

INTRODUÇÃO À COMPUTAÇÃO QUÂNTICA

REALIZAÇÃO FÍSICA DE UM COMPUTADOR QUÂNTICO:
ARMADILHAS DE IÓES

HANDS ON QUANTUM MECHANICS 2016

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29 de Julho, 2016



Requisitos para um computador quântico universal (DiVincenzo, 1996):

- Sistema físico escalável
- Inicialização
- Tempo de coerência \gg Tempo de operação
- Possibilidade de implementação de portas quânticas universais
- Mensurável

TOPICS IN QUANTUM COMPUTERS

D. P. DIVINCENZO

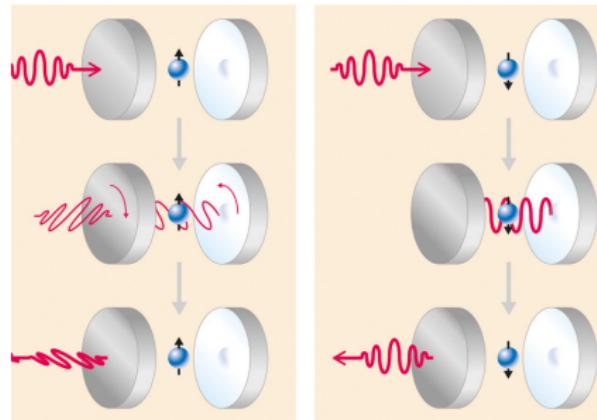
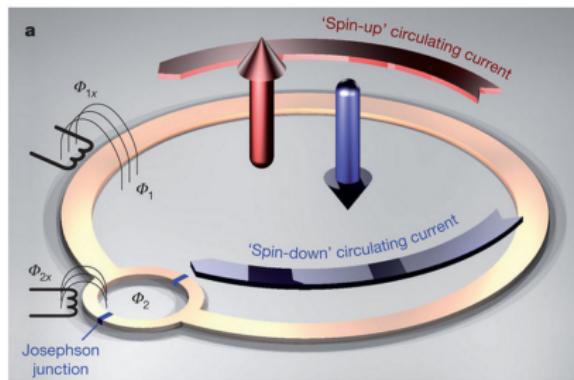
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Abstract. I provide an introduction to quantum computers, describing how they might be realized using language accessible to a solid state physicist. A listing of the minimal requirements for creating a quantum computer is given. I also discuss several recent developments in the area of quantum error correction, a subject of importance not only to quantum computation, but also to some aspects of the foundations of quantum theory.

ALGUMAS IDEIAS PARA COMPUTADORES QUÂNTICOS UNIVERSAIS

Sistemas físicos que cumprem os requisitos:

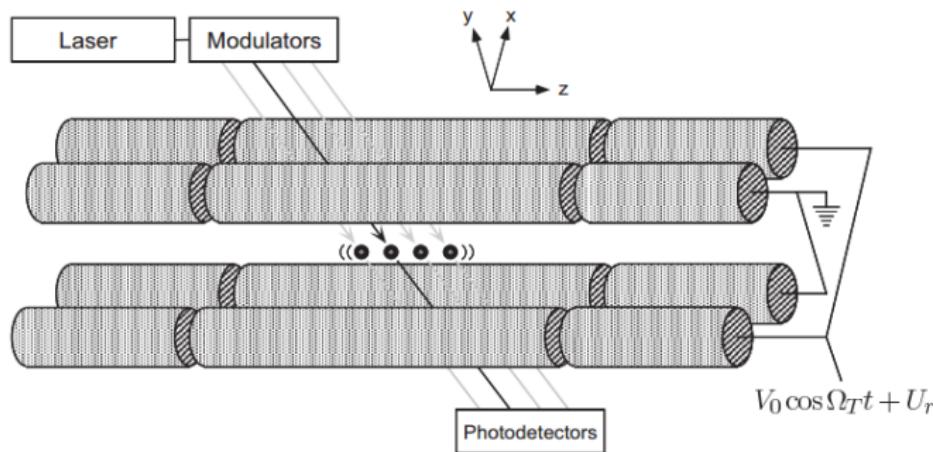
- Fotões
- Armadilhas de iões
- Cavidades ópticas
- Ressonância magnética nuclear
- Pontos quânticos
- Supercondutores



ARMADILHA DE IÓES - MONTAGEM

Principais constituintes:

- Armadilha electromagnética
 - Lasers
 - Fotodetectores
 - Iões
- Eléctrodos criam potencial electromagnético que confina ião
 - Laser incide no ião, alterando o seu estado



