

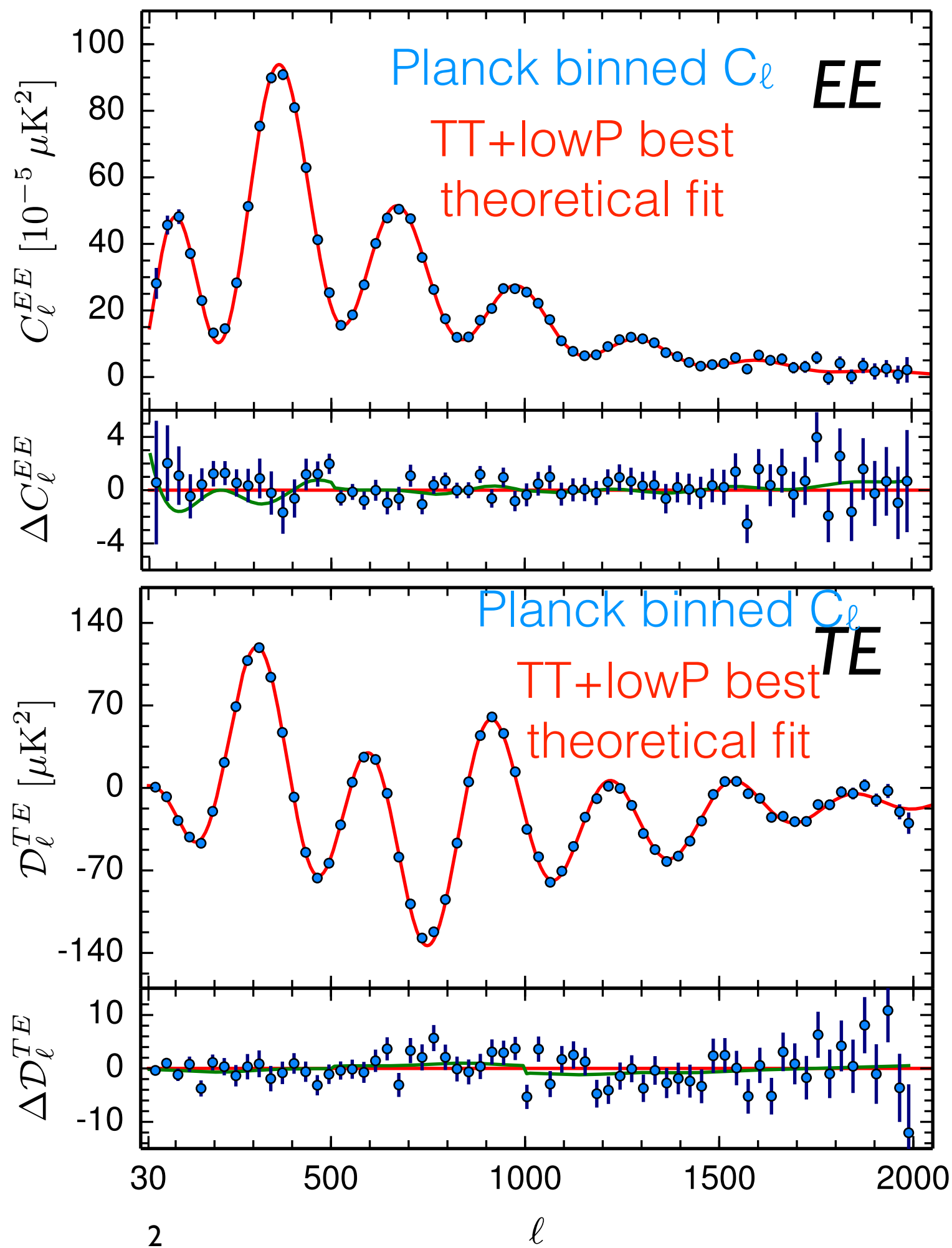
Polarised beam window functions & QUICKPOL

E. Hivon on behalf of the Planck collaboration

A few μK^2 residuals
seen in Planck
EE and TE
 $\ell^2 C_\ell$ spectra !

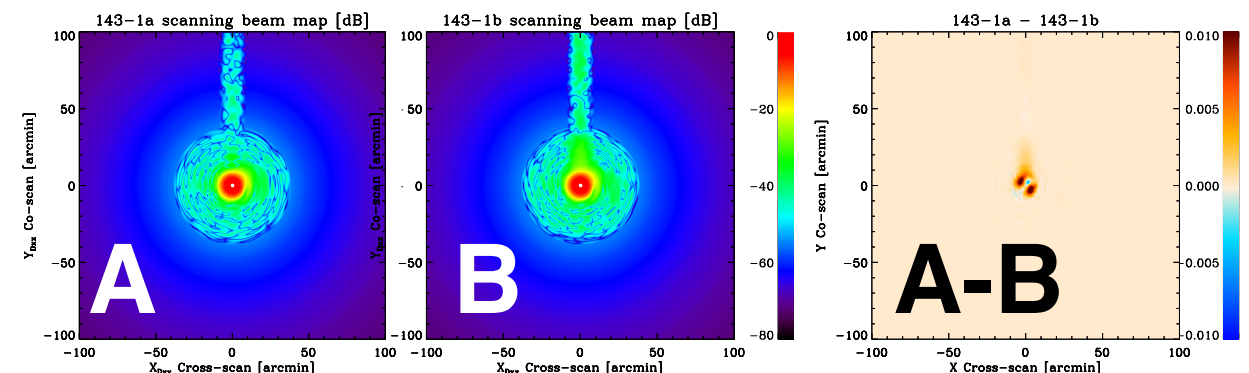
Could this be
related to beams?

*Planck 2015
Cosmological Parameters
paper*



Beam related power leakage

- Since polarisation measurement is differential, and no polarisation modulation (like HWP) in Planck beyond scanning
 - mismatches between a and b effective beams, (different in each sky pixel!) due to **differences** in
 - ▶ **scanning beams**
= optics + TF - deconvolution,
(see B. Crill presentation)
 - ▶ **noise level**
(if individual $1/\text{Noise}$ weighting in map making:
 $0 < \Delta\sigma^{-2}/\sigma^{-2} < 80\%$), and
 - ▶ **number of valid samples or valid rings**
($0 < \Delta n/n < 20\%$),
 - coupling with scanning strategy and NGP map making
 - cause (small scale) Temperature-Polarisation cross talk
- intensely studied (mostly for requirements of B mode measurements)



Challinor++ (2000), Souradeep & Ratra (2001), Fosalba++ (2002),
Hu++ (2003), Mitra++ (2004), O'Dea++ (2007), Smith++ (2007), Shimon++ (2008), Miller++ (2009), Mitra++ (2009),
Hanson++(2010), Rosset++ (2010), Ramamonjisoa++ (2013), Rathaus & Kovetz (2014), Wallis++ (2014), Pant++ (2015)

Different approaches to effect of beam mismatch on polarisation

● Numerical approaches

- ◆ Map deconvolution: PREBEAM (Armitage++ 2009), ARTDECO (Keihanen & Reinecke 2012),...
 - ★ IN: Observed polarised maps
 - ★ OUT: leakage free polarised maps
 - ★ ArtDeco used by LFI in 2015 analysis
- ◆ MC based description: FEBECOP (Mitra++ 2011, extended to polarisation)
 - ★ IN: MC simulated observations of fiducial sky with real beam and scanning
 - ★ OUT: Effective TT, EE, (TE) beam window functions
 - ★ used in 2015 CMB-only map analysis

● Analytical approaches

- ◆ 1) Backward:
 - ★ IN: rough modelling of leakage
 - ★ OUT: templates (with priors) of leakage to be fitted in final EE and TE $C(\ell)$
 - ★ used in 2015 Likelihood
- ◆ 2) Forward: QUICKPOL
 - ★ IN: precise calculation of leakage with real beam ($b_{\ell m}$) and scanning
 - ★ OUT: full beam matrix coupling TT, EE, TE, BB, TB, EB, ...
 - ★ this talk; will be used in 2016 Likelihood

I) Beam leakage in Plik analysis of 2014/2015 maps (DR2)

- backward approach: look in polarised “final” $C(l)$ for contamination templates and remove/marginalise them before cosmological analysis

► leakage model: $E_{\ell m} \mapsto E_{\ell m} + \varepsilon(\ell) T_{\ell m}$

$$\star \Delta C_{\ell}^{\text{TE}} = \varepsilon(\ell) C_{\ell}^{\text{TT}}$$

$$\star \Delta C_{\ell}^{\text{EE}} = \varepsilon(\ell)^2 C_{\ell}^{\text{TT}} + 2 \varepsilon(\ell) C_{\ell}^{\text{TE}}$$

► Templates used: $\varepsilon(\ell) = \varepsilon_0 + \varepsilon_2 \ell^2 + \varepsilon_4 \ell^4$

- because of

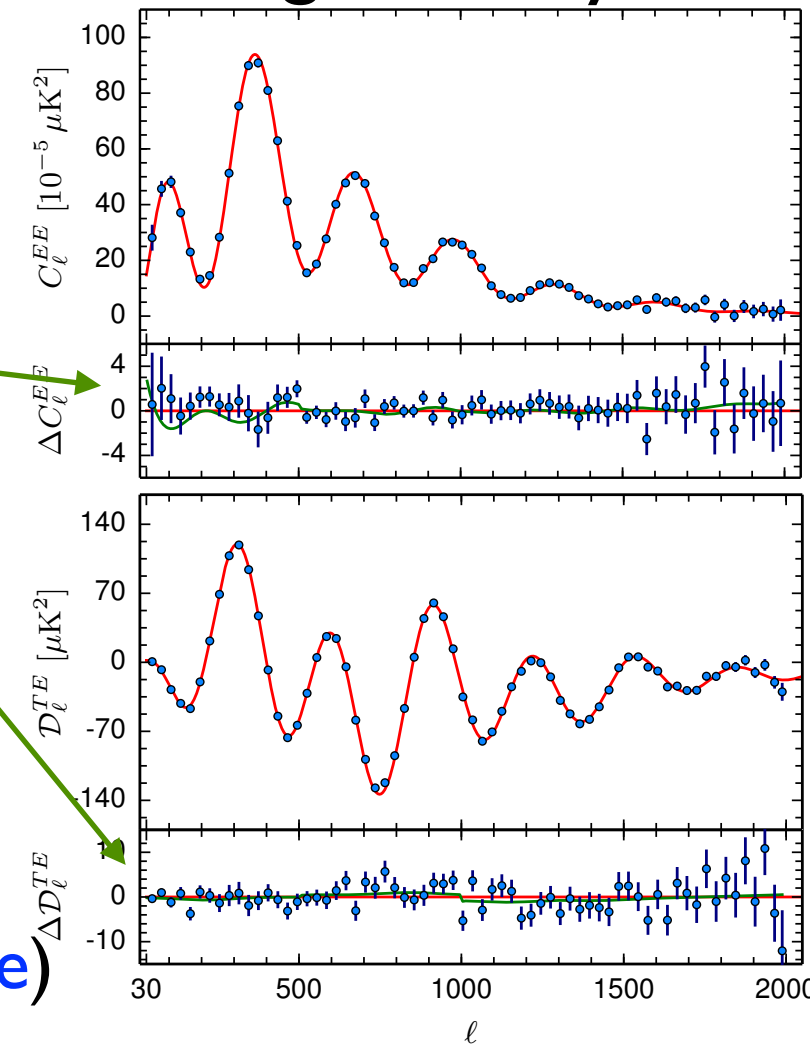
$$\star b_{\ell m} \propto (\theta_{\text{FWHM}} \ell)^m b_{\ell 0}$$

