QCD in Collisions with Polarized Beams

Jianwei Qiu
Brookhaven National Laboratory
Stony Brook University

Annual Hua-Da School on QCD: EIC physics
China Central Normal University (CCNU), Wuhan, China, May 23 – June 3, 2016

Summary of lecture seven

- Single transverse-spin asymmetry in real, and is a unique probe for hadron's internal dynamics – Sivers, Collins, twist-3, … effects

- RHIC data seems to be consistent with the sign change of Sivers function, as predicted by QCD factorization

- But, the evolution of TMDs is still a very much open question! Better approach to non-perturbative inputs is needed!

- JLab12 and EIC should be able to provide much better data to help explore the confined motion of quarks/gluons

Thank you!

Transverse spin phenomena in QCD

Double Transverse-Spin Asymmetry (A_{TT})

Probe the transversity distribution: $\delta q(x)$

Drell-Yan – low rate

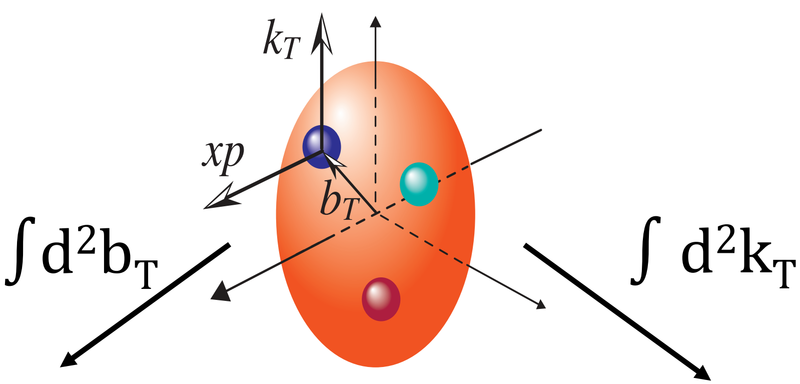
Single Transverse-Spin Asymmetry (SSA)

$$A(\ell, \vec{s}) \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

Chance to go beyond the collinear approximation to explore hadron's 3D structure!

Probe parton's confined transverse motion!

Imagine parton's spatial distribution!



TMDs $f(x,k_T)$

$f(x,b_T)$ **GPDs**

Two scales

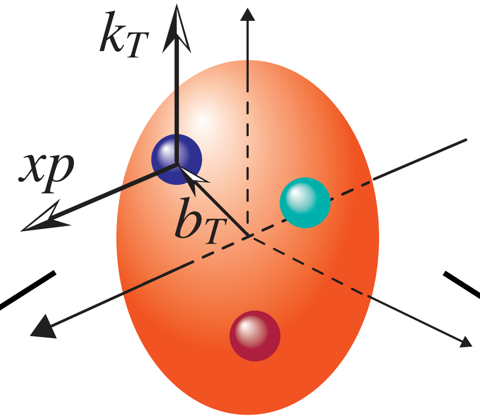
Boosted 3D nucleon structure

- High energy probes "see" the boosted partonic structure:

Momentum Space

TMDs

$\int d^2 b_T$



Coordinate Space

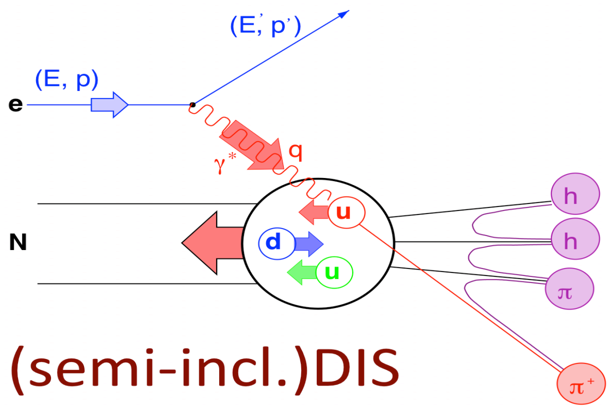
GPDs

$\int d^2 k_T$

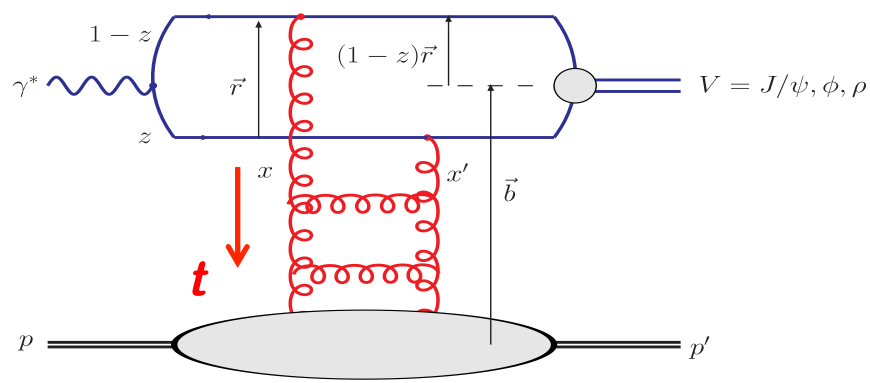
$f(x,k_T)$

$f(x,b_T)$

3D momentum space images



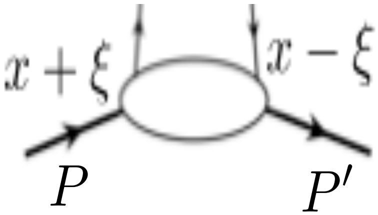
2+1D coordinate space images



Major parts of JLab12's physics program – large x

GPDs – role in solving the spin puzzle

❑ **Quark "form factor":**



$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P' | \bar{\psi}_q(\lambda/2) \frac{\gamma\cdot n}{2P\cdot n} \psi_q(-\lambda/2) | P \rangle$$

$$\equiv H_q(x,\xi,t,\mu^2)\, \left[\bar{\mathcal{U}}(P')\gamma^\mu \mathcal{U}(P)\right] \frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2) \left[\bar{\mathcal{U}}(P') \frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M} \mathcal{U}(P)\right] \frac{n_\mu}{2P\cdot n}$$

with $\xi = (P'-P)\cdot n/2 \;$ and $\; t = (P'-P)^2 \;\Rightarrow\; -\Delta_\perp^2 \;$ if $\; \xi \to 0$

$\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

❑ **Total quark's orbital contribution to proton's spin:** Ji, PRL78, 1997

$$J_q = \frac{1}{2} \lim_{t\to 0} \int dx\, x\, [H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$= \frac{1}{2}\Delta q + L_q$$

❑ **Connection to normal quark distribution:**

$H_q(x,0,0,\mu^2) = q(x,\mu^2)$ **The limit when** $\xi \to 0$

What can GPDs tell us?

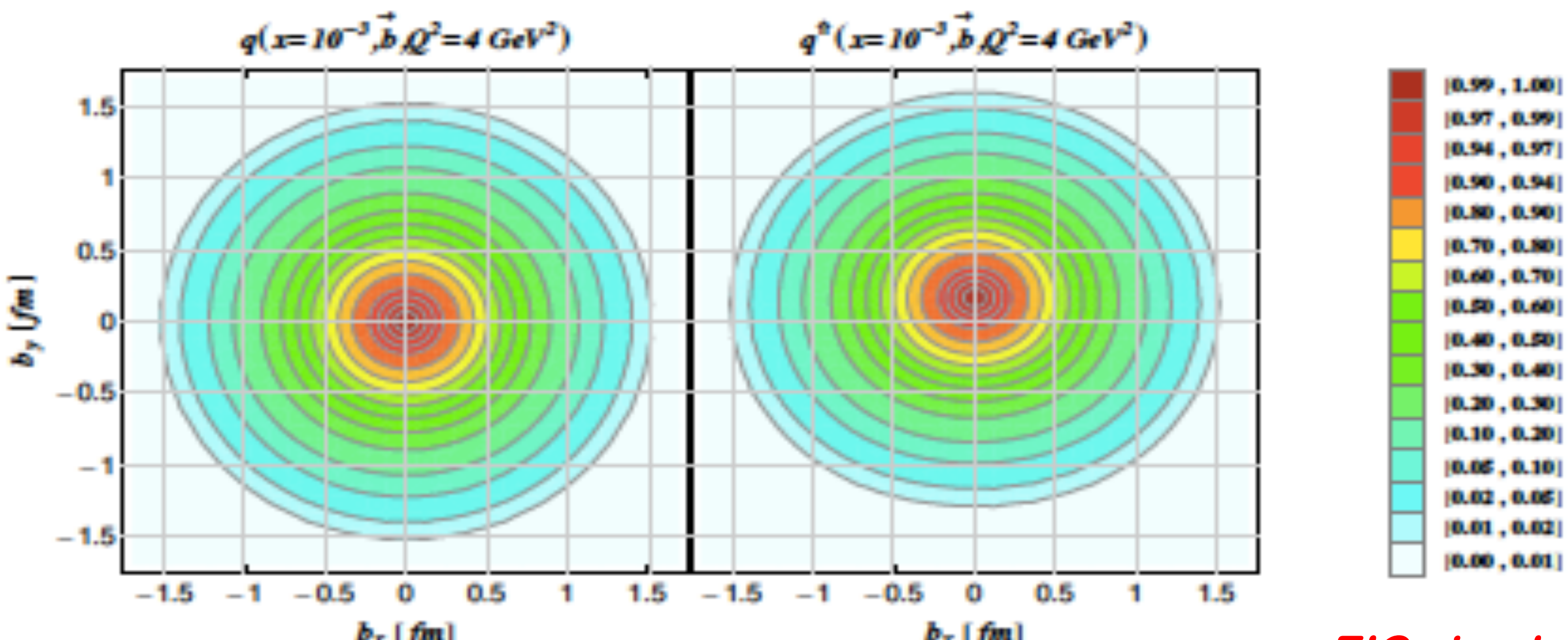
- **GPDs of quarks and gluons:**

$$H_q(x,\xi,t,Q), \quad E_q(x,\xi,t,Q),$$
$$\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$$