O HAMILTONIANO

- ➊ Iões de spin $\frac{1}{2}$ num potencial harmónico de frequência ω_0 :
$$z = z_0 (a + a^\dagger)$$
- ➋ Campo magnético exterior aplicado :
$$\vec{B}(z, t) = B_0 \cos(kz - \omega t + \phi) \hat{x}$$
- ➌ Hamiltoniano de interação $\vec{B} - \vec{S}$:
$$\hat{H} - \vec{\mu} \cdot \vec{B} \text{ com } \vec{\mu} = \mu_m \vec{S}$$
- ➍ Define-se a frequência de Rabi:
$$\Omega = \frac{\mu_m B_0}{2\hbar}$$
- ➎ Define-se o parâmetro de Lamb-Dicke:
$$\eta = kz_0$$
- ➏ Expande-se a exponencial da oscilação no espaço ($e^{k|z|}$) retendo-se apenas 2 termos:
- ➐ Contas chatas...

$$\begin{aligned}\hat{H} = & \frac{\hbar\Omega}{2} [S_+ (e^{i(\phi-\omega t)} + e^{-i(\phi-\omega t)}) + S_- (e^{i(\phi-\omega t)} + e^{-i(\phi-\omega t)})] + \\ & \frac{i\hbar\Omega\eta}{2} (S_+ a + S_- a^\dagger + S_+ a^\dagger + S_- a) (e^{i(\phi-\omega t)} - e^{-i(\phi-\omega t)})\end{aligned}$$

O HAMILTONIANO

$$\hat{H} = \frac{\hbar\Omega}{2} [S_+ (e^{i(\phi-\omega t)} + e^{-i(\phi-\omega t)}) + S_- (e^{i(\phi-\omega t)} + e^{-i(\phi-\omega t)})] + \frac{i\hbar\Omega\eta}{2} (S_+ a + S_- a^\dagger + S_+ a^\dagger + S_- a) (e^{i(\phi-\omega t)} - e^{-i(\phi-\omega t)})$$

- ➊ Escrevem-se os operadores S_+ , S_- , a , a^\dagger na representação de interação:

$$S_+(t) = S_+ e^{i\omega_0 t} \quad S_-(t) = S_- e^{-i\omega_0 t} \\ a^\dagger(t) = a^\dagger e^{i\omega_z t} \quad a(t) = a e^{-i\omega_z t}$$

- ➋ Desprezam-se os termos exponenciais com $\omega_0 + \omega$

- ➌ Define-se $\Delta = \omega - \omega_0$:

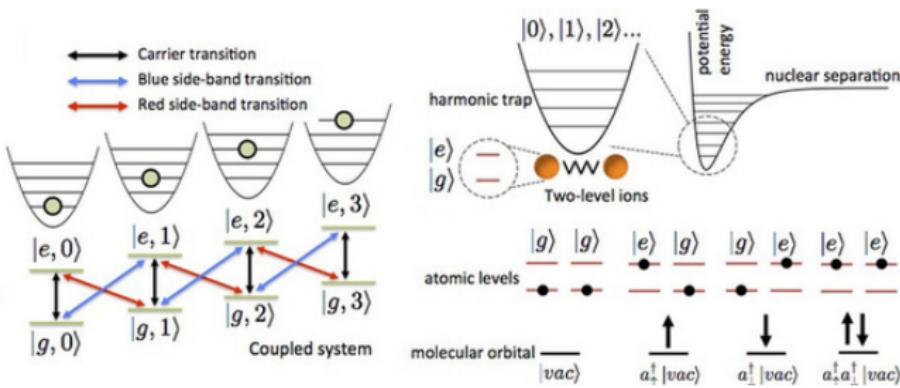
$$\hat{H} = \frac{\hbar\Omega}{2} [S_+ e^{i(\phi-\Delta t)} + S_- e^{-i(\phi-\Delta t)}] + \frac{i\hbar\Omega\eta}{2} (S_+ e^{i(\phi-\Delta t)} - S_- e^{-i(\phi-\Delta t)}) (a e^{-i\omega_z t} + a^\dagger e^{i\omega_z t})$$

O HAMILTONIANO

$$\hat{H} = \frac{\hbar\Omega}{2} [S_+ e^{i(\phi - \Delta t)} + S_- e^{-i(\phi - \Delta t)}] + \frac{i\hbar\Omega\eta}{2} (S_+ e^{i(\phi - \Delta t)} - S_- e^{-i(\phi - \Delta t)}) (a e^{-i\omega_z t} + a^\dagger e^{i\omega_z t})$$

3 frequências de *detuning* são relevantes:

