QCD in Collisions with Polarized Beams

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Summary of lecture two and three

- PQCD factorization approach is mature, and has been extremely successful in predicting and interpreting high energy scattering data with momentum transfer > 2 GeV

- NLO calculations are available for most observables, Many new techniques have been developed in recent years for NNLO or higher order calculations (not discussed here), NNLO are becoming available for the search of new physics

- Leading power/twist pQCD "Factorization + Resummation" allow to have precision tests of QCD theory in the asymptotic regime, and to control the background so well to discover potential "new physics" beyond SM

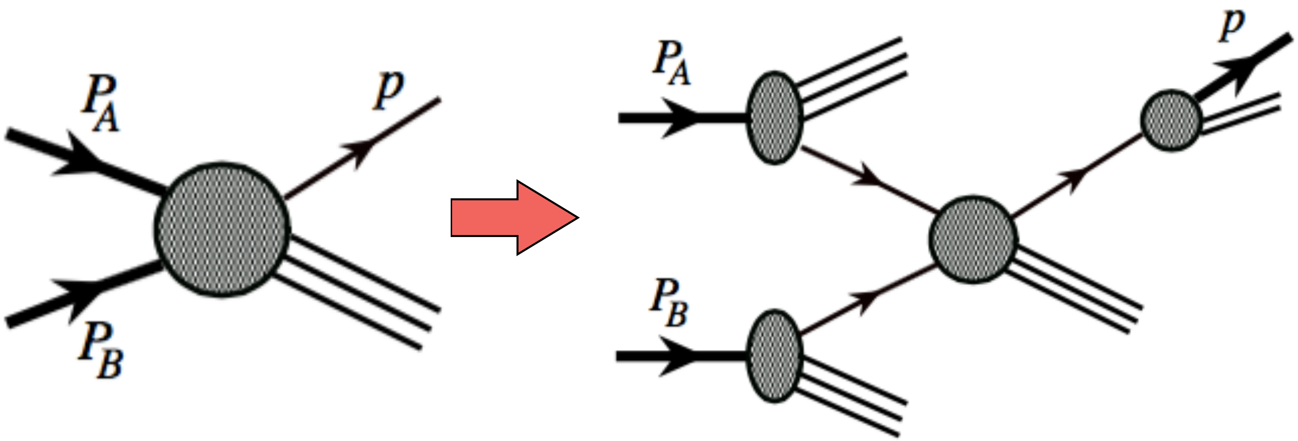
See Yuan's lectures

What about the power corrections, richer in dynamics?

Factorization for more than two hadrons

Nayak, Qiu, Sterman, 2006

- Factorization for high p_T single hadron:



$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O (1/P_T^2)

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2) \otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

✧ **Fragmentation function:** $D_{c\to C}(z, \mu_F^2)$

✧ **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Example: Heavy quarkonium production at high P_T

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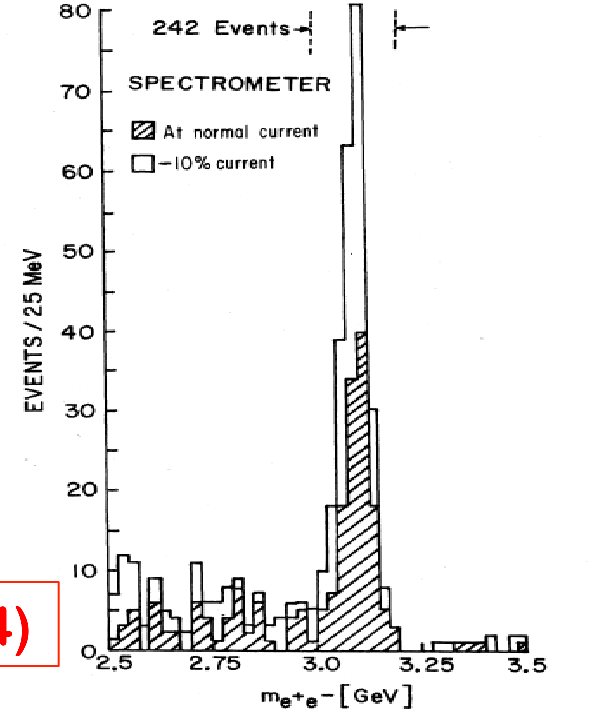
Experimental Observation of a Heavy Particle J†

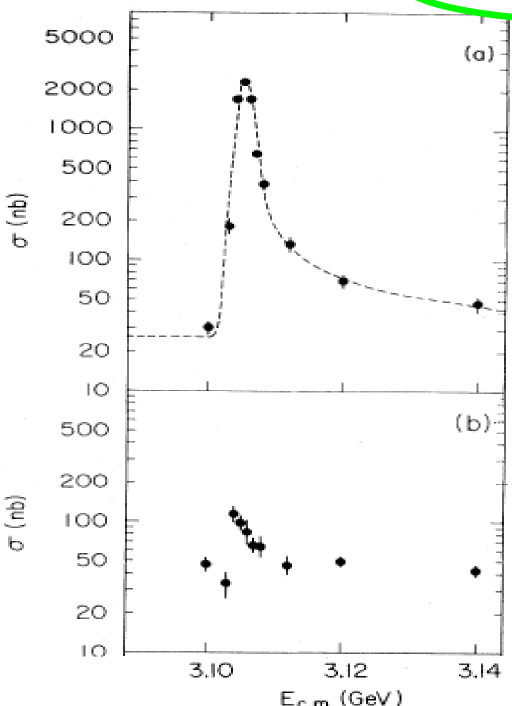
J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973

(Received 12 November 1974)





November Revolution (1974)

Discovery of a Narrow Resonance in e^+e^- Annihilation*

J.-E. Augustin,† A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie,† R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, and F. Vannucci‡
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

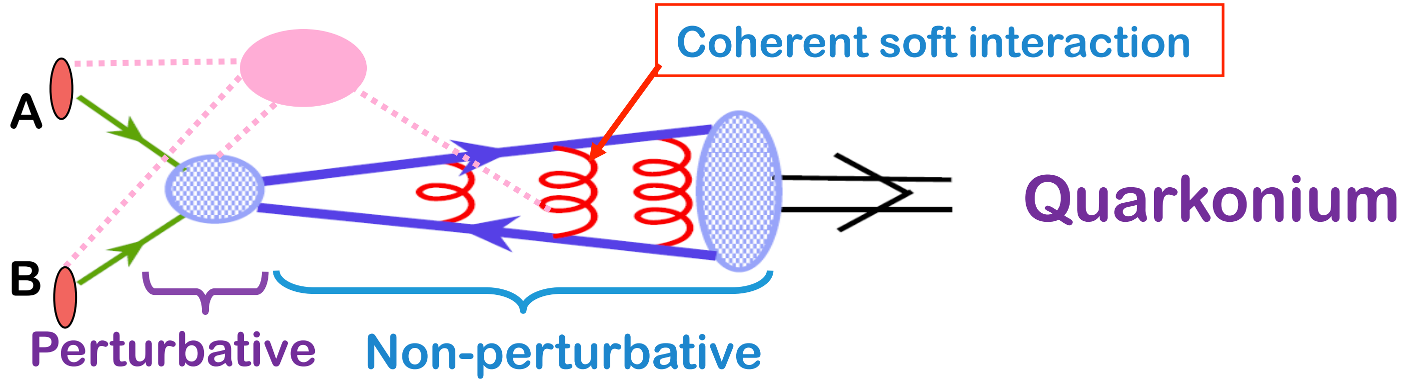
and

G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre,§ G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

(Received 13 November 1974)

Basic production mechanism

❑ Production of an off-shell heavy quark pair:



$$\Delta r \leq \frac{1}{2m_Q}$$

❑ Approximation: production of an on-shell pair + hadronization

$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}}\right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

✧ Factorization – proof?

✧ Different models ⇔ Different assumptions/treatments on how the heavy quark pair becomes a quarkonium?

A long history for the production

- Color singlet model: 1975 –

  Einhorn, Ellis (1975),
  Chang (1980),
  Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- Color evaporation model: 1977 –

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- NRQCD model: 1986 –

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and α_s

- QCD factorization approach: 2005 –

  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …

  $P_T >> M_H$: M_H/P_T power expansion + α_s – expansion

  Unknown, but universal, fragmentation functions – evolution

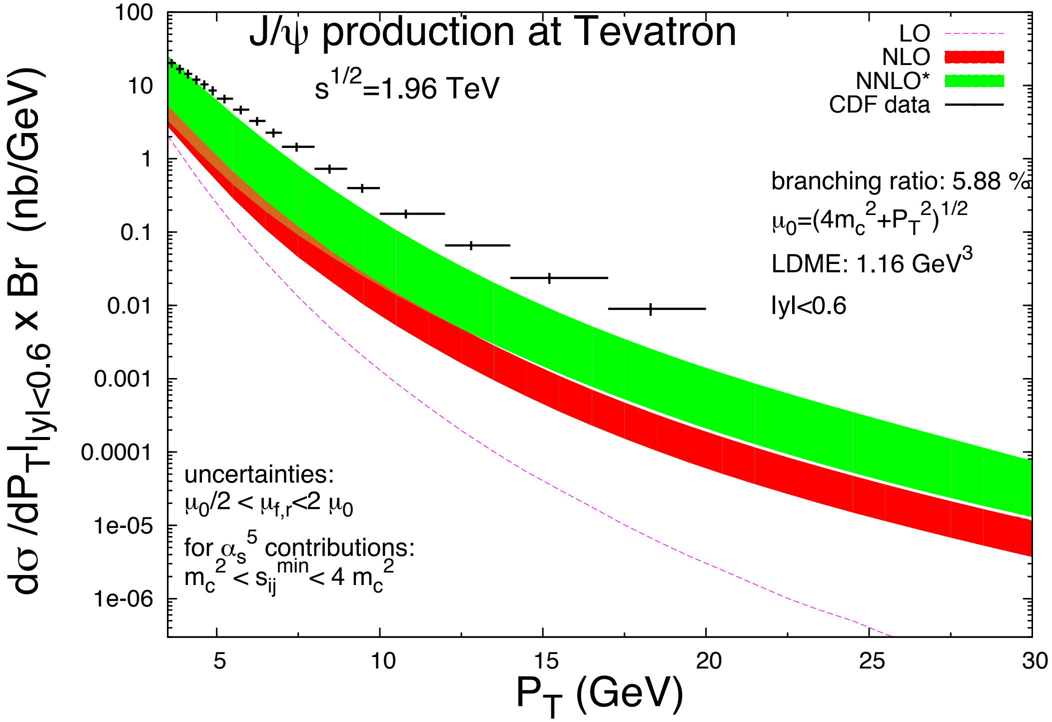
- Soft-Collinear Effective Theory + NRQCD: 2012 –