Evolution in Q
– gluon GPDs

- **Imaging ($\xi \to 0$):** $q(x,b_\perp,Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x,\xi=0,t=-\Delta_\perp^2,Q)$

- **Influence of transverse polarization – shift in density:**



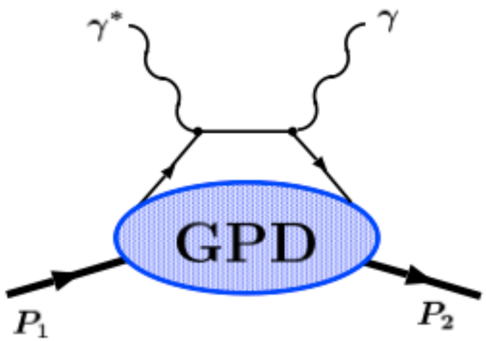
EIC simulation

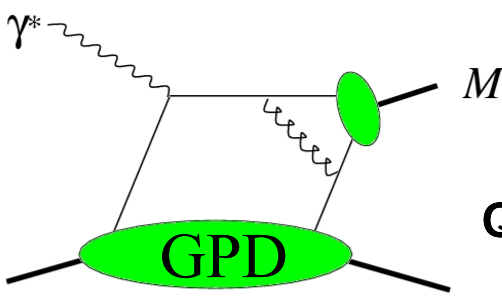
Exclusive DIS: Hunting for GPDs

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

- Experimental access to GPDs:
  - ✧ Diffractive exclusive processes – high luminosity:

    DVCS: Deeply virtual Compton Scattering
    DVEM: Deeply virtual exclusive meson production





Require

$Q^2 \gg (-t), \Lambda^2_{QCD}, M^2$

  - ✧ No factorization for hadronic diffractive processes – EIC is ideal

- Much more complicated – (x, ξ, t) variables:

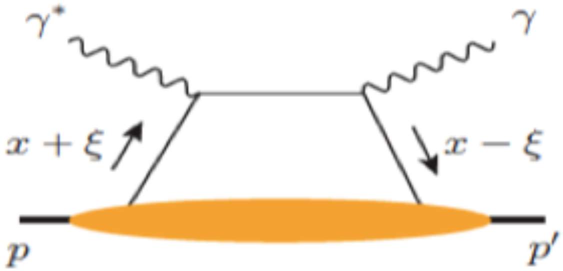
  Challenge to derive GPDs from data

- Great experimental effort:

  HERA, HERMES, COMPASS, JLab → JLab12, COMPASS-II, EIC

Deep virtual Compton scattering

- **The LO diagram:**



$$\xi \;=\; Q^2/(2\bar{P}\cdot q)$$
$$P' = P + \Delta$$

- **Scattering amplitude:**

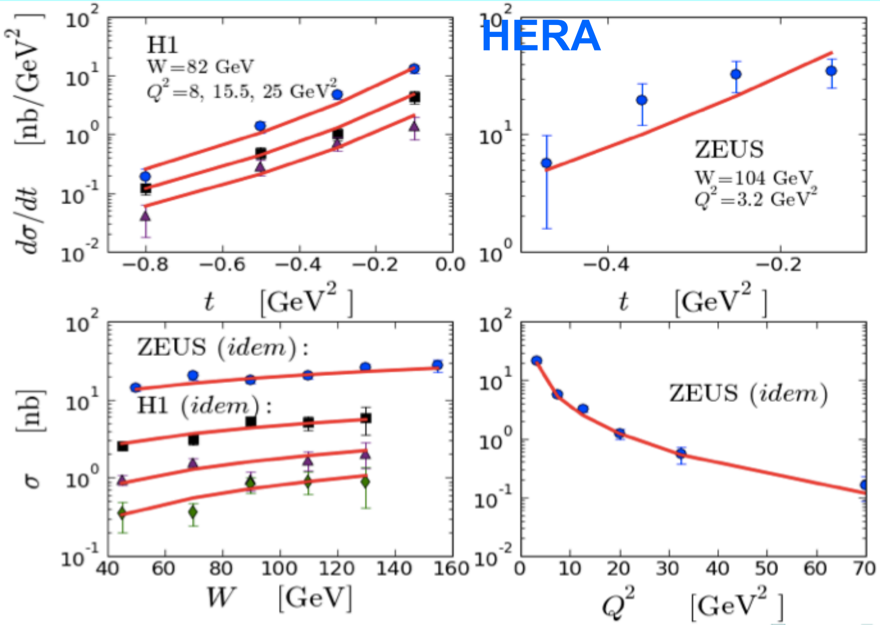
$$\begin{aligned} T^{\mu\nu}(P,q,\Delta) = & -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu}) \int dx \left(\frac{1}{x-\xi/2+i\epsilon} + \frac{1}{x+\xi/2+i\epsilon}\right) \\ & \times \left[H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}U(P) + E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right] \\ & -\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta \int dx \left(\frac{1}{x-\xi/2+i\epsilon} - \frac{1}{x+\xi/2+i\epsilon}\right) \\ & \times \left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5 U(P)\right] \end{aligned}$$

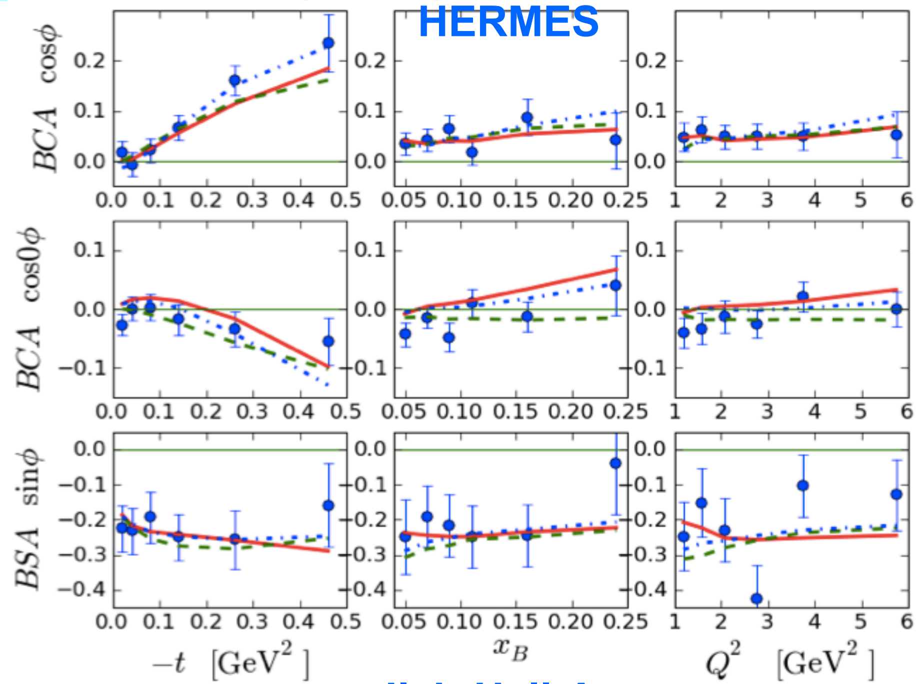
- **GPDs:**

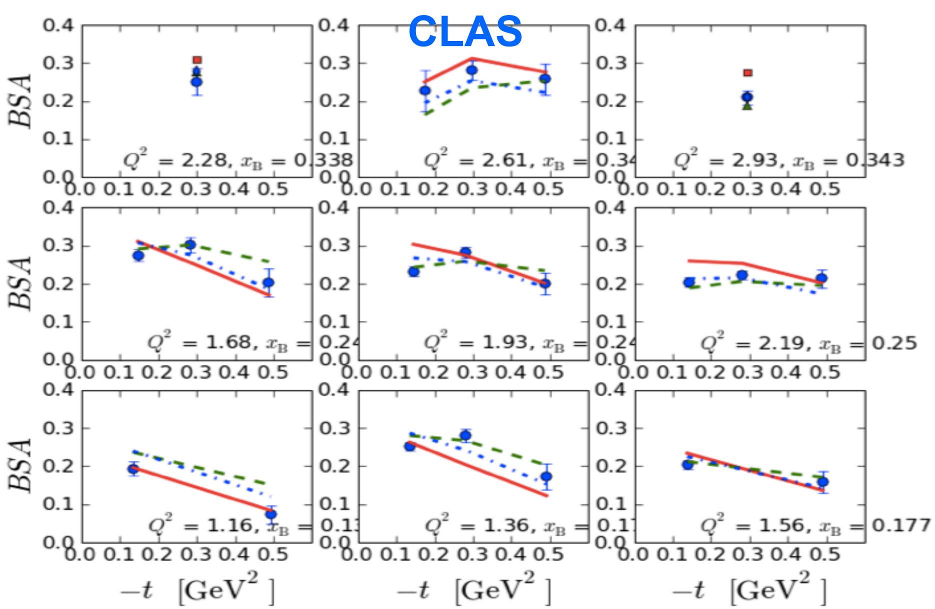
$$\begin{aligned} \int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = & H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P) \\ & + E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + \ldots \end{aligned}$$