- ① $\Delta = 0$: $\hat{H}_c = \frac{\hbar\Omega}{2} (S_+ e^{i\phi} + S_- e^{-i\phi})$
- ② $\Delta = \omega_z$: $\hat{H}_b = \frac{i\hbar\Omega\eta}{2} (S_+ a^\dagger e^{i\phi} - S_- a e^{-i\phi})$
- ③ $\Delta = -\omega_z$: $\hat{H}_r = \frac{i\hbar\Omega\eta}{2} (S_- a^\dagger e^{-i\phi} - S_+ a e^{i\phi})$



Uma armadilha de iões é:

- **Escalável** - Qubits = Níveis internos dos iões com tempo de vida longo.
- **Inicializável** - *Optical pumping*
- **Tempo de coerência** \gg **Tempo de operação** - Verifica-se experimentalmente $\tau_{coer} \gg \tau_{op}$
- **Possibilidade de implementação de portas quânticas universais** - É possível implementar qualquer porta de 1 qubit aplicando um campo electromagnético oscilante
- **Mensurável** - Faz-se incidir laser de frequência capaz de criar excitação de um estado $|n\rangle$ para $|m\rangle$, instável. Quando ocorre desexcitação são emitidos fotões que podem ser detectados. Caso o qubit estivesse no estado $|n\rangle$ vêm-se fotões.

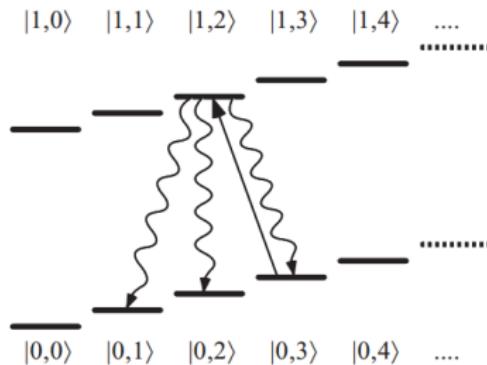
INICIALIZAÇÃO - ARREFECIMENTO

Para poder manipular o ião é necessário tê-lo no seu estado fundamental de vibração. Isto é impossível para $k_B T \geq \hbar\omega$ devido a excitação térmica \Rightarrow é preciso arrefecer o ião:

Sideband Cooling

Arrefecimento de Doppler

- Ião rodeado por 6 lasers, em direcções opostas dois a dois
- Frequência dos lasers ligeiramente inferior à frequência de ressonância
- Efeito de Doppler \Rightarrow ião vê laser para o qual se move emitindo com maior frequência, mais perto da de ressonância
- Estes fotões tornam-se mais importantes \Rightarrow ião perde velocidade \Rightarrow arrefece
- Emissão de fotões pelo ião leva a aumento de velocidade \Rightarrow há limite para o arrefecimento



- Método para levar o ião a temperaturas mais baixas do que é possível com Doppler
- Transições entre $|0, n\rangle$ e $|1, m\rangle$, sendo 0 e 1 os níveis electrónicos e n e m níveis vibracionais
- Laser com frequência tal que as transições permitidas são $|0, n\rangle \rightarrow |1, n-1\rangle$
- Decaimento pode depois acontecer para $|0, n\rangle$, $|0, n-1\rangle$ ou $|0, n-2\rangle$
- O estado $|0, 0\rangle$ não pode ser excitado, pelo que eventualmente o ião fica neste estado

IMPLEMENTAÇÃO DE PORTAS

Aplicando um campo eletromagnético de frequência ω_0 , o termo relevante do Hamiltoniano é:

$$H = \frac{\hbar\Omega}{2} (S_+ e^{i\phi} + S_- e^{-i\phi})$$

Evolução temporal dada por:

$$|\psi(t)\rangle = e^{-i\frac{Ht}{\hbar}} |\psi(t=0)\rangle = e^{-i\frac{\Omega t}{2}(S_+ e^{i\phi} + S_- e^{-i\phi})} |\psi(t=0)\rangle$$

$$|\psi(t)\rangle = [I\cos(\Omega/2) - i(S_x\cos(\phi) - S_y\sin(\phi))\sin(\Omega/2)] |\psi(0)\rangle$$

Operadores de rotação em torno dos eixos e_x e e_y .

$$R_x(\theta) = \cos(\theta/2)I - i\sin(\theta/2)S_x$$

$$R_y(\theta) = \cos(\theta/2)I - i\sin(\theta/2)S_y$$

⇒ É possível fazer qualquer combinação de rotações em torno dos eixos e_x e e_y , o que permite implementar qualquer porta unitária de 1 qubit.

