QCD in Collisions with Polarized Beams

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Summary of lecture four

- PQCD factorization approach is mature, and has been extremely successful in predicting and interpreting high energy scattering data with momentum transfer > 2 GeV

- Theorists have moved beyond the leading power/twist approximation, including saturation phenomena

- Theory had a lot advances in last decade in dealing with observables with multiple observed momentum scales:

  Provide new probes to "see" the confined motion: the large scale to pin down the parton d.o.f. while the small scale to probe the nonperturbative structure as well as the motion

- Proton spin provides another controllable "knob" to help isolate various physical effects

Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

- Cross section:

  Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- Spin-averaged cross section:

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ – Positive definite

- Asymmetries or difference of cross sections:

  – Not necessary positive!

  - both beams polarized A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

  - one beam polarized A_L, A_N

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Two roles of the proton spin program

- **Proton is a composite particle:**

  Spin is a consequence of internal dynamics of the bound state

  For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

  ➡ Decomposition of proton spin in terms of quark and gluon d.o.f.

  helps understand the dynamics of a fundamental QCD bound state

  – Nucleon is a building block all hadronic matter

  (> 95% mass of all visible matter)

- **Use the spin as a tool – asymmetries:**

  Cross section is a probability – classically measured

  Spin asymmetry – the difference of two cross sections
  involving two different spin states

  Asymmetry could be a pure quantum effect!

Spin of a composite particle

❑ Spin:

- ✧ Pauli (1924): two-valued quantum degree of freedom of electron
- ✧ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant ħ)
- ✧ Composite particle = Total angular momentum when it is at rest

❑ Spin of a nucleus:

- ✧ Nuclear binding: 8 MeV/nucleon << mass of nucleon
- ✧ Nucleon number is fixed inside a given nucleus
- ✧ Spin of a nucleus = sum of the valence nucleon spin

❑ Spin of a nucleon – Naïve Quark Model:

- ✧ If the probing energy << mass of constituent quark
- ✧ Nucleon is made of three constituent (valence) quark
- ✧ Spin of a nucleon = sum of the constituent quark spin

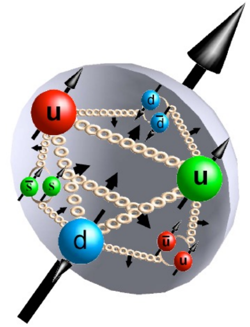
State: $$|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$$

Spin: $$S_p \equiv \langle p\uparrow|S|p\uparrow\rangle = \frac{1}{2}, \qquad S = \sum_i S_i$$

Carried by valence quarks

Spin of a composite particle

- **Spin of a nucleon – QCD:**
  - Current quark mass << energy exchange of the collision
  - Number of quarks and gluons depends on the probing energy



- **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

- **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk}\int d^3x\; M^{0jk}_{\rm QCD} \quad \Leftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

  - Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x}\times(-i\vec{D}))\,\psi_q\right]$$

  - Gluon angular momentum operator:

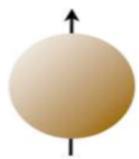
$$\vec{J}_g = \int d^3x \left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

Need to have the matrix elements of these partonic operators measured independently

Current understanding for Proton Spin

- **The sum rule:** $S(\mu) = \sum_f \langle P, S | \hat{J}_f^z(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$
  - **Infinite possibilities of decompositions – connection to observables?**
  - **Intrinsic properties + dynamical motion and interactions**

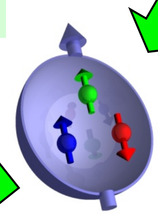
- **An incomplete story:**

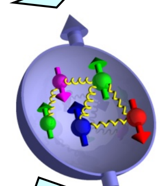


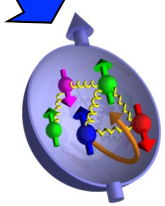
Proton Spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …







Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

Sea quarks?

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

$$\sim 20\% (\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

Net effect of partons' transverse motion?

Some fundamentals about spin

❑ Spin in non-relativistic quantum mechanics:

✧ Spin as an intrinsic angular momentum of the particle

– three spin vector:

$$\vec{\mathcal{S}} = (\mathcal{S}_x\,,\ \mathcal{S}_y\,,\ \mathcal{S}_z\,)$$

– angular momentum algebra:

$$[\mathcal{S}_i\,,\ \mathcal{S}_j\,] = i\epsilon_{ijk}\,\mathcal{S}_k \qquad \epsilon_{123} = +1$$

$$\Longrightarrow \quad \left[\vec{\mathcal{S}}^{\,2},\ \mathcal{S}_j\right] = 0$$