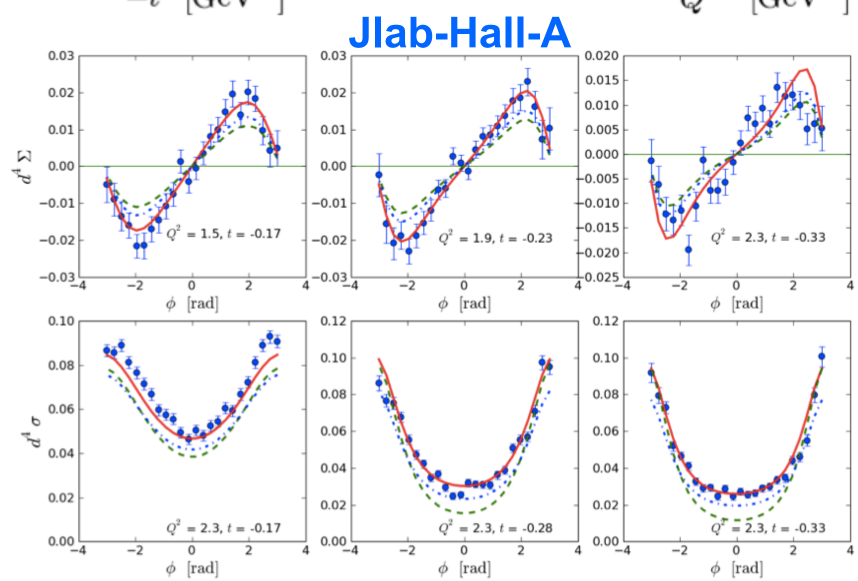
$$\begin{aligned} \int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = & \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5 U(P) \\ & + \tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P) + \ldots \end{aligned}$$

GPDs: just the beginning



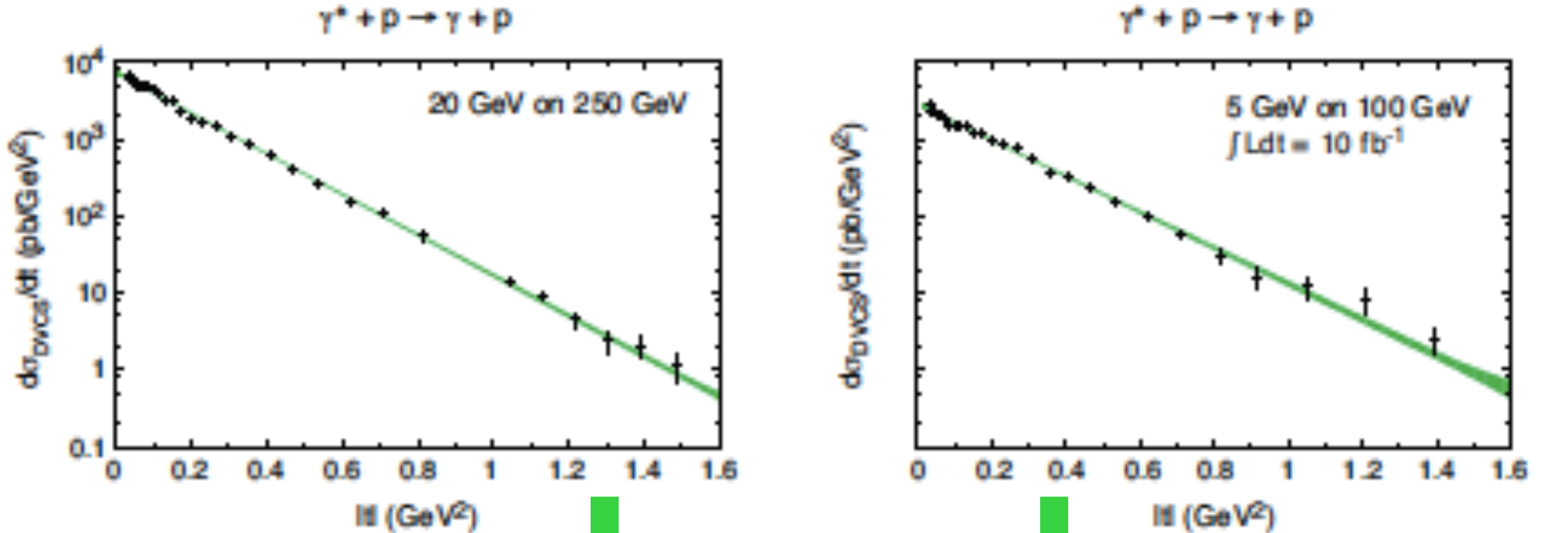




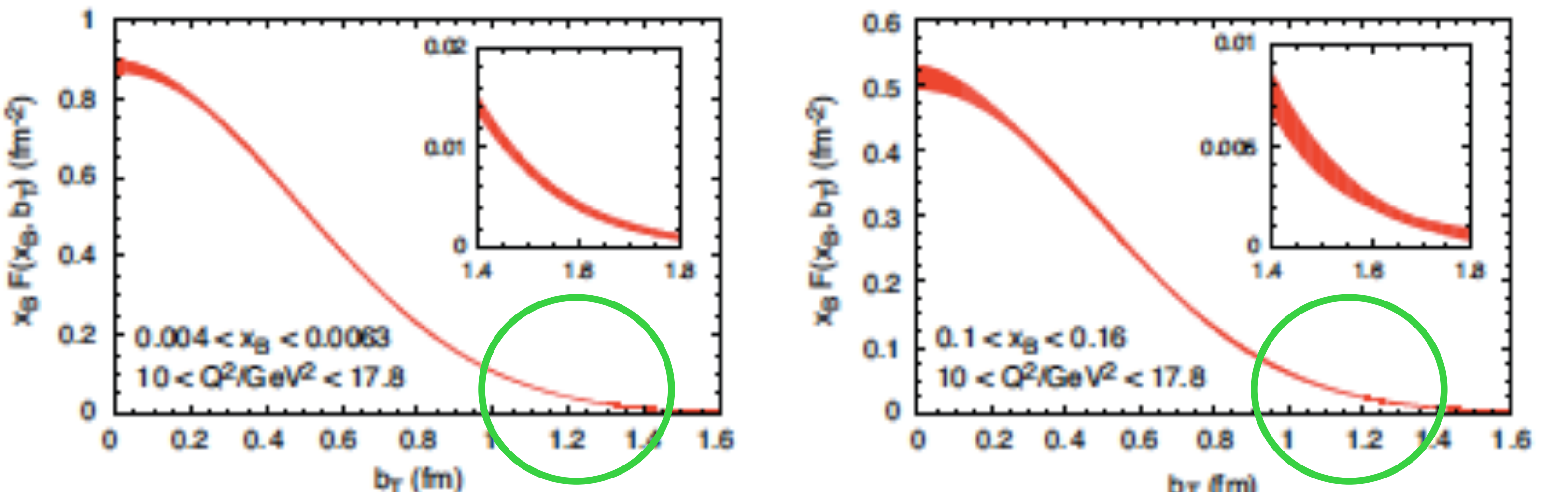


DVCS @ EIC

- ❑ Cross Sections:

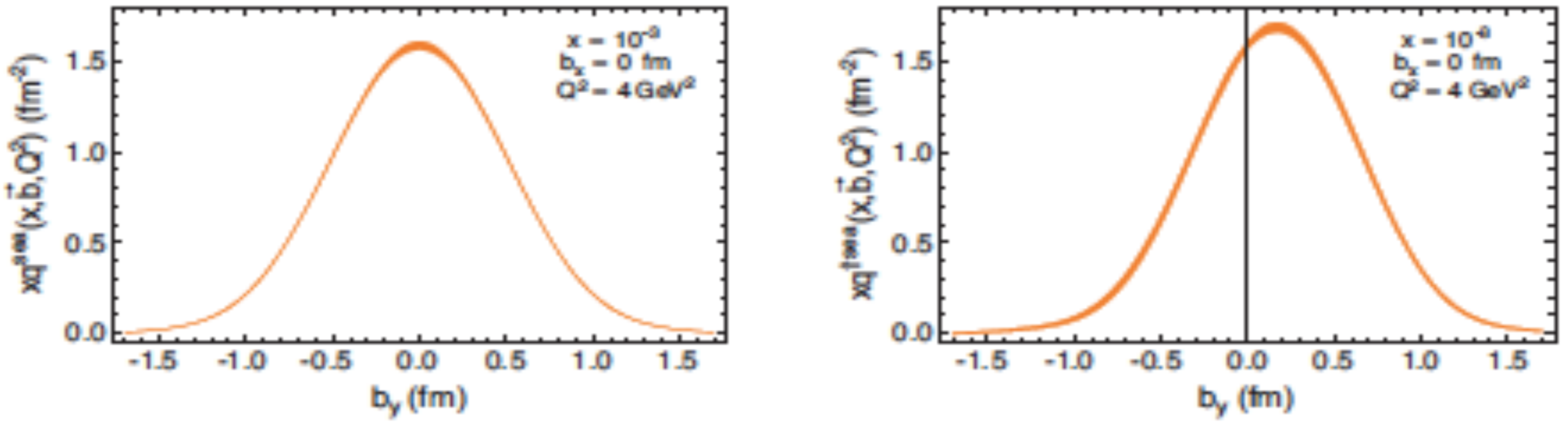


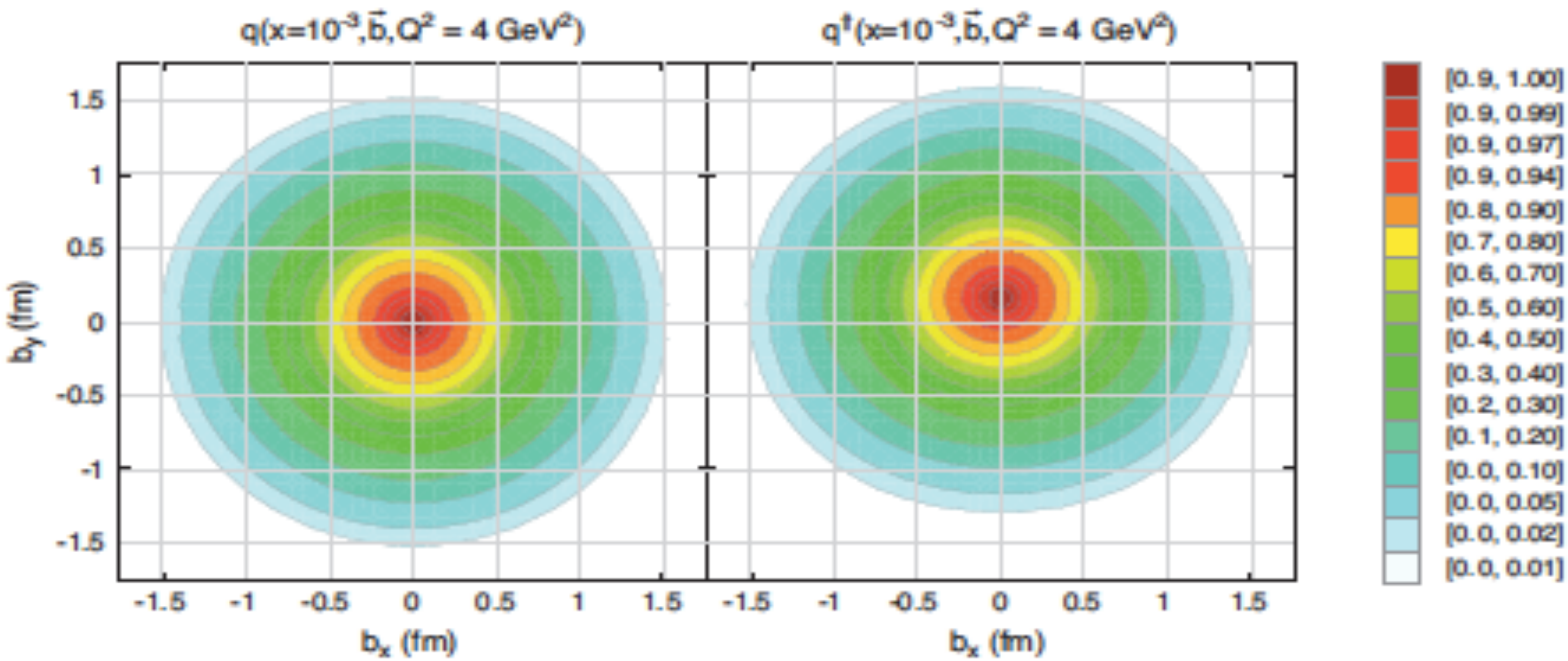
- ❑ Spatial distributions:



Polarized DVCS @ EIC

- Spin-motion correlation:



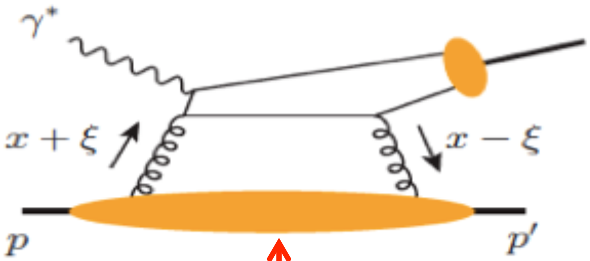


Spatial distribution of gluons

- **Exclusive vector meson production:**

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



t-dep

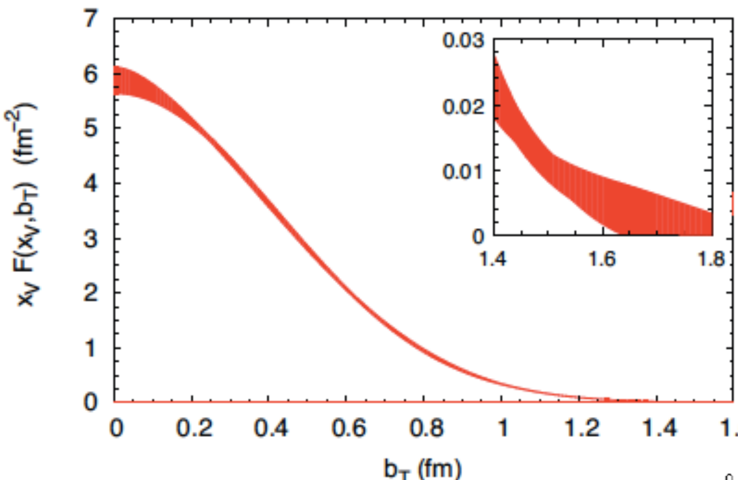
J/Ψ, Φ, ...

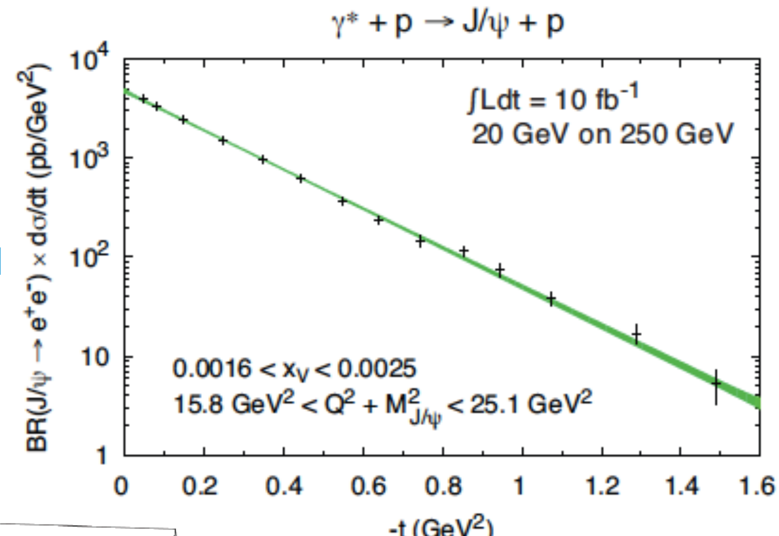
✧ Fourier transform of the t-dep

➡ Spatial imaging of glue density

✧ Resolution ~ 1/Q or $1/M_Q$

- **Gluon imaging from simulation:**

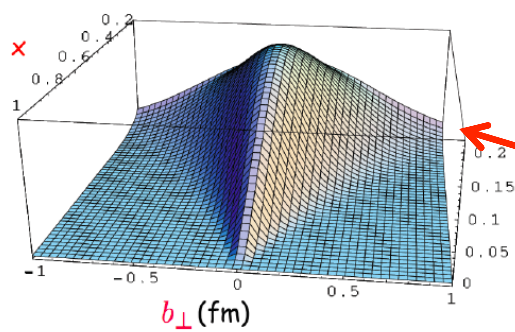




Only possible at the EIC

Gluon radius?

Gluon radius (x)!

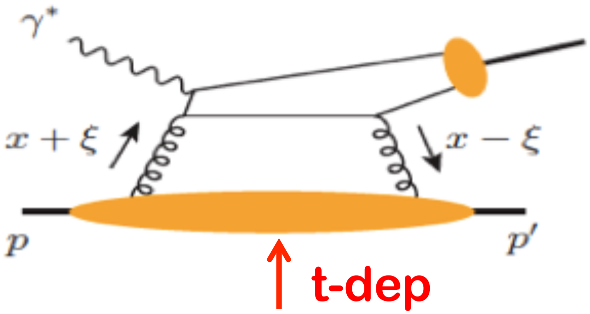


How spread at small-x?

Color confinement

Spatial distribution of gluons

❑ Exclusive vector meson production:



J/Ψ, Φ, …

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

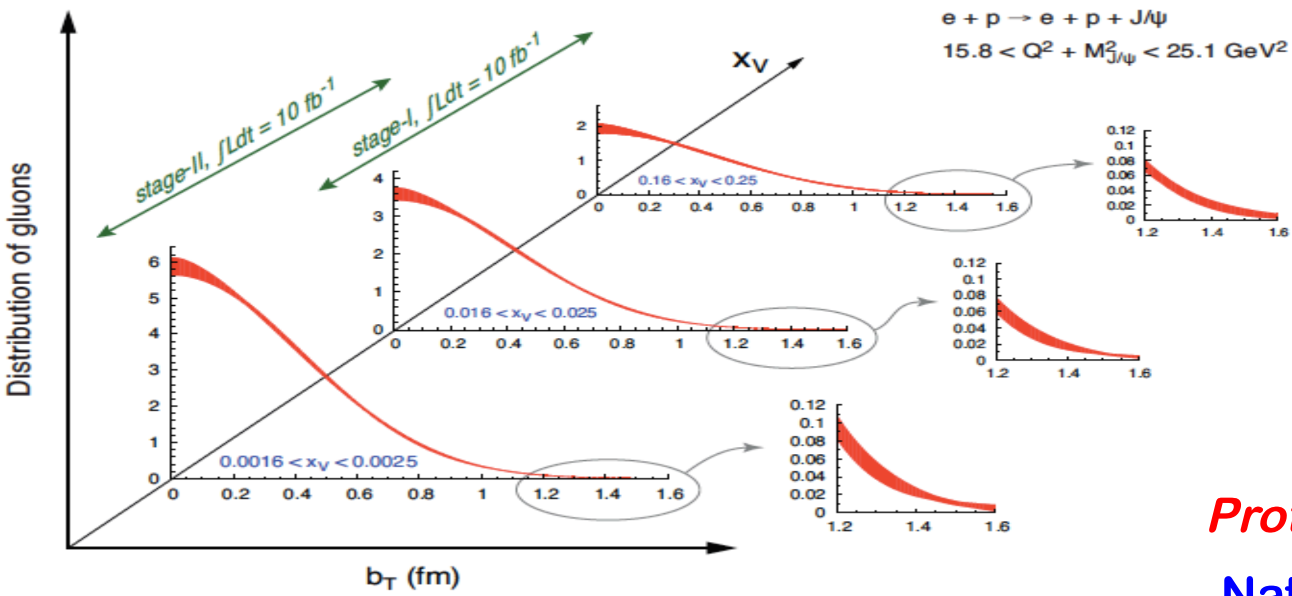
EIC-WhitePaper

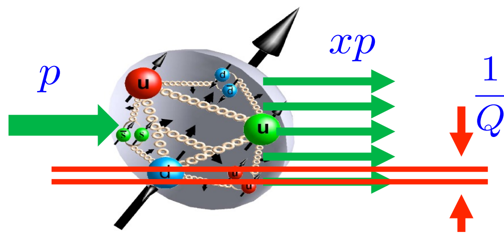
✧ Fourier transform of the t-dep

➡ Spatial imaging of glue density

✧ Resolution ~ 1/Q or 1/M_Q

❑ Gluon imaging from simulation:





Images of gluons from exclusive J/ψ production

Proton's "gluon radius"

Nature of pion cloud?