(the wider the beam, the worse the leakage)

★ scanning strategy (reduces odd degree terms)

- Gaussian priors of ε_m : mean = 0,
 $\sigma_0 = 1 \times 10^{-5}$, $\sigma_2 = 1.25 \times 10^{-8}$, $\sigma_4 = 2.7 \times 10^{-15}$

- See [Likelihood2015 paper](#)



2) QuickPol

- Temperature QuickBeam (used in DR1 and DR2):

$$\diamond C'_\ell{}^{TT} = \sum_s \omega_s^2 b_{\ell s}^* b_{\ell s} C_\ell^{TT}$$

► $b_{\ell s}$: weighted combination of scanning beams in DetSet,

► ω_s^2 : encodes scanning strategy (assumed to vary slowly across the sky)

- Temperature + Polarisation QuickPol (**New!**):

$$\diamond C'_\ell = \sum_{\alpha ij} \Omega_{\alpha ij} \otimes B_{\ell \alpha i}^{*t} \cdot C_\ell \cdot B_{\ell \alpha j}$$

► C : 3x3 $C(l)$ matrix

► B : weighted scanning polarised beams in DetSet

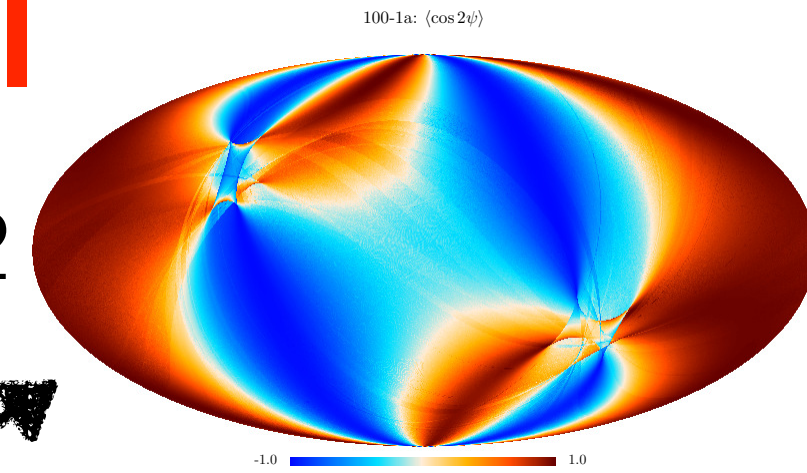
► Ω : encodes scanning strategy weighted by map-making IQU inverse covariance matrix
can be based on a subset of pixels !

♦ provides effective beam window matrix \mathbf{W}_ℓ describing C_ℓ coupling

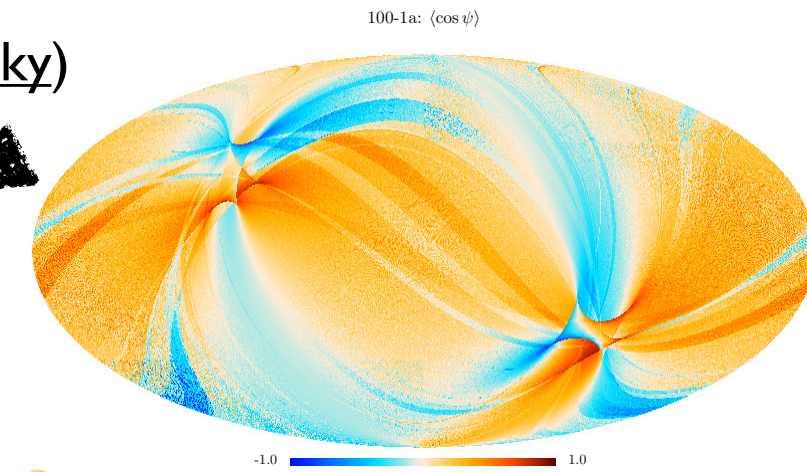
♦ has been extended to gain and polar efficiency uncertainty

♦ Backward $C(l)$ fitting can then still be used as a rain check to detect/catch remaining systematics

$\ell=2$



$\ell=1$



Map(s) Power Spectra

$$\begin{pmatrix} \tilde{C}_\ell^{TT} \\ \tilde{C}_\ell^{EE} \\ \tilde{C}_\ell^{BB} \\ \tilde{C}_\ell^{TE} \\ \tilde{C}_\ell^{TB} \\ \tilde{C}_\ell^{EB} \\ \tilde{C}_\ell^{ET} \\ \tilde{C}_\ell^{BT} \\ \tilde{C}_\ell^{BE} \end{pmatrix}$$

$$= \mathbf{W}_\ell \cdot$$

For each ℓ ,
 \mathbf{W}_ℓ is a 9x6
(diagonal dominated)
matrix

Sky Power Spectra

$$\begin{pmatrix} C_\ell^{TT} \\ C_\ell^{EE} \\ C_\ell^{BB} \\ C_\ell^{TE} \\ C_\ell^{TB} \\ C_\ell^{EB} \end{pmatrix}$$

TT column

EE column

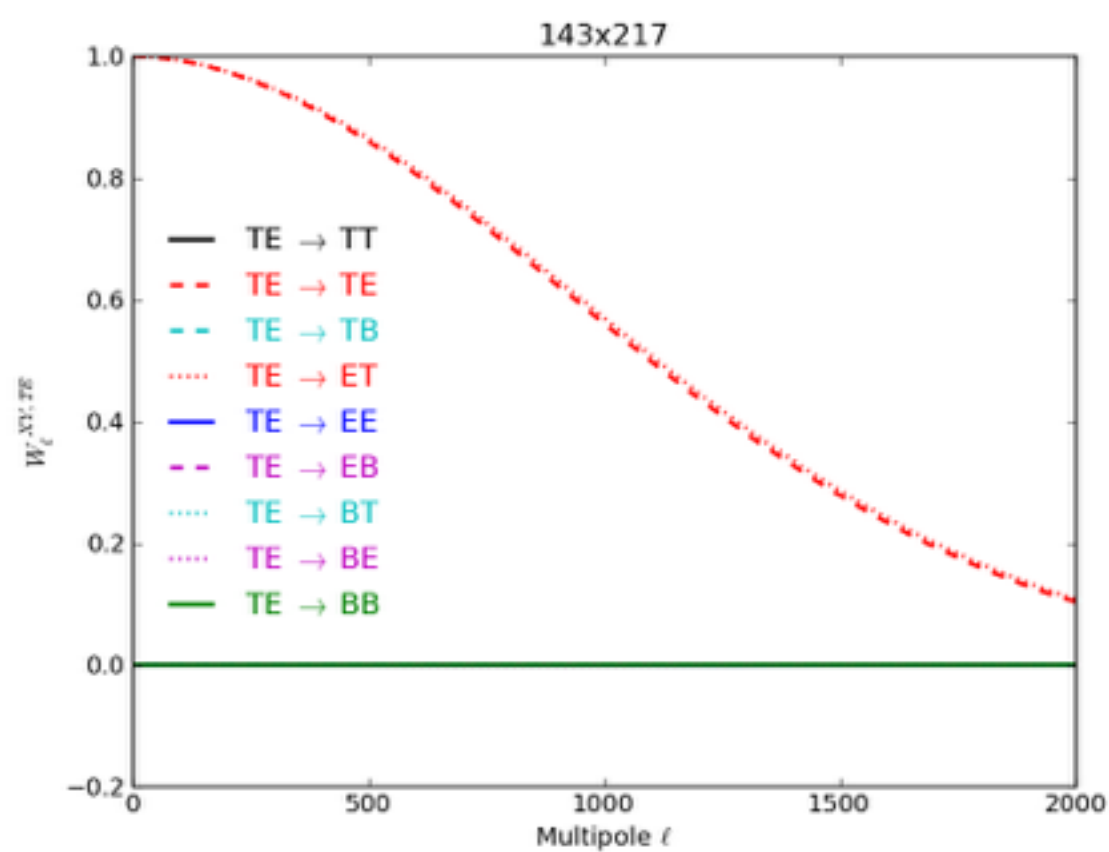
$$W_l^{XY,EE} = \sum_s \sum_{j_1 j_2} \frac{\rho'_{j_1} \rho'_{j_2}}{4}$$