✧ $\vec{\mathcal{S}}^{\,2},\ \mathcal{S}_z$ Have set of simultaneous eigenvectors: $|\,S,\ m\rangle$

$$\vec{\mathcal{S}}^{\,2}\,|\,S,\ m\rangle = S(S+1)\,\hbar^2\,|\,S,\ m\rangle \qquad S = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots$$

$$\mathcal{S}_z\,|\,S,\ m\rangle = m\,\hbar\,|\,S,\ m\rangle \qquad -S \le m \le S$$

✧ Spin d.o.f. are decoupled from kinematic d.o.f.

$$\Psi_{\mathsf{Schr}}(\vec{r}) \longrightarrow \Psi_{\mathsf{Schr}}(\vec{r}) \times \chi_m$$

where χ_m is a (2S+1) – component “spinor”

Some fundamentals about spin

- **Spin–1/2:**
  - **Two component spinors:** $$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$
  - **Operators could be represented by Pauli-matrices:**

$$\mathcal{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathcal{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \mathcal{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  - **Eigenstates to $\vec{\mathcal{S}}^2$ and $\mathcal{S}_z$:**

$$\chi_z^{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_z^{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

  - **Eigenvalues:**

$$\mathcal{S}_z\,\chi_z^{\uparrow} = +\frac{1}{2}\chi_z^{\uparrow} \qquad \mathcal{S}_z\,\chi_z^{\downarrow} = -\frac{1}{2}\chi_z^{\downarrow}$$

Particles in these states are "polarized in z-direction"

Some fundamentals about spin

- **General superposition:**

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\,\chi_z^{\uparrow} + b\,\chi_z^{\downarrow} \qquad (\chi^{\dagger}\chi = 1)$$

$$\Rightarrow \quad \langle \mathcal{S}_z \rangle = \chi^{\dagger}\mathcal{S}_z\chi = \left(+\frac{1}{2}\right)|a|^2 + \left(-\frac{1}{2}\right)|b|^2 = \frac{1}{2}\left[|a|^2 - |b|^2\right]$$

- **Example:** $a = b = 1/\sqrt{2}$

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \langle \mathcal{S}_z \rangle = 0$$

- **Notice:** $\langle \mathcal{S}_x \rangle = \chi^{\dagger}\mathcal{S}_x\chi = +\frac{1}{2}$

- **Eigenstate to $\mathcal{S}_x$:**

$$\chi_x^{\uparrow} = \frac{1}{\sqrt{2}}\left[\chi_z^{\uparrow} + \chi_z^{\downarrow}\right]$$

- **Arbitrary direction $\vec{n}$ with $|\vec{n}| = 1$:**

$$\mathcal{S}_n = \vec{n}\cdot\vec{\mathcal{S}} = n_x\mathcal{S}_x + n_y\mathcal{S}_y + n_z\mathcal{S}_z = \frac{1}{2}\begin{pmatrix} n_z & n_x - in_y \\ n_x + in_y & -n_z \end{pmatrix}$$

A state that is an eigenstate to this operator: "polarized in $\vec{n}$- direction"

$\vec{n}$ **= Polarization vector**   **Eigenvalues =** $\pm 1/2$

Some fundamentals about spin

❑ Spin in the relativistic theory:

Physics is invariant under Lorentz transformation:
boost, rotations, and translations in space and time

✧ **Poincare group – 10 generators:** $\mathcal{P}^\mu\,, \quad \mathcal{M}^{\mu\nu}$

✧ **Pure rotations:** $\mathcal{J}_i = -\frac{1}{2}\epsilon_{ijk}\mathcal{M}^{jk}$, **pure boosts:** $\mathcal{K}_i = \mathcal{M}^{i0}$

Total angular momentum: $[\,\mathcal{J}_i\,,\mathcal{J}_j\,] = i\,\epsilon_{ijk}\,\mathcal{J}_k$

✧ **Two group invariants (fundamental observables):**

$$\mathcal{P}_\mu\,\mathcal{P}^\mu = \mathcal{P}^2 = m^2$$

$\mathcal{W}_\mu\,\mathcal{W}^\mu$ where $\mathcal{W}_\mu = -\dfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{M}^{\nu\rho}\mathcal{P}^\sigma$ Pauli-Lubanski

✧ **Fact:** $[\,\mathcal{W}_\mu\,,\mathcal{W}_\nu\,] = i\,\epsilon_{\mu\nu\rho\sigma}\mathcal{W}^\rho\,\mathcal{P}^\sigma$ ➡ $[\,\mathcal{W}^i\,,\mathcal{W}^j\,] = i\,m\,\epsilon_{ijk}\,\mathcal{W}^k$

