QCD in Collisions with Polarized Beams

Jianwei Qiu
Brookhaven National Laboratory
Stony Brook University

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The plan for my eight lectures

- **The Goal:**

  To understand QCD and the strong interaction dynamics, and to explore hadron structure and its properties by studying high energy collisions with polarized beams

- **The Plan (approximately):**

See also talks by Yuan and Xiao

Electron-Ion Collider

Connecting QCD quarks and gluons to observed hadrons and leptons

Fundamentals of QCD factorization and evolution

Three lectures

Hard scattering processes with longitudinally polarized beams

Three lectures

Hard scattering processes with transversely polarized beams

Two lectures

Summary of lecture one

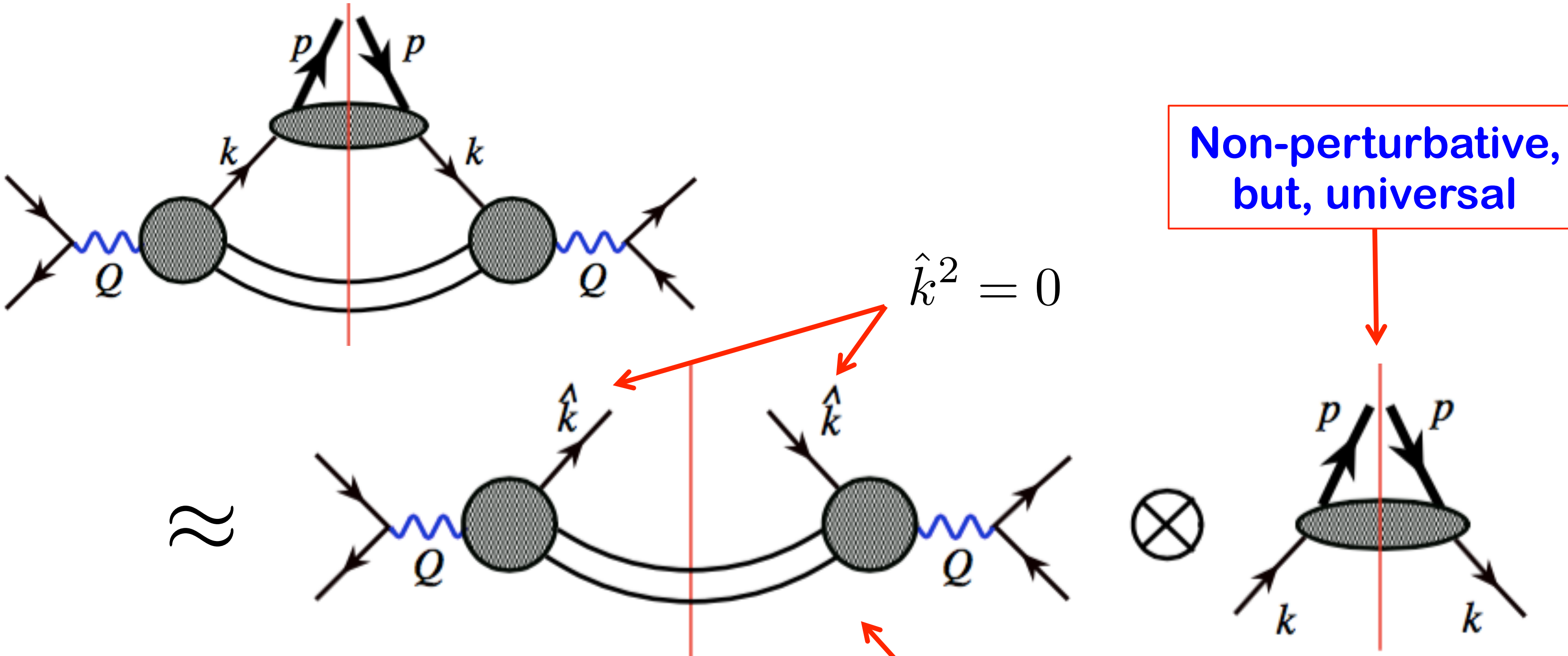
- EIC is a ultimate QCD machine:
  1) to discover and explore the quark/gluon structure and properties of hadrons and nuclei,
  2) to search for hints and clues of color confinement, and
  3) to measure the color fluctuation and color neutralization
- EIC is a tomographic machine for nucleons and nuclei with a resolution better than 1/10 fm
- Cross section with identified hadron(s) is NOT completely calculable in QCD perturbation theory
- QCD Factorization – neglecting quantum interference between dynamics at hard partonic scattering and those at hadronic scales – approximation
- Predictive power of QCD factorization relies on the universality of PDFs (or TMDs, GPDs, …), the calculations of perturbative coefficient functions – hard parts

How to connect QCD quarks and gluons to observed hadrons and leptons?

Fundamentals of QCD factorization and evolution

QCD factorization – approximation

- ❑ **Creation of identified hadron(s):**



$$+ \ \mathcal{O}\left(\left[\frac{\langle k^2 \rangle}{Q^2}\right]^{\alpha}\right)$$

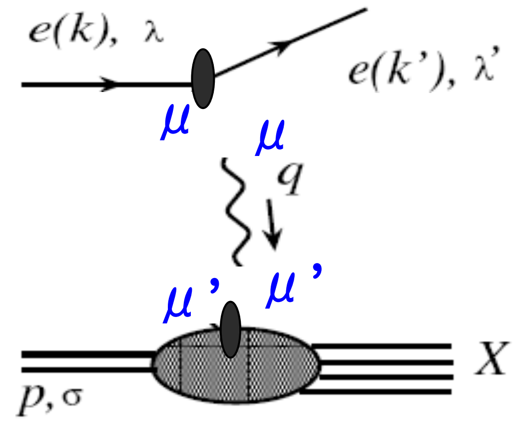
Perturbative!

Factorization: factorized into a product of "probabilities" !

Example: Inclusive lepton-hadron DIS

- **Scattering amplitude:**

$$\mathrm{M}(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k)$$
$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$
$$* \left\langle X\middle|eJ^{em}_{\mu'}(0)\middle|p,\sigma\right\rangle$$



- **Cross section:**

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X} \frac{d^3 l_i}{(2\pi)^3 2E_i}\right] \frac{d^3k'}{(2\pi)^3 2E'} (2\pi)^4 \delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$\Rightarrow \quad E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

- **Leptonic tensor:**

– known from QED

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}\right)$$

DIS structure functions

- **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\; e^{iq\cdot z}\;\langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

- **Symmetries:**

  - Parity invariance (EM current) → $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
  - Time-reversal invariance → $W_{\mu\nu} = W^*_{\mu\nu}$ real
  - Current conservation → $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

$$Q^2 = -q^2 \qquad x_B = \frac{Q^2}{2p\cdot q}$$

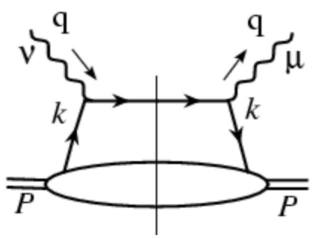
- **Structure functions – infrared sensitive:**

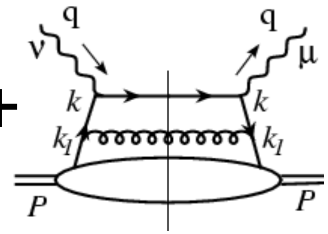
$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

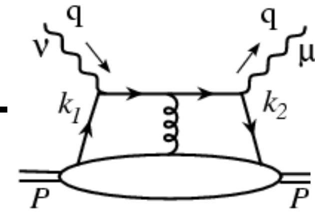
No QCD parton dynamics used in above derivation!

Long-lived parton states

- **Feynman diagram representation:**

$W^{\mu\nu} \propto$ 

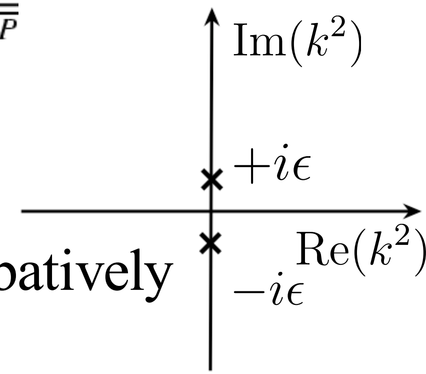
$+$ 

$+$ 

$+ \ldots$

- **Perturbative pinched poles:**

$$\int d^4k \, \mathrm{H}(Q,k) \left(\frac{1}{k^2 + i\varepsilon} \right) \left(\frac{1}{k^2 - i\varepsilon} \right) \mathrm{T}(k, \frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$



- **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2 + k_T^2}{2xp \cdot n} n^\mu + k_T^\mu$$

$$\int \frac{dx}{x} d^2k_T \, \mathrm{H}(Q, k^2 = 0) \int dk^2 \left(\frac{1}{k^2 + i\varepsilon} \right) \left(\frac{1}{k^2 - i\varepsilon} \right) \mathrm{T}(k, \frac{1}{r_0})$$

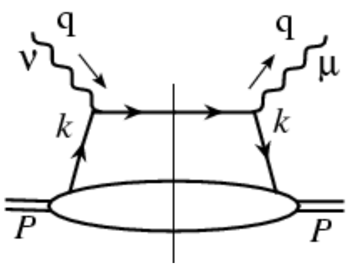
Nonperturbative matrix element

Short-distance

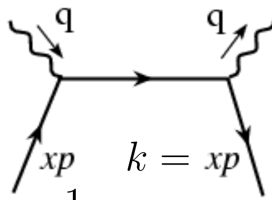
Collinear factorization – further approximation

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T,\ \sqrt{k^2}$

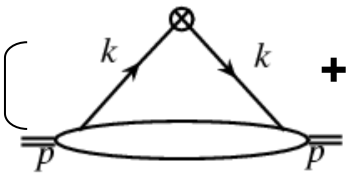
  – **Lowest order:**



$\approx$



$$+\mathcal{O}\left(\frac{k_T^2}{Q^2}\right) \otimes \int \frac{dx}{x} \quad \frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x - \frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$



+UVCT

$\frac{1}{2}\gamma\cdot(xp)$

Same as an elastic x-section

Scheme dependence

Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \longrightarrow \infty$, while x_B fixed

Feynman's parton model and Bjorken scaling

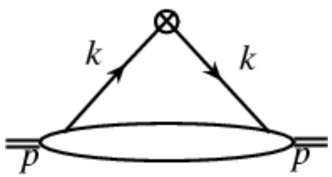
$$F_2(x_B,Q^2) = x_B \sum_f e_f^2\,\phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

Spin-½ parton!

- **Corrections:** $\mathcal{O}(\alpha_s) + \mathcal{O}\left(\langle k^2\rangle/Q^2\right)$

Parton distribution functions (PDFs)

❑ PDFs as matrix elements of two parton fields:



– combine the amplitude & its complex-conjugate

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\, \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

$|h(p)\rangle$ can be a hadron, or a nucleus, or a parton state!

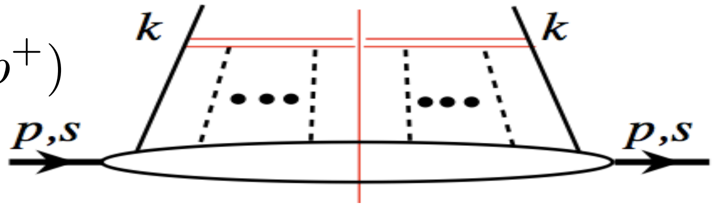
But, it is NOT gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

– need a gauge link:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+}\, \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

– corresponding diagram in momentum space:

$$\int \frac{d^4k}{(2\pi)^4}\, \delta(x - k^+/p^+)$$



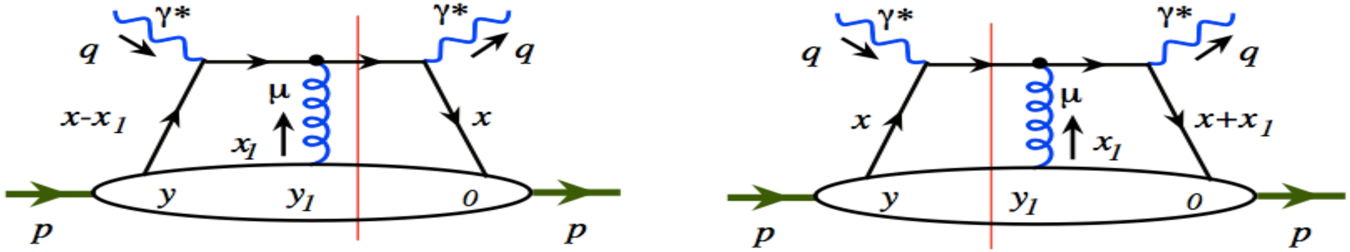
$+\,\mathrm{UVCT}(\mu^2)$

μ -dependence

Universality – process independence – predictive power

Gauge link – 1st order in coupling "g"

❑ Longitudinal gluon:



❑ Left diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_Bp^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{-x_1+i\epsilon}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

❑ Right diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1p^+y_1^-}\, n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_Bp^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-)t^a \left[\int dx_1\, \frac{1}{x_1-i\epsilon}\, \mathrm{e}^{ix_1p^+y_1^-}\right] \mathcal{M} \;= ig\int_0^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

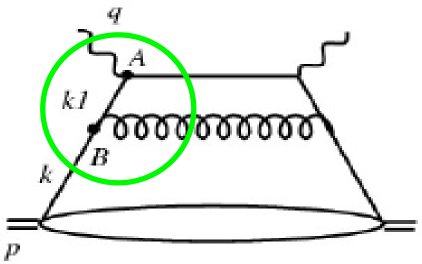
❑ Total contribution:

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$

O(g)-term of the gauge link!