ρ' : polar efficiency

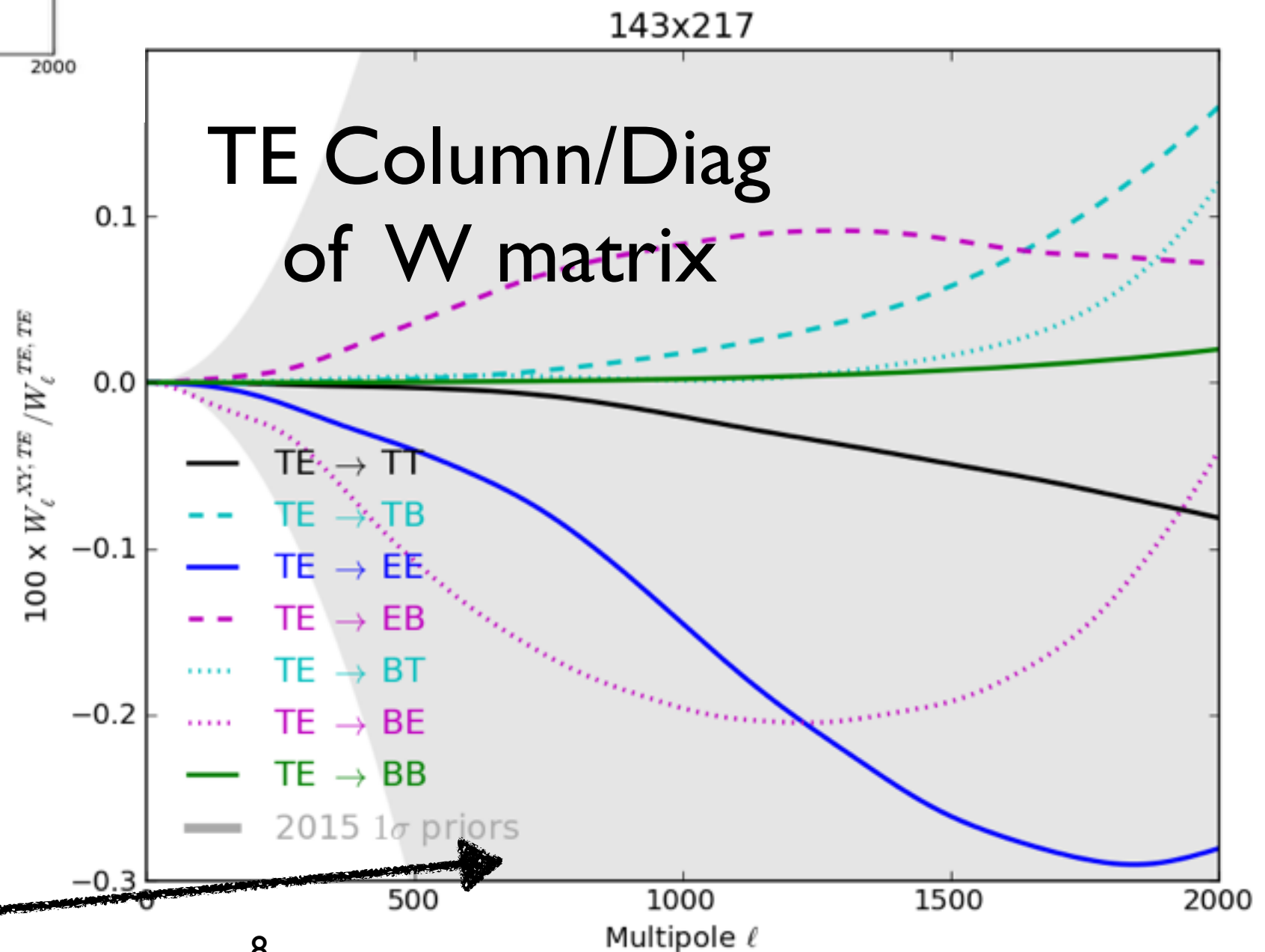
$$\left(\begin{aligned} & \hat{\Omega}_{00}^s (\hat{b}_{l,s-2}^{(j_1)*} + \hat{b}_{l,s+2}^{(j_1)*}) (\hat{b}_{l,s-2}^{(j_2)} + \hat{b}_{l,s+2}^{(j_2)}) \\ & \hat{b}_{l,s}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{-22}^s + \hat{\Omega}_{2-2}^s + \hat{\Omega}_{22}^s) + \hat{b}_{l,s+4}^{(j_2)} (\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{2-2}^s) + \hat{b}_{l,s-4}^{(j_2)} (\hat{\Omega}_{-22}^s + \hat{\Omega}_{22}^s) \right] \\ & + \hat{b}_{l,s+4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{-22}^s) + \hat{\Omega}_{-2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{-22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & + \hat{b}_{l,s-4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{2-2}^s + \hat{\Omega}_{22}^s) + \hat{\Omega}_{2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & \hat{b}_{l,s}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-2-2}^s - \hat{\Omega}_{-22}^s - \hat{\Omega}_{2-2}^s + \hat{\Omega}_{22}^s) + \hat{b}_{l,s+4}^{(j_2)} (\hat{\Omega}_{-2-2}^s - \hat{\Omega}_{2-2}^s) + \hat{b}_{l,s-4}^{(j_2)} (\hat{\Omega}_{22}^s - \hat{\Omega}_{-22}^s) \right] \\ & + \hat{b}_{l,s+4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-2-2}^s - \hat{\Omega}_{-22}^s) + \hat{\Omega}_{-2-2}^s \hat{b}_{l,s+4}^{(j_2)} - \hat{\Omega}_{-22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & + \hat{b}_{l,s-4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{22}^s - \hat{\Omega}_{2-2}^s) - \hat{\Omega}_{2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & - (\hat{b}_{l,s-2}^{(j_1)*} + \hat{b}_{l,s+2}^{(j_1)*}) \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{0-2}^s + \hat{\Omega}_{02}^s) + \hat{\Omega}_{0-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{02}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & - i (\hat{b}_{l,s-2}^{(j_1)*} + \hat{b}_{l,s+2}^{(j_1)*}) \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{02}^s - \hat{\Omega}_{0-2}^s) - \hat{\Omega}_{0-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{02}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & i \hat{b}_{l,s}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (-\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{-22}^s - \hat{\Omega}_{2-2}^s + \hat{\Omega}_{22}^s) - \hat{b}_{l,s+4}^{(j_2)} (\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{2-2}^s) + \hat{b}_{l,s-4}^{(j_2)} (\hat{\Omega}_{-22}^s + \hat{\Omega}_{22}^s) \right] \\ & + i \hat{b}_{l,s+4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-22}^s - \hat{\Omega}_{-2-2}^s) - \hat{\Omega}_{-2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{-22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & + i \hat{b}_{l,s-4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{22}^s - \hat{\Omega}_{2-2}^s) - \hat{\Omega}_{2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & - (\hat{b}_{l,s-2}^{(j_2)} + \hat{b}_{l,s+2}^{(j_2)}) \left[(\hat{\Omega}_{-20}^s + \hat{\Omega}_{20}^s) \hat{b}_{l,s}^{(j_1)*} + \hat{\Omega}_{-20}^s \hat{b}_{l,s+4}^{(j_1)*} + \hat{\Omega}_{20}^s \hat{b}_{l,s-4}^{(j_1)*} \right] \\ & i (\hat{b}_{l,s-2}^{(j_2)} + \hat{b}_{l,s+2}^{(j_2)}) \left[(\hat{\Omega}_{20}^s - \hat{\Omega}_{-20}^s) \hat{b}_{l,s}^{(j_1)*} - \hat{\Omega}_{20}^s \hat{b}_{l,s+4}^{(j_1)*} + \hat{\Omega}_{-20}^s \hat{b}_{l,s-4}^{(j_1)*} \right] \\ & i \hat{b}_{l,s}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{-22}^s - \hat{\Omega}_{2-2}^s - \hat{\Omega}_{22}^s) + \hat{b}_{l,s+4}^{(j_2)} (\hat{\Omega}_{-2-2}^s - \hat{\Omega}_{2-2}^s) + \hat{b}_{l,s-4}^{(j_2)} (\hat{\Omega}_{-22}^s - \hat{\Omega}_{22}^s) \right] \\ & + i \hat{b}_{l,s+4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{-2-2}^s + \hat{\Omega}_{-22}^s) + \hat{\Omega}_{-2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{-22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \\ & - i \hat{b}_{l,s-4}^{(j_1)*} \left[\hat{b}_{l,s}^{(j_2)} (\hat{\Omega}_{2-2}^s + \hat{\Omega}_{22}^s) + \hat{\Omega}_{2-2}^s \hat{b}_{l,s+4}^{(j_2)} + \hat{\Omega}_{22}^s \hat{b}_{l,s-4}^{(j_2)} \right] \end{aligned} \right) \quad (95)$$

