Parallel metric trees for multi-billion bodies simulations

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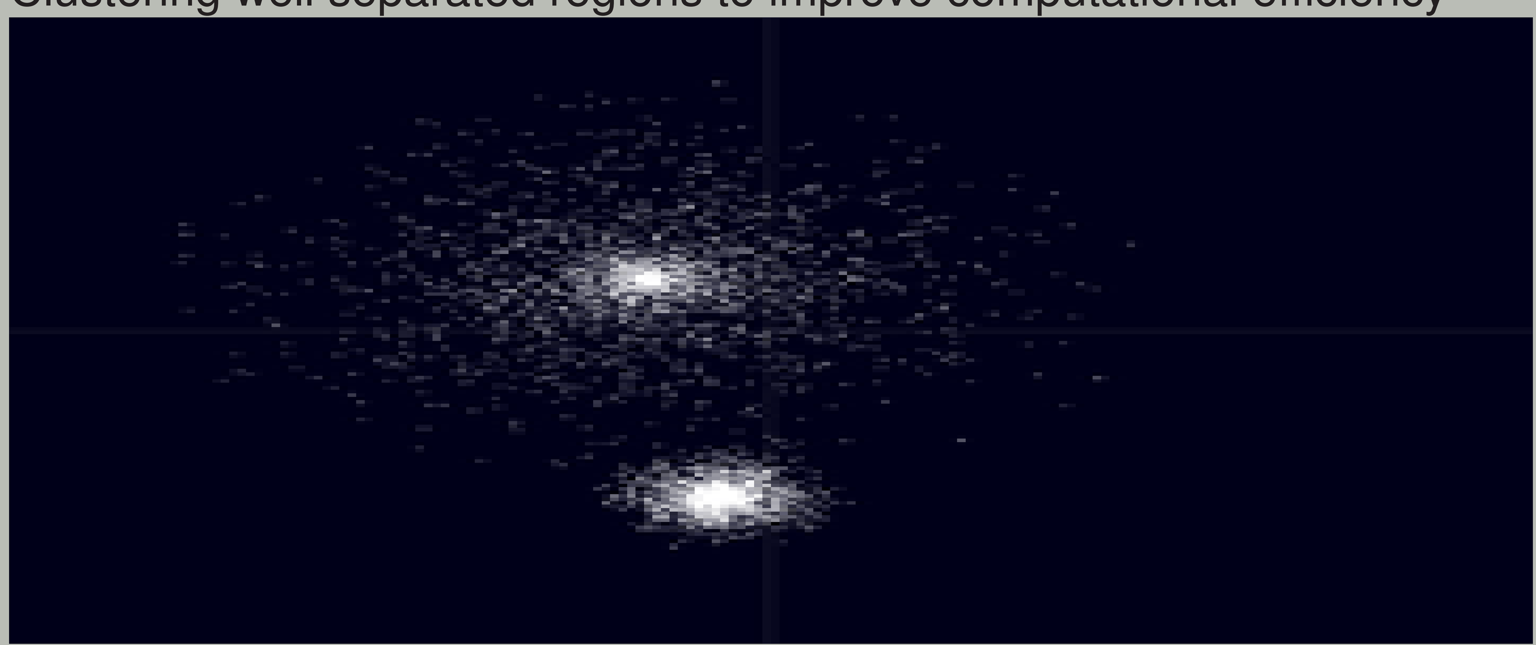
A research question

- Theoretical foundations and tool
  - Computational Geometry
    Vantage point trees
    Metric trees
  - Parallel algorithms
    Contraction algorithms
    Accumulation trees
  - Parallel architectures
    Multi-core
    GPGPU and extended device computing
    Distributed memory archittecures
- **The research question**
  How effective can the algorithms and data structures of theorectical parallel computing be when applied to practical problems in high dimensional domains.