QCD high order corrections

❑ NLO partonic diagram to structure functions:



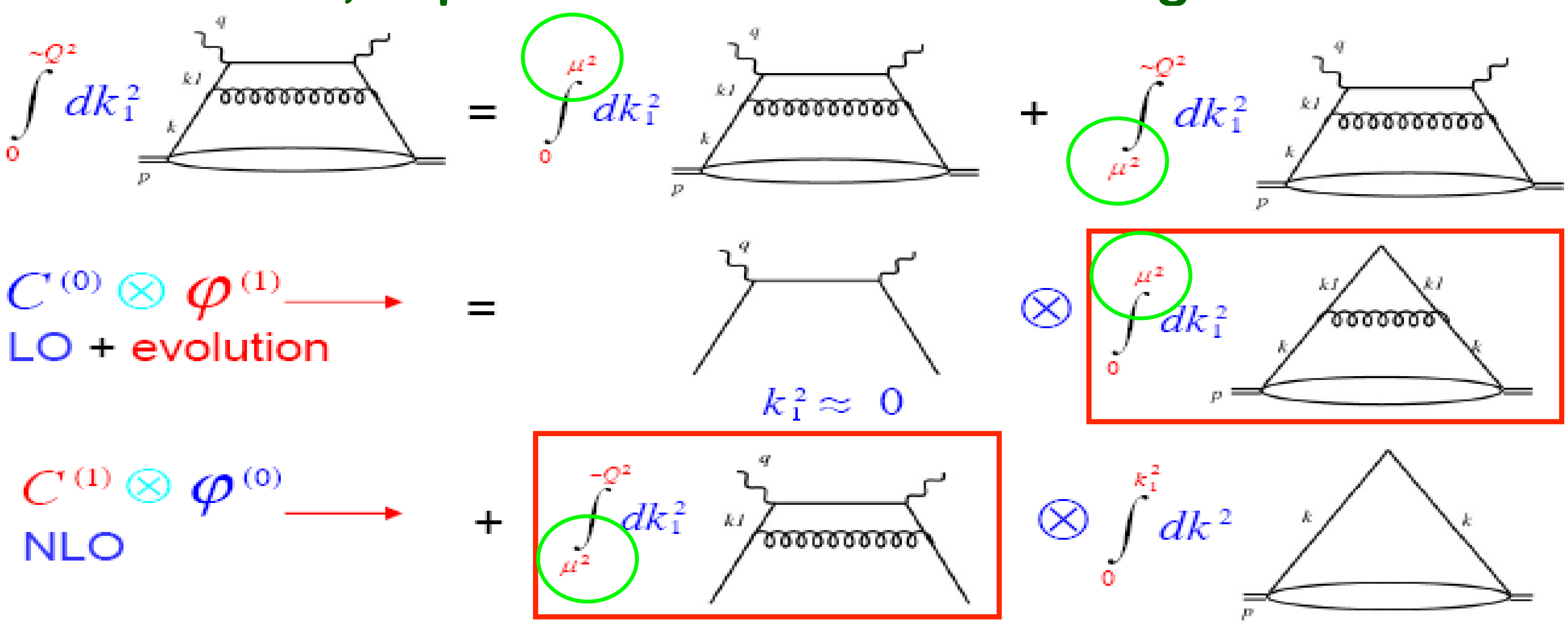
$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by

$$\begin{cases} k_1^2 \sim 0 \\ t_{AB} \rightarrow \infty \end{cases}$$

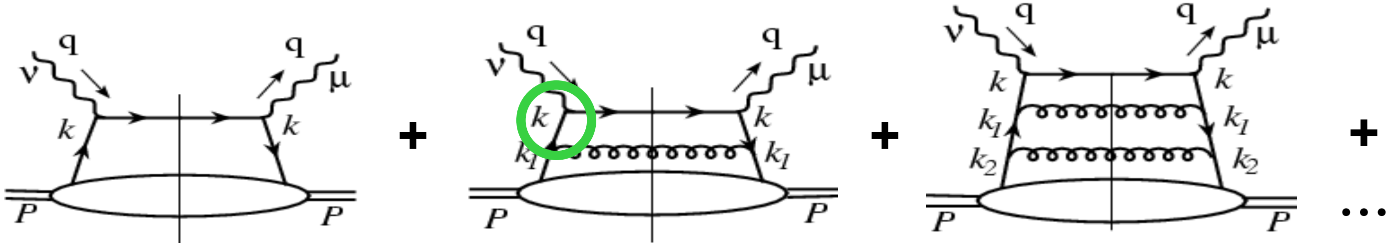
Diagram has both long- and short-distance physics

❑ Factorization, separation of short- from long-distance:

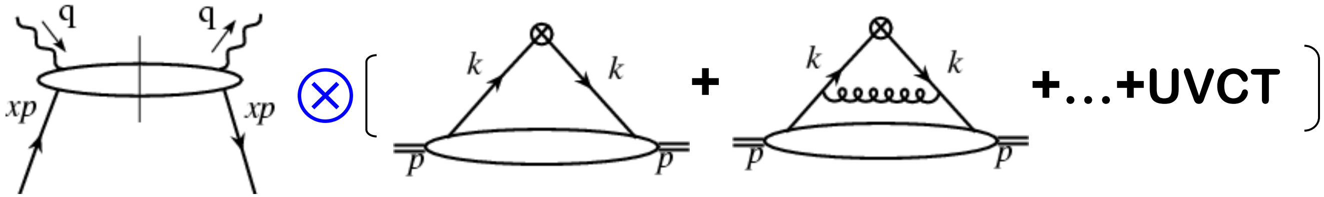


QCD high order corrections

- **QCD corrections: pinch singularities in** $\int d^4 k_i$



- **Logarithmic contributions into parton distributions:**



$$F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

- **Factorization scale:** μ_F^2

→ **To separate the collinear from non-collinear contribution**

Recall: renormalization scale to separate local from non-local contribution

How to calculate the perturbative parts?

❑ Use DIS structure function F_2 as an example:

$$F_{2h}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

✧ Apply the factorized formula to parton states: $h \to q$

Feynman diagrams → $$F_{2q}(x_B,Q^2) = \sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$ ← Feynman diagrams

✧ Express both SFs and PDFs in terms of powers of α_s:

0th order: $$F_{2q}^{(0)}(x_B,Q^2) = C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right)$$

⇒ $$C_q^{(0)}(x) = F_{2q}^{(0)}(x) \qquad \varphi_{q/q}^{(0)}\left(x\right) = \delta_{qq}\delta\left(1-x\right)$$

1th order: $$F_{2q}^{(1)}(x_B,Q^2) = C_q^{(1)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu^2\right) + C_q^{(0)}(x_B/x,Q^2/\mu^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

⇒ $$C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)$$

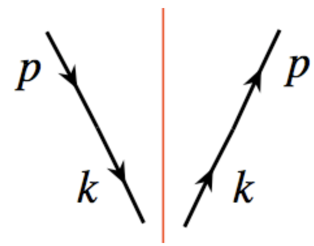
PDFs of a parton

❑ Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\, \psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\text{parton}(p)\rangle$ ⇒ $\phi_{f/q}(x,\mu^2)$ – **given by Feynman diagrams**

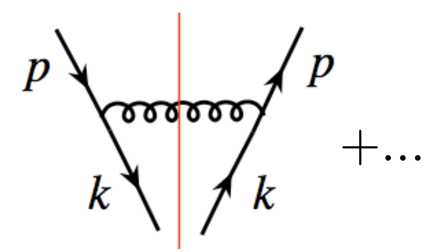
❑ Lowest order quark distribution:



✧ From the operator definition:

$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'} \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

❑ Leading order in α_s quark distribution:



✧ Expand to $(g_s)^2$ – logarithmic divergent:

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \mathrm{UVCT}$$

UV and CO divergence

Partonic cross sections

❑ Projection operators for SFs:

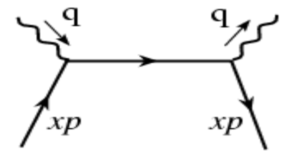
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)F_2\left(x,Q^2\right)$$

$$F_1(x,Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2)$$

$$F_2(x,Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2)$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = xg^{\mu\nu}W_{\mu\nu,q}^{(0)} = xg^{\mu\nu}\left[\frac{1}{4\pi}\right.$$



$$\left.\vphantom{\frac{1}{4\pi}}\right]$$

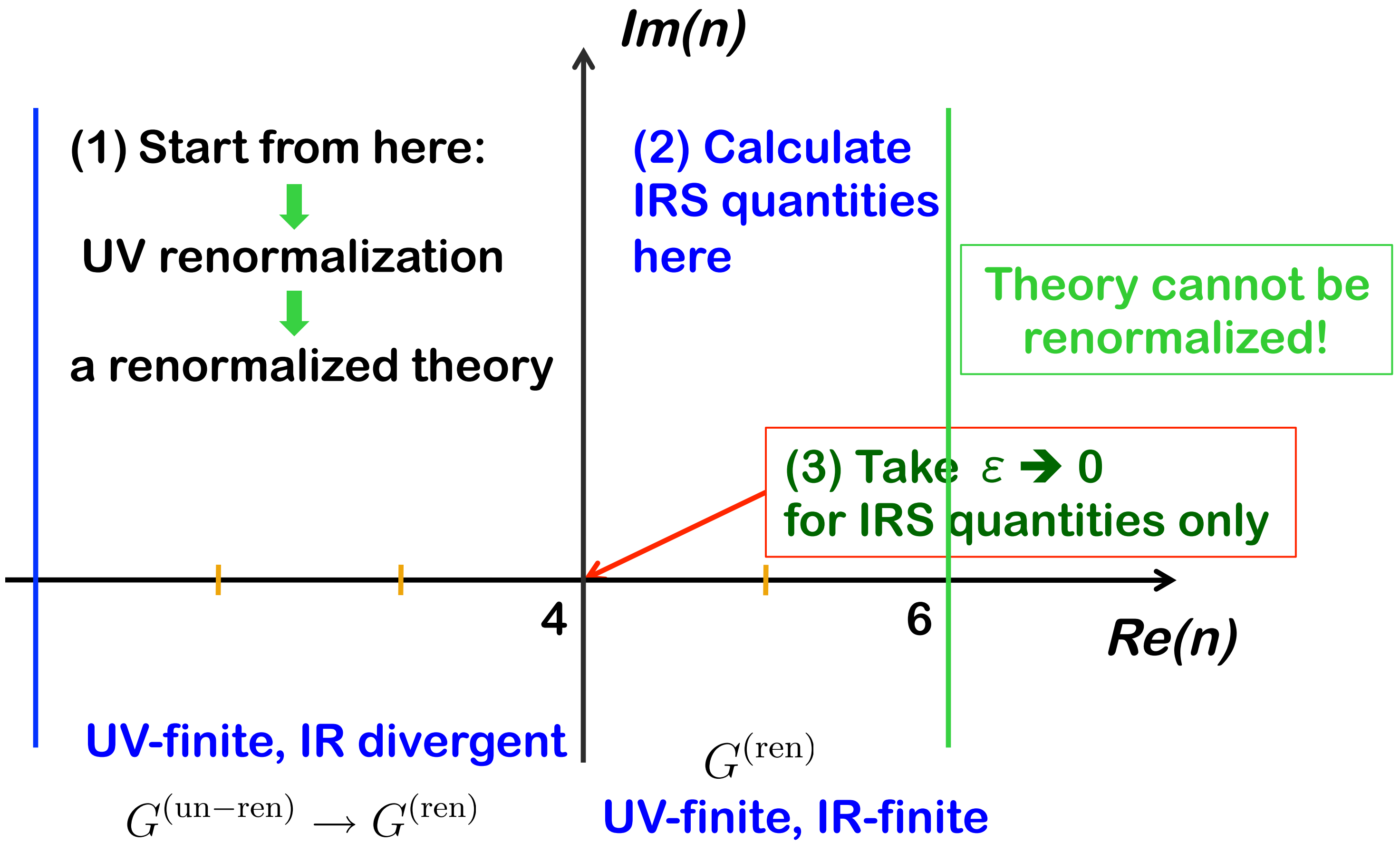
$$= \left(xg^{\mu\nu}\right)\frac{e_q^2}{4\pi}\mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p\gamma_\mu\gamma\cdot\left(p+q\right)\gamma_\nu\right]2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x\delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x\delta(1-x)$$

How does dimensional regularization work?

