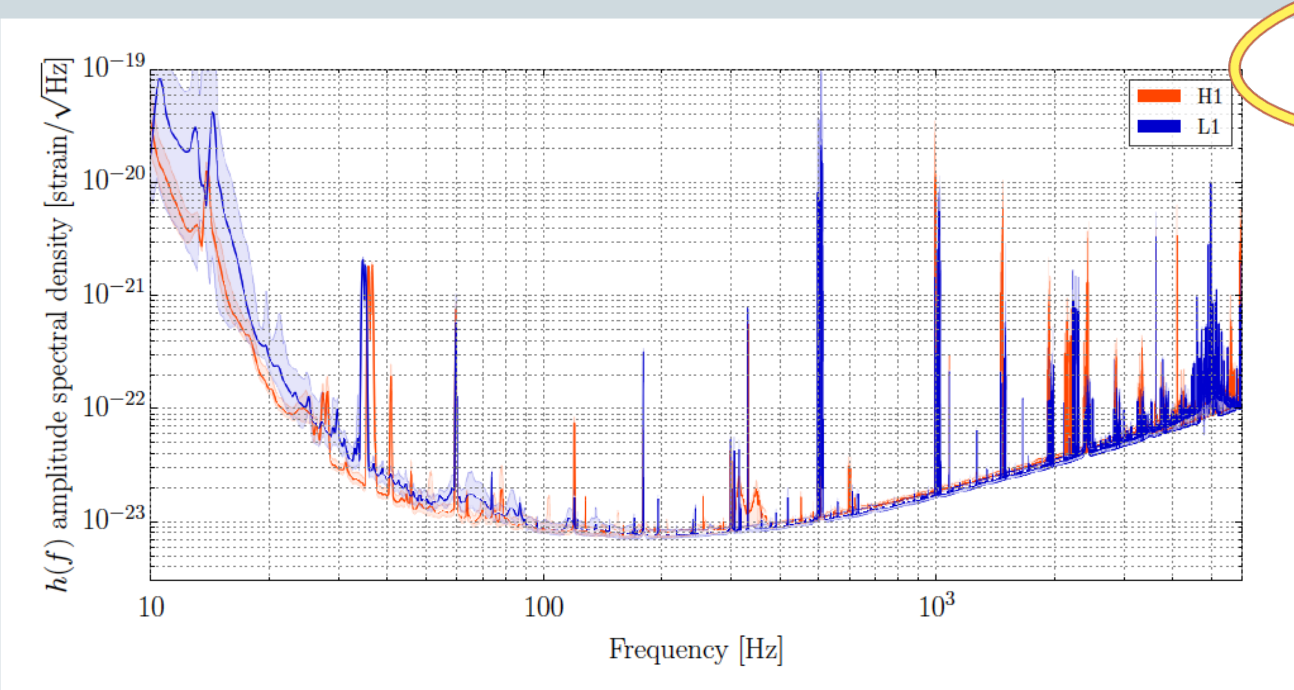

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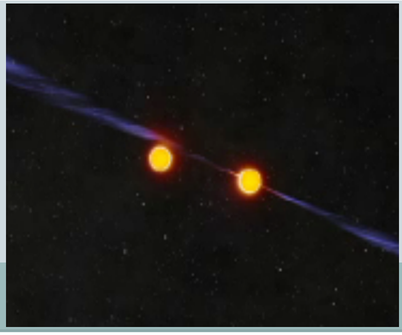
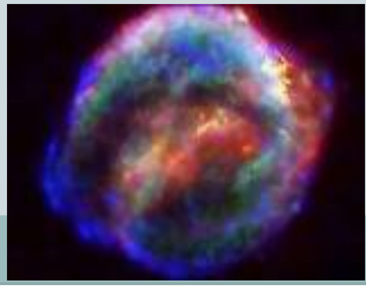
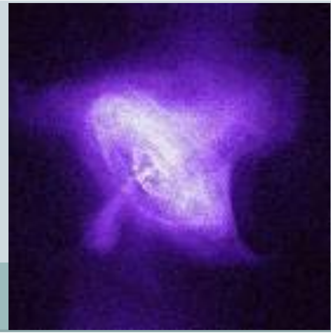
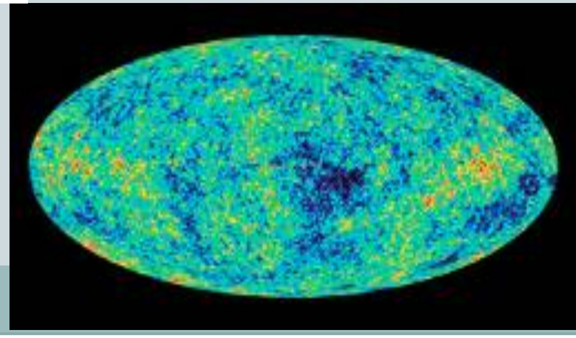


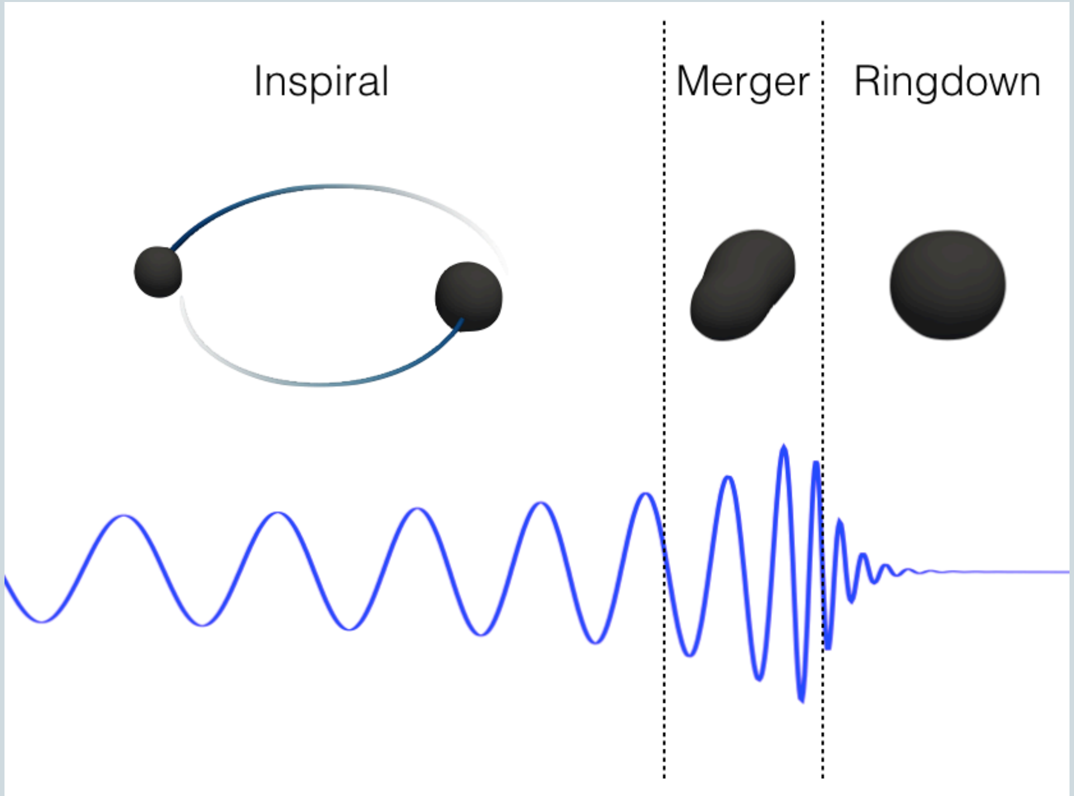




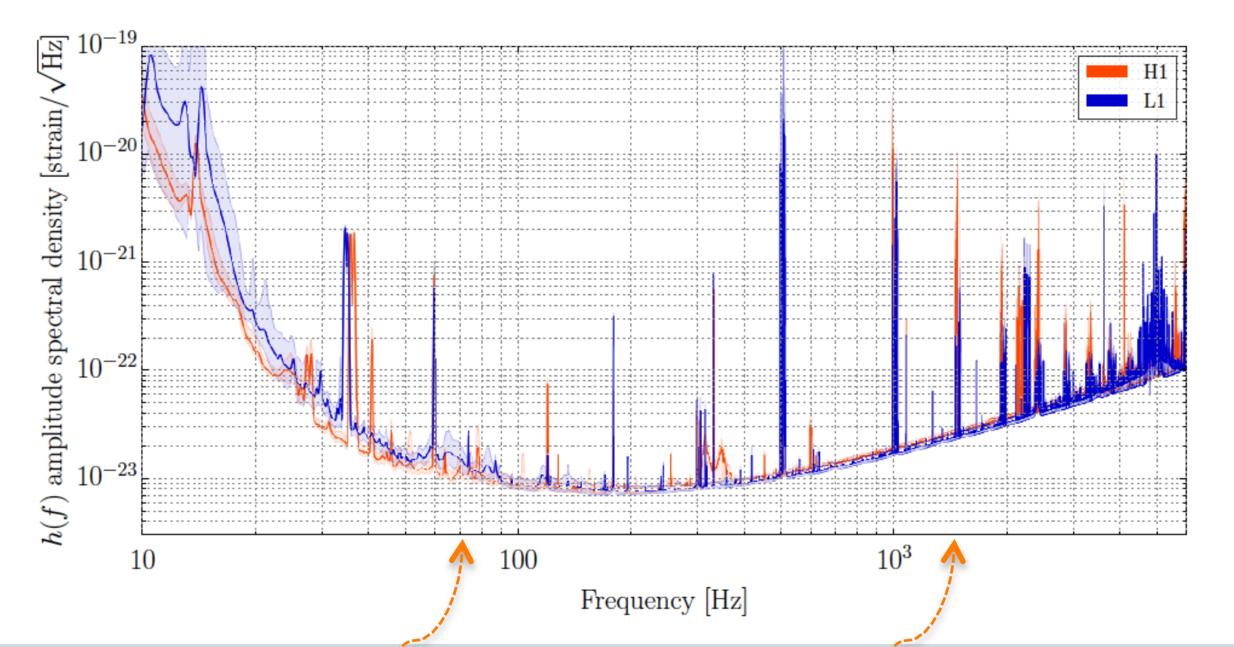


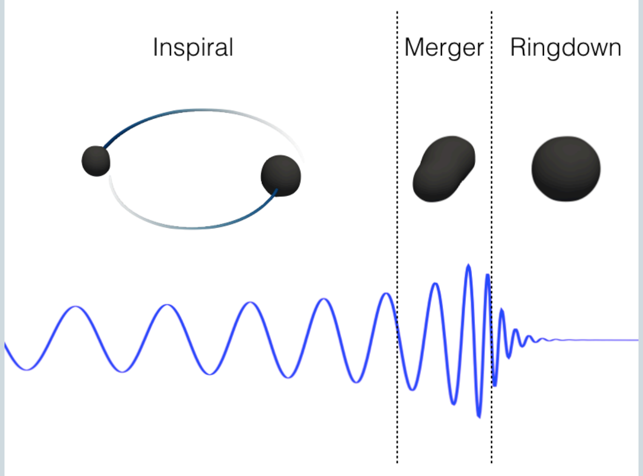
Compact binary coalescences as sources of GW

- Final evolution of compact binary systems involving neutron stars and/or black holes, driven by gravitational radiation



What mass ranges do we see ?