143x217

TE column
of W matrix



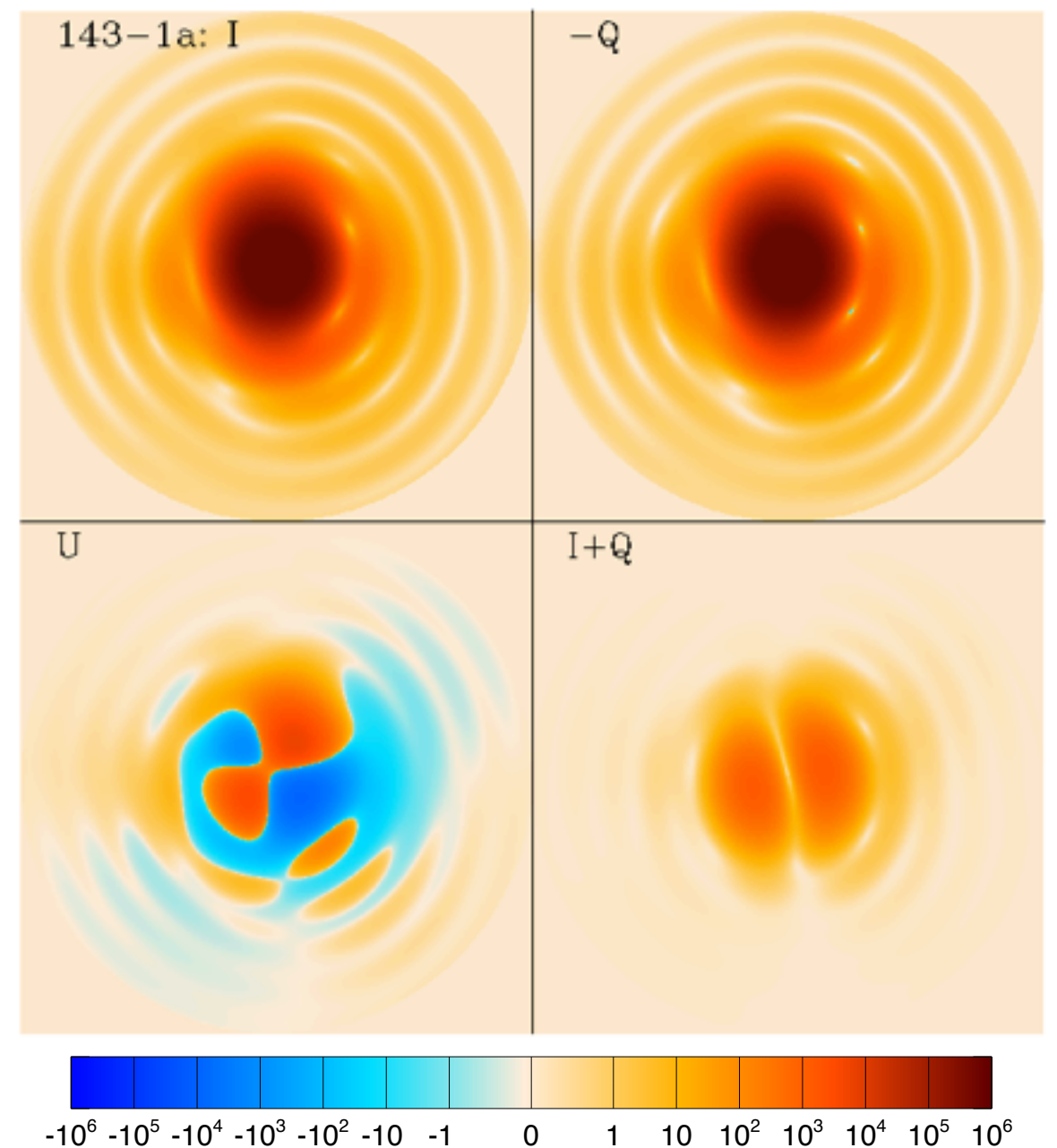
TE Column/Diag
of W matrix



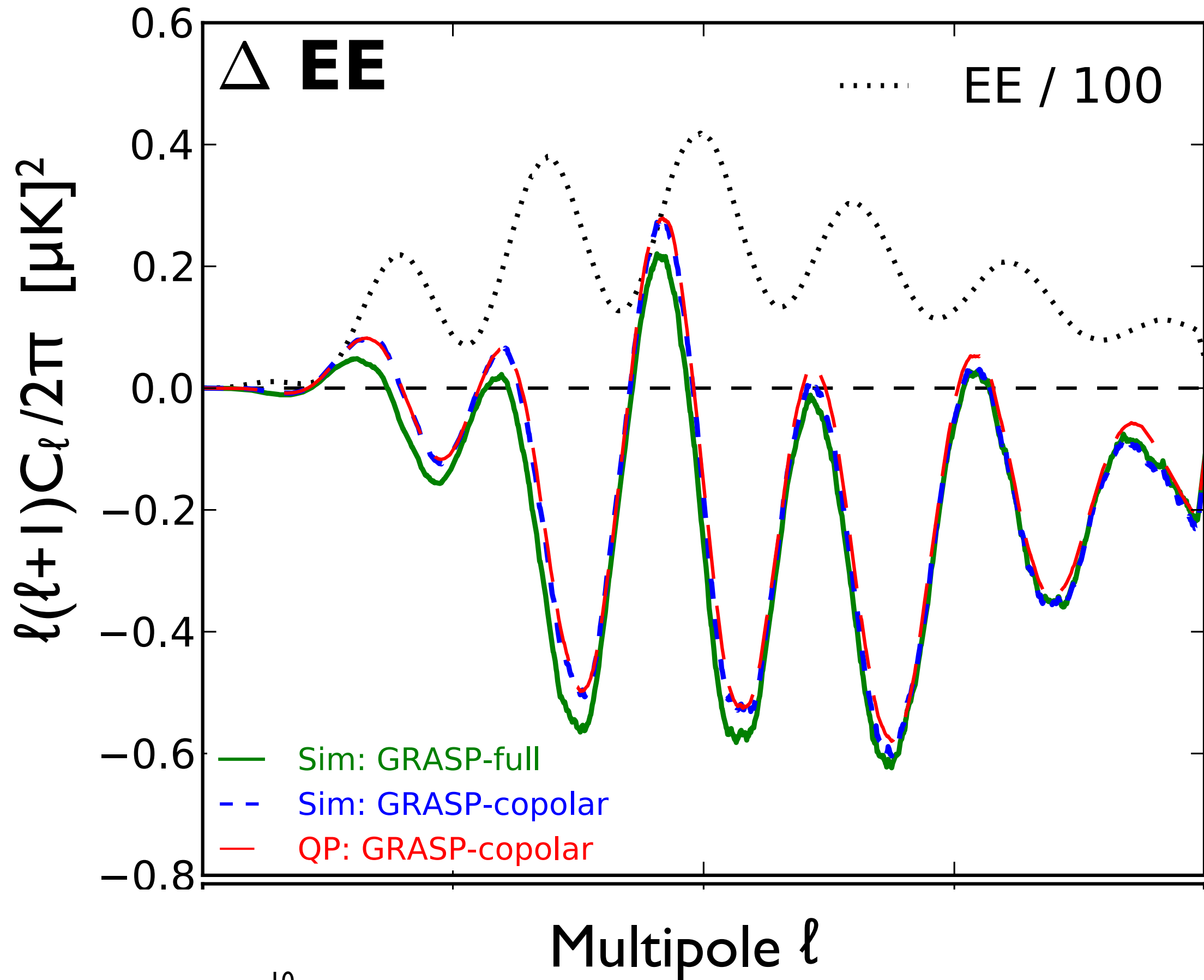
The 2015 prior was
wide enough !

Comparison to simulations

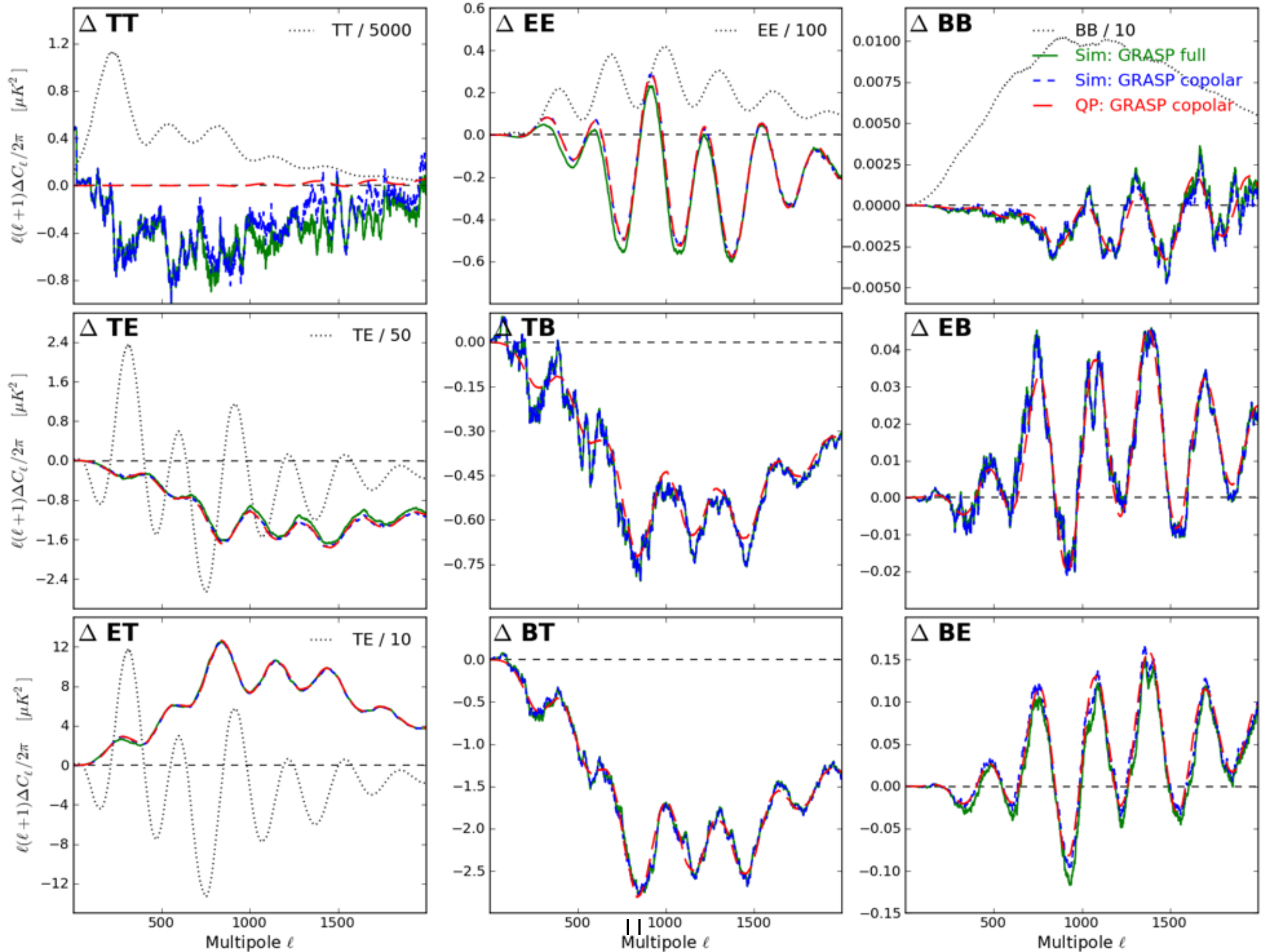
- Simulations using (some of) newly available HFI End-to-End simulation facility
 - ✦ CMB only
 - ✦ 100dsI, 143dsI, 217dsI
 - ✦ with **GRASP 2007** beam maps:
 - ▶ either full IQU maps,
 - ▶ or I maps only, assumed perfectly co-polar (as for actual beams)
 - ✦ imperfect bolometer polar efficiencies (Rosset et al 2010, IMO based)
 - ✦ same flags and bad rings as DR2
 - ✦ TODs generated with LS convicQT + multimod
 - ✦ maps produced with TOI2HPR+Polkapix_projector (assuming perfect calibration)