If acting on states at the rest

✧ **Spin:** $\mathcal{S}_i \equiv \frac{1}{m}\mathcal{W}^i = \mathcal{J}_i$

Note: $\mathcal{W}_\mu\,\mathcal{W}^\mu$ **has eigenvalues** $m^2\,\mathcal{S}\,(\mathcal{S}+1)$

Some fundamentals about spin

✧ **Recall: constructed eigenstates to $\vec{S}^2$ and $\vec{n}\cdot\vec{S}$:**

$$\mathcal{W}_\mu \mathcal{W}^\mu \,|p,S\rangle = m^2 S(S+1)\,|p,S\rangle \qquad S=\tfrac{1}{2}$$

$$-\frac{W\cdot n}{m}\,|p,S\rangle = \pm\frac{1}{2}\,|p,S\rangle \qquad W^\mu = \mathcal{W}^\mu|_{\text{at rest}}$$

✧ **"Polarization operator":**

$$\mathcal{P} \equiv -\frac{W\cdot n}{m}$$

✧ **"Covariant polarization vector":** n^μ **with** $n^2=-1,\quad n\cdot p=0$

✧ **For Dirac particles:** $\mathcal{P} = \frac{1}{2}\gamma_5\,\gamma_\mu n^\mu$

✧ **Transverse polarization:**

➡ **Projection operators to project out the eigenstates of $\mathcal{P}$:**

$$\tfrac{1}{2}\left(\mathbb{1} \pm \gamma_5\, \not{n}\right)$$

✧ **Longitudinal polarization:** $\vec{n} = \vec{p}/|\vec{p}|\,,\quad n^0 = 0$

➡ $\mathcal{P} = \frac{1}{2}\gamma_5\gamma_\mu n^\mu = \frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}$ **with eigenvalues** $\pm\frac{1}{2}$

$$\frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}\,u_\pm(p) = \pm\frac{1}{2}u_\pm(p) \equiv \frac{\lambda}{2}u_\pm(p) \qquad \rightsquigarrow \lambda \text{ "helicity"}$$

Massless particle: $\frac{\vec{J}\cdot\vec{p}}{|\vec{p}|} \to \gamma_5$ helicity = chirality

Some fundamentals about spin

- ✧ **Transverse polarization:** $n^\mu = (0, \vec{n}_\perp, 0)$ (for $\vec{p}$ in z direction)

➡ $\mathcal{P} = \gamma_0 \vec{J} \cdot \vec{n} = \gamma_0 J_\perp \neq J_\perp$

- ✧ **Transversity, not "transverse spin", has the eigenvalue:** $\pm\frac{1}{2}$

$$\gamma_0 J_\perp u_{\uparrow\downarrow}(p) = \pm\frac{1}{2} u_{\uparrow\downarrow}(p)$$

with spinors: $u_\uparrow^{(x)} = \frac{1}{\sqrt{2}}\left[u_+ + u_-\right]$

Same as in non-relativistic theory

➡ **Transverse polarization, or transversity, not "transverse spin", is invariant under the "boosts along $\vec{p}$ "**

- ✧ **Projection operator with both longitudinal and transverse components:**

$$\frac{1}{2}\not{p}\left[\mathbb{1} - s_\parallel \gamma_5 + \gamma_5 \not{s}_\perp\right]$$ **at high energy**

with $s_\parallel \sim \lambda, \quad s_\perp \sim n_\perp$

Some fundamentals about spin

❑ Back to Spin–1/2:

✧ A free spin-1/2 particle obeys Dirac equation

$$(\not{p} - m)\, u(p) = 0 \qquad \text{where } \not{p} = \gamma_\mu p^\mu$$

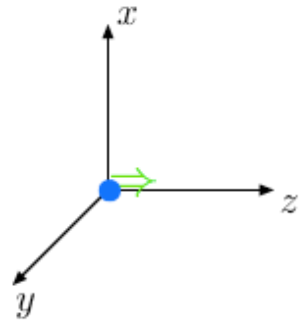
with 4-component solutions:

$$\Psi(x) = \begin{cases} e^{-i\,p\cdot x}\, u(p) & \text{positive energy} \rightarrow \text{particle} \\ e^{+i\,p\cdot x}\, v(p) & \text{negative energy} \rightarrow \textit{anti}\text{particle} \end{cases}$$

Each with "two" solutions: "spin up/down"

✧ If it is at rest,

$$u^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^- = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



They are eigenstates to the spin operator S_z:

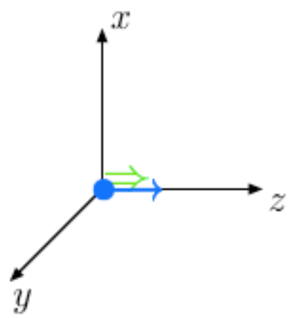
$$S_z\, u^\pm = \pm\frac{1}{2}\, u^\pm$$