Model dependence – parameterization?

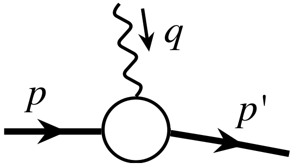
EIC simulation

Proton's radius in color distribution?

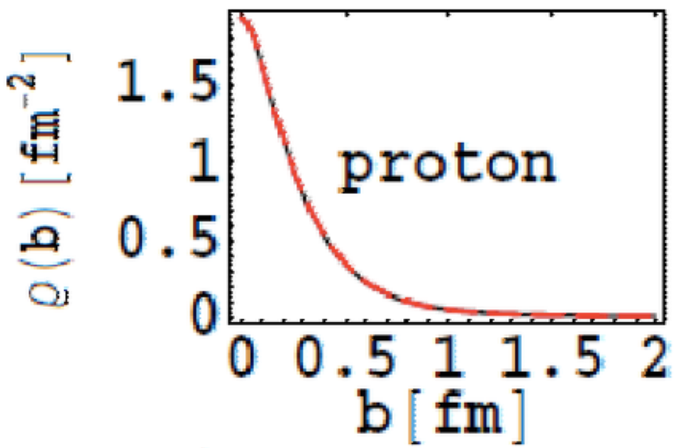
- ❑ **The "big" question:**

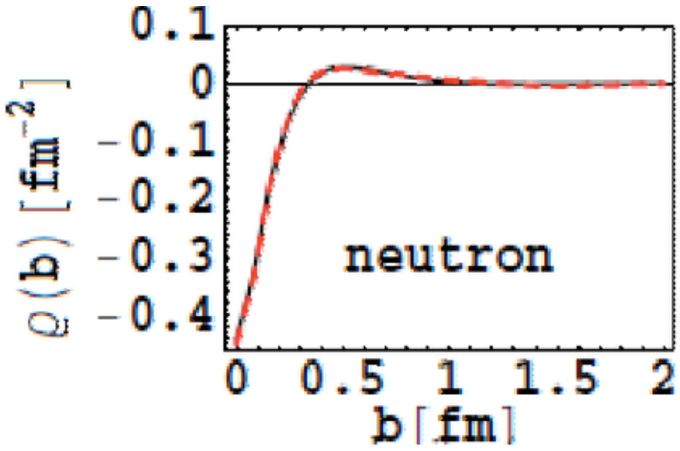
  How color is distributed inside a hadron? (clue for color confinement?)

- ❑ **Electric charge distribution:**



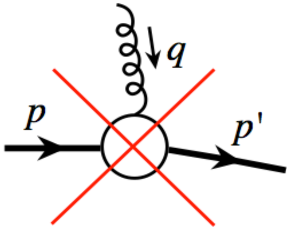
  Elastic electric form factor ➡ Charge distributions





- ❑ **But, NO color elastic nucleon form factor!**

  Hadron is colorless and gluon carries color



➡ *Parton density's spatial distributions – a function of x as well (more "proton"-like than "neutron"-like?) – GPDs*

Unified view of nucleon structure

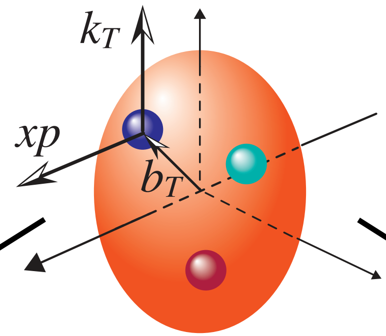
❑ Wigner distributions:

Momentum Space

TMDs

Confined motion

$\int d^2b_T$



$\int d^2k_T$

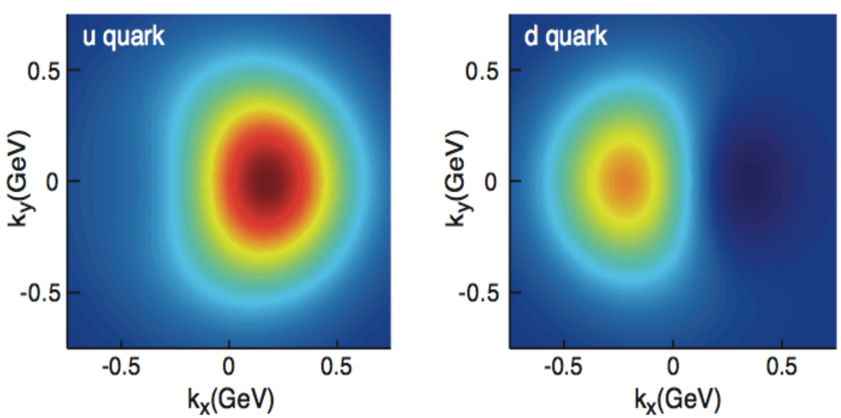
Coordinate Space

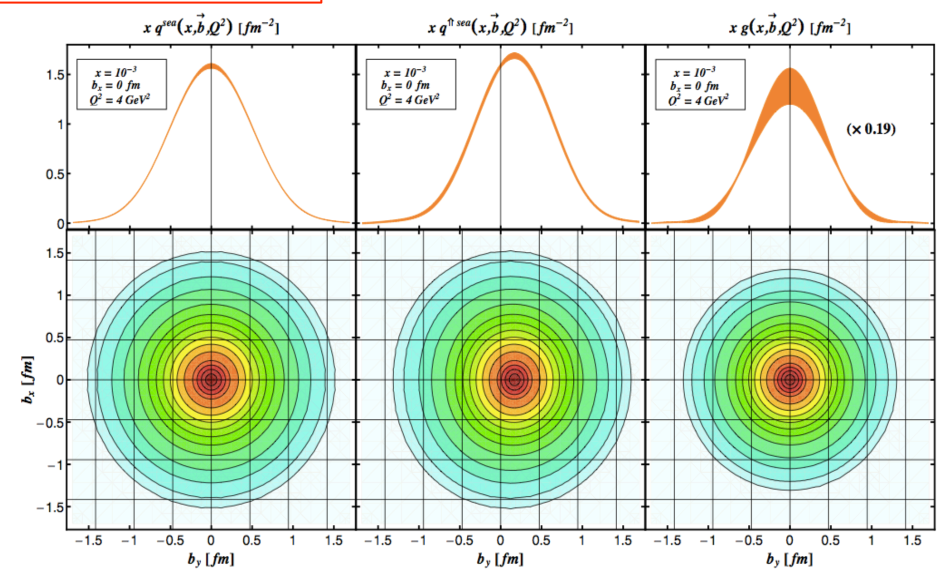
GPDs

Spatial distribution

$f(x,k_T)$ Two-scales observables $f(x,b_T)$

Sivers Functions





Position r x Momentum p → Orbital Motion of Partons

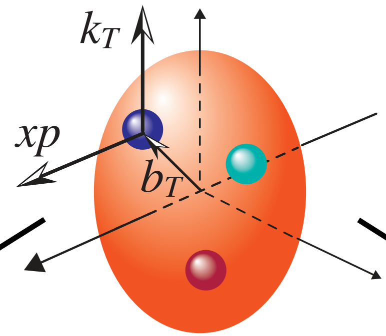
Unified view of nucleon structure

❑ **Wigner distribution:**

Momentum Space | *Coordinate Space*

TMDs | *GPDs*

Confined motion | Spatial distribution

$\int d^2b_T$ ←  → $\int d^2k_T$

$f(x,k_T)$ Two-scales observables $f(x,b_T)$

❑ **Note:**

✧ Partons' confined motion and their spatial distribution are unique – the consequence of QCD

✧ But, the TMDs and GPDs that represent them are not unique!

– Depending on the definition of the Wigner distribution and QCD factorization to link them to physical observables

Position *r* x Momentum *p* → Orbital Motion of Partons

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Lorce, Pasquini, …

✧ compensated by difference between gluon OAM density

✧ represented by different choice of gauge link for OAM Wagner distribution

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x, \vec{b}, \vec{k}_T) \left\{W_q(x, \vec{b}, \vec{k}_T)\right\}$$

with

$$\mathcal{W}_q \left\{W_q\right\}(x, \vec{b}, \vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$

$$\times \langle P'|\, \overline{\psi}_q(0)\, \frac{\gamma^+}{2}\, \underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}}\, \psi(y)\, |P\rangle_{y^+=0}$$

JM: "staple" gauge link

Ji: straight gauge link

between 0 and $y=(y^+=0, y^-, y_T)$

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ generated by a "torque" of color Lorentz force

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0)\frac{\gamma^+}{2}\int_{y^-}^{\infty} dz^- \Phi(0,z^-)$$
$$\times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{"Chromodynamic torque"}} \Phi(z^-,y)\psi(y)|P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

Hunting for orbital angular momentum

Talk by Lorce, C.