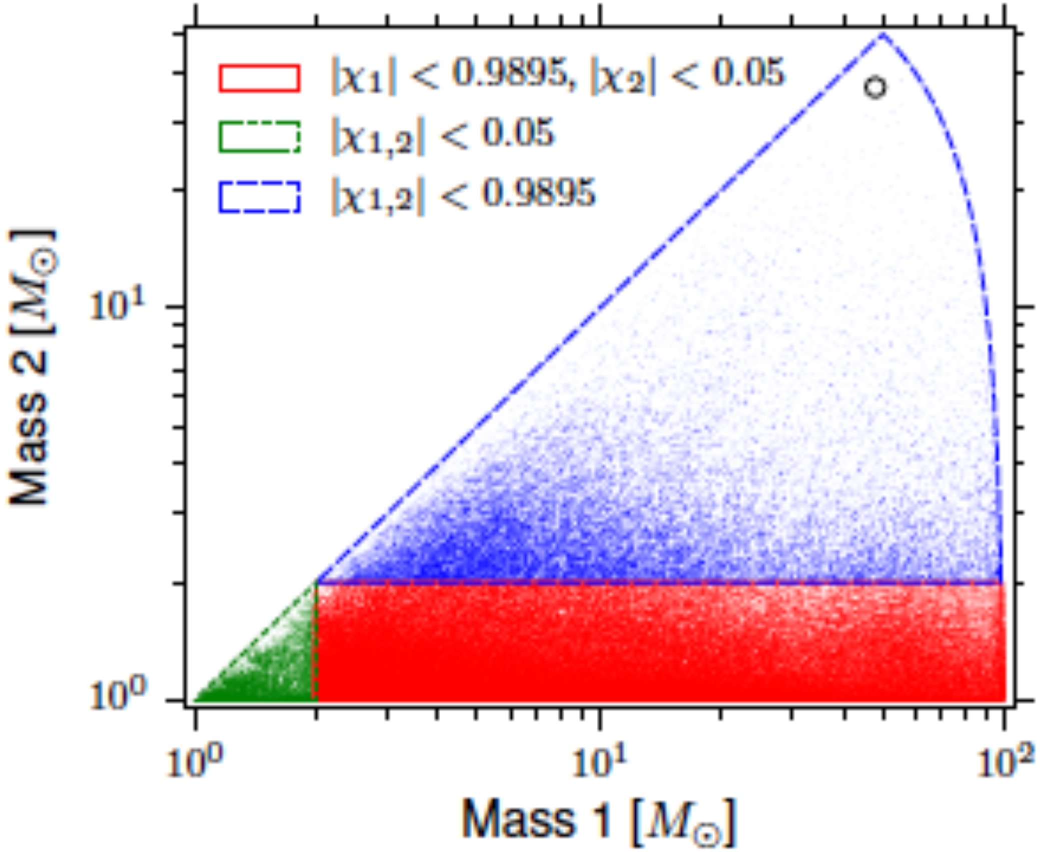


merger freq for $M_{tot} \approx 60\ M_\odot$

merger freq for $M_{tot} \approx 2.4\ M_\odot$

4300 Hz $M_\odot/M_{tot}$

Mass range (O1 search)



- arXiv:1606.04856, arXiv:1607.07456

How do we search for signals ?
Matched filter

- At best you know what you're looking for; then you use a matched filter:

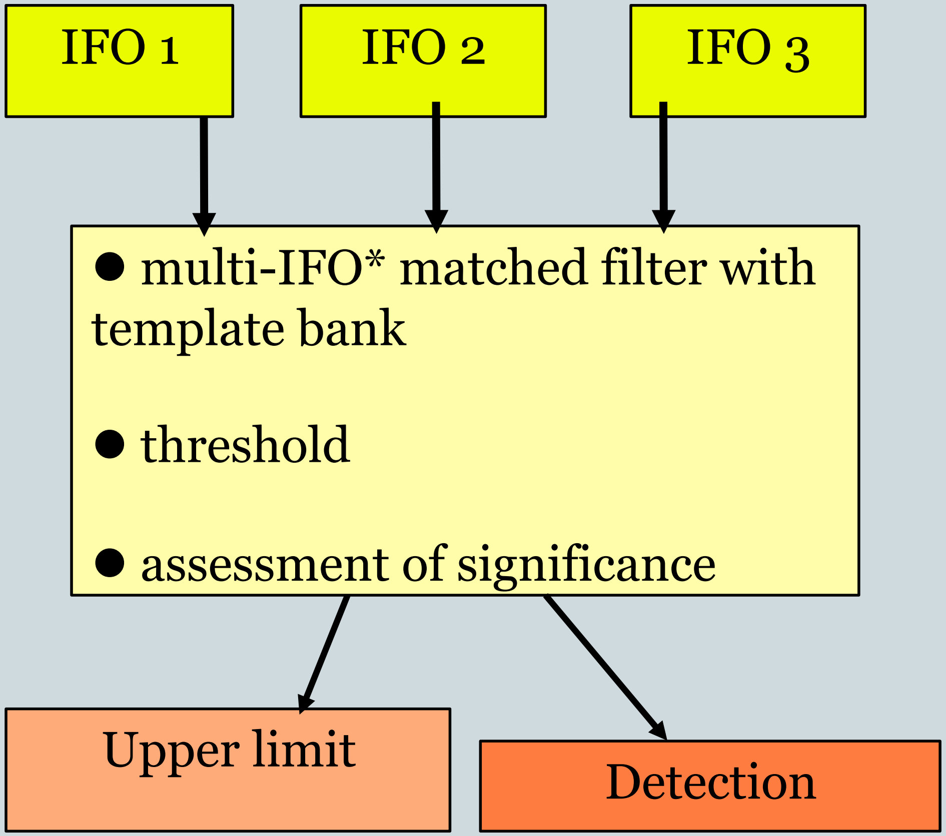
$$\rho(t_0, a) = \int d\omega \frac{H^*(\omega, a, t_0) X(\omega)}{S_h(\omega)}$$

FT of waveform with parameters a, at t_o

FT of data

noise

Idealized pipeline schematics



*IFO : interferometer

but it's more complicated:

- the matched filter is optimal detection statistic for Gaussian stationary noise but our data are neither Gaussian nor stationary:
  - Weed-out spurious noise:
    - Data quality flags
    - Coincidence schemes
    - Signal-based noise rejection techniques
  - Ad-hoc inspection of interesting candidates:
    - Correlations with environmental channel
    - Examine overall status of detectors
  - But the problem remains of assessing the significance
    - Problem of background/noise estimation

The problem with large spurious noise events

Matched filter: is designed to give a large response when the signal waveform matches the template, but it also gives a large response when the instrumental noise has a large glitch. Even if the glitch shape looks nothing like a waveform, it can still drive the filter to give a large response.

Noisy data: the noise of GW detectors presents sporadic prominent non-Gaussian glitches. This is a problem.

The problem with large spurious noise events

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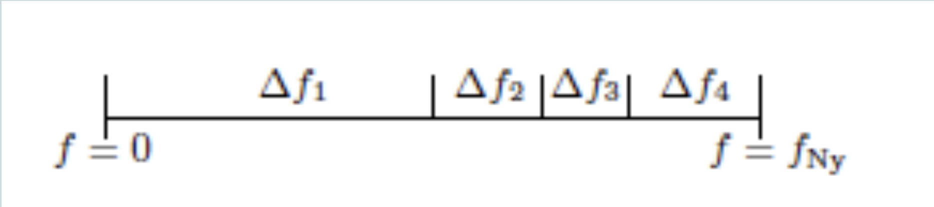
Noisy data: the noise of GW detectors presents sporadic prominent non-Gaussian glitches. This is a problem.

MITIGATION SCHEMES

A counter-measure: signal-based veto, the χ^2 test

If it looks like a duck, quacks like a duck, swims like a duck, then it is a duck.

Does it *really* look like a duck ? the χ^2 test



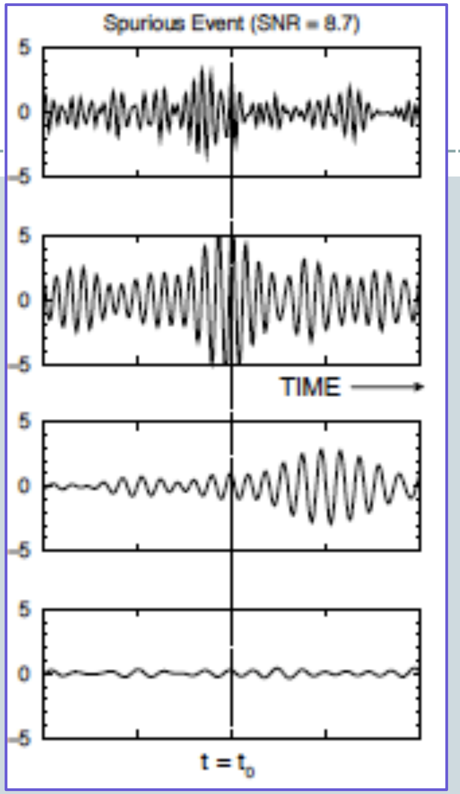
Main idea: consider p frequency sub-bands each contributing the same to the matched filter SNR, z, if a signal is present. Compute the matched filter detection statistic z_j for each of the sub-bands and verify that this is the case:

$$\chi^2 = \frac{p}{2p-2}\sum_{j=1}^{p}\left(z_j - \frac{z}{p}\right)^2 \qquad E[\chi^2] = p - 1 \qquad \mathrm{var}[\chi^2] = 2(p-1)$$

if the hypothesis is correct the residuals are random Gaussian variables and their square sum a chi square variable.