- Complex *n*-dimensional space: $\int d^n k\, F(k, Q)$



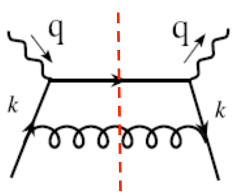
NLO coefficient function – complete example

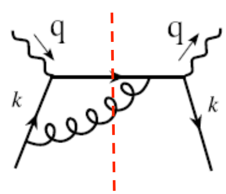
$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

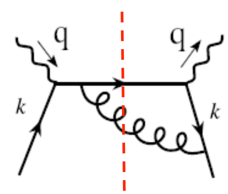
- **Projection operators in n-dimension:** $g_{\mu\nu}g^{\mu\nu} = n \equiv 4-2\varepsilon$

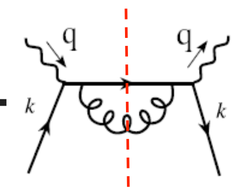
$$\boxed{\left(1-\varepsilon\right)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}}$$

- **Feynman diagrams:**

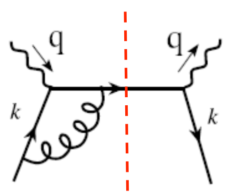
$W_{\mu\nu,q}^{(1)}$ 

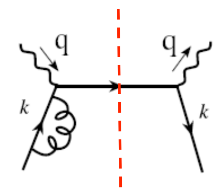
+ 

+ 

+ 

} **Real**

Virtual { + 

+ c.c. (+ 

+ c.c. + UV CT)

- **Calculation:** $-g^{\mu\nu}W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of $W_{\mu\nu}$

- **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx\,\frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx\,\frac{f(x)-f(1)}{1-x} + \ln(1-z)\,f(1)$$

❑ One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu}W^{(1)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon}P_{qq}(x) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$

$$+C_F\left[\left(1+x^2\right)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x)\right.$$

$$\left.\left. +3-x-\left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

- One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

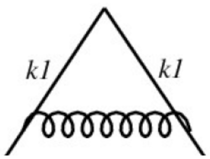
$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

- One loop contribution to F_2 of a quark:

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ln\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$
$$\Rightarrow \quad \infty \quad \text{as} \quad \varepsilon \to 0$$

- One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

❑ Common UV-CT terms:

✧ **MS scheme:** $$\text{UV-CT}\Big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$$

✧ **$\overline{\text{MS}}$ scheme:** $$\text{UV-CT}\Big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}\left(1+\varepsilon \ln(4\pi e^{-\gamma_E})\right)$$

✧ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x, Q^2/\mu^2)\Big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu^2\right)$$

$$C_q^{(1)}(x, Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\Bigg\{ P_{qq}(x) \ln\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right)$$
$$+ C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ln(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\Bigg\}$$

Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

$$F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s)\, \phi_f(x, \mu_F^2)$$

→ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\mathrm{QCD}}^2\right)$

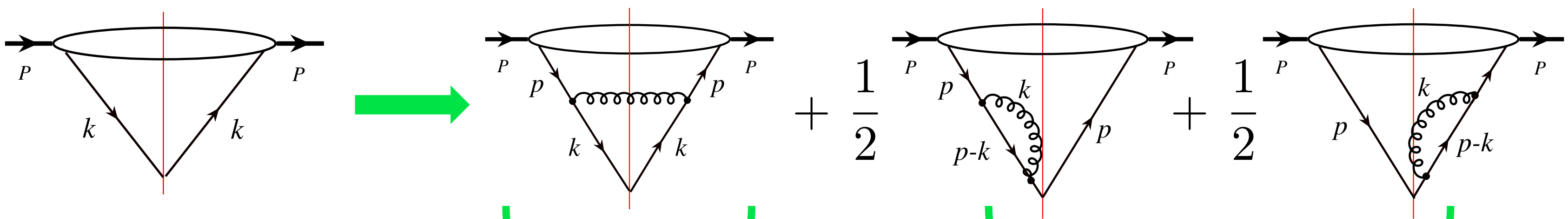
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ DGLAP evolution equation:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

Calculation of evolution kernels

- ❑ **Evolution kernels are process independent**
  - ✧ Parton distribution functions are universal
  - ✧ Could be derived in many different ways
- ❑ **Extract from calculating parton PDFs' scale dependence**



Collins, Qiu, 1989

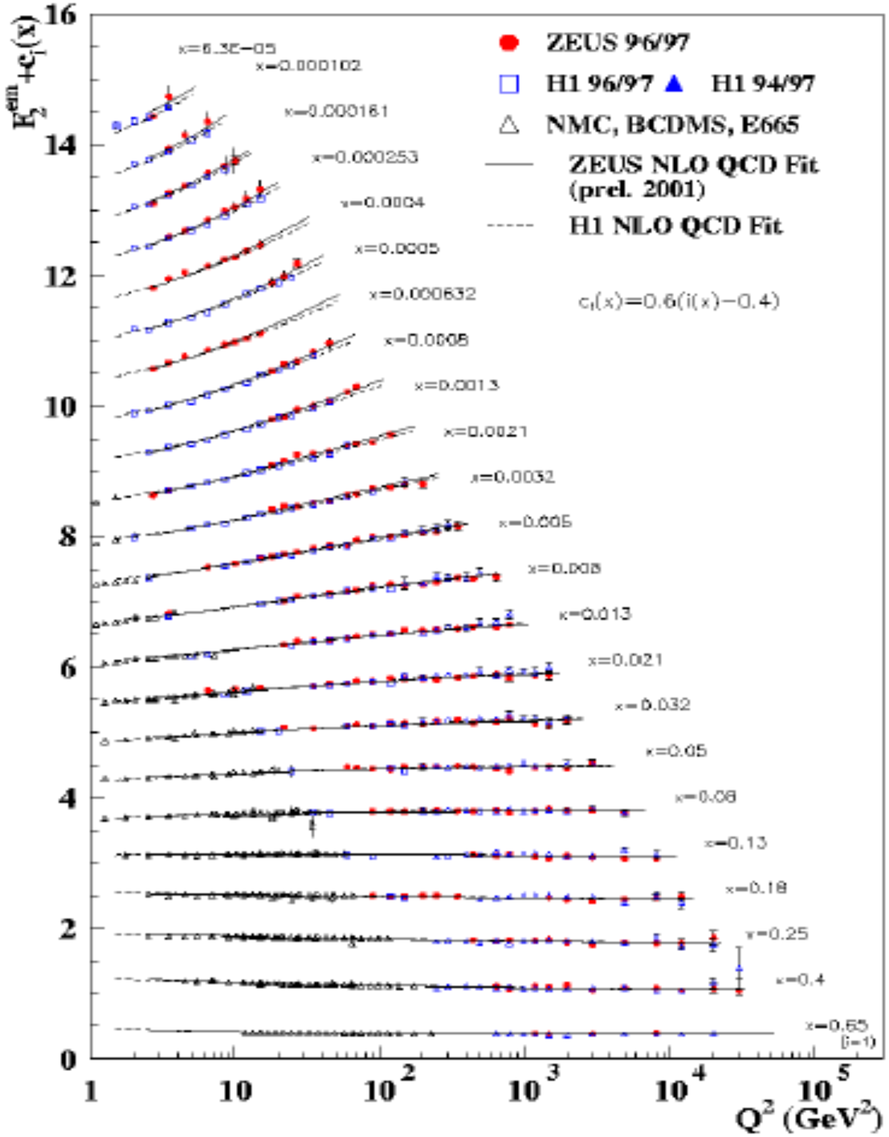
$$Q^2\frac{d}{dQ^2}q_i(x,Q^2)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}q_i(x_1,Q^2)\gamma_{qq}\left(\frac{x}{x_1}\right)-\frac{\alpha_s}{2\pi}q_i(x,Q^2)\int_0^1 dz\,\gamma_{qq}(z)$$

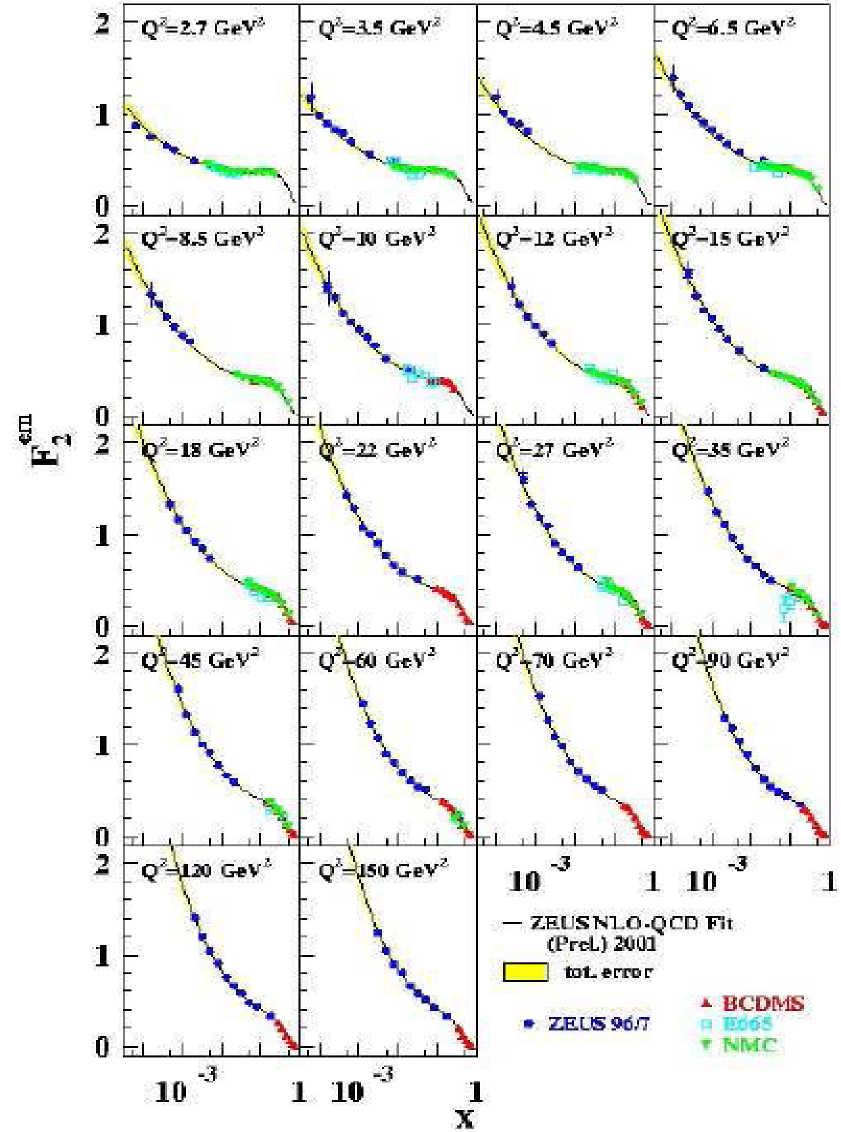
Change "Gain" "Loss"

- ✧ Same is true for gluon evolution, and mixing flavor terms

- ❑ **One can also extract the kernels from the CO divergence of partonic cross sections**

Scaling and scaling violation

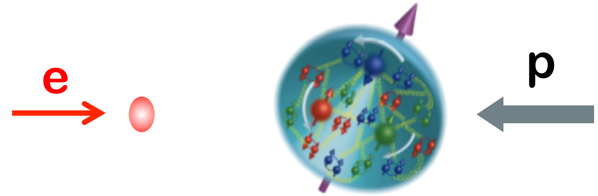




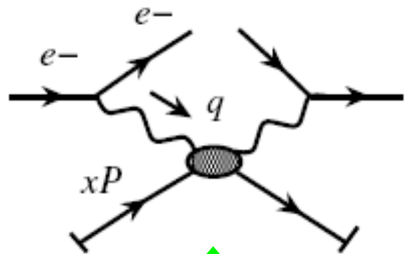
Q^2-dependence is a prediction of pQCD calculation

From one hadron to two hadrons

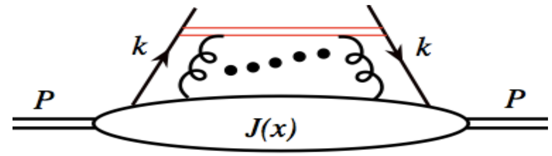
- ❑ **One hadron:**



$\sigma_{\text{tot}}^{\text{DIS}} \sim$



$\otimes$

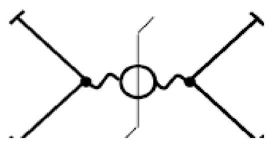


$+\, O\left(\frac{1}{QR}\right)$

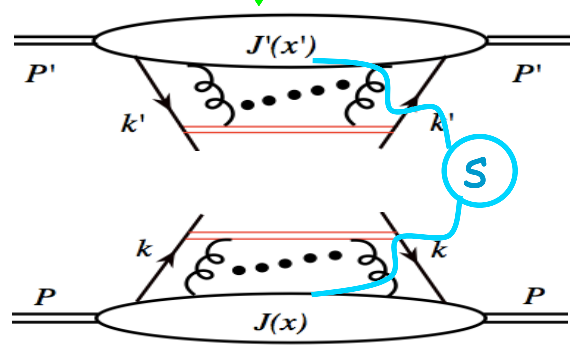
Hard-part Probe	Parton-distribution Structure	Power corrections Approximation

- ❑ **Two hadrons:**

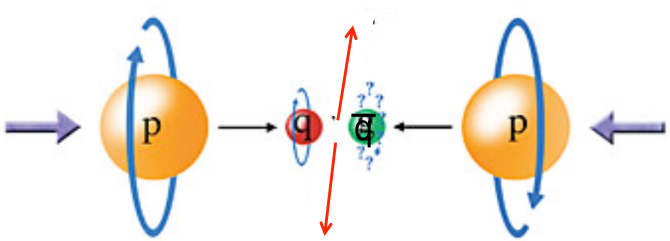
$\sigma_{\text{tot}}^{\text{DY}} \sim$



$\otimes$



$+\, O\left(\frac{1}{QR}\right)$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

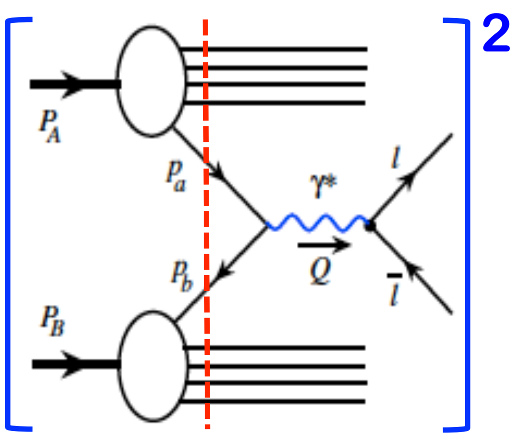
❑ Drell-Yan mechanism:

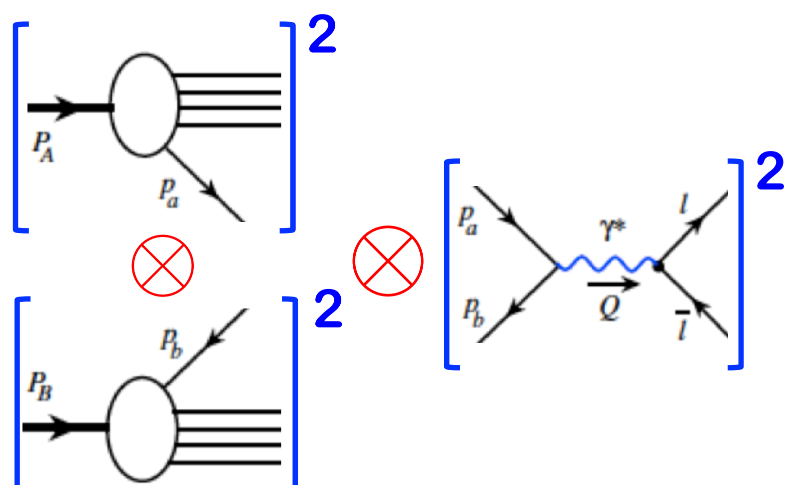
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X$$ **with** $$q^2 \equiv Q^2 \gg \Lambda_{\rm QCD}^2 \sim 1/\text{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general, a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