High dimensional data processing examples

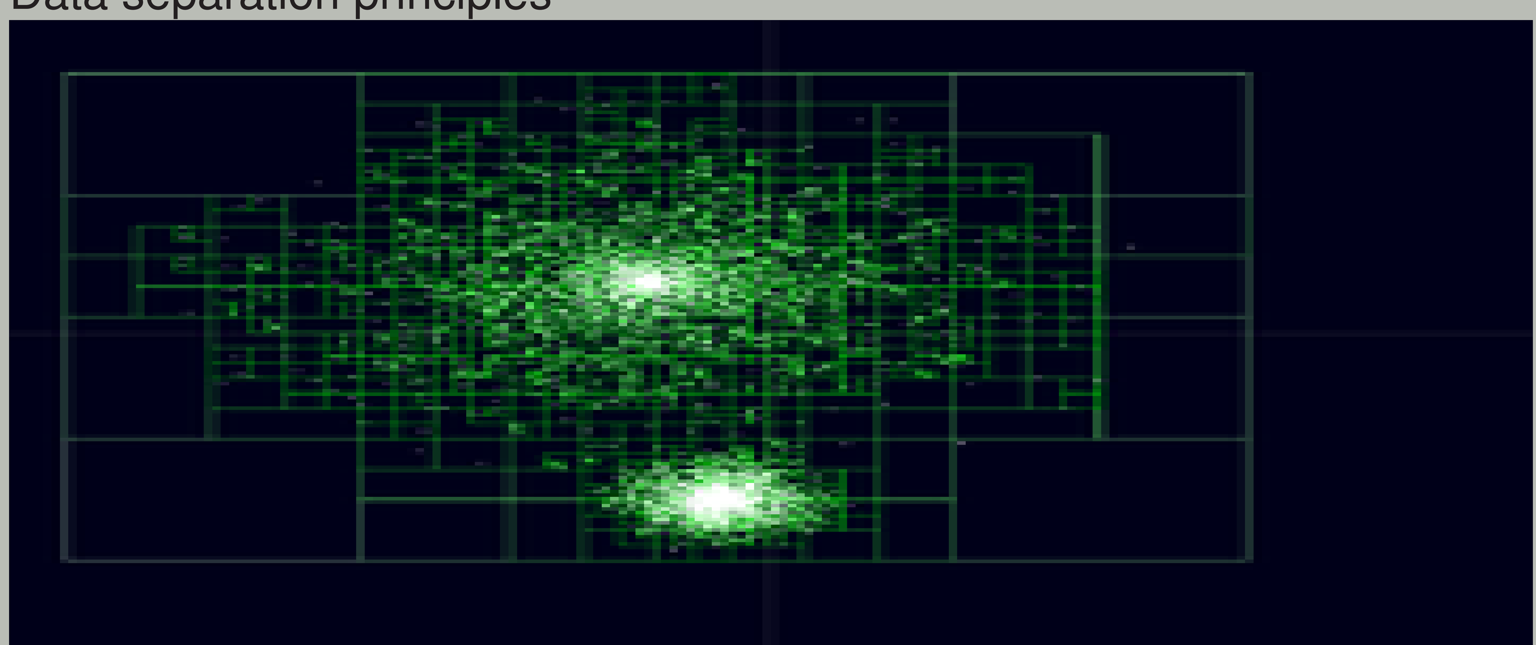
- Clustering well separated regions to improve computational efficiency



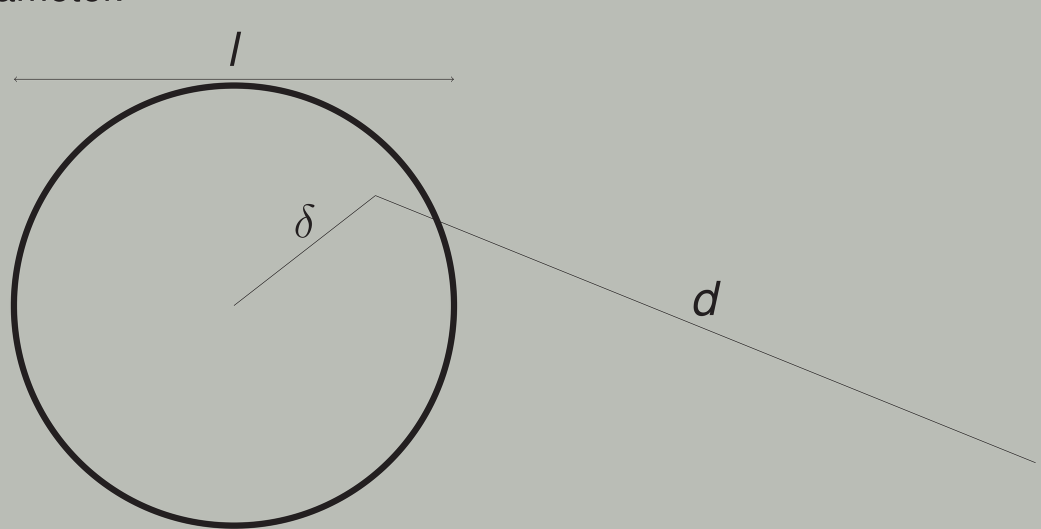
- Clustering well separated blocks of points to improve line detection.
  Possible figure here showing OpenCV HT failure
- Higher order visualization and sequence matching in DNA or data mining in time series.
  Sergey Brin (before Google): *A ball of radius 2 in 20 dimension Euclidean space is a million times larger in volume than a ball of radius*
  Metric trees can reduce partition the space.