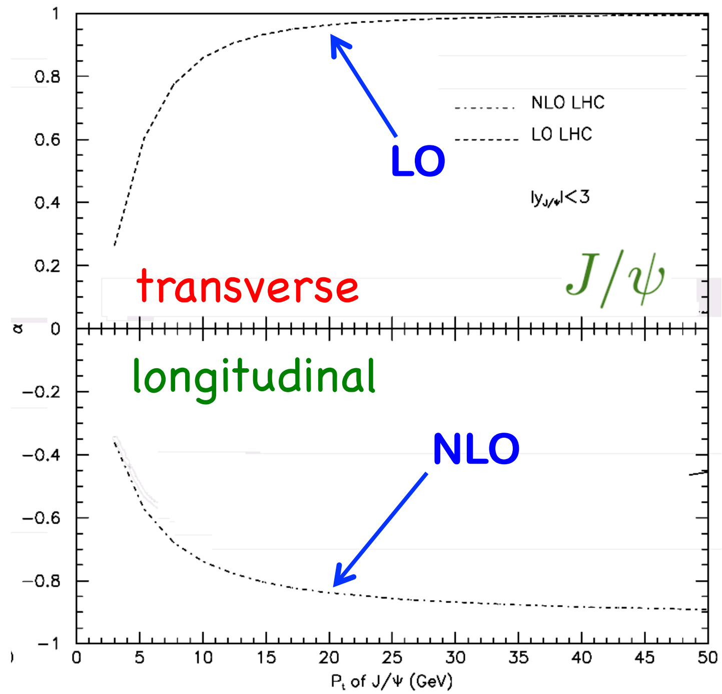
  Fleming, Leibovich, Mehen, …

Color singlet model (CSM)

- Effectively No parameter:

Campbell, Maltoni, Tramontano (2007),
Artoisenet, Lansburg, Maltoni (2007),
Artoisenet, et al. (2008)



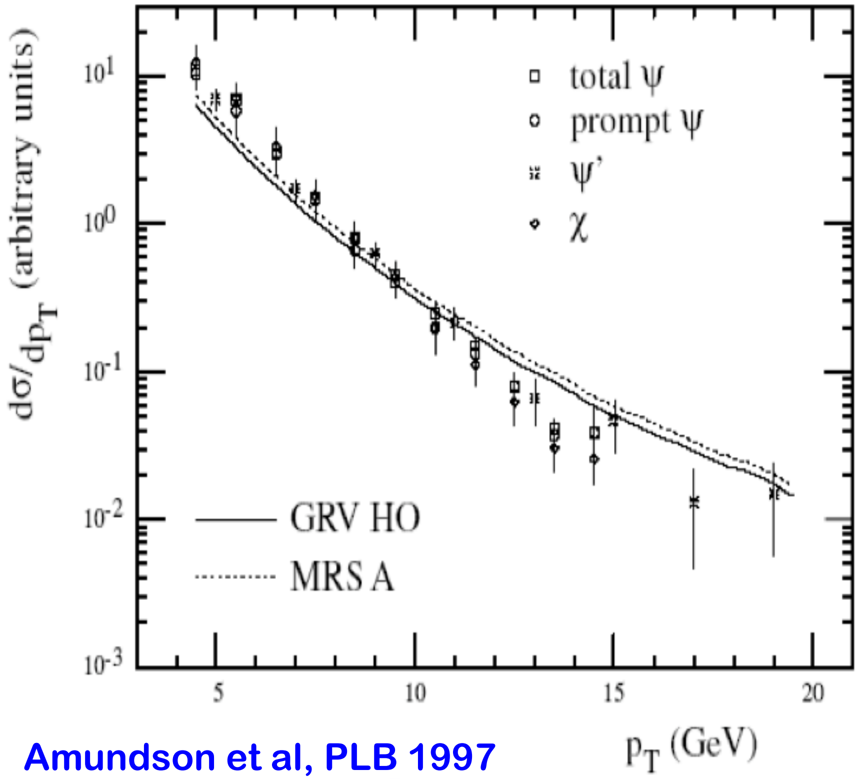


B. Gong et, al. PRL (2008)

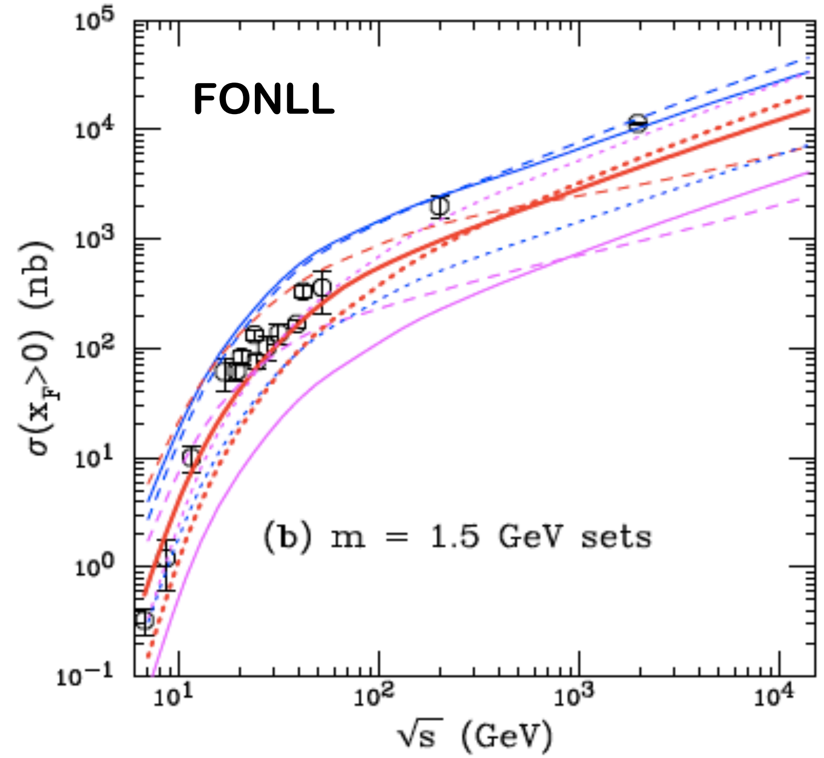
- Issues:
  - ✧ How reliable is the perturbative expansion?
  - ✧ S-wave: large corrections from high orders
  - ✧ P-wave: Infrared divergent – CSM is not complete

Color evaporation model (CEM)

- One parameter per quarkonium:



Amundson et al, PLB 1997



Nelson, Vogt, Frawley, PRC 2013

- Question:
  - Better p_T distribution – the shape?
  - Need intrinsic k_T – its distribution?

NRQCD – most successful so far

Bodwin, Braaten, Lapage, 1995

- **NRQCD factorization:**

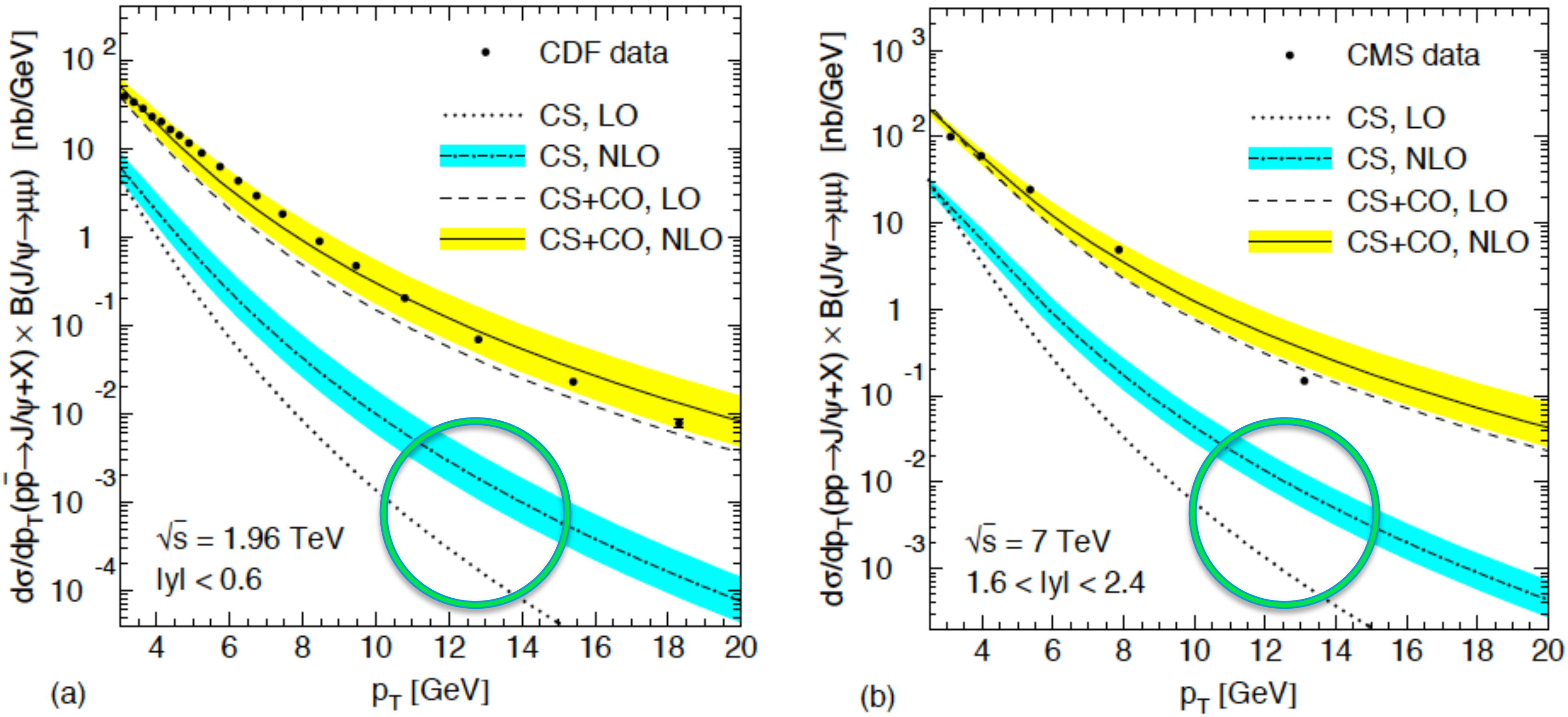
$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n)\rangle$$