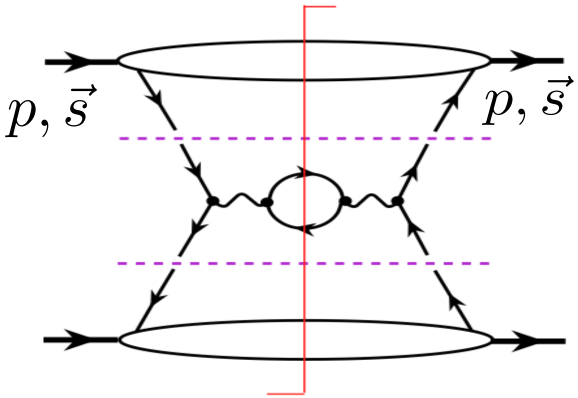
Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

$$x_A = \frac{Q}{\sqrt{S}} e^{y} \qquad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD

❑ Spin decomposition – cut diagram notation:



$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

$\Leftarrow$ all γ structure: $\gamma^\alpha,\ \gamma^\alpha\gamma^5,\ \sigma^{\alpha\beta}(\text{or } \gamma^5\sigma^{\alpha\beta}),\ I,\ \gamma^5$

❑ Factorized cross section:

$$\sigma(Q,\vec{s}) \pm \sigma(Q,-\vec{s}) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle \pm \langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$$

❑ Parity-Time reversal invariance:

$$\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle = \langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle$$

❑ Good operators:

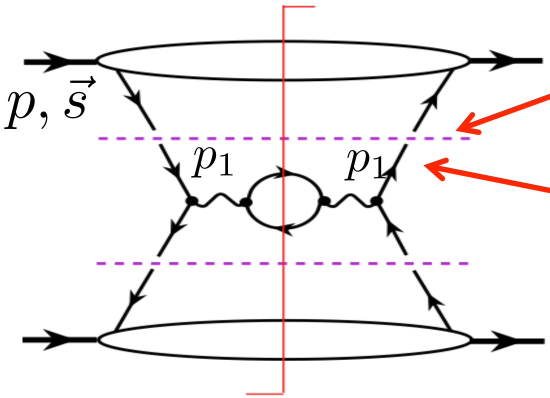
$$\langle p,\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,\vec{s}\rangle = \pm\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

"+" for spin-averaged cross section → PDFs:

$$\langle p,\vec{s}|\overline{\psi}(0)\gamma^+\psi(y^-)|p,\vec{s}\rangle, \qquad \langle p,\vec{s}|F^{+i}(0)F^{+j}|p,\vec{s}\rangle(-g_{ij})$$

Drell-Yan process in QCD – LO

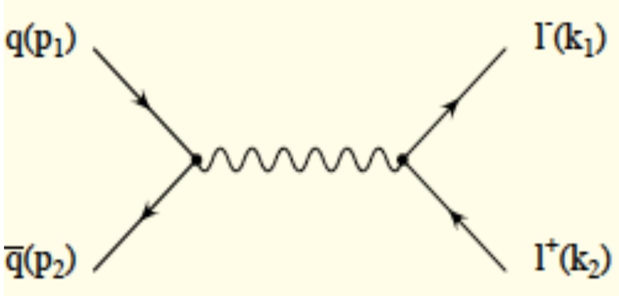
❑ **Spin-averaged cross section – Lowest order:**



$$\frac{1}{2p^+}\gamma^+\,\delta(x - p_1^+/p^+)dx$$

$$\frac{1}{2}\gamma\cdot p = \frac{1}{2}\sum_s u_s(p)\overline{u}_s(p)$$

$$\hat{s} = (p_1 + p_2)^2 = Q^2$$



❑ **Lowest order partonic cross section:**

$$\overline{\Sigma}\,|M|^2 = \frac{e_q^2 e^4}{\hat{s}^2}\left[\frac{1}{2}\right]\left[\frac{1}{2}\right]3\left\{\frac{1}{3}\right\}\left\{\frac{1}{3}\right\}\mathrm{Tr}[\not{p}_1\gamma^\nu\not{p}_2\gamma^\mu]\mathrm{Tr}[\not{k}_1\gamma_\nu\not{k}_2\gamma_\mu] = \left\{\frac{1}{3}\right\}e_q^2 e^4(1+\cos^2\theta)$$

$$PS^{(2)} = \frac{d^2k_1}{(2\pi)^3 2E_1}\frac{d^2k_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2) = \frac{1}{16\pi}d\cos(\theta)$$

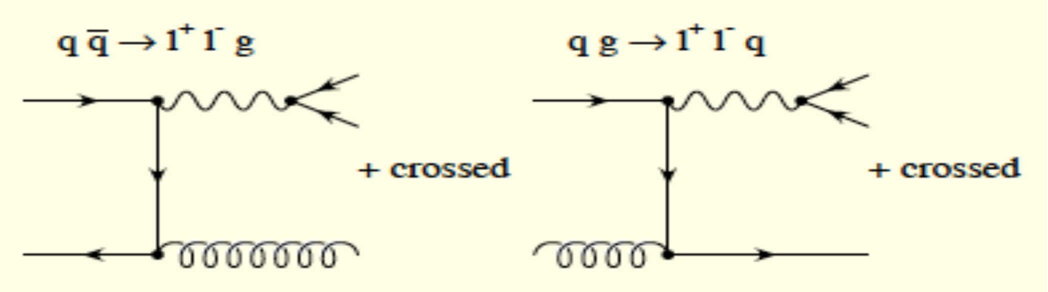
$$\sigma(q\bar{q}\to l^+l^-) = \left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2 \equiv \sigma_0$$

❑ **Drell-Yan cross section:**

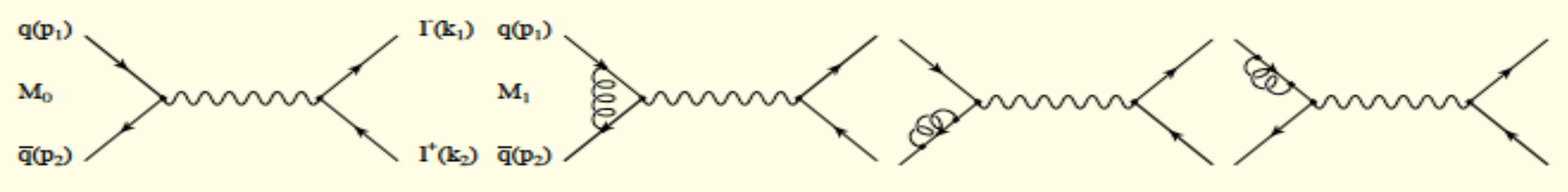
$$\frac{d\sigma}{dQ^2dy} = \Sigma_q\int dx_A\,dx_B\,\phi_{q/A}(x_A)\phi_{\bar{q}/B}(x_B)\left[\left\{\frac{1}{3}\right\}\frac{4\pi\alpha^2}{3\,\hat{s}}e_q^2\right]\delta(Q^2-\hat{s})\,\delta(y-\frac{1}{2}\ln(\frac{x_A}{x_B}))$$

Drell-Yan process in QCD – NLO

- **Real contribution:**



- **Virtual contribution:**



- **NLO contribution:**

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^0}{dQ^2} + \frac{\alpha_s}{2\pi}\left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right) \qquad q\bar{q} \to l^+l^-$$

$$\frac{d\sigma}{dQ^2} = \frac{\alpha_s}{2\pi}\left(-\frac{A}{\epsilon^2} + \frac{B'}{\epsilon} + C'\right) \qquad q\bar{q} \to l^+l^-g$$

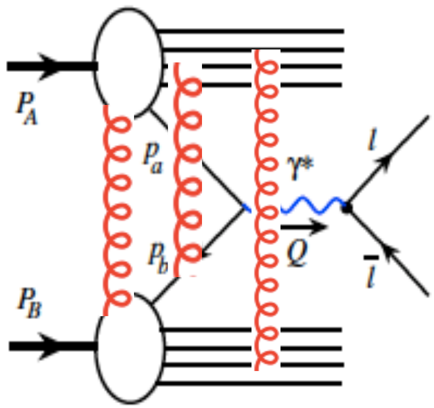
$$\frac{d\sigma}{dQ^2} = \frac{\alpha_s}{2\pi}\left(\frac{B''}{\epsilon} + C''\right) \qquad qg \to l^+l^-q$$

$$\Rightarrow \quad \frac{d\sigma}{dQ^2} = \frac{d\sigma^0}{dQ^2} + \frac{\alpha_s}{2\pi}\left(\frac{B + B' + B''}{\epsilon} + C + C' + C''\right)$$

Absorbed into PDFs – scheme dependence

Drell-Yan process in QCD – factorization

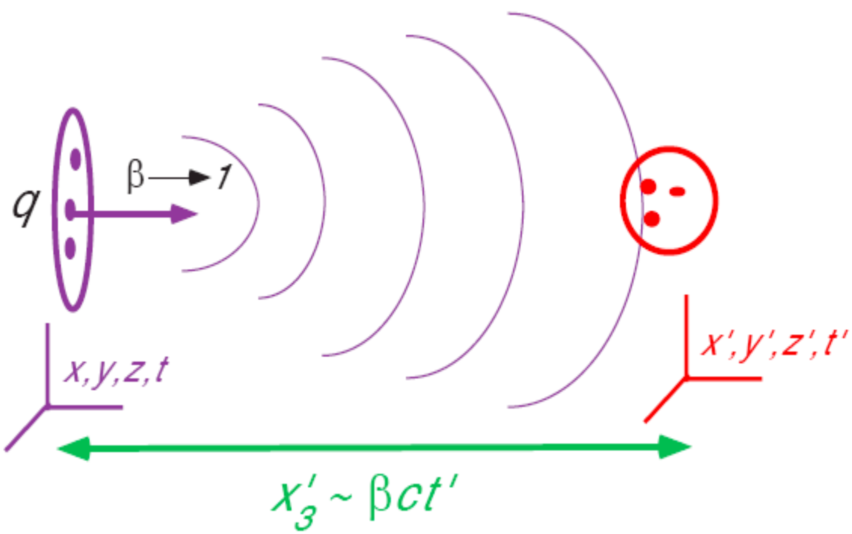
❑ Beyond the lowest order:



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$ $\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$ $\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

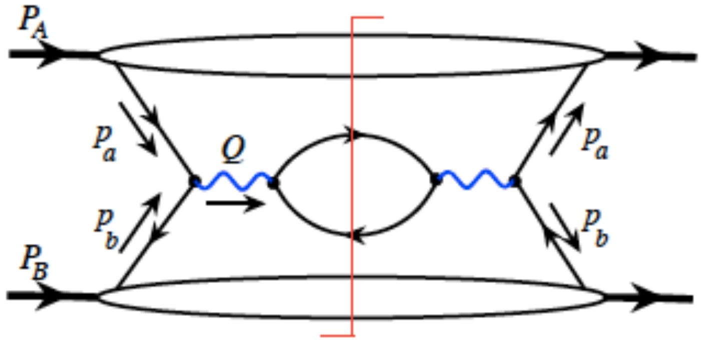
Drell-Yan process in QCD – factorization

❑ **Factorization – approximation:**

Collins, Soper, Sterman, 1988

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

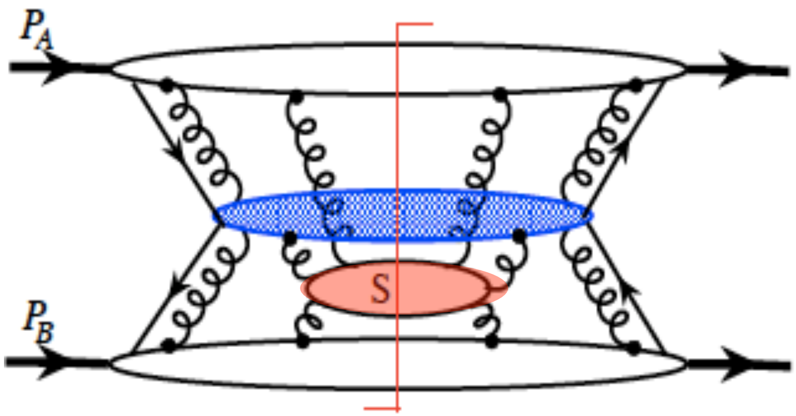
✧ Infrared safe of partonic parts:

Cancelation of IR behavior
Absorb all CO divergences into PDFs

$$\begin{aligned} &\text{on-shell:} && p_a^2,\ p_b^2 \ll Q^2; \\ &\text{collinear:} && p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ &\text{higher-power:} && p_a^- \ll q^-; \quad \text{and} \\ & && p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan process in QCD – factorization