Computational Geometry data structures

- Data separation principles



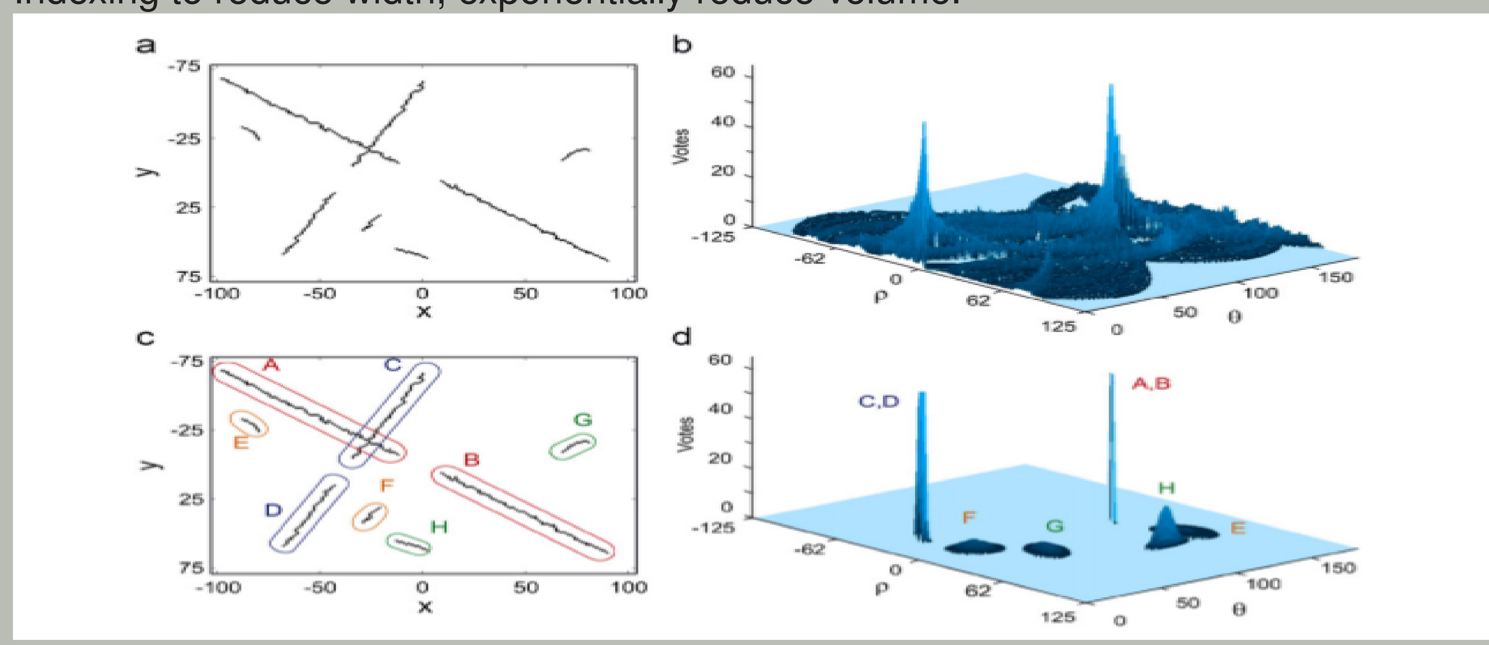
  - Mathematically well founded
  - Implemented using metric trees, or quad-trees.
  - Example: n-body tree codes.
    A particle is well separated from a region when $s/d < \theta$ for
    s, the width of the region
    d, distance between the paricle and the centre of mass of the region
    θ, a given parameter.



