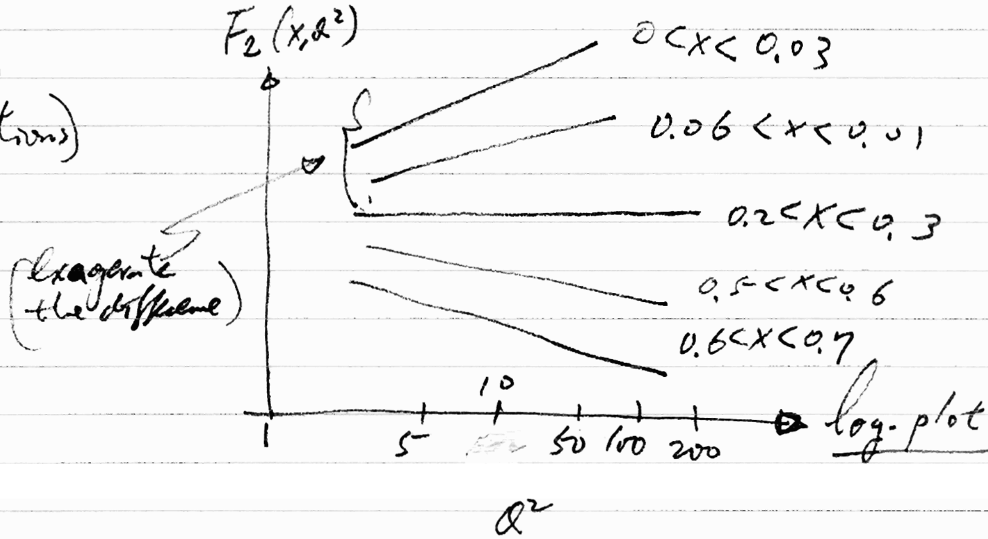
DGLAP Parton Evolution

IX-1/9

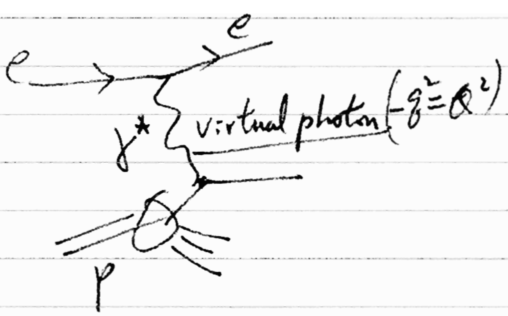
3) Does QCD work?

(1) Experimental data (Scaling violations)



$F_2(x,Q^2) \equiv \sum_i e_i^2\, x f_i(x,Q^2)$

① For larger Q^2, the large-momentum quark component ($x \simeq 1$) is depleted and shifted toward low momenta ($x \simeq 0$).



As Q^2 increases, γ^* sees

in QCD.

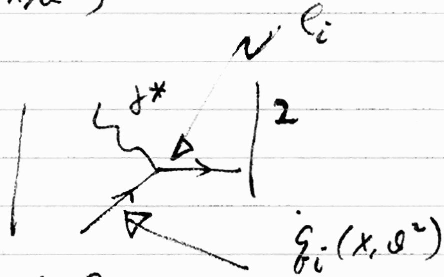
Thus, $\gamma^*(Q^2)$ sees (softer) quarks inside $g(x)$

② $\Delta q(x,Q^2) \sim \ln(Q^2)$

2) Recall the definition of $F_2(x, Q^2)$.

