Symmetries: Standard Model and Slightly Beyond

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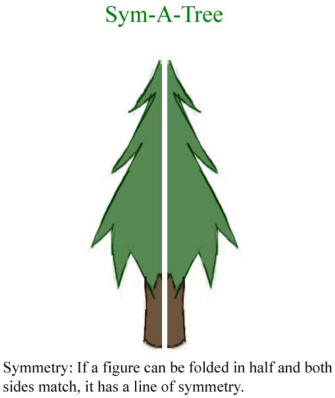
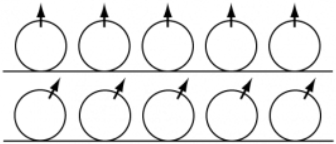
Outline

Symmetries


Sym-A-Tree
Symmetry: If a figure can be folded in half and both sides match, it has a line of symmetry.

Global Symmetries



A global symmetry is a symmetry that holds at all points in the spacetime, relevant to the system.

- U(1) with α as symmetry generator:
  $\psi(x,t) \rightarrow \psi'(x,t) = e^{i\alpha}\psi(x,t) \Rightarrow \left[e^{i\alpha}, H\right] = [\alpha, H] = 0$

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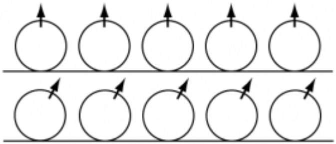

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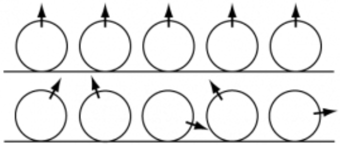
The system/Hamiltonian is invariant under global U(1) transformations.

Local Symmetries



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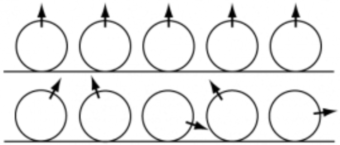

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The system/Hamiltonian is not invariant under local U(1) transformations.

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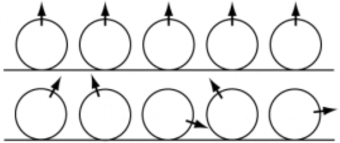

The laws of physics are the same in different locations, let's say Lisbon and London. So measurements done in each location should have the same results, but the states/wavefunctions prepared for the experiences certainly have different phases, which change through spacetime.

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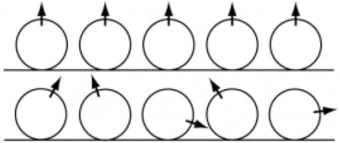

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How to restore the gauge/local invariance?

Local Symmetries

Change the equation of motion adding gauge fields that transform accordingly to the gauge symmetry. These gauge fields are closely related with the classical potentials.

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- U(1) gauge symmetry $(\hbar = c = 1)$: $\psi'(x,t) = e^{iq\alpha(x,t)}\psi(x,t)$
  Changing $i\partial_t\psi = -\frac{1}{2m}\partial_x^2\psi$ to
  $i\left(\partial_t + iq\phi\right)\psi = -\frac{1}{2m}\left(\partial_x - iqA\right)^2\psi$ with the transformations $A \to A' = A + \partial_x\alpha$ and $\phi \to \phi' = \phi - \partial_t\alpha$. The system becomes invariant under the gauge symmetry.

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The interaction between the gauge fields and particles is what we call force, like electromagnetism.

Parity

Parity can be thought as test for chirality since a chiral phenomenon is one that is not identical to its mirror image.

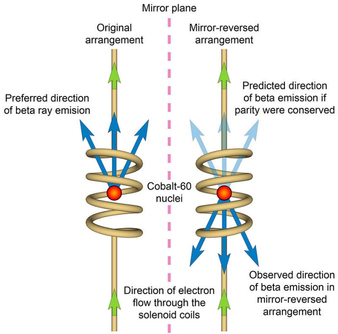
- A system with x^2 have $P = +1$, with x have $P = -1$, but a system with $x^2 + x$ is chiral.