- **Leading singular integration regions (pinch surface):**



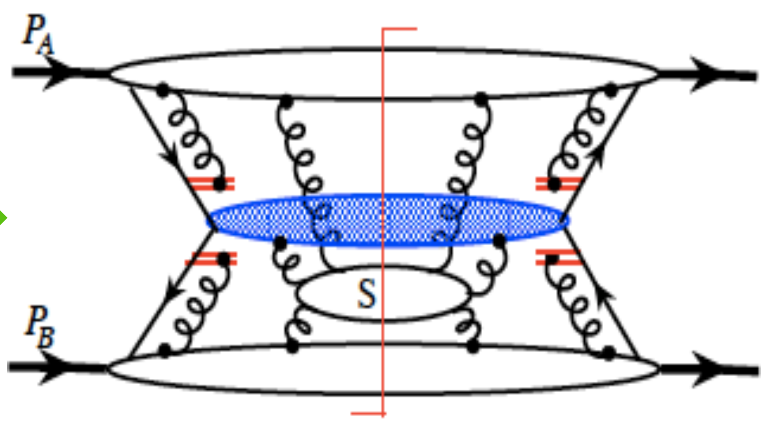
Hard: all lines off-shell by Q

Collinear:
- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

- **Collinear gluons:**

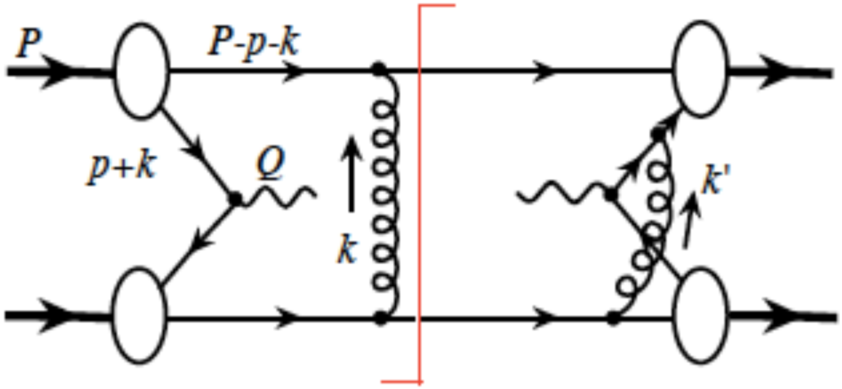
  - ✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
  - ✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

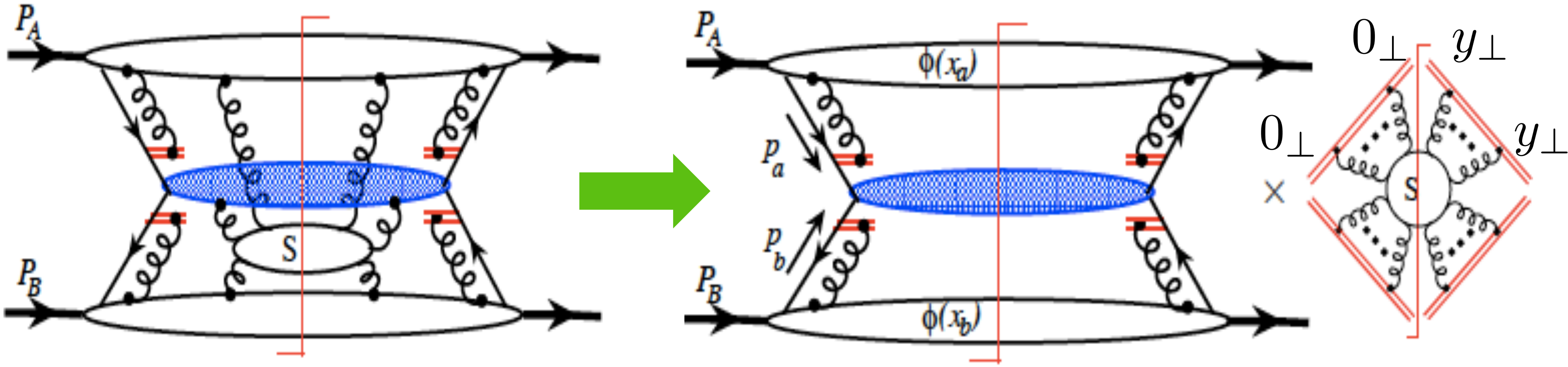
✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

✧ The soft gluon approximations (with the eikonal lines) need $k^{\pm}$ not too small. But, $k^{\pm}$ could be trapped in "too small" region due to the pinch from spectator interaction: $k^{\pm} \sim M^2/Q \ll k_{\perp} \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan process in QCD – factorization

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^{\pm}$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

❑ **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^y \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

❑ **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

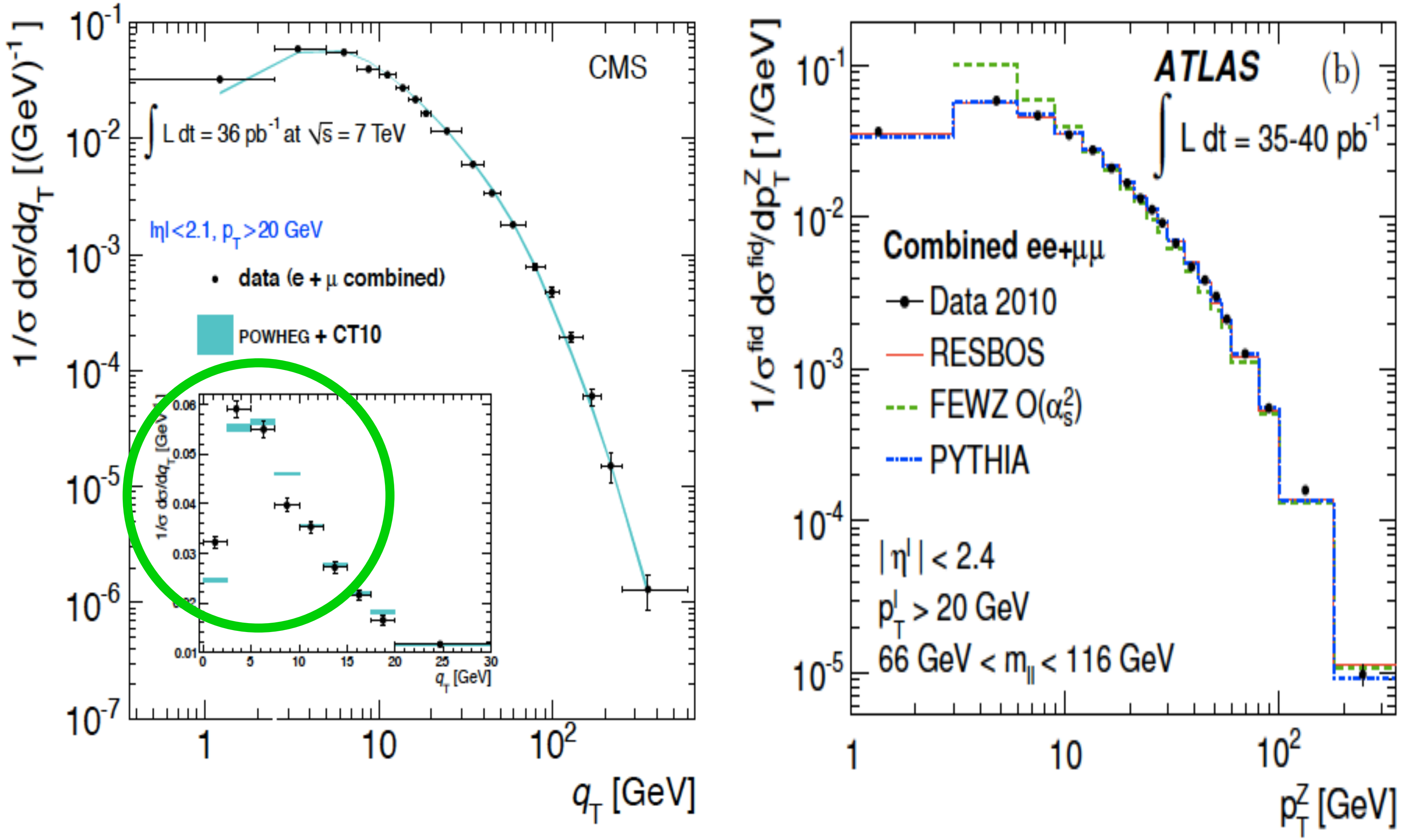
❑ **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

➡ **same formula with polarized PDFs for γ*,W/Z, H^0...**

P_T–distribution (P_T << M) – two scales

❑ **Z^0-PT distribution in pp collisions:**



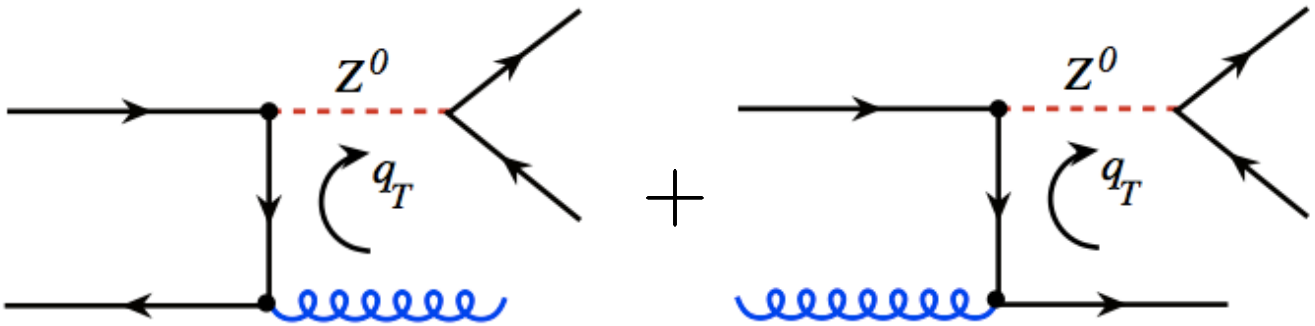
P_T as low as [0,2.5] GeV bin (or about 1.25 GeV)

P_T–distribution (P_T << M) – two scales

- Interesting region – where the most data are:

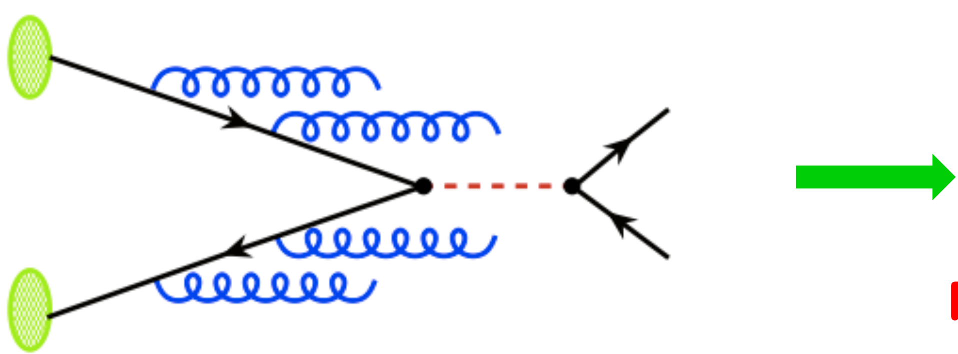
  $P_T << M_Z \sim 91$ GeV   Two observed, but, very different scales

- Fixed order pQCD calculation is not stable!



$$\propto \frac{1}{q_T^2} \to \infty$$

- Large logarithmic contribution from gluon shower:



$$\left[\alpha_s \ln^2\left(\frac{M_Z^2}{q_T^2}\right)\right]^n$$

Resummation is necessary!

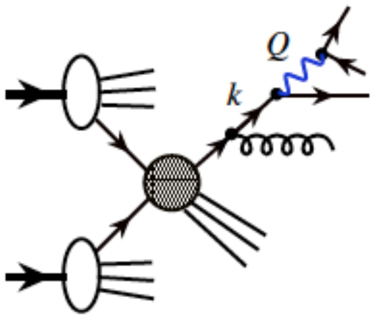
See Yuan's talk

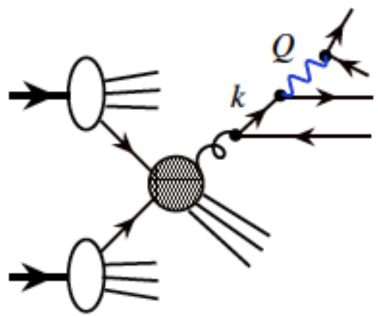
P_T–distribution (P_T >> M) – two hard scales

Beger et al. 2015

- **P_T-distribution – factorizable if M >> Λ_{QCD}:**

$$\frac{d\sigma_{AB}}{dy\,dp_T^2\,dQ^2} = \sum_{a,b}\int dx_a\, f_{a/A}(x_a)\int dx_b\, f_{b/B}(x_b)\,\frac{d\hat{\sigma}_{ab}}{dy\,dp_T^2\,dQ^2}(x_a, x_b, \alpha_s)$$





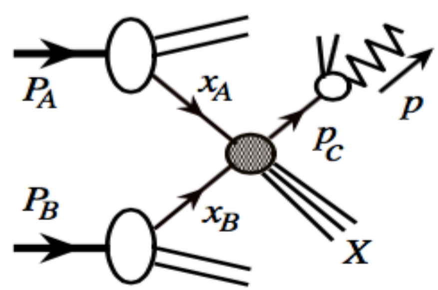
$$\begin{aligned}\hat{\sigma}^{\rm F}/\hat{\sigma}_f^{\rm LO} &\sim \frac{\alpha_s(\mu)}{2\pi} P_{f\to q}(z)\int_{k^2_{\rm min}}^{k^2_{\rm max}}\frac{dk^2}{k^2}\\ &\sim \frac{\alpha_s(\mu)}{2\pi}\ln\left(\frac{p_T^2}{Q^2}\right)\\ &\lesssim \frac{\alpha_s(Q)}{2\pi}\ln\left(\frac{p_T^2}{Q^2}\right) \lesssim 10\%\end{aligned}$$

How big is the logarithmic contribution?