notation note : ρ <--> z here

An example

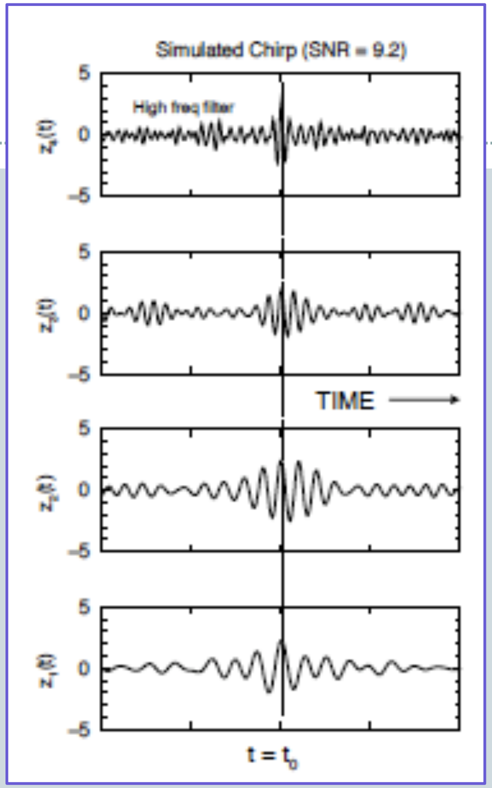


$z_4(t)$

$z_3(t)$

$z_2(t)$

$z_1(t)$



$z_4(t)$

$z_3(t)$

$z_2(t)$

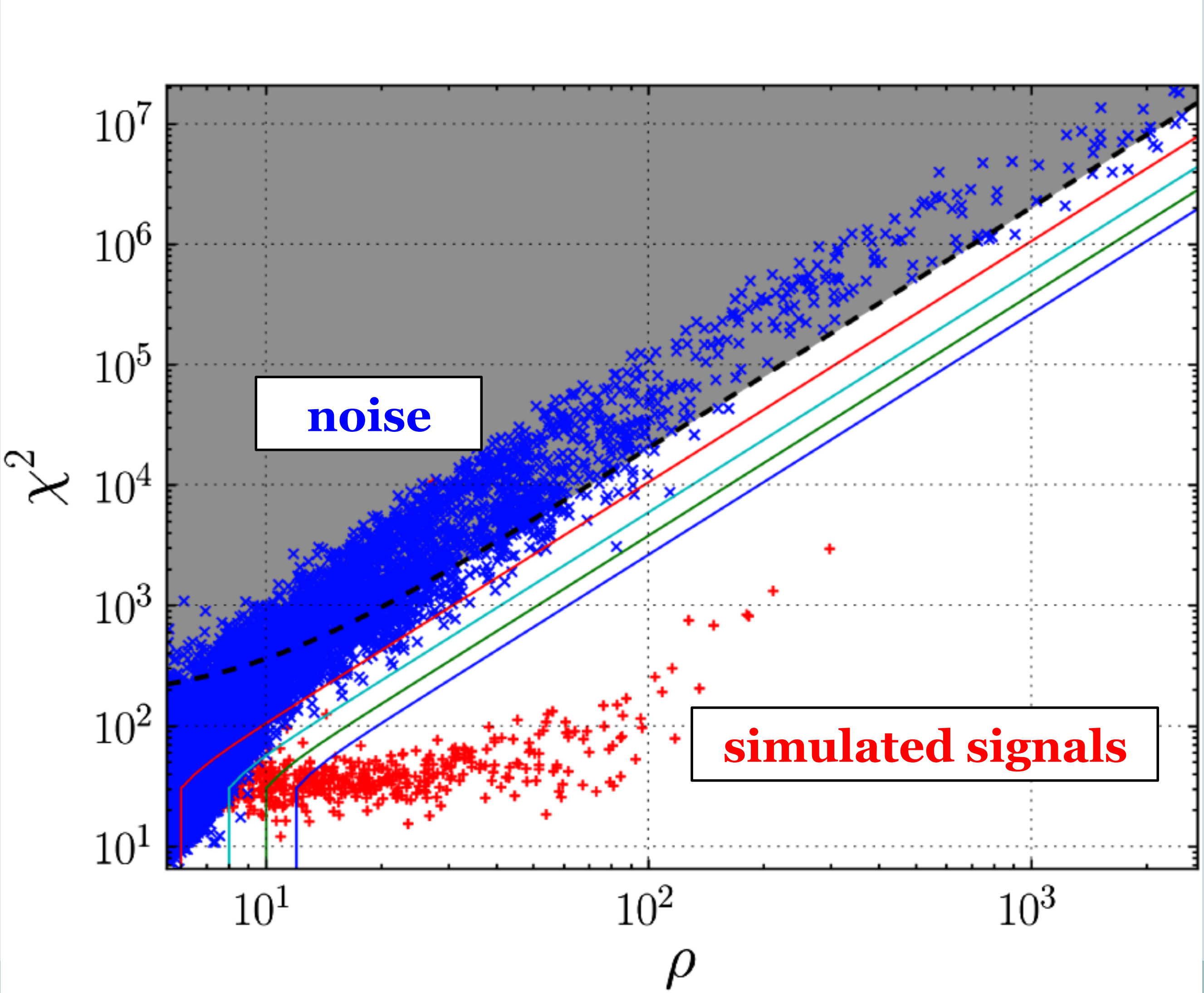
$z_1(t)$

$$
\begin{aligned}
z_1 &= 0.23 \\
z_2 &= 0.84 \\
z_3 &= 5.57 \\
z_4 &= 2.33 \\
z &= z_1 + z_2 + z_3 + z_4 = 8.97 \\
\chi^2 &= 4\sum_{j=1}^{4}(z_j - z/4)^2 = 68.4 \\
P_{\chi^2 \geq 68.4} &= 1 - \frac{\gamma(3/2, 34.2)}{\Gamma(3/2)} = 9.4 \times 10^{-15}.
\end{aligned}
$$

$$
\begin{aligned}
z_1 &= 2.25 \\
z_2 &= 2.44 \\
z_3 &= 1.87 \\
z_4 &= 2.64 \\
z &= z_1 + z_2 + z_3 + z_4 = 9.2 \\
\chi^2 &= 4\sum_{j=1}^{4}(z_j - z/4)^2 = 1.296 \\
P_{\chi^2 \geq 1.296} &= 1 - \frac{\gamma(3/2, 0.648)}{\Gamma(3/2)} = 73\%.
\end{aligned}
$$

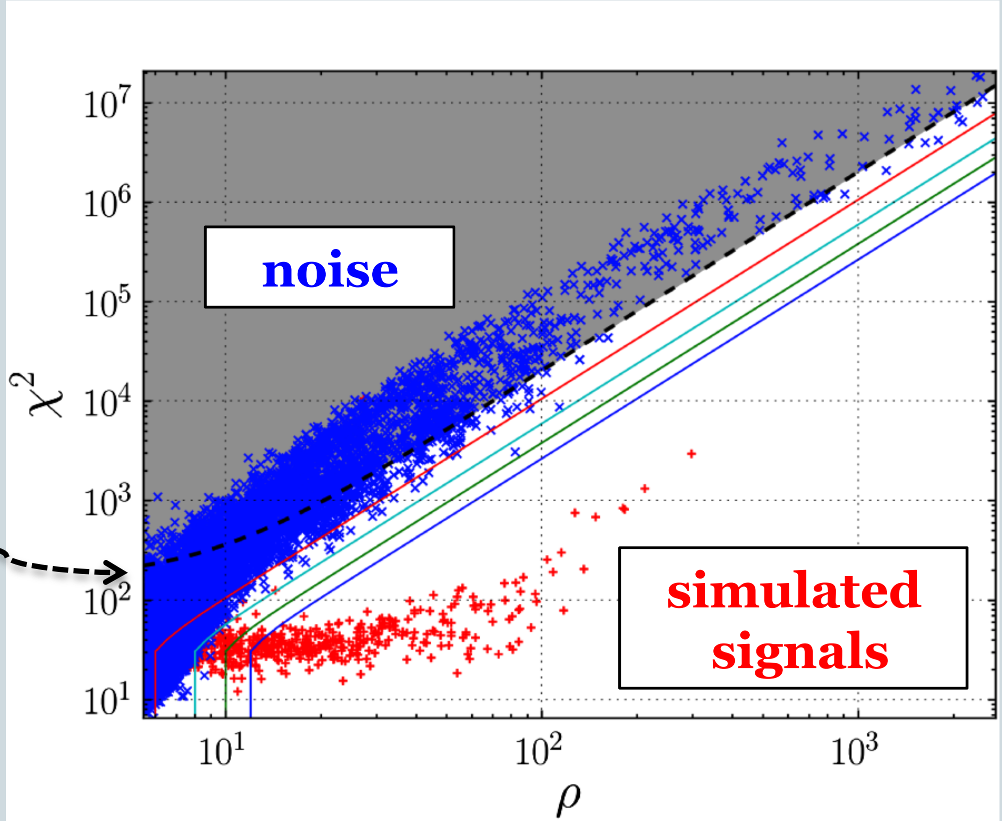
notation note : ρ <--> z here

Cannot just threshold on χ^2



Use of χ^2 : a veto

- in previous example number of freq bins p=4 usually p=16
- $n_{dof} = 2p-2$
- veto all triggers with $\chi^2 > 10\,(p + 0.2\,\rho^2)$



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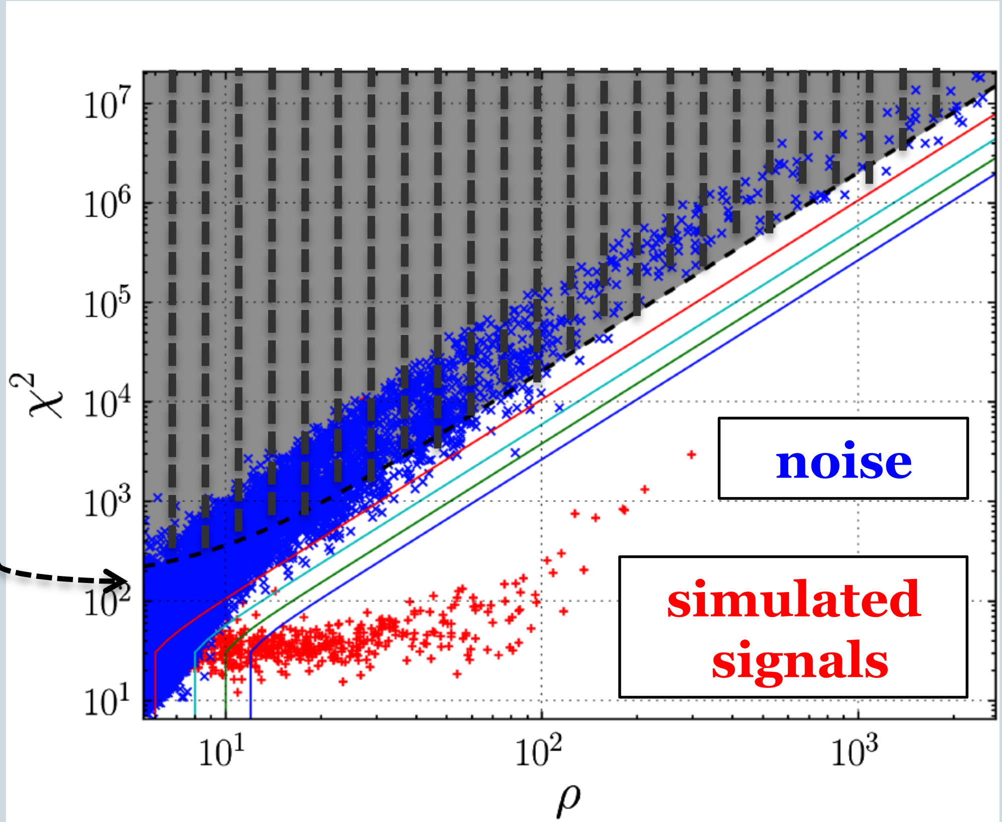
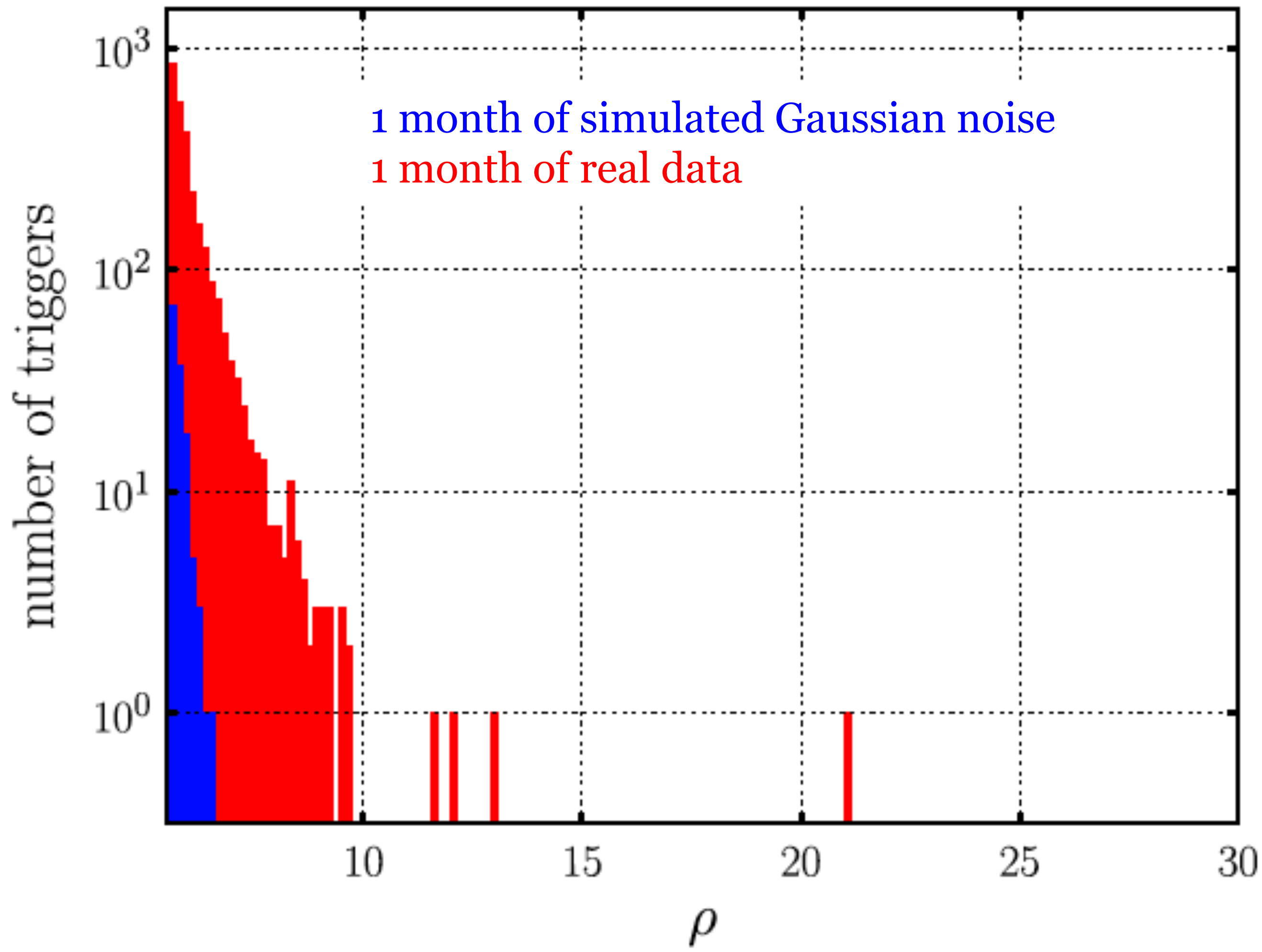