In particle physics, each fermion can be described by its (orthogonal) chiral components/projections:

- $\psi = P_L\psi + P_R\psi \equiv \psi_L + \psi_R$
- $P_L P_R = 0$, $P^\dagger_{L,R} = P_{L,R}$ and $P^2_{L,R} = P_{L,R}$

Gravity, electromagnetism and strong interactions preserve parity, acting equally in ψ_L and ψ_R. But weak interactions do not.

Wu Experiment



Lee and Yang proposed a set of experiments to test if parity is violated by weak interactions. The experiment from Wu and his colleagues showed that the parity is maximally violated in weak decays.

Weak Interactions

It is then establish that weak interactions act solely on left-handed particles. Then right-handed fields of quarks (up: $\psi_{uR} \equiv u_R$ and down: $\psi_{dR} \equiv d_R$) and the right-handed component of electron ($\psi_{eR} \equiv e_R$) are singlets under the gauge symmetry associated with weak interactions: $\mathrm{SU}(2)_L$.

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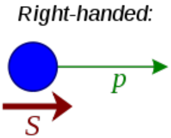

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When the Standard Model was constructed, neutrinos were thought to be massless and pure left-handed particles: $\psi_\nu = \psi_{\nu L} \equiv \nu_L$

Neutrinos

Neutrino Basics

- Neutrinos interact very weakly and only through the weak force and gravity ($Q(\nu) = 0$).
- Neutrinos appear in three flavours, electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ).
- In the SM (only ν_L) neutrinos are strictly massless. No gauge invariant mass term ($m_\nu \overline{\nu_L} \nu_R$), due to the absence of ν_R.
- Experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidences for the existence of neutrino oscillations.

Neutrino Oscillations

The fact that neutrinos oscillate imply non-vanishing masses and non-vanishing mixing angles.

$$P_{\nu_e \to \nu_\mu} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{2E}L\right)$$

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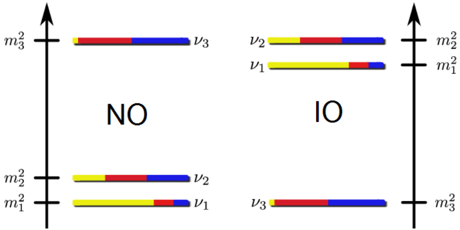
Like in the quarks case with the CKM matrix, for neutrinos, PMNS matrix represents the mismatch between mass states and states that take part in weak interactions.

$$\nu_i = \sum_j \mathbf{U}^{ij}_{PMNS} \, \nu_j, \;\; i = e, \nu, \tau, \;\; j = 1, 2, 3.$$

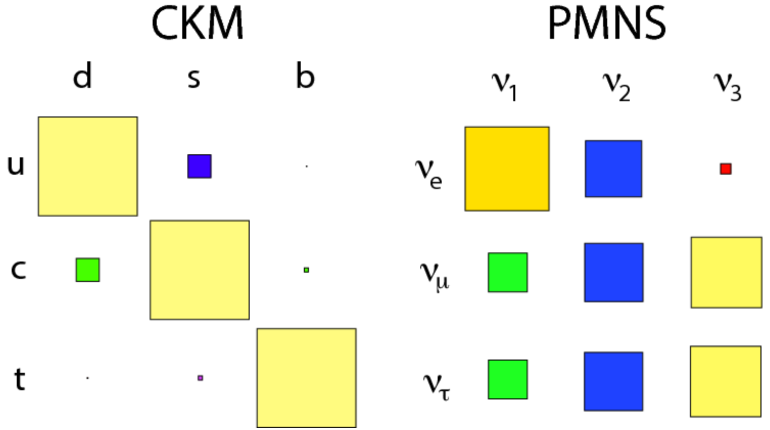
Neutrino Oscillations

The hints for the masses squared differences do not provide an absolute scale of neutrino mass and the nature of neutrinos is yet to be confirmed: Dirac ($\nu \neq \bar{\nu}$) or Majorana ($\nu \neq \bar{\nu}$) particles.