143ds1x217ds1

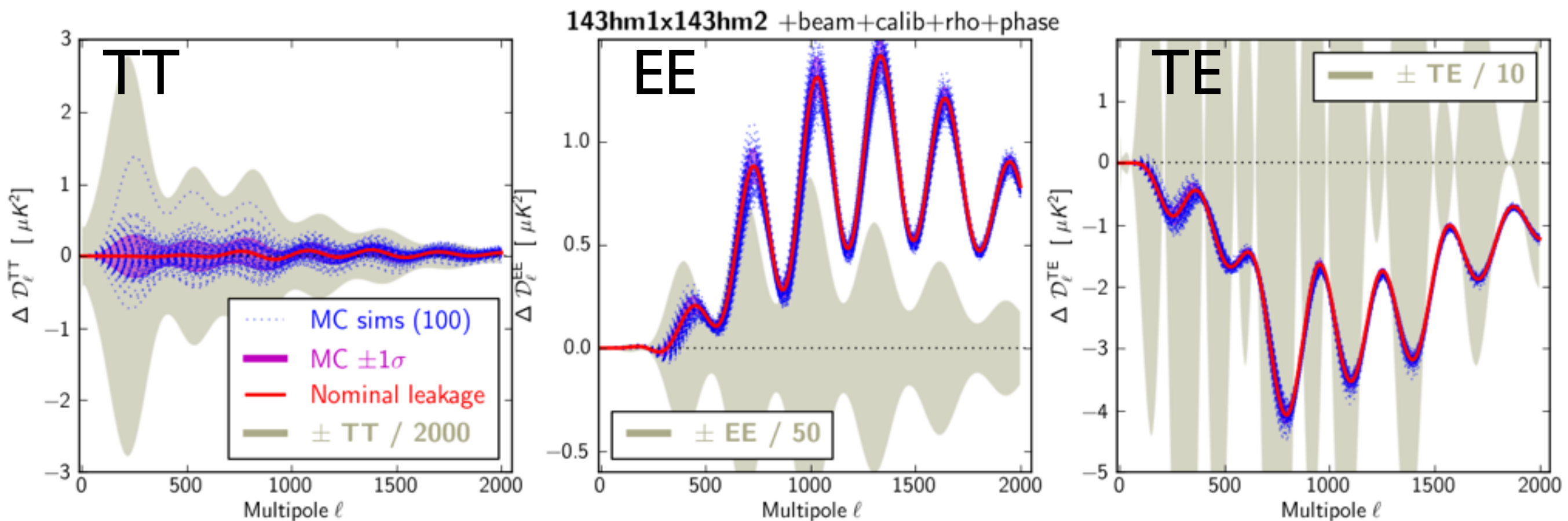


143ds1x217ds1



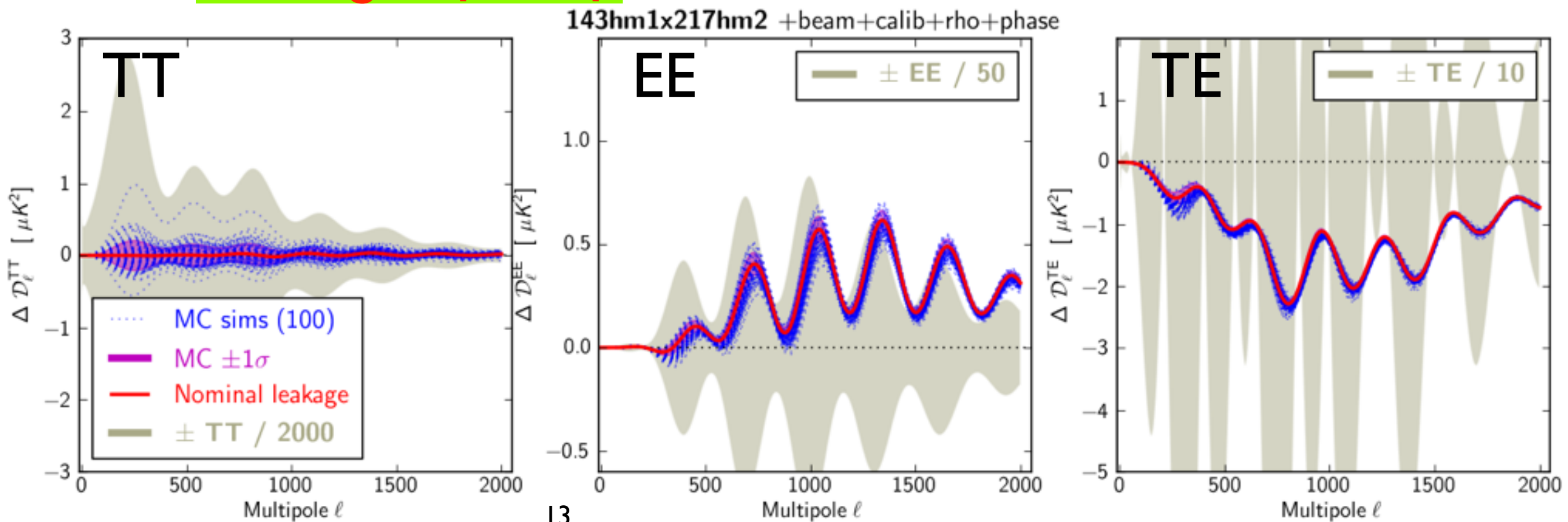
Error propagation

- MonteCarlo simulations of QuickPol are run quickly with the following uncertainties on each detector
 - ▶ beam measurements:
 - ★ detector scanning $b_{\ell m}$ from MC observation of planets,
 - ▶ gain calibration (g):
 - ★ Gaussian distributed (GD) around nominal value (1.0),
 - ★ $\delta g = 0.1\%$ @ 100-217GHz,
 - ▶ polar efficiency (ρ), $0 < \rho_{\text{SWB}} < \rho_{\text{PSB}} < 1$
 - ★ GD around IMO value,
 - ★ $\delta \rho = \text{a few } 0.1\%$ (read from Rosset+2010),
 - ▶ polarisation orientation (ψ):
 - ★ GD around IMO value,
 - ★ $\delta \psi = 1\text{deg}$ for PSB, 5deg for SWB (adapted from Rosset+2010).



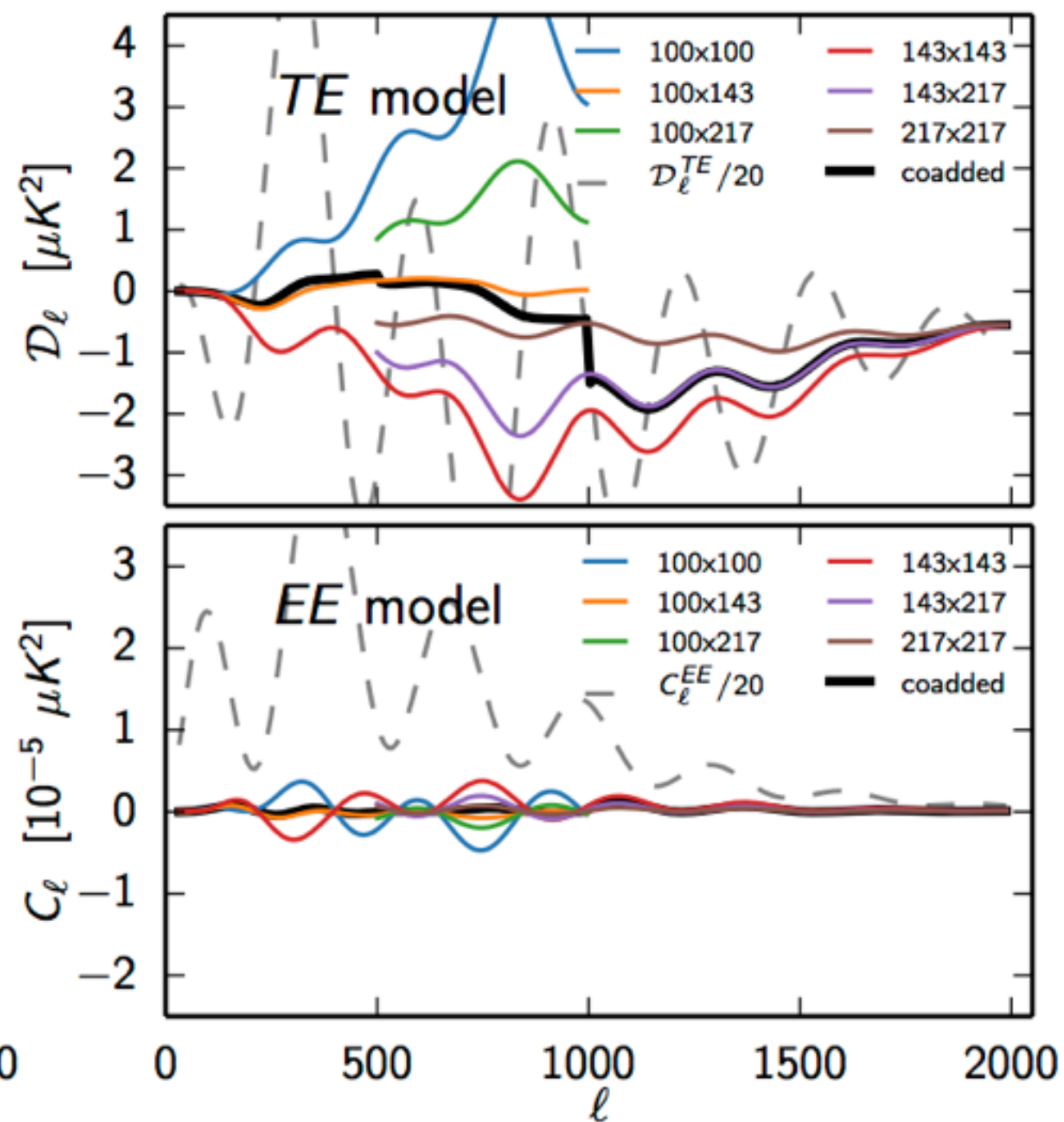
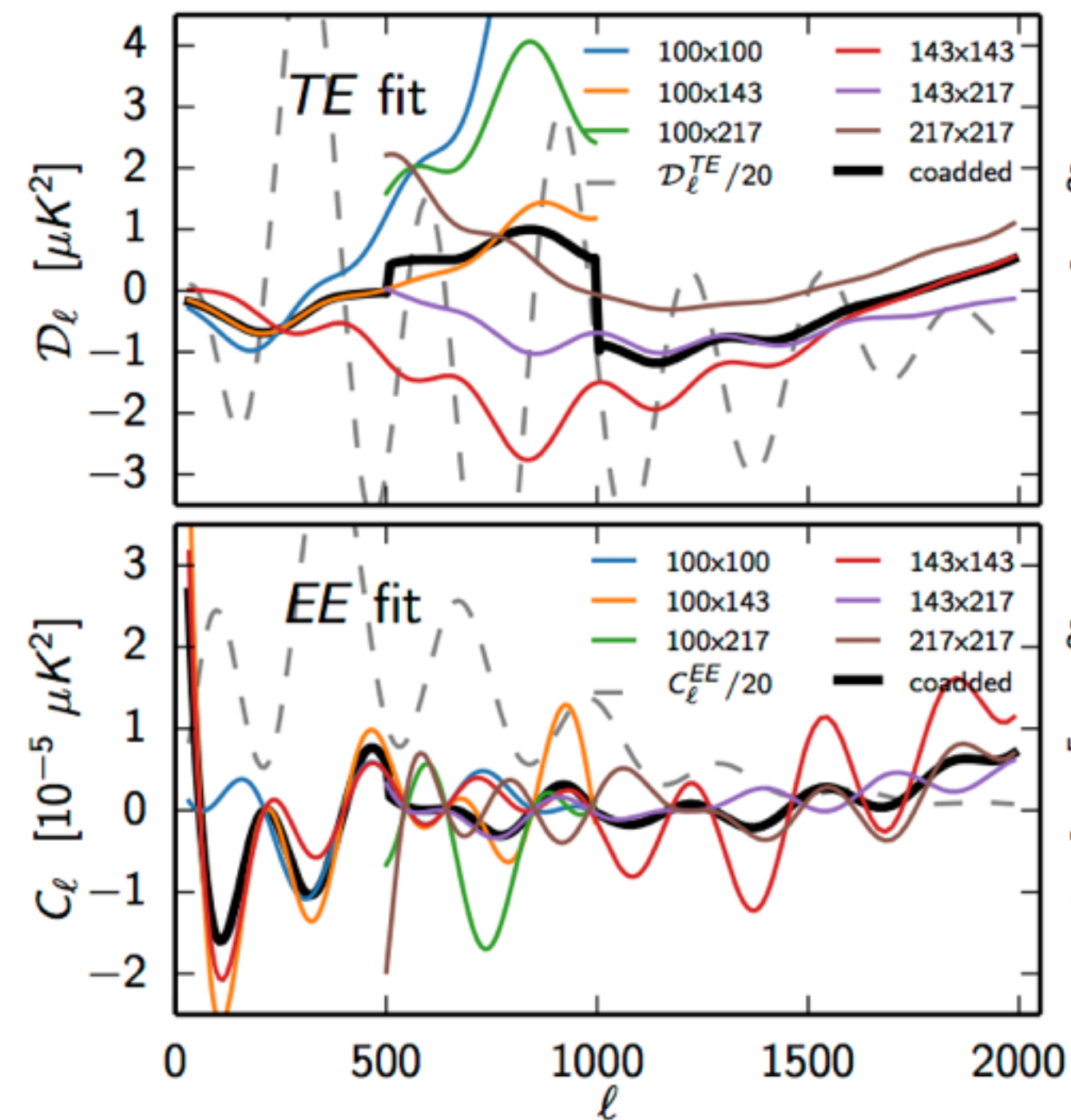
$b_{\ell m} + \delta g + \delta \rho + \delta \psi$