- **Improved factorization:**

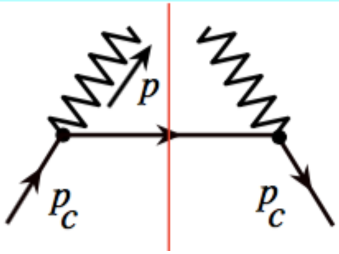
$$\frac{d\sigma_{AB\to V(Q)X}}{dp_T^2\,dy} \equiv \frac{d\sigma^{\rm Dir}_{AB\to V(Q)X}}{dp_T^2\,dy} + \frac{d\sigma^{\rm Frag}_{AB\to V(Q)X}}{dp_T^2\,dy}$$

$$\frac{d\sigma^{\rm Frag}_{AB\to V(Q)X}}{dp_T^2\,dy} = \sum_{a,b,c}\int dx_1 f_a^A(x_1,\mu)\int dx_2 f_b^B(x_2,\mu) \times\int\frac{dz}{z^2}\left[\frac{d\hat{\sigma}^{\rm Frag}_{ab\to cX}}{dp_{c_T}^2\,dy}(x_1,x_2,p_c;\mu_D)\right] D_{c\to V}(z,\mu_D^2;Q^2)$$



P_T–distribution (P_T >> M) – two hard scales

- **Fragmentation functions of elementary particles:**



$$D^{(0)}_{g\to V}(z,\mu_D^2;Q^2)=0$$

$$D^{(0)}_{q\to V}(z,\mu_D^2;Q^2)=\frac{(|g_L^{Vq}|^2+|g_R^{Vq}|^2)}{2}\left(\frac{\alpha_{em}}{2\pi}\right)\left[\frac{1+(1-z)^2}{z}\ln\left(\frac{z\mu_D^2}{Q^2}\right)-z\left(1-\frac{Q^2}{z\mu_D^2}\right)\right]$$

- **Evolution equations:**

$$\mu_D^2\frac{d}{d\mu_D^2}D_{c\to V}(z,\mu_D^2;Q^2)=\left(\frac{\alpha_{\rm em}}{2\pi}\right)\gamma_{c\to V}(z,\mu_D^2,\alpha_s;Q^2)+\left(\frac{\alpha_s}{2\pi}\right)\sum_d\int_z^1\frac{dz'}{z'}P_{c\to d}\left(\frac{z}{z'},\alpha_s\right)D_{d\to V}(z',\mu_D^2;Q^2)$$

$$D_{c\to V}(z,\mu_D^2\le Q^2/z;Q^2)=0$$

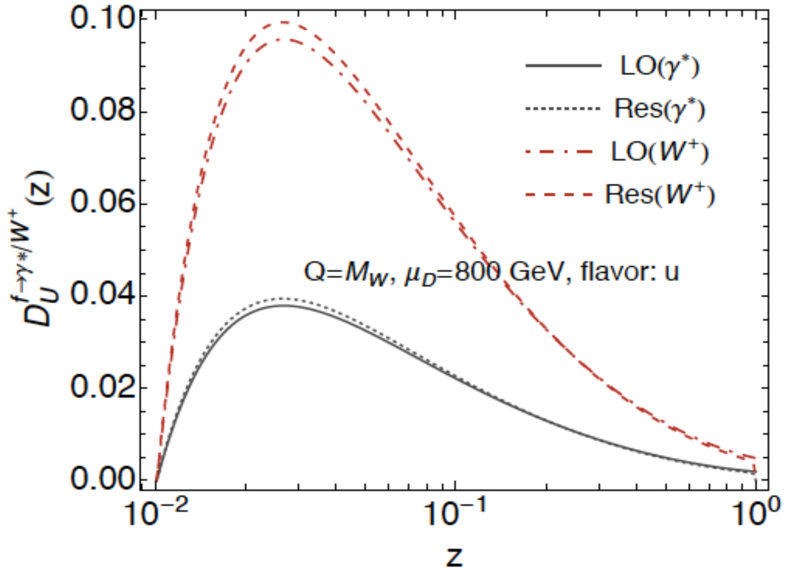
- **Evolution kernels:**

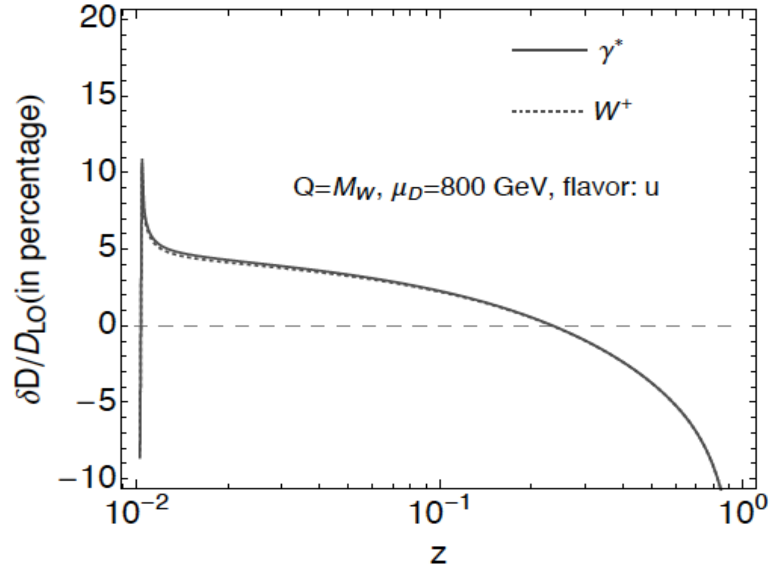
$$\gamma^{(0)}_{q\to V}(z,k^2;Q^2)\;=\;\frac{(|g_L^{Vq}|^2+|g_R^{Vq}|^2)}{2}\left[\frac{1+(1-z)^2}{z}-z\left(\frac{Q^2}{zk^2}\right)\right]\theta\left(k^2-\frac{Q^2}{z}\right)$$

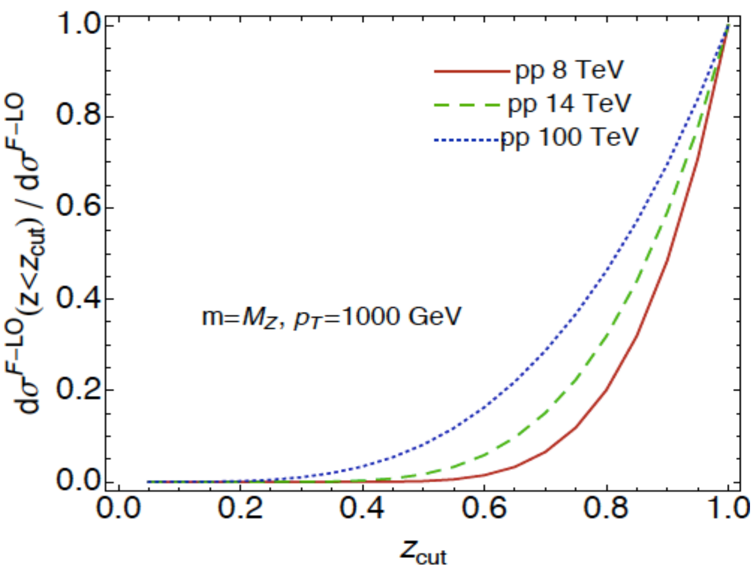
$$\gamma^{(0)}_{g\to V}(z,k^2;Q^2)=0$$

If $Q\gg\Lambda_{\rm QCD}$, reorganization of perturbative expansion to remove all logarithms of hard parts

P_T–distribution (P_T >> M) – two hard scales







$\sigma_{p_T>800\,\mathrm{GeV}}$	VB	C-LO	C-NLO	NLO[modified]
13 TeV [fb]	Z	74.1	$117.4^{+12.0}_{-11.5}$	$120.5^{+9.2}_{-10.4}$
	W^+	126.2	$199.4^{+20.1}_{-19.3}$	$204.4^{+15.4}_{-17.3}$
	W^-	55.8	$90.2^{+9.6}_{-9.2}$	$92.7^{+7.4}_{-8.2}$
100 TeV [pb]	Z	11.48	$19.68^{+1.53}_{-1.30}$	$20.16^{+1.00}_{-0.98}$
	W^+	15.08	$26.23^{+2.14}_{-1.79}$	$26.86^{+1.41}_{-1.35}$
	W^-	10.50	$18.18^{+1.47}_{-1.23}$	$18.61^{+0.96}_{-0.92}$

Fragmentation logs are under control!

Summary of lecture two

- PQCD factorization approach is mature, and has been extremely successful in predicting and interpreting high energy scattering data with momentum transfer > 2 GeV

- NLO calculations are available for most observables, Many new techniques have been developed in recent years for NNLO or higher order calculations (not discussed here), NNLO are becoming available for the search of new physics

- Leading power/twist pQCD "Factorization + Resummation" allow to have precision tests of QCD theory in the asymptotic regime, and to control the background so well to discover potential "new physics" beyond SM

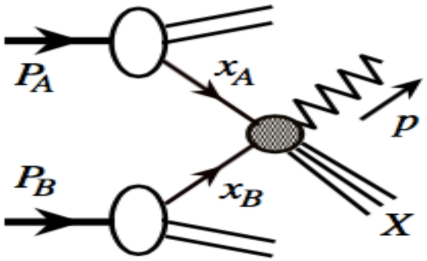
See Yuan's lectures

What about the power corrections, richer in dynamics?

Backup slides

A complete example – "Drell-Yan"

- **Heavy boson production in hadronic collisions:**



$$A(P_A) + B(P_B) \to V[\gamma^*, W/Z, H^0, ...](p) + X$$

- **Cross section with single hard scale:** $p_T \sim M_V$

$$\frac{d\sigma_{AB\to V}}{dydp_T^2}(p_T \sim M_V), \qquad \frac{d\sigma_{AB\to V}}{dy}(M_V), \qquad \sigma_{AB\to V}(M_V)$$

$$\sigma_{AB\to V}(M_V) = \sum_{ff'} \int dx_A\, f(x_A, \mu^2) \int dx_B\, f(x_B, \mu^2)\, \hat{\sigma}_{ff'\to V}(x_A, x_B, \alpha_s(\mu); M_V)$$

– Fixed order pQCD calculation

- **Cross section with two different hard scales:**

$$\frac{d\sigma_{AB\to V}}{dydp_T^2}(p_T \gg M_V)$$

– Resummation of single logarithms:

$$\alpha_s^n \ln^n(p_T^2/M_V^2)$$

$$\frac{d\sigma_{AB\to V}}{dydp_T^2}(p_T \ll M_V)$$

– Resummation of double logarithms:

$$\alpha_s^n \ln^{2n}(M_V^2/p_T^2)$$

Same discussions apply to production of Higgs, and other heavy particles

Total cross section – single hard scale

- **Partonic hard parts:**

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots\right]$$

LO NLO NNLO

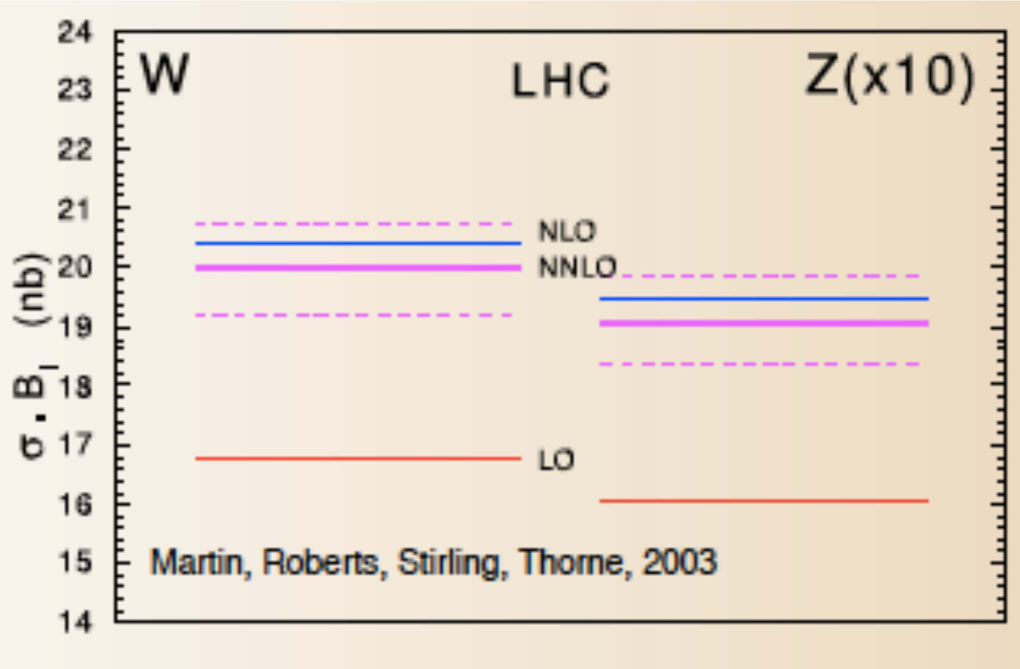
- **NNLO** total x-section $\sigma(AB \to W, Z)$:

(Hamberg, van Neerven, Matsuura; Harlander, Kilgore 1991)