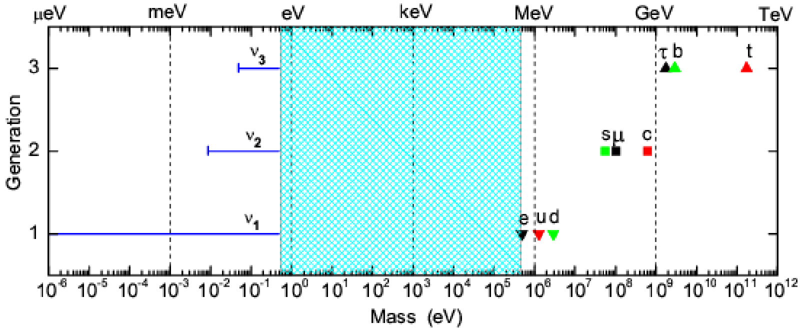
Two possibles orderings:



CKM vs PMNS



Neutrino Masses



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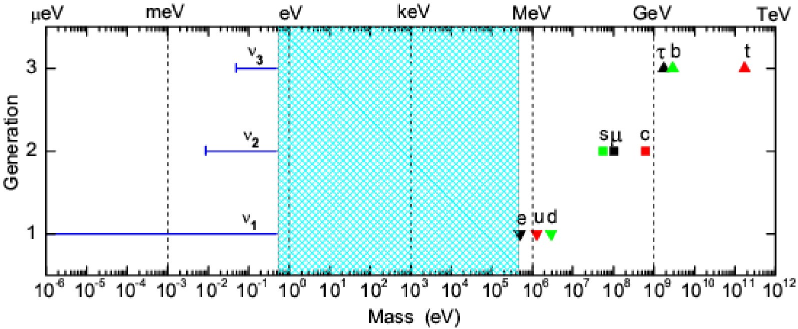

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We can extend the SM with ν_R like other fermions, but it is natural that neutrino masses have the same origin?

Masses


ν_e
Neutrino ν_e
Mosquito
e
Electron
2 apples
u
Up quark
Laptop
t
Top quark
Aircraft

Invariant Mass Terms

The mass term for a given fermion, like every other element present in the Lagrangian of the theory, has to be invariant.

- $\mathcal{L}_m = -m_\psi \overline{\psi}\psi$, for an electron: $\mathcal{L}_m = -m_e \overline{e} e$

Where $\overline{\psi} = \psi^\dagger \gamma^0$, and γ^0 is a matrix with the properties:

- $\left(\gamma^0\right)^2 = 1$, $\left(\gamma^0\right)^\dagger = \gamma^0$ and $P_{L,R}\gamma^0 = \gamma^0 P_{R,L}$

Then the mass term can be written as:

- $\mathcal{L}_m = -m_\psi \overline{\psi_L}\psi_R + \text{h.c.}$, for an electron: $\mathcal{L}_m = -m_e \overline{e_L} e_R + \text{h.c.}$

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Clearly with $\nu_\mathbf{R}$ a similar mass term for ν appears, but it is the only one?

Neutrino Mass Terms

Lepton (e and ν) related terms are invariant under strong/color force, but maybe charged under $\mathrm{U}(1)_{em}$:

- $Q(\psi) = q$, $Q(e) = -1$, $Q(\nu) = 0$ and $\psi' = e^{iq\alpha}\psi$

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Including $\nu_{\mathbf{R}}$ we have

$$-\mathcal{L}_m = m_e \overline{e_L} e_R + m_D^* \overline{\nu_L} \nu_R + \frac{1}{2} m_L^* \overline{\nu_L} C \overline{\nu_L}^T + \frac{1}{2} m_R \overline{\nu_R} C \overline{\nu_R}^T + \text{h.c.}$$

$$-\mathcal{L}_m = m_e \overline{e_L} e_R + \frac{1}{2} \begin{bmatrix} \nu_L^T C & \overline{\nu_R} \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L & C \overline{\nu_R}^T \end{bmatrix} + \text{h.c.}$$

The electron has a definite mass, but the neutrino masses are the eigenvalues of the mass matrix.