At leading order,

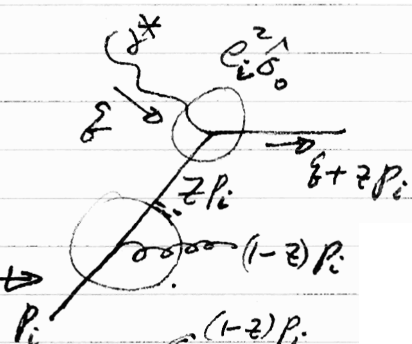
$$\frac{F_2(x,Q^2)}{x} = \left| \quad \right|^2$$



$$= \sum_i e_i^2 \, q_i(x, Q^2)$$

In QCD, beyond the leading order, at α_s, the probability for producing a gluon with momentum fraction $(1-z)$ and transverse momentum p_T in the process $\gamma^* q \to q g$ is given by

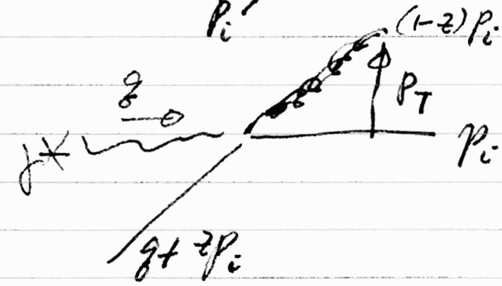
$$\frac{d\hat{\sigma}}{dz\,dp_T^2} = (e_i^2 \hat{\sigma}_0) \underbrace{\left[\frac{\alpha_s}{2\pi} \frac{1}{p_T^2} P_{qq}(z) \right]}$$



out-going massless quark:

$(q + z p_i)^2 = 0$

$\Rightarrow q^2 + 2 z q \cdot p_i + z^2 p_i^2 = 0$



$$p_i^2 = 0 \Rightarrow \quad z = \frac{Q^2}{2 p_i \cdot q} = \frac{Q^2}{(p_i+q)^2 - q^2}$$

$$= \frac{Q^2}{\hat{s} + Q^2}$$

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

the probability of a quark emitting a gluon and so becoming a quark with momentum reduced by a fraction z.

The $z \to 1$ singularity is associated with the emission of a "soft" massless gluon. This ~~[illegible]~~ infrared divergence is canceled by virtual gluon diagram.

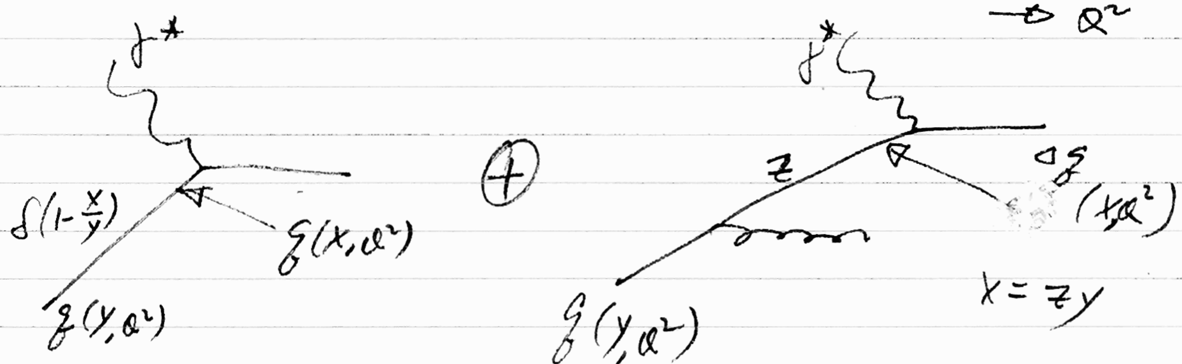
$$\frac{d\hat{\sigma}}{dz} = \left(e_i^2 \hat{\sigma}_0\right) \cdot \int_{\mu^2}^{(p_T^2)_{max}} dp_T^2 \left[\frac{\alpha_s}{2\pi} \frac{1}{p_T^2} P_{qq}(z) \right]$$