✧ **4 leading channels in v**

$${}^3S_1^{[1]},\ {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

✧ **Full NLO in α_s**

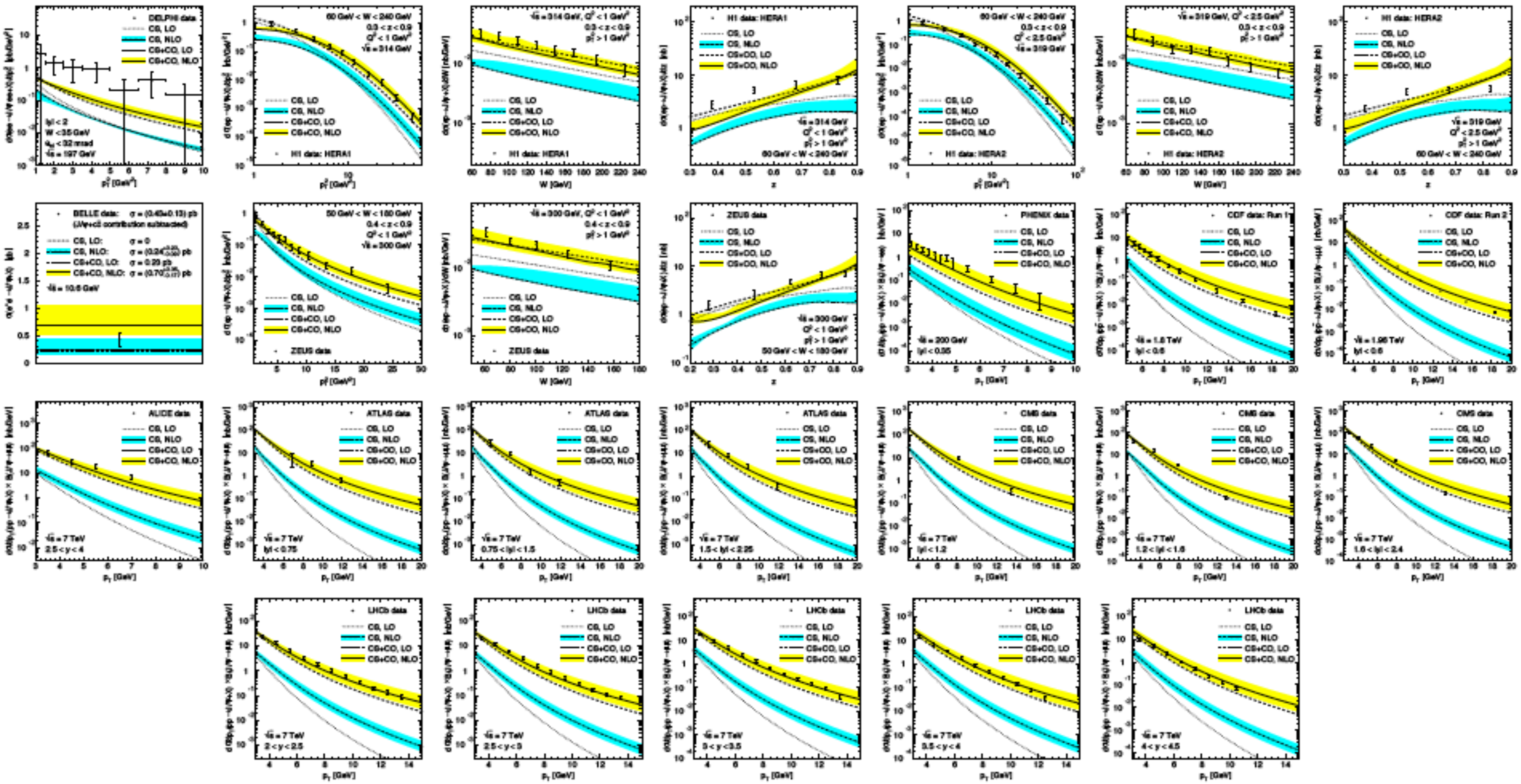
- **Phenomenology:**



- **Fine details – shape – high at large p_T?**

PRL 106, 022003 (2011)

NRQCD – global analysis



194 data points from 10 experiments, fix singlet $\langle O[^3S_1^{[1]}]\rangle = 1.32\ \mathrm{GeV}^3$

$\langle O[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\quad \langle O[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

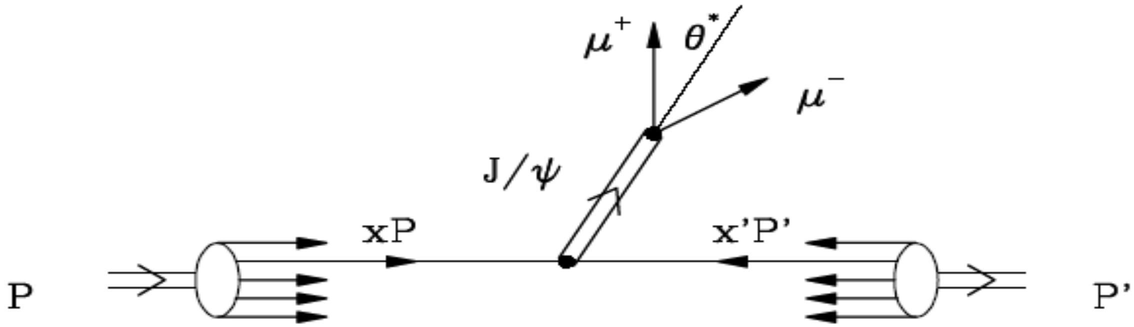
$\langle O[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

$\chi^2/d.o.f. = 857/194 = 4.42$

Butenschoen and Kniehl, arXiv: 1105.0820

Heavy quarkonium polarization

- Measure angular distribution of $\mu^+\mu^-$ in J/ψ decay



- Normalized distribution – integrate over φ:

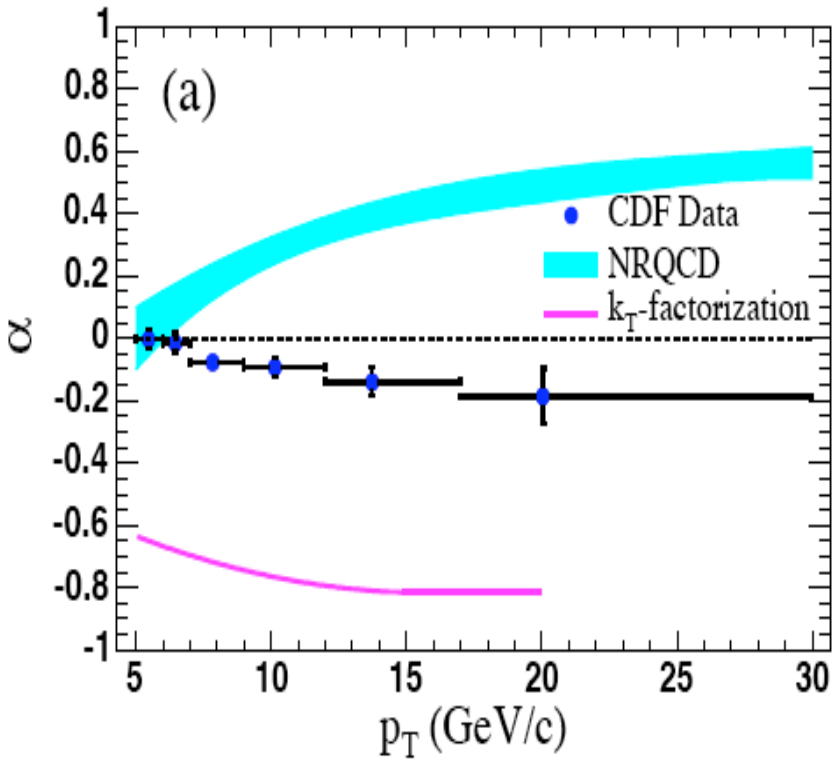
$$I(\cos\theta^*) = \frac{3}{2(\alpha+3)}\left(1+\alpha\cos^2\theta^*\right)$$

$$\alpha = \begin{cases} +1 & \text{fully transverse} \\ 0 & \text{unpolarized} \\ -1 & \text{fully longitudinal} \end{cases}$$

Also referred as λ_θ by LHC experiments

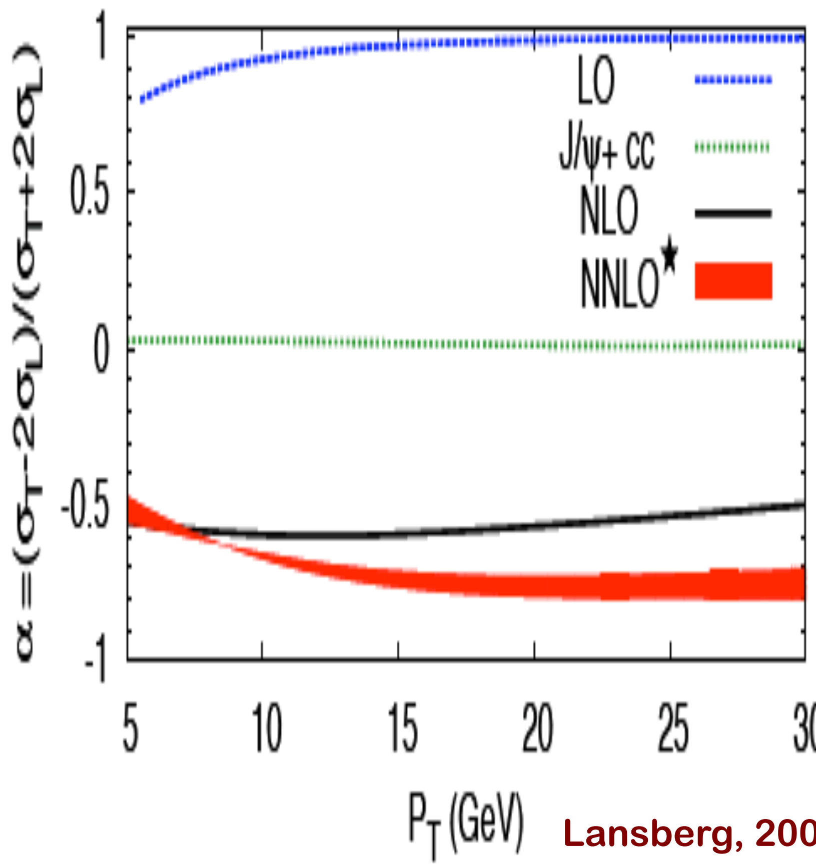
Surprises from J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