14 qubit entanglement: creation and coherence

14-qubit entanglement: creation and coherence

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(Dated: March 24, 2011)

We report the creation of Greenberger-Horne-Zeilinger states with up to 14 qubits. By investigating the evolution of such states over time, we observe a decay proportional to the square of the number of qubits. The decay is suppressed when the initial state is created with a system affected by correlated Gaussian phase noise. This model holds for the majority of current experimental systems developed towards quantum computation and quantum metrology.

PACS numbers: 03.67.Lx, 32.10.Yn, 32.30.Qk

Quantum states can show non-classical properties, for example, their superposition allows for (classically) counter-intuitive phenomena such as a particle being in two places at the same time. Entanglement can extend this paradox even further, e.g., the state of one subsystem can be affected by a measurement on another subsystem without any apparent interaction [1]. These concepts, although experimentally well-verified, conflict with our classical perception and lead to several questions. What is the transition from the classical regime to the quantum? Under which condition does that transition take place? And why? The creation of large-scale multi-particle entangled quantum states and the investigation of their properties will provide a better understanding of this transition [2,3].

Usually, decoherence mechanisms are used to describe the evolution of a quantum system into the classical regime. One prominent example is the spontaneous decay of an excited atom. If two atoms interact with each other, the decay of each would be expected to be independent of the others. Therefore, the number of decay processes in a fixed time window would intuitively be proportional to the number of particles. The situation, however, can be inaccurate. Decoherence effects can act collectively and produce "superdecoherence", a regime in which the rate of spontaneous decay is proportional to the square of the number of excited atoms [4]. Such collective decoherence can also occur in multi-qubit register, as known as "superdecoherence" [5]. This particular effect is often unwanted and must be suppressed or canceled to a energetically non-degenerate state. In these systems, a phase reference (PR) is required to perform coherent operations on a quantum register. Noise in the PR can affect the quality of the register.

In the following we introduce a model describing a quantum register in the presence of correlated phase noise. More specifically, we investigate N -qubit

Greenberger-Horne-Zeilinger (GHZ) states of the form $|00\dots0\rangle - \frac{1}{\sqrt{2}}|0\dots0-1\dots-1\rangle$. These states are of the multi-particle entanglement and play an important role in the field of quantum metrology [6] for quantum mechanically enhanced sensors. This special quantum state, however, has only been generated with up to 6 qubits so far [7]. Increasing up to 8 genuinely multi-particle-entangled ion-qubits in a GHZ state, we predict and verify the presence of superdecoherence effects scaling with the number of qubits N . In general, any system operating with correlated phase noise is affected by this accelerated GHZ-state decoherence.

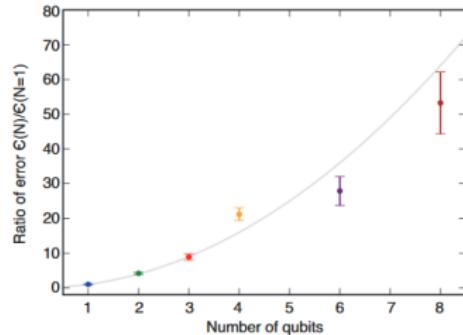
The model collective phase fluctuations acting on the quantum register with a Hamiltonian of the form $H_{\text{noise}} = -\Delta\delta\Omega\sum_{k=1}^N\hat{\sigma}_k^{(2)}$, where $\Delta\delta\Omega$ denotes the strength of the fluctuations, and $\hat{\sigma}_k^{(2)}$ a phase flip on the k -th ion. Under this Hamiltonian, the initial state $|\psi(0)\rangle = |\psi_0\rangle\otimes|0\rangle^{\otimes N}$ decays to $|\psi(t)\rangle = \exp(-i\int_0^tH_{\text{noise}}(\tau)d\tau)|\psi(0)\rangle$. As a measure of state preservation, we use the fidelity $F(t) = \langle|\psi(t)\rangle|\psi(t)\rangle^*$, where the bar refers to an average over all realizations of random phase fluctuations. The decay of the fidelity can be conveniently described by

$$F(t) = \langle 1 + \exp(-2\chi(N,t)) \rangle,$$

where the effective error parameter for a stationary Gaussian random process is derived to be

$$\langle N(t) \rangle = N^2 \frac{1}{N!} \int_0^t d\tau (t-\tau) \langle \Delta\delta\Omega(\tau) \Delta\delta\Omega(0) \rangle. \quad (1)$$

Given homogeneous Gaussian noise (white Gaussian noise), the similar result is found within the spin-boson model [8]. The intuition of an error probability can be recovered in the limit of small $\langle N(t) \rangle$ since the fidelity decays as



- 2011 - Entanglement de 14 qubits
- Crescimento do erro relativo com o número de qubits

