The objective of this work is to solve numerically solve the Gross-Pitaevskii equation

$$(i-\gamma)\frac{\partial\psi}{\partial t} = -\frac{1}{2}\Big(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\Big)+\frac{x^2+y^2}{2}\psi+G|\psi|^2\psi-i\Omega\Big(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\Big)\psi \quad (1)$$

Note the different parameters: γ is dissipative coeficient, G is the non-linear coefficient and Ω is the angular velocity of the super-fluid.

- Discretize it in a box of dimensions $[-10,10]\times[-10,10]$ and 128 points in each direction. To start, use the parameters

$$\begin{aligned} \gamma &= 0.0 && (2)\\ G &= 1000 && (3)\\ \Omega &= 0.85 && (4) \end{aligned}$$

  You are advised to use the provided code. There is the class template `Field2D<T>` as a two-dimensional array. Code for visualization with the SDL 2.0 library https://www.libsdl.org/ is also provided. Compile with `-std=c++11 -lSDL2`

- Set the initial condition

$$\psi(x,y,0)=\frac{1}{\sqrt{2\pi}}\exp\Big(-\frac{1}{2}(x^2+\frac{y^2}{4})\Big) \quad (5)$$

  and draw the wave-function using the provided code

- Discretize the spatial derivatives as

$$\begin{aligned} \frac{\partial\psi}{\partial x} &\to \frac{\psi_{i+1}-\psi_{i-1}}{2\Delta x} && (6)\\ \frac{\partial^2\psi}{\partial x^2} &\to \frac{\psi_{i+1}-2\psi_i+\psi_{i-1}}{(\Delta x)^2} && (7) \end{aligned}$$

- and use the Euler method to time evolve the system

$$\psi^{n+1}=\psi^n+F(\psi)\Delta t \quad (8)$$

- Use a Δt sufficiently small to ensure stability

- Now, solve the equation, using the fourth order Runge-Kutta method

$$\begin{aligned} k_1 &= F(\psi^n)\\ k_2 &= F(\psi^n+\frac{\Delta t}{2}k_1)\\ k_3 &= F(\psi^n+\frac{\Delta t}{2}k_2)\\ k_4 &= F(\psi^n+\Delta t\,k_3)\\ \psi^{n+1} &= \psi^n+\frac{\Delta t}{6}\big(k_1+2k_2+2k_3+k_4\big) \end{aligned}$$

- Parallelize the code using OpenMP
- We now, want to find the ground state of the system. For that consider a small dissipative term $\gamma = 0.1$ and, at each step, impose the normalization condition

$$\int dx\, dy\, \psi(x,y)^* \psi(x,y) = 1 \tag{9}$$