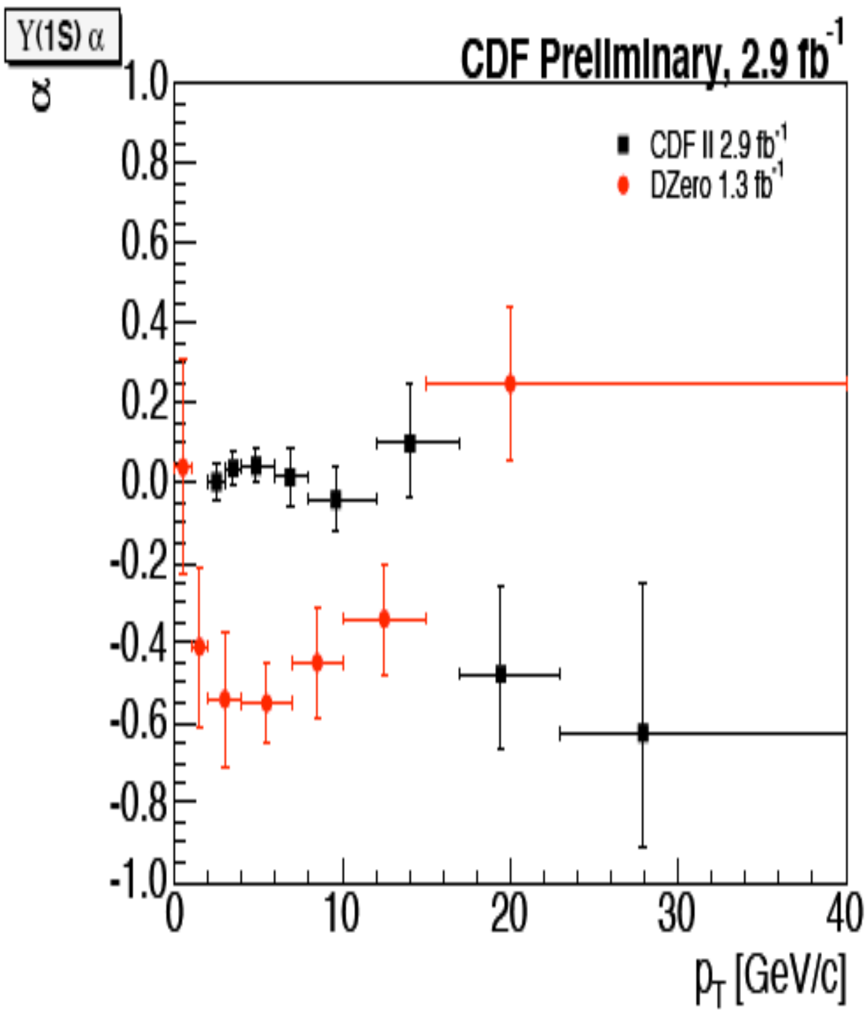
CSM



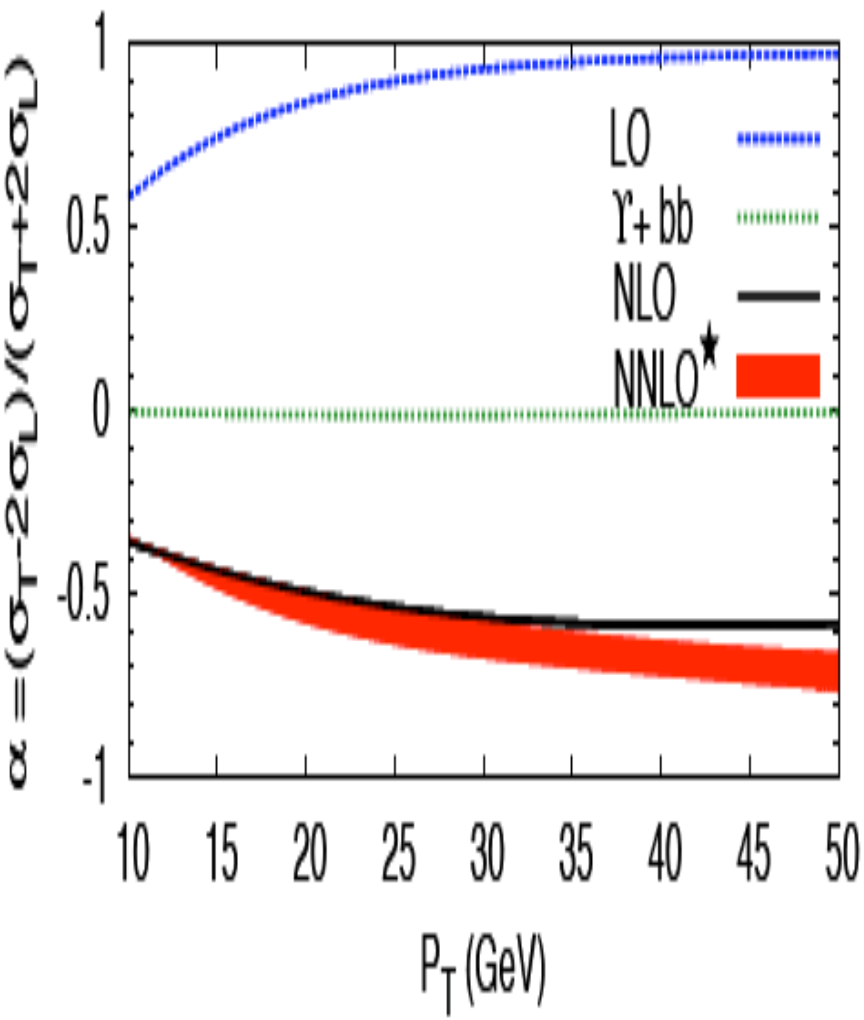
Lansberg, 2009

- ✧ **NRQCD:** Dominated by color octet – NLO is not a huge effect
- ✧ **CSM:** Huge NLO – change of polarization?

Confusions from Upsilon polarization



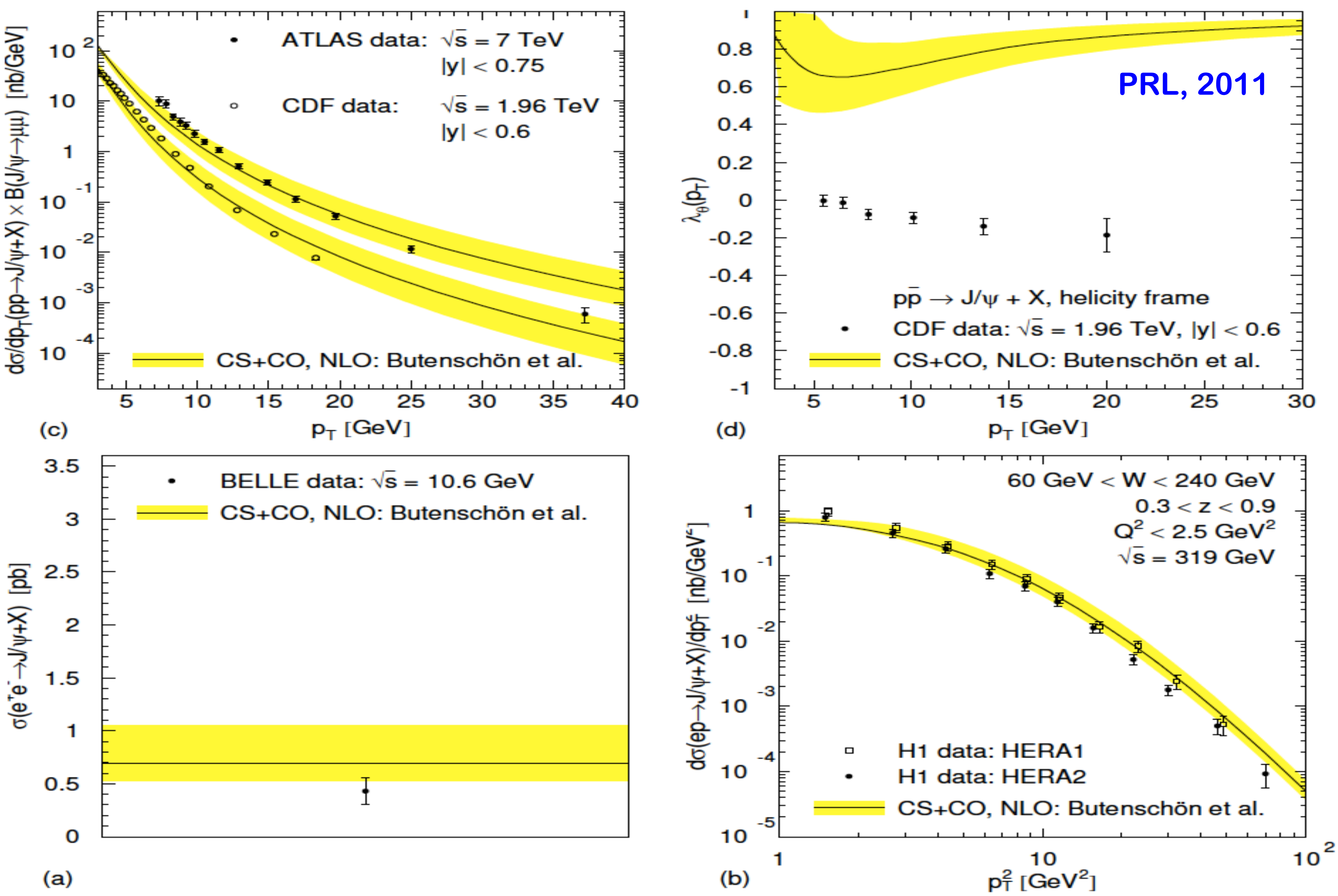
Theory - CSM



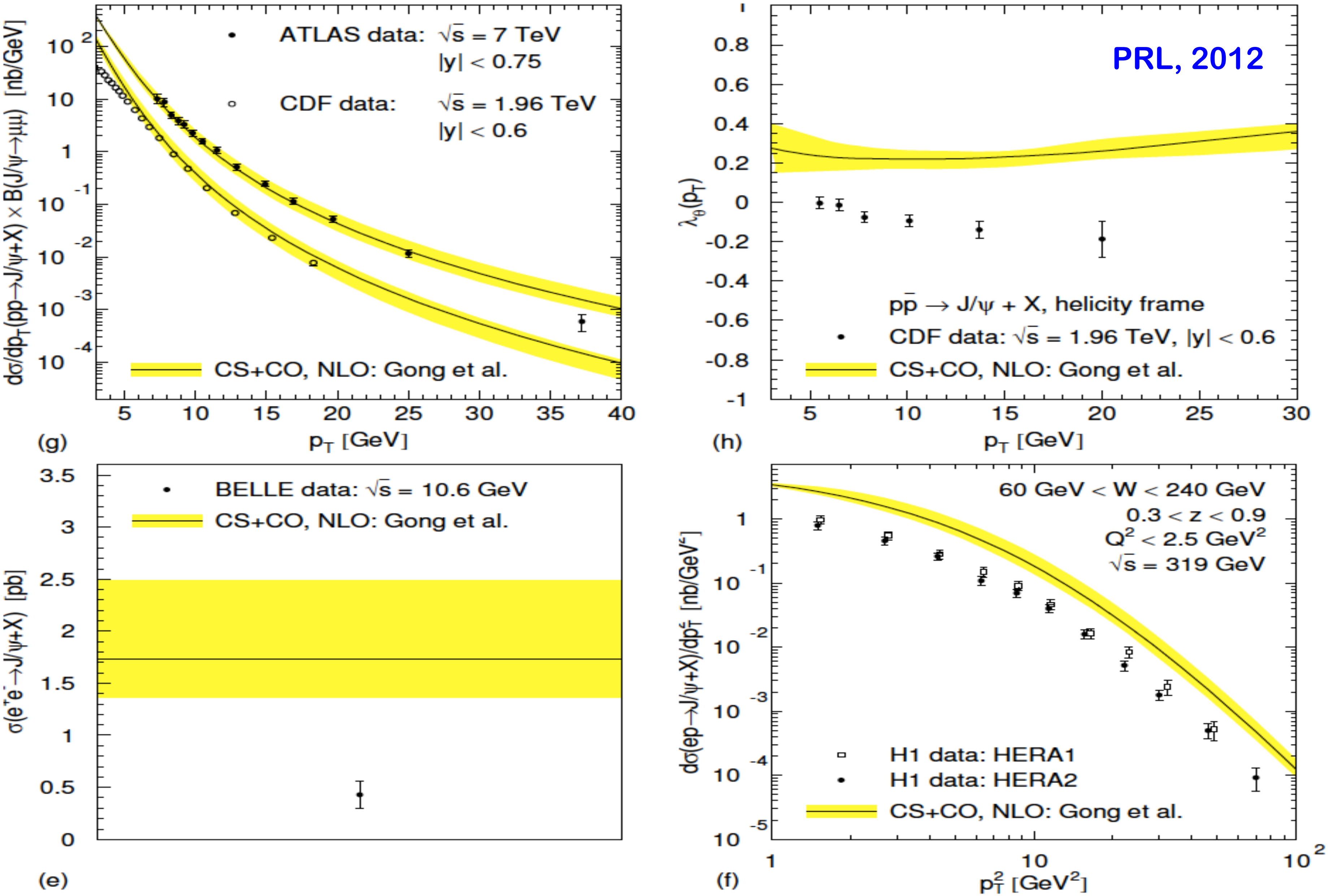
Resolution between CDF and D0?