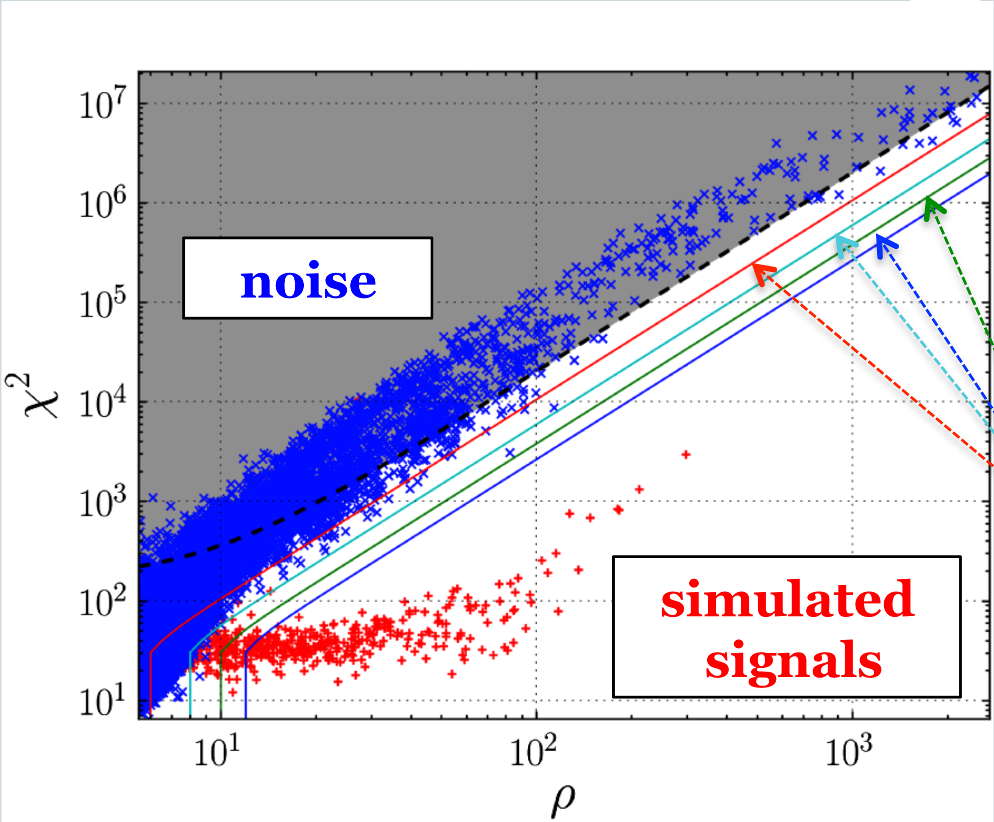
Coincidence requirement

- After the χ^2 veto we also veto triggers in one detector that do not have a consistent counterpart trigger in the other detector
  - Consistency in waveform parameters
  - Close enough in time

But still many spurious triggers



Go back to single-detector triggers and construct a new detection statistic

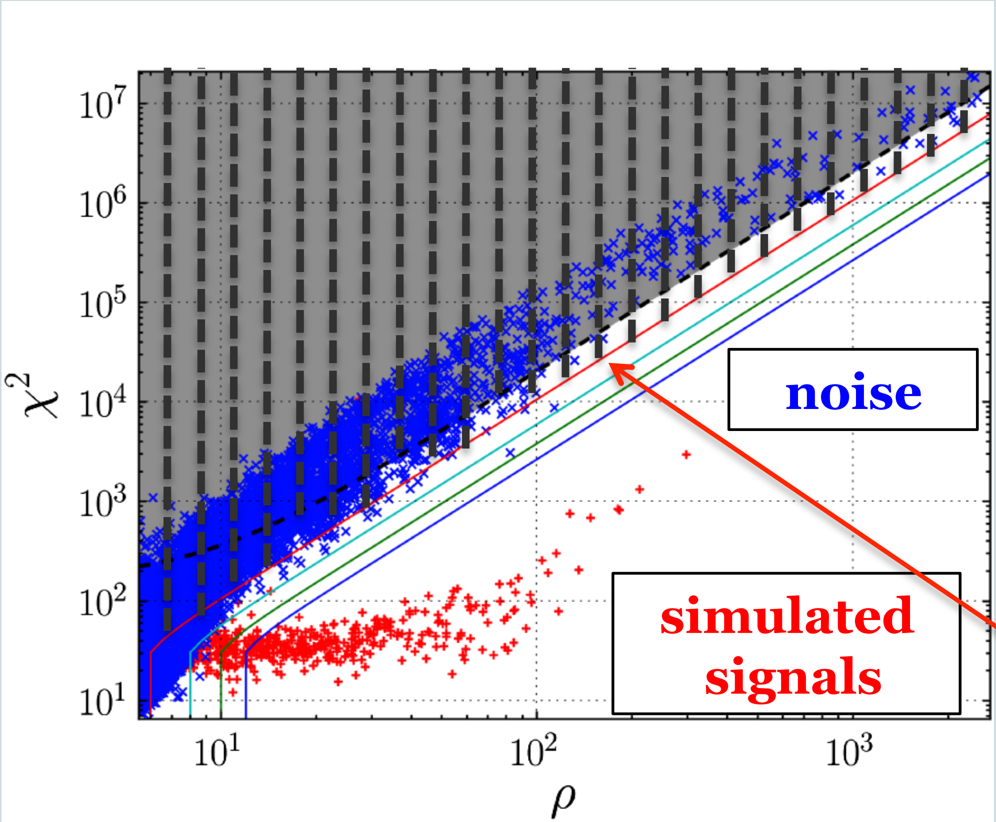


$$\rho_{new}^2 = \begin{cases} \rho^2 & \text{if } \chi_r^2 \le 1 \\ \dfrac{\rho^2}{\left\{\frac{1}{2}\left[1+(\chi_r^2)^3\right]\right\}^{1/3}} & \text{if } \chi_r^2 > 1 \end{cases}$$

with $\chi_r^2 = \chi^2 / n_{dof}$

This statistic matches the constant false alarm contours and discriminates well between signals and noise.

A new detection statistic



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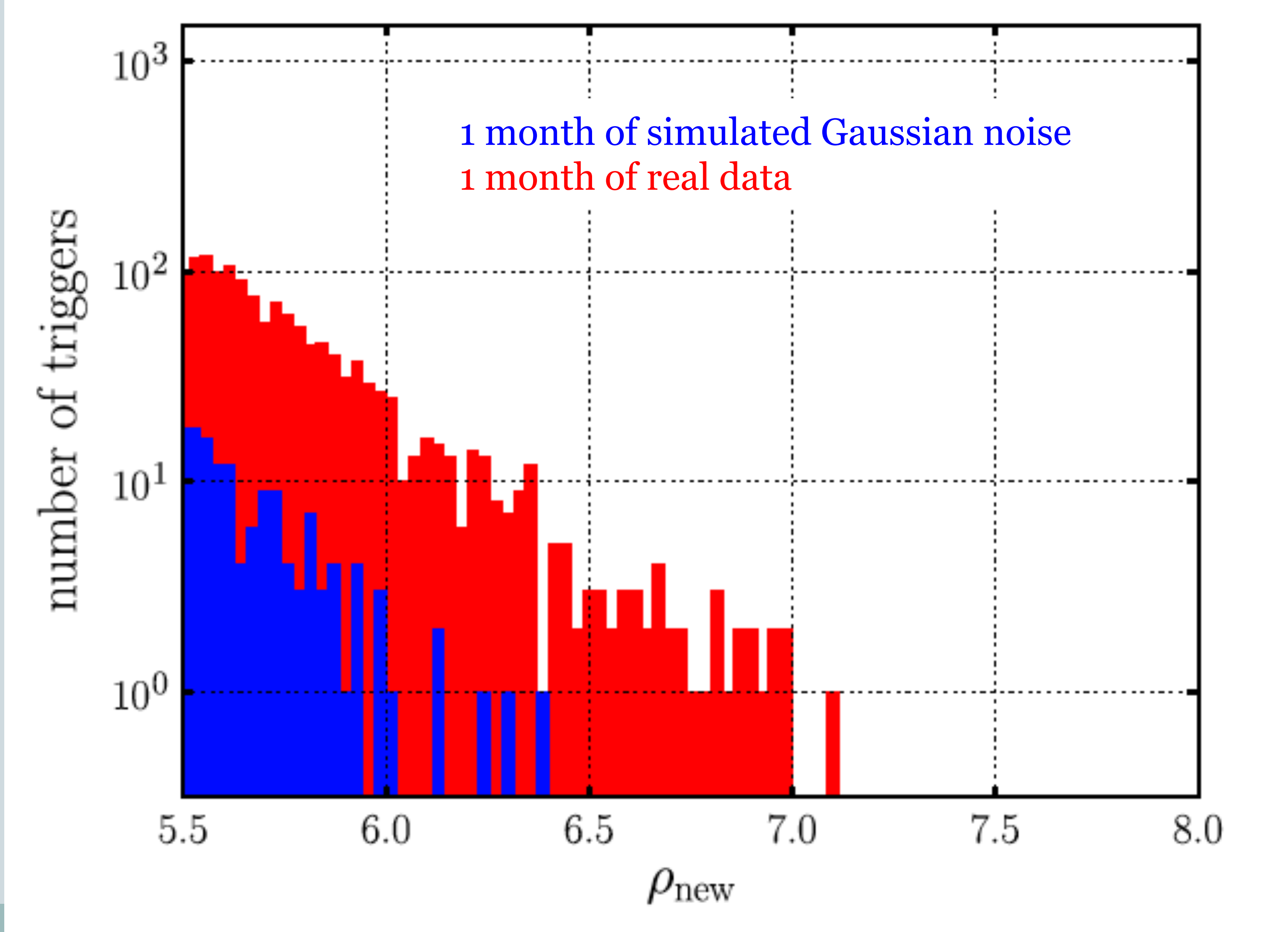
with $\chi_r^2 = \chi^2 / n_{dof}$

This statistic matches the false alarm contours and discriminates well between signals and noise.