Realization of a scalable Shor algorithm

Realization of a scalable Shor algorithm

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(Dated: August 8, 2015)

Quantum computers are able to perform classical computations. This was long recognized by the visionary Richard Feynman who pointed out in the 1980's that quantum mechanical problems were better solved with quantum machines. It was only in 1994 that Peter Shor came up with an algorithm that is able to calculate the prime factors of a large number much more efficiently than any known classical algorithm [1]. This has triggered a lot of research in quantum computation and the following research in quantum information processing and the quest for an actual implementation of a quantum computer. Over the last fifteen years, using shifting of paradigms, several instances of a Shor algorithm have been realized with different physical systems, ranging from ion traps to quantum optics [2–5]. We general scalability, though, a different approach has to be pursued [6]. Here, we report the realization of a fully scalable algorithm as proposed by Kitaev [6]. For this, we developed a quantum circuit for the factorization by efficiently combining the concepts of “phase kick-back” and “four-controlled phase”, together with the implementation of generalized arithmetic operations, known as modular multiple. The scalable algorithm has been realized with an ion trap quantum computer enabling fault-tolerant error correction of QFT.

PACS numbers: 03.67.Lx, 03.67.Ty, 32.80.Qk

Shor's algorithm for factoring integers [2] is one of the most efficient known classical algorithms. Experimentally, its implementation is highly demanding as it requires a large number of qubits and high-fidelity control. Clearly, such challenging requirements raise the question whether optimizations and experimental short cuts are possible. While optimizations are possible, the question of what is physically feasible is more difficult. For a demonstration of Shor's algorithm to be scalable special care has to be taken not to overestimate the implementation – for instance by employing knowledge of the specific hardware or experimental implementation – as pointed out in Ref. [6].

In order to elucidate the general task at hand, we first explain and exemplify Shor's algorithm for factoring the number 15 in a (quantum) circuit model. Subsequently, we show how this circuit can be transferred for an implementation with an ion-trap quantum computer.

How does Shor's algorithm work? There is a classical recipe that finds the factors of a large number. As an example, let us consider the problem of finding the factors of $N = 15$. Then pick a random integer $a \in [1, N - 1]$ (which we call the base in the following), say $a = 7$. Check if the greatest common divisor $\text{gcd}(a, N) = 1$, which is the case for $a = 1, 2, 4, 7, 8, 11, 13$, but not for the case for $a = \{3, 5, 6, 9, 10, 12\}$. Next, calculate the modular representations $a^x \bmod N$ for $x = 0, 1, 2, \dots$ and find its period r : the first $x > 0$ such that $a^x \equiv 1 \pmod{N}$. Given the period r , finding the factors requires calculating the greatest common divisors of $a^{r/2} \pm 1$ and N ,

which is classically efficiently possible – for instance using Euclid's algorithm. For our example ($N = 15, a = 7$) the modular exponentiation yields $1, 7, 4, 13, 1, \dots$, where the last part 4 is the greatest common divisor of $a^r - 1 = 16$ and $N = 15$ (and $r = 4$). The non-trivial factors of 15 are 3 and 5. For the chosen example $N = 15$, the cases $a = \{4, 11, 14\}$ have periodicity $r = 2$ and would require a single multiplication step ($a^2 \equiv 1 \pmod{N}$; $r = 2$). See also Ref. [6]. Note that the periodicity for a chosen a can not be predicted.

How can this recipe be implemented in a QC? A QC has also to calculate N in a computational register, x_0, x_1, \dots, x_{n-1} , and then measure x_n , respectively, using the quantum Fourier transform (QFT). This can be done with high probability in a single step (compared to r steps classically). Here, x is stored in a quantum register x_0, x_1, \dots, x_{n-1} or x_{n-1}, \dots, x_0 , which is a superposition of 0 to $2^n - 1$. The superposition in the period-register on its own does not provide a speedup compared to a classical computer. Measuring the period register would destroy the state and only return a single value, say x_1 , and the algorithm would have to start again with x_0 read in the computational register. However, if the QFT is applied to the period-register, the period of $a^x \bmod N$ can be extracted from $O(1)$ measurements.

What are the major challenges to implement Shor's algorithm? First, we focus on the period register, to subsequently address modular exponentiation in the computational register. Factoring N , an $n = \lceil \log_2(N) \rceil$ -bit number requires a summation of n qubits in the computational register (to store the re-

- Implementação de uma variante do algoritmo de Shor mais facilmente escalável (Kitaev, 1995)
- Usado computador de 11 qubits
- Factorização de 15 em números primos... Não muito impressionante :)