P. Artoisenet, et al. 2008

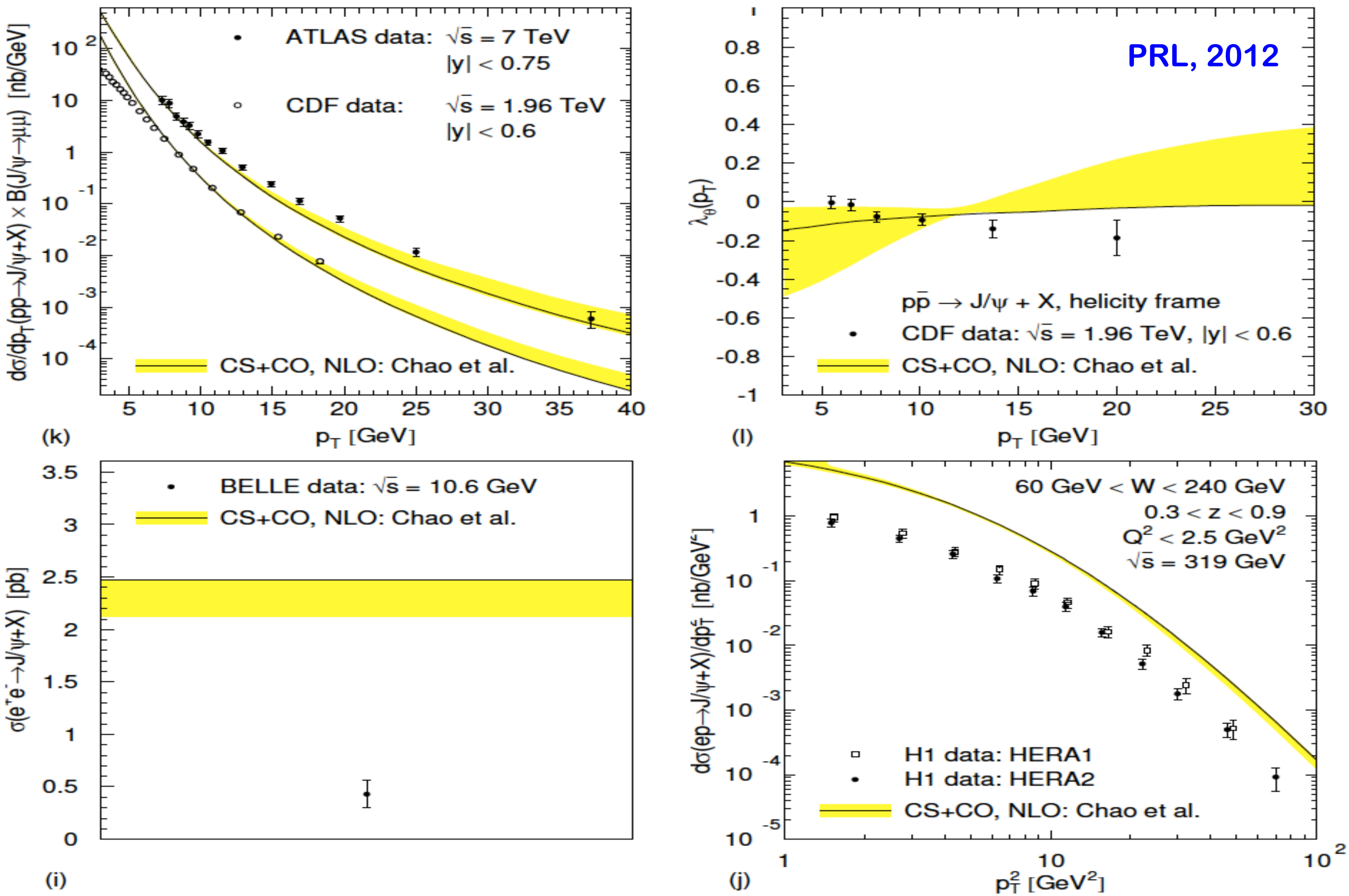
NLO theory fits – Butenschoen et al.



NLO theory fits – Gong et al.



NLO theory fits – Chao et al.



40 years after the November Revolution?

- Theory – the state of arts – NLO (NRQCD):
  - ✧ Very difficult to calculate, no analytical expression

    ➡ hard to obtain a clear physical picture on how various states of heavy quark pair are actually produced?

  - ✧ For some channels, NLO corrections are orders larger than LO

    ➡ questions whether higher order contributions are negligible, while it is extremely difficult, if not impossible, to go beyond the NLO

- Comparison with data:
  - ✧ Quarkonium polarization – "ultimate" test of NRQCD!

    ➡ Clear mismatch between theory predictions and data

  - ✧ Universality of NRQCD matrix elements – predictive power!

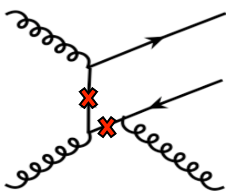
    ➡ Clear tension between different data sets, e+e-, ep, pp, …

Why high orders in NRQCD are so large?

Kang, Qiu and Sterman, 2011

- **LO in α_s but higher power in $1/p_T$:**

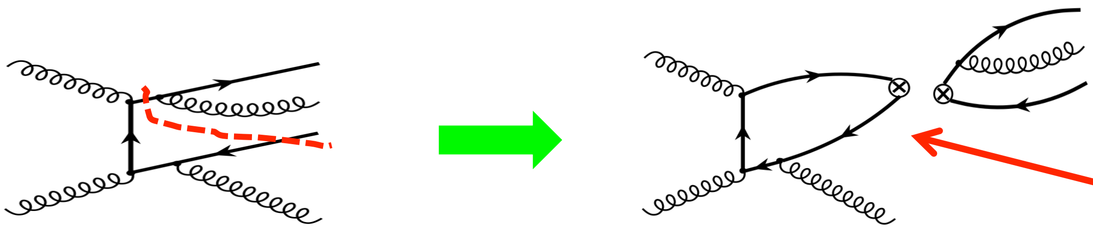
LO in α_s:



$$\hat{\sigma}^{\rm LO} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

CSM and NRQCD spin-1 projection NNLP in $1/p_T$!

- **NLO in α_s but lower power in $1/p_T$:**

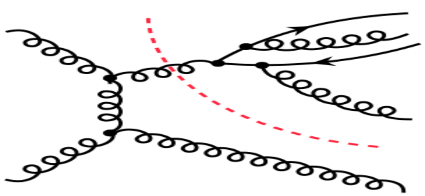


Relativistic projection to all "spin states"

$$\hat{\sigma}^{\rm NLO} \to \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu) \log(\mu^2/\mu_0^2)$$

$$\mu_0 \gtrsim 2m_Q$$

- **NNLO in α_s but leading power in $1/p_T$:**



$$\hat{\sigma}^{\rm NNLP} \to \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu) \log^m(\mu^2/\mu_0^2)$$

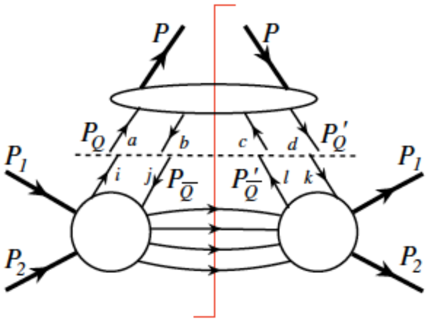
Leading order in α_s-expansion =|= leading power in $1/p_T$-expansion!

QCD factorization beyond leading power

- **Factorization formalism:**

Nayak, Qiu, and Sterman, 2005
Kang, Qiu and Sterman, 2010, …

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$



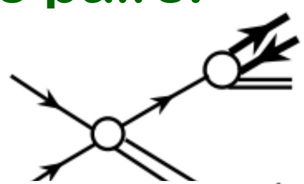
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

$$+ \mathcal{O}(m_Q^4/p_T^4)$$

$$\hat{p}_Q = \frac{1+\zeta}{2z}\hat{p}\,, \quad \hat{p}_{\bar{Q}} = \frac{1-\zeta}{2z}\hat{p}$$

- **Production of the pairs:**

✧ at $1/m_Q$:

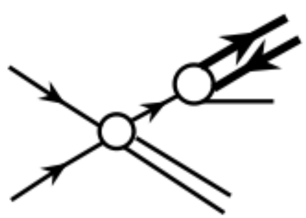


$$D_{i\to H}(z, m_Q, \mu_0)$$

✧ at $1/P_T$:

$$d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]}(\kappa), \mu)$$

✧ between: [$1/m_Q$, $1/P_T$]

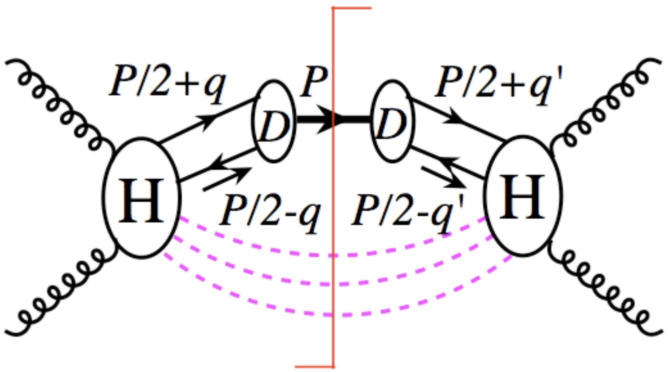


$$\frac{d}{d\ln(\mu)} D_{i\to H}(z, m_Q, \mu) = \ldots + \frac{m_Q^2}{\mu^2}\Gamma(z)\otimes D_{[Q\bar{Q}(\kappa)]\to H}(\{z_i\}, m_Q, \mu)$$

Long-lived heavy quark pairs

Kang, Ma, Qiu and Sterman, 2013

❑ Perturbative pinch singularity:



$$P^\mu = (P^+, 4m^2/2P^+, 0_\perp)$$

$$q^\mu = (q^+, q^-, q_\perp)$$

$$q \neq q'$$

$$D_{ij}(P,q) \propto \langle \mathrm{J}/\psi | \psi_i^\dagger(0)\, \chi_j(y) | 0 \rangle$$

✧ **Scattering amplitude:**

$$\mathcal{M} \propto \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}(P,q,Q)\, \frac{\gamma\cdot(P/2-q)+m}{(P/2-q)^2-m^2+i\epsilon}\, \hat{D}(P,q)\, \frac{\gamma\cdot(P/2+q)+m}{(P/2+q)^2-m^2+i\epsilon}\right]$$

✧ **Potential poles:**

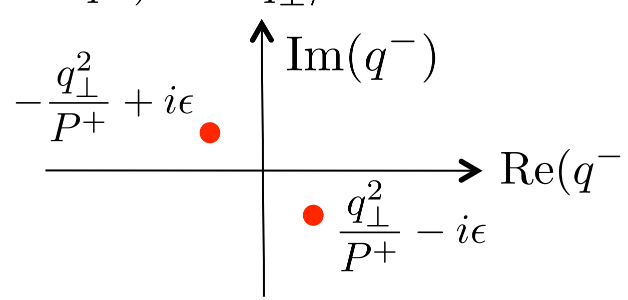
$$q^- = [q_\perp^2 - 2m^2(q^+/P^+)]/(P^+ + 2q^+) - i\epsilon\theta(P^+ + 2q^+) \to q_\perp^2/P^+ - i\epsilon$$

$$q^- = -[q_\perp^2 + 2m^2(q^+/P^+)]/(P^+ - 2q^+) + i\epsilon\theta(P^+ - 2q^+) \to -q_\perp^2/P^+ + i\epsilon$$

✧ **Condition for pinched poles:**

$$P^+ \gg q^+(2m^2/q_\perp^2) \geq 2m$$

At High P_T

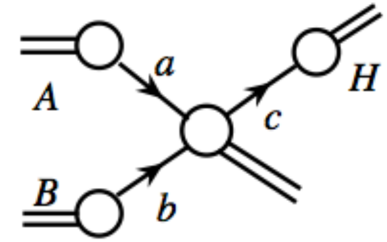


Why such power correction are important?