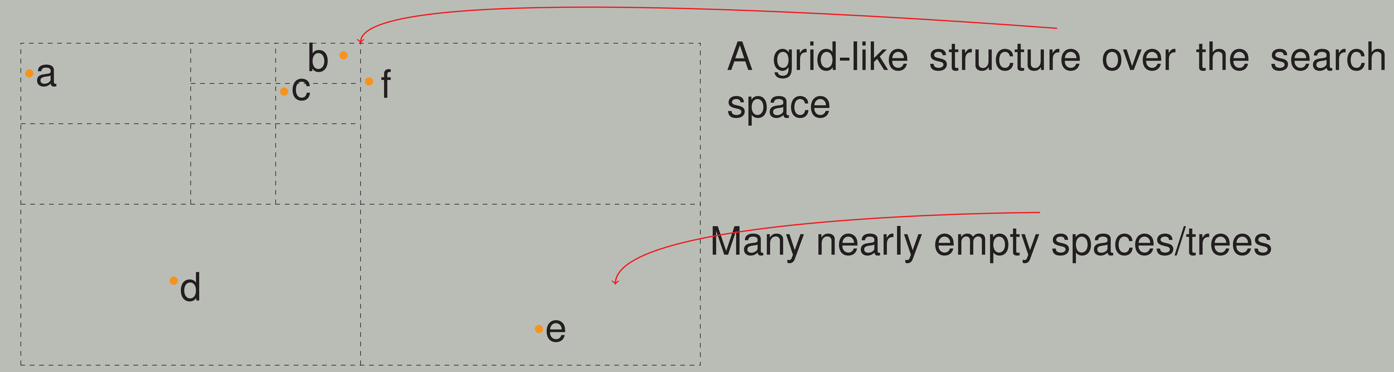
- Problems
  - Parallel algorithm to extract pairs separated regions and particles.
  - Parallel accumulation algorithms. Accumulation can be:
    FMM Taylor expansion;
    Accumulation of force and mass in Barnes-Hut;
    Binning (vote counting) in algorithms for variations of Hough Transform.

Trees for high dimensional data

- Target
  - Indexing to reduce width, exponentially reduce volume.



  - Balancing of construction to optimize sharing of parallel workload.
- Data structures to separate high dimensional data
  - Octrees, and quadtrees tend to be inefficient in space and parallel work balancing;
  - k-d-trees can become unbalanced or have irregularly split regions;



  - Metric and vantage-point tree might offer round regions and binary trees good for upward and downward accumulation.

Metric trees

- Binary search tree.
- Indexed by distance.
- A node defines a sphere of the points living within a distance from its centre.
- Axioms that regulate the construction of a mertic tree
  - Each node is define by a central point C, a radius r , left and right subtrees.
  - Points on the left subtree are within a distance r from C.
  - Points on the right subtree are outside distance r from C.
  - Central point is any point in the indexed set.
  - Radius is the median of distances to that central point.
- Observations:
  - Each node is weight balanced.
  - Each node can be split with linear work, if we assume that a median can be computed with linear work.

Point separation

- Two sets of points P and Q are separated when
$$\frac{max(diam(Q), diam(R))}{d(Q, R)} < \theta$$
- A separated pairs decomposition of a set P is a set of pairs $(A_i, B_i)\}$ where
  - each $A_i, B_i \subset P$
  - A_i and B_i are θ-separated.
- Parallel complexity
  An $O(n \lg n)$ work parallel algorithm with depth $O(\lg n)$ can be defined based the observation that:
  - if sets P and Q are not separated then the separation set of P and Q is given by the separation sets of P_0 and Q and P_1 and Q, where P_0 and P_1 are children of Q.

N-body tree code

- Algorithm: each Physical time step simulated by a sequence of three paralle bulk steps.
  - In parallel: build a metric tree.
    Complexity: Computation with depth $\lg n$.
  - In parallel: build separated sets.
  - In parallel: for each pair (A_i, B_i), apply cell to cell computation to evalute force, centre of mass, position.

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