2 CPU.min per realisation!



a posteriori fit
(2015 likelihood paper)

QuickPol
a priori model



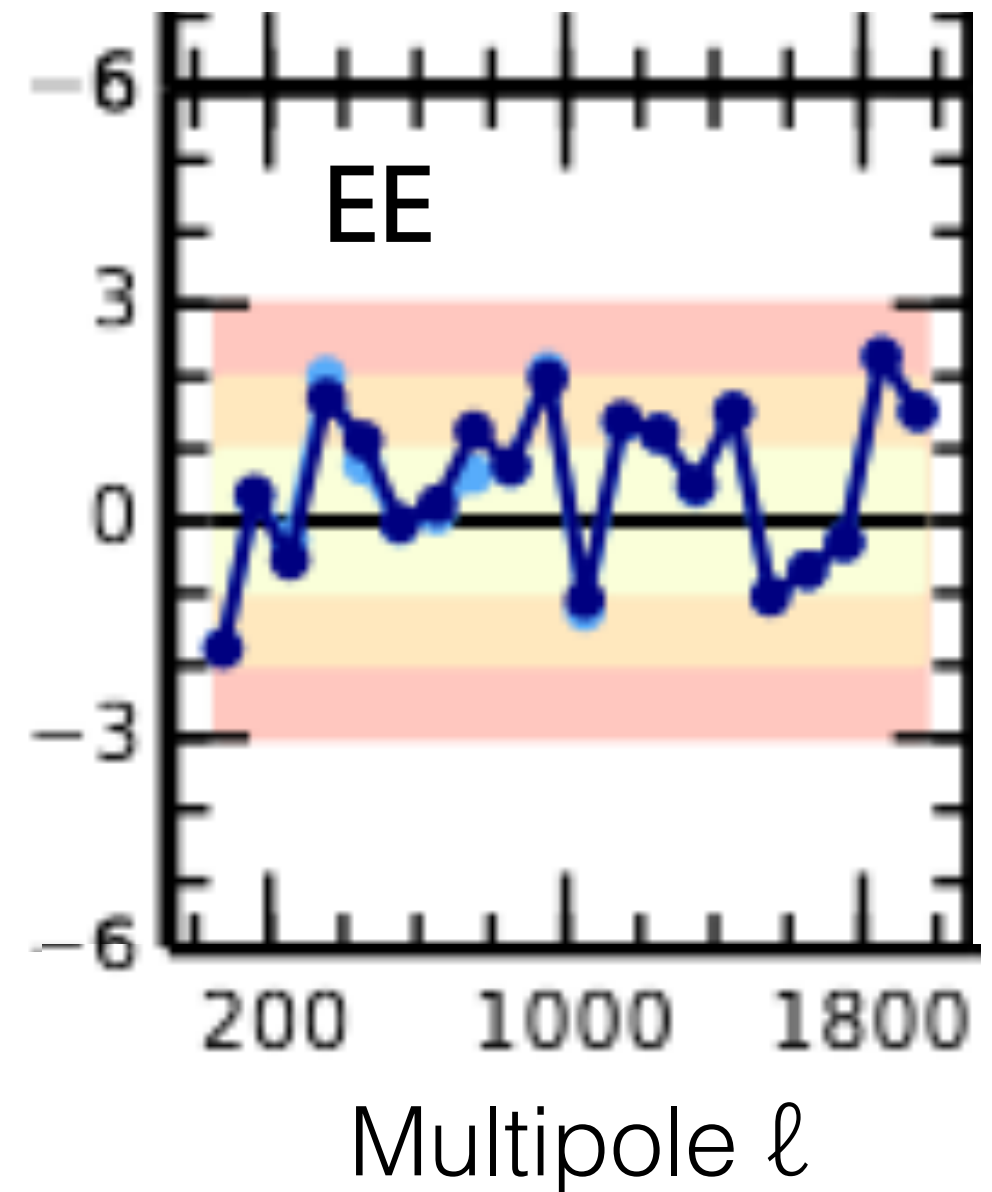
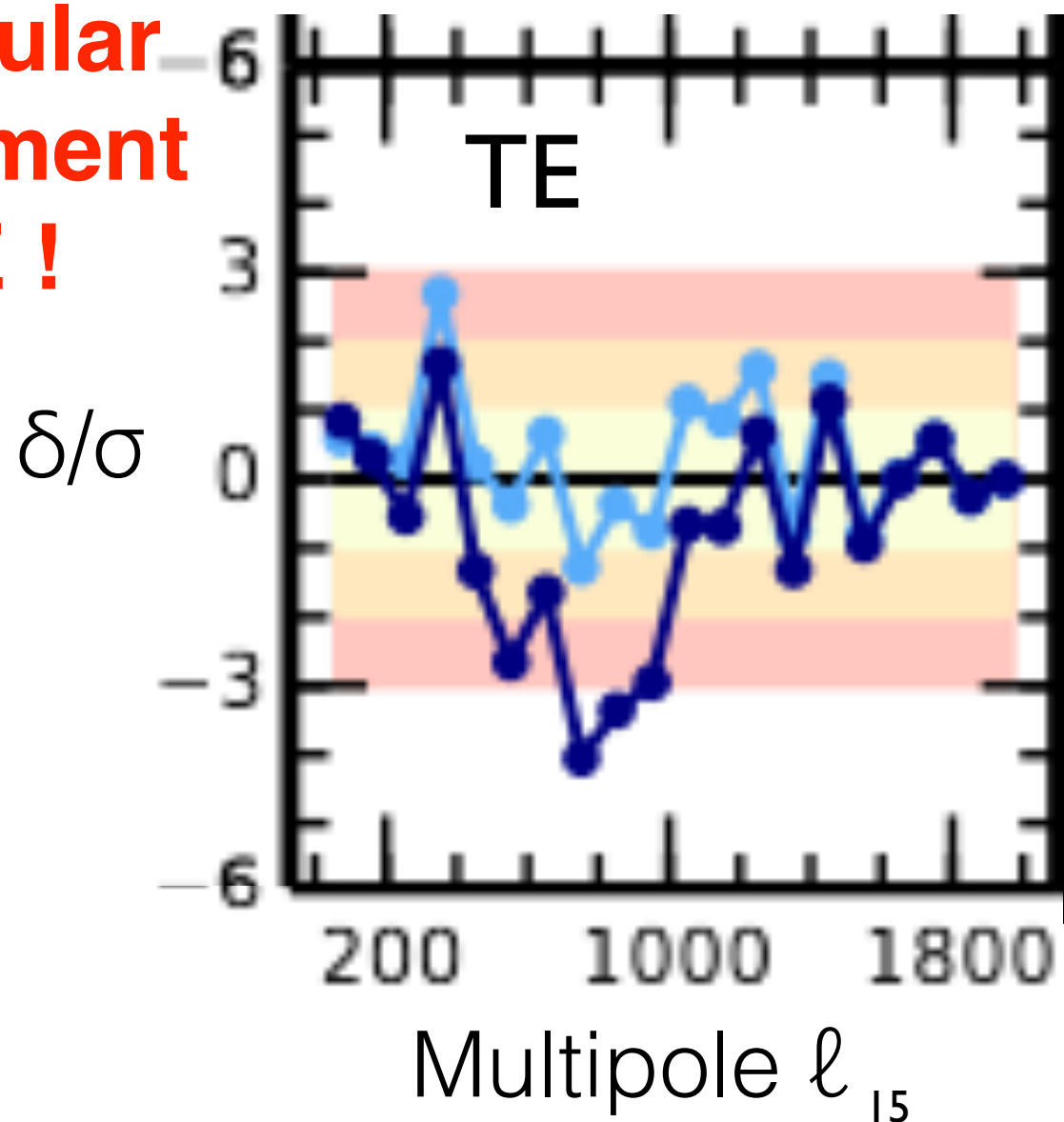
Inter-frequency consistency:
fg corrected $C(l)$
143x143 - 100x100

Preliminary!