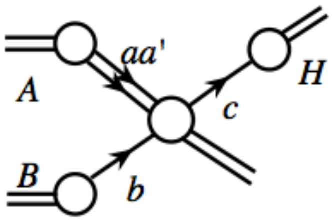
- **Leading power in hadronic collisions:**

Kang, Ma, Qiu and Sterman, 2013

$$d\sigma_{AB\to H} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to cX} \otimes D_{c\to H}$$



- **1st power corrections in hadronic collisions:**



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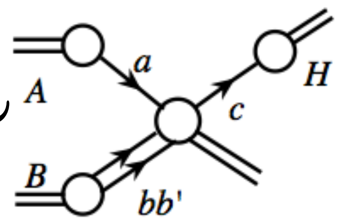

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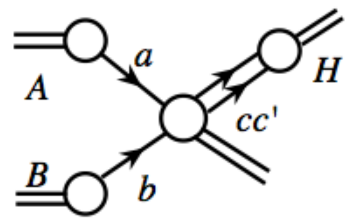
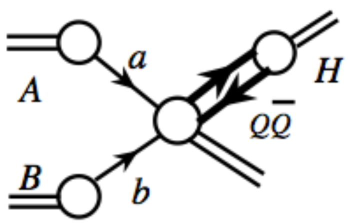


$$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$$

or $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes \mathcal{D}_{[ff]\to H}$

- **Dominated 1st power corrections:**



$$\sim \mathcal{O}\left(\frac{(2m_Q)^2}{P_T^2}\right) \otimes D^{(2)}_{[Q\bar{Q}]\to H}$$

Key: competition between $P_T^2 \gg (2m_Q)^2$ **and** $D^{(2)}_{[Q\bar{Q}]\to H} \gg D_{c\to H}$

Evolution of fragmentation functions

Kang, Ma, Qiu and Sterman, 2013

❑ Independence of the factorization scale:

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$$

✧ at Leading power in $1/P_T$: DGALP evolution

$$\frac{d}{d\ln\mu^2}D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z, m_Q, \mu)$$

✧ next-to-leading power in $1/P_T$ – New non-linear evolution!

$$\frac{d}{d\ln\mu^2}D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z, m_Q, \mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')$$
$$\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

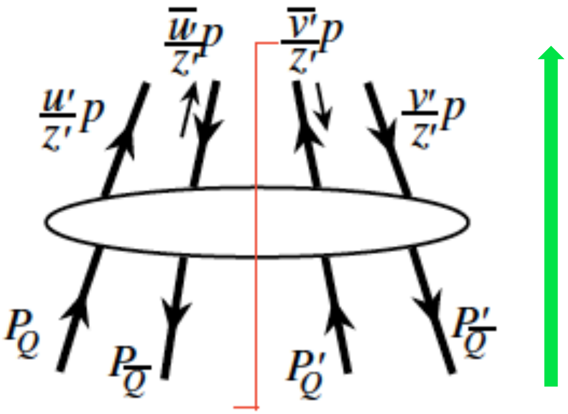
❑ Evolution kernels are perturbative:

✧ Set mass: $m_Q \to 0$ with a caution

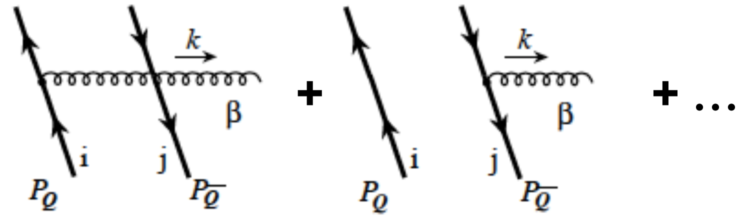
Evolution kernels

Kang, Ma, Qiu and Sterman, 2013

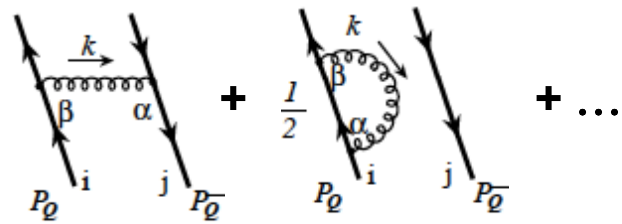
❑ Kernel for $Q\bar{Q} \to Q\bar{Q}$ at (α_s):



Real:



Virtual:



✧ **Projection – Spin: vector, axial vector, tensors – Color: Singlet, Octet**

$$\tilde{\mathcal{P}}^{(v)}(p)_{ji,kl} = (\gamma \cdot p)_{ji}(\gamma \cdot p)_{kl},$$

$$\tilde{\mathcal{P}}^{(a)}(p)_{ji,kl} = (\gamma \cdot p\gamma_5)_{ji}(\gamma \cdot p\gamma_5)_{kl},$$

$$\tilde{\mathcal{P}}^{(t)}(p)_{ji,kl} = \sum_{\alpha=1,2}(\gamma \cdot p\gamma_\perp^\alpha)_{ji}(\gamma \cdot p\gamma_\perp^\alpha)_{kl}$$

$$\mathcal{P}^{(v)}(p)_{ij,lk} = \frac{1}{4p \cdot n}(\gamma \cdot n)_{ij}\frac{1}{4p \cdot n}(\gamma \cdot n)_{lk},$$

$$\mathcal{P}^{(a)}(p)_{ij,lk} = \frac{1}{4p \cdot n}(\gamma \cdot n\gamma_5)_{ij}\frac{1}{4p \cdot n}(\gamma \cdot n\gamma_5)_{lk},$$

$$\mathcal{P}^{(t)}(p)_{ij,lk} = \frac{1}{2}\sum_{\beta=1,2}\frac{1}{4p \cdot n}(\gamma \cdot n\gamma_\perp^\beta)_{ij}\frac{1}{4p \cdot n}(\gamma \cdot n\gamma_\perp^\beta)_{lk}$$

✧ **Example:** “ $[Q\bar{Q}(v8)] \to [Q\bar{Q}(v1)]$ ”

$$K^{(1)}_{v8\to v1}(z,u,v;u'v') = \frac{\alpha_s}{2\pi}\left[\frac{1}{2N_c}\right]\frac{z}{2(1-z)}\left(\frac{u}{u'}+\frac{\bar{u}}{\bar{u}'}\right)\left(\frac{v}{v'}+\frac{\bar{v}}{\bar{v}'}\right)$$

$$\times\left[\delta(u-zu')-\delta(\bar{u}-z\bar{u}')\right]\left[\delta(v-zv')-\delta(\bar{v}-z\bar{v}')\right]$$

All channels are calculated

Short-distance hard parts

Kang, Ma, Qiu and Sterman, 2014

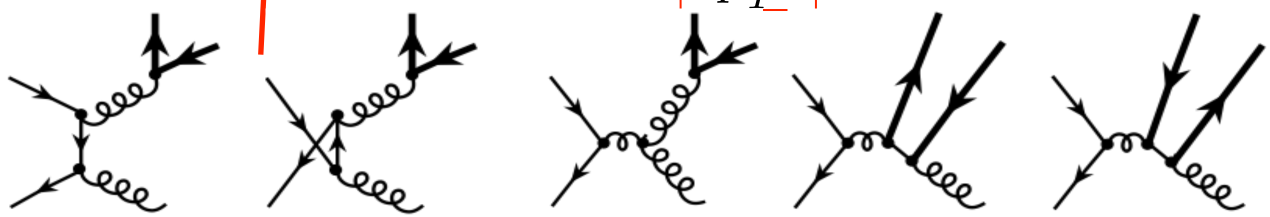
❑ Separation of different powers:

$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

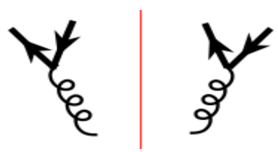
$$\Rightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$\frac{\alpha_s^3(\mu)}{p_T^6}$ $\qquad$ $\frac{\alpha_s^2(\mu)}{p_T^4}$ $\qquad$ $\frac{\alpha_s(2m_Q)}{(2m_Q)^2}$

$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$:



$D^{(1)}_{g\to[Q\bar{Q}]}$:



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$E_p \frac{d\hat{\sigma}^{(3)}_{q+\bar{q}\to[Q\bar{Q}(n)](p)}}{d^3p} \equiv \left[\frac{4\pi\alpha_s^3}{\hat{s}}\right] \frac{1}{\bar{u}u\bar{v}v} H_{q\bar{q}\to[Q\bar{Q}(n)]}(\hat{s},\hat{t},\hat{u})\,\delta(\hat{s}+\hat{t}+\hat{u})$$

$$H_{q\bar{q}\to[Q\bar{Q}(a8)]}(\hat{s},\hat{t},\hat{u}) = 2\left[\frac{N_c^2-1}{8N_c}\right]\left[1+\zeta_1\zeta_2-\frac{4}{N_c^2}\right]\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^3}\right]$$

All channels are calculated

Predictive power and status

- **Calculation of short-distance hard parts in pQCD:**

  Power series in α_s, without large logarithms

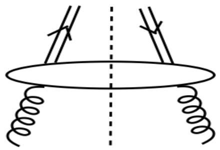
  LO is now available for all partonic channels — Kang, Ma, Qiu and Sterman, 2014

- **Calculation of evolution kernels in pQCD:**

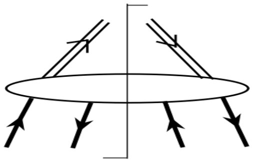
  Power series in α_s, without large logarithms — Kang et al. 2013, Fleming et al. 2013

  LO is now available for both mixing kernels and pair evolution kernels of all spin states of heavy quark pairs

- **Input FFs at μ_0 – non-perturbative, but, universal**



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

Ma et al 2014

- **Physics of the input scale: $\mu_0 \sim 2m_Q$ – a parameter:**

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

  Different quarkonium states require different input distributions!

Non-perturbative input distributions

- **Sensitive to the properties of quarkonium produced:**

  Should, in principle, be extracted from experimental data

- **Large heavy quark mass and clear scale separation:**

  $\mu_0 \sim m_Q \gg m_Q\, v$ ➡ **Apply NRQCD to the FFs – *a conjecture!***

  ✧ **Single parton FFs – valid to two-loops:**

  Nayak, Qiu and Sterman, 2005

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\mathrm{NRQCD}}$$

  Braaten, Yuan, 1994
  Ma, 1995, …
  Braaten, Chen, 1997
  Braaten, Lee, 2000,
  Ma, Qiu, Zhang, 2013
  …

  Complete LO+NLO for S, P states & NNLO for singlet S state

  ✧ **Heavy quark pair FFs – valid to one-loop:**

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi}(z,\zeta,\zeta',\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle_{\mathrm{NRQCD}}$$