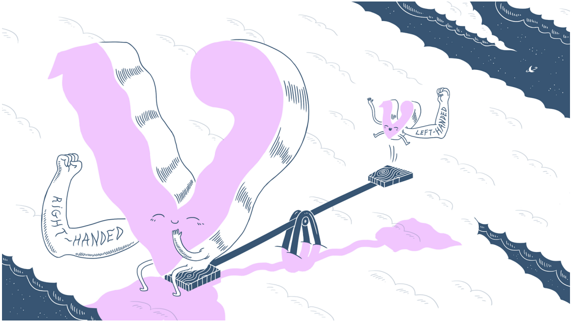
Seesaw Mechanism

$$-\mathcal{L}_m = m_e \overline{e_L} e_R + \frac{1}{2} \begin{bmatrix} \nu_L^T C & \overline{\nu_R} \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L & C\overline{\nu_R}^T \end{bmatrix} + \text{h.c.}$$

Usually m_L is taken to be zero (or very small) since is precisely the effective mass of left-handed neutrinos that we want to find. On the other hand, m_R can be arbitrarily large compared to m_D (and other mass terms like $m_\psi \overline{\psi_L} \psi_R$ that originate from the Higgs mechanism).

Compute the eigenvalues considering $m_L, m_D \ll m_R$

Seesaw Mechanism



There are two different neutrinos mass states:

- Light neutrinos: $m_\nu \simeq m_L - \frac{m_D^2}{m_R} \simeq -\frac{m_D^2}{m_R}$
- Heavy neutrinos: $m_N \simeq m_R + \frac{m_D^2}{m_R} \simeq m_R$

Within experimental bounds, we can predict m_R as:

- $m_\nu \simeq 0.1\,\text{eV}, m_D \simeq 100\,\text{GeV} \Rightarrow m_R \simeq 10^{14}\,\text{GeV}$

Three Families of Fermions

There are three generations of each fermion in the SM, so for charged leptons we have

$$-\mathcal{L}_m = m_e \overline{e_L} e_R + m_\mu \overline{\mu_L} \mu_R + m_\tau \overline{\tau_L} \tau_R + \text{h.c.}$$

But e, μ and τ have the same quantum numbers so terms like $m_{e\tau} \overline{e_L} \tau_R$ or $m_{\tau\mu} \overline{\tau_L} \mu_R$ are also viable.

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General Lagrangian for three generations of leptons:

$$-\mathcal{L}_m = \overline{e_L} \mathbf{m_e} e_R + \overline{\nu_L} \mathbf{m_D^\dagger} \nu_R + \frac{1}{2} \overline{\nu_L} \mathbf{m_L^\dagger} C \overline{\nu_L}^T + \frac{1}{2} \overline{\nu_R} \mathbf{m_R} C \overline{\nu_R}^T + \text{h.c.}$$

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All mass matrices have to be diagonalized.

Seesaw with Three Families

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Find the seesaw mechanism in this case considering again $\mathbf{m_L}, \mathbf{m_D} \ll \mathbf{m_R}$. Note that $\mathbf{m_L}$ and $\mathbf{m_R}$ are symmetric matrices as well as the matrix to be diagonalised, use the Takagi's factorization: $\mathbf{U}^\dagger\mathbf{m_R}\mathbf{U}^* = \mathbf{diag}$.

- Light neutrinos: $\mathbf{m}_\nu \simeq \mathbf{m_L} - \mathbf{m_D}\mathbf{m_R}^{-1}\mathbf{m_D}^T \simeq -\mathbf{m_D}\mathbf{m_R}^{-1}\mathbf{m_D}^T$
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With three families there are more physical observables but much more free complex matrices entries. Let's increase the predictability of the theory with symmetries.

Masses from Symmetries: Democratic Matrix

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The democratic matrix $\Delta = c \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ has very hierarchical eigenvalues: $3c$, 0, 0. We are still left with two massless fermions with this symmetry.

The next step is to assume a breaking term that can accommodate them: $P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}$, $a, b \ll 1$

Masses from Symmetries: Democratic Matrix

Let's apply this symmetry and the breaking matrix to all the mass matrices we studied before and see if there are any constraints in $\mathbf{m}_\nu$.

$$-\mathcal{L}_m = \overline{e_L}\mathbf{m_e} e_R + \overline{\nu_L}\mathbf{m_D}^\dagger \nu_R + \frac{1}{2}\overline{\nu_R}\mathbf{m_R} C\overline{\nu_R}^T + \text{h.c.}\,,$$

with

$$\mathbf{m_k} = c_k\left[\Delta + P_k\right]\,,\ P_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_k & 0 \\ 0 & 0 & b_k \end{bmatrix}\,,\ \ k = \text{e,D,R}$$