Ignoring beam leakage (2015 analysis)

With beam leakage prediction+correction (2016 analysis)

**Spectacular
improvement
for TE !**



Application of QUICKPOL to LiteCore

- <http://coresat.planck.fr/index.php?n=E2ESims.QuickPol2>

- Mission models

- ◆ 2 different scannings: very close to the one used by Ranajoy simulations

- Duration: 380 days (95 precession periods)
 - Angle of precession axis α : 45 or 50 deg
 - Precession period : 4 days
 - Angle of spin axis β : 45 deg
 - Spin period : 60 seconds
 - Sampling frequency : 200Hz
 - Continuous scan
 - Single detector, no HWP

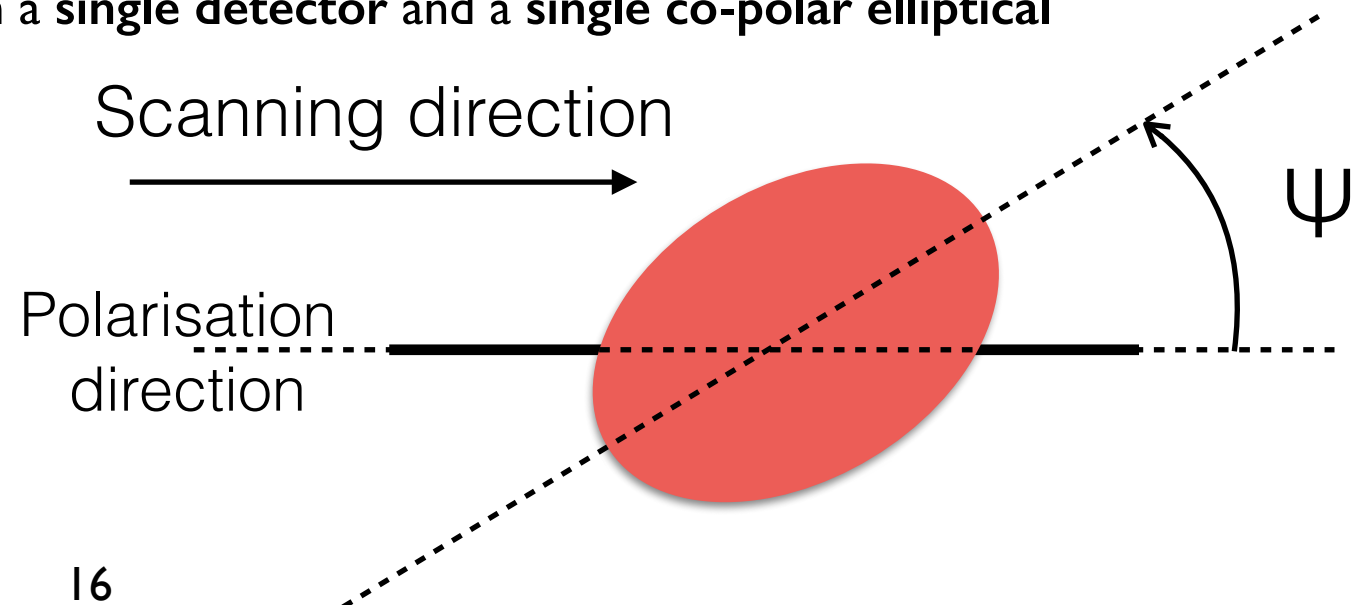
- ▶ Hit map and spins maps are computed, at Nside=1024, assuming a perfect polariser aligned with the fast motion of the detector on the sky (ie, co-scan). This is the longest step of the process.

- ◆ 4 Beam models

- ▶ One assumes the observation to be done with a **single detector** and a **single co-polar elliptical Gaussian beam** chosen among

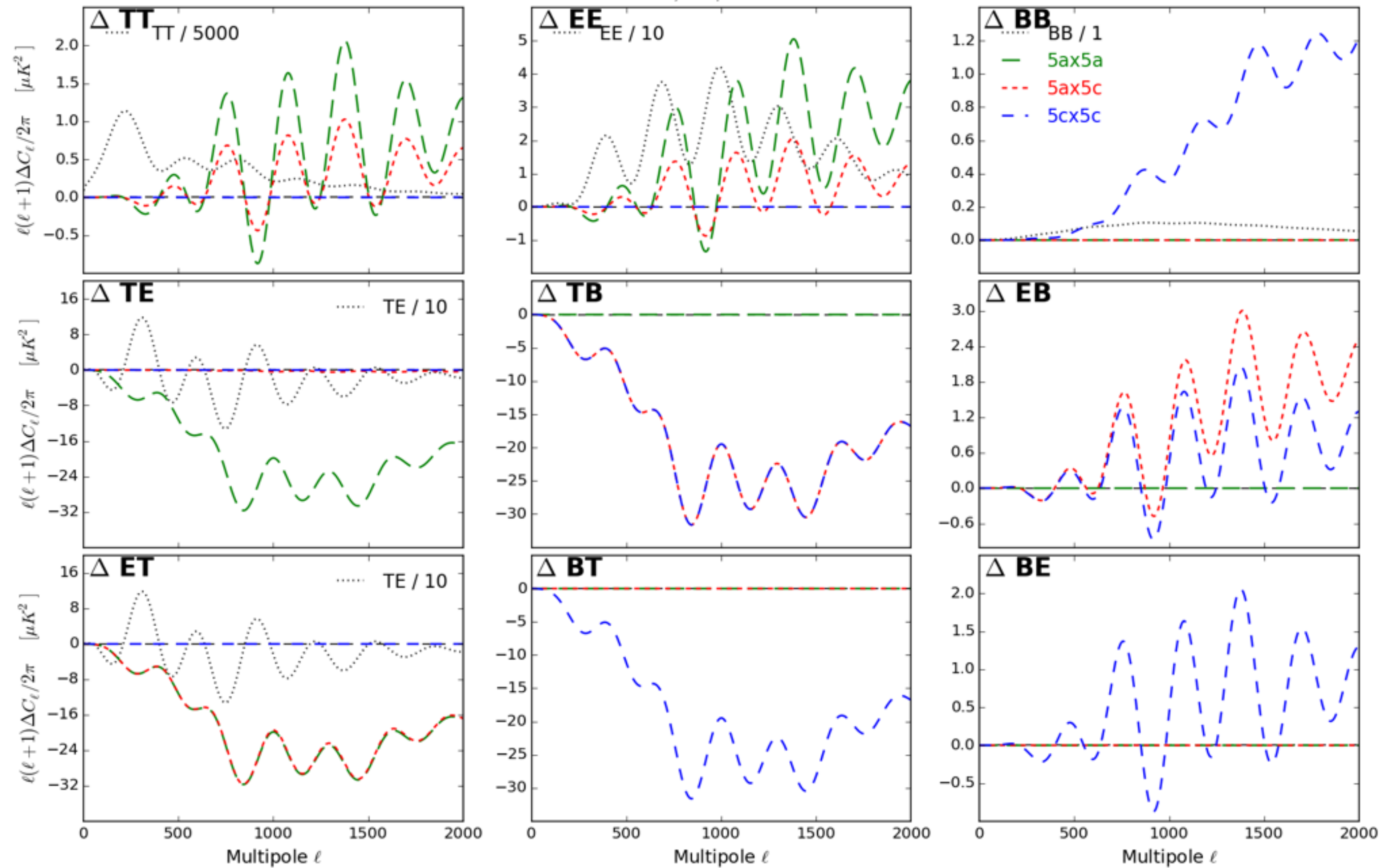
- **5a**: FWHM=5 and 5.5 arcmin, $\psi=0$
 - **5c**: same FWHMs, $\psi=45\text{deg}$
 - **7a**: FWHM=7 and 7.7 arcmin, $\psi=0$
 - **7c**: same FWHMs, $\psi=45\text{deg}$

- ▶ and the corresponding b_{lm} are computed.



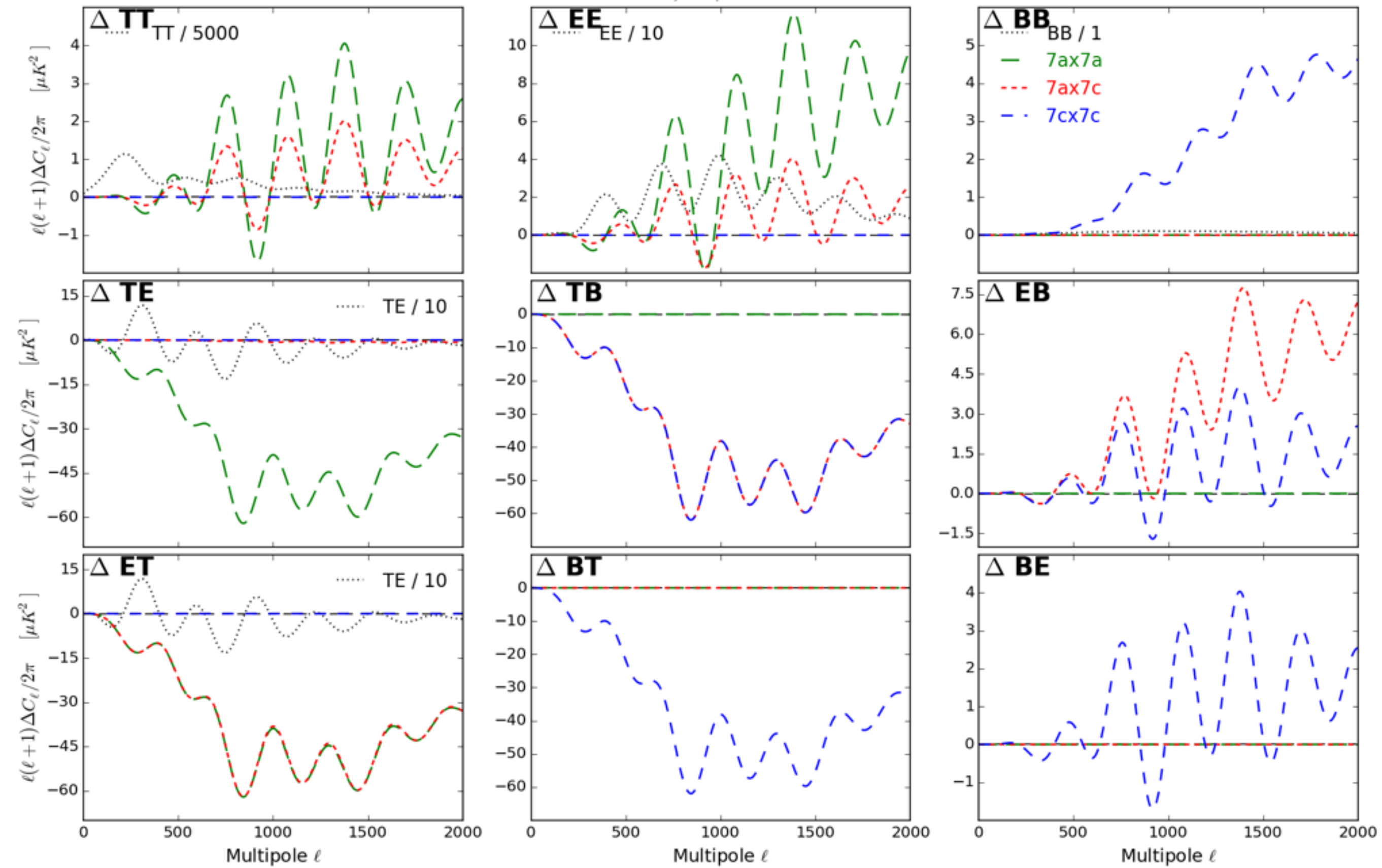
5' x 5.5'