- **Orbital angular momentum:** $L_q \equiv J_q - \frac{1}{2}\Delta\Sigma$

Ji '96

$$J_q = \int_0^1 x\,(H_q + E_q)\,dx$$ = **Hadronic matrix element of local operator**

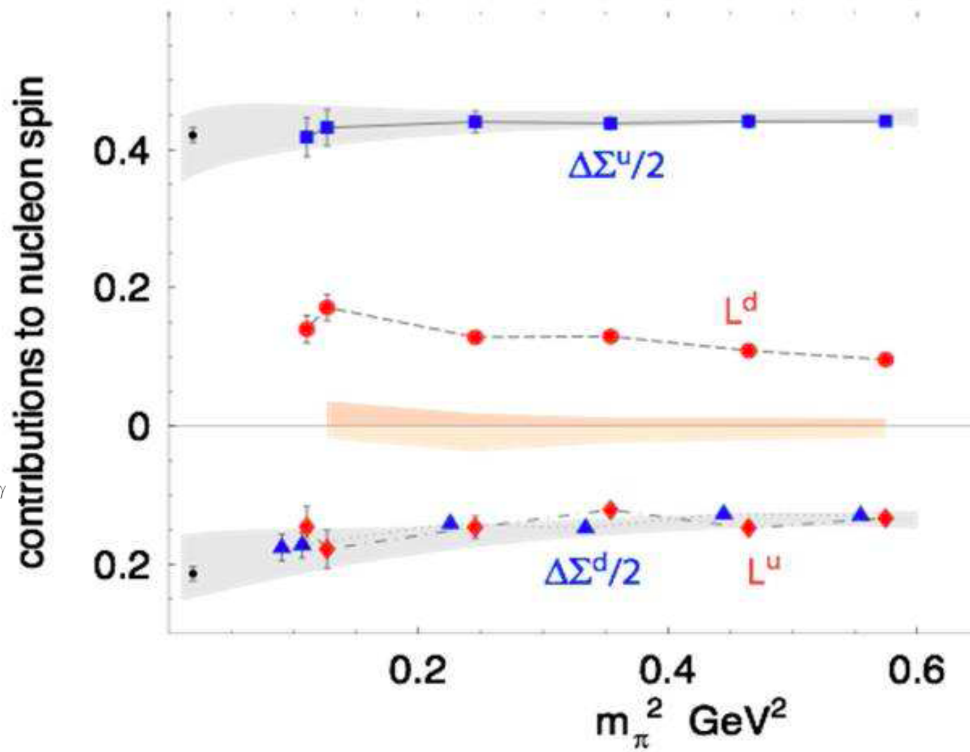
Experimental Extraction of GPDs

Calculable in Lattice QCD!

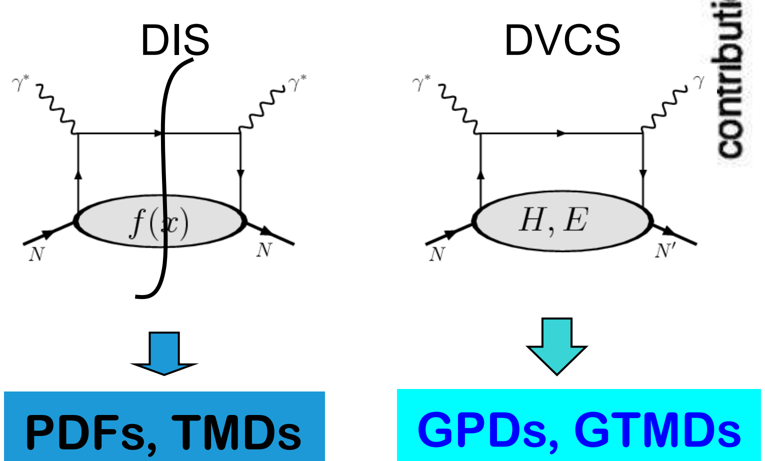
Negele et al

Both Lu amd L_d large

But, $L_u + L_d \sim 0$



- **GPDs:**

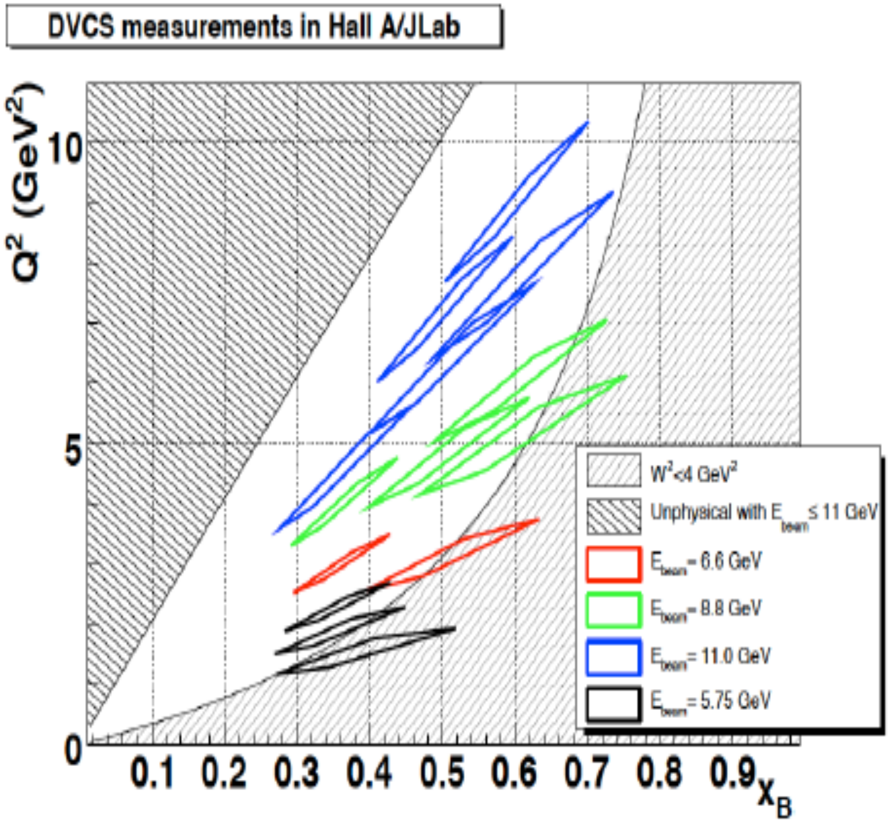


Quark/gluon transverse profile

❑ **DVCS at JLab12**

✧ **Establish scaling of σ_{DVCS} in Hall A**

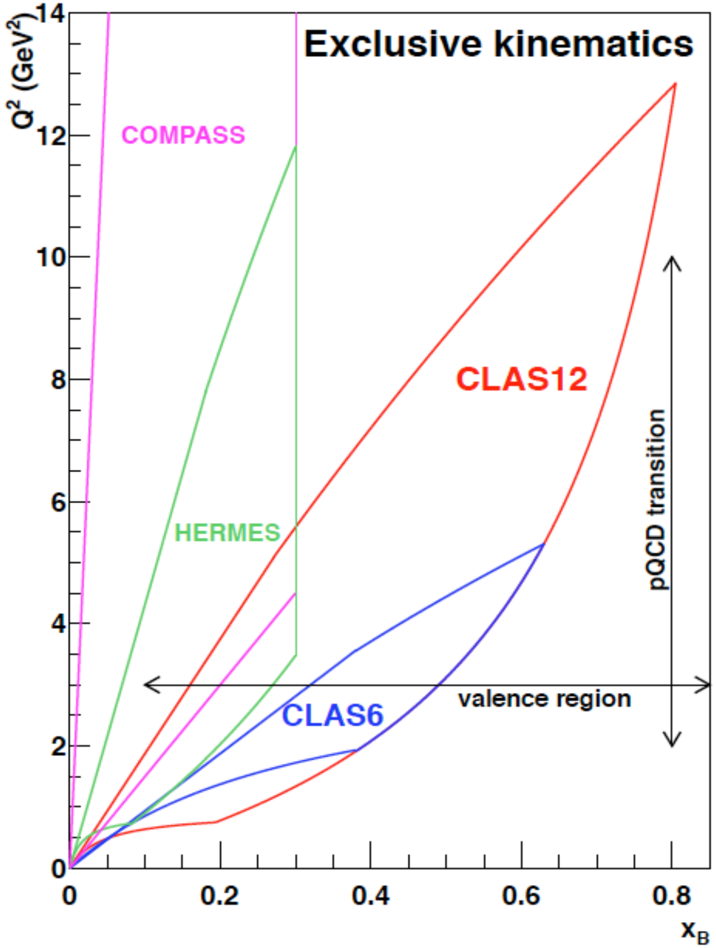
Run earlier



Theory: global fitting to extract GPDs

✧ **Measure DVCS at CLAS**

***broad kinematic range with polarized & unpol** observables*

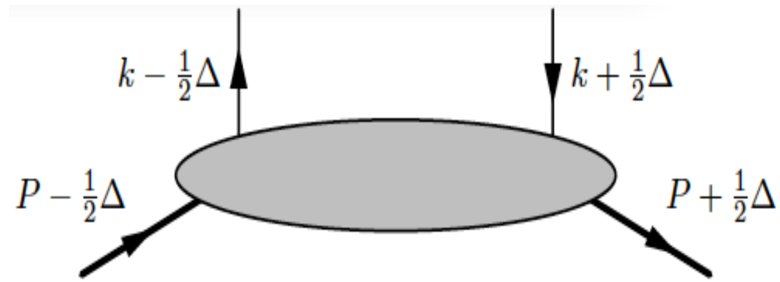


Partonic motion seen by a hard probe

- **Fully unintegrated distribution:**

Meissner, Metz, Schiegel, 2009

$$W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\Gamma\,\mathcal{W}(-\tfrac{1}{2}z,\tfrac{1}{2}z\,|\,n)\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle$$