✧ Scale dependence:

a few percent

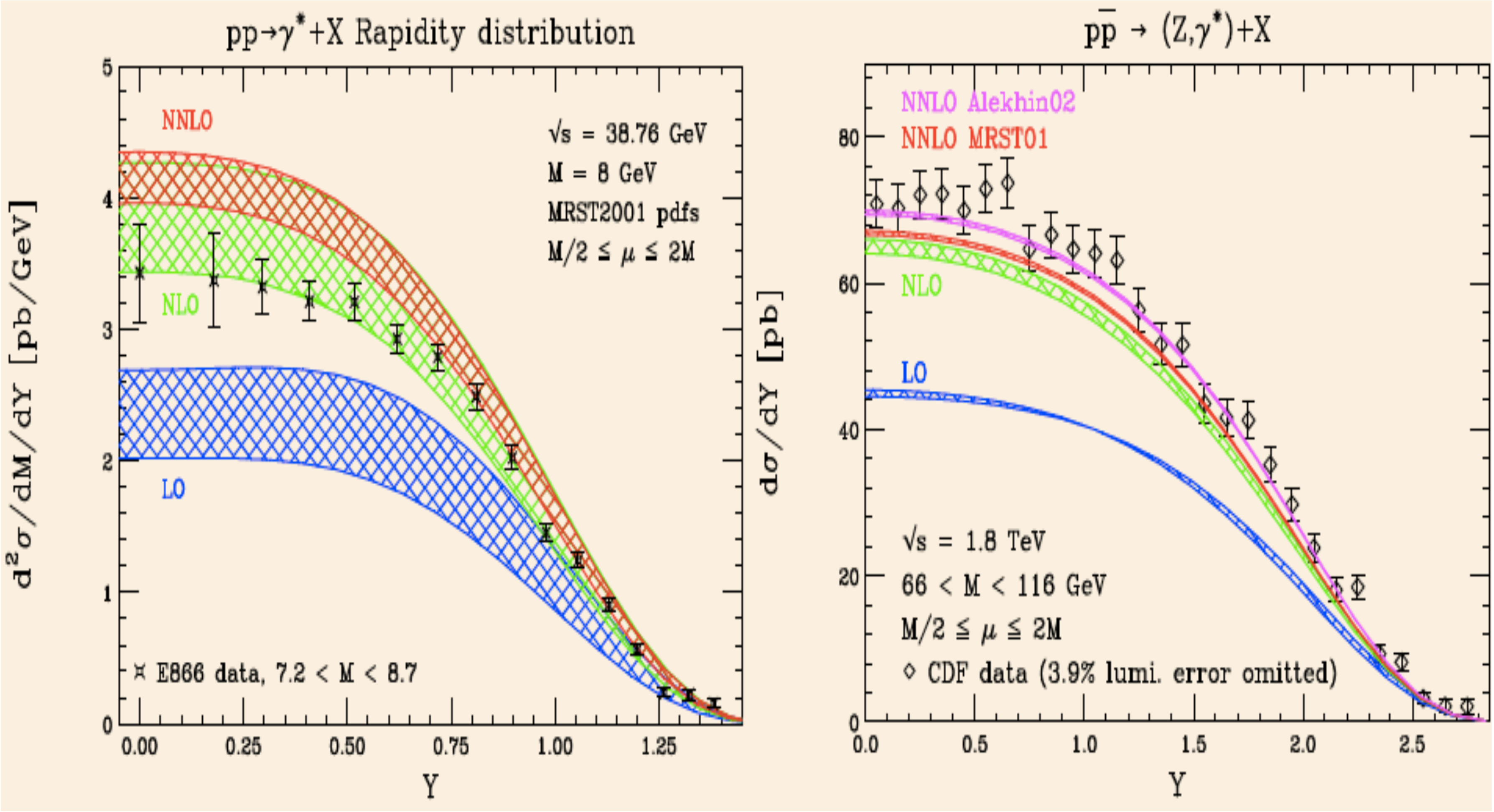
✧ NNLO K-factor is about 0.98 for LHC data, 1.04 for Tevatron data



Rapidity distribution – single hard scale

- ❑ **NNLO differential cross-section:**

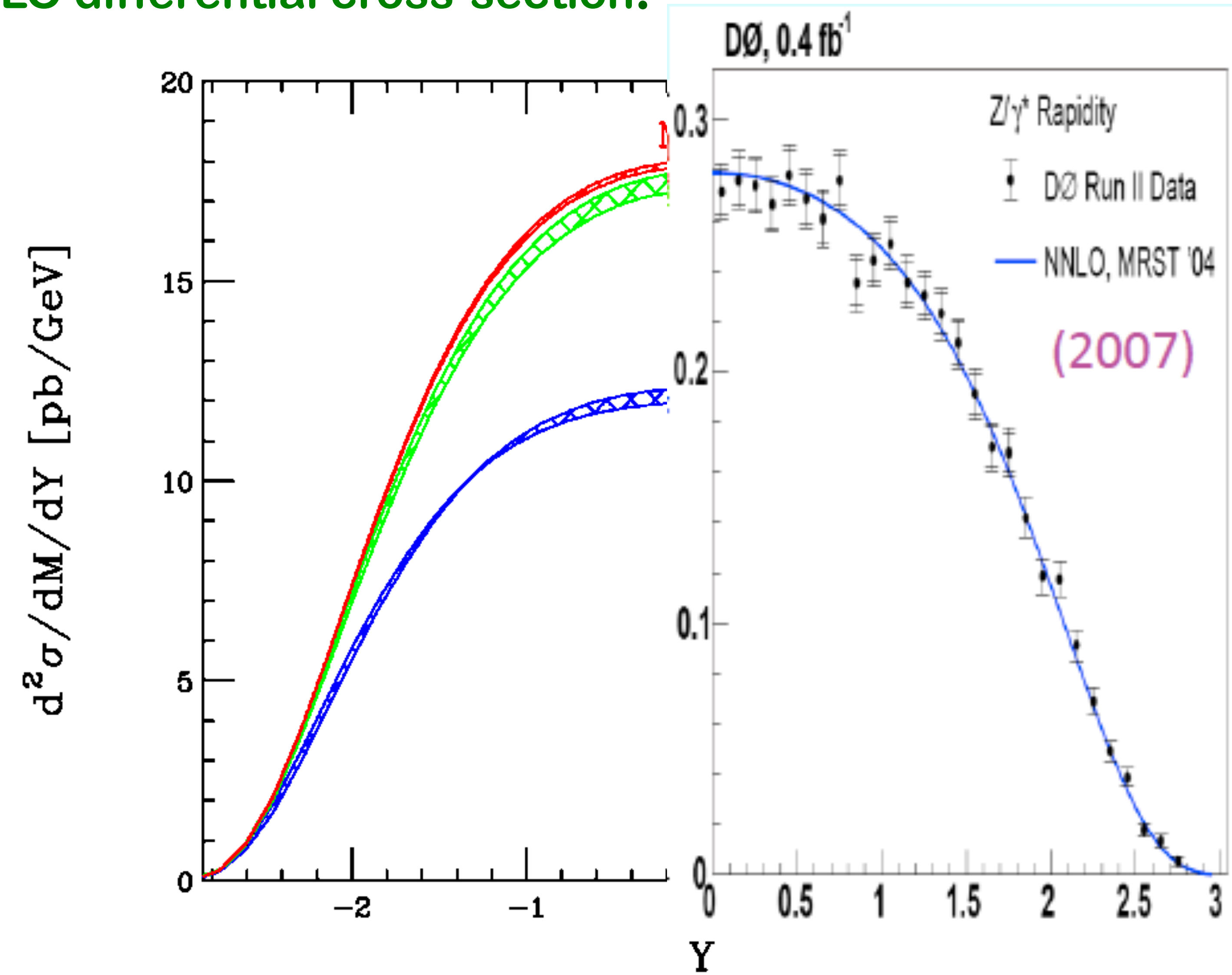
Anastasiou, Dixon, Melnikov, Petriello, 2003-05



Rapidity distribution – single hard scale

❑ NNLO differential cross-section:

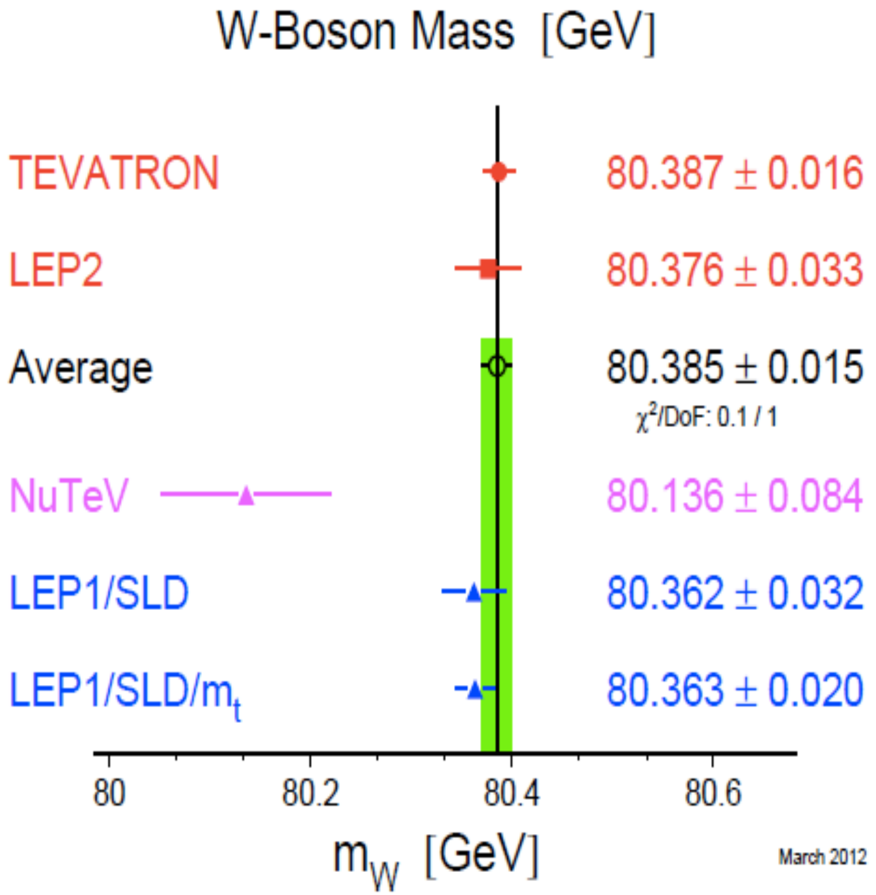
Anastasiou, Dixon, Melnikov, Petriello, 2003-05