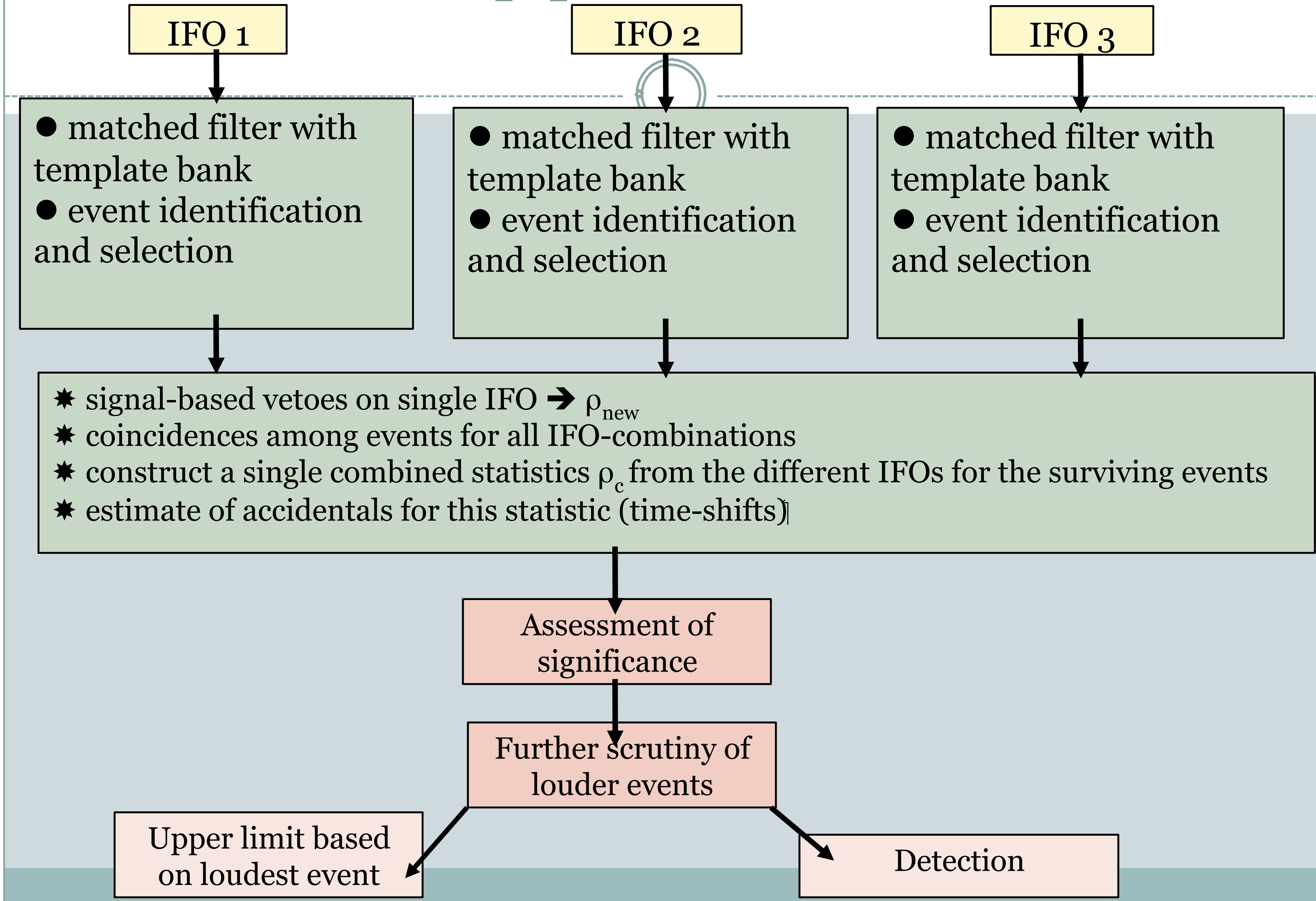
Triggers with $\rho_{new}^2 < 5.5$ are vetoed
Then again coincidences

The final combined detection statistic: $\rho_c^2 = \sum \rho_{new,i}^2$

Remarkable improvement



Actual pipeline schematics



Assessment of significance

- The analysis produces a list of coincident triggers, each with an associated combined SNR, ρ_c .

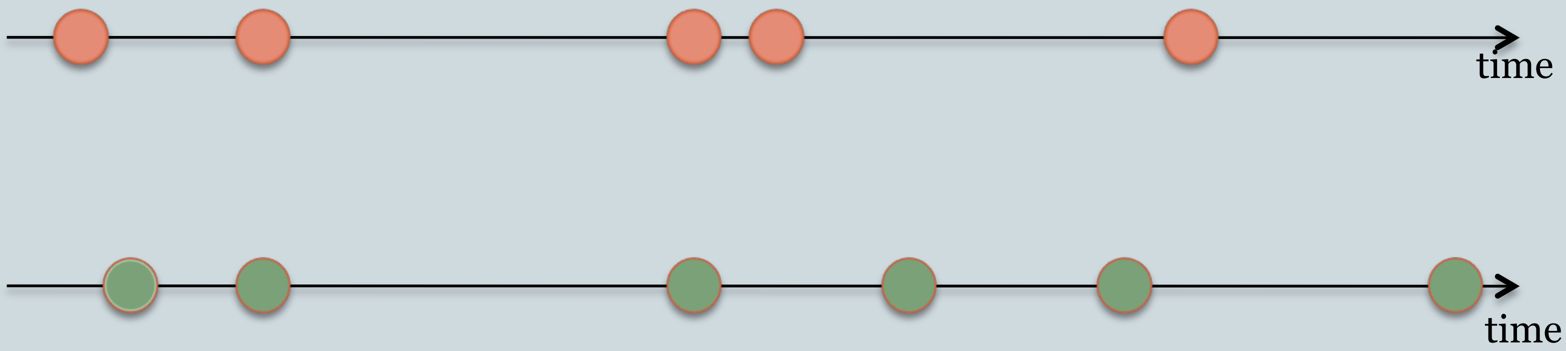
- These triggers need to be compared with those that one would obtain by chance, i.e. the accidentals, the background. We do this by comparing the distributions.

- How do we estimate the background ? We repeat the analysis on off-source data (by time-shifting the data streams).

- If an on-source coincidence trigger is significantly above the estimated background, then it is a candidate event that warrants further inspection.

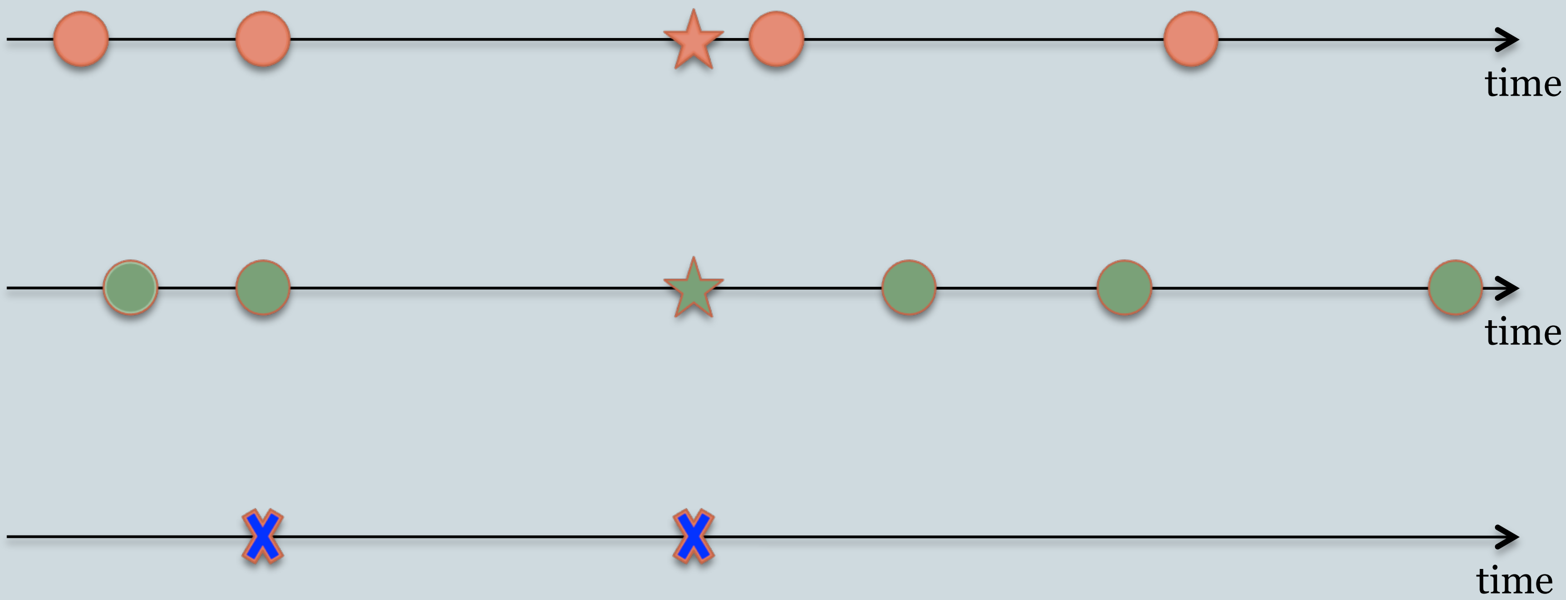
Time-slides

- triggers from detector 1
- triggers from detector 2



Time-slides

- triggers from detector 1
- triggers from detector 2
- coincident triggers
- trigger due to GWs
- trigger due to GWs



Time-slides

- triggers from detector 1
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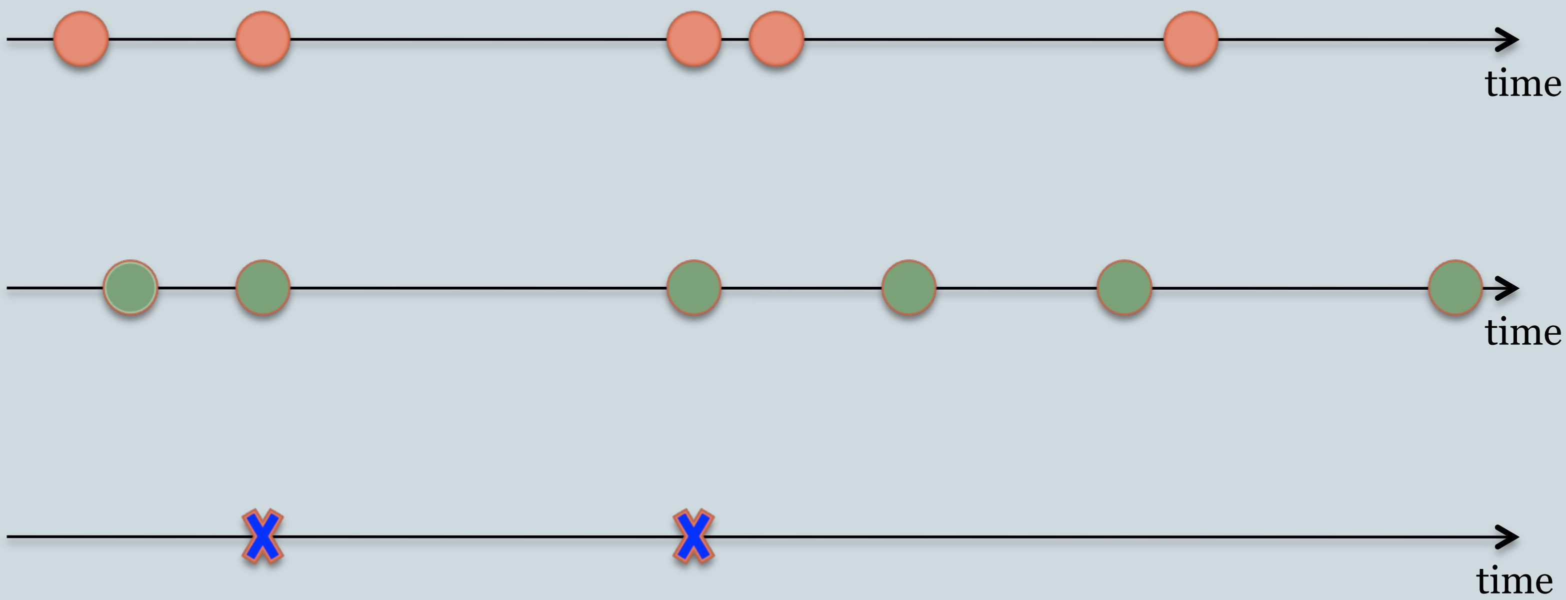
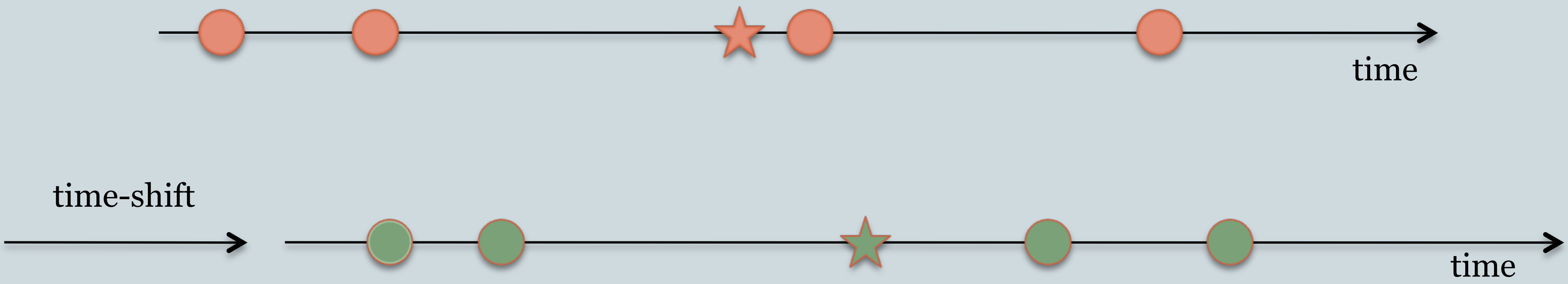