– not factorizable in general

- **Generalized TMDs – hard probe:**

$$\mathcal{W}(x,k_T,\Delta)_\Gamma = \int dk^2\, W(P,k,\Delta)_\Gamma$$

– could be factorized assuming on-shell parton for the hard probe

- **Wigner function:**

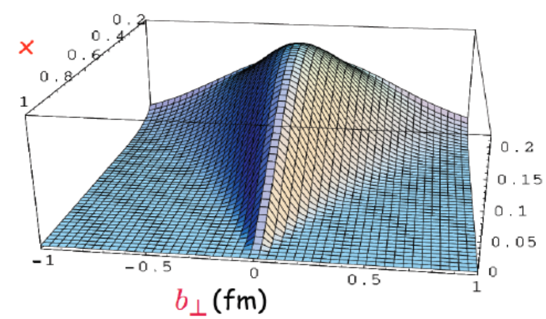
Belitsky, Ji, Yuan

$$W(x,k_T,b) \propto \int d^3\Delta\, e^{i\vec{b}\cdot\vec{\Delta}}\, \mathcal{W}(x,k_T,\Delta)_{\Gamma=\gamma^+}$$

- **Connection to all other known distributions:**

$W(x,k_T,b) \;\Rightarrow$ Tomographic image of nucleon

$$q(x,b_\perp) = \int d^2k_T db^-\, W(x,k_T,b)_{\gamma^+}$$



Burkardt, 2002

$\mathcal{W}(x,k_T,\Delta)_\Gamma \Rightarrow$ TMDs $(\Delta=0)$, GPDs $(\int d^2k_T)$, PDFs $(\Delta=0, \int d^2k_T)$

Connect OAM to observables

❑ Difference between two OAM definitions:

Burkardt, 2008

$$\mathcal{L}^q - L^q = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \left[\vec{x}\times \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

Caused by the work done by the torque along the trajectory of q

Color Lorentz force: $\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v}\times\vec{B}\right)^y$ for $\vec{v} = (0,0,-1)$

❑ Connection to GPDs:

Ji, 96
Burkardt, 2001, 2005

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0)\right] x$$

❑ Quark canonical OAM to TMDs, GTMDs – model dependent:

$$\mathcal{L}_z = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\vec{k}_\perp^2}{2M^2} h_{1T}^\perp(x,\vec{k}_\perp)$$

$$\ell_z = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[Lorce, Pasquini (2012)]

Model	LCCQM			χQSM		
q	u	d	Total	u	d	Total
ℓ_z^q	0.131	−0.005	0.126	0.073	−0.004	0.069
L_z^q	0.071	0.055	0.126	−0.008	0.077	0.069
$\mathcal{L}_z^q$	0.169	−0.042	0.126	0.093	−0.023	0.069

Note:

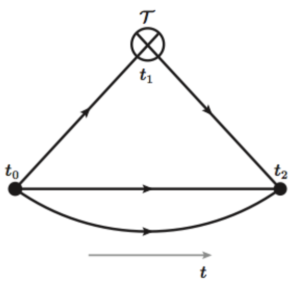
No gluons and not QCD EOM !

[Lorce, Pasquini (2011)]
[Lorce, et al (2012)]
[Kanazawa, et al (2014)]

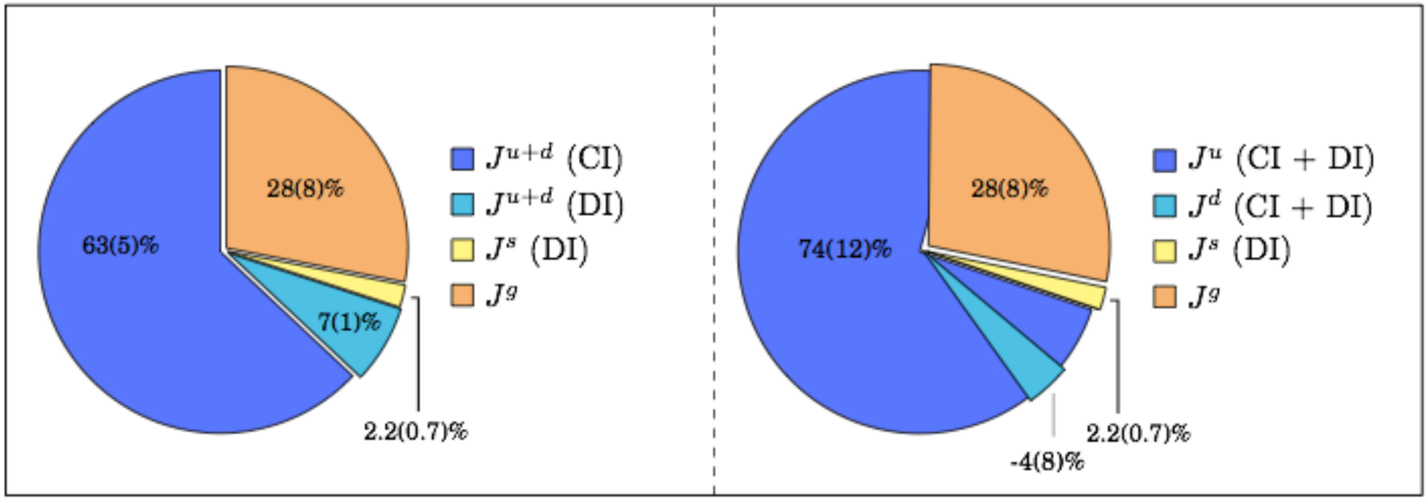
Nucleon spin and OAM from lattice QCD

❑ χQCD Collaboration:

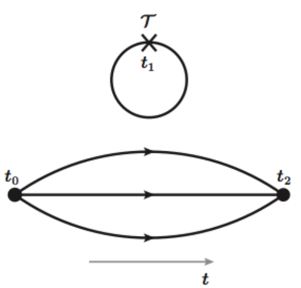
[Deka *et al.* arXiv:1312.4816]



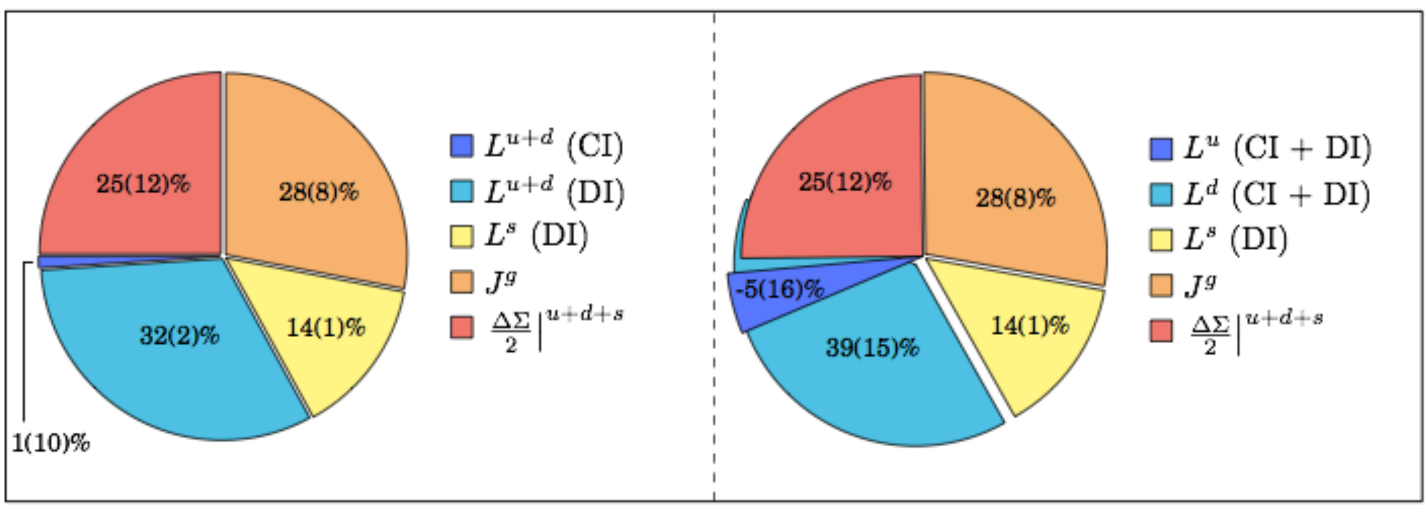
Connected Interaction (CI)



(b)



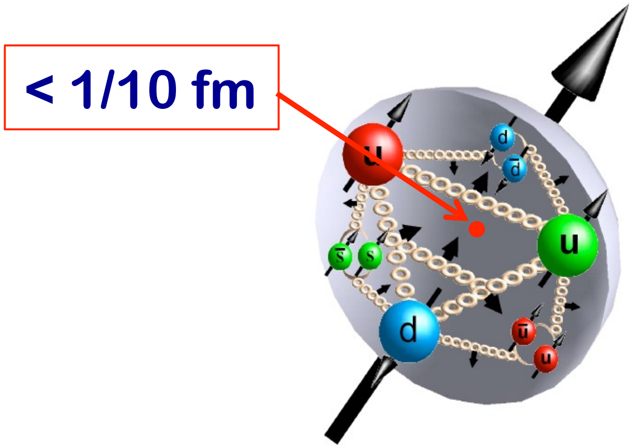
Disconnected Interaction (DI)



(c)

Summary

- QCD has been extremely successful in interpreting and predicting high energy experimental data!