(μ is a cutoff scale to regularize the divergence as $p_T \to 0$)

$$\simeq \left(e_i^2 \hat{\sigma}_0\right) \cdot \frac{\alpha_s}{2\pi} P_{qq}(z) \cdot \ln\left(\frac{Q^2}{\mu^2}\right)$$

for $Q^2 \to \infty$

$$(p_T^2)_{max} = \frac{\hat{s}}{4} = Q^2 \frac{1-z}{4z} \to Q^2$$



the definition of $F_2(x,Q^2)$ measures $\left(\frac{F_2(x,Q^2)}{x} \equiv \sum_q e_q^2 \cdot [q(x) + \Delta q(x,Q^2)] \right)$

$$q(x,Q^2) + \Delta q(x,Q^2) = \int_0^1 dy \int_0^1 dz\, q(y,Q^2) \left[\delta(1-z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \ln \frac{Q^2}{\mu^2} \right] \cdot \delta(x - zy)$$

$$= \int_x^1 \frac{dy}{y}\, q(y,Q^2) \left[\delta\left(1-\frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) \ln \frac{Q^2}{\mu^2} \right]$$

Note: ① Strictly speaking, $P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)_+ = \frac{4}{3} \frac{(1+z^2)}{(1-z)_+} + 2\delta(1-z)$

The virtual diagrams give $\delta(1-z)$. We demand that

$$\int_0^1 dz\, P_{qq}(z) = 1$$

because finding a quark inside a quark, integrated over all z must add up to 1.

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z) - f(1)}{1-z}.$$

② Because $\int_x^1 \frac{dy}{y}$, so, we know nothing about the PDF for x range outside of data.

Hence,

$$\Delta q(x,Q^2) = \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{dy}{y}\, q(y,Q^2)\, P_{qq}\left(\frac{x}{y}\right).$$

Or,

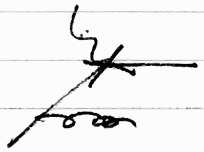
$$\frac{d}{d\ln Q^2}\, q(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y}\, q(y,Q^2)\, P_{qq}\left(\frac{x}{y}\right)$$

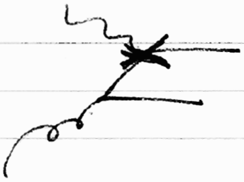
This is a "Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation".

QCD indeed verifies that $\Delta q(x,Q^2) \sim \ln(Q^2)$

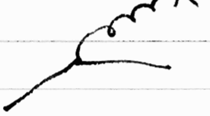
3) Complete Evolution for the parton distribution functions:

$$\frac{d\, q_i(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left\{ q_i(y,Q^2) \cdot P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)\, P_{qg}\left(\frac{x}{y}\right) \right\}$$

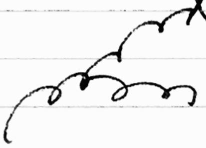




$$\frac{d\, g(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left\{ \sum_i q_i(y,Q^2)\, P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2)\, P_{gg}\left(\frac{x}{y}\right) \right\}$$



(doesn't depend on index i)



$i = 1, \cdots 2n_f$

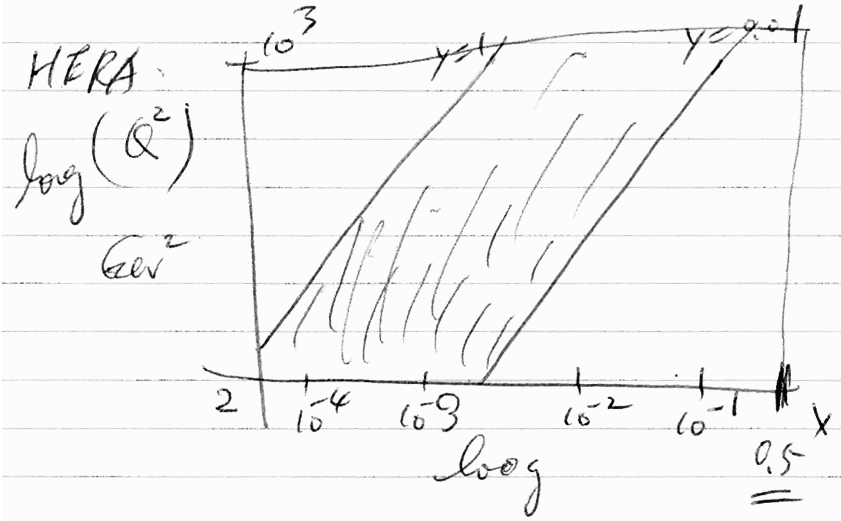
run over quarks & antiquarks for all flavors