"polarized in z-direction"

Some fundamentals about spin

✧ **Boost the particle to momentum** $p = (E, 0, 0, p_z)$

$$\Rightarrow \quad u^+ = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix} \qquad u^- = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_z}{E+m} \end{pmatrix}$$



✧ **Eigenstates of the helicity operator:**

$$\frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} u^\pm = \pm \frac{1}{2} u^\pm$$

✧ **Also eigenstates of the Pauli-Lubanski (polarization) operator:**

$$\frac{1}{2} \gamma_5 \not{n} \, u^\pm = \pm \frac{1}{2} u^\pm$$

where the polarization vector $n = (p_z, 0, 0, E)/m$

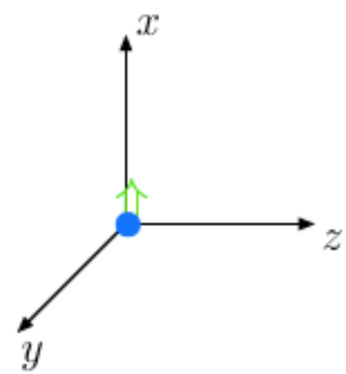
✧ **At high energy,** $E \approx p_z$ **also become eigenstates to chirality** γ_5 **:**

$$\gamma_5 \, u^\pm = \pm \frac{1}{2} u^\pm$$

Some fundamentals about spin

❑ Back to rest frame:

✧ Construct eigenstates to the spin operator $\mathcal{S}_x$:



$$\mathcal{S}_x\, u^{\uparrow} = +\frac{1}{2}\, u^{\uparrow} \qquad \mathcal{S}_x\, u^{\downarrow} = -\frac{1}{2}\, u^{\downarrow}$$

with $u^{\uparrow} = \frac{1}{\sqrt{2}}\left[\, u^{+} + u^{-} \,\right] \qquad u^{\downarrow} = \frac{1}{\sqrt{2}}\left[\, u^{+} - u^{-} \,\right]$

"polarized along x-direction"

✧ Boost the particle to momentum $p = (E,\, 0,\, 0,\, p_z)$

$$\Rightarrow \quad u^{\uparrow} = \frac{N}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ \frac{p_z}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad u^{\downarrow} = \frac{N}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ \frac{p_z}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}$$

Still has $u^{\uparrow} = (u^{+} + u^{-})/\sqrt{2}$

✧ Still the eigenstates of the Pauli-Lubanski (polarization) operator:

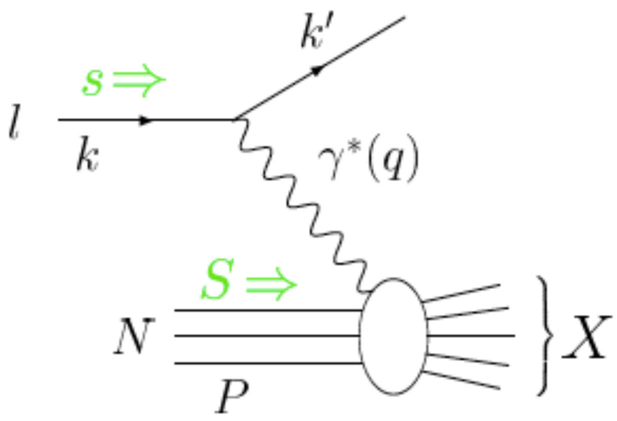
$$\frac{1}{2}\gamma_5\, \not{n}\, u^{\uparrow\downarrow} = \pm\frac{1}{2}\, u^{\uparrow\downarrow} \qquad \text{where} \quad n = (0,\, 1,\, 0,\, 0)$$

✧ But, no longer eigenstates of the transverse-spin operator:

$$\mathcal{S}_x\, u^{\uparrow} \neq +\frac{1}{2}\, u^{\uparrow}$$

Polarized deep inelastic scattering

❑ DIS with polarized beam(s):



"Resolution" $Q \equiv \sqrt{-q^2}$

$$\frac{\hbar}{Q} = \frac{2 \times 10^{-16}\text{m}}{Q/\text{GeV}} \lesssim 10^{-16}\text{m} = 1/10\text{fm}$$

"Inelasticity" – known as Bjorken variable

$$x_B = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ **Recall – from lecture 2:**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p \cdot q}\left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

$$+ iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B, Q^2\right) + \frac{(p.q) S_\sigma - (S.q) p_\sigma}{(p.q)^2} g_2\left(x_B, Q^2\right)\right]$$