  Kang, Ma, Qiu and Sterman, 2014

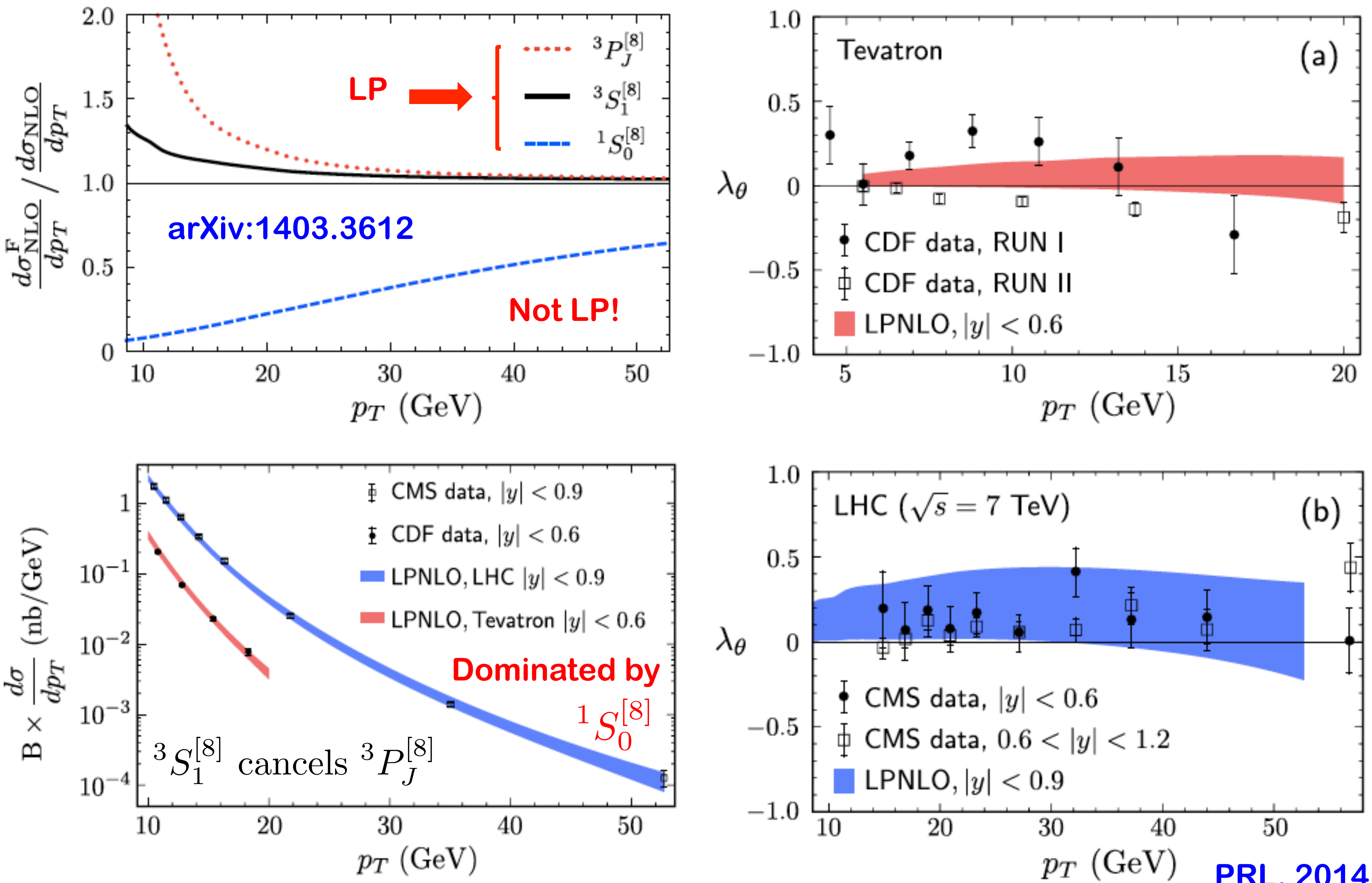
  Full LO+NLO for S, P states is now available

  Ma, Qiu, Zhang, 2013

- **No all-order proof of such factorization yet!**

  Reduce "many" unknown FFs to a few universal NRQCD matrix elements!

Leading power fragmentation – Bodwin et al.



Next-to-leading power fragmentation – Ma et al.

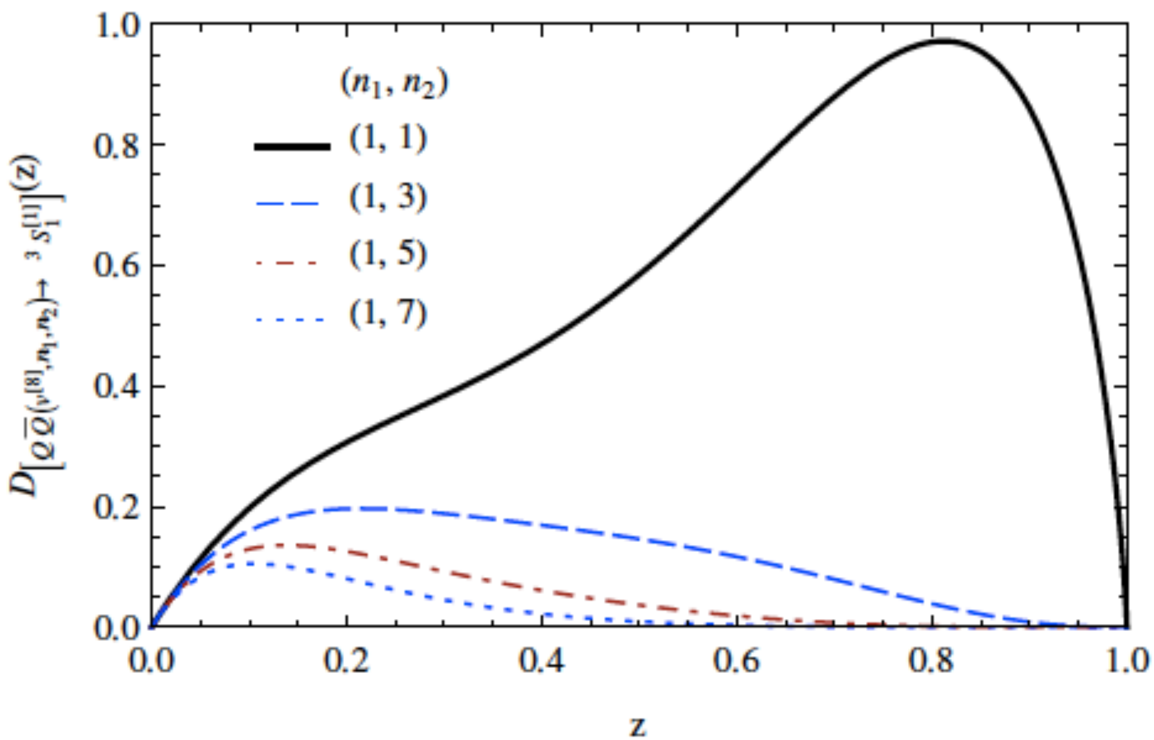
Ma, Qiu, Zhang, 2013

❑ Heavy quark pair FFs:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \right\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

❑ Moment of the FFs:

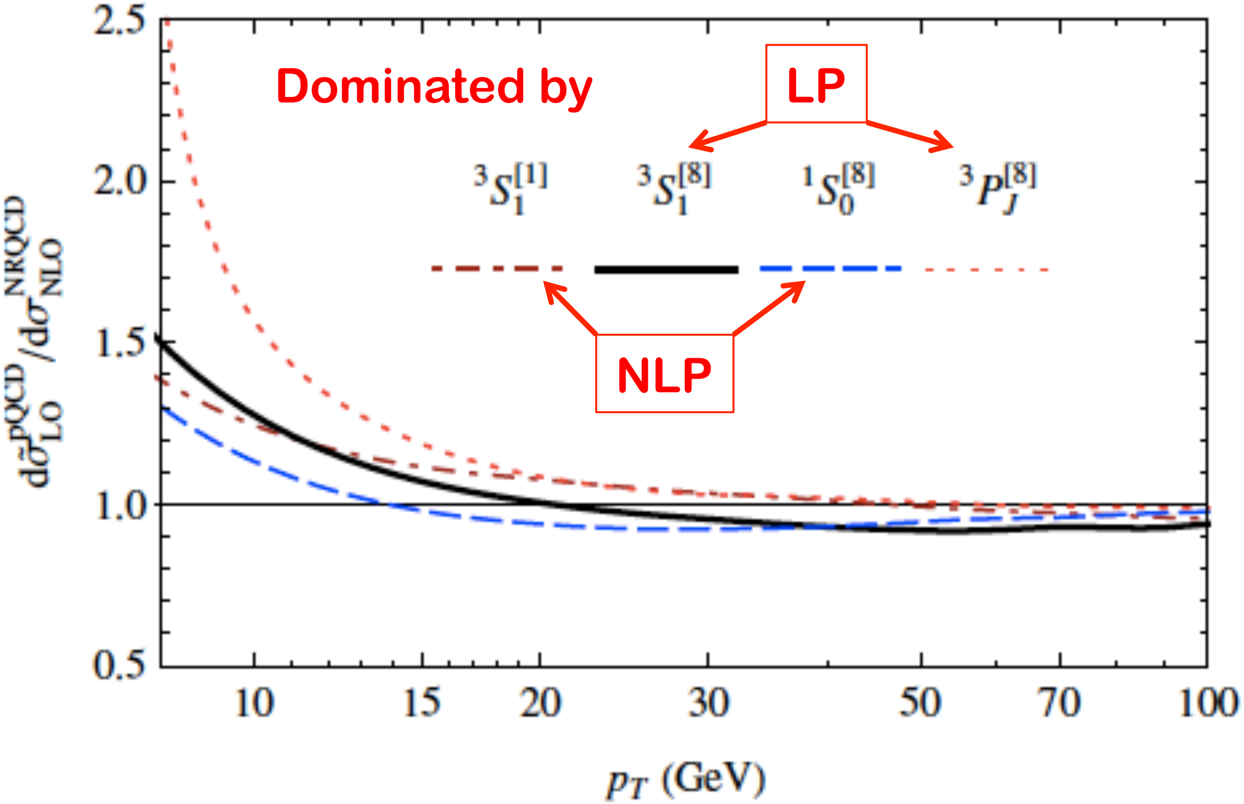
$$\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$$



Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- Channel-by-channel comparison:



independent of NRQCD matrix elements

LO analytical results reproduce NLO NRQCD calculations (numerical)

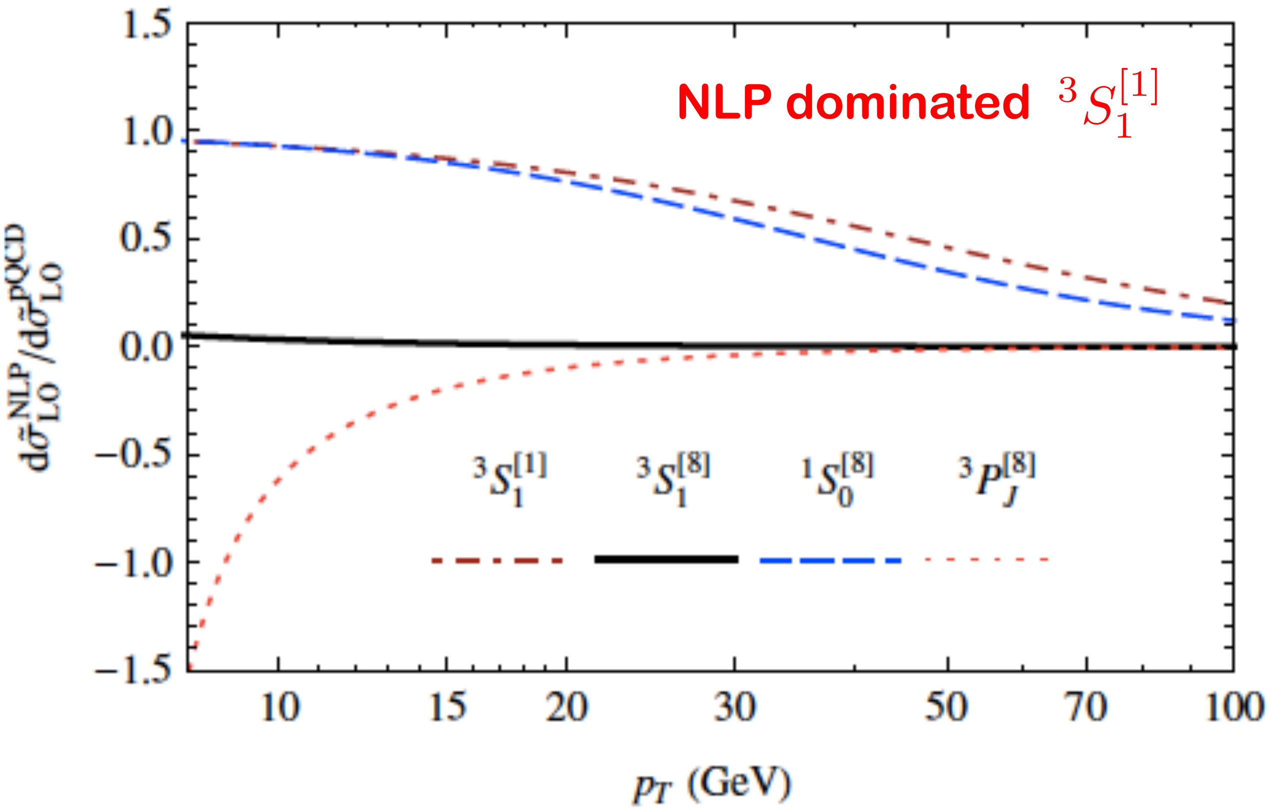
PRL, 2014

Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

- LP vs. NLP (both LO):