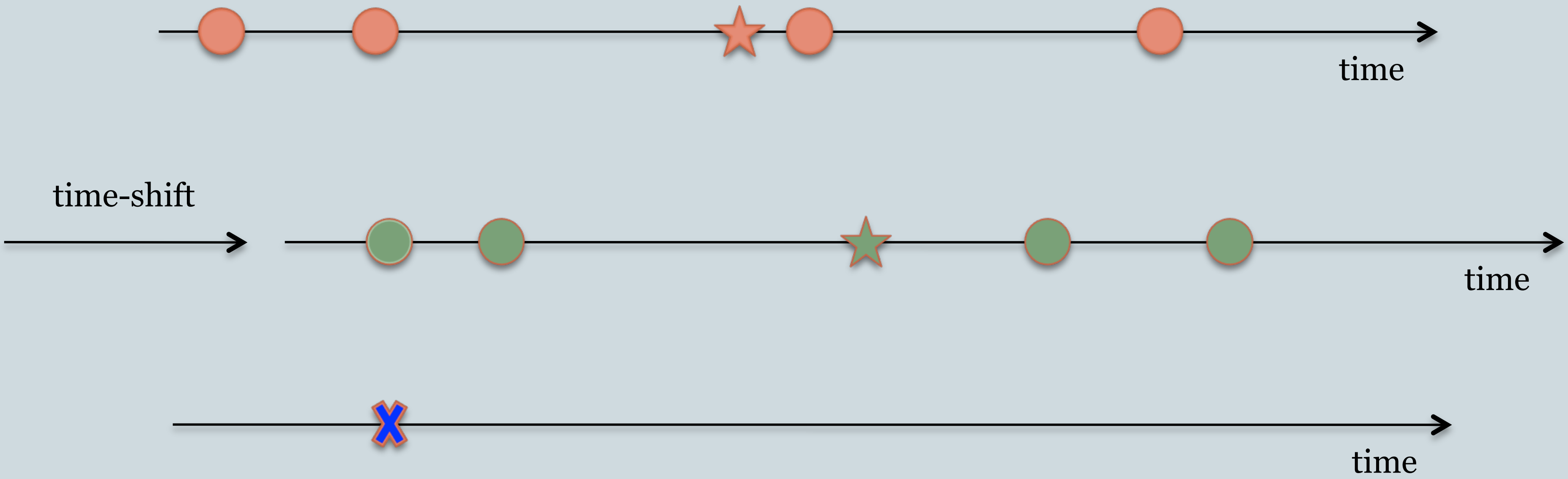
Time-slides

- ★ trigger from detector 1 due to a GW
- ★ trigger from detector 2 due to a GW



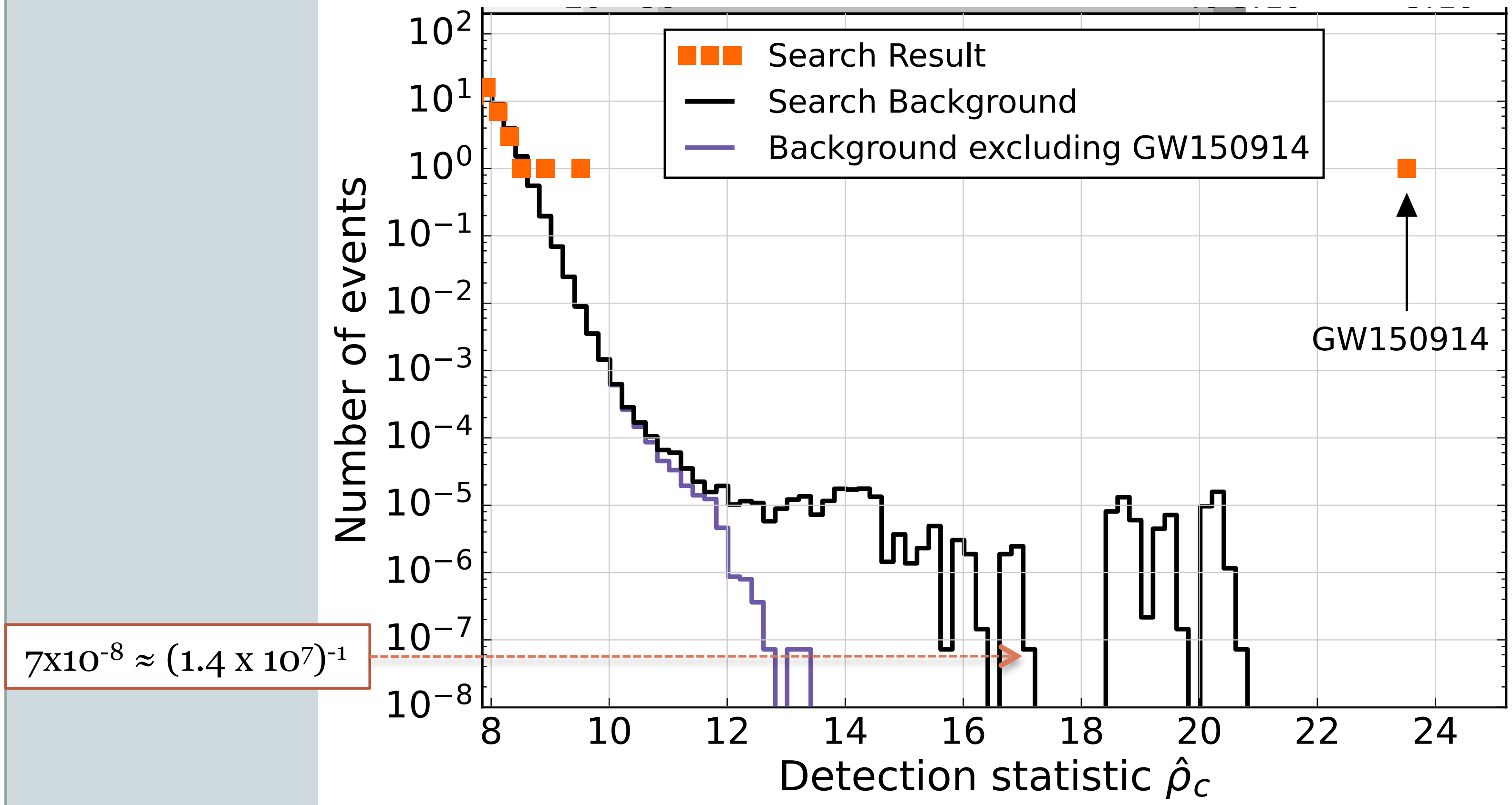
Time-slides

- ☆ trigger from detector 1 due to a GW
- ☆ trigger from detector 2 due to a GW
- X coincident triggers



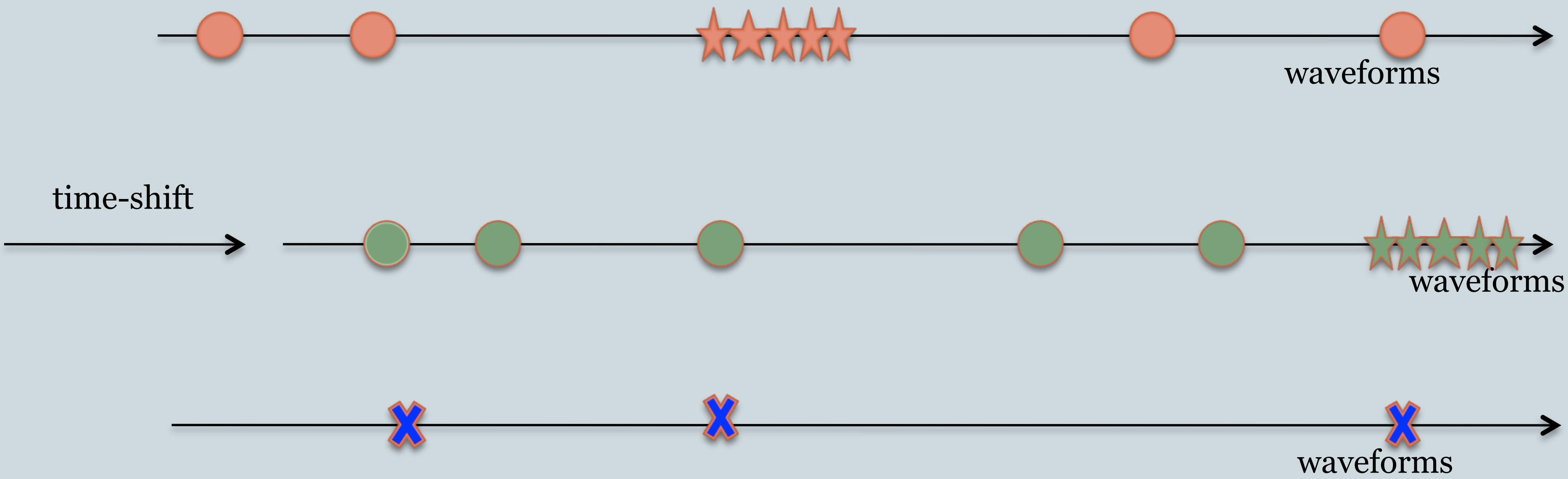
The first GW signal (GW150914)

1.4 x 10⁷ time slides corresponding to 608 000 yrs of simulated background.



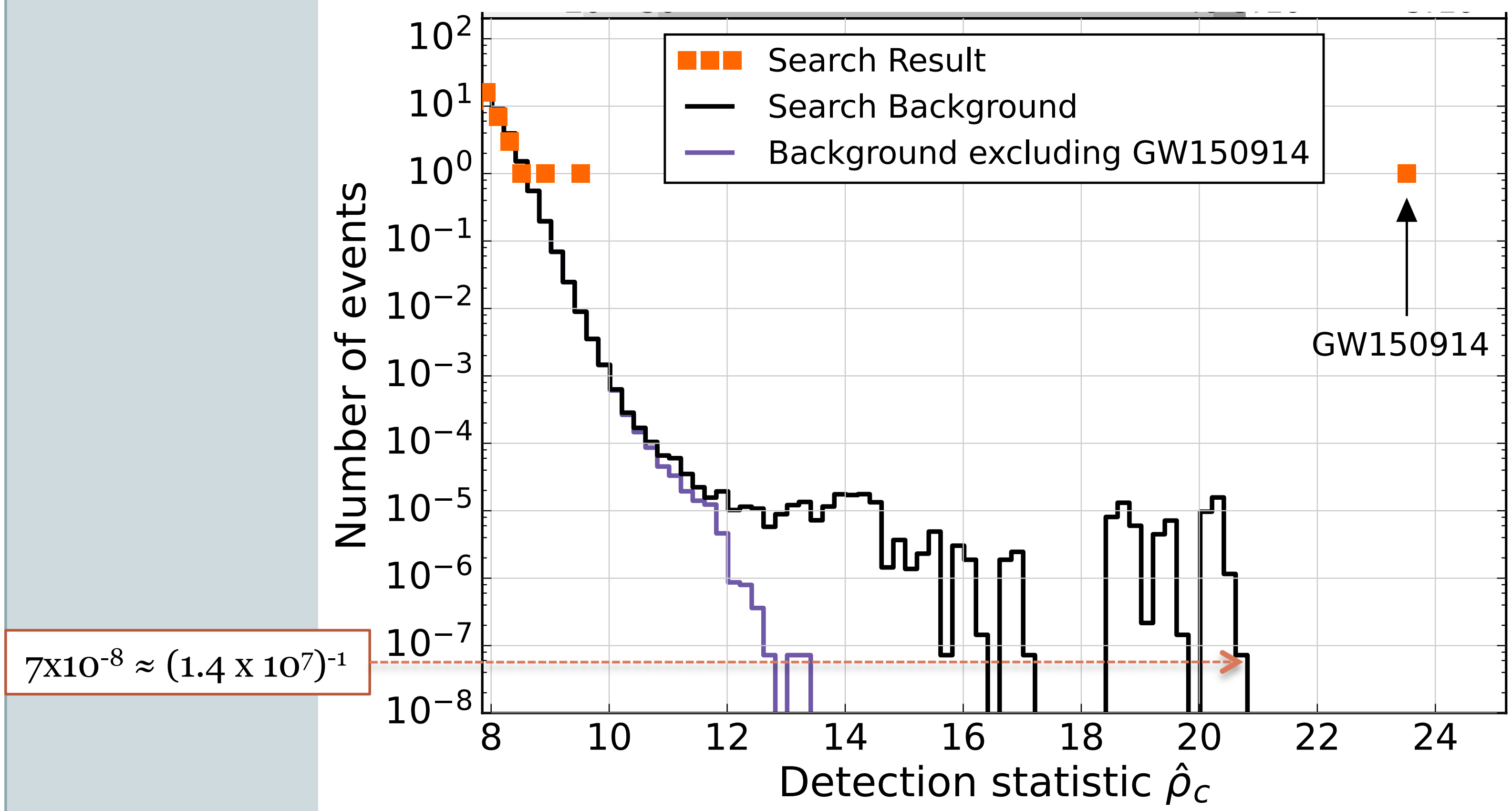
Time-slides: a conservative estimate of the background