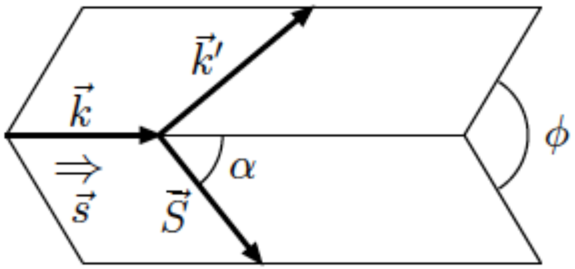
✧ **Polarized structure functions:**

$$g_1(x_B, Q^2), \;\; g_2(x_B, Q^2)$$

Polarized deep inelastic scattering

- **Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) - \mathcal{W}^{\mu\nu}(P,q,-S)$$



- **Define:** $\angle(\hat{k},\hat{S}) = \alpha$ **,**
  and lepton helicity λ

- **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2}\times$$

$$\times\left\{\cos\alpha\left\{\left[1-\frac{y}{2}-\frac{m^2x^2y^2}{Q^2}\right] g_1(x,Q^2) - \frac{2m^2x^2y}{Q^2} g_2(x,Q^2)\right\}\right.$$

$$\left. -\sin\alpha\cos\phi\,\frac{2mx}{Q}\sqrt{\left(1-y-\frac{m^2x^2y^2}{Q^2}\right)}\left(\frac{y}{2}\,g_1(x,Q^2) + g_2(x,Q^2)\right)\right\}$$

- **Spin orientation:**

$$\alpha = 0 : \Rightarrow \quad g_1$$

$$\alpha = \pi/2 : \Rightarrow \quad y\,g_1 + 2\,g_2$$, suppressed m/Q

Polarized deep inelastic scattering

❑ Spin asymmetries – measured experimentally:

✧ **Longitudinal polarization –** $\alpha = 0$

Known function

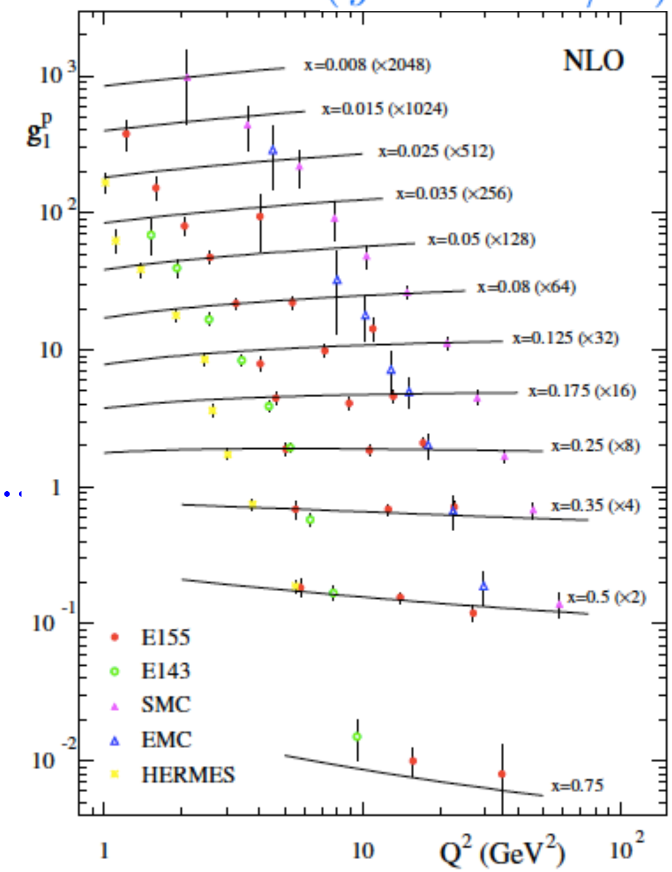
$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

$(\, y = 1 - E'/E \,)$

✧ **So far only "fixed target" experiments:**

CERN: EMC, SMC, COMPASS
SLAC: E80, E130, E142, E143, E154
DESY: HERMES
JLab: Hall A,B,C with many experiments