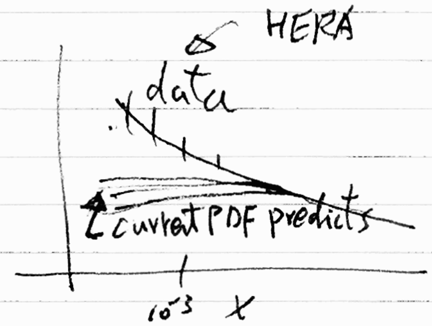
4) For a given set of PDF (parton distribution function) at Q_0, we know how to get

$q_i(x, Q^2)$ from $q_i(x, Q_0^2)$ and $g(x, Q_0^2)$

Then comparing with $F_2(x, Q^2) = \sum_i e_i^2 x q_i(x, Q^2)$, we can determine PDF.

So far, all the experimental data agrees with QCD prediction.





$ep \to eX$,

Q^2: $4 \to 1000$ GeV^2

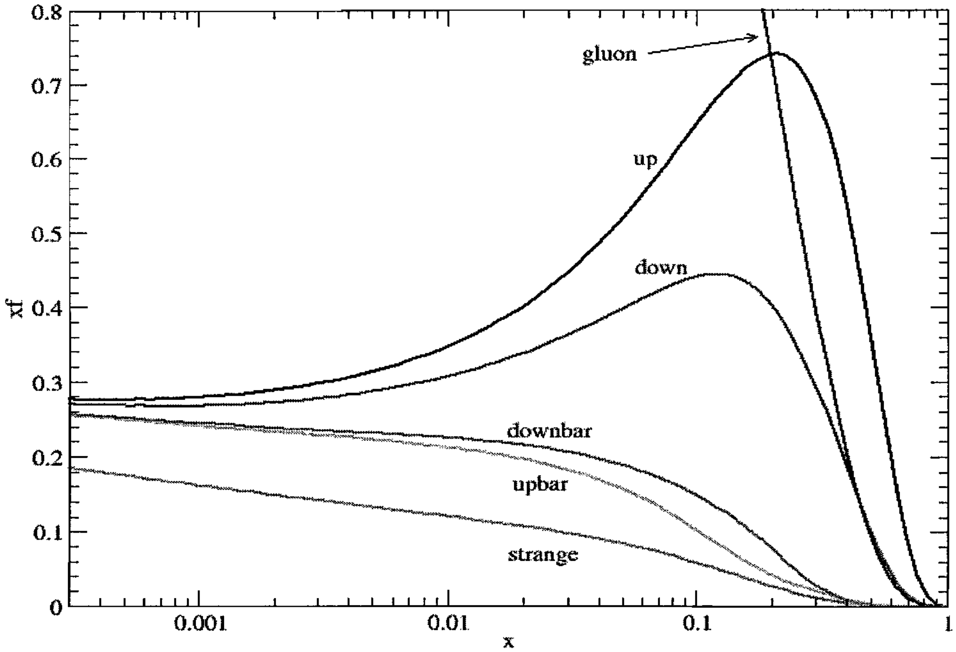
x: down to 10^{-4} ($Q^2 \sim 10$)

10^6 events

Note: Because QCD does not "predict" PDF for x range outside of data, therefore, it's dangerous to use PDF that extrapolates into the smaller-x region which has not been measured.
(The latest HERA data verified this worry.)