- But, we still do not know much about hadron structure – a lot of work to do!

- Since the "spin crisis" in the 80th, we have learned a lot about proton spin – but, still a long way to go!

- TMDs and GPDs, accessible by high energy scattering with polarized beams, encode important information on hadron's 3D structure – distributions as well as motions of quarks and gluons

Thank you!

Backup slides

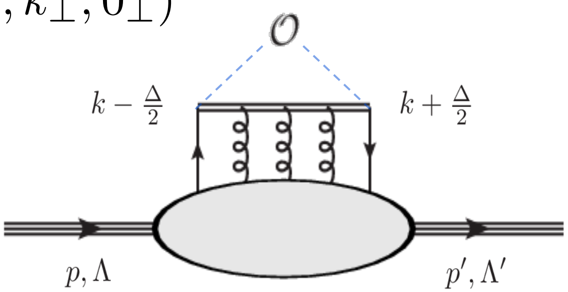
OAM from Generalized TMDs?

- **Canonical OAM:**

GTMD

Meissner, et al. 2008
See talk by Metz
Also, Lorce at ECT*

$$l_q^3 = \int d^2 b_\perp \, [\vec{b}_\perp \times \langle \vec{k}_\perp \rangle (\vec{b}_\perp)] = -\int dx \, d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)$$



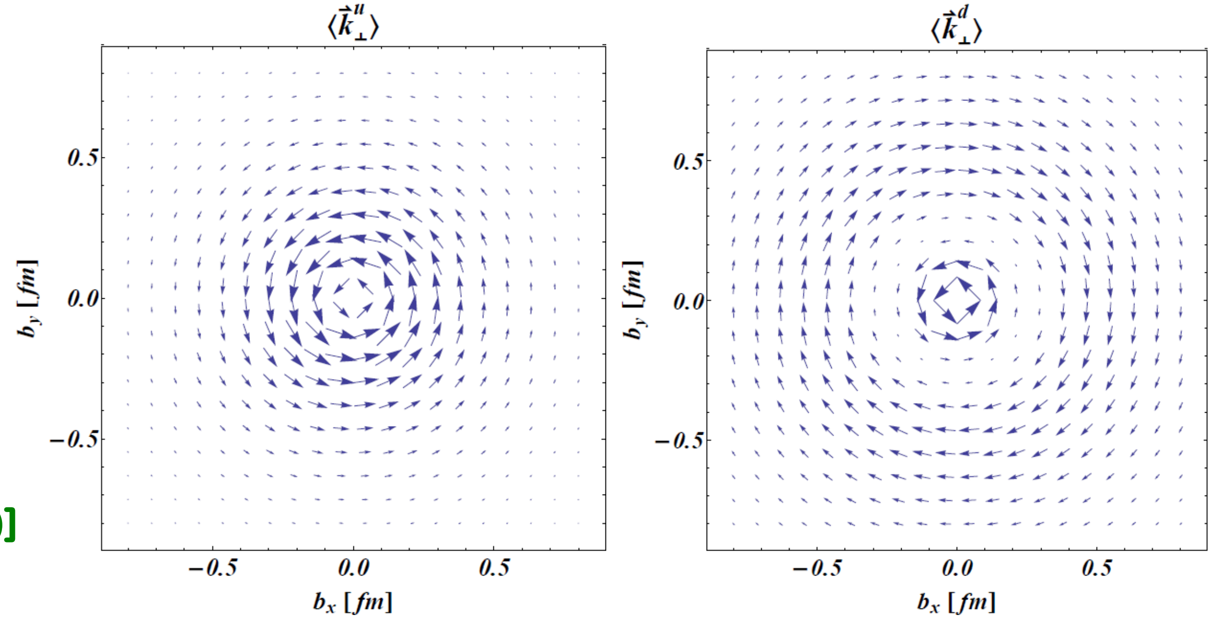
- **Connection to the Wigner distribution:**

2D FT of GTMDs ($\Delta_\perp \to b_\perp$)

$$\rho^{\mathcal{O}}_{\Lambda'\Lambda}(P, k, \vec{b}_\perp; \mathcal{W}) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} W^{\mathcal{O}}_{\Lambda'\Lambda}(P, k, \Delta; \mathcal{W})\Big|_{\Delta^+ = 0}$$

- **Spatial distribution of $\langle \vec{k}_\perp \rangle$:**

$$\langle \vec{k}_\perp \rangle (\vec{b}_\perp) = \int d^4 k \, \vec{k}_\perp \, \rho^{\gamma^+}_{\Lambda'\Lambda}(P, k, \vec{b}_\perp; \mathcal{W})$$



[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (2012)]
[Hatta (2012)]
[Kanazawa, et al. (2014)]

Orbital angular momentum contribution

Ji, Xiong, Yuan, PRL, 2012
Lorce, Pasquini, PRD, 2011
Lorce, et al, PRD, 2012

❑ The definition in terms of Wigner function:

✧ **Gauge invariant:**

$$L_q \equiv \frac{\langle P,S|\int d^3r\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)\psi(\vec{r})|P.S\rangle}{\langle P,S|P,S\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)\,dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

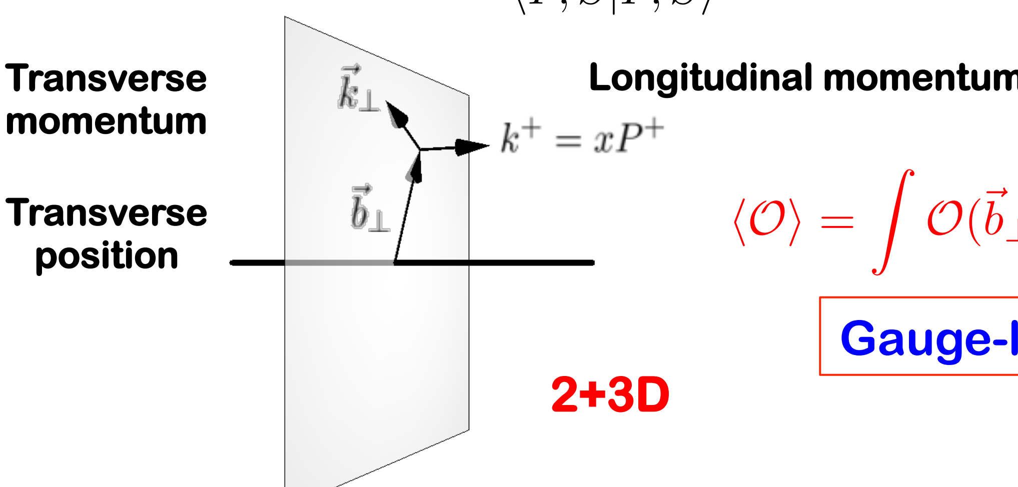
✧ **Canonical:**

$$l_q \equiv \frac{\langle P,S|\int d^3r\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|P.S\rangle}{\langle P,S|P,S\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x,\vec{b}_\perp,\vec{k}_\perp)\,dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

✧ **Gauge-dependent potential angular momentum – the difference:**

$$l_{q,pot} \equiv \frac{\langle P,S|\int d^3r\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times (-g\vec{A}_\perp))\psi(\vec{r})|P.S\rangle}{\langle P,S|P,S\rangle} = L_q - l_q$$

Quark-gluon correlation



$$\langle \mathcal{O} \rangle = \int \mathcal{O}(\vec{b}_\perp,\vec{k}_\perp)\,W_{GL}(x,\vec{b}_\perp,\vec{k}_\perp)\,dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

Gauge-link dependent Wigner function

Same for gluon OAM

Orbital angular momentum contribution

Ji, Xiong, Yuan, PRL, 2012
Lorce, Pasquini, PRD, 2011
Lorce, et al, PRD, 2012

❑ The Wigner function:

✧ **Quark:**

$$W^q_{GL}(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int \frac{dz^- d\vec{z}_\perp}{(2\pi)^3} e^{ik\cdot z} \left\langle P+\frac{\vec{\Delta}_\perp}{2} \middle| \overline{\Psi}_{GL}\left(-\frac{z}{2}\right)\gamma^+\Psi_{GL}\left(\frac{z}{2}\right) \middle| P-\frac{\vec{\Delta}_\perp}{2} \right\rangle$$

GL: gauge link dependence

Gauge to remove "GL"

$$\Psi_{FS}(z) = \mathcal{P}\left[\exp\left(-ig\int_0^\infty d\lambda\, z\cdot A(\lambda z)\right)\right]\psi(z)$$ Fock-Schwinger

$$\Psi_{LC}(z) = \mathcal{P}\left[\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(\lambda n+z)\right)\right]\psi(z)$$ Light-cone

✧ **Gluon:**

$$W^g_{GL}(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int \frac{dz^- d\vec{z}_\perp}{(2\pi)^3} e^{ik\cdot z} \left\langle P+\frac{\vec{\Delta}_\perp}{2} \middle| \mathbf{F}^{i+}_{GL}\left(-\frac{z}{2}\right)\mathbf{F}^{+i}_{GL}\left(\frac{z}{2}\right) \middle| P-\frac{\vec{\Delta}_\perp}{2} \right\rangle$$

❑ Gauge-invariant extension (GIE):

$$i\vec{\partial}^\alpha_\perp = i\vec{D}^\alpha_\perp(\xi) + \int^{\xi^-} d\eta^-\, L_{[\xi^-,\eta^-]}\, gF^{+\alpha}(\eta^-,\xi_\perp)\, L_{[\eta^-,\xi^-]}$$ Twist-3 correlators

Fixed gauge local operators ⟺ gauge invariant non-local operators

Note: the 2+3D Wigner distributions are not "physical"
But, their reduced distributions could be connected to observables

QCD and hadrons