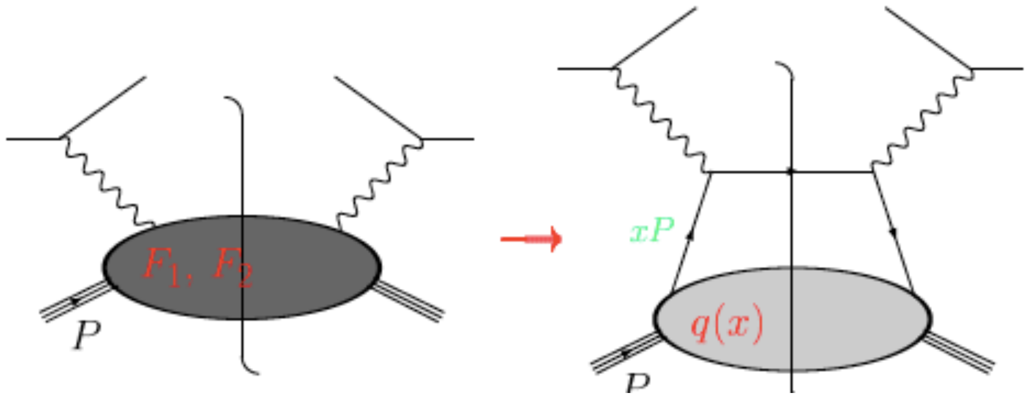
with various polarized targets: $p,\ d,\ {}^3\mathrm{He}, \ldots$



✧ **Future: EIC**

Polarized deep inelastic scattering

❑ Parton model results – LO QCD:



✧ Structure functions:

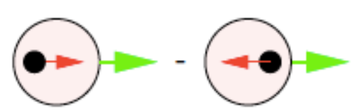
$$F_1(x) = \frac{1}{2}\sum_q e_q^2\,[\,q(x) + \bar{q}(x)\,]$$

$$g_1(x) = \frac{1}{2}\sum e_q^2\,[\,\Delta q(x) + \Delta\bar{q}(x)\,]$$

$$g_1 = \frac{1}{2}\left[\frac{4}{9}\,(\Delta u + \Delta\bar{u}) + \frac{1}{9}\,(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s})\right]$$

✧ Polarized quark distribution:

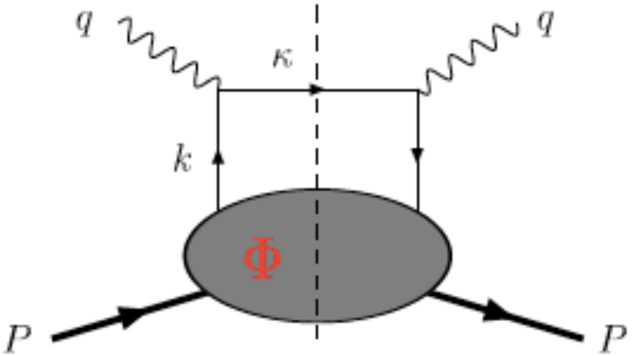
$$\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)$$

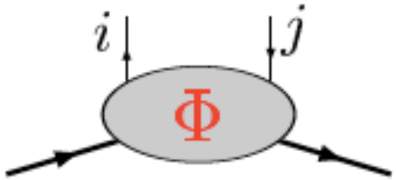


Information on nucleon's spin structure

Polarized deep inelastic scattering

❑ Systematics polarized PDFs – LO QCD:





✧ **Two-quark correlator:**

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X)\, \langle PS|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\, e^{ik\cdot z}\, \langle PS\,|\,\bar{\psi}_j(0)\,\psi_i(z)\,|\,PS\,\rangle$$

✧ **Hadronic tensor (one –flavor):**

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4}\, \delta\left((k+q)^2\right)\, \mathrm{Tr}\left[\,\Phi\, \gamma^\mu (\not{k}+\not{q})\gamma^\nu\,\right]$$

Polarized deep inelastic scattering

❑ Collinear expansion – PM kinematics:

– proton momentum : $P = (p, 0, 0, p)$,

– parton : $k^\mu \sim \xi P^\mu$

– virtual photon : $q^\mu = (P \cdot q)\, n^\mu - \xi P^\mu$

where $n = (1, 0, 0, -1)$

✧ **Light-cone frame:**

$$P^+ = p\,,\ k^+ = \xi\, p\,,\ P^- = k^- = 0\,,\ n^+ = 0\,,\ n^- = 1$$

✧ **On-shell condition:**

$$\delta\big((k+q)^2\big) = \frac{1}{2P\cdot q}\,\delta(x-\xi) = \frac{1}{2P\cdot q}\,\delta\left(x - \frac{k^+}{P^+}\right)$$

✧ **Approximated hadronic tensor:**

$$\mathcal{W}^{\mu\nu} = \frac{e^2}{2} \underbrace{\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\delta\left(x - \frac{k^+}{P^+}\right) \mathrm{Tr}\Big[\Phi}_{\equiv\,\phi(x)}\ \gamma^\mu \not{n} \gamma^\nu\Big]$$

Polarized deep inelastic scattering

✧ **General expansion of $\phi(x)$:**

must have general expansion in terms of P, $\not{n}$, $\not{s}$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma \cdot P + s_{||}\Delta q(x)\gamma_5\gamma \cdot P + \delta q(x)\gamma \cdot P\gamma_5\gamma \cdot S_\perp\right]$$