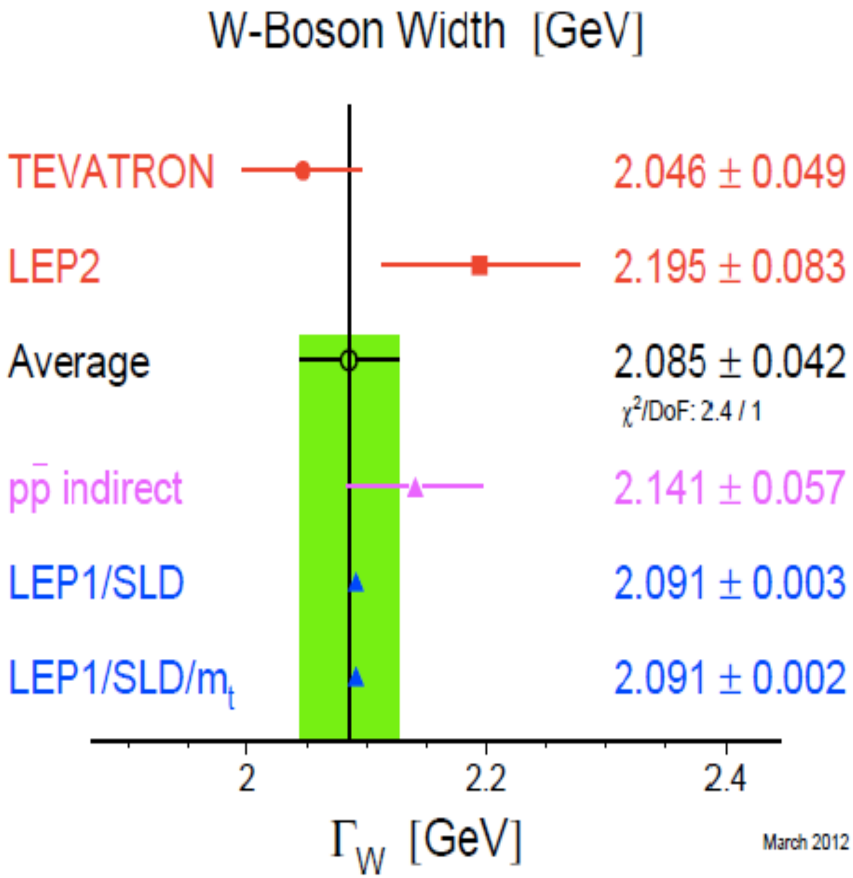


Determination of mass and width

Fernando Febres Cordero, CTEQ SS2012

- **W mass & width:**

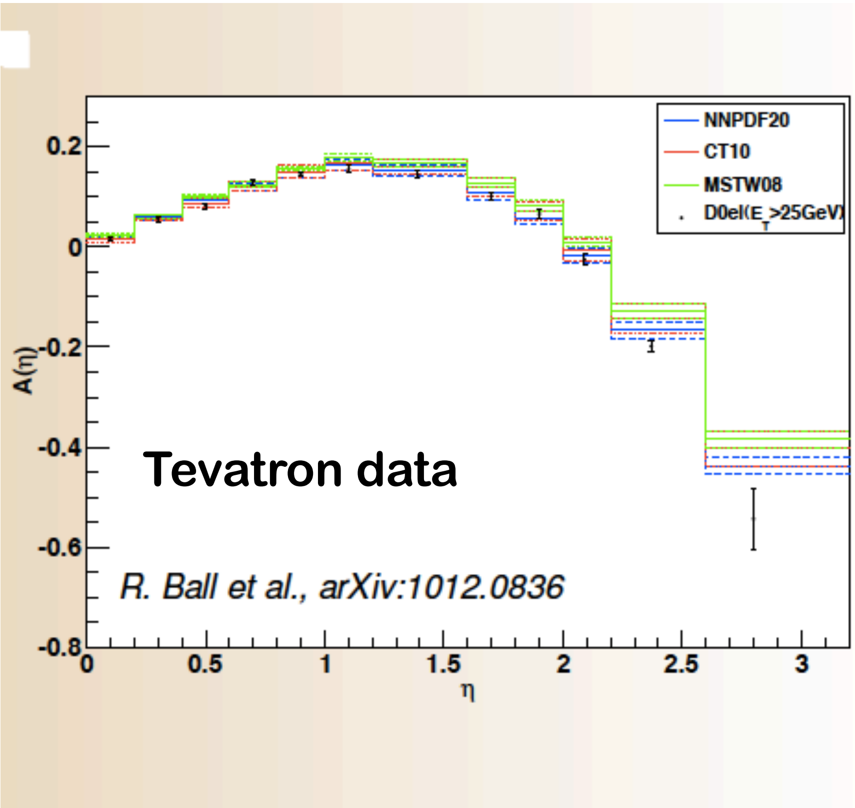


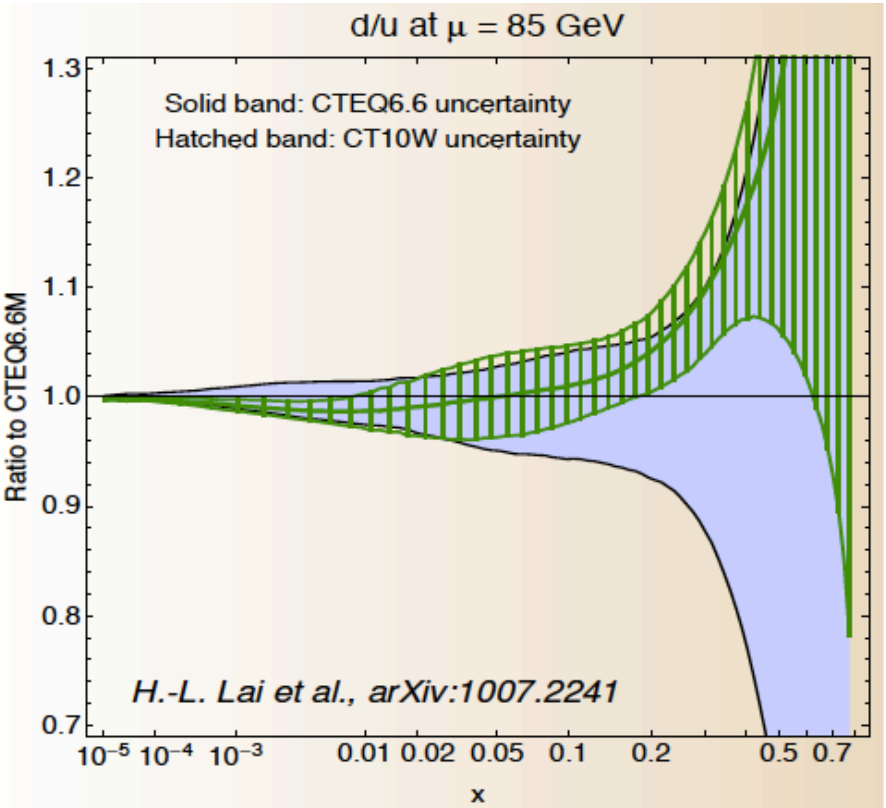


Charge asymmetry – single hard scale

- **Charged lepton asymmetry:** $y \to y_{\max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B, M_W)/u(x_B, M_W) - d(x_A, M_W)/u(x_A, M_W)}{d(x_B, M_W)/u(x_B, M_W) + d(x_A, M_W)/u(x_A, M_W)}$$





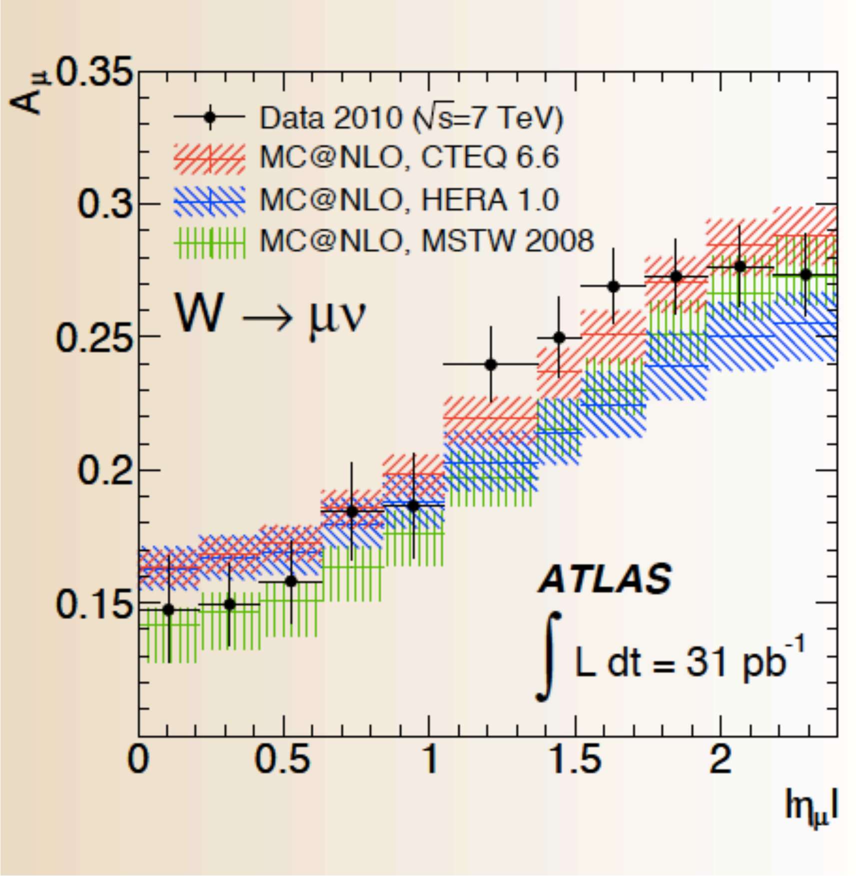
The A_{ch} data distinguish between the PDF models, reduce the PDF uncertainty

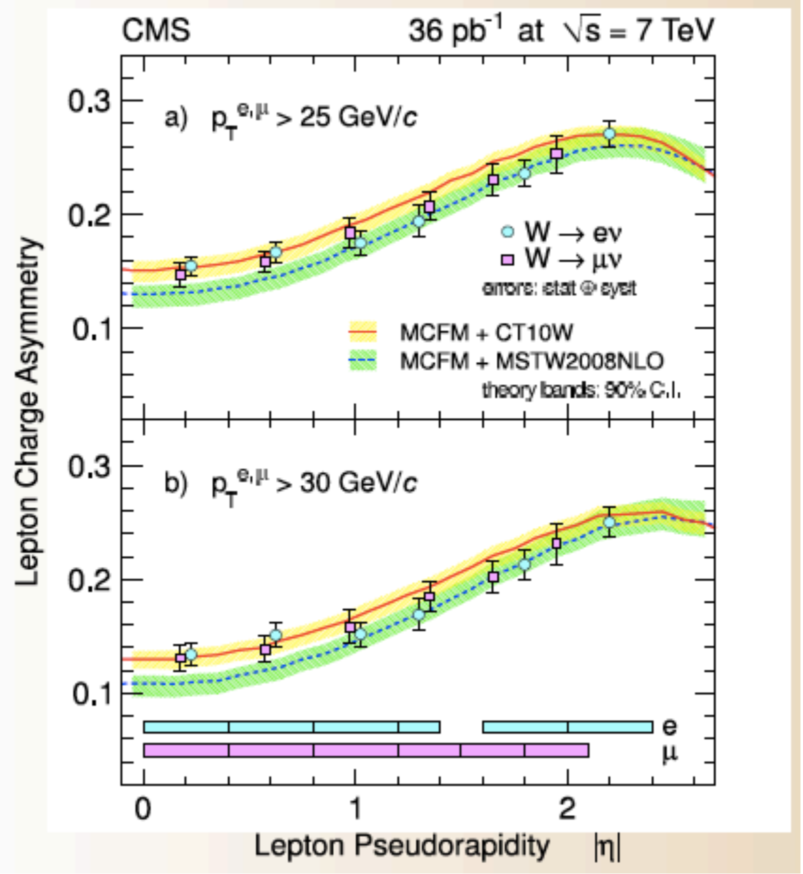
D0 – W charge asymmetry

Charge asymmetry – single hard scale

- **Charged lepton asymmetry:** $y \to y_{\max}$

$$A_{ch}(y_e) = \frac{d\sigma^{W^+}/dy_e - d\sigma^{W^-}/dy_e}{d\sigma^{W^+}/dy_e + d\sigma^{W^-}/dy_e} \longrightarrow \frac{d(x_B,M_W)/u(x_B,M_W) - d(x_A,M_W)/u(x_A,M_W)}{d(x_B,M_W)/u(x_B,M_W) + d(x_A,M_W)/u(x_A,M_W)}$$



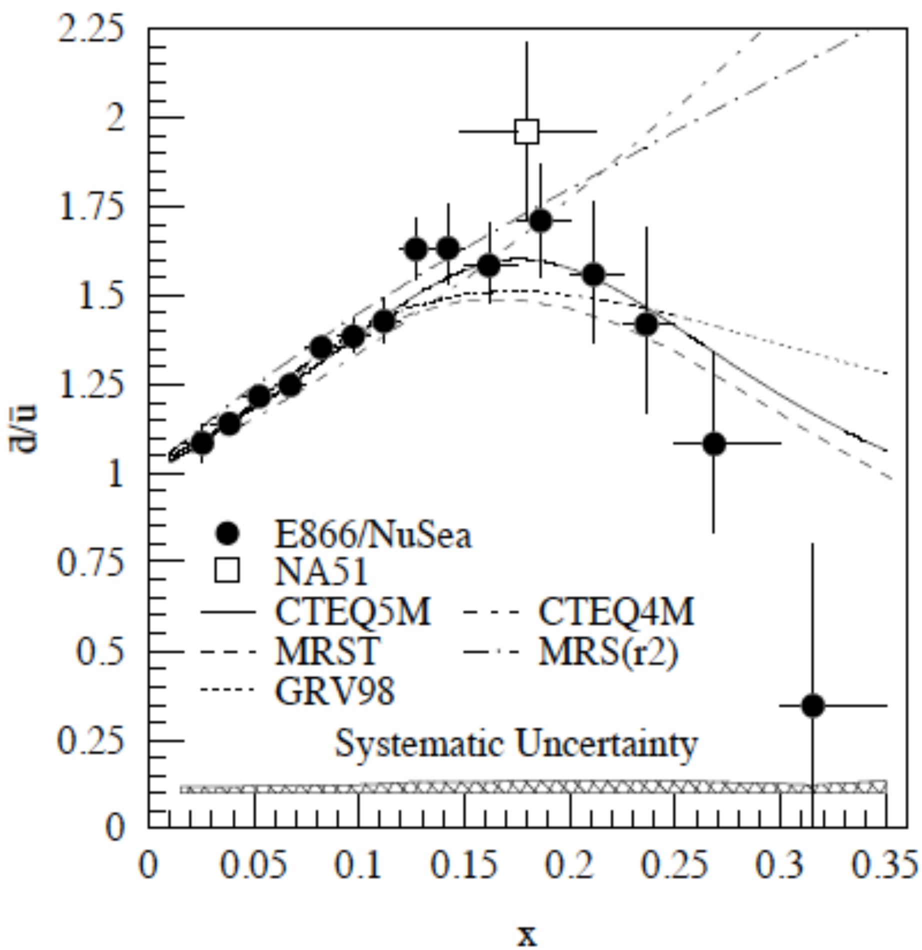


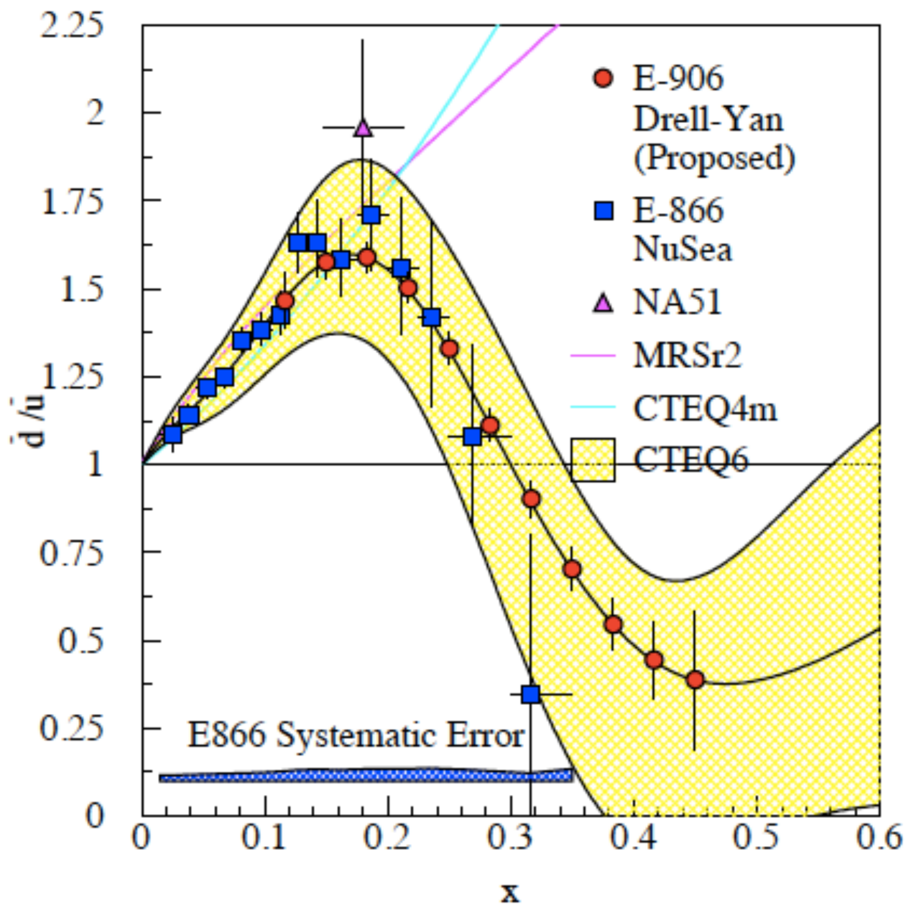
Sensitive both to d/u at x > 0.1 and u/d at x ~ 0.01

Flavor asymmetry – single hard scale

❑ **Flavor asymmetry of the sea:**

$$\sigma_{DY}(p+d)/2\sigma_{DY}(p+p) \simeq \left[1+\bar{d}(x)/\bar{u}(x)\right]/2$$





Could QCD allow ubar(x) > dbar(x)?