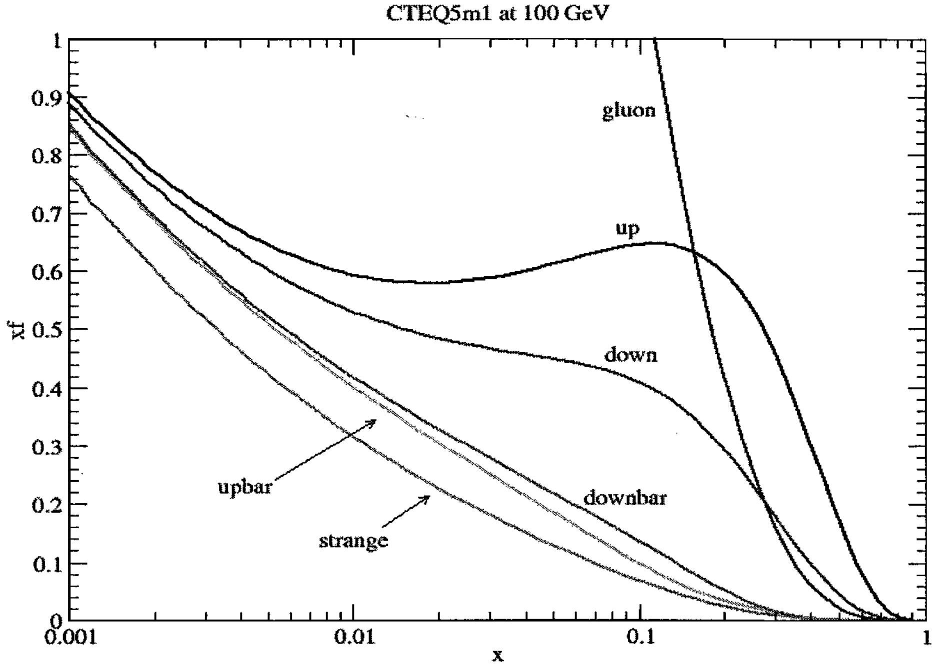
Parton Distribution Function

CTEQ5M1 at 2GeV



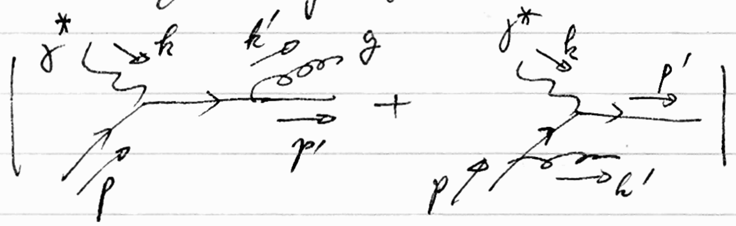
Parton Distribution Function

CTEQ5m1 at 100 GeV



4) Derive $\dfrac{d\sigma}{dz\,dp_T^2} = (e_i^2\hat\sigma_0)\cdot\left[\dfrac{\alpha_s}{2\pi}\cdot\dfrac{1}{p_T^2}P_{qq}(z)\right]$

Consider $\gamma^* q \to q g$ process $\qquad \left(\hat\sigma_0 = \dfrac{4\pi\alpha^2}{s}\right)$

$\mathcal{M}^2 = \Big|$  $\Big|^2$

$$\overline{|\mathcal{M}|^2} = 32\pi^2\left(e_i^2\alpha\alpha_s\right)\cdot\frac{4}{3}\left\{\frac{-t}{s}-\frac{s}{t}+\frac{2uQ^2}{st}\right\}$$