NLP dominated
$^1S_0^{[8]}$
for wide pT

LP dominated
$^3S_1^{[8]}$ and $^3P_J^{[8]}$

PT distribution is consistent with distribution of
$^1S_0^{[8]}$

PRL, 2014

QCD factorization vs NRQCD factorization

- **QCD factorization – not always true:**
  - Expand physical cross section in powers of $1/p_T$
  - Expand the coefficient of each term in powers of α_s
  - Factorization is valid for all powers of α_s of the 1st two terms in $1/p_T$

- **NRQCD factorization – conjectured:**
  - Expand physical cross section in powers of relative velocity of HQ
  - Expand the coefficient of each term in powers of α_s
  - Verified to NNLO in α_s for the leading power term in the *v*-expansion

- **Connection:**

  If NRQCD factorization for fragmentation functions is valid,

$$E_P \frac{d\sigma_{A+B\to H+X}}{d^3P}(P, m_Q) \equiv E_P \frac{d\sigma^{\rm QCD}_{A+B\to H+X}}{d^3P}(P, m_Q = 0) + E_P \frac{d\sigma^{\rm NRQCD}_{A+B\to H+X}}{d^3P}(P, m_Q \neq 0) - E_P \frac{d\sigma^{\rm QCD-Asym}_{A+B\to H+X}}{d^3P}(P, m_Q = 0)$$

Mass effect + connection to lower p_T region

Heavy quarkonium polarization

Ma et al. 2014

- **Polarization = input fragmentation functions:**
  - ✧ Partonic hard parts and evolution kernels are perturbative
  - ✧ Insensitive to the properties of produced heavy quarkonia

- **Projection operators – polarization tensors:**

$$\mathcal{P}^{\mu\nu}(p) \equiv \sum_{\lambda=0,\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}$$

Unpolarized quarkonium

$$\mathcal{P}_T^{\mu\nu}(p) \equiv \frac{1}{2}\sum_{\lambda=\pm1} \epsilon_\lambda^{*\mu}(p)\epsilon_\lambda^{\nu}(p) = \frac{1}{2}\left[-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n}\right]$$

Transversely polarized quarkonium

$$\mathcal{P}_L^{\mu\nu}(p) \equiv \mathcal{P}^{\mu\nu}(p) - 2\mathcal{P}_T^{\mu\nu}(p) = \frac{1}{p^2}\left[p^\mu - \frac{p^2}{2p\cdot n}n^\mu\right]\left[p^\nu - \frac{p^2}{2p\cdot n}n^\nu\right]$$

Longitudinally polarized quarkonium

for produced the quarknium moving in +z direction with

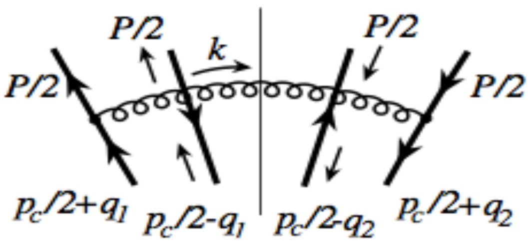
$$p^\mu = (p^+, p^-, p_\perp) = p^+(1, 0, \mathbf{0}_\perp) \qquad p^2 = n^2 = 0$$

$$n^\mu = (n^+, n^-, n_\perp) = (0, 1, \mathbf{0}_\perp) \qquad p\cdot n = p^+$$

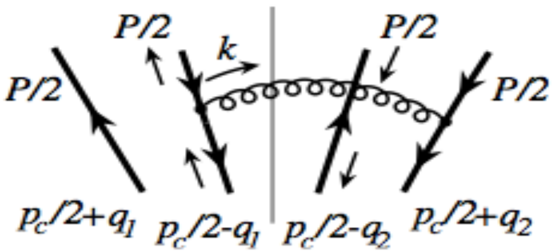
Polarized fragmentation functions

Kang, Ma, Qiu and Sterman, 2014
Zhang, Ph.D. Thesis, 2014

❑ Color singlet as an example:



+



+ …

All channels are calculated

✧ A axial vector pair to a singlet NRQCD pair:

$$\mathcal{D}^{L,CR}_{[Q\bar{Q}(a8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_+(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^{T,CR}_{[Q\bar{Q}(a8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_+(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

✧ A vector pair to a singlet NRQCD pair:

$$\mathcal{D}^{L,CR}_{[Q\bar{Q}(v8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_-(u,v)\times\frac{\alpha_s}{2\pi}\frac{z}{1-z}\left[\ln\left(r(z)+1\right)-\left(1-\frac{1}{1+r(z)}\right)\right]$$

$$\mathcal{D}^{T,CR}_{[Q\bar{Q}(v8)]\to J/\psi}(z,u,v;m_Q,\mu) = \frac{1}{2N_c^2}\frac{\langle\mathcal{O}^H_{1(^3S_1)}\rangle}{3m_Q}\Delta_-(u,v)\times\frac{\alpha_s}{2\pi}z(1-z)\left[1-\frac{1}{1+r(z)}\right]$$

where

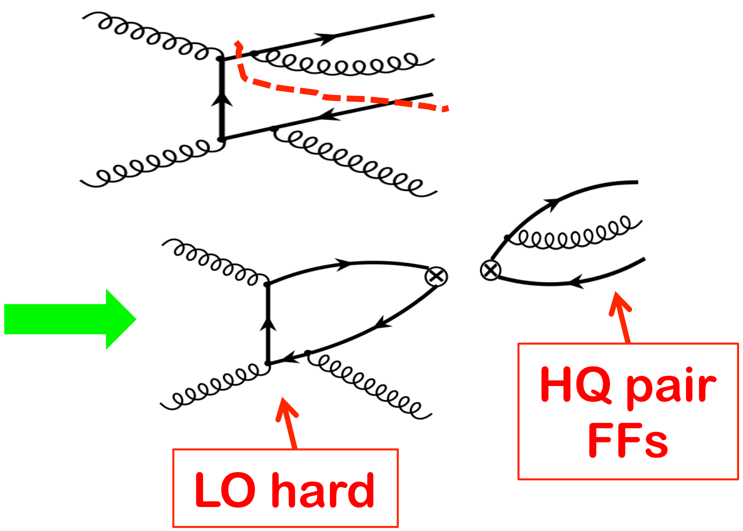
$$\Delta_+(u,v) = \frac{1}{4}\left[\delta\left(u-\frac{z}{2}\right)+\delta\left(\bar{u}-\frac{z}{2}\right)\right]\left[\delta\left(v-\frac{z}{2}\right)+\delta\left(\bar{v}-\frac{z}{2}\right)\right]$$
$$\Delta_-(u,v) = \frac{1}{4}\left[\delta\left(u-\frac{z}{2}\right)-\delta\left(\bar{u}-\frac{z}{2}\right)\right]\left[\delta\left(v-\frac{z}{2}\right)-\delta\left(\bar{v}-\frac{z}{2}\right)\right]$$

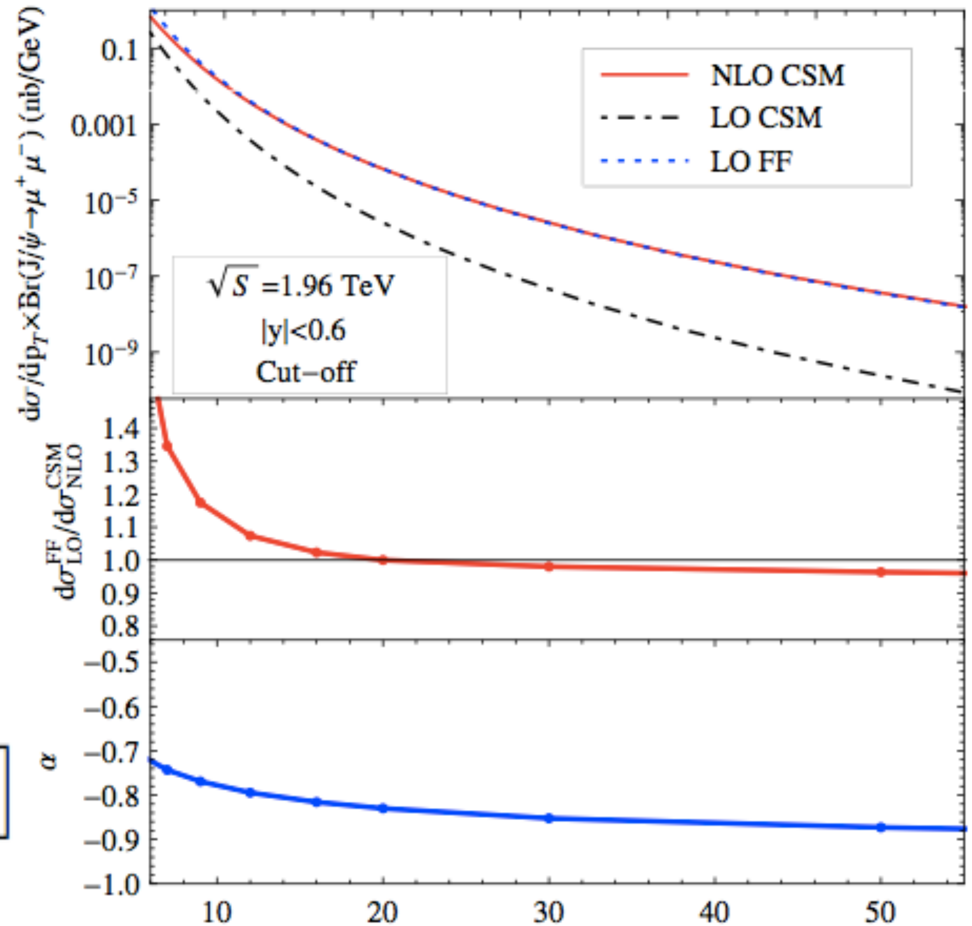
$$r(z)\equiv\frac{z^2\mu^2}{4m_c^2(1-z)^2}$$

Production and polarization

Kang, Ma, Qiu and Sterman, 2014

- Color singlet as an example:





$$\sigma^{(NLO)}_{NRQCD} \propto \Big[d\hat{\sigma}^{A(LO)}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(LO)}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S(LO)}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(LO)}_{[Q\bar{Q}(a8)]\to J/\psi} \Big]$$

Reproduce NLO CSM for p_T > 10 GeV!

Cross section + polarization

QCD Factorization = better controlled HO corrections!

Summary of lecture four

- PQCD factorization approach is mature, and has been extremely successful in predicting and interpreting high energy scattering data with momentum transfer > 2 GeV

- Theorists have moved beyond the leading power/twist approximation, including saturation phenomena

- Theory had a lot advances in last decade in dealing with observables with multiple observed momentum scales:

  Provide new probes to "see" the confined motion: the large scale to pin down the parton d.o.f. while the small scale to probe the nonperturbative structure as well as the motion

- Proton spin provides another controllable "knob" to help isolate various physical effects

Backup slides