- ★ trigger from detector 1 due to a GW
- ★ trigger from detector 2 due to a GW
- X coincident triggers



The first GW signal (GW150914)

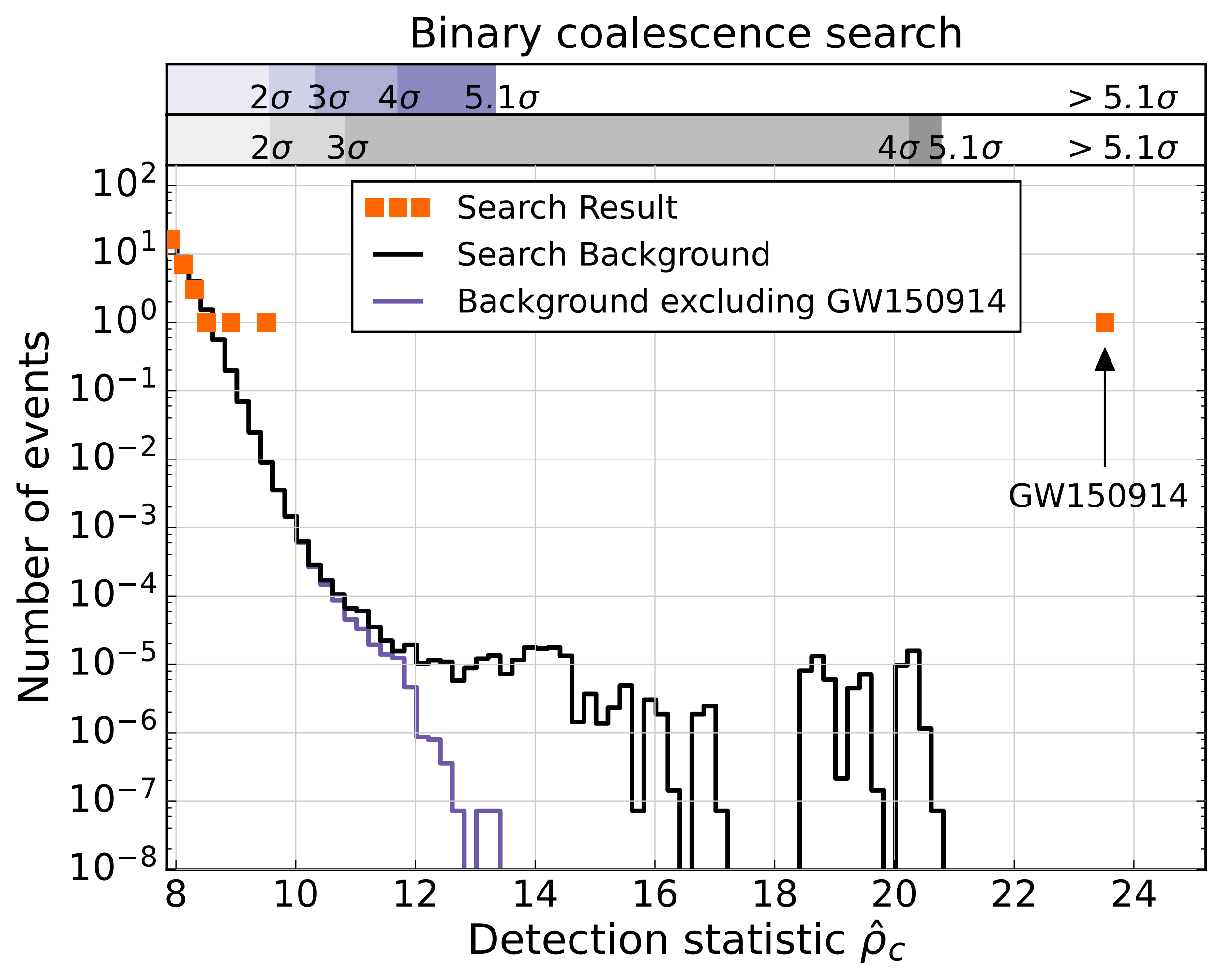
1.4×10^7 time slides corresponding to 608 000 yrs of simulated background.



Significance numbers for GW150914

- Analysis time (livetime): 14.14 days
- Could not get as significant detection statistic value in 1.375×10^7 realizations of the experiment.
- This corresponds to an analysis background time of 1.375×10^7 x 14.14 days / 365 days=608 000 yrs
- Let's take the most significant event at $\rho_c \sim 21$
- False alarm rate (FAR) = 1 event / background-time = $1.6 \times 10^{-6}\ yr^{-1}$
- FAR ➔ 3 x FAR X because 3 independent searches were performed (trials factor). FAR = $4.9 \times 10^{-6}\ yr^{-1}$
- Poisson process with λ = FAR*livetime =2×10^{-7}
- The probability to measure one event or more in a Poisson process with that average rate λ is FAP = 2×10^{-7} (it's the same as λ because $\lambda << 1$)
- The Gaussian sigma level corresponding to such FAP is 5.1

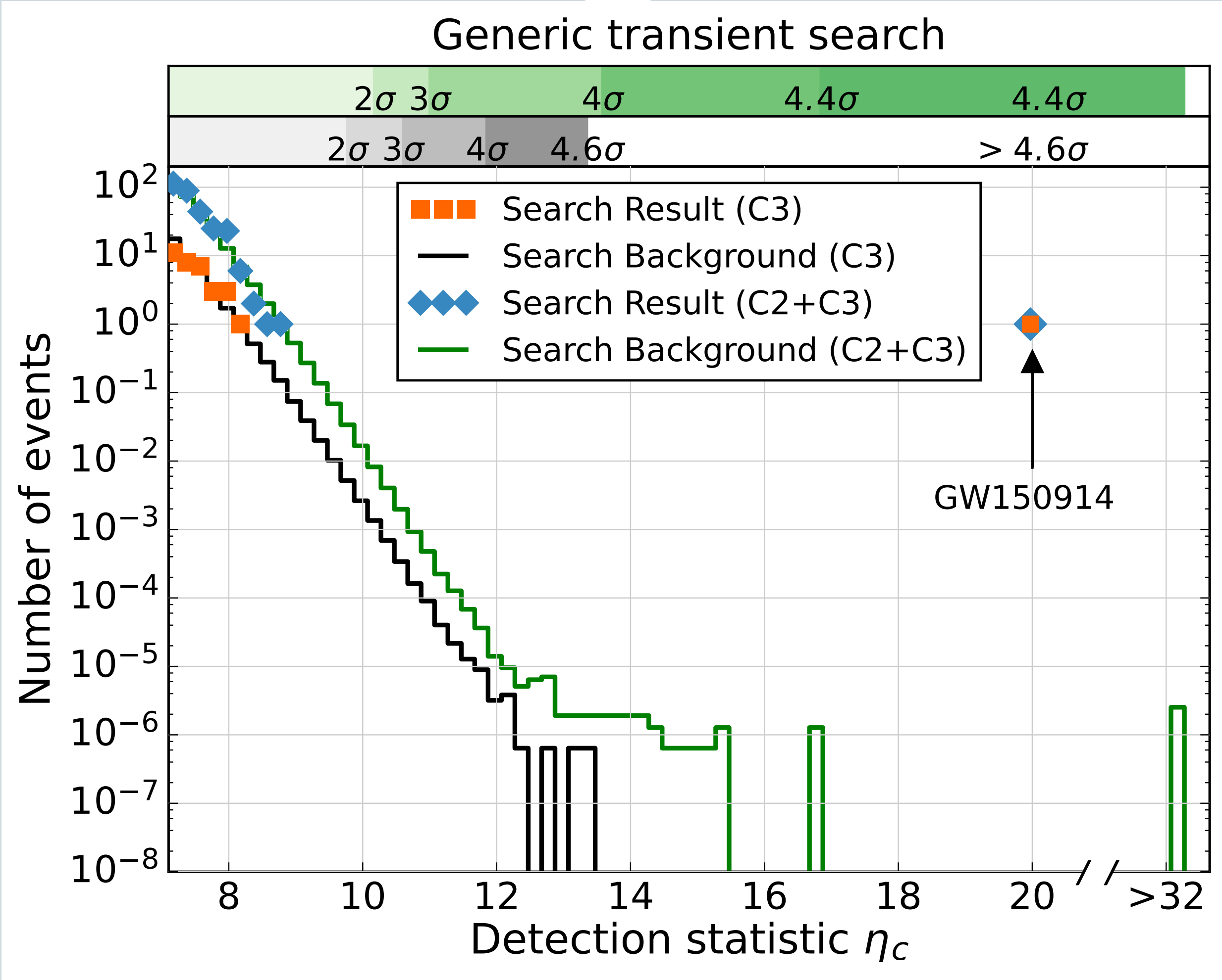
The first GW signal (GW150914)



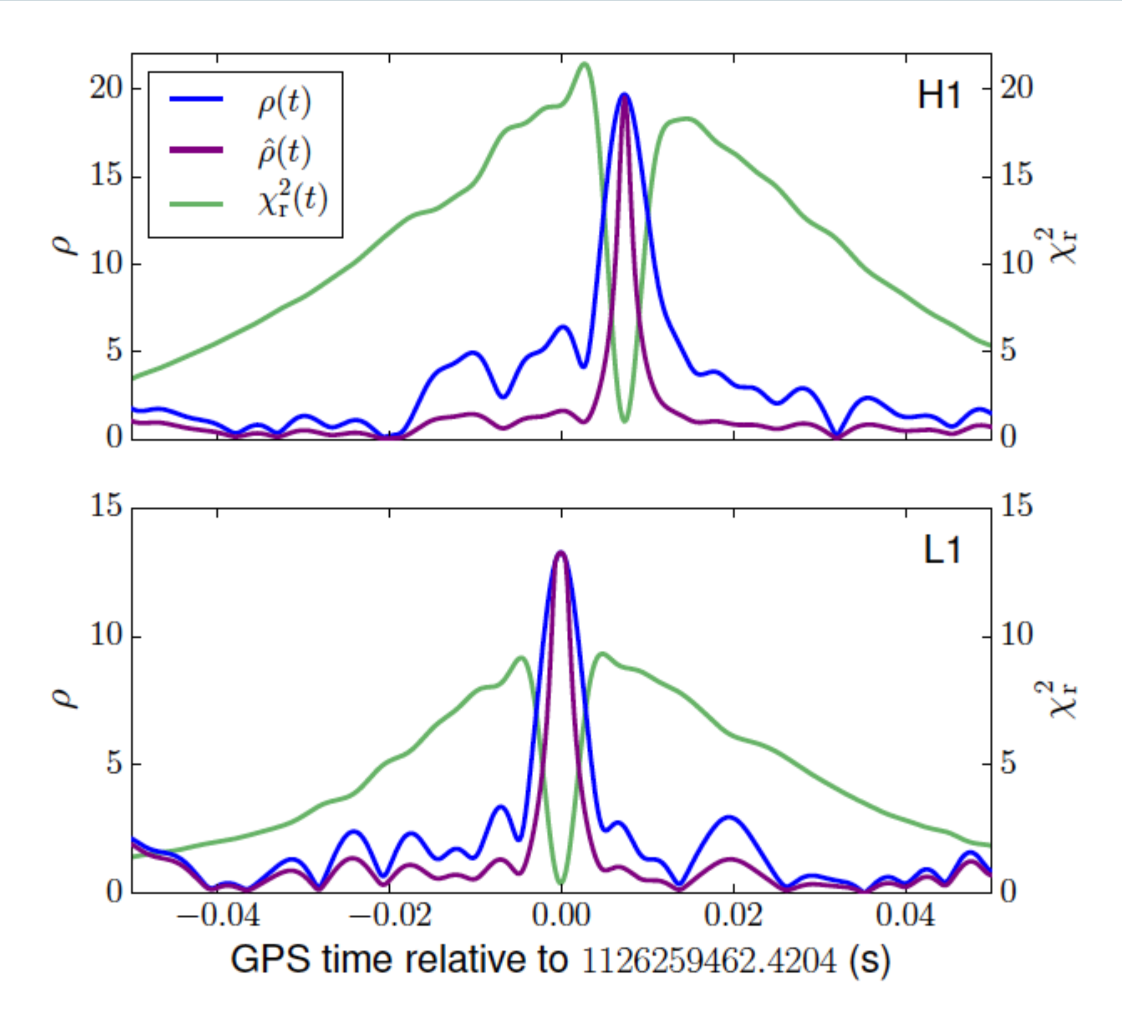
Further inspection means

- Statistical significance of the candidate (cross check with other pipelines)
- Status of the interferometers
- Check for environmental or instrumental causes
- Check intermediate stages of the analysis
- Check for coincidences with non-GW searches: other E/M or particle detectors when relevant