$s=(k+p)^2 = 4|k'|^2$

$t=(p-k')^2 = -2|k||k'|(1-\cos\theta)$, $\quad s+t+u=-Q^2$

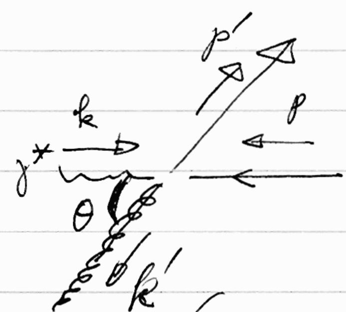
$u=(k-k')^2 = -2|k||k'|(1+\cos\theta)$

$\left(\begin{matrix} k^2=-Q^2 \\ p^2=p'^2=k'^2=0\end{matrix}\right)$

$p_T = |\vec{k}'|\sin\theta$,

Since $\quad ut = 4|\vec{k}|^2|\vec{k}'|^2\cdot(1-\cos^2\theta)$

$\qquad\qquad = 4|\vec{k}|^2 p_T^2$



So

$$p_T^2 = \frac{ut}{4|\vec{k}|^2} = \frac{stu}{(s+Q^2)^2}$$

(in c.m. frame)

$$\begin{bmatrix} k' = (|\vec{k}'|, \vec{k}') \\ p' = (|\vec{k}'|, -\vec{k}') \\ k = (q_0, \vec{k}) \\ p = (|\vec{k}|, -\vec{k}) \end{bmatrix}$$

$$t = -2|\vec{k}||\vec{k}'|(1-\cos\theta)$$

$$\frac{dt}{d\cos\theta} = 2|\vec{k}||\vec{k}'| = \frac{1}{2}(s+Q^2)$$

$$\frac{dp_T^2}{dt} = \frac{-s}{s+Q^2}$$

$$\Rightarrow \quad d\Omega = d\phi\, d\cos\theta = 2\pi\, d\cos\theta$$

$$= 2\pi\cdot\left|\frac{d\cos\theta}{dt}\frac{dt}{dp_T^2}\right|\cdot dp_T^2$$

$$= 2\pi\cdot\left(\frac{2}{s}\right)dp_T^2$$

$$= \frac{4\pi}{s}\,dp_T^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\overline{|\mathcal{M}|^2}$$

As $p_T \to 0 \Rightarrow |t| \to 0$,

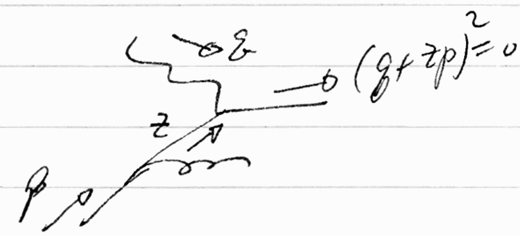
then

$$\frac{d\sigma}{dp_T^2} = \frac{4\pi}{s}\cdot\left(\frac{1}{64\pi^2 s}\right)\overline{|\mathcal{M}|^2}$$

$$\xrightarrow[|t|\to 0]{} \frac{8\pi e_i^2 \alpha\alpha_s}{3s^2}\left(\frac{1}{-t}\right)\left(s - \frac{2(s+Q^2)}{s}Q^2\right)$$

Define $z \equiv$ momentum fraction of quark after emission

$$z = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{(p+q)^2 - q^2} = \frac{Q^2}{s+Q^2}$$



$$\Rightarrow Q^2 = \frac{zs}{1-z}$$

$$p_T^2 \xrightarrow[|t|\to 0]{} \frac{-st}{s+Q^2} \Rightarrow \left(\frac{1}{-t}\right) = \frac{1}{p_T^2}\left(\frac{s}{s+Q^2}\right) = \frac{s}{Q^2 p_T^2} z$$

$$\Rightarrow \frac{d\sigma}{dp_T^2} \xrightarrow[|t|\to 0]{} \left(e_i^2 \hat{\sigma}_0\right)\frac{1}{p_T^2}\frac{\alpha_s}{2\pi}P_{qq}(z)$$

$$\hat{\sigma}_0 = \frac{4\pi^2\alpha}{s}$$

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

($z \to 1$ $\Rightarrow$ soft-gluon emission.)

Note: $\sin\theta \to 0 \Rightarrow |t| \to 0$ & $p_T \to 0$

collinear singularity