✧ **3-leading power quark parton distribution:**

$$q\,(x) \;=\; \frac{1}{4\pi}\int dz^- \,\mathrm{e}^{iz^- xP^+}\,\langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\,(0,z^-,\mathbf{0}_\perp)\,|P,S\rangle$$

$$\Delta q\,(x) \;=\; \frac{1}{4\pi}\int dz^- \,\mathrm{e}^{iz^- xP^+}\,\langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\,(0,z^-,\mathbf{0}_\perp)\,|P,S\rangle$$

$$\delta q\,(x) \;=\; \frac{1}{4\pi}\int dz^- \,\mathrm{e}^{iz^- xP^+}\,\langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\gamma_5\,\psi\,(0,z^-,\mathbf{0}_\perp)\,|P,S\rangle$$

"unpolarized" – "longitudinally polarized" – "transversity"

Polarized deep inelastic scattering

❑ Physical interpretation:

$$q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right)$$

$$\times \left[\left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \lambda = \frac{1}{2}\rangle \right|^2 + \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \lambda = \frac{1}{2}\rangle \right|^2 \right]$$

$$\Delta q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right)$$

$$\times \left[\left| \langle X | \mathcal{P}^+ \psi_+(0) | P, \lambda = \frac{1}{2}\rangle \right|^2 - \left| \langle X | \mathcal{P}^- \psi_+(0) | P, \lambda = \frac{1}{2}\rangle \right|^2 \right]$$

$$\delta q(x) = \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+)$$

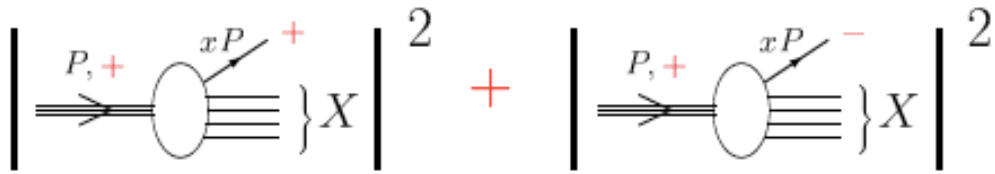
$$\times \left[\left| \langle X | \mathcal{P}^\uparrow \psi_+(0) | P, S_\perp = \frac{1}{2}\rangle \right|^2 - \left| \langle X | \mathcal{P}^\downarrow \psi_+(0) | P, S_\perp = \frac{1}{2}\rangle \right|^2 \right]$$

Spin projection: $\mathcal{P}^\pm \equiv \frac{1\pm\gamma_5}{2}$ **and** $\mathcal{P}^{\uparrow\downarrow} \equiv \frac{1\pm\gamma_\perp\gamma_5}{2}$

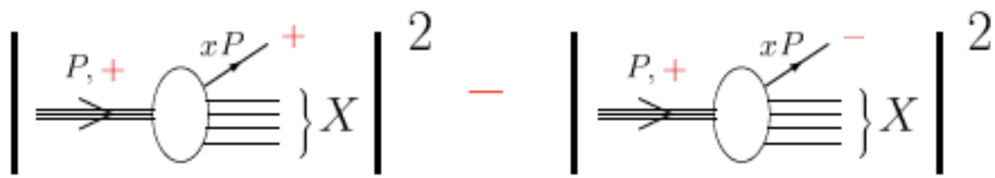
Polarized deep inelastic scattering

- **Pictorially:**

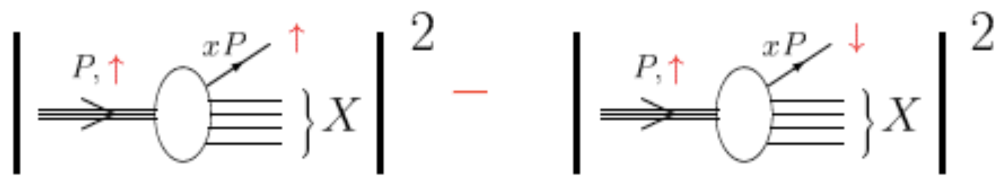
$q(x) =$



$\Delta q(x) =$



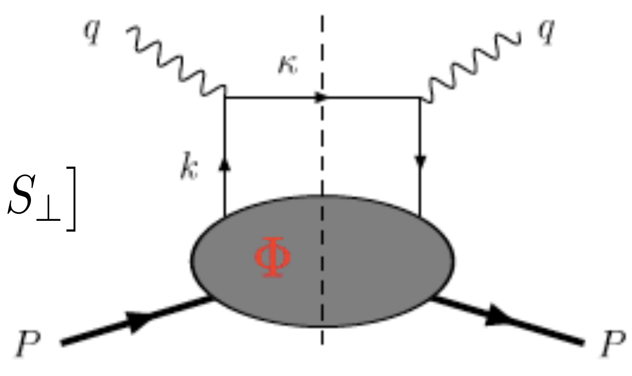
$\delta q(x) =$



- **Note:**

No transversity contribution to inclusive DIS!