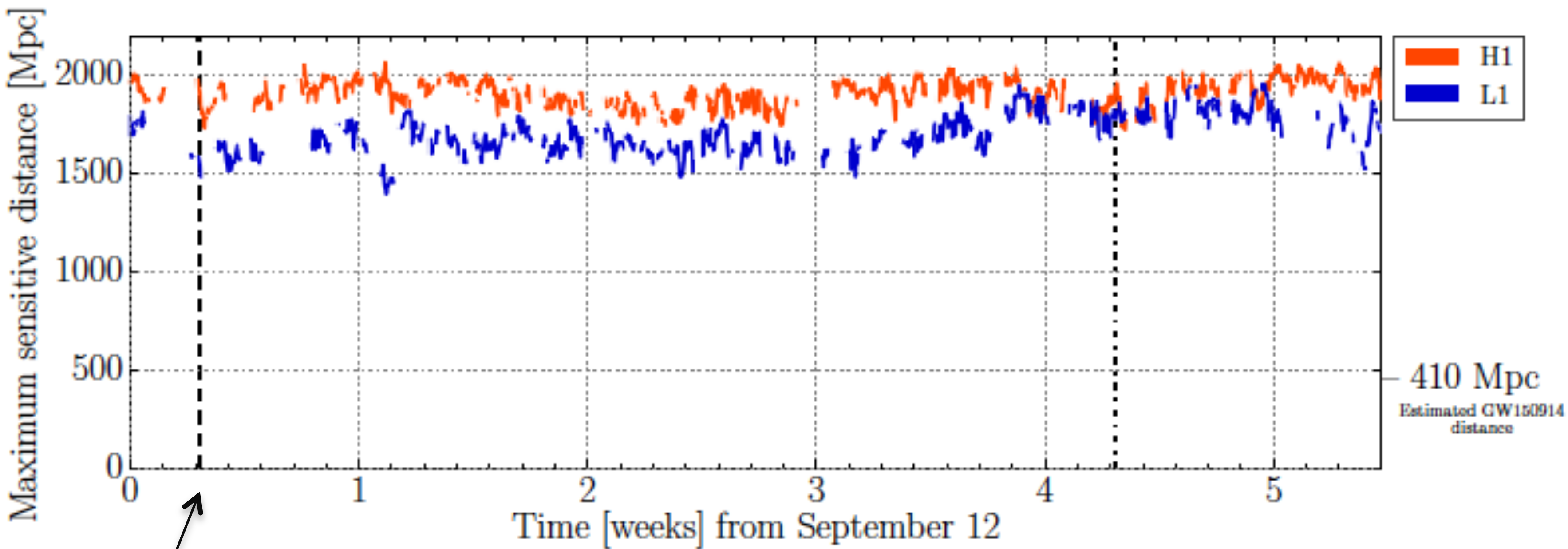
GW150914 as seen by the generic short-transient signals search



Intermediate analysis products



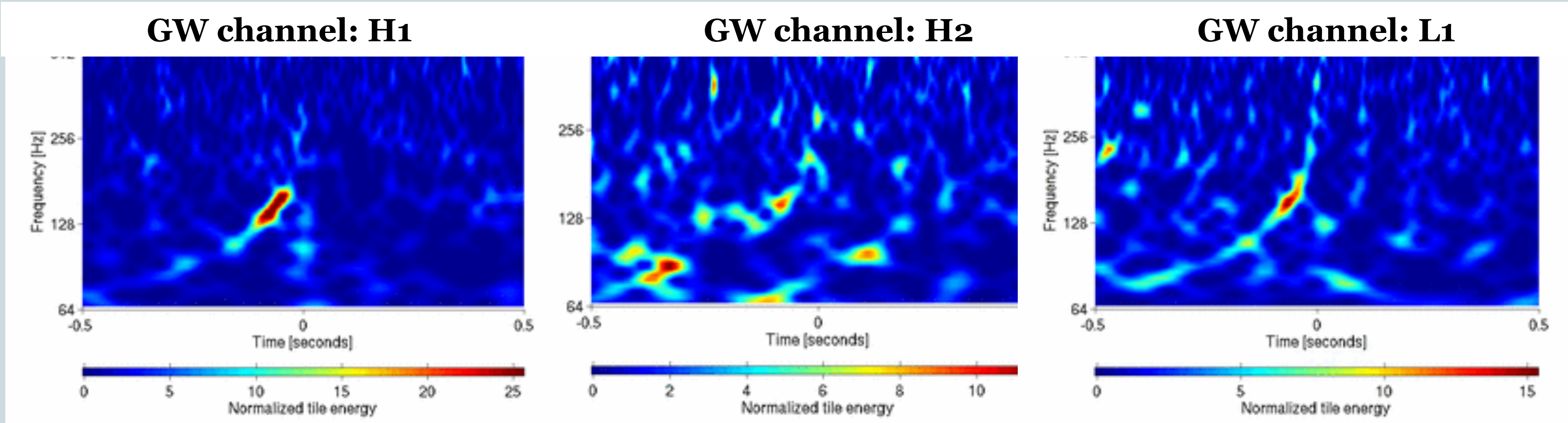
State of the interferometers: stability



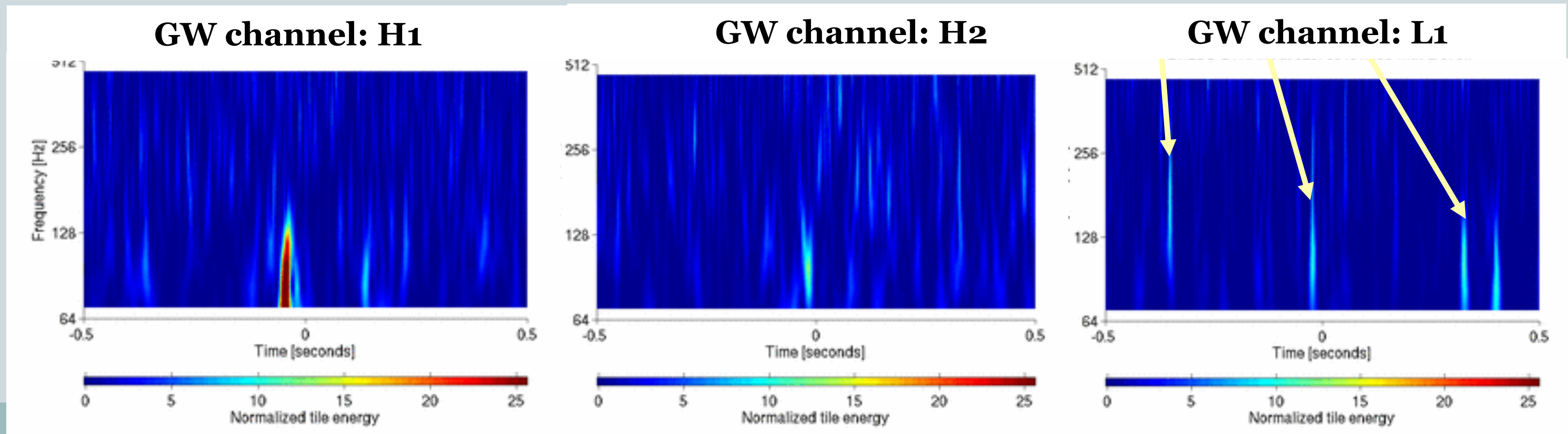
GW150914

Candidate appearance, examples from LIGOI science runs

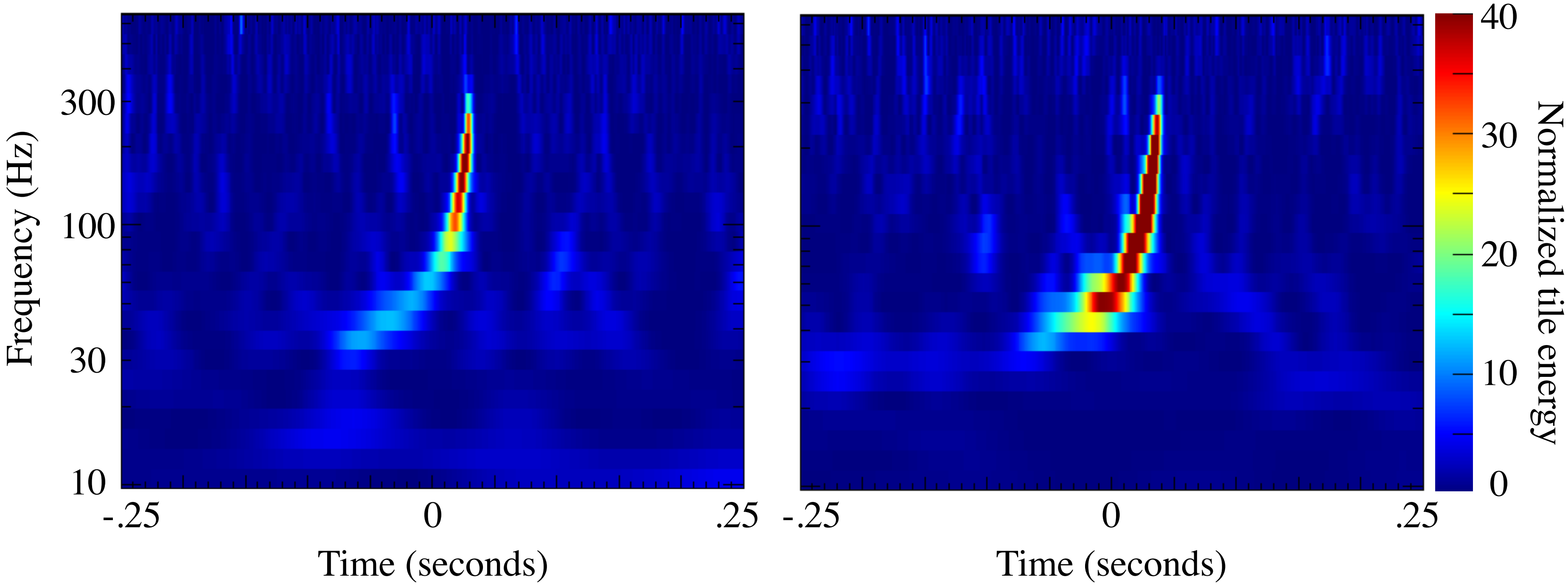
Trigger from simulated signal



Background trigger



GW150914 candidate appearance



GW150914, the first GW detection

Washington, 12 February 2016