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$$\mathbf{m}_\nu = -\mathbf{m_D}\mathbf{m_R}^{-1}\mathbf{m_D}^T = -c_{\text{eff}}\left[\Delta + P_D\right]\left[\Delta + P_R\right]^{-1}\left[\Delta + P_D\right]^T$$

Masses from Symmetries: Democratic Matrix

$$\mathbf{m}_\nu = -\mathbf{m_D}\mathbf{m_R}^{-1}\mathbf{m_D}^T = -c_{\text{eff}}\left[\Delta + P_D\right]\left[\Delta + P_R\right]^{-1}\left[\Delta + P_D\right]^T$$

$$Z \equiv \left[\Delta + P_R\right]^{-1} = \frac{1}{a_R\, b_R}\begin{bmatrix} a_R + b_R + a_R b_R & -b_R & a_R \\ -b_R & b_R & 0 \\ -a_R & 0 & a_R \end{bmatrix}$$

So we have $\mathbf{m}_\nu = -c_{\text{eff}}[\Delta Z \Delta + \Delta Z P_D + P_D Z \Delta + P_D Z P_D]$.

Remarkably, $\Delta Z \Delta = \Delta$, the second and third term vanish and the fourth yields $P_D Z P_D = P_{\text{eff}}$.

Masses from Symmetries: Democratic Matrix

Then $\mathbf{m}_\nu = -c_{\text{eff}}[\Delta + P_{\text{eff}}]$, with $P_{\text{eff}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_{\text{eff}} & 0 \\ 0 & 0 & b_{\text{eff}} \end{bmatrix}$, and $a, b, c_{\text{eff}} = a, b, c_D^2/a, b, c_R$.

Check the calculations to obtain $\mathbf{m}_\nu$

Our assumption is preserved by the seesaw mechanism since $\mathbf{m}_\nu$ has the same form as other mass matrices in which the symmetry were imposed.

Lepton Democratic Mass Matrix

Instead of computing all optimal values of a, b, c to fit the data let's just focus on charged leptons masses which are known very well experimentally.

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Calculate (using PC) the eigenvalues of $c_e[\Delta + P_e]$ and then, using perturbation/expansion series, find the values for a_e, b_e, c_e

- $m_e = \frac{1}{2}a_e c_e \simeq \frac{1}{2}a_e c_e \rightarrow a_e = 6\frac{m_e}{m_\tau}$
- $m_\mu = \frac{2}{3}b_e c_e + \frac{1}{6}a_e c_e \simeq \frac{2}{3}b_e c_e \rightarrow b_e = \frac{9m_\mu}{2m_\tau}$
- $m_\tau = \left(3 + \frac{a_e}{3} + \frac{b_e}{3}\right) c_e \simeq 3c_e \rightarrow c_e = \frac{m_\tau}{3}$

Masses from Symmetries: Family U(1)

Another common example to eliminate entries and make the theory more predictable is using U(1) symmetries that act differently according to each generation.
Let's say the symmetry $L_e - L_\mu$, under which the terms $m_{e\tau}\overline{e_L}\tau_R$ or $m_{\tau\mu}\overline{\tau_L}\mu_R$ are forbidden.

$Q(e,\nu_e) = +1\,,\ \ Q(\mu,\nu_\mu) = -1\,,\ \ Q(\tau,\nu_\tau) = 0$, then:

$$\mathbf{m_D} = \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}, \text{ and } \mathbf{m_R} = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & * \end{bmatrix}.$$

This example certainly does not accommodate but then we could break the symmetry softly. And the game could go on and on...

Summary and Conclusions

- Symmetries are extremely important in particle physics, they are the foundations of interactions and appear in almost every extension of the SM.
- Neutrinos are the most elusive particles of the SM, having tiny masses. The seesaw mechanism is one economical and simple way of solving this puzzle (and others).
- We discussed the interesting possibility of having predictive matrices arising from a symmetry principle deeply connected with quantum properties. Besides the obvious reduction in the parameter space, it can shed some light in the right path to construct theories even more compelling than the ones we know today (neutrinos as dark matter, matter-antimatter asymmetry, multi-Higgs models...)