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{\parallel}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_{\perp}\right]$$



Basics for spin observables

- **Factorized cross section:**

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I,\gamma_5,\gamma^\mu,\gamma_5\gamma^\mu,\sigma^{\mu\nu}$$

- **Parity and Time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

- **IF:** $\langle p,-\vec{s}|\mathcal{P}\mathcal{T}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

  or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

  Operators lead to the "+" sign ⟹ spin-averaged cross sections

  Operators lead to the "-" sign ⟹ spin asymmetries

- **Example:**

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

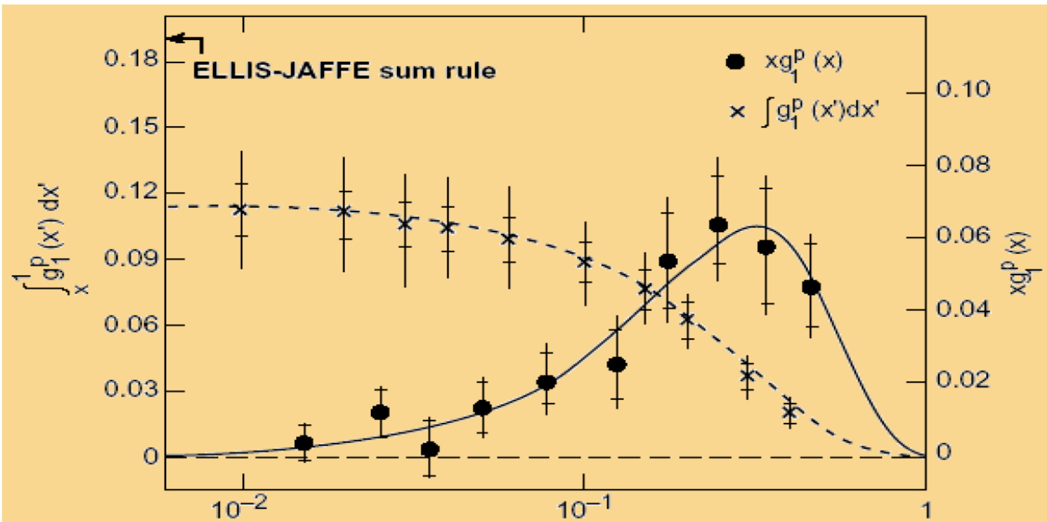
$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x) \to h(x)$$

$$\mathcal{O}(\psi,A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

Proton "spin crisis" – excited the field

❑ EMC (European Muon Collaboration '87) – "the Plot":



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}\left(1/Q\right)$$

✧ **Combined with earlier SLAC data:** $\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$

✧ **Combined with:** $g_A^3 = \Delta u - \Delta d$ **and** $g_A^8 = \Delta u + \Delta d - 2\Delta s$

from low energy neutron & hyperon β decay

➡ $\Delta\Sigma = \sum_q\left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$

❑ "Spin crisis" or puzzle:

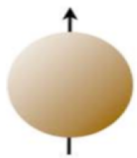
✧ **Strange sea polarization is sizable & negative**

✧ **Very little of the proton spin is carried by quarks**

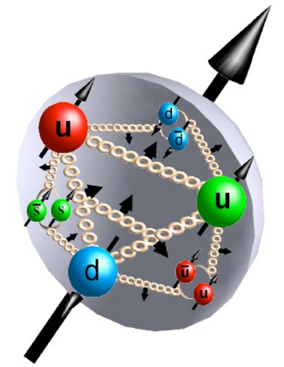
➡ ***New era of spin physics***

Summary of lecture five

❑ Key for a good proton spin decomposition – sum rule:



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$



- ✧ Every term can be related to a physical observable with controllable approximation – "independently measurable"

  DIS scheme is ok for F2, but, less effective for other observables

  Additional symmetry constraints, leading to "better" decomposition?

- ✧ Natural physical interpretation for each term – "hadron structure"

- ✧ Hopefully, calculable in lattice QCD – "numbers w/o distributions"

❑ Since the "spin crisis" in the 80^{th}, we have learned a lot about proton spin – there is a need for orbital contribution

Thank you!

Backup slides

QCD and hadrons