$\alpha=50, \beta=45$



$7' \times 7.7'$

$\alpha=50, \beta=45$

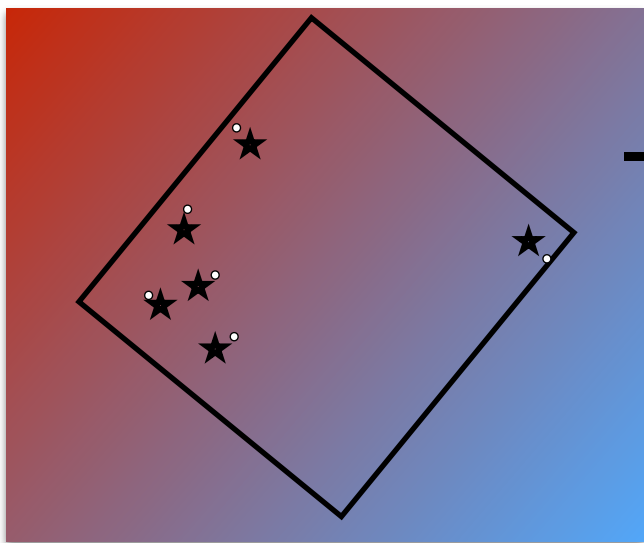


QUICKPOL + LiteCore summary

- The impact of the beam elongation on the T and P power spectra can be computed very fast.
- One observes that
 - ◆ at this level of idealism:
 - ▶ a single detector is enough to get I,Q,U and all the spectra simultaneously over the whole sky
 - ▶ the precession angle (α) makes no difference;
 - ◆ as expected, the leakages are larger for larger FWHM, at constant long to short axis ratio;
 - ◆ the leakage of temperature toward EE or BB depends on the angle (ψ) of the beam long axis with the polarizer (green and blue curves). It is therefore possible to mitigate it by cross correlating $\psi=0$ detectors with $\psi=45$ detectors (red curves).

Sub-pixel effects and pointing error

- 2 effects due to non-uniform sky signal at scales $<$ pixel size, both described as extra “noise” terms = offset * gradient of signal, (same formalism as Gravitational Lensing + leakage $T \rightarrow P$)
 - ▶ **Sub-pixel effects and pixelized map:**
 - signal *usually* assumed uniform in pixel during map making (NGP),
 - but samples distributed all over pixel, far ($\sim 60''$) from pixel nominal center ,
 - for Planck-HFI frequency maps (averaged over many samples, several detectors):
 - ★ hits center of mass $\sim 6''$ from pixel center,
 - ★ offset weakly correlated between pixels (\sim white noise)
 - ▶ **Pointing error:**
 - small ($\sim 3''$) offset between real and measured sample position,
 - how does it averages in each pixel over samples and detectors ?



Sub-pixel effects and pointing error

Measured power spectra (X,Y in {T,E,B}):

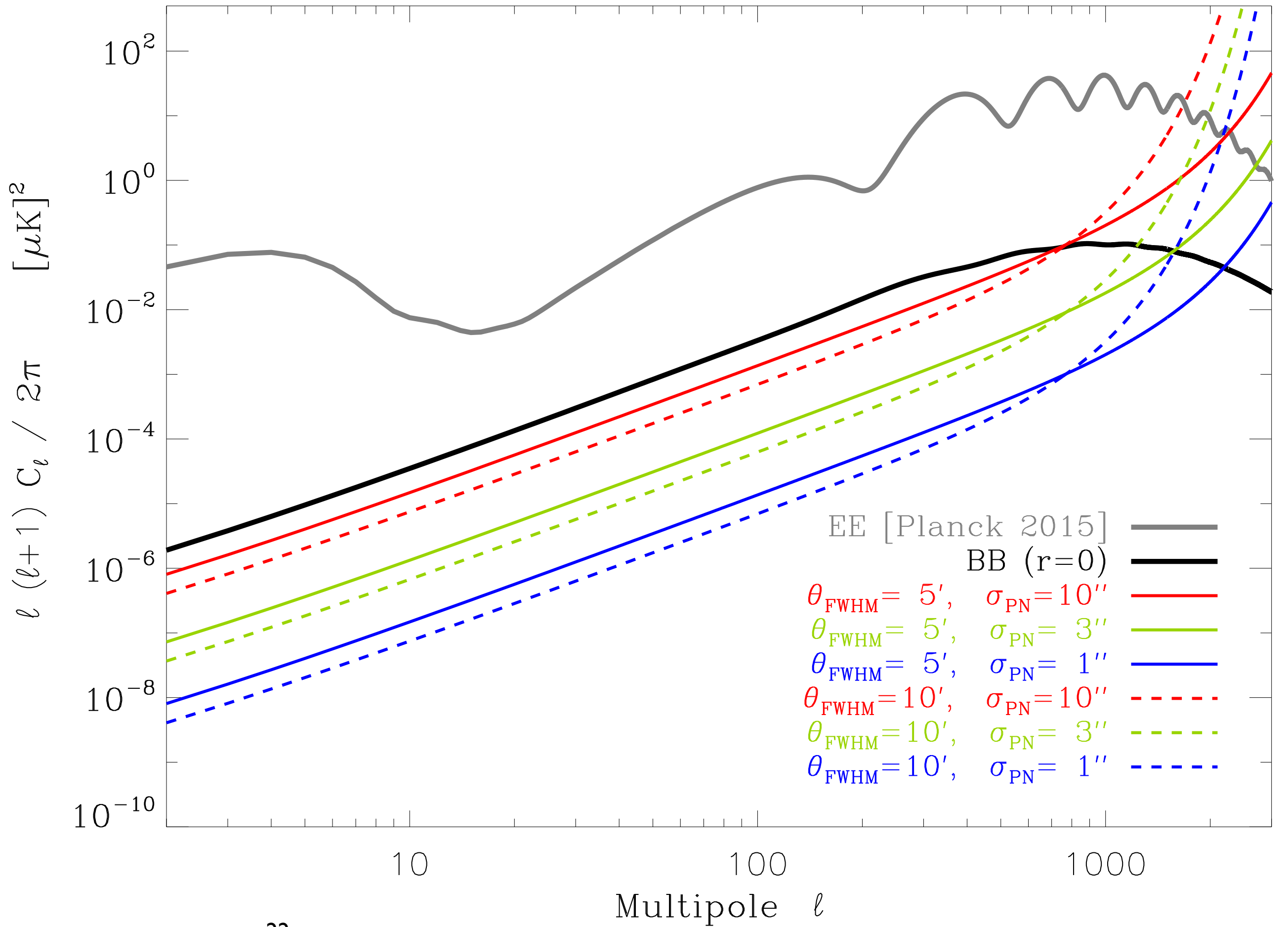
$$\tilde{C}_\ell^{XY} = \underset{\substack{\text{pixel} \\ \text{smearing}}}{W_\ell^{\text{pix}}} \sum_{X'Y'} \underset{\substack{\text{(Non circular)} \\ \text{beam}}}{W_\ell^{XY, X'Y'}} \underset{\substack{\text{Sky spectra}}}{C_\ell^{X'Y'}} + \underset{\substack{\text{sub-pixel} \\ \text{"noise"}}}{N_\ell^{XY}}$$

one finds

$$N_\ell^{\text{TT}} \sim N_\ell^{\text{EE}} \sim N_\ell^{\text{BB}} \gg N_\ell^{\text{TE}} \sim N_\ell^{\text{TB}} \sim N_\ell^{\text{EB}}$$

If Pointing Noise is white with variance/pixel σ_{PN}^2 then

$$N_\ell^{\text{EE}} = N_\ell^{\text{BB}} \simeq \sigma_{\text{PN}}^2 \sum_{\ell'} \ell'(\ell' + 1) \frac{2\ell' + 1}{4\pi} C_{\ell'}^{\text{TT}} B_{\ell'}^2$$



Conclusions

- Make identical circular small beams, and modulate polarisation by other means than scanning only ! (eg, front-end rotating Half Wave Plates)
- **Otherwise:**
 - ✦ **T→P leakage** and **P↔P cross-talk** due to beam mismatch (and polar efficiency and inter calibration inaccuracy) **can not be ignored** (at least in Planck)
 - ✦ **Analytical tool to model them fully now available (QUICKPOL),**
 - ▶ validated with simulations,
 - ▶ allowing extensive error propagation (no need for full focal plane simulations),
 - ▶ which seems to greatly improve TE inter-frequency consistency in Planck-HFI data (**preliminary**).
 - ✦ Applicable to other problems ?
 - ▶ HPW specific systematic problems
 - ▶ data mosaicking (heterogeneous data processing)